# Machine learning enabled digital compensation of phase-to-amplitude distortion in fibre-optical parametric amplifier based transmission links

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**Abstract.** We numerically demonstrate an advanced digital signal processing method for compensating the phase-to-amplitude distortion conversion caused by the interaction of the phase noise induced by pump dithering with the dispersive fibre channel in fibre-optical parametric amplifier based transmission systems.

## 1 Introduction

Fibre-optical parametric amplifiers (FOPAs) have been a research interest for decades owing to their many advantageous features for optical communications, including virtually unconstrained wavelength operation [1], wide gain bandwidth [2] and ultra-fast response, amongst the others. From a practical perspective, however, the mitigation of stimulated Brillouin scattering (SBS) still remains a significant challenge for their development. The SBS effect limits the pump power that can be delivered to the highly nonlinear fibre (HNLF), thereby restricting the device's gain performance. A common way to mitigate SBS relies on broadening the line-width of the pump source, most often via external modulation of the pump phase at a relatively high rate  $(f_m > 1 \text{ GHz})$  [3]. However, a drawback of this technique is that the pump phase modulation causes temporal variation of the parametric gain as a result of the induced instantaneous pump frequency modulation [4], which is primarily a source of phase distortion for coherently detected complex-amplitude signals. Being of a relatively high frequency, the dithering-induced phase distortion can not be tracked and sufficiently suppressed by the conventional carrier phase recovery schemes [5] employed in commercial coherent receivers.

Our recent work has utilised machine learning (ML) to develop advanced digital signal processing (DSP) techniques that can effectively address the issue of phase distortions caused by pump dithering in fibre-optical parametric devices. In [6], we used a kernel-based algorithm to suppress the phase distortion of an optical phase conjugation (OPC) device after a conventional phase noise (PN) compensation stage. In [7], we presented a scheme that was also able to mitigate the phase-to-amplitude noise transfer resulting from the interaction of the pump dithering with the dispersive fibre channel in a transmission link with mid-span OPC. In this paper, we further develop our approach to encompass FOPA amplified transmission links, enabling the removal of the amplitude and phase distortions that are contributed independently by each FOPA in the link. The technique is numerically demonstrated in a 28-Gbaud 16 quadrature amplitude modulation (QAM) FOPA-assisted transmission, achieving significant bit-error-rate (BER) improvement over conventional PN compensation.

### 2 Methods

We considered an optical transmission system of multiple fibre spans with inline FOPAs providing gain for compensating the propagation losses (Fig. 1(a)). In our numerical model of the FOPA, we used a single-pump design and assumed a sinusoidal phase modulation of the pump wave,  $\xi(t) = m \sin(\omega_m t)$ , where m is the modulation index, and the typical modulation frequency  $f_m = \omega_m/(2\pi) = 2 \,\text{GHz}$  is sufficient for SBS mitigation in most FOPAs [2]. In the undepleted pump regime, the signal gain can be expressed as [1, 4]  $\mu$  =  $\left\{\cosh(gL) + i\left[(\kappa + \delta\kappa)/(2g)\right]\sinh(gL)\right\}e^{i\left[2\gamma P - (\kappa + \delta\kappa)/2\right]L},$ where  $\gamma = 10 (W \cdot km)^{-1}$  is the nonlinear coefficient of the fibre, P = 1.5 W is the pump 200 m is the fibre length, g =power, L = $\sqrt{(\gamma P)^2 - (\kappa + \delta \kappa)^2/4}$  is the parametric gain coefficient,  $\kappa = 2\gamma P + \beta_2 \Omega^2 + (\beta_4/12)\Omega^4$  is the standard phase mismatch,  $\delta \kappa(t) = \beta_2 \phi_t^2 - \beta_3 (\xi_t \Omega^2 + \xi_t^3 / 3) + \beta_4 (\xi_t^4 + 6\xi_t^2 \Omega^2) / 12$ is the instantaneous phase mismatch induced by the pump phase modulation,  $\Omega = \omega_p - \omega_s$  is the frequency deviation from the pump, and  $\beta_n$  is the nth-order dispersion coefficient. We used the parameters:  $\lambda_p = 2\pi c/\omega_p = 1563.7 \text{ nm}, \ \lambda_0 = 1562.9 \text{ nm}, \ \beta_3 = 1.2 \times 10^{-40} \text{ s}^3/\text{m}, \ \beta_4 = -2.85 \times 10^{-55} \text{ s}^4/\text{m}, \ \text{and we}$ operated the FOPA at  $\lambda_s = 1538.7$  nm, where its power gain  $|\mu|^2$  had a peak of 20 dB. The amplifier's noise figure

To study the impact of the pump dithering on the channel memory, we considered an equivalent base-band model (Fig. 1(b)), where the fibre channel is represented by a linear filter  $h_f(t)$ , the dithering-induced phase distortion introduced at the *n*th FOPA stage is  $\phi_n(t) = \Phi + \delta \phi_n(t)$ ,  $n = \overline{1, N}$ ,  $\Phi$  is a constant phase shift, and  $\delta \phi_n$  is the tem-

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was 4.5 dB.

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poral variation due to dithering. Because of its relatively low speed (at the kHz level), the contribution to the PN due to non-zero laser line-width can be neglected. Considering first a transmission scenario with N = 1, by using linear back-propagation, we can recover the transmitted signal x(t) from the received signal y(t) as:  $x(t) = [y(t)e^{-j\phi_1(t)}] * h_f^*(t)$ , where  $h_f^*(t)$  is the complex conjugation of  $h_f(t)$ . Because  $\delta\phi_1$  is relatively small, we can approximate  $e^{-j\delta\phi_1}$  by its first-order Maclaurin polynomial and get  $x(t) \simeq e^{-j\Phi} \left[ y(t) * h_f^*(t) - [y(t)j\delta\phi_1(t)] * h_f^*(t) \right]$ . By a similar approach, we can generalise the expression above to the scenario of N > 1 spans and, after ignoring the beating terms, obtain

$$\begin{aligned} x(t) &\simeq e^{-j\Phi} \left[ y(t) * h_e^{(N)}(t) \\ &- \sum_{n=1}^N \left[ [y(t) * h_e^{(N-n)}(t)] j \delta \phi_n(t) \right] * h_e^{(n)}(t) \right], \end{aligned}$$
(1)

where  $h_e^{(n)}(t)$  is the accumulated filter after *n* spans of  $h_e(t)$ , and  $h_e(t) = h_{\epsilon}^*(t)$  is the equalisation filter.



**Figure 1.** (a) Schematic diagram of a transmission system with N cascaded FOPA stages. (b) Equivalent base-band model. (c) Principle of the proposed DDC algorithm.

The proposed dithering-induced distortion compensation (DDC) algorithm (Fig. 1(c)) is based on Eq. (1). The received signal u[k] is sampled at the rate  $T_s/2$ , with  $T_s$  the symbol duration, and the equalisation filter  $h_e[k]$  is implemented as a finite impulse response filter [8]. The phase fluctuation at the *n*th FOPA stage  $\delta \hat{\phi}_n[k]$  is estimated by learning  $\boldsymbol{\theta}_n$  in the linear equation  $\delta \hat{\boldsymbol{\phi}}_n[k] = \boldsymbol{\theta}_n \boldsymbol{X}_n$ , where  $\boldsymbol{X}_n = [\sin(\omega_{m,n}k), \cos(\omega_{m,n}k)],$  and the dithering frequency  $\omega_{m,n}$  is assumed to be known. We can then create a design matrix **X** consisting of all the N + 1 branches in Fig. 1(c) and form the linear regression equation:  $\hat{x}'[k] = \theta X$ , where the coefficient vector is given by  $\boldsymbol{\theta} = e^{-j\Phi}[\boldsymbol{1}, \boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_N].$ We solved the equation for  $\theta$  using the first 1000 received symbols, then we predicted the recovered signal  $\hat{x}'[k]$  on the remaining symbol set. The algorithm was followed by a blind phase search (BPS) block [5] compensating for the laser PN.

#### 3 Results and discussion

The numerical model was implemented for the transmission of a 28-Gbaud 16-QAM Nyquist shaped signal with a roll-off factor of 0.1 and simulated with  $2^{16}$  symbols over a channel containing multiple fibre spans of 100 km. Each span had the dispersion parameter  $D = 17 \text{ ps/(nm \cdot km)}$ ,  $\gamma = 1.2 (W \cdot \text{km})^{-1}$  and the loss coefficient  $\alpha = 0.2 \text{ dB/km}$ . The laser line-widths for the transmitter/receiver and the FOPAs were 50 kHz and 30 kHz respectively. The directcount BER was used as a performance metric.

In Fig. 2, we compare the performances of the proposed DSP scheme and a scheme with only electronic chromatic dispersion compensation and BPS phase recovery, as functions of the transmission distance in the link. The BPS filter's length and launch power were optimised for each scheme. The latter scheme is equivalent to the uppermost branch in Fig. 1(c). By including lower branches, the DDC algorithm accounts for the interaction between the dithering-induced phase distortion and the dispersive channel. This capability yields a consistent BER improvement of around an order of magnitude over conventional PN compensation across a wide range of transmission lengths.



**Figure 2.** BER after conventional PN compensation (orange) and the proposed DDC (blue) versus length of the fibre link. The inset shows the constellation diagrams after 1500-km transmission.

#### 4 Conclusion

We have developed a DSP scheme to tackle the phaseto-amplitude noise transfer due to the interaction of the pump dithering PN with the fibre dispersion in FOPA amplified transmission systems. By using a ML algorithm to assist the conventional BPS method, we have shown significant BER benefit in 28-Gbaud 16-QAM signal transmission over long distances.

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