MICROWAVE RADIO COMMUNICATIONS THROUGH A TURBULENT FLAME USING A FAST CORRELATION METHOD WITH FEEDBACK

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by

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To my dear parents

MIHAILO and VIDOSAVA

SUMMARY

The research had two objectives:-

(1) To obtain further insight into the $1/\Delta f$ noise which is observed near the signal frequency when the characteristics or parameters of the system or medium through which the signal is transmitted fluctuate in a random manner (Δf is the difference between the signal frequency and the frequency at which measurements are carried out in a bandwidth df equal to or less than Δf), and also

(2) To devise and test a means for transmitting signals through such a medium and detecting them in such a manner that the retrieval of the signal is improved.

In the first group of experiments the time varying system consisted of a laboratory flame operating in an erratic or turbulent manner in a resonator tuned to approximately 9 GHz, i.e. to a wavelength of about 3 cm. An appreciable degree of attention was given to the development of self stabilising bridges for the separation and measurement of signal and noise powers from the mixture of signal and noise received after the signal was passed through the resonator. The measuring system could also be used to measure the autocorrelation function of the system over a short period of time, i.e. with a short averaging time.

It was shown theoretically and demonstrated experimentally that within the coherence bandwidth a convolution of the signal component and the Fourier transform of the system correlation function is a suitable and adequate model to describe the action of physical systems of which the parameters vary fast.

Starting from this and the fact that there is no mathematical or physical interpretation for an inverse convolution operation, it has been proposed that signals and noise can be unscrambled only in the real time

domain and provided the bandwidth is limited to that in which the coherence between various frequency components of the signal and the noise are preserved. It has been demonstrated that such unscrambling can be carried out by a rapidly acting control system operating at the receiver on information brought in by the mixture of signals and noise whereby the amplitude and phase changes due to the fluctuating transmission medium are corrected.

A simple form of signal in which there are two orthogonal signals, one of which served as a reference signal, was used. Simultaneously, information was recorded on the initial fluctuations in the form in which they were received and also after they had been corrected by the rapidly acting control system. The experiments covered a range of signal frequencies different from the reference frequency, and the signals were subjected to analysis by the use of computer programs as well as in real time by analog measurements.

The second part of the research was therefore concerned with experiments and analysis of the coherence between two signals when rapidly-acting automatic gain control (R.A.A.G.C.) and rapidly-acting automatic phase control (R.A.A.P.C.) are applied.

It was found that an improvement in the signal to noise ratio of at least 20 db is possible when both gain and phase controls are used, even with very modest electronic apparatus, much less comprehensive and precise than that developed in the first part of the investigation.

Signals requiring wide frequency bands for transmission should be split into several channels of such a bandwidth that there is coherence over the entire band, and with the simultaneous transmission of a "comb" of reference or pilot signals.

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CONTENTS

CHAPTER 1

1.1 Introduction

It has long been known that some part of noise, for example, shot noise and thermal noise, due to various physical processes taking place, is present even in the absence of signals, and many investigations have been made on this (Nyquist, 1928; Rice, 1944). It is, however, only in the past few years (Bull, 1959; Bozic 196) that it has been demonstrated both theoretically and experimentally that there are types of noise which are present only when a signal is applied to the system. Earlier theoretical methods had not predicted and experimental methods had not revealed this type of noise.

On the other hand, it is for some time a well recognized fact that during all rocket launching, the quality of communications to and from rockets is greatly degraded. Very severe signal perturbations and loss of information have been attributed to fluctuations of the parameter of the transmission path, due to the presence of highly turbulent and unstable plasma in the rocket exhaust trail. (Harper and El y, 1964). Furthermore, similar effects to the latter in ionospheric and tropospheric transmission have been known and studied in detail, for a relatively longer time (Brooker and Gordon, 1950; Lawrence, Little and Chivens,1964). One of the common features of all the phenomena mentioned is the similarity of the corresponding power density spectra, all being approximately of $(1/\Delta f)^{\alpha}$ type.

The basic postulate underlying the study of the noise in the vicinity of the signal is that parameters which define the relation between the input and the output of the system fluctuate. In some cases the fluctuation of parameters is brought about by the signal itself while in the others it is present at all times but revealed only when a signal is applied (Bull, 1968) ,which fluctuates then in phase and amplitude(Fig.1)

It is a purpose of this research to examine experimentally in more detail this type of noise, and to give more insight into its nature, using already well established (Zadeh, 1950) and almost conventional mathematical methods.

Let us start with the description of a communication system defining some of the terms to be used. At the same time an outline of the suggested method for suppressing $1/\Delta f$ noise will be given.

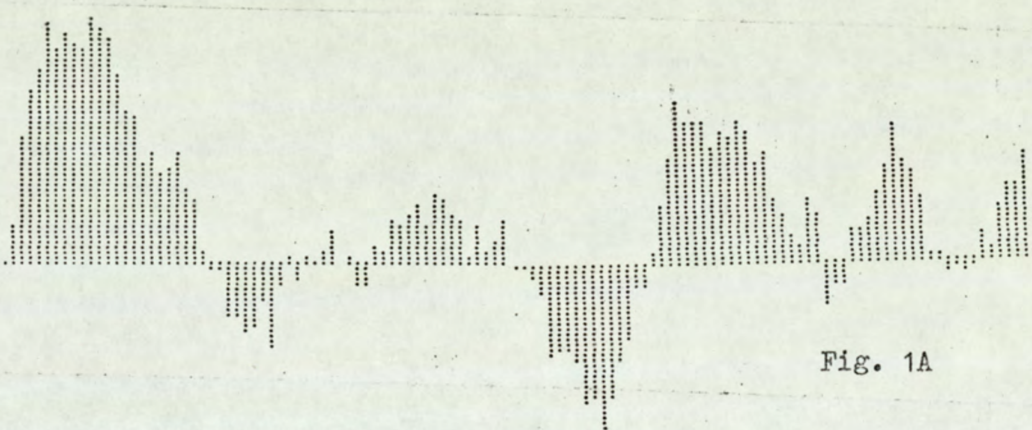


Fig. 1A

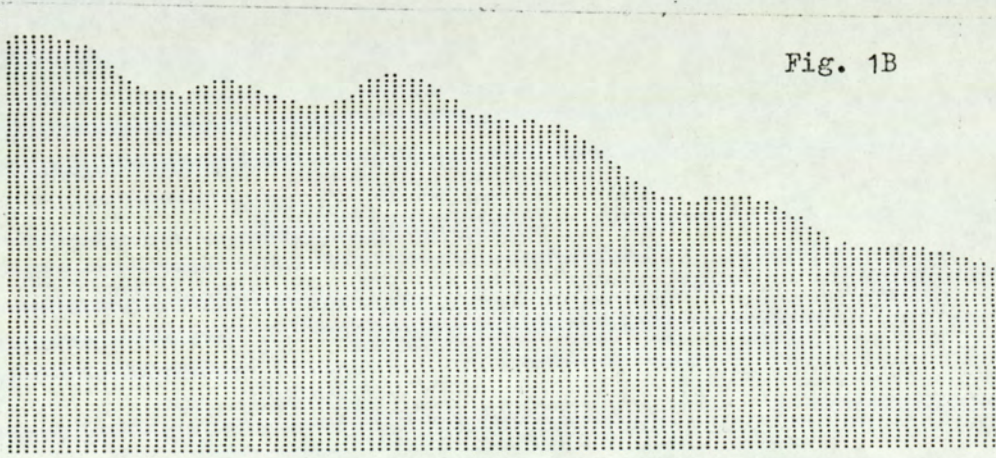


Fig. 1B

A communication system contains a transmitter, a transmission path and a receiver (Fig.1.1).

The source selects one message out of a set of possible messages to be transmitted to the receiver terminals.

The transmitter operates on the message and produces a signal suitable for transmission to the receiving point over the transmission paths.

The channel is merely the medium used to transmit the signal from the transmitting to the receiving point. In the simplest case it is a pair of wires, with time invariant transmission characteristics, more often it comprises some physical system, suitable for radio or optical communication, in general with time varying parameters.

The signal is almost always perturbed by additive noise N. During transmission the signal, being operated on by fluctuating transmission parameters and mixed with additive noise, is so modified that at least a part of information sent out is lost. Note here that some of the information a signal conveys is not at all relevant. There is usually a high degree of redundancy in information transmission.

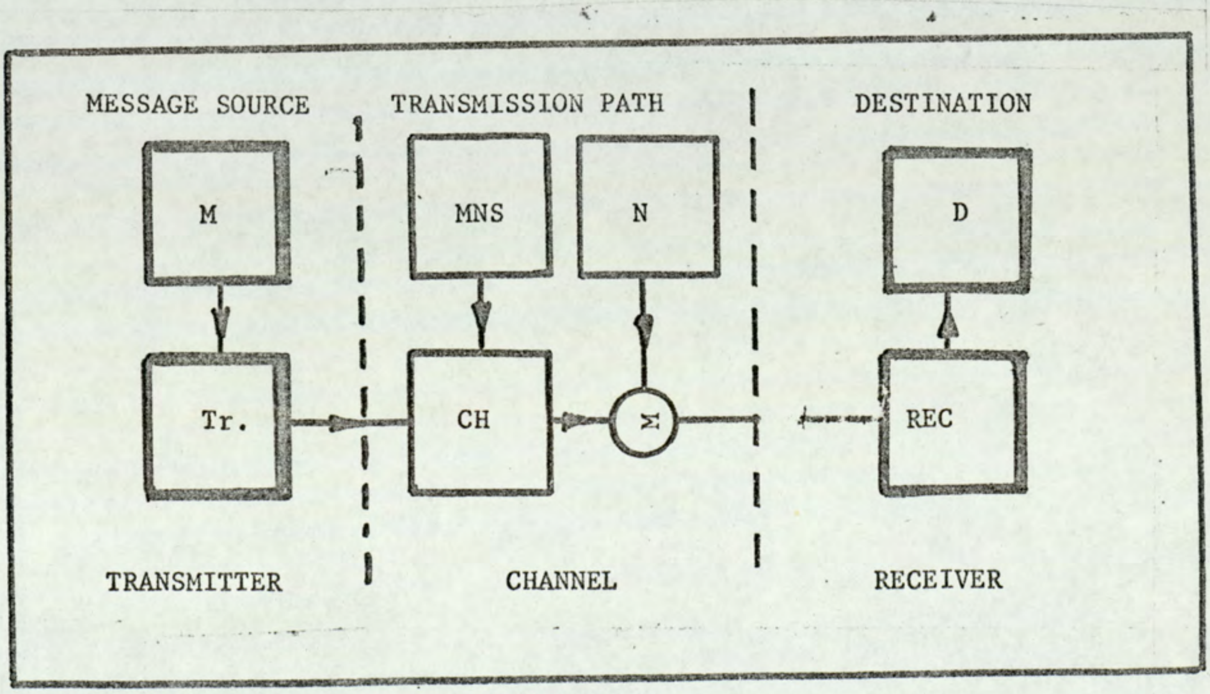


Fig.1.1

Fluctuation of parameters is due to the change in the geometrical, physical or chemical entities of the physical system, introduced here as the multiplicative noise source, MNS).

The destination is the person, device or equipment for which the message is intended.

In either case noise involves statistical and unpredictable perturbations. Fluctuating parameters of physical systems and their influence on signal transmission is of the central interest in this research.

The receiver operates on the received signal and attempts to reproduce from it the original message. Ordinarily it will perform approximately the mathematical inverse of the operations at the transmitter. More sophisticated receivers may differ somewhat in their complexity in order to combat stationary and nonstationary perturbations, fluctuations of the parameters of the transmission path, etc.

If the transmission path has constant parameters and is sufficiently short, the signal received is merely a reduced and slightly delayed version of that transmitted. The noise added is the thermal and shot noise together with some picked up unwanted interference. The thermal noise defines and is proportional to the effective temperature of the radiation resistance of the aerial and thermal resistances of other passive components, mainly in the input stage of the receiver, and the shot noise is due to the currents flowing in the active components of the system. Thermal and induced noises set then a limit to the sensitivity and S/N define channel capacity (App.3).

On the other hand, signal transmission may be disturbed by moving objects, winds, dust, clouds, atmospheric or ionospheric turbulence etc., so that the transmission parameters of the signal path fluctuate. The signal and hence the message is then modified by the fluctuation of these parameters. In other words, the desired signal information appears along with unwanted information.

If the parameters vary slowly in relation to the lowest frequency in the band of frequencies in the signal modulation, then it is well known that automatic gain control, A.G.C. can be used to reduce the effect of varying transmission parameters. In using A.G.C. the assumption is that the transmitted carrier is of constant amplitude. The A.G.C. signal is obtained by passing a part of the detected signal current through a circuit with a time constant longer than the period of the lowest signal frequency. The modulation is removed thereby and a D.C. voltage is produced which is used to vary the gain of the receiver inversely as the received signal strength. The level of signal output is set by comparing the A.G.C. voltage with a standard level, set at the receiver, which represents the constant level transmitted.

If the parameters of the transmission path vary more quickly as for example in the communication link to and from rockets, the effect cannot be removed in this way, since if a shorter time constant were used, the A.G.C. voltage would vary at signal modulation frequency and so remove at least some portions of the message spectrum.

As a result of the investigations to be described later, it was proposed to reduce these defects by transmitting a signal which contains modulation due to the signal to be transmitted and also a reference signal of constant magnitude and phase in relation to the signal modulation. This reference signal need only be of nominally constant frequency. It is desirable that the reference signal should not increase the power to be transmitted appreciably.

It is also necessary that the reference signal should be separated very easily and completely from the information bearing signal. The reference signal can then be used to control the gain of the amplifier producing it together with that amplifying the information carrying signal in such a manner as to bring the reference signal to a constant level. The variations in transmission parameters can then take place at a rate faster than the lowest frequency in the signal, and their effect will be removed by the rapid acting automatic gain control, R.A.A.G.C.

The R.A.A.G.C. signal is obtained by using an amplifier having a broad bandwidth on the reference signal, in fact, the reference signal and the information signal may be amplified by the same amplifier and separated at the output. The operation of the R.A.A.G.C. will then be able to remove some of the effects due to fading at frequencies in the signal band. (Chapter 9).

Even if a constant frequency local oscillator be used to detect the signal modulation, the R.A.A.G.C. will bring about a reduction in the effects of the transmission path.

It has also been found that the use of even very fast A.G.C. alone, does not remove all the defects in the signal on account of the variations in the transmission characteristics. There are still left effects due to phase shifts, differential delays, selective fading, which over considerable distances may be appreciable, causing distortion of the modulation on detection.

The remaining defects in the signal can be removed, at least over a coherence bandwidth, if the local oscillator is replaced by the reference signal. The possibility of doing this rests on the properties of the distortion produced by variations in the fluctuations in the transmission path.

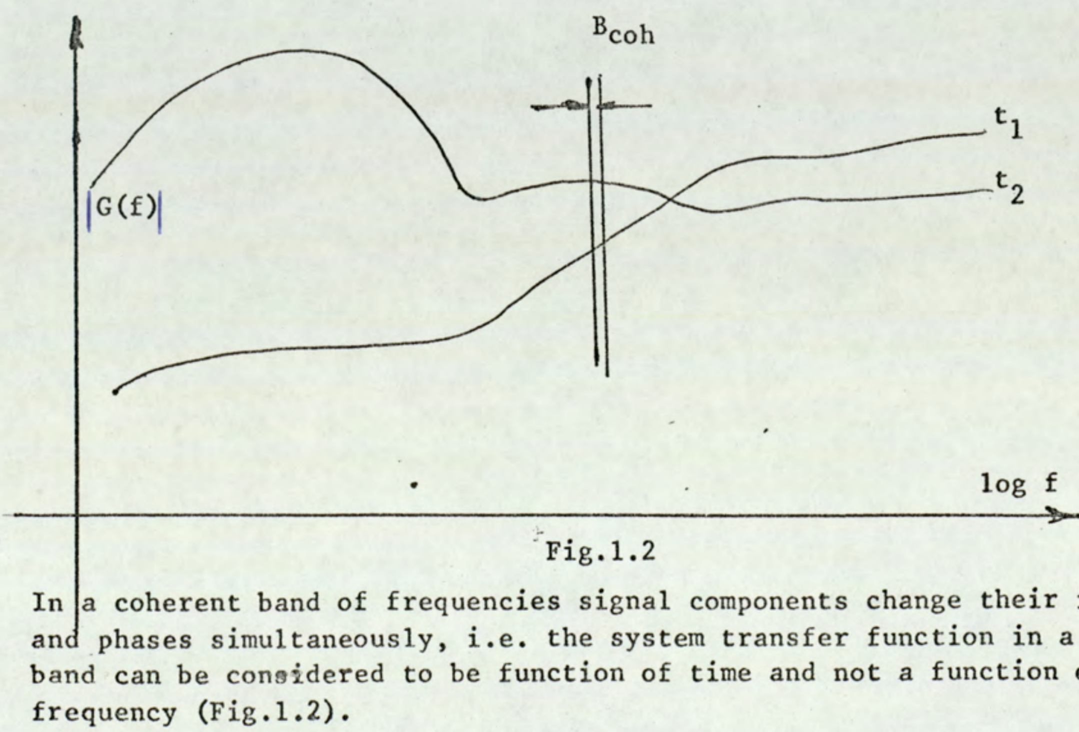


Fig.1.2

In a coherent band of frequencies signal components change their intensities and phases simultaneously, i.e. the system transfer function in a narrow band can be considered to be function of time and not a function of frequency (Fig.1.2).

At any one frequency, the total received signal intensity is a vector sum of randomly and individually attenuated and delayed components. Thus, a frequency component of a signal is a stochastic function of time. In a narrow band of frequencies there exists between these functions, a degree of coherence ranging from the value of 1, down to say 0.95. Over wider bands, the coherence falls to values near zero. Therefore, (Fig.1.2) for a particular time, the amplitudes (and phases) of different frequency components are functions of frequency. Each signal component is scattered into a spectrum, whose shape is dependent both on the rate and on the intensity of fluctuations in parameters (Table 3,p.194). Hitherto some parts of the problems presented by these fluctuations have been corrected in various ways, which will now be mentioned briefly.

When the transmission parameters change slowly with time, this phenomenon is known as selective fading. Space and/or frequency diversity reception, combined with the automatic gain control, and the use of narrow-beam anti-fading antennas are the usual remedy. For relatively low speed signalling a system utilizing correlation techniques in combination with delay compensation, known as the Rake system, has proved to be suitable (Price and Green 1958). More sophisticated systems, using multiple sub-carriers and AGC for each channel separately made possible reliable long distance digital data transmission (Zimmerman and Kirsch, 1967). When the transmission parameters change quickly, the frequency spread is relatively wide and the phenomenon is similar to a noise problem, and therefore has been approached similarly. It was pointed out (Bull and Bozic, 196), that this type of noise is found also in components normally regarded as passive such as resistors and also in active circuit components. The phenomena encountered in transmission systems differ from those in amplifying and detecting circuits only in that the actual time delays and phase shifts are more striking in the former.

1.2 Stochastic Signals.

Practically all signals containing information are to some extent random in nature, i.e. they are stochastic.

A stochastic signal, since it is generated by a system which has an element of chance, contains an element of chance as regarded from the receiver, and it cannot therefore be represented by a deterministic function, such as for example, a sine wave.

Also, if a deterministric signal is transmitted through a system having fluctuating parameters it will bring to the output of the system some information about the fluctuating process taking place in the system, since it will have acquired to some extent random characteristics.

In both the case of a stochastic signal and that of a deterministic signal which has become to some extent random, one cannot say that the stochastic signal generated in this way will have a specified value at a particular time, but only that it has a certain probability of lying within certain ranges of values, i.e. in multi-dimensional signal space, the termination of the vector representing the signal will lie within a small closed space, S.

For a completely predictable or deterministic function the idea of function implies a definite dependence of the variable upon its argument.

In a sense the wholly predictable signal is the limiting case of a stochastic signal which the probability distributions have been strongly peaked so that the uncertainty of the location of the signal in signal space for a particular value of the argument is zero.

On the other hand, after the value of a stochastic variable has been observed it is known that for very close future, its probability distribution is very strongly peaked and the uncertainty concerning the position of the signal in signal space tends to zero. Starting from this point one can predict the future values. The probability distribution widens with time (Fig.3.7a)in a space C_i of possible outcomes, ($C_i \subset S$, i.e. C_i

is a subset of a closed space S).

A signal of almost any kind can be represented by the real and the imaginary parts of

$$\underline{S}(t) = \underline{A}(t) \exp\left[j(\omega(t)t + \phi(t)\right] \qquad 1.2.1$$

where the amplitude $\underline{A}(t)$, the frequency $f(t) = \omega(t)/2\pi$, and the phase $\phi(t)$ in general can be functions of time. The vector notation such as $\underline{S}(t)$ indicates that S(t) can be a space vector. However, if we limit ourselves to its scalar quantity, we can write $\underline{S}(t)$ as

$$S(t) = A(t) \exp j\psi(t) \qquad 1.2.2$$

where $\psi(t) = \omega(t).t + \phi(t)$ 1.2.3

Both A(t) and/or $\psi(t)$ can be constant periodic or vary randomly. In the case of a signal that has to convey a message and the exact nature of the message is not known "a priori", the signal is random for there would otherwise be no point in transmitting it.

On the other hand, even if A(t) and $\psi(t)$ are very complex functions of time, but their past and future behaviour are known or could be predicted exactly, the signal is deterministic. There must be some element of unpredictability if new information is to be obtained, or, in other words, all signals carrying information are random.

Note that signal carrier being deterministic, at least at the transmitting end of an information transmission system, could often be suppressed, thus making the system more economical.

When transmitted through a physical system with time varying parameters, the carrier or some other kind of transmitted reference signal may be of great use as it will be shown in this work.

When a signal is transmitted through a system which is noisy and has randomly fluctuating parameters, the degree of randomness observed in the signal is increased, and obviously the desired information is masked.

Denoting the entropy of the source of messages as H(x) and that of the signal received as H(y), it is possible to write, (Shannon and Weaver, 1948/1949 ,book published 1963).

$$H(x) - H_y(x) = H(y) - H_x(y) \qquad 1.2.4$$

where $H_y(x)$ is the uncertainty of the message source if the received signal is known, and

$H_x(y)$ is the uncertainty of the received message if the message transmitted is known.

In order to be able to study more thoroughly the mechanism of the signal perturbations and therefore the corresponding loss of information, random processes should be discussed first (Chapter 2).

For an amplitude modulated signal sent from the transmitting end of a system, we may write,

$$^{k}s(t) = {}^{k}a(t) \ \underline{/\omega_c t + {}^{k}\psi_o} \qquad 1.2.5$$

and for the receiver,

$$\{z(t)\} = a(t)\ \{y_1(t)\}\ \underline{/\omega_c t + \psi_o \cdot \{y_2(t)\}} + \{n(t)\} \qquad 1.2.6$$

where , using matrix notation, (see p. 63 et seq.)

$\{y_1(t)\}$ is the uncertainty due to the fluctuating characteristics of the system or medium on the amplitude $x(t)$ of the signal

$\{y_2(t)\}$ is the uncertainty due to the fluctuating characteristics operating on the total phase of the signal, (t).

$\{n(t)\}$ is the additive noise due to the same system, this being assumed to be present even in the absence of any signal.

In general, there can exist some correlation between particular column matrices ${}^k y_1(t)$ and ${}^k y_2(t)$, but not necessarily between either of these and $\{n(t)\}$. One of the aims of the work carried out for this thesis is to what correlation there is between the functions ${}^k y_1(t)$ and ${}^k y_2(t)$.

It should be noticed that $\{y_1(t)\}$ and $\{y_2(t)\}$ constitute or contain information about the fluctuations in the characteristics of the system. Experiments carried out by Bozic (1965, 1967*) show that they have as a rule different power spectra than that of $\{n(t)\}$. Unlike the latter their frequency spectra are found to be concentrated in the neighbourhood of the signal frequencies.

In other words, the noise spectra of $y_1(t)$ and $y_2(t)$, unlike that most often assumed for $n(t)$, are not "white", and depend not only on the signal frequency but also on its magnitude.

In order to determine the properties of the fluctuations of the system characteristics, an experimental method was developed in which the signal and noise were measured over short periods of time and studied theoretically (Chapter 7).

* Bull and Bozic 1967

CHAPTER 2

2.1 Stochastic Processes

The word stochastic is used by mathematicians and physicists to describe processes in which there is an element of chance, but in which also the outcome is to some extent predictable. The word random is used often in the same sense, but it has been in use for a much longer time and has acquired connotations which are not always sufficiently precise. For example, it is often implied that when the statement has been made that a succession of events occur at random, that the word random defines some feature of the succession of events. Alternatively, the words "completely random" are sometimes used, which implies that all successive events defined in this way have the same properties, but are quite independent from the succession of previous events.

The fluctuating parameters of a physical system belong to a group of stochastic processes known as Markov processes. In the zero order Markov processes, each value observed at successive intervals of time has no dependence on recent previous events. Such processes do, in fact, occur, as for example when counting the number of electrons given off by a photo-cell when illuminated by a constant light source. They have often been referred to as the "completely random" processes mentioned in the last paragraph. Thus, a Markov process of zero order is not often in mind when a process is defined as being a Markov process.

The first order Markov process is one in which the present value of the function depends on the value observed at the preceding or a previous observation. A second order Markov process is one in which the present value depends on the last value and the last but one, or, expressed alternatively, on the last value and the first derivative of the function.

Since the process in this research can be considered to be Markovian, and the results of observations can to some extent be predicted at least for the very close future, the use of the word stochastic is more precise than the use of the word random in this connection. In what follows the word stochastic is used when a degree of predictability exists, or, in other words there is some dependence on previously observed values. The word random then defines the process as being a Markov process of zero order.

It has been shown (Bull, 1966) that a Markov process of the first order can be constructed theoretically by adding together two processes. One of the two is a zero process, a new occurrence, while the other is a decayed remnant from the previous events. It is not possible, however, by simple observations to separate out the two processes from the observations constituting the first order process.

In the experiments on sampling the output from the process generating the stochastic function, the example when a deterministic signal is transmitted through a system with fluctuating parameters, a new fluctuating waveform will be obtained which exhibits some regularities in a wide statistical sense. There will be more or less well defined probability distributions but also transition probabilities and the degree of coherence between input and output will be observable.

Some physicists, particularly those concerned with quantum mechanics and noise, take the view that all physical processes are discontinuous and entirely undeterministic when examined in sufficiently fine detail to reveal the atomic constitution of the system. (Born, M. 1946; Bull, 1966). The use of continuous functions is regarded as being acceptable only as and when they can successfully be related to the influences and interdependence of macroscopic parameters of the system under investigation.

The main common feature of a random process is the indeterminancy of its behaviour. Knowledge of the past behaviour of a completely random function or of a set of functions does give only a statistical indication of the values to be expected at some future time.

Any single stochastic or random function is an accident, never likely to occur again. The best one can do to define it is to make a particular set of measurements over a large collection or set or ensemble of functions, and determine various average properties of the sets so as to define some of the statistical properties of the ensemble.

In many particular cases, estimates of the mean value and the mean square values constitute two kinds of statistical information from which reasonable conclusions about the spread of values which might be obtained in future observations.

Statistical analysis for stochastic signals can be carried out in terms of probability density functions and other statistical characterisations such as the average values and correlation functions.

In view of its statistical properties a stochastic signal is often conveniently treated as a member of a family of signals, each generated by a statistically identical process. This family of signals is termed an ensemble or set, and can be represented in mathematical form by a matrix.

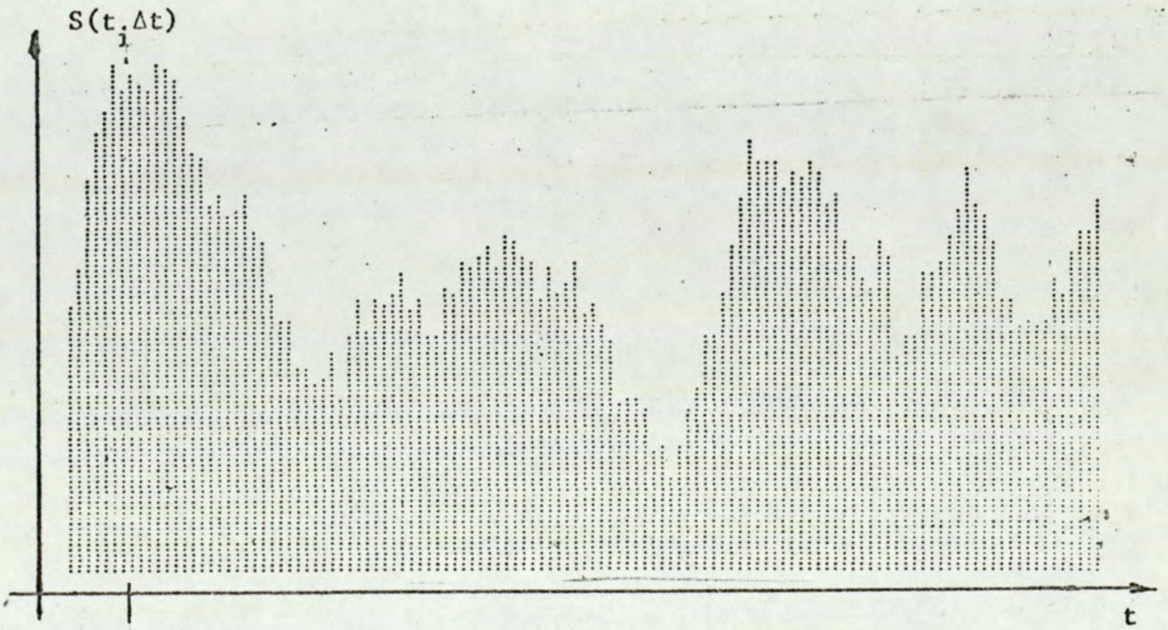


Fig.2.1 An actual signal magnitude fluctuation obtained in an experiment made for this research (Δt_s= 4msec). (No phase lock).

2.2 Discrete representation of random functions.

A random function can be defined as an ensemble or set of deterministic functions of time, each defined for a prescribed range from t_1 to t_2 of real time, or, if the reference time is taken to be $t_1 = 0$ from the times 0 to $t_2 = T$.

The notation x(t) will be used to denote an infinite dimensional column matrix listing all the values of the variable x along an infinitesimally divided or discrete set of values of the real time t. The use of a linear scale of t will be assumed, so that x(t) can alternatively be understood as a numerical representation of a graph of x(t) against time on an axis extending down the column matrix. (Fig.1.2)

The sampling theorem (Shannon, 1949) states that it is possible to define a continuous function signal(or fluctuation)with any desired degree of accuracy in terms of the sequence of values which the function assumes at discrete intervals of the independent variable, which in the case of signals, is the time, t (Appendix 1). The instants at which samples are taken must be made close enough to reproduce as much of the fine structure of the signal and/or fluctuation, as is required for any particular analysis.

In general x(t) can be transmitted by a process of modulation in many ways, either in terms of A(t) (amplitude) or $\psi(t)$ (phase). In any case we are interested in reproducing the random function information signal at the receiver of the communication system in a form resembling as closely as possible the transmitted signal ${}^k x(t)$. Whatever the process used, the initial signal ${}^k x(t)$ and the final output obtained, represented by ${}^k z(t)$, can be represented by a time varying amplitude of current or voltage.

A set of functions x(t) can be represented by the use of successive indices, 0 to n, in matrix notation by

$$\{x(t)\} = \{{}^1x(t), {}^2x(t), \ldots\ldots {}^kx(t), \ldots {}^nx(t)\} \qquad 2.2.1$$

where in general $n \to \infty$, but is in practice large but finite.
Written in extended form the matrix appears as

$$\{x(t)\} = \begin{Bmatrix} {}^{1}x(0) & {}^{2}x(0) & \ldots\ldots & {}^{k}x(0) & \ldots. & n^{n}x(0) \\ {}^{1}x(1) & {}^{2}x(1) & \ldots\ldots & {}^{k}x(1) & \ldots. & {}^{n}x(1) \\ .. & .. & & .. & & .. \\ .. & .. & & .. & & .. \\ {}^{1}x(t) & {}^{2}x(t) & \ldots\ldots & {}^{k}x(t) & \ldots. & {}^{n}x(t) \\ {}^{1}x(T) & {}^{2}x(T) & \ldots\ldots & {}^{k}x(T) & \ldots. & {}^{n}x(T) \end{Bmatrix} \qquad 2.2.2$$

Any column of $\{x(t)\}$ in a particular possible realisation of the random function. When sending a signal we select one function or column from the set.

Any row of $\{x(t)\}$ for say $t = t_j$ is a random variable i.e. a set of values which the random variable can assume for any particular selected time $t = t_j$ in different realisations of the signal.

A noise added to the signal, which we will refer to as an additive noise to distinguish it from other types of noise to be discussed elsewhere, can be represented in the same time interval in a similar manner as $\{n(t)\}$. When the signal is transmitted through a physical medium, which may be for example, a turbulent atmosphere for radio signals, or a semiconductor device in a receiver, it is modified by the parameters which define the properties of the medium, or what may be called the characteristics of the medium. Only in an idealised case are the values of these parameters (defining the transmission) constant and unchanging with time. Both the signal amplitude (Fig.2.1) and phase are changed and made uncertain by the unwanted modulation due to the fluctuation in the parameters of the medium or system through which the signal is transmitted. A discussion of the fluctuations of the parameters or characteristics of a system is given by Bull in Chapter VI, pp 81 - 102, of his book (Bull, 1966).

2.3 The State Variable approach to Stochastic Processes.

Any stochastic stationary process $x(t)$ with a finite spectrum, i.e. limited at the high frequency end of the band covered, can be represented also by the Ito stochastic differential equation (K. Ito, 1951)

$$\frac{d^m}{dt^m}\, x(t) \; + \; \psi_1 \frac{d^{m-1}}{dt^{m-1}}\, x(t) + \ldots\ldots + \psi_m x(t) = \qquad 2.3.1$$

$$\lambda_1 \frac{d^{m-1}}{dt^{m-1}}\, \xi\,(t) + \lambda_2 \frac{d^{m-2}}{dt^{m-2}}\, \xi(t) + \ldots\ldots + \lambda_m \xi(t)$$

where $\psi_1 \ldots\ldots \psi_m$ and $\lambda_1 \ldots\ldots \lambda_m$ are coefficients and $\xi(t)$ is formally a random function with a white spectrum, or, in other words, it consists of a Markovian process of zero order.

Thus $x(t)$ can be realised by the passage of $\xi(t)$ through a low-pass filter or a band pass filter. A particular representation originated by Zadeh and Desoer (Linear System Theory) McGraw Hill, N.Y., 1963, p.281) is made by introducing the following substitutions:-

$$x(t) = x_1(t)$$

$$\frac{d}{dt}\, x_1(t) = -\psi_1\, x_1(t) + x_2(t) + \lambda_1 \xi(t)$$

$$\frac{d}{dt}\, x_2(t) = -\psi_2\, x_1(t) + x_3(t) + \lambda_2 \xi(t) \qquad 2.3.2$$

$$\ldots\ldots\ldots$$

$$\ldots\ldots\ldots$$

$$\frac{d}{dt}\, x_{m-1}(t) = -\psi_{m-1} x_1(t) + x_m(t) + \lambda_{m-1} \xi(t)$$

$$\frac{d}{dt}\, X_m(t) = -\psi_m x_1(t) + \lambda_m \xi(t)$$

or, in matrix form

$$\frac{d}{dt}\, \{x(t)\} = \{F\}\, \{x(t)\} + \{G\}\, \{\xi(t)\} \qquad 2.3.3$$

where

$$\{F\} = \begin{Bmatrix} -\psi_1 & 1 & 0 & 0 & \ldots & 0 \\ -\psi_2 & 0 & 1 & 0 & \ldots & 0 \\ & \ldots\ldots\ldots \\ -\psi_{m-1} & 0 & 0 & 0 & \ldots & 1 \\ -\psi_m & 0 & 0 & 0 & \ldots & 0 \end{Bmatrix} \qquad 2.3.4$$

$$\{G\} = \{\lambda_1, \lambda_2, \lambda_3, \ \ldots\ldots \ \lambda_m\}T \qquad 2.3.5$$

Note that {F} contains all the denominators and {G} all the nominator coefficients.

For a stationary process the coefficients are invariant with time, and therefore {F} and {G} are independent of time. Using Laplace transform we may write the Ito stochastic differential equation in the form,

2.3.6

$$(\lambda_1 s^{m-1} + \lambda_2 s^{m-2} + \ \ldots \ \lambda_m)x(s) = (s^m + \psi_1 s^{m-1} + \psi_2 s^{m-2} + \ldots \psi_m)\xi(s)$$

from which we may immediately write a corresponding transformer function of the so called Markovian filter, which memorizes the random process $\xi(t)$ generating a Markovian process $x(t)$.
In the time domain this can be expressed by

$$\xi(t) \rightarrow \boxed{\text{Markovian filter}} \rightarrow x(t)$$

In fact, if we process a random function with a white spectrum through even the simplest filter we shall get at the output a Markovian process the order of which will depend on the filter characteristic.

Using the state variable approach, and merely by the inspection of {F} and {G} we may draw the following diagram, Fig.2.2.

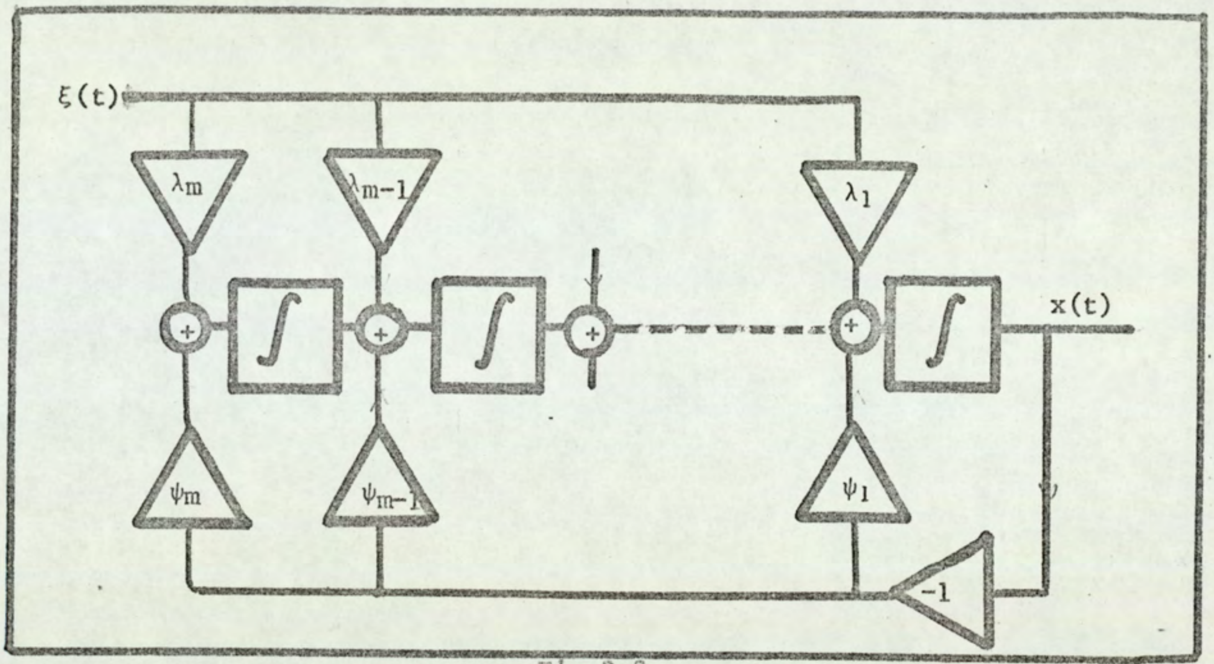


Fig.2.2

which represents the system transfer function of a Markovian filter.

Obviously from the Laplace Transform of the Ito stochastic differential equation

$$H_m(s) = \frac{\lambda_1 s^{m-1} + \lambda_2 s^{m-2} + \ \dots \ + \lambda_m}{s^m + \psi_1 s^{m-1} + \ \dots \ + \psi_m} \qquad 2.3.7$$

We can expand this expression as a partial fraction into the form

$$H_m(s) = \sum_1^m \frac{\alpha_i}{s + s_i} + D_0 \qquad 2.3.8$$

where the s_i are the roots of the denominator that are assumed to be distinct and real, and the α_i are the corresponding residues. Note that $D_0 = \lim_{s \to \infty} H(s)$. (See P. M. Derusso, R. J. Roy and C. M. Close, State Variables for Engineers, John Wiley and Sons, Inc. N.Y., pp.329-338)

The matrices {F} and {G} may then be written

$$F = \begin{Bmatrix} s_1 & 0 & \dots & \\ 0 & s_2 & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ & & & s_m \end{Bmatrix} \qquad 2.3.9$$

$$\{G\} = \{\alpha_1, \alpha_2, \alpha_m\}^T \qquad 2.3.10$$

and the transfer function diagram decomposed (Fig.2.3).

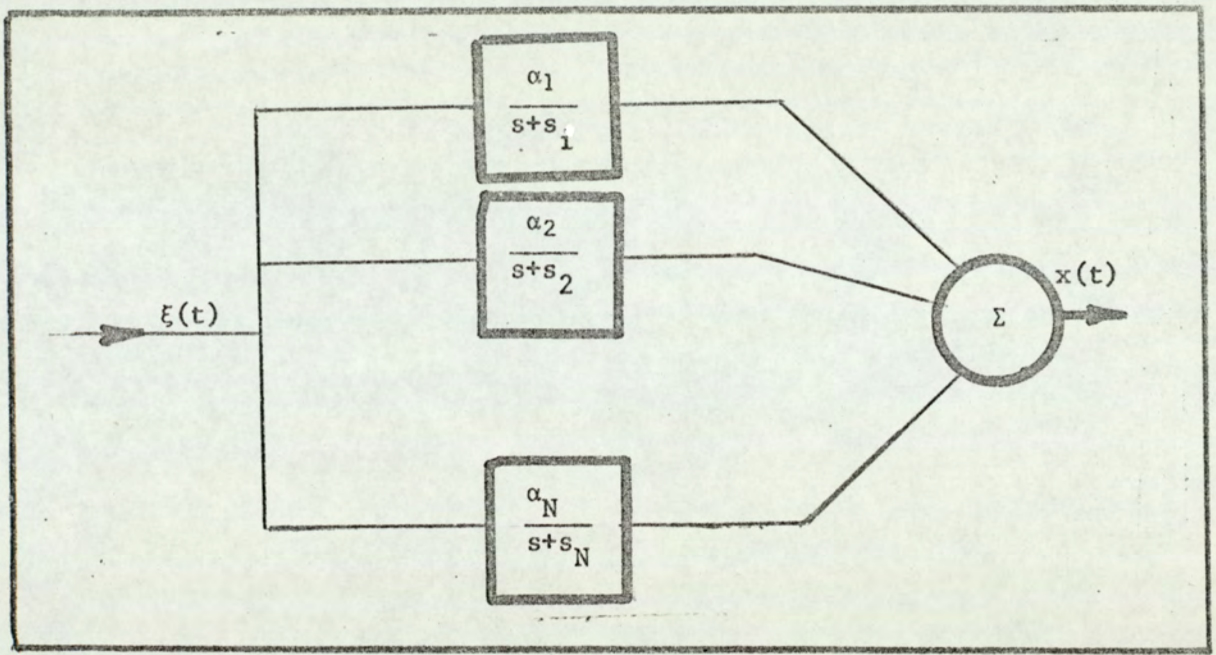


Fig.2.3

In general the roots may be repetitive and also appear in complex conjugate pairs (See P. M. Derusso et al. loc. cit. pp.329-338)

The first and second order Markovian processes.

A filter to generate a higher order Markovian process can be decomposed or at least approximated by several simultaneously operating first and second order Markovian filters. Thus the simplest first order filter may be represented by

$$H_0(s) = \frac{\alpha_1}{s + s_1} = \frac{\lambda_1}{s + \psi_1} \qquad 2.3.11$$

where $\alpha_1 = \lambda_1$ and $s_1 = \psi_1$ in the diagram (Fig.2.4) below

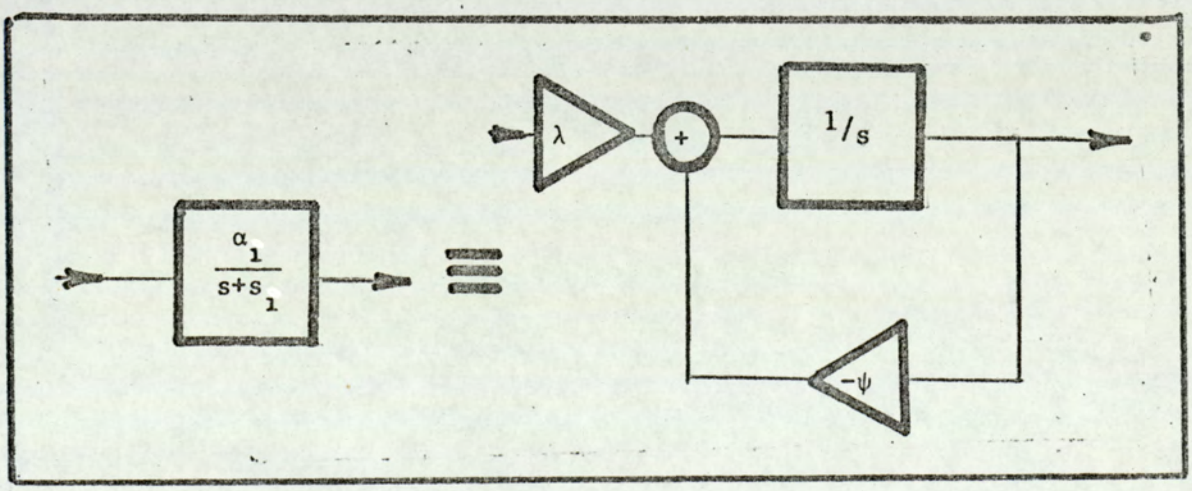


Fig.2.4

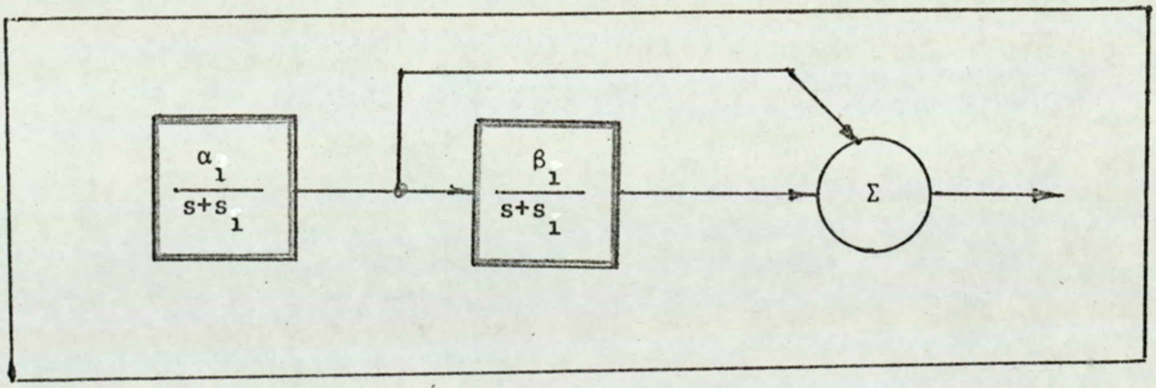
For either distinct or repetive roots of the second order Markovian filter we may use the already well known partial fraction expansion and represent it by

$$H(s) = D_o + \frac{\alpha_{i1}}{(s + s_i)^2} + \frac{\alpha_{i2}}{(s + s_i)} \qquad 2.3.12$$

where s_i is a repetitive root and α_{i1} and α_{i2} the corresponding residues. The given expression for a second order filter transfer function may also be obtained from the residue theorem for repeated roots. Thus, in general, for a k^{th} order root, (see Guillimin, pp.303-310)

$$\alpha_{IJ} = \frac{1}{(I - J)} \frac{d^{(k - J)}}{ds^{(k - J)}} \left[(s + s_i)^k . H(s)\right]\Bigg|_{s = s_i} \qquad 2.3.13$$

For a second order repetitive root, we may draw the following diagram:-



2.5

from which it is clear that the output from the first order Markovian filter is processed once again by the second first order Markovian filter.

This and even higher order Markovian processes could be realised physically as a microwave resonator partially or completely filled by a dielectric having fluctuating parameters, for example, a slightly ionised flame.

The power density spectrum of the fluctuating process at the output is a sum, if the output is of higher order, of the several power density spectra.

2.4 Generating a Markovian process which has an exponential cosine correlation function using a digital computer.

It can be shown that an exponential cosine correlation function (Sec.3.5).

$$\psi(\tau) = A\, e^{-\alpha\tau} \cos \omega_o \tau \qquad 2.4.1$$

has a Fourier transform

$$P(\omega) = |Y(j\omega)|^2 = \frac{2A\alpha}{\pi} \frac{\omega^2 + \alpha^2 + \omega_o^{\,2}}{\omega^4 + 2(\alpha^2 - \omega^2)\omega_o^2 + (\alpha^2 + \omega_o^{\,2})^2}$$

$$= \frac{2A}{\pi} \frac{j\omega + (\alpha^2 + \omega_o^{\,2})^{\frac{1}{2}}}{(j\omega + \alpha + j\omega_o)(j\omega + \alpha - j\omega_o)} \cdot$$

$$= \frac{-j\omega + (\alpha^2 + \omega_o^{\,2})^{\frac{1}{2}}}{(-j\omega + \alpha + j\omega_o)(-j\omega + \alpha - j\omega_o)} \qquad 2.4.2$$

substituting $s = \sigma + j\omega$ and evaluating $|Y(j\omega)|$ on the imaginary axis

$$Y(s) = \left(\frac{2A\alpha}{\pi}\right)^{\frac{1}{2}} \frac{s + (\alpha^2 + \omega_o^{\,2})^{\frac{1}{2}}}{s^2 + 2\alpha s + \alpha^2 + \omega_o^{\,2}} \qquad 2.4.3$$

which is the second order Markovian filter transfer function.

Putting at the input of this filter a random function with a white power spectrum $\xi(t)$, at the output we shall obtain $x(t)$ with an exponential cosine correlation function. For this reason consider a simple feedback system with a forward transfer function

$$G_1(s) = A_1/(s + s_1) \qquad 2.4.4$$

and a feedback path with a transfer function

$$G_2(s) = A_2/(s + s_2) \qquad 2.4.5$$

where A_1 and A_2 are the amplification coefficients and s_1 and s_2

the poles of the corresponding transfer functions (Fig.2.6)

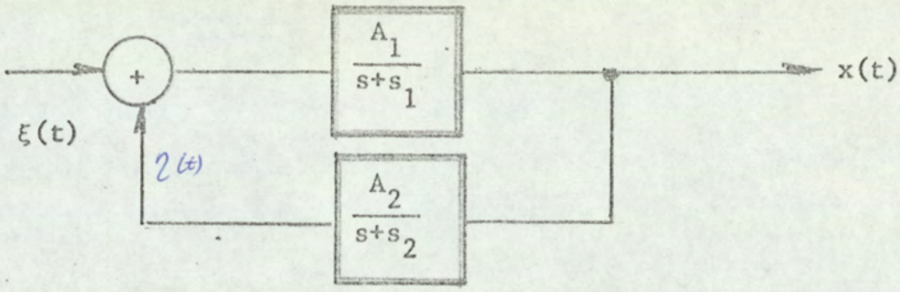


Fig.2.6

In the time domain this can be represented by (Fig.2.7)

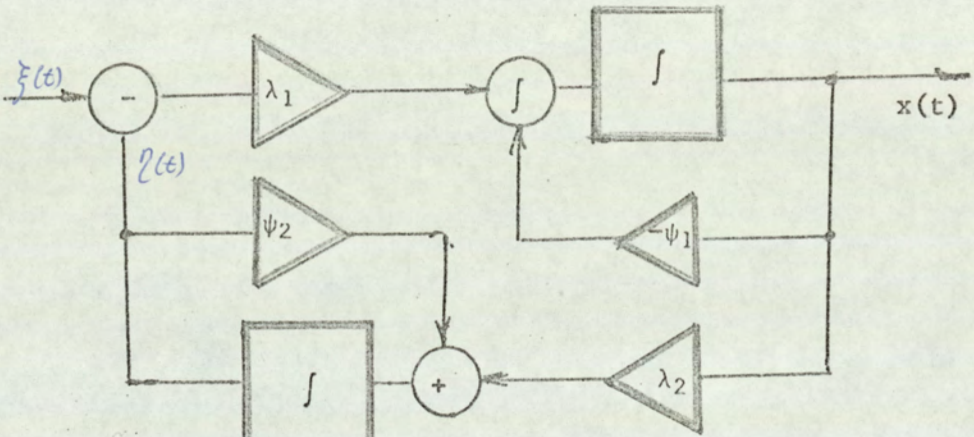


Fig.2.7

where as we have already shown,

$$\lambda_1 = A_1 \quad ; \quad \lambda_2 = A_2 \quad ; \quad \psi_1 = s_1 \quad ; \quad \psi_2 = s_2$$

By inspection we may write

$$x(s) = G_1(s) \left[\xi(s) - \eta(s)\right] \qquad 2.4.6$$

$$\eta(s) = G_2(s) \, . \, x(s) \qquad 2.4.7$$

Substituting the latter relation in the previous one, we obtain

$$x(s) = G_1(s) \left|\xi(s) - G_2(s)x(s)\right|$$

so that

$$x(s) = \frac{G_1(s)}{1 + G_1(s).G_2(s)} \, . \, \xi(s) \qquad 2.4.8$$

which is the familiar expression for a feedback system with a single feedback loop.

The closed loop transfer function $Y(s)$ will be given by

$$Y(s) \quad = \quad \frac{x(s)}{\xi(s)} \quad = \quad \frac{A_1/(s + s_1)}{1 + A_1A_2/\ (s + s_1)(s + s_2)}$$

$$= \quad \frac{A_1(s + s_2)}{s^2 + (s_1 + s_2)s + (s_1s_2 + A_1A_2)} \qquad 2.4.9$$

If we compare this expression with the previously obtained expression for a Markovian shaping filter for generating a stochastic process having an exponential cosine correlation function, we find by inspection

$$s_1 \quad = \quad (\alpha^2 \quad + \quad \omega_o{}^2)^{\frac{1}{2}} \qquad 2.4.10$$

$$A_1 \quad = \quad \left(\frac{2A\alpha}{\pi}\right)^{\frac{1}{2}} \qquad 2.4.11$$

$$s_1 + s_2 = 2\alpha \qquad 2.4.12$$

$$A_2A_1 + s_1s_2 \quad = \quad \alpha^2 + \omega_o{}^2 \qquad 2.4.13$$

It is necessary therefore to solve four equations to determine appropriate parameters. Note that for $\omega_o \; = \; 0$,

$$S_1 \quad = \quad s_2 \quad = \quad \alpha$$

$$A_1 \quad = \left(\frac{2A\alpha}{\pi}\right)^{\frac{1}{2}} \quad ; \quad A_2 \quad = \quad 0$$

so that the shaping filter and therefore the Markovian process also degenerates to the first order filter and process respectively, represemted diagrammatically by (Fig.2.8)

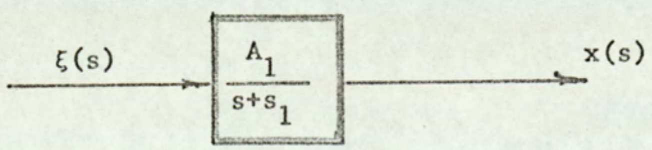


Fig.2.8

Bearing in mind these derived relations a simple computer programme for generating a Markovian generating process with exponential cosine correlation function has been written. (Procedure FLICKER) (See Fig 2.9) An analogue simulation could be made experimentally and directly using the circuit diagram Fig.2.7 or Fig.2.8.

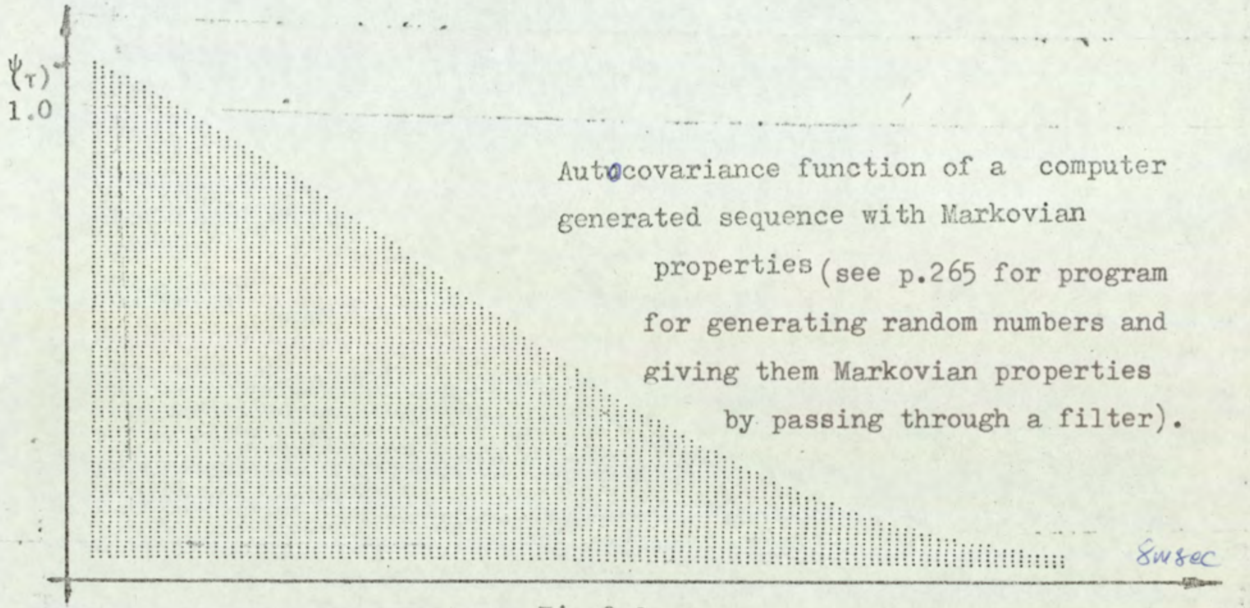


Fig.2.9

Compare the correlation functions obtained from a sequence of computer generated numbers with Markovian properties (Program FLICKER) and the experimentally obtained (Fig.2.10), when a test signal was transmitted through a laboratory ionised flame.

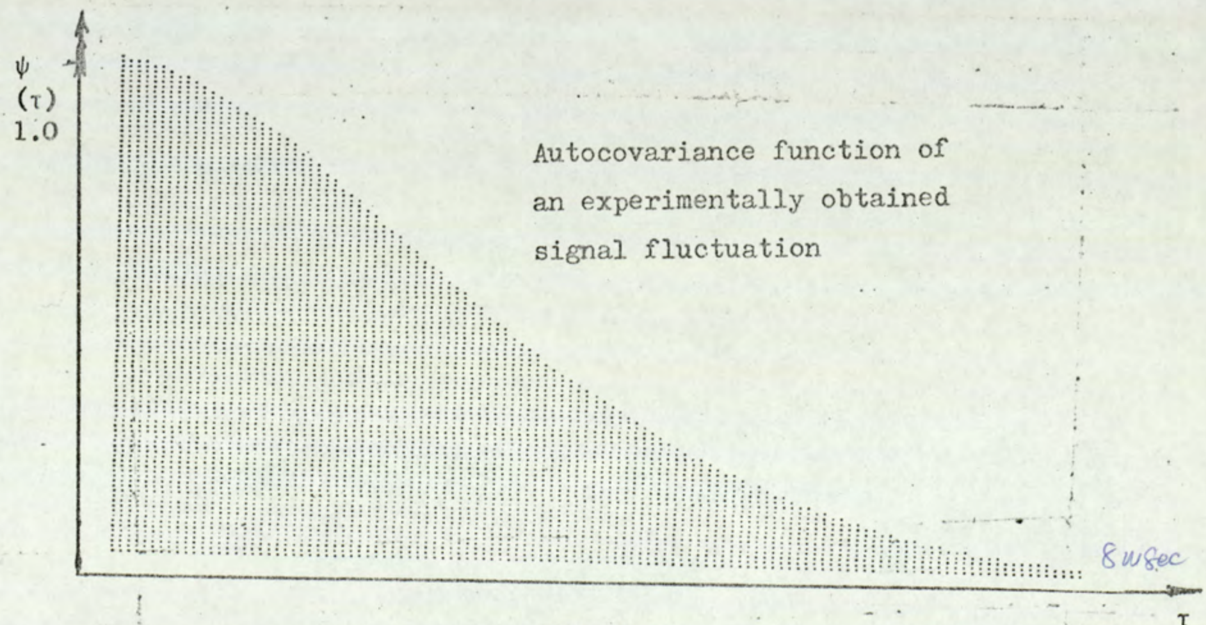


Fig.2.10

CHAPTER 3

Analysis of stationary and non-stationary processes.

A stationary process is one of which the statistical properties remain constant when the time origin is shifted, i.e. when the ensemble averages are independent of time.

Since for a stationary process the probability density functions are independent of the time origin, it follows that the statistics of a stationary process are revealed by an examination either of a set of functions for any selected time t, or by taking a particular example of sufficiently long duration, which may be mathematically approximated to a sample of infinite duration.

We can define T as the total time range through which a particular function from a set of stationary functions is taken. The element of time ΔT is the time during which the function lies between the value x_1 and $(x_1 + \Delta x_1)$ during the total time T as t changes from 0 to T.(Fig.3.1) From this the first probability density function can also be defined as

$$p_1 (x) = \lim_{\substack{T \to \infty \\ \Delta x \to o}} \frac{\dfrac{\Delta T_1(x_1,\Delta x_1;T)}{T}}{\Delta x} = \lim_{\substack{T \to \infty \\ \Delta x \to o}} \frac{1}{T} \cdot \frac{\Delta T}{\Delta x} \qquad 3.0.1$$

The second probability density function.

Let $\Delta N_2(x_1,t_1,\Delta x_1;\ x_2,t_2,\Delta x_2)$ be the number of values lying in the range Δx_1 at x_1 at time t_1 and in the range Δx_2 at x_2 at time t_2. Then the second probability density function is defined by

$$p_2(x_1,t_1;x_2,t_2) = \lim_{\substack{N \to \infty \\ \Delta x_1 \to o \\ \Delta x_2 \to o}} \frac{\dfrac{\Delta N_2}{N}}{\Delta x_1 . \Delta x_2} \qquad 3.0.2$$

For any stationary process we expect that the fraction $(\Delta N_2/N)/\Delta x_1 \Delta x_2$ tends to a finite value as Δx_1 and Δx_2 tend to zero.

For stochastic processes the second probability density function gives a more detailed picture of the ensemble of stochastic signals than does the first. This follows because the second probability density function is a function of time and the variable x. The first probability density function can be derived as a special case of the second as $t_2 \rightarrow t_1$.

It is almost obvious that for a stationary process the second probability density function will be independent of the particular values t_1 and t_2 chosen, provided the difference,

$$\tau = t_2 - t_1 \qquad 3.0.3$$

which is the delay variable, remains constant. The second probability density function can then be defined by

$$p_2(x_1,x_2;\tau) = \lim_{\substack{\Delta x_1 \to 0 \\ \Delta x_2 \to 0}} \Delta N_2/N\Delta x_1 . \Delta x_2 \qquad 3.0.4$$

As was done for the first probability density function, we may take out of an ensemble one particular stochastic function and obtain the second probability density function defined by

$$p_2(x_1,x_2;\tau) = \lim_{\substack{T \to \infty \\ \Delta x_1 \to 0 \\ \Delta x_2 \to 0}} \Delta T_2/T\Delta x_1 \Delta x_2 \qquad 3.0.5$$

where ΔT_2 is the total time from the duration of the selected signal, T, for which the values of x is in the range Δx_1 at x_1 at time t and the value of x is in the range Δx_2 at x_2 at the time $(t + \tau)$.

For a fixed value of x_1, if the process is stationary,

$$p_1(x_1) = \int_{-\infty}^{+\infty} p_2(x_1,x_2;\tau)dx_2 \qquad 3.0.6$$

and furthermore

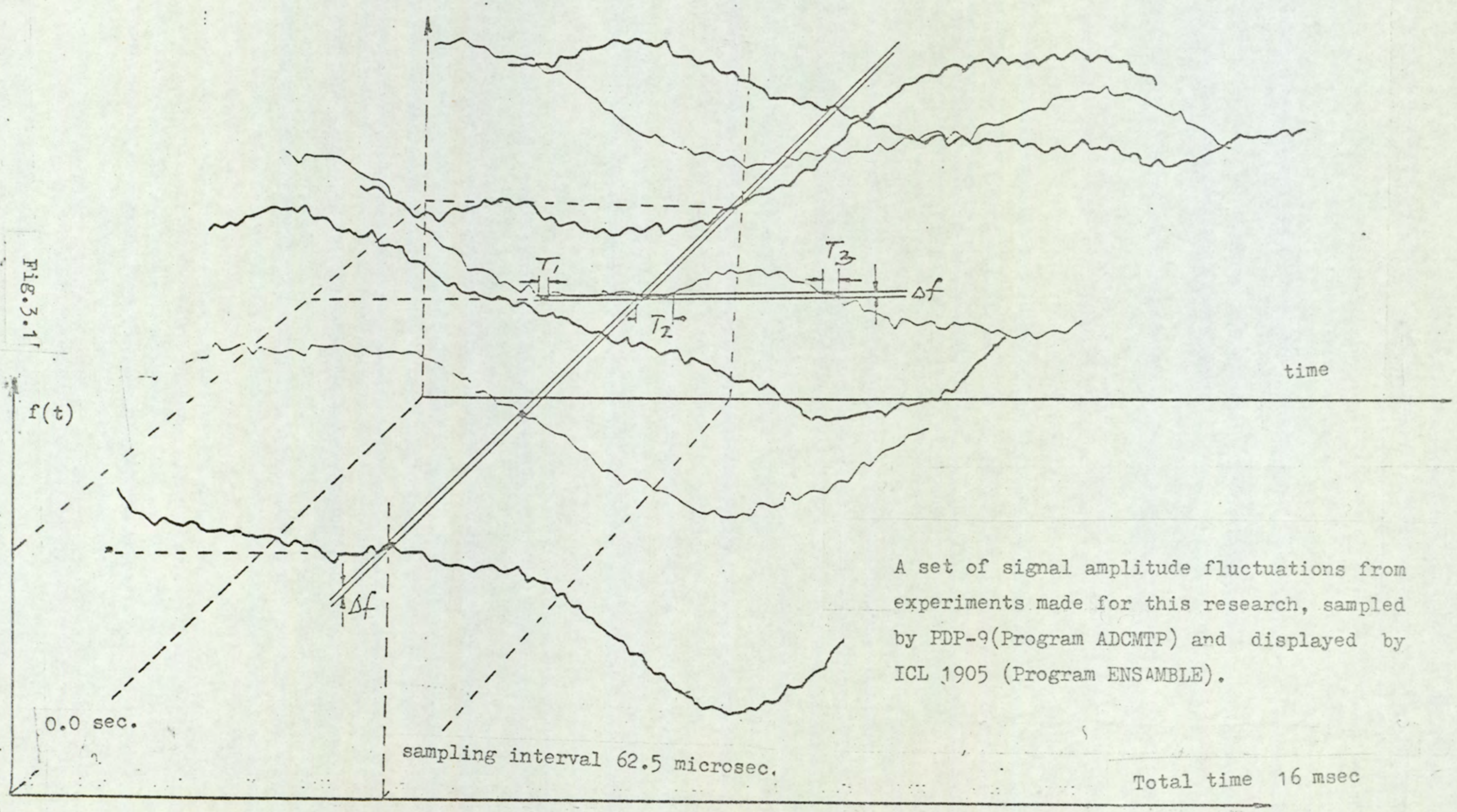


Fig.3.1

A set of signal amplitude fluctuations from experiments made for this research, sampled by PDP-9(Program ADCMTP) and displayed by ICL 1905 (Program ENSAMBLE).

$$\int_{-\infty}^{+\infty} p_1(x_1)\, dx_1 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_2(x_1,x_2;\tau)\, dx_2 dx_1 = 1 \qquad 3.0.7$$

A diagram representing this equation is given in Fig.3.2
The area under the second probability density function taken for all values of x_2 is the value of $p_1(x_1)$.

The second probability density function is closely connected with the correlation function, and this connection will be discussed later.

From the diagram in Fig.3.2 it will be seen that the occurrence of the value x_1 at time t, or when $\tau = 0$, leads to a probability distribution for the values of x_2 at a time $(t + \tau)$. The second probability density is therefore connected with Markov processes.

In order to be able to apply analysis pertinent to finite matrices, which is suitable for numerical computation, we shall divide continuous variables into a finite number of discrete values, such that $x_i \sqsubset S$ where S is the total and closed space of the variable x.

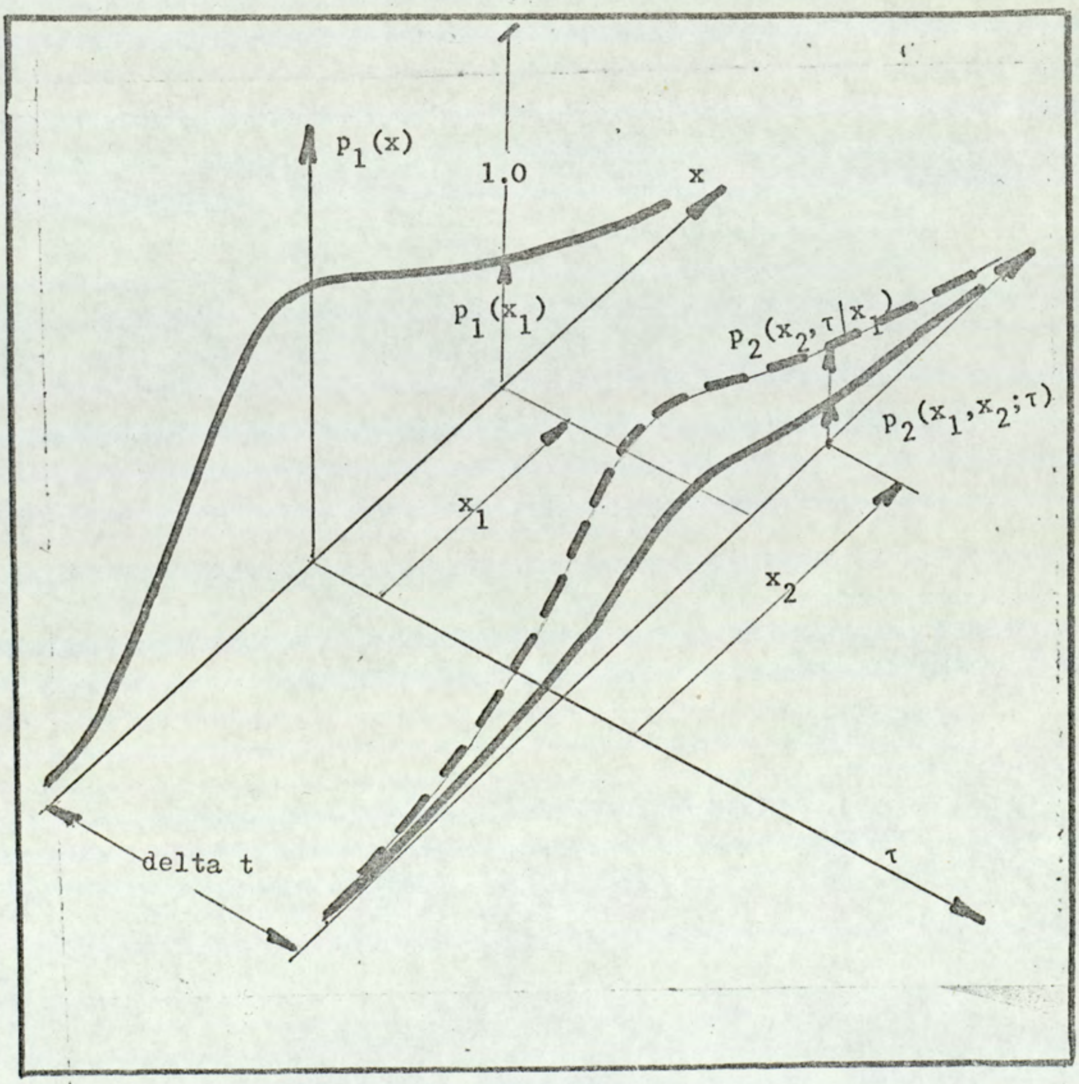


Fig.3.2

3.1 Probability density functions.

In general an infinite set of probability density function is required to describe completely a stochastic process. Simplifications can be made when the statistics of the process are independent of the particular time interval over which the stochastic process is investigated. The process is then said to be stationary.

The first or initial probability density function may be approximated as the ratio of the function of the number of signals lying in the range between x_1 and $(x_1 + \Delta x_1)$ at time t_1 (Fig.3.1)

Thus if

N = the total number of signals in the ensemble

$N_1\ (x_1,\ x_1 + \Delta x_1,\ t_1;N)$

= the number of signals lying between x_1 and $(x_1 + \Delta x_1)$ at time t_1

then

$$p_1(x_1,t_1;N) = \lim_{N \to \infty} \lim_{x \to o} \frac{\dfrac{N_1(x_1,x_1+\Delta x_1,t_1;N)}{N}}{\Delta x} \qquad 3.1.1$$

$$\therefore \qquad p_1 = \lim_{\substack{N \to \infty \\ \Delta x \to o}} \frac{1}{N} \cdot \frac{\Delta N}{\Delta x}$$

The first probability density function is a function of the location x, and for a stationary process, is independent of time.

In general a single sample of a stochastic function (k th sample) does not have a sufficiently significant average value when taken with respect to time. It will be an average pertaining to the particular period of time over which the signal was observed, and may be different from an average calculated from an ensemble of similar signals, or from an average taken at another time.

To obtain a significant ensemble average value it is necessary to take the average over an ensemble at a particular time. Thus, in

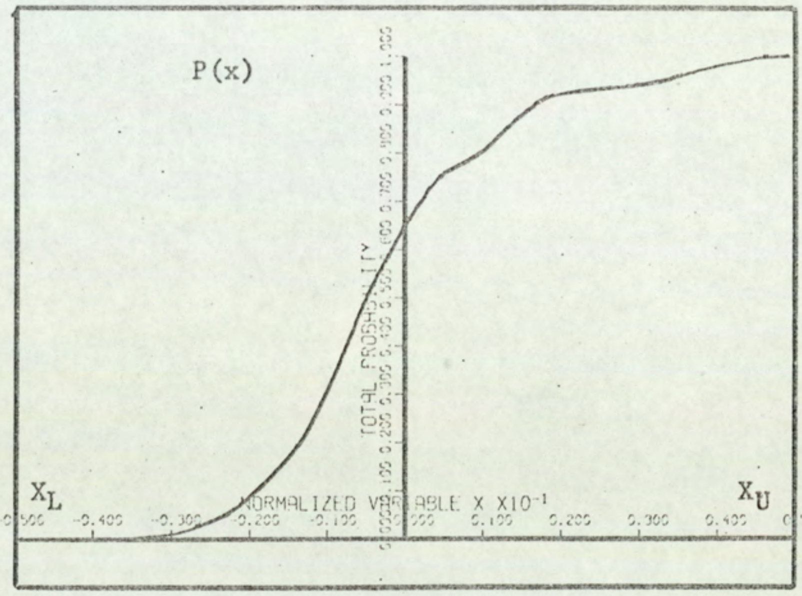


Fig.3.3

An experimentally obtained cumulative probability P(s) where signal is transmitted through a laboratory flame (T = 0.1 sec)

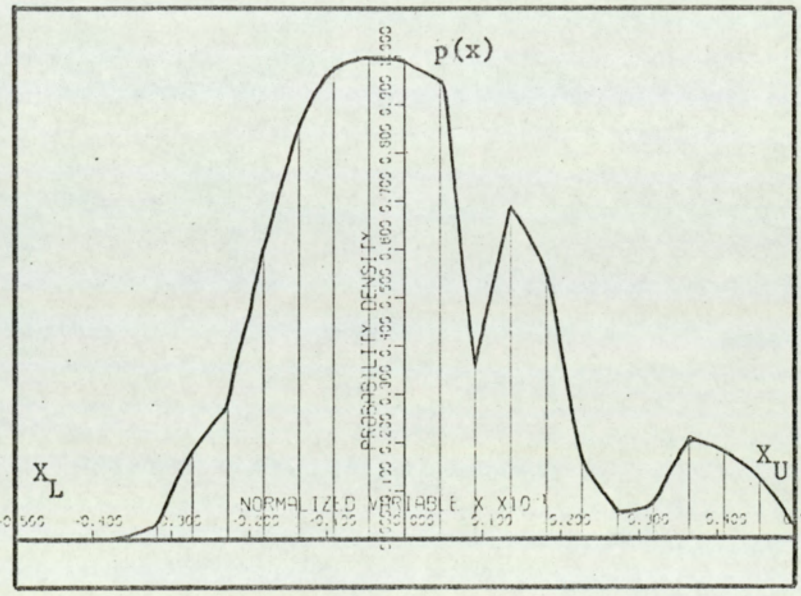


Fig.3.4 a

An experimentally obtained probability density functin p(x) corresponding to Fig.3.3.

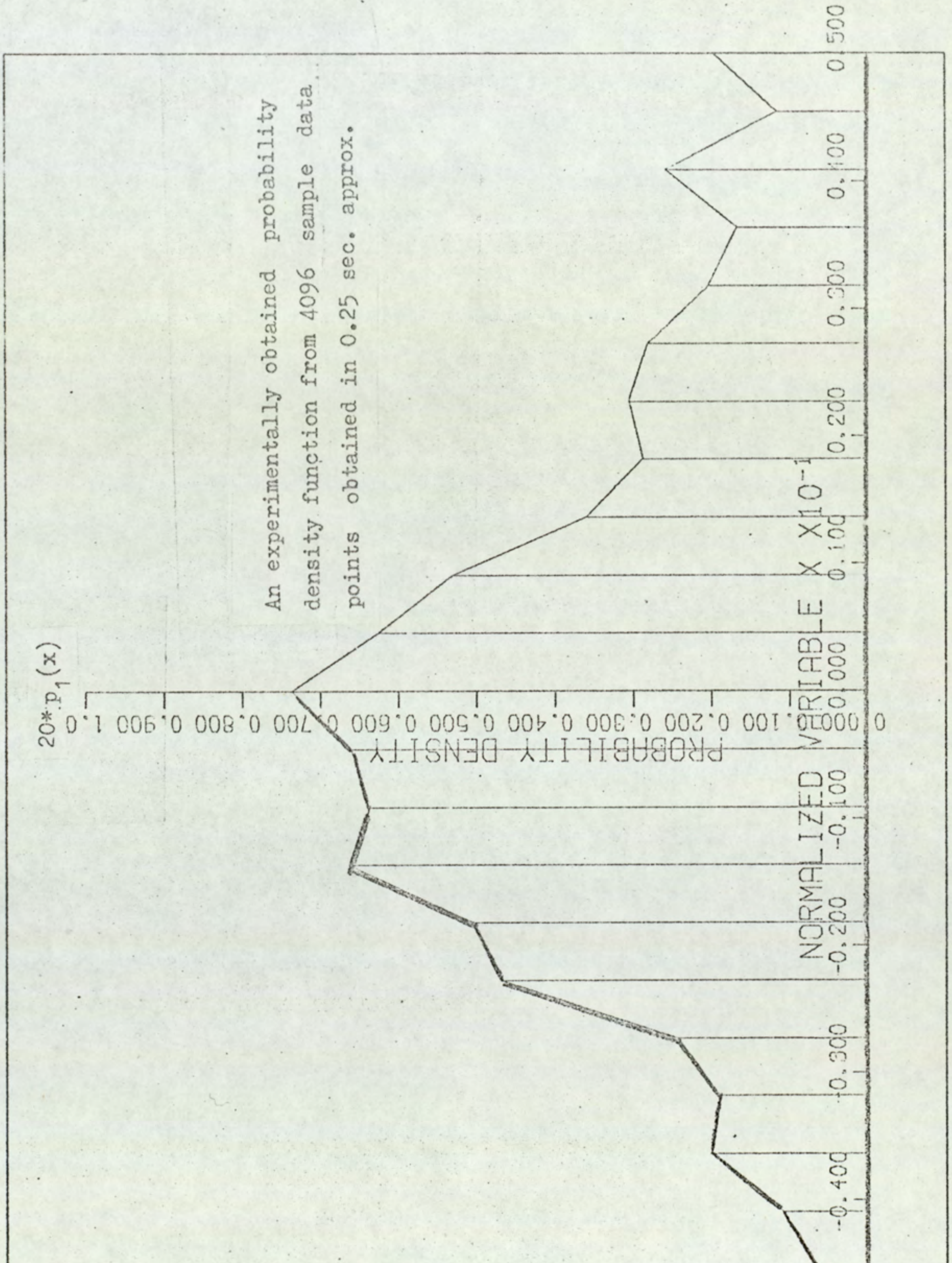

An experimentally obtained probability
density function from 4096 sample data
points obtained in 0.25 sec. approx.
20*p_1(x)
PROBABILITY DENSITY
0 0.100 0.200 0.300 0.400 0.500 0.600 0.700 0.800 0.900 1.0
NORMALIZED VARIABLE X X10-1
-0.500 -0.400 -0.300 -0.200 -0.100 0.000 0.100 0.200 0.300 0.400 0.500

Fig.3.5

general, the average value will be a function of the time at which the average is taken, i.e.

$$\text{ave } \langle x_t \rangle = t_1 = \text{ave } \langle x(t_1) \rangle$$

$$= \frac{{}^1x(t_1) + {}^2x(t_1) + {}^3x(t_1) \ldots + {}^Kx(t_1) + \ldots {}^Nx(t_1)}{N}$$

$$= \frac{1}{N} \sum_{K=1}^{N} {}^Kx(t_1) \qquad 3.1.2$$

Using the first probability density function we have

$$\text{ave } \langle x(t_1) \rangle = \sum_{1}^{N} x_K . p(x_K, t_1)$$ or, for a continuous function

$$\text{ave } \langle x(t_1) \rangle = \int_{-\infty}^{+\infty} x.p(x,t)\,dx \qquad 3.1.3$$

If the value of x is limited or bounded, the limits of integration may be changed to these limits without changing the value of the average.

For a discrete variable, which moves in discrete steps, and more closely resembles the action taking place physically, this probability density function may be calculated by grouping the signals together (Fig.3.6) according to a sequence of signal levels, for example x_i. The ratio of these signals to the total number of signals in the ensemble is in the limit $N \to \infty$, the probability

$$p_1(x\ ,t_1)\ dx_1$$

The area under the probability density function for any given time is

$$\int_{x_L}^{x_U} p_1(x\ ,t_1)\ dx = 1 \qquad 3.1.4$$

which expresses the fact that there is certainty that the signal lies between the two limits x_L and x_U. (Fig.3.1)

If we take the lower limit constant at the value $-\infty$ or x_L, and vary the upper limit we obtain the cumulative distribution function,

$$P(x_i) = \int_{x_L}^{x_i} p_1(x,t_1)\,dx \leq 1; \text{ where } x_L < x_i < x_u$$

which is the probability that the value lies between the values x_L and x_i.

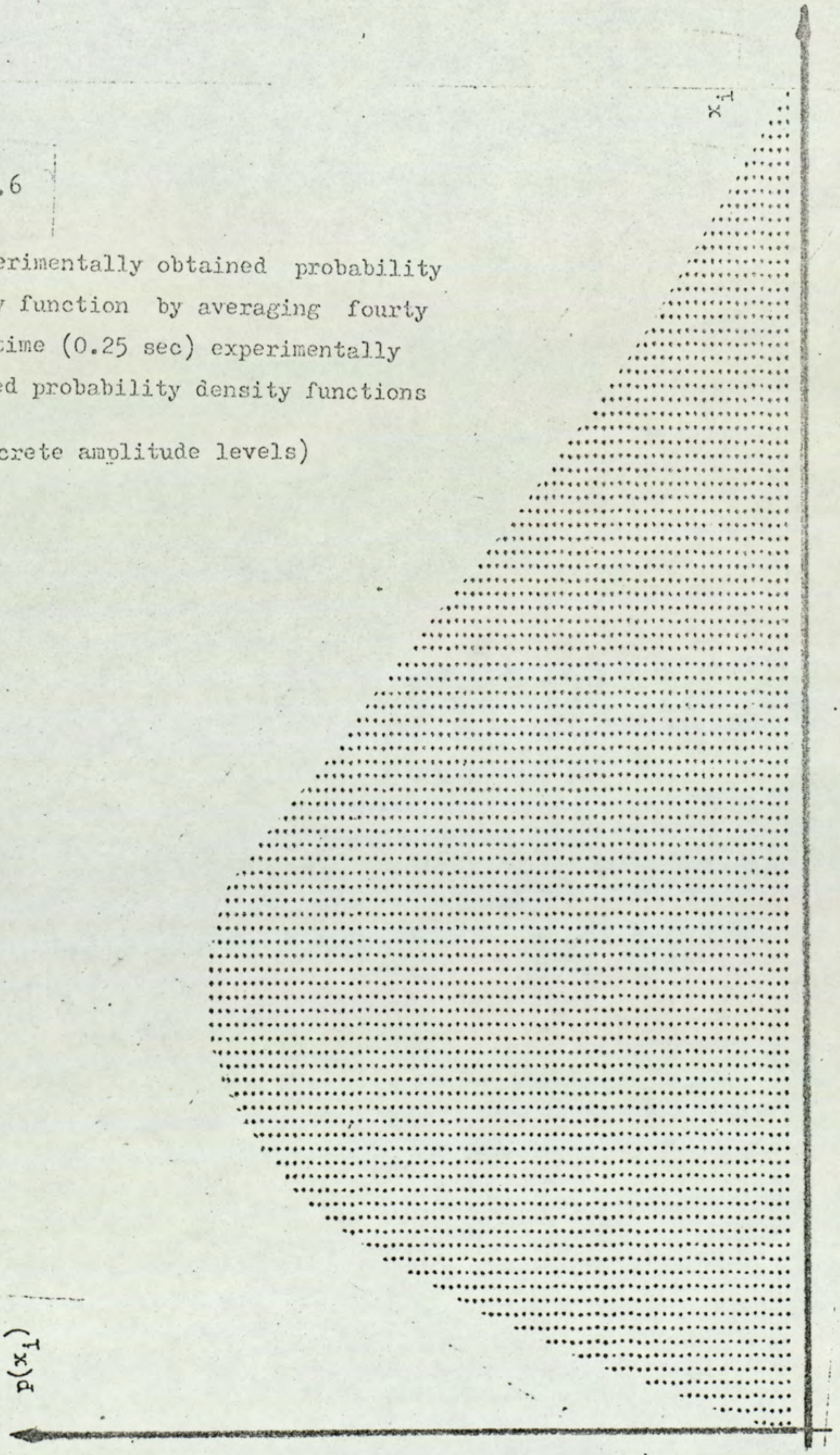


Fig 3.6

An experimentally obtained probability density function by averaging fourty short time (0.25 sec) experimentally obtained probability density functions (99 discrete amplitude levels)

3.2 Transition Probabilities.

Let us consider a system which starts in the state x_i with probability $p(i)$. The probability that the system is in x_i is given by the i-th component on the initial probability vector $\{{}^{o}p\}$.

Similarly the probability that the same system will be in the state x_j after say k consecutive steps is given by the j-th component of the vector $\{{}^{k}p\}$, i.e. ${}^{(k)}p(j)$.

Here we suppose that the system can be statistically described by its probability vector, a row matrix, and a tensor, i.e. a corresponding stochastic matrix of transition probabilities

$$\pi = \{p(i,j)\} \qquad 3.2.1$$

If the system before the last k-th step of observation was in the state x_l, for $l = 1, 2, 3, \ldots\ldots N$,
with probability ${}^{(k-1)}p(l)$, then for the next step to be in $x(j)$ the transition probabilities are ${}^{(1)}p(l,j)$.

In general some of the transition probabilities could be zero, but the total probability that in the k-th step is in j, a particular j, is

$$^{(k)}p(j) = \sum_{l=1}^{N} {}^{(k-1)}p(l) \,.\, {}^{(1)}p(l,j) \qquad 3.2.2$$

for different $j = 1, 2, 3, \ldots\ldots N$

we can write

$$^{(k)}p(1) = {}^{(k-1)}p_1 . p(1,1) + {}^{(k-1)}p_2 . p(1,2) + \ldots\ldots + {}^{(k-1)}p_N . p(1,N)$$

$$^{(k)}p(2) = {}^{(k-1)}p_1 . p(2,1) + {}^{(k-1)}p_2 . p(2,2) + \ldots\ldots + {}^{(k-1)}p_N . p(2,N)$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$^{(k)}p(N) = {}^{(k-1)}p_1 . p(N,1) + {}^{(k-1)}p_2 . p(N,2) + \ldots\ldots + {}^{(k-1)}p_N . p(N,N)$$

or, in more compact matrix form

$$\left\{{}^{(k)}{}_{p}\right\} = \left\{{}^{(k-1)}{}_{p}\right\}\pi \qquad 3.2.3$$

Similarly, going a step back.

$$\left\{{}^{(k-1)}{}_{p}\right\} = \left\{{}^{(k-2)}{}_{p}\right\}\cdot\pi \qquad 3.2.4$$

and so on until

$$\left\{{}^{(2)}{}_{p}\right\} = \left\{{}^{(1)}{}_{p}\right\}\pi \qquad 3.2.5$$

$${}^{(1)}{}_{p} = {}^{o}{}_{p}.$$

By subsequent substitution

$$\begin{aligned}
\left\{{}^{(2)}{}_{p}\right\} &= \left\{{}^{(o)}{}_{p}.\right\}\pi^2 \\
\left\{{}^{(3)}{}_{p}\right\} &= \left\{{}^{(o)}{}_{p}.\right\}\pi^3 \\
\ldots &= \ldots \qquad\qquad 3.2.6 \\
\left\{{}^{(k)}{}_{p}\right\} &= \left\{{}^{(o)}{}_{p}.\right\}\pi^k \\
\ldots &= \ldots \\
\left\{{}^{(m)}{}_{p}\right\} &= \left\{{}^{(o)}{}_{p}.\right\}\pi^m
\end{aligned}$$

This simply means that by multiplying the initial probability vector $\left\{{}^{(o)}{}_{p}\right\}$ by the k-th power of the stochastic matrix which defines the first order transistion probabilities we will obtain the probability distribution after k steps from an initial state.

Usually it is not possible to reach a state x_j from any other x_p state in only one step, but in a few steps. If every state can be reached from any other state, not necessarily in one step, the process is called irreducible. (Takacs, L. Stochastic Processes, John Wiley, 1960). In an irreducible Markov chain the set of all possible states forms a closed set, and no subset which belongs to that set is closed. Normally for Markov chains which are not irreducible from one particular state, say x_z, we can go to a statistically defined subset

$$s_i \;\varepsilon\; s_{01}$$

where s_0 is the total available space of states.

To all other states, $x_j \not\varepsilon s_i$ the transitional probabilities must by definitions be equal to zero. We shall deal exclusively with irreducible Markov chains.

If the Markov process were not of zero order, i.e. completely random, the transition probabilities would be peaked , quickly decreasing to zero for remote states in the same subset s_i. If no subset s_i is closed, i.e. the chain is irreducible, then after say k steps we could reach any state $x_j \;\varepsilon\; S_o$ with a joint probability distribution independent of the initial state, i.e.

$$\begin{aligned} \{{}^{(k)}p(i,j)\} &= \{{}^{(o)}p(j)\}\,\pi^k \\ &= {}^{(o)}p(j) \end{aligned} \qquad 3.2.7$$

so that

$$\pi^k \rightarrow 1 \text{ for } k \geq N \text{ when N is a large number of steps}$$

For this case the Markov chain is said to be ergodic, and the initial probabilities ${}^{(o)}p$ are therefore called also stationary probabilities.

Note that for a stationary Markov process, transitional probabilities will depend on the chosen unit of time between two successive steps of observation, (Fig.3.7a).

The diagram shows how the transition probabilities are related to one another in an actual case when signal fluctuation was analysed.

It is almost intuitively clear that if the time difference is made small enough, the transition probability density will be a very peaked and narrow function and the corresponding subset of states which could be reached even in several successive time intervals would be very limited.

By increasing the unit of difference time, or taking more units of the shorter time, the corresponding accessible subset of states becomes larger and consequently the corresponding transition probability distribution less peaked.

In order to distinguish between the different transition probabilities they should be described and noted as functions of the unit of time difference, $t_1 - t_o = \tau$, where t_o is the time at which the initial state was observed and t_1 is the next subsequent time of observation, which may be taken arbitrarily close to t_o if necessary.

If the time τ were chosen to be longer than the natural time constant determined by the inertia or the memory incorporated in the system, then the transition probabilities become almost completely independent of the initial state. In other words, all row vectors in the transition probability matrix become the same and equal to the initial probability vector.

If, on the other hand, the time interval τ is chosen too small, the transition probabilities

$$\begin{aligned} p(i,j) &\doteq 1.0 \quad \text{for } i = j \\ p(i,j) &\doteq 0.0 \quad \text{for } i \neq j \end{aligned} \qquad 3.7.8$$

become almost certainties, i.e. each is almost equal to 1.0, and very poor statistical descriptions for the future spread of values is obtained. However, this property can be utilised for obtaining a theory applicable to short time intervals consisting of several periods τ in succession, resulting in the retrieval of information transmitted through a system with fast fluctuating parameters.

On the other hand, the observed inertia of signal magnitude and phase fluctuations is utilised in a special technique of suppressing noise in the vicinity of a signal transmitted through a system with time varying transmission parameters (see Aston Filtering).

c

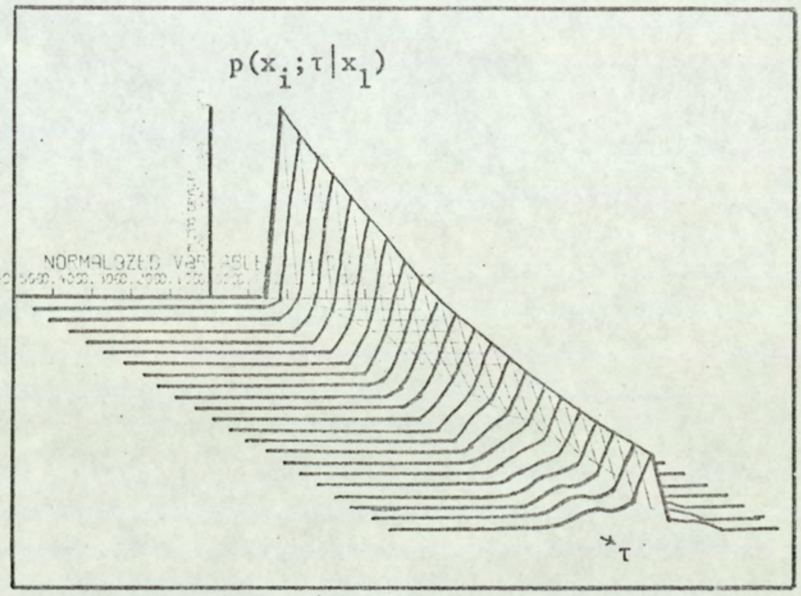


Fig.3.7a.

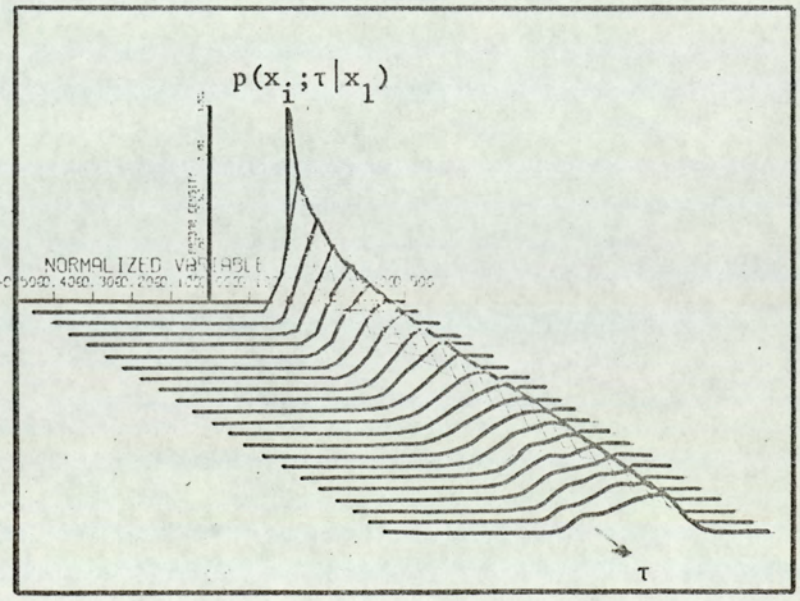


Fig.3.7b

Fig.3.7 Samples of transitional probabilities of fluctuations when a signal is transmitted through a system with fluctuating parameters. Fig.3.7a is obtained directly from a fluctuating signal amplitude and Fig.3.7b, after the same signal has been processed by the Aston Filter (see Chapter 10).

3.3 Correlation Functions.

Similar conclusions could be obtained by a study of the correlation function which, by definition, is the expectation of the product of two values or states which the stochastic process assumes for two instants of time, i.e. t_o and t_1. For a statistically stationary process

$$\rho(\tau) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1 . x_2 \ {}^{(1)}p(x_1, x_2; \tau) \ {}^{(o)}p . dx_1 . dx_2 \qquad 3.3.1$$

where ${}^{(1)}p(x_1, x_2; \tau)$ is the probability that in time $t_o + \tau$ we can reach a state x_2 when the initial state is x_1, and $\rho(\tau)$ is the correlation coefficient for time τ. For a closed space of discrete values of x_i, i.e. $x_i \in S$, $(1 \leq i \leq N)$

$$\rho(\tau) = \sum_{I=1}^{N} \sum_{J=1}^{N} x_I . x_J . p(I,J;\tau) \ {}^{(o)}p \qquad 3.3.2$$

where as before, x_I and x_J are two consecutive states and τ is the time difference of the transition. Thus $p(I,J;\tau)$ is the transition probability for the time difference, τ and ${}^{(o)}p$ is the initial probability distribution.

We can write in the matrix form

$$\left\{ p\,[I,J;\tau] \right\} = \left\{ p(I,J) \right\} \left\{ {}^{(o)}p(k) \right\} \qquad 3.3.3$$

joint = transition*initial

For a stationary and ergodic process

$$\rho(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t).x(t+\tau)\ dt. \qquad 3.3.4$$

Note that here the integration is taken in the time domain, while in the previously given expression for $\rho(\tau)$ it was summed or integrated in the domain of the variable x.

The initial value, $\rho(0)$, of the correlation function $\rho(\tau)$ is equal to the normalised power of the time function if the time function represents a voltage or current passing through and dissipating in a normalised resistor of 1 ohm.

For $\tau = 0$,

$$\rho(0) = \frac{1}{N} \sum_{I=1}^{N} \sum_{J=1}^{N} x_I, x_J \, p\,[I,J;0] \qquad 3.3.5$$

$$= \frac{1}{N} \sum_{I=1}^{N} (x_I)^2 p(I)$$

since

$$p\,[I,J;0] = \delta_{IJ} ,$$

i.e. $\delta_{IJ} = 0$ for $I \neq J$

$\delta_{IJ} = 1$ for $I = J$

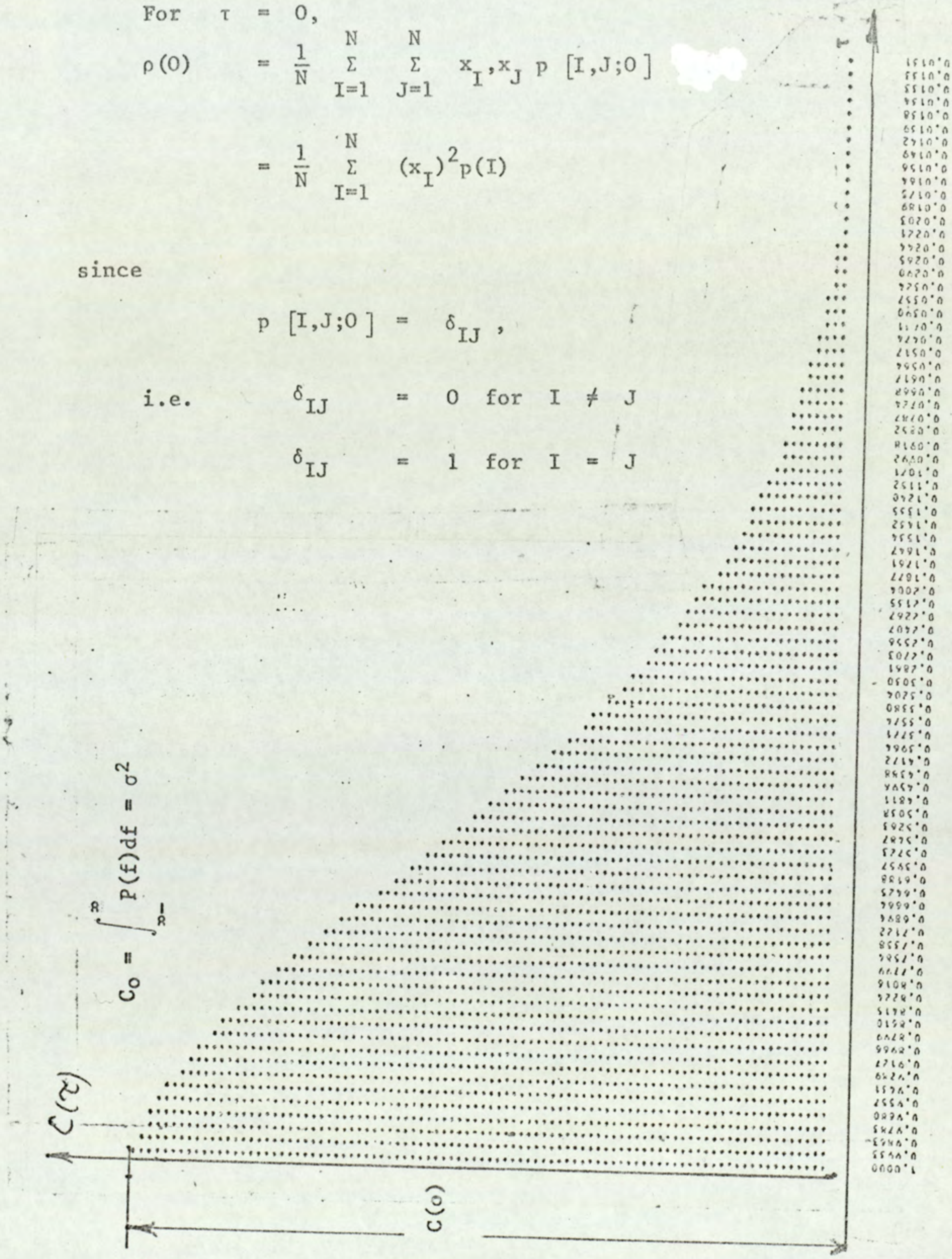


Fig.3.8 A sample correlation function obtained from experiments in Chapter 8.

Corrections due to finite time of observation.

As all measurements have to be carried out in some finite length of time, it is clear that we cannot meet the mathematical definition of integration for limits $T \to \infty$, neither in the real time domain nor forthe time delays or lags. We usually have at our disposal a truncated time function or time function of finite length, with the time "window" extending from T_1 to T_2, T_1 being the start and T_2 the end of the time of observation.

If we are concerned with the computation of the estimate of the covariance function, then it is obvious that the maximum delay of interest must be much shorter than the maximum time available. Otherwise, in the digital computation, for example, we would be summing a smaller number of products for the shorter time delays than for the larger.

In order not to have the results modified too greatly by this effect, the maximum delay must be only a small fraction of the total time of the record, say for example T/20, i.e. 5% of the total observation time.

The apparent autocovariance functions can then be defined by

$$C_{xxapp}(\tau) = \frac{1}{(T_2-T_1) - |\tau|} \int_{T_1+\tau/2}^{T_2-\tau/2} x(t - \tau/2).x(t + \tau/2)\, dt \qquad 3.3.6$$

where $|\tau| \leq \tau_{max} \ll (T_2 - T_1)$

The corresponding apparent power density spectrum can be obtained as the Fourier transform of the apparent autocovariance function.

As the time function is multiplied by a square wave time function, * the corresponding frequency spectrum is the convolution of the true spectrum and the spectrum of the square wave time window function.

The inverse transform of this convolution is the result we observe as the apparent autocovariance function. (R. B. Blackman and J. W. Tukey).

* representing the time "window",

The true estimate or the corrected autocovariance function can be obtained by modifying the apparent covariance function. This could be done by multiplying it by a prescribed even function of τ, say $D(\tau)$, usually called a "lag window" correcting function. Thus

$$C_{xx}(\tau) = D(\tau).C_{xxapp}(\tau) \qquad 3.3.7$$

which expresses the fact that the true estimate can be obtained by multiplying each co-ordinate of $C_{xxapp}(\tau)$ as measured by the corresponding co-ordinate value of $D(\tau)$. (PROCEDURE CORRFUN) (List of Programs p.263)

There are known three different lag windows correcting functions due to Parzen, Hamming and Hann (Blackman and Tukey, 1958). For analysis made in this research, Hamming method was adopted and included in the program SHORTBITS. When no smoothing of data is preferred, Hamming is "FALSE". The Hamming spectral window, i.e. the Fourier transform of $D(\tau)$, is given in Fig.3.9.

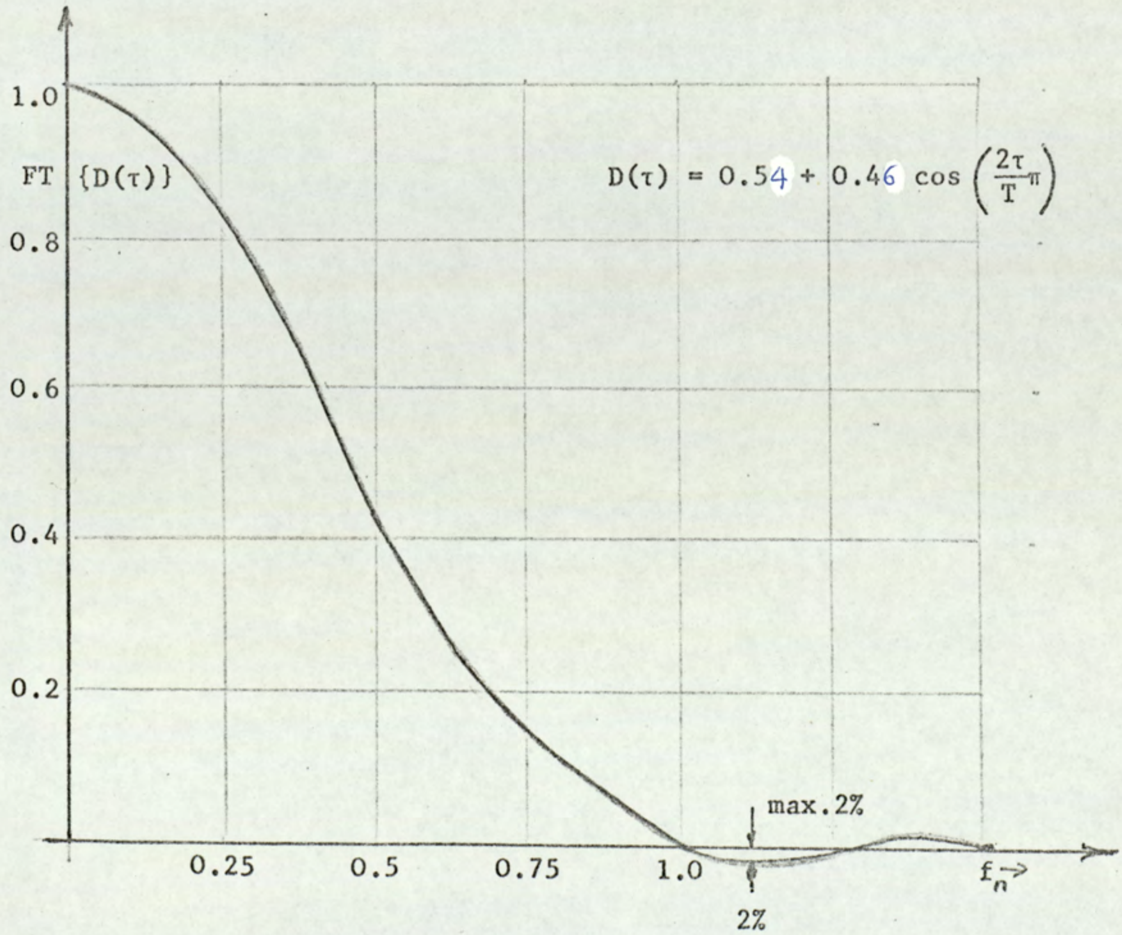


Fig.3.9

3.4 Autocorrelation Functions and the Power Spectra.

We shall study the two functions, the autocovariance functions and the autocorrelation function, both of which are the averaged values of the product of two time functions, a time delay or lag being applied to one of the two functions.

In terms of ensembles the average product of two functions with a lag τ is defined as

$$\rho_{xx}(\tau) = \text{ave} <\{\underset{\sim}{x}(t)\} . \{\underset{\sim}{x}(t+\tau)\}> \qquad 3.4.1$$

where < > as before stands as the notation for a set or ensemble. In terms of single functions defined in the time domain

$$^{k}\rho_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} {}^{k}x(t) . {}^{k}x(t+\tau) \; dt$$

$$= \overline{{}^{k}x(t) . {}^{k}x(t+\tau)} \qquad 3.4.2$$

where the superscript k denotes the k th sample function from the ensemble, i.e. the k th column of a random variable function matrix.

The autocovariance function is defined as

$$C_{xx}(\tau) = \text{ave} <\{\underset{\sim}{x}(t) - \text{ave} <\underset{\sim}{x}(t)>\} . \{\underset{\sim}{x}(t+\tau) - \text{ave} <\underset{\sim}{x}(t)>\}> \qquad 3.4.3$$

where ave $<\underset{\sim}{x}(t)>$ is the mean or ensemble average of a random variable x(t). For a stationary random process

$$\text{ave} <\underset{\sim}{x}(t)> \quad = \quad \text{ave} <\underset{\sim}{x}(t+\tau)>$$

i.e. the ensemble average does not change with time.

For a single k th function, i.e. one of the possible realisations of the random function

$$^{k}C_{xxx}(\tau) = \lim_{T \to \infty} \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} [{}^{k}x(t) - \langle{}^{k}x(t)\rangle] \; [{}^{k}x(t+\tau) - \langle{}^{k}x(t)\rangle] \; dt \qquad 3.4.4$$

Note also that $\overline{\overline{x(t)}} = \overline{\overline{x(t+\tau)}}$ for a stationary process, the double bar indicating the long term average.

Under the term autocorrelation function we shall understand

$\psi_{xx}(\tau) = C(\tau)/C(0)$,

i.e. the normalised autocovariance function. $C(0)$ is the variance and $(C(0))^{\frac{1}{2}} = \sigma$ is the standard deviation of the random function. Furthermore

$$\psi\ (0) \equiv 1.0$$

Whenever we deal with the actual recorded data, we manipulate with the averaged lagged product. Since as a rule the averages of a random process or signals which generate random processes which operate on them have mean values zero, we usually have to deal with the autocovariance function.

Using the Wiener-Khintchine theorem the autocorrelation function may be reduced to the form

$$C_{xx}(\tau) = \int_{-\infty}^{+\infty} P(f).\ e^{-j2\pi f\tau}\ df \qquad 3.4.5$$

where

$$P(f) = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{+T/2} x(t)\ e^{-j2\pi ft}\ dt \right|^2 \qquad 3.4.6$$

The function of frequency $P(f)$ describes the power density spectrum of the stationary random process considered. Expressed more precisely $P(f)\ df$ represent the contribution to the variance from frequencies between f and $(f + df)$. In other words, this is the variance for a narrow frequency band of the fluctuating signal. This can be visualised in the diagram, Fig.3.10.

The pure mathematician might prefer to refer to this spectrum as the covariance spectrum because the d.c. power is either zero, or for the ease of calculations made to be zero by subtracting the mean value.

Otherwise both spectra are the same.

As the autocovariance function is expressed as the Fourier transform of the power spectrum density, the inverse Fourier transform of ${}^{k}C_{xx}(\tau)$ should give ${}^{k}P_{xx}(f)$ in this way:-

$$P_{xx}(f) = \int_{-\infty}^{\infty} C_{xx}(\tau) \; e^{-j2\pi f\tau} \, d\tau \qquad 3.4.7$$

As $C_{xx}(\tau)$ is an even function

$$P_{xx}(f) = 2\int_{-\infty}^{\infty} C_{xx}(\tau) \, . \cos 2\pi f\tau . d\tau \qquad 3.4.8$$

integrating or averaging for all delays from $-\infty$ to $+\infty$. It should particularly be noted that setting $\tau = 0$ in expression for $C(\tau)$

$$C(0) = \int_{-\infty}^{\infty} P(f).df, = \sigma^2 \qquad 3.4.9$$

i.e. the mean power or variance of the random function. (Fig.3.8)

Being the Fourier transforms of each other, the covariance function and its corresponding power spectrum, from the mathematical point of view represent equally well certain important properties of random functions. Depending on the available instrumentation, either one or the other might be the easier to determine in practice.

For analogue spectrum computation it has long been practice to use spectrum analysis. In recent years autocovariance measurements have been extensively used. The disadvantage of the latter lies in the necessity to have delay circuits which are independent of frequency, i.e. the delay circuits should have a flat response right up to the highest frequency of interest. While for a simple filter there is a definite relation between the phase or delay and the amplitude of the frequency characteristic, for a more complicated filter or delay network, the phase can become appreciably affected even when the amplitude is only slightly influenced.

For digital computation either of the two methods is equally easy to use, with a preference for the use of the power density spectrum measurement. When "on line" computation is performed the Fast Fourier Transform method (FFT) is preferred. For a recorded sample function the computation of the correlation function requires a less complicated programme, and so often would be preferred, even if the computation time is longer.

In the interpretation of the results, the power density spectrum has certain advantages over the use of the autocovariance fuction. In almost all practical situations the data to be analysed do not represent the actual output of the system under investigation. In such cases, the data have been modified, sometimes quite radically, by the transmission characteristics of the devices used to detect, amplify and record the data. Whether this modification is intentional or unavoidable, or even due to ignorance, corrections can be obtained to correct the effect of using the modified data if one performs the computation on the power density spectrum rather than on the autocovariance function.

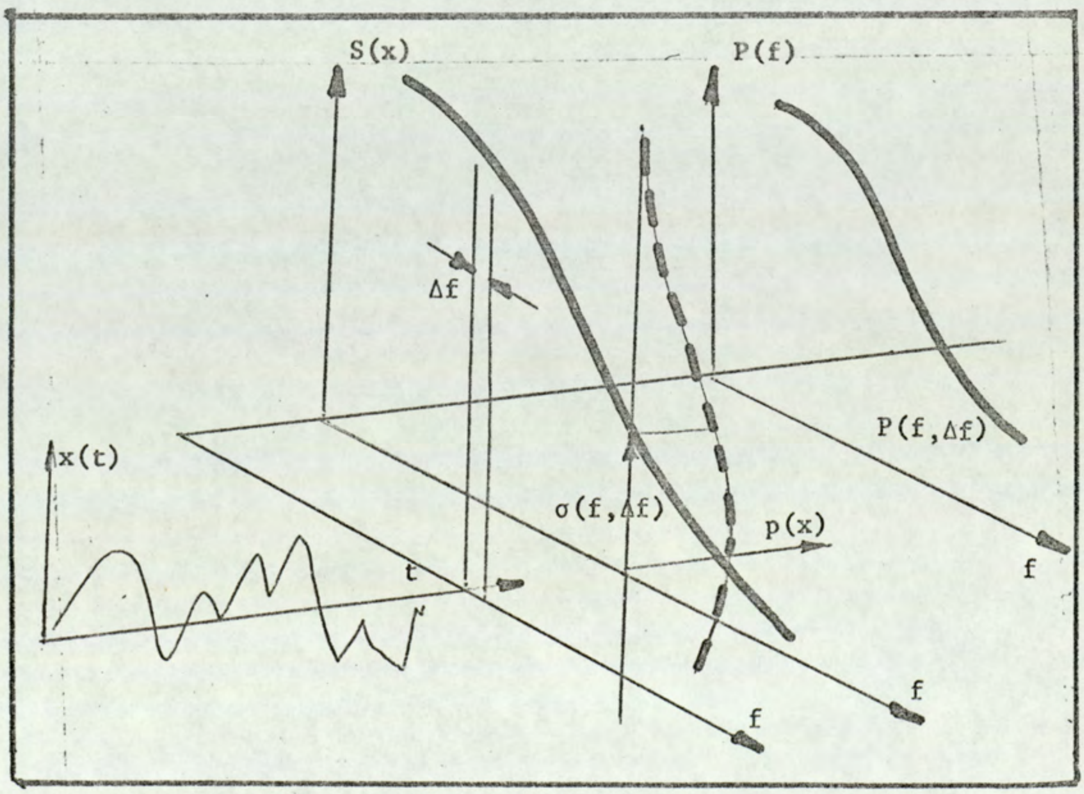


Fig.3.10.

3.5 Analysis of the Power Density Function.

Let us assume that we may make an approximation to the autocovariance function (obtained by computing the experimentally recorded data) is an exponential cosine function or by a sum of such functions.

$$C_{xx}(\tau) = \sigma^2 e^{-\alpha|\tau|} \cos \omega_o \tau \qquad 3.5.1$$

The parameter ω_o has the dimensions radian $\sec^{-1}$, α has dimensions $\sec^{-1}$, and τ, seconds. σ^2 has dimensions of power and is always positive, i.e. $\sigma^2 > 0$. The dimensions of σ^2 can be eliminated by introducing, as previously, the autocorrelation function. Thus, by putting $\tau = 0$,

$C_{xx}(0) = \sigma^2$, so that

$$\chi_{xx}(\tau) = C_{xx}(\tau)/C_{xx}(0) = e^{-\alpha|\tau|} \cos \omega_o \tau \qquad 3.5.2$$

The corresponding normalised power density spectrum evaluated for all $\tau \geq 0$, is given by the Fourier transform of χ_{xx}, as

$$G_{xx}(\omega) = \frac{2}{\pi}\int_0^\infty \chi_{xx}(\tau)\cos \omega\tau . d\tau = \frac{2}{\pi}\int_0^\infty e^{-\alpha\tau} \cos\omega_o\tau . \cos\omega\tau . d\tau$$

$$= \frac{1}{\pi}\int_0^\infty e^{-\alpha\tau} \left\{ \cos(\omega_o - \omega)\tau + \cos(\omega_o + \omega)\tau \right\} . d\tau \qquad 3.5.3$$

For convenience in writing put

$$\beta = \omega_o - \omega \quad ; \quad \gamma = \omega_o + \omega$$

from which, by substituting exponential forms for the cosine terms we find

$$G_{xx}(\omega) = \left[\frac{\alpha}{\pi} \quad \frac{1}{\beta^2 + \alpha^2} + \frac{1}{\gamma^2 + \alpha^2}\right] \qquad 3.5.4$$

which on substituting back for β and γ, we get

$$G_{xx}(\omega) = \frac{\alpha}{\pi}\left[\frac{1}{(\omega_o - \omega)^2 + \alpha^2} + \frac{1}{(\omega_o + \omega)^2 + \alpha^2}\right] \qquad 3.5.5$$

The expression in the square bracket could be simplified as the ratio of two polynominals in ω, getting finally after some reduction,

$$G_{xx}(\omega) = \frac{2\alpha}{\pi} \cdot \frac{N(\omega)}{D(\omega)} = \frac{2\alpha}{\pi} \cdot \frac{\omega^2 + (\omega_o{}^2 + \alpha^2}{\omega^4 + 2(\alpha^2 - \omega_o{}^2)\omega^2 + (\alpha^2 + \omega_o{}^2)^2} \qquad 3.5.6$$

From this it immediately follows that,

as $\omega \to 0$, $G_{xx}(0) \to \frac{2\alpha}{\pi} \cdot \frac{1}{\alpha^2 + \omega_o{}^2}$

for large ω $G_{xx}(\omega) \doteq \frac{2\alpha}{\pi} \cdot \frac{1}{\omega^2}$

and $G_{xx}(\omega) \to 0$ as $\omega - \infty$

Differentiating $G_{xx}(\omega)$ with respect to ω and re-arranging terms we get

$$G'_{xx}(\omega) = \frac{-2\omega\left[\omega^4 + 2(\alpha^2 + \omega_o{}^2)\omega^2 + (\alpha^2 - \omega_o{}^2)^2 - 4\omega_o{}^4\right]}{\left[\omega^4 + 2(\alpha^2 - \omega_o{}^2)\omega^2 + (\alpha^2 + \omega_o{}^2)^2\right]^2} \qquad 3.5.7$$

By equating $G'_{xx}(\omega)$ to zero we can find the extrema of $G_{xx}(\omega)$. One root of $G_{xx}(\omega)$ is obviously at $\omega = 0$, and has the value zero. To find the roots of the polynomial in the square brackets in the numerator we find, putting $\eta = \omega^2$

$$\eta = \omega^2 = \frac{-(\alpha^2 + \omega_o{}^2) \pm \left[(\alpha^2 + \omega_o{}^2)^2 - (\alpha^4 - 2\alpha^2\omega_o{}^2 + 3\omega_o{}^4)\right]^{\frac{1}{2}}}{} \qquad 3.5.8$$

Real roots of ω could exist only if the following conditions are met:-

(1) $D \geqq 0$

where $D = (\alpha^2 + \omega_o{}^2)^2 - (\alpha^4 - 2\alpha^2\omega_o{}^2 - 3\omega_o{}^4) \geqq 0$

or $D = 4\omega_o{}^2(\alpha^2 + \omega_o{}^2) \geqq 0$ 3.5.9

which is a condition met essentially for all values of α and ω_o.

(2) $$D^{\frac{1}{2}} \geqq |\alpha^2 + \omega_o^2| \text{ , or } \eta \geq 0 \qquad 3.5.10$$

so that

$$\therefore \quad 4\omega_o^2 (\alpha^2 + \omega_o^2) \geqq (\alpha^2 + \omega_o^2)^2$$

or

$$-\alpha^4 + 2\omega_o\alpha^2 + 3\omega_o^4 \geqq 0 \qquad 3.5.11$$

To find the limiting case we shall equate the polynomial in α to zero, first substituting $\xi = \alpha^2$.

Obviously ω real are obtained when

$$\xi = \alpha^2 \leqq 3\omega_o^2 \qquad 3.5.12$$

For $\alpha^2 > 3\omega_o^2$ no real maxima of $G_{xx}(\omega)$occur, other than that at $\omega = 0$, and the power density spectrum decreases monotonically to zero as $\omega \to \infty$.

For $3\omega_o^2 \geqq \alpha^2$, a single maximum occurs at

$$\omega_1 = + \sqrt{\xi_1} \qquad 3.5.13$$

which is given by

$$\omega_1^2 = \xi_1 = \left[(\alpha^2 + \omega_o^2)^2 - (\alpha^2 - \omega_o^2)^2 + 4\omega_o^4\right]^{\frac{1}{2}} - (\alpha^2 + \omega_o^2)$$

which, on reduction becomes

$$\omega_1^2 = 2\omega_o(\alpha^2 + \omega_o^2)^{\frac{1}{2}} - (\alpha^2 + \omega_o^2) \qquad 3.5.14$$

The value of the new maximum of $G_{xx}(\omega)$ could be obtained if we substitute ω_1^2 in the expression for $G_{xx}(\omega)$, i.e. first in the expressions for $N(\omega)$ and $D(\omega)$. Thus

$$N(\omega_1) = 2\omega_o(\alpha^2 + \omega_o^2)^{\frac{1}{2}} \qquad 3.5.15$$

$$D(\omega_1) = 8\omega_o^2 \{ - \omega_o (\alpha^2 + \omega_o^2)^{\frac{1}{2}} + (\alpha^2 + \omega_o^2)\}$$

$$G_{xx}(\omega_1) = \frac{2\alpha}{\pi} \cdot \frac{N(\omega_1)}{D(\omega_1)} \qquad 3.5.16$$

$$= \frac{1}{2\pi.\omega_{01}} \cdot \frac{1}{(\alpha_1^2 + \omega_{01}^2)^{\frac{1}{2}}} \qquad 3.5.17$$

If we form the ratio of $G_{xx}(\omega_1)$ and $G_{xx}(0)$ we have

$$\frac{G_{xx}(\omega_1)}{G_{xx}(0)} = \frac{1}{4\omega_o(\alpha^2 + \omega_o^2)^{\frac{1}{2}} - 4\omega_o^2} > 1.0 \qquad 2.5.18$$

The following conclusions and remarks can be made:-

For all values of $\omega_o^2 > \alpha^2/3$, i.e. when the new maximum exists, the extremum at $\omega = 0$ becomes a minimum of the power density spectrum, or in other words, the minimum of the power density spectrum of the assumed exponential cosine type autocorrelation function.

The suppressed low frequency end of the power density spectrum can be expected to be obtained when the fast frequency dependent feedback system is in operation.

On the other hand, when the record is too short, for the fluctuating process under study, the low frequency part of the spectrum could also be defective, giving rise to the exponential-cosine autocorrelation function, which has already been obtained. (See Chapter 8).

experimentally

3.6 The Approximation to the experimentally obtained power density spectrum by means of analytic functions.

The aim of this Section is to represent a power density spectrum $P(\omega)$ obtained experimentally by an analytic function $P_1(\omega)$. To do this the first step is to change the variable ω introducing ψ, given by

$$\psi = 2 \arctan \omega \qquad 3.6.1$$

which transforms the domain $-\infty<\omega < \infty$ into the domain $-\pi<\psi <\pi$. The transformed function $P(\psi)$ is periodic with the period 2π. It is easy to expand this function in the domain of ψ. Being symmetrical, all the sine functions will have coefficients zero.

Thus

$$P(\psi) = c_0 + c_1 \cos\psi + \dots \; c_k \cos k\psi + \dots \; c_n \cos n\psi. \qquad 3.6.2$$

This may be done by any of the known methods, based equally spaced or unequally spaced discrete values of $P(\omega)$.

The variable now can be substituted by

$$u = \cos\psi \quad \text{where} \quad \psi = \text{arc cos } u \qquad 3.6.3$$

so that terms $c_k \cos k_\psi$ can be expressed as

$$c_k \cos k\psi = \cos(k \text{ arc cos } u) \qquad 3.6.4$$

It is known that a polynomial of the form

$$T_k(u) = (\cos k\psi)/2^{k-1} \quad \text{where} \quad u = \cos\psi \quad \text{and} \quad k = 1,2,\dots.$$

is called a Chebychev polynomial of the first kind. Consequently, we can put (Solodovnikov,1965)

$$P(u) = C_o + C_1 T_1(u) + C_2 T_2(u).2 + \dots + C_k T_k(u).2^{k-1} + \dots$$
$$+ C_n T_n(u).2^{n-1}. \qquad 3.6.5$$

a series whose terms are Chebychev polynomials of the first kind.

Recalling that $T_1(u) = u$,

$$T_2(u) = \cos\frac{2\psi}{2} = (2\cos^2\psi - 1)/2 = u^2 - \tfrac{1}{2} \qquad 3.6.7$$

$$T_3(u) = u^3 - \frac{3}{4}u$$

$$T_4(u) = u^4 - u^2 + \frac{1}{8}$$

and the recurrence formula

$$T_{k+1}(u) = uT_k(u) - \frac{1}{4}T_{k-1}(u) \quad ; \quad k = 2,3,4, \ldots.$$

Having calculated the polynomials $T_k(u)$ and substituting them into $P(u)$ we obtain

$$P(u) = \alpha_0 + \alpha_1 u + \alpha_2 u^2 + \ldots. \quad + \alpha_k u^k + \ldots. \quad + \alpha_n u^n \qquad 3.6.8$$

The coefficients α_k are obtained by grouping coefficients of u^k together as is shown in the corresponding computer program (ANALYSIS).

The next step is to go back to the domain of ω. Since

$$u = \cos\psi = 2\cos^2\frac{\psi}{2} - 1 = \frac{2}{1 + \tan^2\frac{\psi}{2}} - 1 = \frac{2}{1 + \omega^2} - 1$$

$$= (1 - \omega^2)/(1 + \omega^2) \qquad 3.6.9$$

The analytical function in the ω domain may be written as a hypergeometric function

$$P_i(\omega) = \alpha_o + \alpha_1 \frac{1 - \omega^2}{1 + \omega^2} + \alpha_2 \left(\frac{1 - \omega^2}{1 + \omega^2}\right)^2 + \alpha_k \left(\frac{1 - \omega^2}{1 + \omega^2}\right)^k + \ldots.$$

$$+ \alpha_n \left(\frac{1 - \omega^2}{1 + \omega^2}\right)^n \qquad 3.6.10$$

The expansion in series makes it clear that the very complicated spectra are in fact composed of high order spectra with different weights, α_k.

From the same expression it is possible to find out the order of the Markovian process which generates the stochastic function. For this purpose it is even more convenient to express $P_i(\omega)$ in the form of ratio of two polynomials

$$P_i(\omega^2) = \frac{\beta_0 + \beta_2\omega^2 + \dots + \beta_{2k}\omega^{2k} + \dots + \beta_{2n}\omega^{2n}}{(1 + \omega^2)^n} \qquad 3.6.11$$

which is obtained from the previous equation by reducing the series to a common denominator and grouping the nominator terms in ascending order of powers of ω^2. This is done in the second part of the computer program (ANALYSIS). The coefficients α_k and β_k enable us to compare the different spectra without looking at their graphical representations. (See p. 192: Table 1).

CHAPTER 4

4.0 System identification

System identification is defined as the determination of the transfer function of the system. (Appendix 12).

The necessary characteristics for identifying a system,depends on whether the system can be considered as a time invariant system or not. In the first case it is sufficient to determine the system transfer function, whereas in the second it is of more importance to find the transfer function of the Markovian filter through which the randomness of the process is impressed on the signal. From the experimental results it is obvious that the knowledge of coherence bandwidths is also necessary.

It is well-known that a system with time invariant parameters can be mathematically described in the time domain by its impulse response $h(\tau)$, or by its Fourier transform, i.e. system function, in the frequency domain.

A particular input signal ${}^{k}x(t)$ will give rise to a particular output signal ${}^{k}y(t)$. The output, in the time domain is obtained by convolution of the system weighting function and the input signal. In the frequency domain, the corresponding output $Y(j\omega)$ is given as the multiplication of the input signal $X(j\omega)$ by the system transfer function $H(j\omega)$.

For systems with time varying parameters the input output relationship is not so simple. It has been shown (Zadeh 1961) that this relation can be expressed mathematically in a variety of ways, starting with the classical differential equation representation, to the more recent state space variable approach. Although the mathematical rigour and validity of such methods cannot be suspected, it is dubious if any of these methods give easily real insight into the physical properties of systems with fluctuating parameters.

In the course of this research, it was found that the simplest and therefore clearest picture of a time varying system could be given, by the system correlation function defined for a coherent band of frequencies. This approach leads to the input-output signal relation which is expressed by the convolution in the frequency domain and by an ordinary multiplication in the time domain.

Furthermore, this simple approach leads to a relatively simple solution. In the following few pages the philosophy of this approach is explained, starting from the time invariant system and its tranfer function identification. As the system with time varying parameters is specified by the Markovian filter transfer function, for the coherent band of frequencies, identification of its parameters is necessary.

In the course of this research this has been done experimentally but utilising ICL computer for numerical calculations (Program SYSTEM). Note that Markovian filter characteristics are also functions of time, but may be approximated by their average values for the time interval of interest. In the case of a turbulent but otherwise stable laboratory flame, it was found that these parameters do not change appreciably.

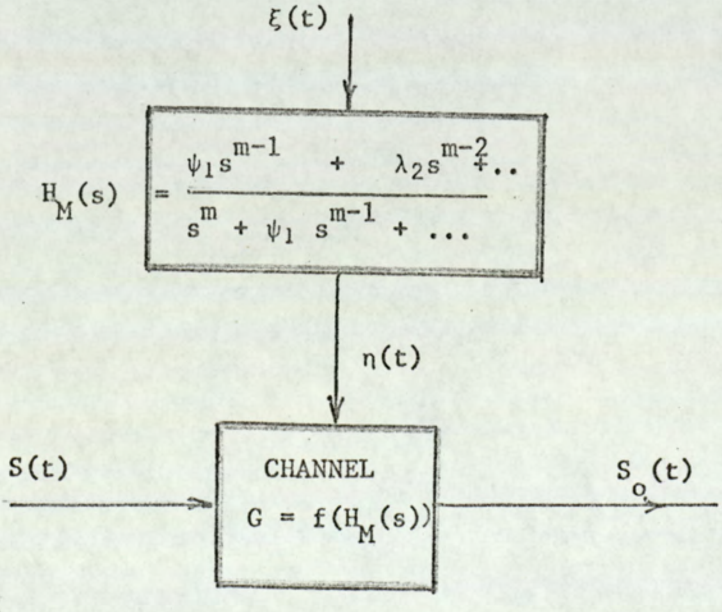


Fig.4.1 Mathematical model of a single path signal transmission through a physical system with time varying transmission parameters.

4.1 Identification of time invariant systems.

In control engineering it is usually accepted that a complete statistical description of a system may be given either in the frequency domain or in the time domain.

If we know the amplitude characteristic $A(j\omega)$ and the phases $\chi(j\omega)$ as functions of frequency for the system we can find the system output spectrum $S_y(j\omega)$ for any input spectrum $S_x(j\omega)$ merely by multiplying the input amplitude spectrum by the system amplitude characteristic and adding to the input phase the system phase characteristic. Such methods are also widely used in electrical engineering practice including telecommunications.

The corresponding description of a system in the time domain is the impulse response or its weighting function, $h(\lambda)$, which, on applying a Fourier transformation becomes the system transfer characteristic,

$$FT\{h(\lambda)\} = H(j\omega) = A(j\omega).e^{\phi(j\omega)}$$

If we know the system weighting function for a linear system, $h(\lambda)$, we can calculate the response $y(t)$ to any time function $x(t)$ applied at the input by the use of the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\lambda).x(t - \lambda)\, d\lambda$$

For physical systems, i.e. those in which cause and effect bring about a future effect, the present output of the system does not depend on future events. The upper limit of integration should therefore be changed from ∞ to the present time, t. If also, the input signal has been applied only since the time $t = 0$, the lower limit of the integration becomes zero. Therefore the output is expressed more particularly by

$$y(t) = \int_{0}^{t} h(\lambda).x(t - \lambda)\, d\lambda$$

This expression remains a mathematical description which lacks physical realisability, as will be shown. An impulse is by definition of infinitely small duration, while the area under the curve is finite. Even if we try to approximate to an ideal input impulse function by attributing an extremely large value or amplitude of the input quantity for a very short time, we are likely to find that we have exceeded the limits within which the system can be regarded as being linear, and the system may even have been overloaded to saturation.

In evaluating the impulse response we may consider the response to all inputs which is only a rough approximation in the frequency domain of interest. Thus, if we have a rectangular pulse of width t centred at $t = 0$, and which satisfies the condition that the product of its amplitude and the time width Δt is equal to unity (and at the same time that $2\Delta t \leqslant 1/f_{max}$, i.e. that the pulse duration is short enough in comparison to the duration of the highest frequency of interest) then the useful part of the pulse spectrum so defined is white up to f_{max}.

An approximation to the impulse response of a linear system can be determined by applying a suitable noise signal to the system input and measuring the cross correlation between the input and the output. The advantage of the method is that the test noise signal can be applied at a very low level, so that no disturbance is made to the system. It can also be done without interfering with the normal operation of the system while it is under investigation. (Appendix 12).

The disadvantage is, one which will become clearer later, that it can be utilised only with linear systems having constant or characteristics which vary only slowly with time, for the cross correlation, which is essentially a process of averaging, builds up the result only in a considerable time.

In this latter case we can use, instead of real white noise as an input signal, a repetitive pseudorandom noise which has an autocorrelation function very close to that of an impulse.

4.2 Relationship between input and output in noiseless systems

As has already been shown, (Shannon, 1949) it is possible to define a continuous function with any desired degree of accuracy in terms of the sequence of values the function assumed at discrete intervals of time. The requirement is that the intervals must be sufficiently short to ensure that as much as we require of the fine structure of the function is reproduced by the samples for the particular analysis we intend to use.

Let us write these values as a vector

$$\{\underset{\sim}{x}\} = \{x_1, x_2, x_3, \ldots \quad x_N\}^T$$

$$\equiv \{x_1(t_1), x_2(t_1 + \Delta t), x_3(t_1 + 2\Delta t), \ldots.. \quad x_N(t_1 + (N-1)\Delta t)\}^T \qquad 4.2.1$$

i.e. as the transpose of a column matrix.

Suppose that the sequence so defined represents a sampled signal at the input of a physical system. N.B. Many physical and engineering systems can be represented for analysis as having only one input and one output.

The corresponding output values depend to some extent on all the previous values of the samples taken at the input, but in such a manner that the more recent values have more weight than those which occurred previously. If we denote the weighting factors in sequence by w_j, we can write the output of the system in the form

$$y_n = \sum_{j=1}^{N} w_{n-j+1} \cdot x_j \qquad 4.2.2$$

where y_n is the n^{th} output value, often called the response of the system at the time $(t_1 + n\Delta t)$.

For a first order Markov process where the output depends on the response at the present moment and also on the response for the immediately preceding moment (and not at all on all previous)

$$y_n = w_1 x_n + w_2 x_{n-1} \qquad 4.2.3$$

In general, for the n^{th} order Markov process equation 2 is valid, but there will be a number of terms greater by unity than the order of the process.

Each input value is in fact the value of a continuous function multiplied by the Dirac delta function $\delta(t_1 - n\Delta t)$, and the weighting function can be regarded as the response of the system to a Dirac delta function at the input.

For a system of which the parameters do not vary with time, the weighting functions do not change with time. The system always behaves in the same way, and the weighting functions w_j are functions of $n_j\Delta t$, and not of t, i.e.

$$w = w(\tau) \quad \text{where we have written} \qquad \text{for } n\Delta t, \qquad 4.2.4$$

If one or more basic parameters of the system vary or fluctuate with time, then the weighting functions are functions of t and , i.e.

$$w = w(t,\tau) \qquad 4.2.5$$

It was found that for a clear understanding of differencies between time varying and time invariant systems, it is necessary to use the matrix representation of their input output relationship. From this representation it becomes obvious that in the case of time varying systems the convolution of $w(\tau,t)$ and $x(t)$ is not so simple as in the case of time invariant systems. The big simplification is obtained in the former case when the weighting function tail may be neglected, i.e. in the case of very slow rate of change of signal in the comparison to the longest time constant of the system. Recall also that the longest time delay in the signal transmission defines also the band of coherence of signal components.

4.3 Matrix representation of time invariant system.

The relation between the input and the output may be written

$$\{\underset{\sim}{y}\} = \{w\}\{\underset{\sim}{x}\}$$

where

$$[w] = \left\{\begin{matrix} w_1 & 0 & 0 & 0 & \cdots\cdots & 0 & 0 \\ w_2 & w_1 & 0 & 0 & \cdots\cdots & 0 & 0 \\ w_3 & w_2 & w_1 & 0 & \cdots\cdots & 0 & 0 \\ & & \text{etc} & & & & \\ & & & & & w_1 & 0 \\ w_n & w_{n-1} & w_{n-2} & & \cdots\cdots & w_2 & w_1 \end{matrix}\right\} \qquad 4.3.1$$

in which each separate column is the same but shifted one row downwards for each period of time Δt.

If the input and the output are of the same dimensions, then $[w]$ represents a dimensionless number.

For a no-memory system $[w]$ degenerates into a diagonal matrix,

$$[w] = \left\{\begin{matrix} w_1 & 0 & 0 & 0 & & \\ 0 & w_1 & 0 & 0 & & \\ 0 & 0 & w_1 & 0 & & \\ & & & w_1 & & \\ & & & & w_1 & 0 \\ & & & & 0 & w_1 \end{matrix}\right\} \qquad 4.3.2$$

If the discrete values of the input and output samples are held at constant value until the next in the sequence arrives, the input and output functions will have a step-like form. Let us describe the operation of holding the successive values constant by an operator "rect". Then

$$\text{rect}(w_j) = \Delta t . g_j$$

and

$$\{\underset{\sim}{y}\} = \Delta t . [G] \{\underset{\sim}{x}\} \qquad 4.3.3$$

in which $[G]$ is the matrix

$$[G] = \begin{Bmatrix} g_1 & 0 & \dots & \dots & \dots & 0 \\ g_2 & g_1 & \dots & & & \\ & \text{etc} & & & g_1 & 0 \\ g_n & \dots & \dots & & g_2 & g_1 \end{Bmatrix} \qquad 4.3.4$$

Referring back to equations 1 and 2, we see that equation can be expressed as the sum of terms of the form

$$g_i . x_j . \Delta t$$

which, in the limit, for $\Delta t \rightarrow 0$, becomes

$$y(t) = \int_0^t g(t - \tau) . x(\tau) . d\tau \qquad 4.3.5$$

which is an integral relationship known as the convolution integral.

Multiplication of the square semi-matrix $[G]$ with a column matrix $\{\underset{\sim}{x}\}$ in extended form gives

$$[G]\ \{\underset{\sim}{x}\} = \begin{Bmatrix} g_1 & o & & x_1 \\ g_2 & g_1 & & x_2 \\ & & g_1 & x_3 \\ g_n & g_{n-1} & \dots & x_n \end{Bmatrix} \qquad 4.3.6$$

which can be transformed for systems which are invariant in time,

$$[G]\ \{\underset{\sim}{x}\} = \begin{Bmatrix} x_1 & 0 & 0 & & g_1 \\ x_2 & x_1 & 0 & & g_2 \\ x_3 & x_2 & x_1 & & . \\ .. & .. & .. & 0 & . \\ x_n & x_{n-1} & & x_1 & g_n \end{Bmatrix} \equiv [x]\ \{g\} \qquad 4.3.7$$

Similarly we can transform the convolution integral,

$$\int_o^t g(t - \tau) . x(\tau)\ d\tau = \int_o^t x(t - \tau) . g(\tau)\ d\tau \qquad 4.3.8$$

4.4 Noisy Channels having parameters varying with time.

Let us again, as before denote the input and output of a system in which there is noise and of which the parameters fluctuate from time to time, and of which the impulse response is therefore represented by equation

$$\{w_{f1}\} \quad = \quad \{w(t,\tau)\} \qquad 4.4.1$$

as $\{\underset{\sim}{x}(t)\}$ and $\{\underset{\sim}{y}(t)\}$. If additive noise is introduced in the system output it may be represented by

$$\{\underset{\sim}{n}(t)\} \quad = \quad \{n_1,\ n_2,\ n_3,\ \ldots . \qquad n_N\}^T \qquad 4.4.2$$

then the output may be represented in matrix notation as

$$\{\underset{\sim}{z}(t)\} \quad = \quad \{\underset{\sim}{y}(t)\} \ + \ \{\underset{\sim}{n}(t)\} \qquad 4.4.3$$

where $z(t)$ is the resultant representation of the output.

Engineering systems are usually designed so that only the zero'th, the first and the second order Markovian processes are necessary in their representation. In general the time taken for the transmission of the signal through the system may be subject to very long delays. There may also be more than one transmission path, and the delays arising from them may not be either constant from time to time nor the same along one path as another. Consequently, it may be necessary to include the considera-
along
tion of Markovian processes higher than the first or second.

For a system with randomly fluctuating parameters the weighting function may be represented by the matrix

IX)
$$\begin{Bmatrix}
w_{11} & 0 & 0 & & & \\
w_{21} & w_{12} & 0 & & & \\
w_{31} & w_{22} & w_{13} & & & \\
\cdot & w_{32} & & & & \\
\cdot & & & & & \\
w_{n1} & & & & & w_{1N} \\
0 & w_{n2} & & & & w_{2N} \\
 & & & & & w_{3N} \\
 & & & & & w_{4N} \\
 & \ldots\ldots & & & \ldots\ldots & \cdot \\
0 & \ldots\ldots & & & \ldots\ldots & w_{nN}
\end{Bmatrix} \qquad 4.4.4$$

The highest order of the Markov process which is considered to be of importance in defining the action of the system on the input signal is denoted by n. Also, since the system parameters fluctuate at random, within the limits allowed by the system, some of the weighting functions, w_{ji} $(0 \leq j \leq n)$ may have zero or even negative values.

The relationship between the output and the input is then represented by a banded diagonal matrix $\{\omega(t,\tau)\}$

$$\begin{Bmatrix} y_o \\ \vdots \\ y(t) \\ \vdots \\ y(T) \end{Bmatrix} = \begin{Bmatrix} \ddots & & \\ & \omega(t,\tau) & \\ & & \ddots \end{Bmatrix} \begin{Bmatrix} x_o \\ \vdots \\ x(t) \\ \vdots \\ x(T) \end{Bmatrix} \qquad 4.4.5$$

The output $y(t)$ for a particular time $t = t_{j+n}$ can be obtained by multiplying the $(j+n)^{th}$ row of $w(t,\tau)$ by all the values of $x(t)$. Thus

$$y(t_{j+n}) = w_{nj}x_j + w_{(n-1)(j+1)}x_{j+1} \; \ldots\ldots \; w_{1(j+n)}x_{j+n}$$

$$= \sum_{i=j}^{j+n} w_{(n-j+1)(j+i-1)}x_{(j+i-1)} \qquad 4.4.6$$

It will be seen that the output, $y(t_i)$ occurring at the time $t_i = t_{j+n}$ is the sum of n input terms multiplied by randomly varying weighting functions values. From this representation it is clear that the output function $y(t)$ is a blurred version of the input function $x(t)$. The blurring of the input function being due to the fluctuating weighting function (or impulse response) of the system. In the simplest case, namely that of a Markov process of zero order, the weighting function degenerates to a diagonal matrix. This approach could be extended to sampled signal values (Hancock, J.C. and Wintz, P.A., 1966). Following this notation used by these authors, the output signal may be written as

$$Z = A \cdot S \qquad 4.4.7$$

where S is the signal vector

$$S = \{s_1, s_2, \ldots\ldots \quad s_N\}^T \tag{4.4.8}$$

and A is a time varying vector

$$A = \{A(t)\} = \left\{\begin{array}{llll} a_{11} & 0 & 0 & \\ a_{21} & a_{12} & 0 & \\ & a_{22} & & \\ a_{n1} & & & \\ & a_{n2} & & \\ & & & a_{1K} \\ & & & a_{2K} \\ & & & a_{nK} \end{array}\right\} \tag{4.4.9}$$

The output from a fluctuating system with randomly varying parameters which operates on a signal and adds noise can then be represented by

$$Y = Z + N = AS + N \tag{4.4.10}$$

where the capital letters denote matrices.

The analog representation of these relations is given in Fig.4.2. The output signal is the sum of randomly delayed input signal components. This summation is vectorial with equally probable phase angles, resulting in a Rayleigh distribution. (Lord Rayleigh, 1880, 1889). Experimentally obtained probability density functions(Fig. 3.4a p.31 and Fig. 3.4b p.32) show that the skew coefficient (Program PROBAB&PROBABPLOT) is negative.

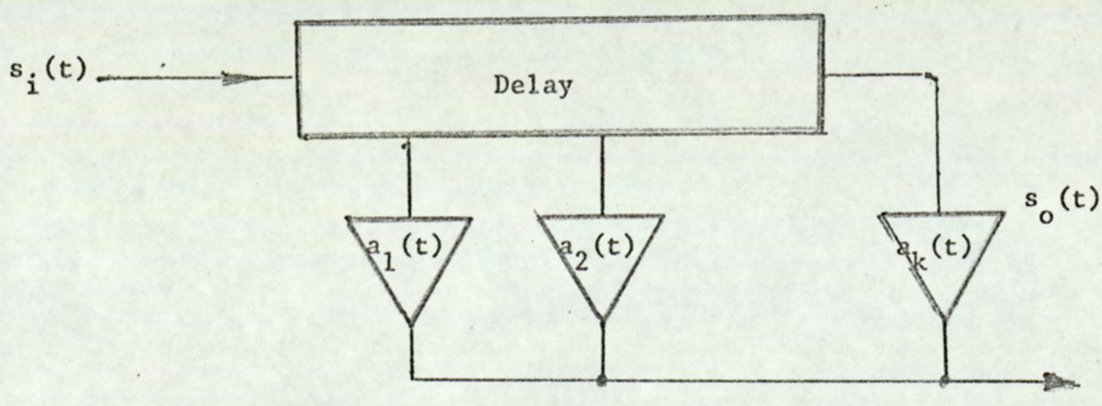


Fig.4.2

4.5 The Correlation Function and Power Spectra of Signals Transmitted Through Systems with Fluctuating Parameters.

Let us first consider the effect of very slow fluctuations of the system parameters, i.e. when the parameters are almost constant even during the transmission of the longest message element. Then any single message element is received unperturbed and there is either no or inappreciable loss of information. The usual method of correcting the variations in intensity of the signal as the system parameters vary slowly is the application (at the receiver) of the automatic volume control, AVC or automatic frequency control, AFC, according to the requirements and type of modulation encountered. Such automatic methods, often both, are employed in all types of receiver.

Suppose that, on the other hand, the fluctuations of the parameters take place at a very high rate when compared with the fastest rate at which the signal elements may fluctuate. Then, if the long term fluctuations are negligible, the average value of a fluctuating parameter may be regarded as having a constant value during the signal elements.

When it is not sufficient to or possible to find an average value of a parameter during a signal element, then the procedure generally adopted is to slow the rate of transmission of information, and to apply methods involving the use of correlation at the receiver. In this case also any particular signal element is received without perturbation, and almost all the information sent is retrieved.

When the fluctuations of the mean value can be observed in periods of time in the range from that of the longest to that of the shortest signal element, then this will cause signal changes that will be indistinguishable from the transmitted signal. Using either, or a combination of the two methods already mentioned will not correct the errors arising from the fluctuations. For example, if the AVC operates at a rate in the signal frequency range, then it will tend to remove the signal, even though it may reduce the fluctuations.

Before explaining the methods used in this research for the retrieval of information, we must study in more detail the statistical properties of systems which vary with time, distinguishing between:-

a) systems with a single transmission path

b) systems in which two or more transmission paths exist.

Strictly speaking, it is impossible to find any physical system which corresponds exactly to a system with a single transmission path. Case a) is encountered when the differences on the delays and attenuations for two or more paths do not differ very appreciably, so that to a sufficiently good approximation, the system may be treated as having one transmission path. Such will be the case when the differences between the delays for the various paths are negligible when compared with the period of the highest signal frequency. Such systems are often referred to as "no memory" systems. The best representation is one in which the parameters suffer a simple amplitude modulation. It should be pointed out that even though the system is referred to as a no memory system, the modulated high frequency signals at the output can still be Markovian of any order, depending on the character and order of the modulating signals. If the modulating signal is a sample Markovian function of the second order, so is the envelope of the R.F. signal at the output of the memoryless A.M. modulator.

The response of a system with time varying parameters to an input consisting of a Dirac delta function $\delta(t - t_0)$ applied at time t_0 differs from the commonly used output function for a system with constant parameters in that it is a function of both the absolute time t at which the response is being observed and the absolute time ξ at which the impulse was applied. Since the system has been assumed to be linear, i.e. the parameters are independent of the magnitude of the signals applied, the response of the system to any input, $x(t)$, can be found by superposing the responses of the system to a weighted series of impulses. This could be very simply visualised using the discrete absolute and lag time matrix representation discussed earlier.

The output $y(t)$ is a function of time defined usually by

$$y(t) = \int_0^t h(t,\xi).x(t-\xi).\, d\xi \qquad 4.5.1$$

where $h(t,\xi) = 0$ for $\xi < 0$

$x(t-\xi) = 0$ for $t < 0$

i.e. t and ξ are absolute times, so that the age variable,

$$\tau = |\xi - t| \qquad 4.5.2$$

Using the Fourier transform we can represent $x(t - \xi)$ as

$$x(t - \xi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega).\, e^{j\omega(t - \xi)}\, d\omega \qquad 4.5.3$$

so that

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \; e^{j\omega t} \left[\int_0^t h(t,\xi) \; e^{-j\omega\xi} d\xi \right] .d\omega \qquad 4.5.4$$

The expression in the square brackets is usually called the system function $H(j\omega,t)$ (L.A.Zadeh, 1961).

Note that this definition implies that t in $h(t,\xi)$ is a parameter which must be kept in mind especially when making the Fourier transform.

We will not follow the Zadeh procedure, which becomes mathematically very involved and great caution is consequently necessary to ensure that physical conclusions are not reached which would not conform with the mathematical assumptions made.. In order to simplify the analysis Zadeh often takes either t or ω to be parameters; thus, taking t as a parameter, he introduces a "frozen system function" which can be treated with the same mathematical methods as would be used for the ordinary time invariant systems. This approach was shown to be very useful for systems with slowly varying parameters, and can be used extensively in control engineering and with the analysis of slowly fading radio transmission.

Let us start from Zadeh's definition of the system function and follow a slightly different path which is consistent with the experimental results which have been obtained in this work. Thus the system function of a variable network or system could be defined as the response of the network to the exponential input $x(t) = e^{j\omega t}$. Denoting the output by $y(t)$ we can write

$$H(j\omega,t) = y(t)/x(t) \Big]_{x(t) \equiv e^{j\omega t}} \qquad 4.5.5$$

Note that the definition implies that ω is a parameter and $H(j\omega,t)$ is a function of time defined by the ratio of the two time functions on the r.h.s. of the equation 4.5.5 It would be more correct to adopt the notation $H(t,\omega)$. At a later stage after defining the coherence bandwidth the same function will be written

$$H(t;\omega, B_{coh}) \text{ or } H(t;\omega_0 \pm \Delta\omega) \qquad 4.5.6$$

Now the output $y(t)$ of the system with fluctuating parameters may be written as

$$y(t) = H(t;\omega) \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega)\, e^{j\omega t}\, d\omega \qquad 4.5.7$$

and the autocorrelation function of the output as

$$\psi_{yy}(\tau) = \int_0^t y(t).y(t+\tau).dt = \overline{y(t).y(t+\tau)} \qquad 4.5.8$$

substituting $y(t)$ and $y(t+\tau)$ written in the form given by the expression in equation 4.5.4

$$\psi_{yy}(\tau) = (2\pi)^{-2} \int_0^t H(t;\omega).H(t+\tau;\omega)\left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(j\omega).x(j\omega')e^{j\omega t}.e^{j\ (t+\tau)} d\omega.d\omega'\right] dt$$

$$= (2\pi)^{-2} \overline{H(t;\omega).H(t+\tau;\omega)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{x(j\omega).x(j\omega')e^{j\omega t}\ e^{j\omega'(t+\tau)}} d\omega.d\omega' \qquad 4.5.9$$

For an exponential input function

$$\overline{x(j\omega).x(j\omega')} = G_{xx}(\omega).\delta(\omega-\omega') \qquad 4.5.10$$

where $\delta(\omega - \omega')$ denotes a unit impulse or Dirac delta function in the frequency domain. The mean value of the product of the two exponential expressions is zero unless $\omega + \omega' = 0$.

Therefore

$$\psi_{yy}(\tau) = (2\pi)^{-2}\left[\overline{H(t;\omega).H(t+\tau;\omega)}\right]\int_{-\infty}^{\infty} G_{xx}(\omega)e^{-j\omega\tau}d\omega \qquad 4.5.11$$

the Fourier transform of the auto power spectrum gives the autocorrelation function

$$\psi_{yy}(\tau) = (2\pi)\left[\overline{H(t;\omega).H(t+\tau;\omega)}\right]\psi_{xx}(\tau) \qquad 4.5.12$$

The expression in the square bracket could be regarded as the autocorrelation function of the system function. Thus

$$\psi_{H}(\tau) = (2\pi)\overline{H(t;\omega).H(t+\tau;\omega)} \qquad 4.5.13$$

and the output autocorrelation function may be written as

$$\psi_{yy}(\tau) = \psi_{H}(\tau).\psi_{xx}(\tau) \qquad 4.5.14$$

If we take the Fourier transform of both sides we shall obtain the input and output power density relationship

$$G_{yy}(\omega) = G_{H}(\omega) \otimes G_{xx}(\omega) \qquad 4.5.15$$

where, as before, $\otimes$ denotes the convolution. It should also be noticed that, as occurred previously, multiplication in the time domain becomes the convolution in the frequency domain and v.v.

We see thus that a simple spectral line in the input signal spreads into a spectrum whose properties, i.e. width, etc., depends on the fluctuations of the characteristics of the system. If the correlation function of the system is an exponential function, then the output autocorrelation function becomes an exponential cosine function the properties of which have already been discussed.

For a system with invariant parameters we have already seen that the output spectrum was obtained as the ordinary product of the system transfer function and the input spectrum. The output spectrum can differ from this product only if some additive noise is present or produced in the system. Otherwise the output spectrum is defined by (Appendix 12).

$$G_{yy}(\omega) = |H(\omega)|^2 . \; G_{xx}(\omega) \qquad 4.5.16$$

which should be compared with

$$G_{yy}(\omega) = G_H(\omega) \otimes G_{xx}(\omega) \qquad 4.5.17$$

for a system with time dependent parameters.

It becomes obvious that contrary to the identification of system with time invariant parameters; in the case of fluctuating system parameters, white noise approach is not applicable.

The identification in this case is done by applying a deterministic sinusoidal signal whose spectrum is defined by one discrete line. At the output, its position becomes indeterminate. When two signals very close in frequency are transmitted simultaneously through the same system, although their absolute positions are still indeterminate, their relation is pertained, which is proved experimentally by correlating their fluctuations. (Chapter 8).

4.6 Determination of transfer function of Markovian Filter from experiments.

Let us start from the existence of a function $W(j\omega)$ which represents the data points of the desired filter response, obtained, for example, from the experimental power density spectrum and its decomposition.

Using Hilbert Transform or Bode's Technique it is possible to find the real and imaginary parts of this function (Procedure BODE)

$$W(j\omega) = X(\omega) + jY(\omega) \qquad 4.6.1$$

The filter transfer function can be written in terms of complex frequency variable s

$$F(s) = \frac{b_o + b_1 s + b_2 s^2 + b_3 s^3 + \ldots}{1 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots} \qquad 4.6.2$$

substituting $s = j\omega$ and separating the real and imaginary parts in the numerator and the denominator

$$F(j\omega) = \frac{(b_o - b_2\omega^2 + b_4\omega^4 \ldots) + j\omega(b_1 - b_3\omega^2 + b_5\omega^4 - \ldots)}{(1 - a_2\omega^2 + a_4\omega^4 \ \ldots) + j\omega(a_1 - a_3\omega^2 + a_5\omega^4 - \ldots)}$$

$$= \frac{\alpha_n + j\omega\beta_n}{\alpha_d + j\omega\beta_d} = \frac{N(\omega)}{D(\omega)} \qquad 4.6.3$$

At any specific frequency ω_k, the error in fitting experimentally obtained data points by analytical function is

$$\varepsilon(\omega_k) = W(j\omega_k) - F(j\omega_k)$$

$$= W(j\omega_k) - \frac{N(\omega k)}{D(\omega k)} \qquad 4.6.4$$

and the problem is to minimise this error at each sampling point on the curve, which is usually done by summing up $|\varepsilon(\omega k)|^2$ over all sampling frequencies and setting the partial derivatives of the summation with respect to each of the coefficients equal to zero. This would correspond to a least square fit and would result in a set of linear simultaneous algebraic equations.

By multiplying both sides of the last equation by $D(\omega_k)$ and squaring them, a weighted error function E is obtained, which is summed up over the sampling frequencies

$$D(\omega_k)\varepsilon(\omega_k) = A(\omega_k) + jB(\omega_k) \qquad 4.6.5$$

$$E = \sum_{k=o}^{m} |D(\omega_k)\varepsilon(\omega_k)|^2 = \sum_{k=o}^{m} \left[A^2(\omega_k) + B^2(\omega_k)\right] \qquad 4.6.6$$

where

$$A(\omega_k) = \mathrm{Re}\left[D(\omega_k)\ W(j\omega_k) - N(\omega_k)\right]$$

$$= \alpha_{dk}X_k - \omega_k\beta_{dk} - \alpha_{nk}$$

and

$$B(\omega_k) = \mathrm{Im}\left[D(\omega_k)\ .\ W(j\omega_k) - N(\omega_k)\right] \qquad 4.6.7$$

Substituting in E, and differentiating with respect to each of the coefficients, (Levy, 1959)

$$\frac{\partial E}{\partial b_o} = \sum_{k=o}^{m} 2(\alpha_{dk}X_k - \omega_k\beta_{dk}Y_k - \alpha_{nk})\ (-A)$$

$$\frac{\partial E}{\partial b_1} = \sum_{k=o}^{m} 2(\omega_k\beta_{dk}X_k + \alpha_{dk}Y_k - \omega_k\beta_{nk})\ (-\omega_k)$$

$$\frac{\partial E}{\partial b_2} = \sum_{k=o}^{m} 2(\alpha_{dk}X_k - \omega_k\beta_{dk}Y_k - \alpha_{nk})\ (+\omega_k^{\ 2})$$

$$\frac{\partial E}{\partial b_3} = \sum_{k=o}^{m} 2(\omega_k\beta_{dk}X_k + \alpha_{dk}Y_k - \omega_k\beta_{nk})\ (+\omega_k^{\ 3}) \qquad 4.6.8$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{\partial E}{\partial a_1} = \sum_{k=o}^{m} \Big[2(\alpha_{dk}X_k - \omega_k\beta_{dk}Y_k - \alpha_{nk})\ (-\omega_k Y_k)$$

$$+\ 2(\omega_k\beta_{dk}X_k + \alpha_{dk}Y_k - \omega_k\beta_{nk})\ (\omega_k X_k)\Big]$$

$$\frac{\partial E}{\partial a_2} = \sum_{k=o}^{m} \Big[2(\alpha_{dk}X_k - \omega_k\beta_{dk}Y_k - \alpha_{nk})\ (-\omega_k{}^2X_k)$$

$$+\ 2(\omega_k\beta_{dk}X_k + \alpha_{dk}Y_k - \omega_k\beta_{nk})\ (-\omega_k{}^2Y_k)\Big]$$

Some further algebra (after equating all derivatives to zero) and introducing

$$\Omega_h = \sum_{k=o}^{m} \omega_k^h \quad ; \quad T_h = \sum_{k=o}^{m} \omega_k^h Y_k$$
$$S_h = \sum_{k=o}^{m} \omega_k^h X_k \quad ; \quad U_h = \sum_{k=o}^{m} \omega_k^h (X_k + Y_k) \tag{4.6.9}$$

leads to the following set of linear algebraic equations, which could be written in the more compact matrix form (PROCEDURE TRANSFIT)

$$\left|\begin{array}{c} S_o \\ T_1 \\ S_2 \\ T_2 \\ \cdot \\ \vdots \\ \hline 0 \\ U_2 \\ 0 \\ U_4 \\ \\ \end{array}\right| = \left|\begin{array}{cccccc|ccccc} \Omega_o & 0 & -\Omega_2 & 0 & \Omega_4 & \cdot & T_1 & S_2 & -T_3 & -S_4 & \cdot \\ 0 & \Omega_2 & 0 & -\Omega_4 & 0 & \cdot & -S_2 & T_3 & S_4 & -T_5 & \cdot \\ \Omega_2 & 0 & -\Omega_4 & 0 & \Omega_6 & \cdot & T_3 & S_4 & -T_5 & -S_6 & \cdot \\ 0 & \Omega_4 & 0 & -\Omega_6 & 0 & \cdot & -S_4 & T_5 & S_6 & -T_7 & \cdot \\ \Omega_4 & 0 & -\Omega_6 & 0 & \Omega_8 & \cdot & T_5 & S_6 & -T_7 & -S_8 & \cdot \\ \vdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline T_1 & -S_2 & -T_3 & S_4 & T_5 & \cdot & U_2 & 0 & -U_4 & 0 & \cdot \\ S_2 & T_3 & -S_4 & -T_5 & S_6 & \cdot & 0 & U_4 & 0 & -U_6 & \cdot \\ T_3 & -S_4 & -T_5 & S_6 & T_7 & \cdot & U_4 & 0 & -U_6 & 0 & \cdot \\ S_4 & T_5 & -S_6 & -T_7 & S_8 & \cdot & 0 & U6 & 0 & -U_8 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}\right| \left|\begin{array}{c} b_o \\ b_1 \\ \\ \\ b_4 \\ \\ \hline a_1 \\ a_2 \\ a_3 \\ a_4 \\ \\ \end{array}\right| \tag{4.6.10}$$

where b_i and a_i being unknown can be easily solved by digital computer (PROCEDURE PIVOT).

4.7 Measuring the coherence of two functions.

The estimation of the transfer function of time invariant systems is influenced when there is additive noise present, and the coherence between the output and input signals is a useful quantity to measure.

Suppose that in the output spectrum there is a part S_n which is not correlated with the input spectrum S_x. The cross power spectrum G_{zx} may be written

$$G_{zx} = (S_y + S_n)S_x^{*} = G_{yx} + G_{nx} \qquad 4.7.1$$

where * denotes the complex conjugate

G_{yx} the cross correlated power spectrum between the input signal x(t) and the output signal y(t)

and G_{nx} the cross power spectrum of the input signal x(t) and the internally generated noise n(t).

If the internally generated noise is independent of the signals, i.e. the noise is of additive type, then the averaged cross power spectrum $\overline{G_{nx}} = 0$, and then

$$\overline{G_{zx}} = \overline{G_{yx}} \qquad 4.7.2$$

The measure of coherence sometimes called the coherence function, is by definition (J. S. Bendat and A.G. Piersol)

$\gamma^2(\omega)$ is written as $\overline{\gamma^2}$ and $G_{zx}(\omega) = G_{zx}$ to simplify notation until later.

$$\overline{\gamma^2} = \overline{G_{zx}} \cdot \overline{G_{zx}^{*}} \,/\, \overline{G_{zz}} \cdot \overline{G_{xx}} \qquad 4.7.3$$

Recalling that

$$\overline{G_{zz}} = \overline{(S_y + S_n)\,(S_y + S_n)^{*}}$$

$$= \overline{G_{yy}} + \overline{G_{yn}} + \overline{G_{ny}} + \overline{G_{nn}}$$

$$= \overline{G_{yy}} + \overline{G_{nn}} \qquad 4.7.4$$

If S_n and S_y are uncorrelated.

$$\overline{G_{zx}} = \overline{G_{yx}} = \overline{S_y . S_x^*}$$

$$G_{yx} = \overline{(A_j + iB_y)\ (A_x - iB_x)} \qquad 4.7.5$$

$$= \overline{(A_y A_x + B_y B_x) + i(B_y A_x - B_x A_y)}$$

$$\overline{G_{yx}^{\ 2}} = \overline{G_{yx} * G_{yx}^*}$$

$$= \overline{(A_y A_x)^2 + (B_y B_x)^2 + (A_x B_y)^2 + (B_x A_y)^2} \qquad 4.7.6$$

since

$$\overline{G_{yy}} = \overline{(A_y + jB_y)(A_y - jB_y)} = \overline{A_y^{\ 2}} + \overline{B_y^{\ 2}} \qquad 4.7.7$$

and similarly

$$G_{xx} = \overline{A_x^{\ 2}} + \overline{B_y^{\ 2}} \qquad 4.7.8$$

$$\therefore \qquad \overline{G_{yy}}.\overline{G_{xx}} = \overline{A_x^{\ 2} A_y^{\ 2} + A_x^{\ 2} B_y^{\ 2} + A_y^{\ 2} B_x^{\ 2} + B_x^{\ 2} B_y^{\ 2}} \qquad 4.7.9$$

which is identically the same results as that obtained for $\overline{|G_{yx}|^2}$.

Thus

$$\overline{|G_{yx}|^2} = \overline{G_{yy}}.\overline{G_{xx}} \qquad 4.7.10$$

On the other hand, when n(t) and y(t) are uncorrelated

$$\overline{G_{zx}} = \overline{G_{yy}} + \overline{G_{nn}} \qquad 4.7.11$$

and $\overline{\gamma(\omega)^2}$ may be written

$$\gamma(\omega) = \overline{G_{yy}(\omega)} \ . \ \overline{G_{xx}(\omega)} / [(\overline{G_{yy}(\omega)} + \overline{G_{nn}(\omega)}.\overline{G_{xx}(\omega)}] \qquad 4.7.12$$

$$\frac{\overline{G_{yy}(\omega)}}{\overline{G_{yy}(\omega)} + \overline{G_{nn}(\omega)}} = \frac{\overline{G_{yy}(\omega)} / \overline{G_{nn}(\omega)}}{1 + \overline{G_{yy}(\omega)} / \overline{G_{nn}(\omega)}}$$

The invariable ω has been introduced again in order to emphasise that the power densities are functions of frequency.

When there is no additive noise the correlation function is independent of frequency, i.e.

$$\overline{\gamma^2(\omega)} \equiv 1.0 \qquad 4.7.13$$

When there is noise present the coherence measure as a function of frequency will depend on the spot signal to noise ± ratio, i.e. on the signal to noise ratio for individual spectral lines.

If the total signal to noise power is denoted by

$$(S/N) = \int_{-\infty}^{\infty} G_{yy}(\omega).\, d\omega \Big/ \left(\int_{-\infty}^{\infty} G_{nn}(\omega)\, d\omega \right) \qquad 4.7.14$$

then the cumulative coherence measure

$$\overline{\gamma_c^{\ 2}} = (S/N)/(1 + (S/N) = 1/(1 + (N/S)); \quad 0 < \overline{\gamma_c^{\ 2}} < 1.0 \qquad 4.7.15$$

and where $\overline{\gamma_c^{\ 2}}$ is not now a function of frequency.

When the noise power spectrum is independent of the signal power spectrum then the coherence measure and the cumulative coherence both tend to unity as the signal power is increased.

For systems with parameters which fluctuate with time the behaviour is not so simple and we should consider only the equation

$$\overline{\gamma(\omega)^2} = \overline{|G_{zx}(\omega)^2|} / (\overline{G_{zz}(\omega)}.\overline{G_{xx}(\omega)}) \qquad 4.7.16$$

which can be obtained as a function of frequency by recording x(t) and y(t) and finding the Fourier transform of $\psi_{zx}(t)$, $\psi_{zz}(t)$ and $\psi_{xx}(t)$ and then evaluating the ratio for each frequency separately.

Note that for this case x(t) and y(t) represent two fluctuations. Bring just a normalised cross spectrum of two fluctuations, the coherence function gives, therefore, a very informative picture of correlation of different frequency components of two fluctuations.

Much fuller picture is obtained if the coherence function is separated in its real (Fig.4.3) and imaginary part (Fig.4.4).

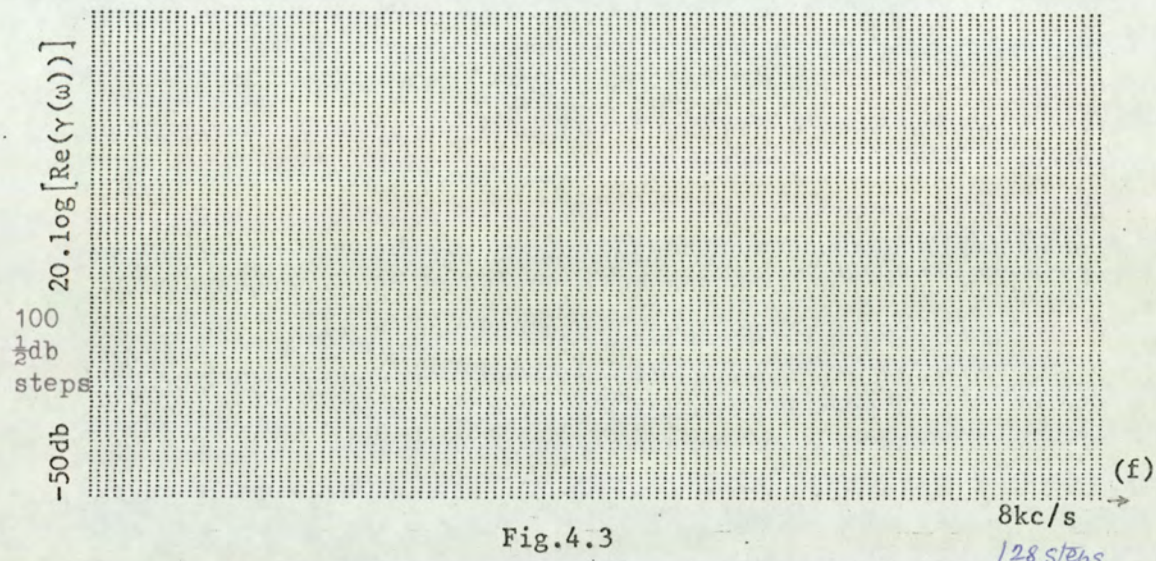


Fig.4.3

For the particular use of the coherence property of two fluctuations in the filtering of signals, as it is described in Chapter 8, it is of the utmost importance that only the real part of coherence of fluctuation exist. The imaginary part, as it will become more clear later could only deteriorate the retrieval of information if used in a feedback loop.

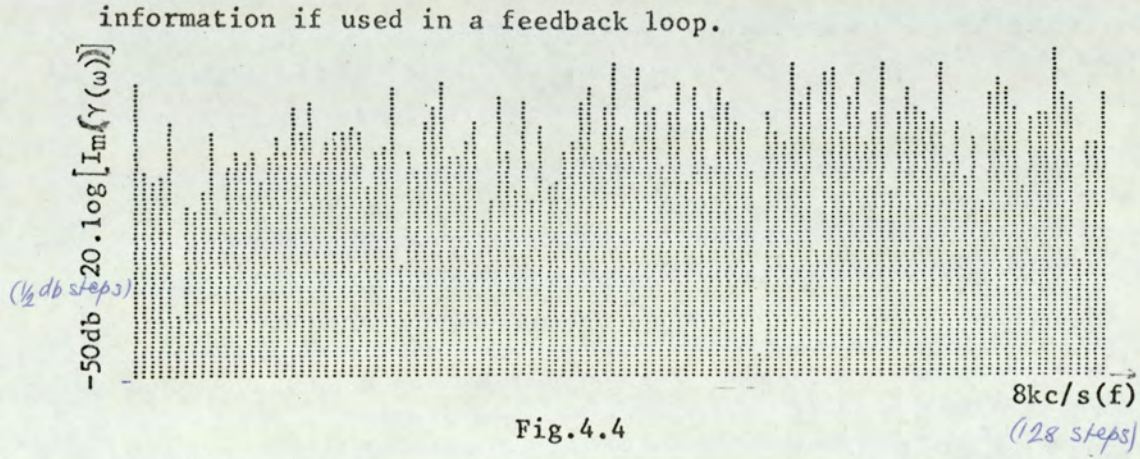


Fig.4.4

Due to the inevitable phase shift in a feedback loop, even highly coherent signal fluctuations (Figs.4.3 and 4.4), become less so after being processed by the described filtering (Figs.4.5 and 4.6)

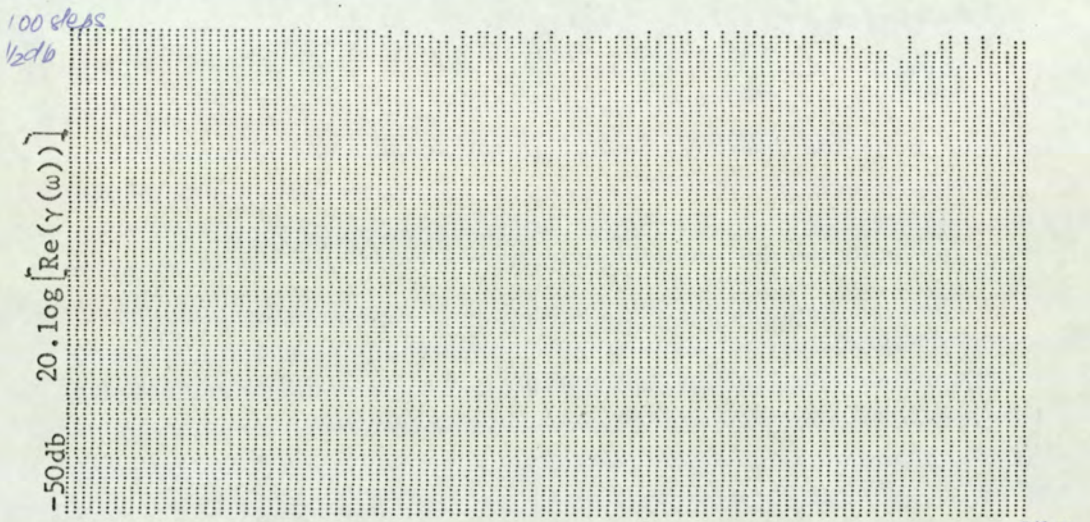


Fig.4.5

Taking into account that both fluctuations are suppressed for at least 20 db, and that the high frequency end of both spectra is anyway low, the deterioration is not really important, so that its effect is merely a change in the spectrum shape (Fig.19.2 and 19.3).

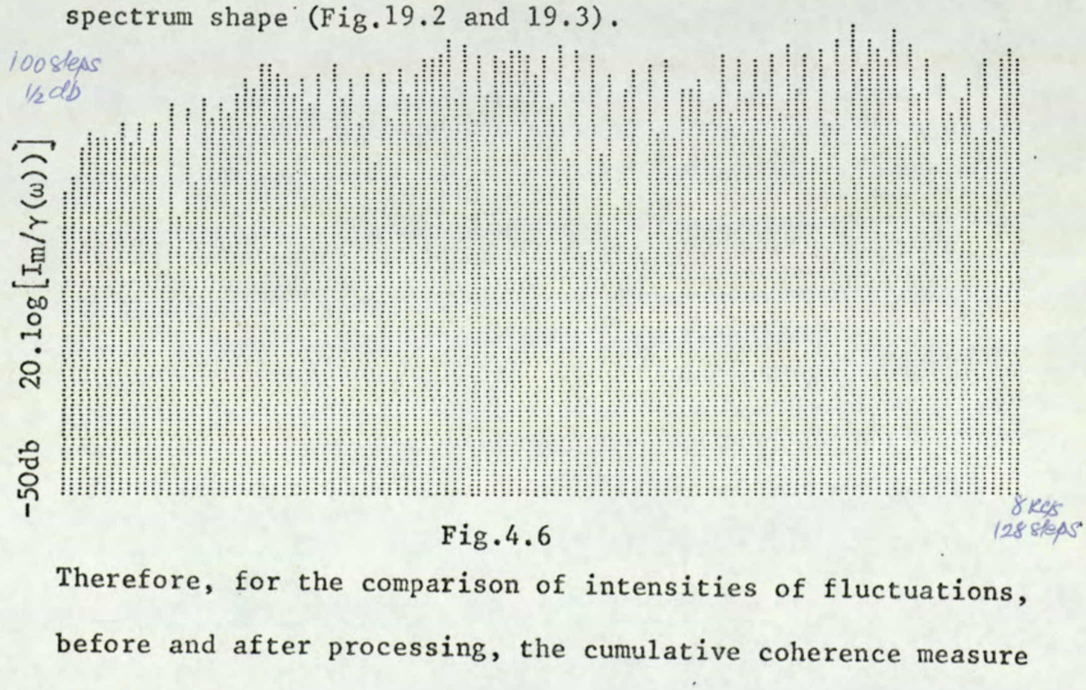


Fig.4.6

Therefore, for the comparison of intensities of fluctuations, before and after processing, the cumulative coherence measure must be introduced.

4.8 <u>Coherence of two signals of equal mean power.</u>

Consider two signals a(t) and c(t) represented by the signal vectors $\{a(t)\} = \sum_{1}^{2BT} a_i$ and $\{b(t)\} = \sum_{1}^{2BT} b_i$. The scalar product will give the correlation between them.

If we introduce the Schwartz inequality

$$\left[\sum_{1}^{2BT} a_n b_n\right]^2 \leq \left(\sum^{2BT} a_n^2\right).\left(\sum^{2BT} b_n^2\right) \qquad 4.8.1$$

since

$$a_i^2 b_j^2 - 2a_i a_j b_i b_j + a_j^2 b_i^2 - (a_i b_j)^2 \geq 0 \qquad 4.8.2$$

In terms of continuous functions,

$$\left[\int_0^T a(t)b(t)dt\right]^2 \leq \left(\int_0^T a(t)^2 dt\right)\left(\int_0^T b(t)^2 dt\right) \qquad 4.8.3$$

so that cumulative coherence measure, Γ may be defined:

$$\Gamma = \frac{\int_0^T a(t).b(t)\ dt}{\left[\left(\int_0^T a(t)^2 dt\right)\left(\int_0^T b(t)^2\ dt\right)\right]^{\frac{1}{2}}} \leq 1 \qquad 4.8.4$$

Equality is realised only when

$$a(t) \equiv b(t) \qquad 4.8.5$$

i.e. when the two functions are completely correlated or are identical at all times.

Γ value, resulting from the scalar product of two vectors, is not dependent on the mutually orthogonal vector components. Furthermore, as standard deviations of both signal fluctuations can be normalised, Γ value can be computed numerically by evaluating the area under the real part of the corresponding cross-spectrum. (Procedure COHERENCE).

CHAPTER 5

5.1 DATA ACQUISITION, REDUCTION AND ANALYSIS

From the sampling theorem (Appendix 1) it is known that to represent a band limited signal, in the time interval 0 - T, it is necessary to specify its values at N points spaced at equal intervals of time, ΔT

$$N = 2.f_{max}.T \qquad 5.1.1$$

where f_{max} is the highest frequency component of the signal

and $$\Delta T = \frac{1}{2.f_{max}} = \frac{1}{2.B} \qquad 5.1.2$$

where B is the necessary bandwidth of the channel to transmit this signal.

When a white additive noise with Gaussian distribution is present there is an uncertainty of signal detection. The maximum number of distinct signals, M_{max}, which then could be transmitted (Appendix 3) is given by

$$\log_2 M_{max} = B.T.\log_2(1 + S/N) \qquad 5.1.3$$

where S is the signal power

N is the noise power

The channel capacity C, which is by definition the amount of information which can be transmitted and interpreted without ambiguity (Appendix 3) is also a function of S/N ratio:

$$C = B \log_2(1 + S/N) \qquad 5.1.4$$

It follows that for a signal in a physical system with invariant parameters and contaminated by additive noise, the signal to noise ratio is an important figure of merit in connection with the transmission of signals.

For a time varying system, where the signal power is partially converted to noise power, the coherent portion of the received signal must be reduced (Hathaway, 1970). Hence, by dividing the power of the coherent part of the signal by the sum of noise power and incoherent signal power, being considered as noise too, an even smaller S/N ratio is obtained. This ratio depends strongly on the intensity of the transmission parameter fluctuation which becomes more important for low additive noise levels.

For an ionised flame, for example, the ratio N/S, when no additive noise is present, is a measure of its turbulence.

Whatever the origin of the noise in a system, transmitted information is masked, in one way or another, by the noise, so that its presence is undesirable. A physical system is always noisy, although in some cases, the noise power is hardly measurable.

Although a lot of information about the suitability of a system for signal propagation can be deduced from the S/N ratio, this ratio itself does not disclose the way the noise power is distributed in the frequency spectrum. In order to devise methods and design systems to combat the effects of noise is an optimum way, knowledge of noise spectra and signal power distribution versus frequency is necessary. (Fig.5.1).

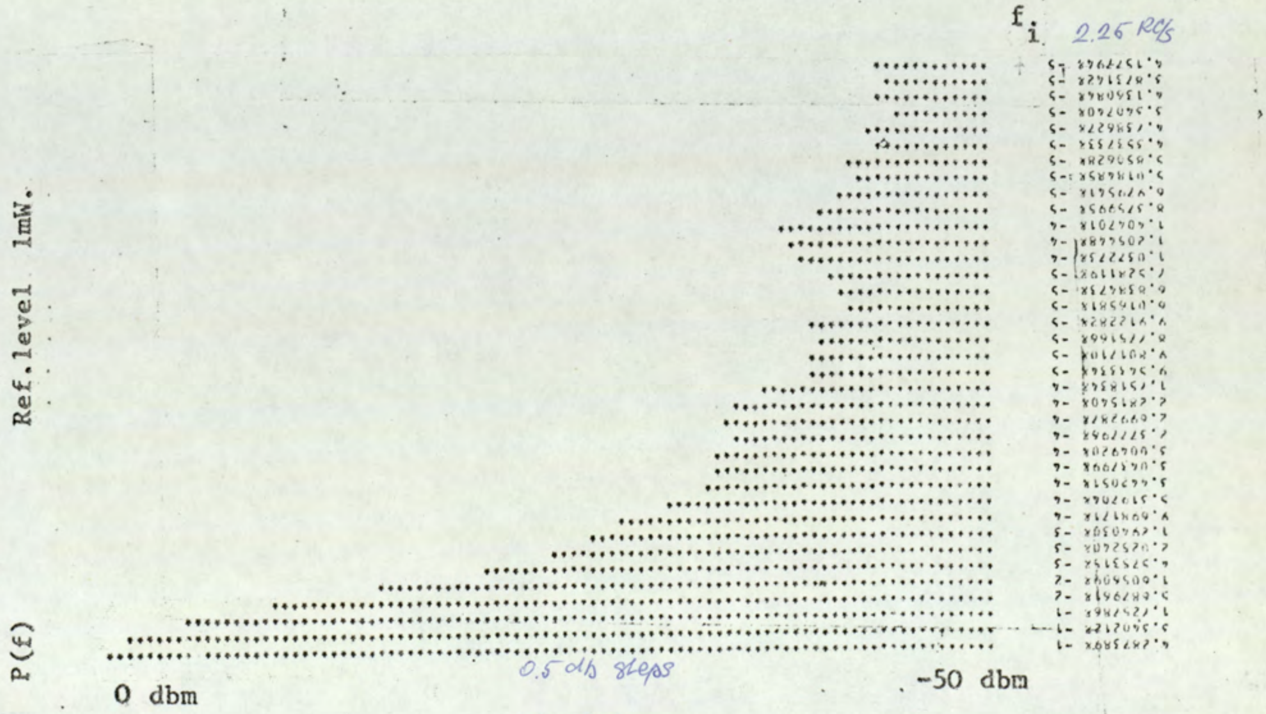


Fig.5.1.

A physical system with time varying parameters is characterised by (see Chapter 4)

a) system correlation function $\psi_H(\tau)$

b) coherence bandwidth Bcoh

The Fourier transform of the system correlation function $G_H(\omega)$ convoluted with the input power density spectrum $G_{xx}(\omega)$ results in the output power density spectrum:

$$G_{yy}(\omega) = G_H(\omega) \circledast G_{xx}(\omega) \qquad 5.1.5$$

If the input power density spectrum is reduced to a δ function, the output density spectrum becomes proportional to the $FT\{\psi_H(\tau)\}$.

By changing the signal power, the output power density shifts accordingly. In general, $\psi_H(\tau)$ and its Fourier transforms are functions of time. For turbulent but otherwise stable flame, with constant fluid rate and mixture ratio, $\psi_H(\tau)$ is statistically stable for averages taken in $\Delta T \gtrsim 0.5$ sec (τ max 80msec). If the fluid rate and mixture ratio are functions of time, so is obviously the short term system correlation function (Fig.5.2), and the short term S/N ratio.

Initial measurements of S/N ratio were made using the equipment arrangement described in Sec.5.3 (Fig.5.6). In all experiments, the same additive $CsC\ell$ in a 100% solution was used. The atomiser and its calibration curve is described by E. R. Miller, Ph.D. thesis, University of Aston, 1969. A typical power density spectrum is given in Fig 9.11a Depending on the fluid rate and the size of the burners used, the slope of the power spectrum varied from 10-16 dB/oct, being less steep when the nozzle aperture was reduced. (See Table 1 in Chapter 8).

The influence of different fuels, seeds and other parameters were not studied, as these effects are either already very well known or extensively studied elsewhere (for example, G.Roberts, University of Aston, Department of Chemistry). For the present research the phenomenon of system parameters fluctuations and their effect on signal fluctuation was of the main interest.

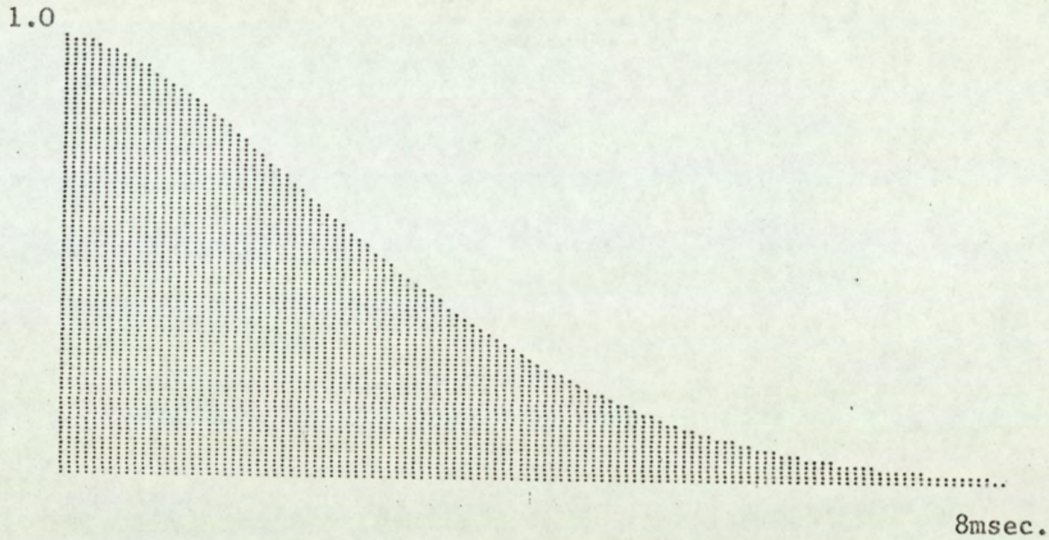


Fig.5.2a. Autocovariance function of a fluctuating signal transmitted through a laboratory flame (gas nozzle speed 0.7 m/sec).

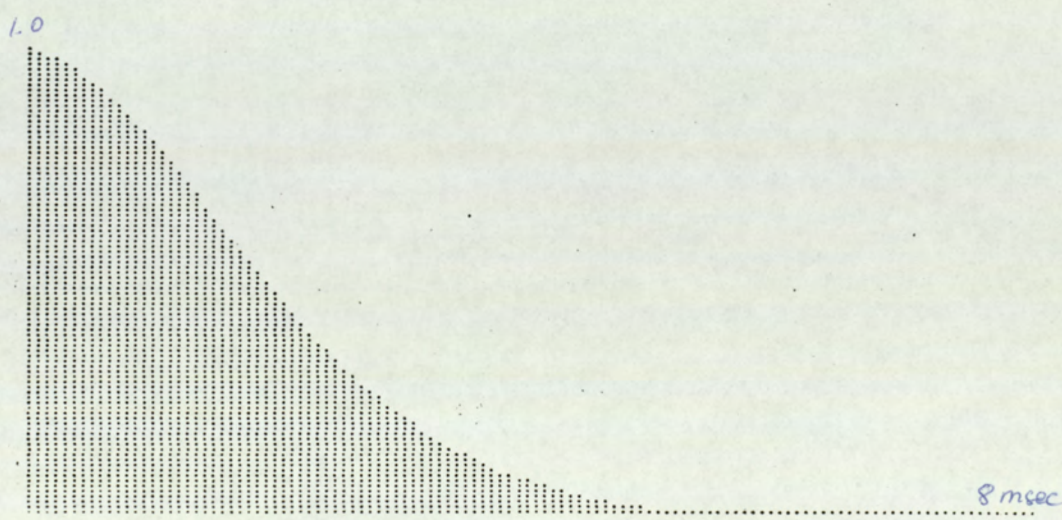


Fig.5.2b. Autocovariance function of a fluctuating signal transmitted through the same flame as on Fig.5.2a, but for gas nozzle speed 1.0 m/sec.

5.2 Determination of Transmission Parameters of a Physical System.

In a coherent band of frequencies a system with time varying parameters is defined by its system correlation function, from which, as it was shown, coefficients of an associated Markovian filter could be found. The first step in identifying such a system is, therefore, to find the magnitude and phase fluctuations of an initially deterministic sinusoid signal, after being transmitted through the physical system under investigation.

When "on line" analysers become available, data reduction and statistical analysis of data can be achieved as soon as an experiment is finished, otherwise tape recordings must be made first and analysed later.

There are obvious disadvantages of the latter method, especially in the inherent delay in preparing paper tapes with the digital data punched on them from the initial magnetic tape recording. However, the recorded data can be analysed more thoroughly, making repeating analysis of even a single recorded portion as many times as necessary, in a variety of ways, using, when necssary, different interval durations and starting times.

The real and more obvious "off line" advantage, especially when financial resources are limited, is that a common available digital computer can be utilised.

The process of reduction and data analysis for this research is described in outline using a flow diagram (Fig. 5.3).

Signal magnitude and phase fluctuations are recorded using a portable seven channel FM recorder (Pambeco). The tape speed was chosen to be 32 inches per second. The recorded portion of an experimental fluctuation was usually only a few seconds. The tape was then replayed using a speed sixteen times slower, i.e. $1\frac{7}{8}$ inches per second. Sampling was done at time intervals of lmsec, corresponding to 62.5 microseconds in the real time. The data was stored in digital form (4096 Levels or 12 bits) on the PDP-9 magnetic tape (Program ADCMTP).

Unfortunately this tape is not compatible with the ICL 1905 magnetic tape, so that it was necessary to transfer data to a paper tape (Program PUNSUB).

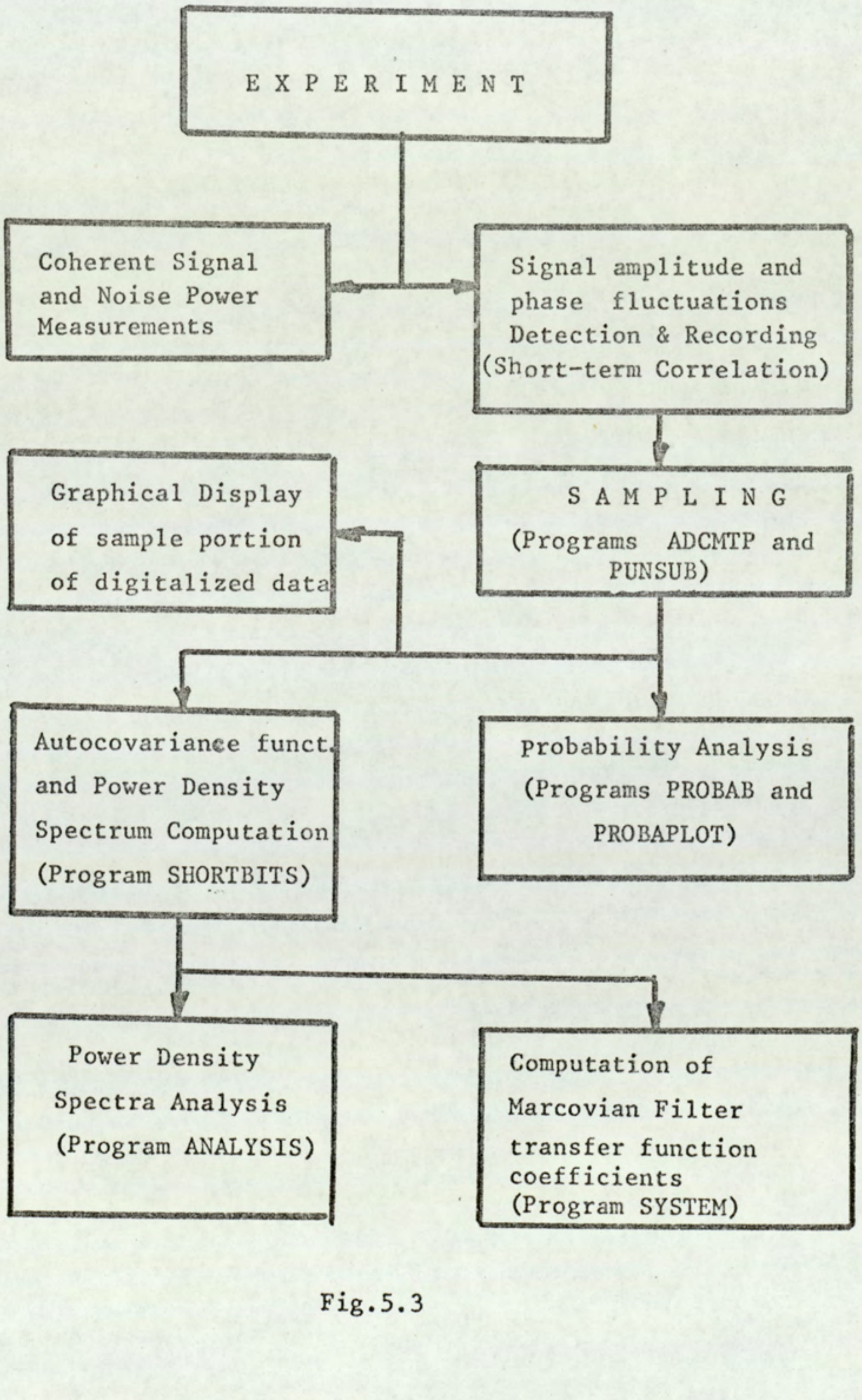


Fig.5.3

The largest but still easily manipulated tape roll size is 5 to 6 inches in diameter with approximately 5,500 to 6,500 digital data. In most cases 4096 to 6144 data were put on one tape, making approximately three tapes necessary for each duration of one second in the actual experiment.

At the very beginning of this research, when digital method of data analysis was adopted, from each particular experiment lasting up to ten seconds, a portion of 4 seconds was digitalized resulting in 64,000 data points per experiment. It was very soon found that even only 16,000 give statistically stable results.

When short term auto and cross-correlation were studied, recorded fluctuations of approximately 63 to 315 msec. were found to be quite satisfactory, producing 1024 to 5120 digital data. For practical reasons, even when longer records were necessary, not more than 6144 data were used on one reel of paper tape.

For checking numerical contents on tapes visually, a simple program was used (STARS) to display sample portions using computer lineprinter (Fig. 1A and 1B).

Based on theoretical considerations outlined in Chapter 2, several computer programs were made and utilised.

Programs PROBAB and PROBABPLOT compute and plot respectively first and second probability distributions, one step transition probability matrix and several step transition probabilities, starting from a defined variable value.

As the distribution differs from a Gaussian, the skew and excess coefficients were also computed (Program PROBAB).

Program SHORTBITS computes 4 to 20 autocovariance functions, each corresponding to 1024 real time data, then averages results and produces also a cumulative autocovariance function, taking into account the influence of fluctuating variance, calculated for each particular short intervals of time.

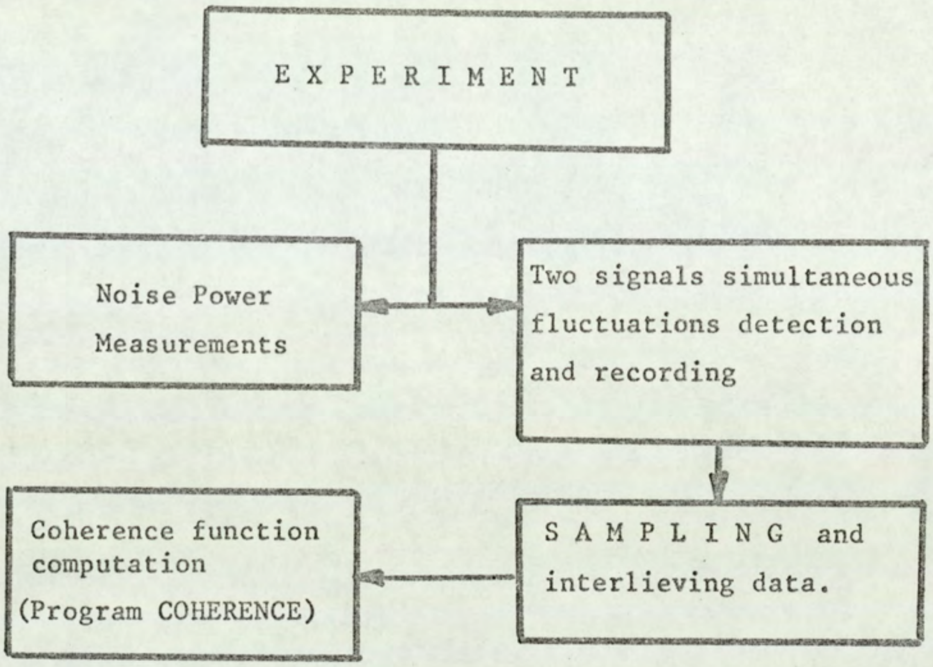


Fig.5.4

Simultaneously, a Fast Fourier Transform of each autocovariance function is computed and the corresponding power density spectra are obtained. The same program displays graphically results using lineprinter and outputs tape for eventually plotting (Program SPECTRPLOT) or further analysis (Programs ANALYSIS & SYSTEM).

Program ANALYSIS takes the obtained cumulative power density spectrum data and computes analytical meromorphic expressions, using Wiener-Lee technique and Chebyshev polynomials. Thus several thousand real data are finally reduced in 128 data of an autocovariance function or corresponding 128 data points of power density spectrum, which are eventually compressed to four-five coefficients of a polynomial in meromorphic representation.

To obtain the approximate "frozen" (i.e. slowly time varying) coefficients of the Marcovian Filter transfer function, the Program SYSTEM is used.

Coherence Function Computation and Coherence Bandwidth Estimation.

Here two sinusoidal signals are simultaneously sent through the same physical system and corresponding fluctuations recorded. Each fluctuation is sampled and obtained data interlieved. Using the Program COHERENCE, the auto spectra, cross spectrum, coherence function and cross correlation function are computed. The coherence bandwidth is estimated as to be equal to the frequency difference of two signals when cumulative coherence factor drops to, say, 0.95 or 0.9, depending on the desired quality of signals after being processed by the R.A.A.G.C. (See ASTON Filtering).

5.3 The Experimental Arrangement.

The experimental arrangement is shown in simplified schematic form in Fig.5.5. It consists of a microwave transmitting-receiving equipment (1-20), H.F. modulator and transmitter (21), H.F. receiver, demodulator unit and a dual channel RMS voltmeter-correlator (22).

The same apparatus can be used for microwave plasma diagnostic purpose, when open-ended microwave resonators (TM 040) should be substituted by a pair of horn antennas, or in the case of low variation densities by a high Q Fabry-Perot resonator, (Primich 1964, Chaffin 1968) which is particularly suitable especially at higher microwave, millimeter, and optical frequencies.

The disadvantage of using Fabry-Perot resonator for fluctuation phenomena studies is in the necessity to make experiments only in the vicinity of a very critically adjustable resonant frequency of the resonator. Klystron frequency tracking could be accomplished with the available Microwave Instrument Equipment model MOT, but would mask to a great extent the low frequency components of the measured power density spectra. This fact was not clearly understood at the preparatory stage of Fabry-Perot resonator experiments, but was discovered at a later stage in the research. Horn antennas, on the other hand, having comparatively low sensitivity for fluctuation phenomena of slightly ionised small laboratory flame plasma, were found unsuitable too. For this research it was found that the open ended circular resonators (TM 040 and TM 050) were more convenient, especially for two signal fluctuation coherence measurements, when it was possible, by using lower resonator Q factors in the range 120-150, to measure coherence up to 20 Mc's for single and double path transmission. Theoretical background for application of this resonator as a random amplitude and phase modulator are given in

Appendices 6-10. The resonant frequency shifts by

$$\Delta f(t) = K \cdot P(t) \cdot f_o \cdot N(t) \qquad 5.3.1$$

where K is proportionality constant
P(t) is filling factor
N(t) electron density

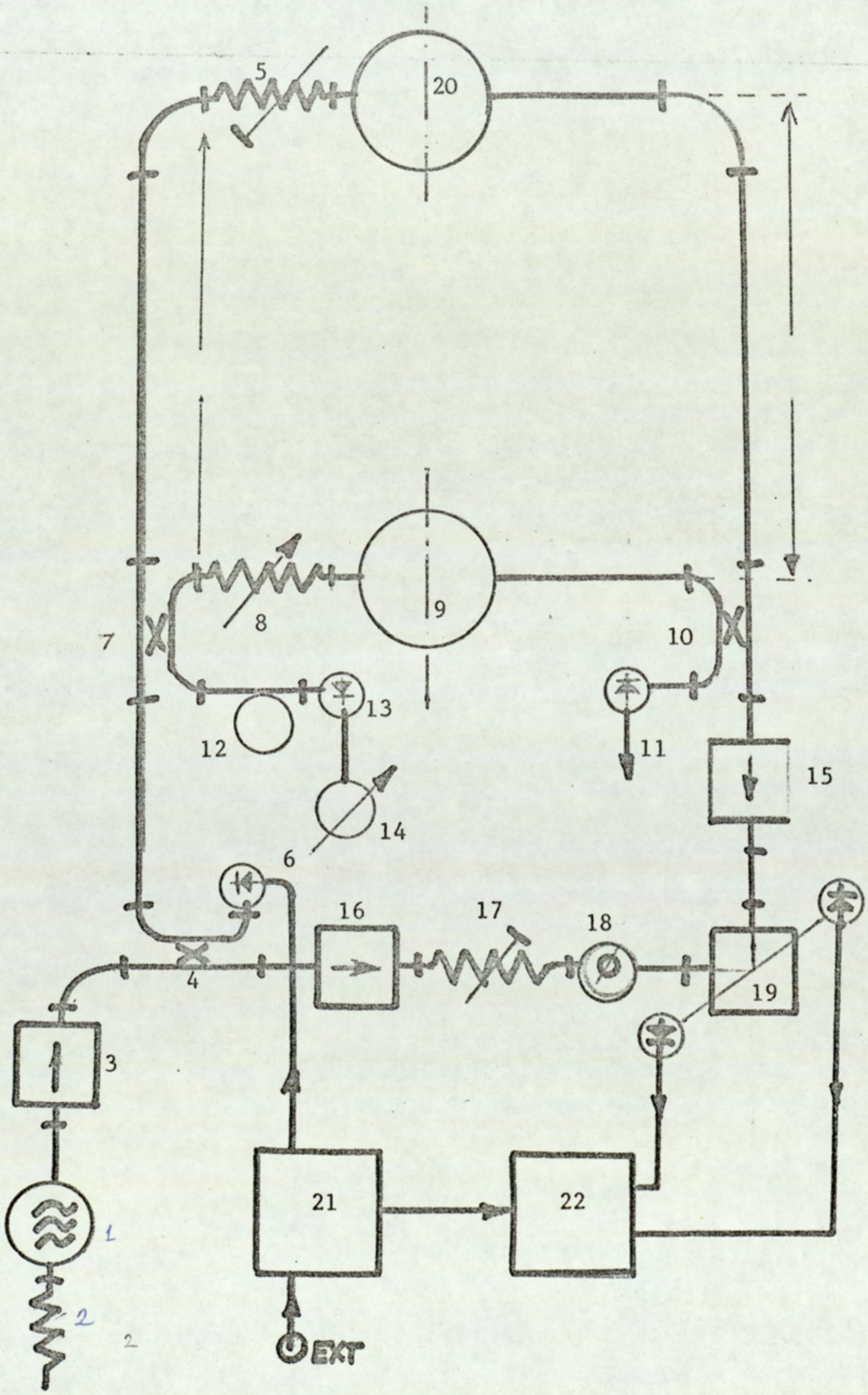


Fig.5.5.

For a turbulent and unstable flame plasma, both P(t) and N(t) are stochastic time functions.

Also, from the Appendix 10 it could be seen that the real part of Transmission coefficient is a function of the effective Q factor and this in turn depends on filling factors, collision frequency and ionisation density.

As the complex transmission coefficient of the resonator depends on randomly time varying quantities of the perturbing plasma sample, it is possible to obtain a very simple and useful random amplitude and phase modulation of transmitted microwave signals. For single path transmission one resonator suffices. When difference delay simulation is necessary at least two resonators must be used. Adjustment procedure for more than two resonators in an arrangement becomes a little involved, but does not necessarily improve the validity of the experiment for simulating a multipath transmission of signals through a flame. The microwave frequency signal carrier is generated by a KS9-20A Klystron (1) with a nominal output power of 40mw at 9.35 Gc/s, which was adopted as the centre of the microwave frequency band used in all experiments in this research.

The resonant probe in the Klystron valve (1) waveguide was adjusted for maximum radiation in direction of the isolator unit (3) and the opposite waveguide end was properly terminated (2). The total microwave energy which reached 3dB coupler unit was split in two parts. One part, after attenuation (17) and necessary mean phase adjustment, was applied for mixing purposes to the "Magic T" device (19). The other part was amplitude modulated by an HF signal (4Mc/s) generated and processed in the HF modulator and transmitting unit (21).

For the single path transmission simulation only the lower resonator of the described arrangement was utilised. (Fig. 8, A - p. 161)

A signal randomly pertubed in amplitude and phase was applied also to the magic T (E plane). At the output of the same device, among others, an IF signal of 4Mc/s was obtained, and applied to the HF

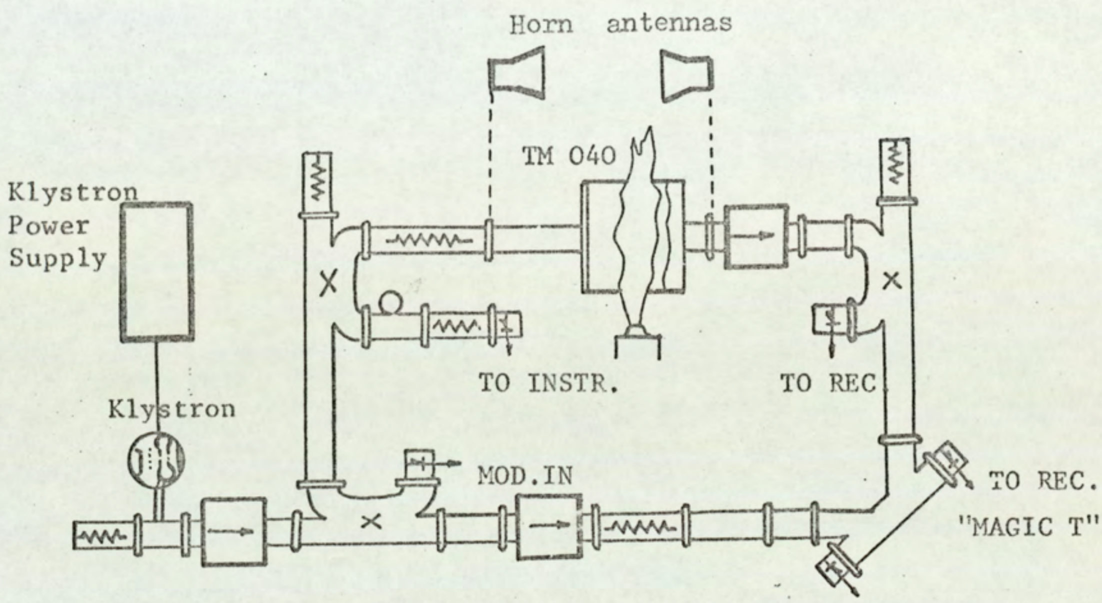


Fig.5.6.

receiving, demodulating and measuring equipment (22).

For the double path transmission both resonators have been utilised. The distance between resonators and the propagation velocity in the wave-guide define the maximum time delay difference, and consequently the maximum differential phase delay. As it was already stated the coherence bandwidth is inversely proportional to the maximum differential phase delay, so that increasing the delay difference, coherent bandwidth is reduced.

HF Modulator and Transmitter

The HF modulator consists of an orthogonal 4Mc/s signal generator, which can be either amplitude or angle, i.e. phase or frequency, modulated.

For the majority of experiments 4Mc/s signal was amplitude modulated and carrier was suppressed. For coherence bandwidth measurements two signals were used i.e. either a fixed 16Mc/s signal generated by the master oscillator or an externally applied signal of variable frequency from a standard Laboratory Signal Generator.

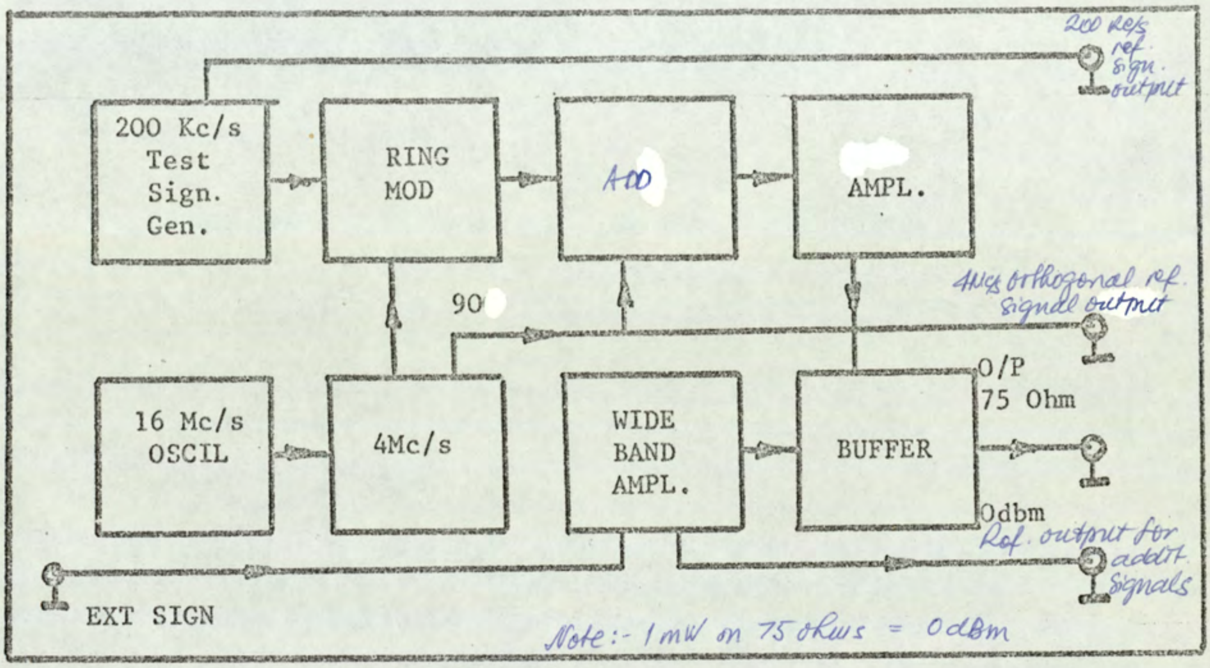
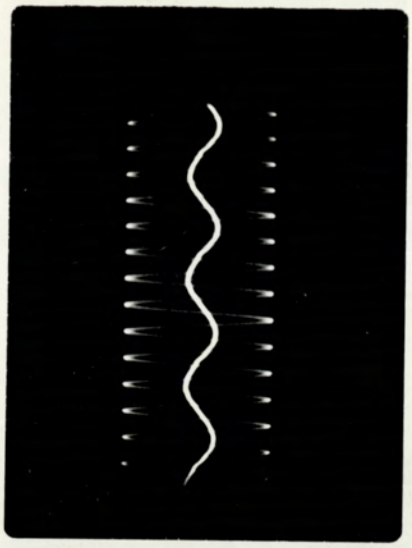


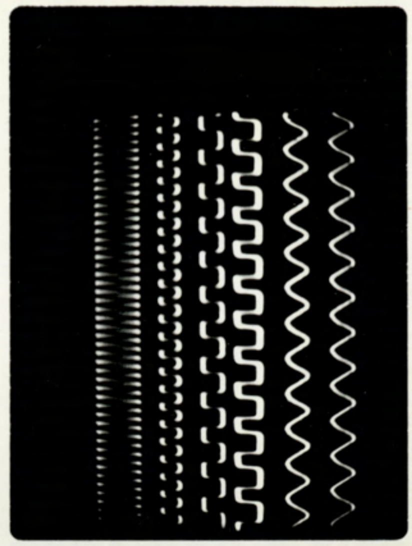
Fig.5.7.

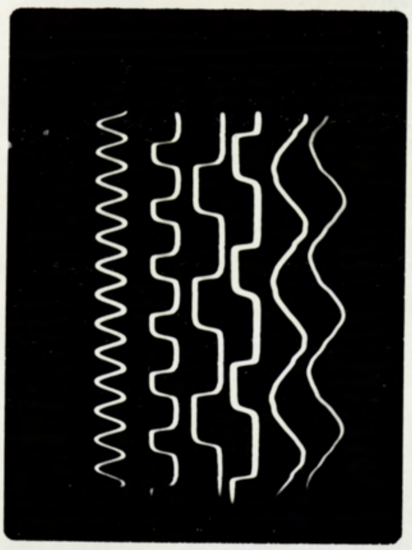
Orthogonal Signal Generation

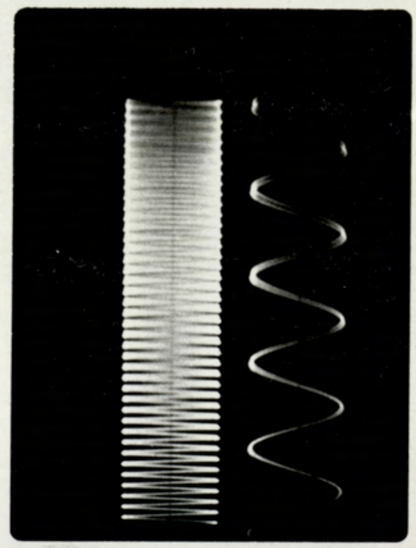
The simplest method of obtaining the same frequency orthogonal signals, quite independent of possible oscillator frequency changes, is by employing fast flip flops, FF, in "ripple carry" mode. Thus to obtain a 4Mc/s signal from 16Mc/s signal, it is necessary to use two divide by two stages. Each stage, in any binary counter (FF), (Fig.5.8) has its outputs to its 'AND' inputs, so that input trigger pulses are routed alternately to the two inputs. Each output waveform, therefore, is a square wave at one half the frequency of its input. By connecting the output of one stage as the input to the next, the frequency is successively halved.

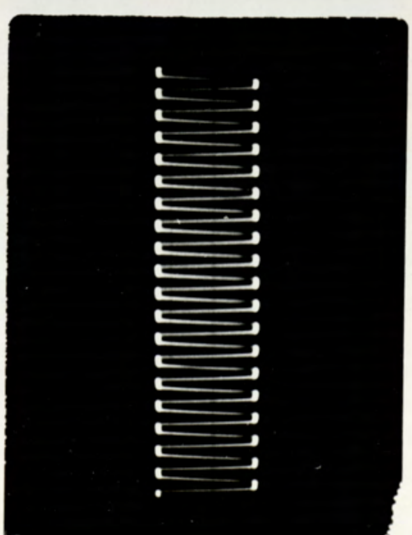
The sinusoidal signal (16Mc/s) is first shaped by a Schmidt trigger stage.

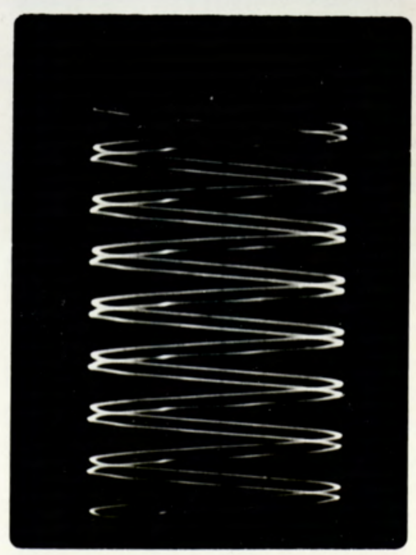












The waveform of the output is shown on Fig. 5.a First FF halves the frequency using both outputs (Q_1 and $\bar{Q}_1$) to trigger the next corresponding FF's, two 4Mc/s square waveforms are obtained at their outputs (Fig. 5 b,c)

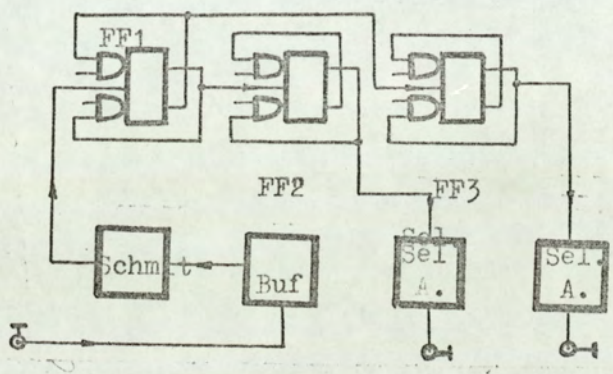


Fig.5.8

The next step is to get rid of higher order harmonics present in squared waveforms. This is accomplished by two separate 4Mc/s selective amplifiers. These amplifiers, in order not to introduce unwanted phase shifts, must have relatively wide bands* and flat phase vs frequency characteristics.

As it is also necessary to maintain the proper sequence of phases (i.e. that the first 4Mc/s signal leads 90^o) an inhibit pulse from the second FF (Q_2) is applied to C 'AND' circuit of the third FF.

All frequency changes of 16Mc/s oscillator are instantaneously followed by the change in frequency of both 4Mc/s output signals. Thus if 16Mc/s signal is frequency modulated, both 4Mc/s signals will be also, with the same modulation frequency and the same modulation index. For FM modulation, 16Mc/s master oscillator is provided with a varicap diode (BA102).

<u>200 Kc/s Signal Oscillator</u>

This is an ordinary amplitude stabilized LC oscillator, with a PMC loop (Fig.5.9). The output signal taken from AMPL1 is used as the

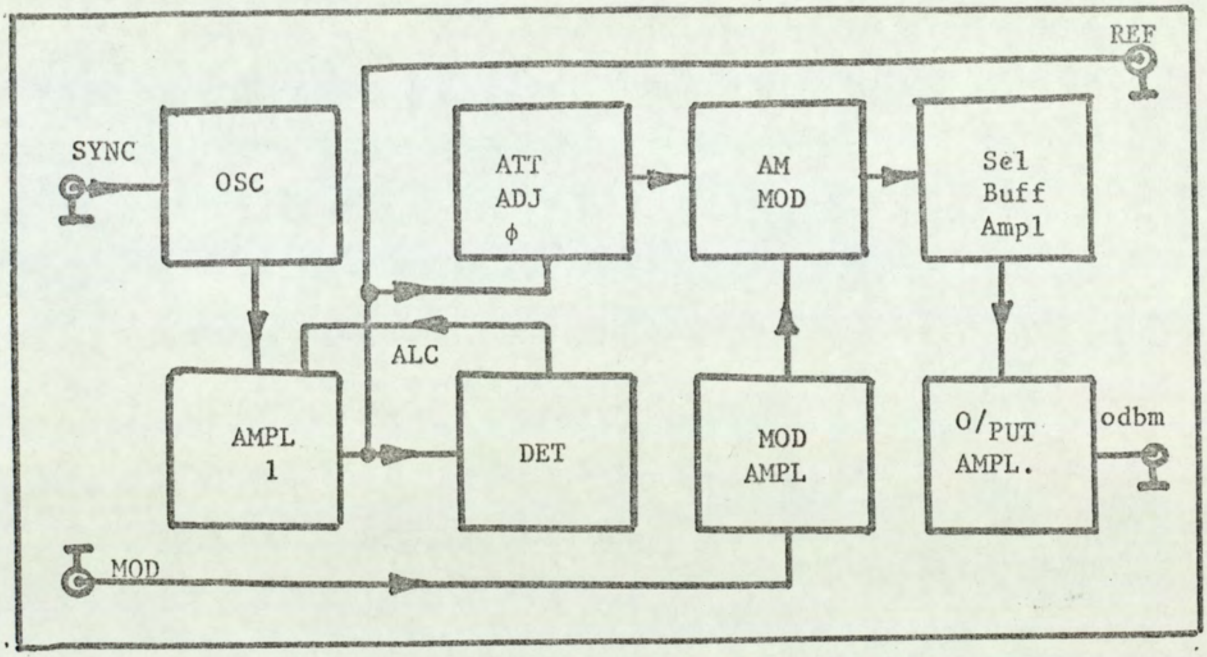


Fig.5.9

transmitted reference signal. After necessary attenuation the same signal is applied to an AM modulator stage, and after this through a buffer stage to the 75 ohms output. Facility with AM is provided in order to enable fast and easy checking of the receiver shifting correlator-detector circuits, as well as for ASTON FILTER (see Chapter 9) testing.

Generating AM test signal with 90^{o} rotated carrier

4Mc/s signal (0^{o}) is applied to the ring modulator (Fig.5.7) at which output double sided band suppressed carrier (DSBSC) signal is obtained. This signal is added to the second 4Mc/s signal (90^{o}) and then amplified with buffer and output amplifier (75 ohms - Odbm).

For the two signal fluctuation coherence vs frequency measurements, an external signal could be added, so that the output contains 4Mc/s AM with rotated carrier, plus a variable frequency (0.1 - 20 Mc/s) testing signal.

HF Receiver - Demodulator circuit

For the initial measurements a very simple receiver arrangement (Fig.) was used. From the 'magic T' detector a IF signal (4Mc/s) was obtained, which was amplified by 4Mc/s (40dB) amplifier. This signal then was coherently detected, by a reference signal, using a balanced diode ring. The resulting output (200 kc/s) signal was applied to the dual channel root mean square, RMS, voltmeter-correlator, so that the coherent portion of the signal as well as the remaining incoherent portion, i.e. noise level, were measured. Simultaneously, a short term shifting interval correlator was used in order to detect fluctuations of either the signal amplitude or the signal phase. At the shift-correlator output, the detected fluctuation signal level was of the order of 1V RMS, which was suitable for FM tape recording necessary for further processing (PDP-9) and analysing (ICL 1905).

The RMS voltmeter and shifting interval correlator will be described in more detail in sections 6.6 and 7.3 respectively.

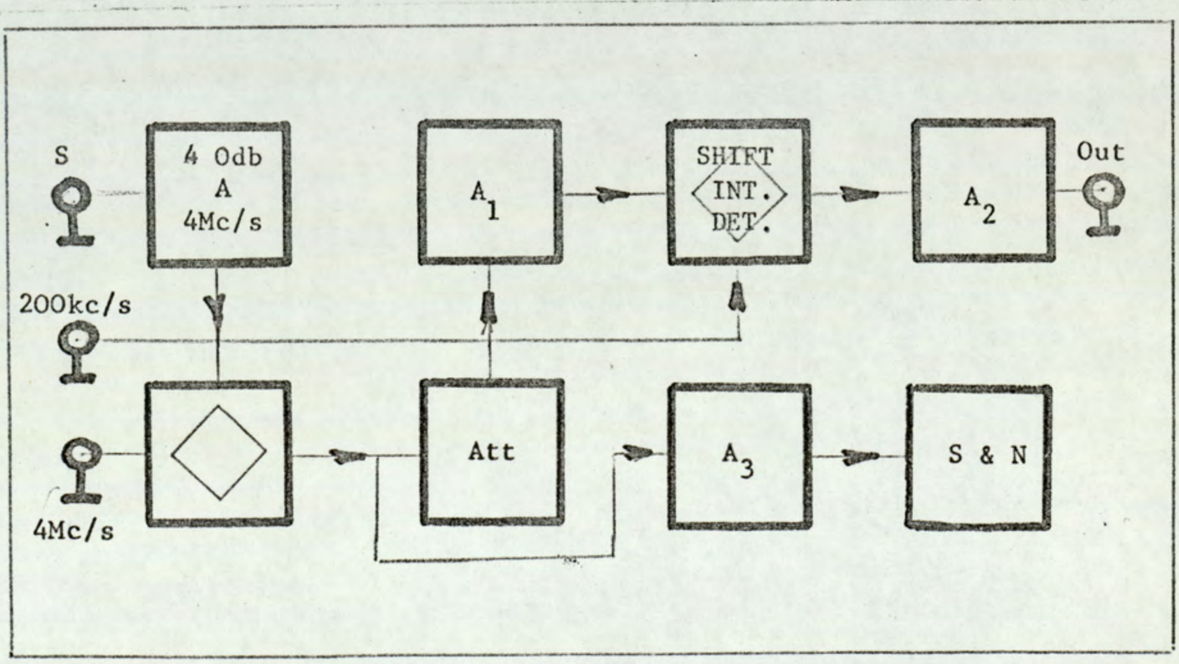


Fig. 5.10.

CHAPTER 6

A. Signal and Noise Power Measurements

Signal and noise powers are measured during all experiments. As the signal fluctuates both in magnitude and phase, its power spreads over a spectrum on either side of the signal frequency, and which varies inversely as Δf, i.e. it forms a "$1/\Delta f$" spectrum of noise. The coherent portion of the signal is extracted by a double bridge arrangement and then measured spearately.

The sensing elements are two balanced indirectly heated thermistors, the temperatures of which are held constant by means of a sampled data servo system.

The dominant time constant of the feedback loop is determined by the heat inertia of the thermistors, which is about 4 seconds (B 21). Current pulses obtained from the sampled servo are integrated and applied to a Field Effect Transistor (F.E.T) which is connected as voltage regulated resistor. An A.C. current, which is not necessarily of the same frequency as the signal is fed back. The apparatus can be used in several ways, which will be explained in the following pages. As the methods are based on the mutual orthogonality of the noise, the signal and the D.C. currents, a three dimensional signal space can be used to represent the actions taking place. The three dimensional representation is a reduction of a multidimensional signal space representation which is reviewed in the next section 6.1 (See also Appendices 1 - 4).

For the initial experiments, separate but simultaneous signal and noise measurements were necessary. From these the signal to noise ratio was determined.

For the second part of this research, when noise suppression due to feedback was of interest, the mean signal level at the receiver was held constant, and the simultaneous measurements of the observed noise with and without feedback were made.

I_o DC current.

σ_N Noise current RMS value.

S Signal current RMS value.

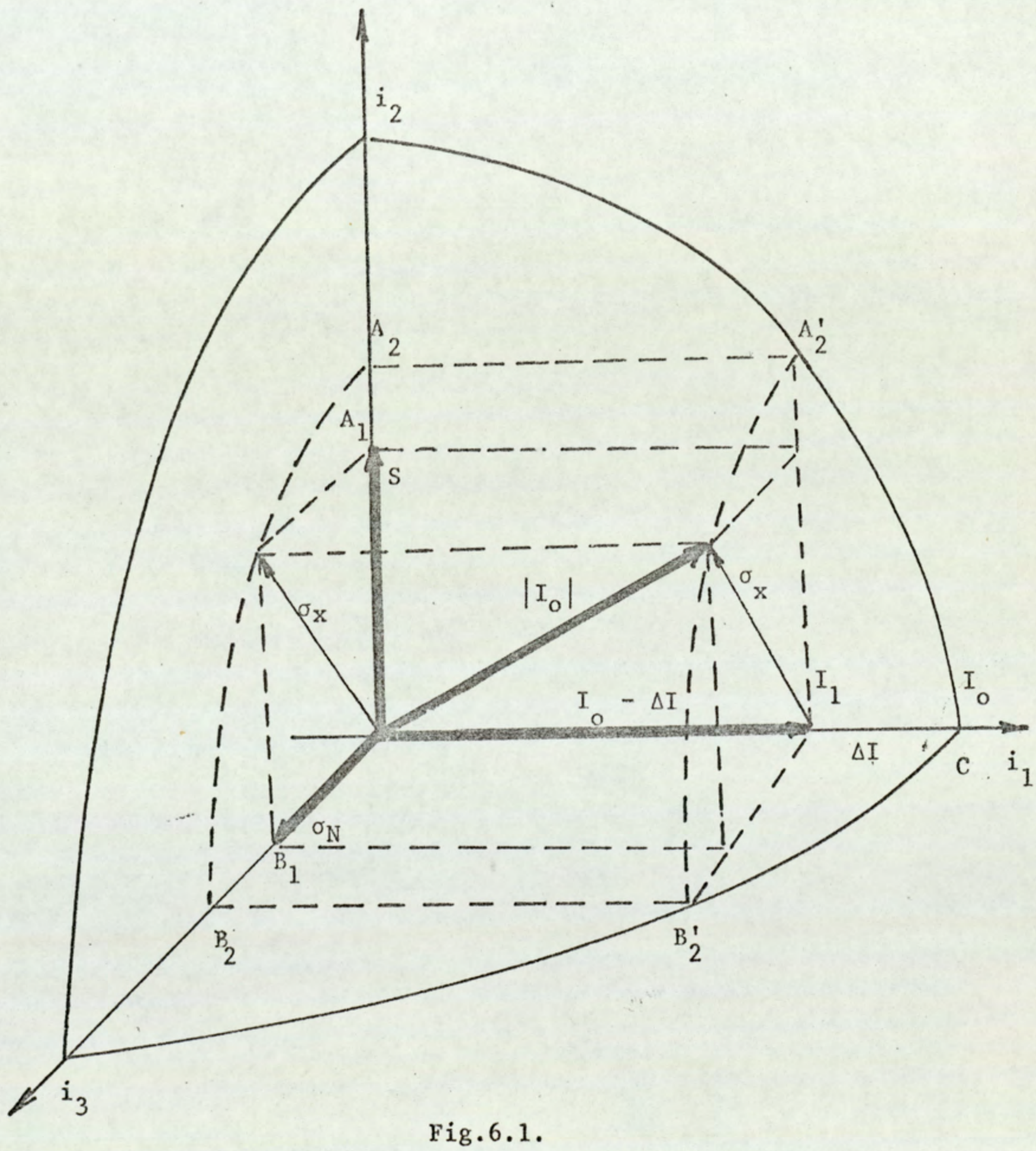


Fig.6.1.

For signal and/or noise power measurements, the circuit is found to be suitable up to 65 Mc/s with approximately 0.2 db error. For measuring signal and noise separately, reliable results were obtained up to 400 kc/s, by a method involving cancelling the coherent portion of the signal received.

6.1 Signal energy and the mean power.

The distance from the origin to a point in the signal space with co-ordinates x_n, $n = 1,2,3, \ldots\ldots 2BT$, is (App.2 and 3)

$$d = \left(\sum_{n=1}^{2BT} a_n^2 \right)^{\frac{1}{2}} \qquad 6.1.1$$

As has been shown a band limited continuous function $x(t)$ in the interval $0 - T$ can be represented by (App.1) 2BT sample value taken at equally spaced intervals of time. Assume that the sampling time interval is taken as a reference unit of time and that the signal is dissipated in a resistance the value of which is taken to be a normalised unit of resistance

$$\int_0^T a^2(t).dt = \frac{1}{2B} \sum_{n=1}^{2BT} (a_n)^2 = d^2/2B = P.T. \qquad 6.1.2$$

which expresses the signal energy expended in a time T, or, if the signal power is P, then

$$d = (2BTP)^{\frac{1}{2}} \qquad 6.1.3$$

The same results will be obtained from the analytical expression for the interpolation function, equation A1.11, when

$$\int_0^T a^2(t).dt = \int_0^T \left[\sum_{n=1}^{2BT} a_n \frac{\sin \pi(2Bt + n)}{\pi(2Bt+n)} \right]^2 .dt \qquad 6.1.4$$

The cross products (occurring in the integral) are given by

$$I_{mn} = \int_0^T \frac{\sin\pi(2Bt + n)}{\pi(2Bt+n)} \cdot \frac{\sin\pi(2Bt + m)}{\pi(2Bt+m)} .dt \qquad 6.1.5$$

$$I_{mn} = 0 \text{ for } n \neq m \; ; \; I_{mn} = 1/2B \text{ for } n = m,$$

so that

$$\int_0^T a^2(t), dt = \frac{1}{2B} \cdot \sum_{n=1}^{2BT} (a_n)^2 \qquad 6.1.6$$

The area under the interpolation curve (sinx/x) is equal to unity. The average power in the signal is equal to the sum of the squares of the sample values obtained divided by the total number of samples, or

$$P = \frac{1}{T}\int_0^T a^2(t).dt = \frac{1}{2BT}\sum_{n=1}^{2BT}(a_n)^2 \qquad 6.1.7$$

Similarly, the distance between two points in the 2BT space is $(2BT)^{\frac{1}{2}}$ times the r.m.s. of difference of values of two corresponding signals. If two signals, both band limited, averaged in the same time intervals of the same duration T have the same energy, or equal mean powers, they could be represented in the 2BT space of N = 2BT dimensions by two vectors of the same magnitude. The points at the terminations of the two vectors lie on the same hypersphere with radius (Fig.6.2)

$$\left|\vec{r}_1\right| = \left|\vec{r}_2\right| = d = (2BTP)^{\frac{1}{2}} \qquad 6.1.8$$

The difference between the two vectors becomes zero only if the two signals are totally correlated. (Sec.4.8)

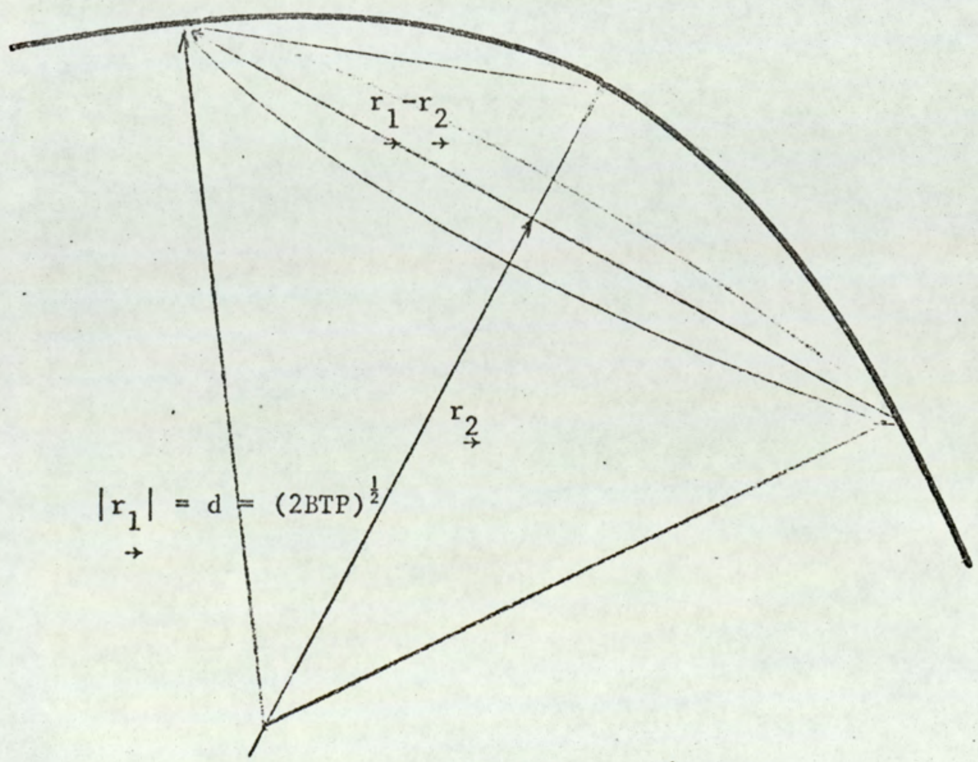


Fig.6.2

B. Description of r.m.s.measuring methods

6.2 Total signal and noise power measurement using d.c. feedback only.

This mode of operation is very useful for very complex signal and noise mean power measurements. A simplified diagram of the circuit is shown in Fig.6.3. This principle already has been utilised for several years. Campbell (1950) used the filament of a temperature limited diode as the sensing device for the input voltage. The circuit has been mentioned in several text books on noise measurements. (Benet 1950)

Further improvements were suggested by Burgess (1951) , Widdis (1956) and Bozic (1966). Some d.c. drift was observed even in versions such as Bozic's in which an a.c. servo action was used. In order to minimise the drift still further thermistor bridges were used in this research, in conjunction with a sampling data servo system. For ease of explaining the basic principles of such a bridge the complicated sampling systems and pulsed supply to the bridge have been (Fig.6.3) replaced by simple operational amplifier OA1 and a floating battery. (F.B)

Through the thermistor heater an initial current I_o defined by

$$I_o = I_{oo} - \Delta I_o \qquad 6.2.1$$

where $I_{oo} = U_b/(R_2 + R_3 + Th1)$ 6.2.2

ΔI_o = initial offset current going into the operational amplifier for balancing the bridge which consists of the equal resistances $R_{1/1} = R_{1/2} = R_{1/3} = R_{Th1}$

The power dissipated in the thermistor heater is

$$p_o = R_{th1} . I_o^2 \qquad 6.2.3$$

Normalising all the resistances in the bridge circuit so that $R_{th1} = 1$, the initial dissipation of power in the thermistor heater may be written

$$p_o = I_o^2 \qquad 6.2.4$$

The servo action of the operational amplifier can be arranged to hold the total power dissipated in the thermistor heater constant. If the current I_o is replaced by the sum of d.c. and a.c. components, say i_1, i_2, i_3, etc. for which if the servo action is perfect we obtain (Fig.6.1)

$$P_o = I_o^2 = i_1^2 + i_2^2 + i_3^2 + \text{etc} \qquad 6.2.5$$

$$= \text{constant.}$$

If the currents i_1, i_2, i_3 etc are plotted on orthogonal axes, then the constancy of the power P_o is indicated by the constancy of the vector I_o, which is the vector sum of the separate independent currents. I_o therefore represents the constant radius of a sphere on which the operating point is held by the action of the operational servo amplifier. There is another current due to U_b which will in further descriptions be neglected, since it consists of a very short sampling pulse in the actual arrangement used. It gives negligible heating of the bridge and thermistor elements and provides information on the state of balance of the bridge under the currents already mentioned.

A unit gain, high input and low output resistance operational amplifier is used in order to enable complicated a.c. voltages to be applied to the bridge without loading the source of these signals appreciably. For matching purposes (mean power measurements) a variable resistor R_4 is provided. For matched conditions

$$P_x = E_x^2/4R_4 = \overline{e_x(t)^2}/4R_4 \qquad 6.2.6$$

where P_x is the mean power,

E_x is the r.m.s. value of the complicated voltage waveform $e_x(t)$, which is a function of time.

The voltage across R_4 and R_{th1} will be on account of the unit gain of the amplifier OA2 equal, and the corresponding a.c. power will be

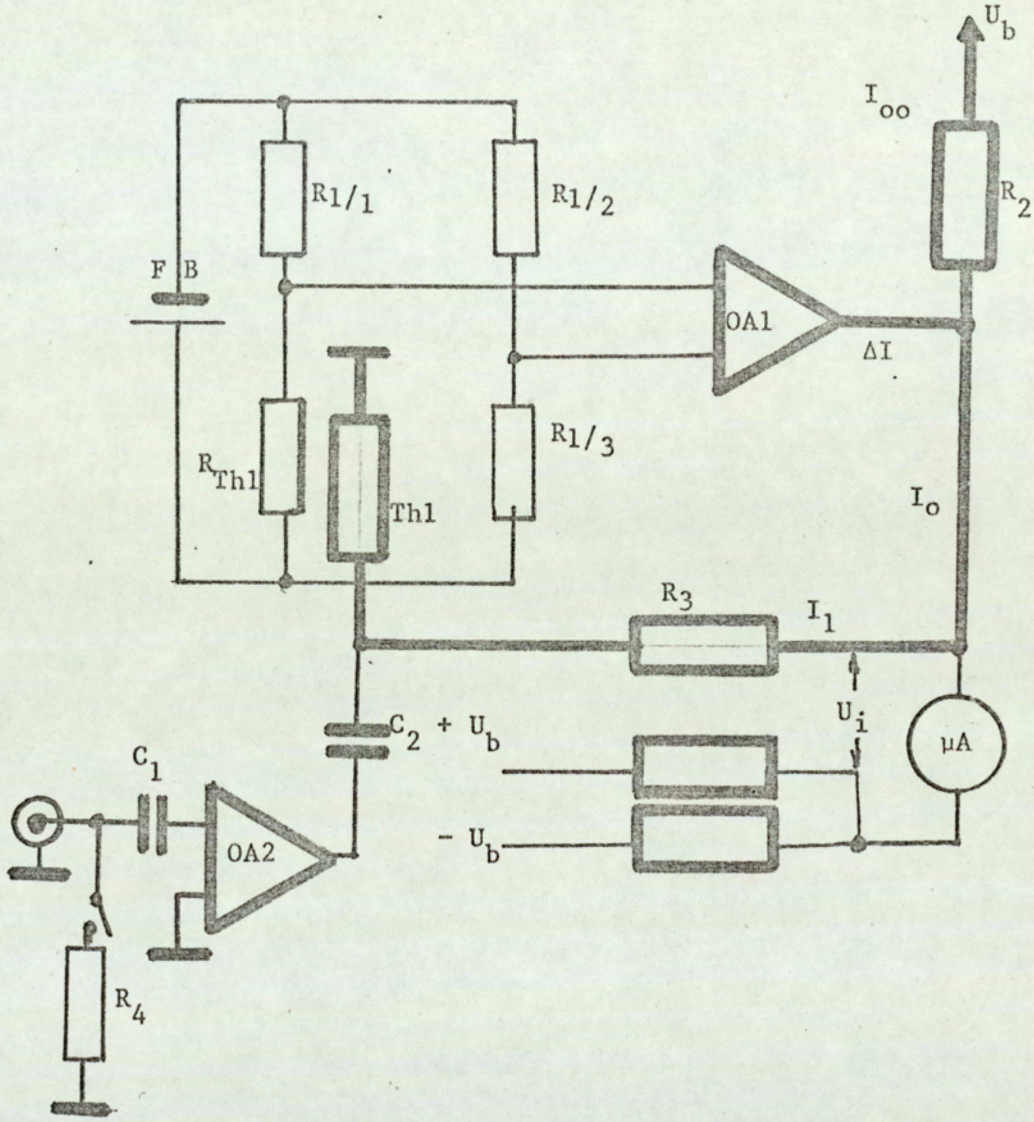


Fig.6.3

Initially the DC current through Th_1 is I_o. When an AC current is added, in order to keep the bridge in balance DC current is reduced to I_1 ($I_o - \Delta I$).

$$P_{acm} = E_x^2/4R_{th1}, \text{ for matched conditions} \quad 6.2.7$$

or $P_{acm} = E'^2_x/R_{th1}$, when the switch 1 is open, i.e.

where $P_{acm} = \sigma_x^2$ since $R_{th} = 1$. 6.2.8

If the dissipation in Th1 is increased, the temperature of R_{Th1} is increased and so the bridge will go out of balance. This will increase the current into OA1, thus decreasing the d.c. current until a new balance is reached, when the total dissipation due to a.c. and d.c. currents are almost exactly the same as originally. There will be a small error voltage, E_1, across the bridge, but this can be made as small as necessary by increasing the gain of the operational amplifier. The system is inherently stable being a second order servo with a dominant time constant defined by the thermistor itself. For the new balance

$$P_o = I_o^2 = (I_o - \Delta I)^2 + \sigma_x^2 \quad 6.2.9$$

$$\sigma_x^2 = 2I_o\Delta I - \Delta I^2 \quad 6.2.10$$

When

$$\Delta I << I_o \quad 6.2.11$$

$$\sigma_x^2 \approx 2I_o\Delta I \quad 6.2.12$$

It is possible to use this "mean power to d.c." converter even for $\Delta I_{max} = I_o/2$, when a scale calibration is necessary.

For a fixed value of σ_x^2 independently of the ratio S/N, the locus of the total root mean power dissipation will be the arc $A_2'B_2'$ in Fig.6.1

To indicate change in I_1 a sensitive high impedance microammeter should be inserted in the circuit, and balanced to zero.

6.3 Correlation Measurements using only d.c. feedback.

For detecting signal partially masked by noise Hathaway (1969) and Arthur (1968) used a mean square voltmeter almost identical with Bozic's in very similar arrangements of their circuits.

The measurements were based on the definition of the cross-correlation function

$$\psi_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int x(t).y(t + \tau)\, dt \qquad 6.3.1$$

where $x(t)$ and $y(t)$ are two records of the way a voltage (or current) has been varying with time, and also

$x(t)$ is the complex record of the signal and noise at the input to the measuring system after transmission through the physical system under investigation.

and $y(t + \tau)$ is a reference signal voltage delayed by the time τ.

Suppose that the deflection of the calibrated microammeter is exactly proportional to the mean square value of the applied voltage or current signal,

$$\theta_2 = \left[x(t) + y(t + \tau)\right]^2 = 2I_o.I_2 \qquad 6.3.2$$

$$\theta_1 = \left[x(t) + y(t + \tau)\right]^2 = 2I_o\ I_1 \qquad 6.3.3$$

$$\theta_2 - \theta_1 = 4\ \overline{x(t).y(t - \tau} = 4\tau_{xy}(\tau) \qquad 6.3.4$$

Since in the actual "correlator" system constructed there was no facility for varying τ, it was in fact not possible to measure the correlation function, but only the correlation coefficient $\psi_{xy}(0)$, which represents the coherent component of the signal partially masked by noise.

To carry out the operations indicated by equations 6.3.2 - 6.3.4 a double bridge shown in Fig.6.4 was constructed. This was almost the equivalent to duplicating the single bridge already described. Three more operational amplifiers are included, one being an inverter with gain

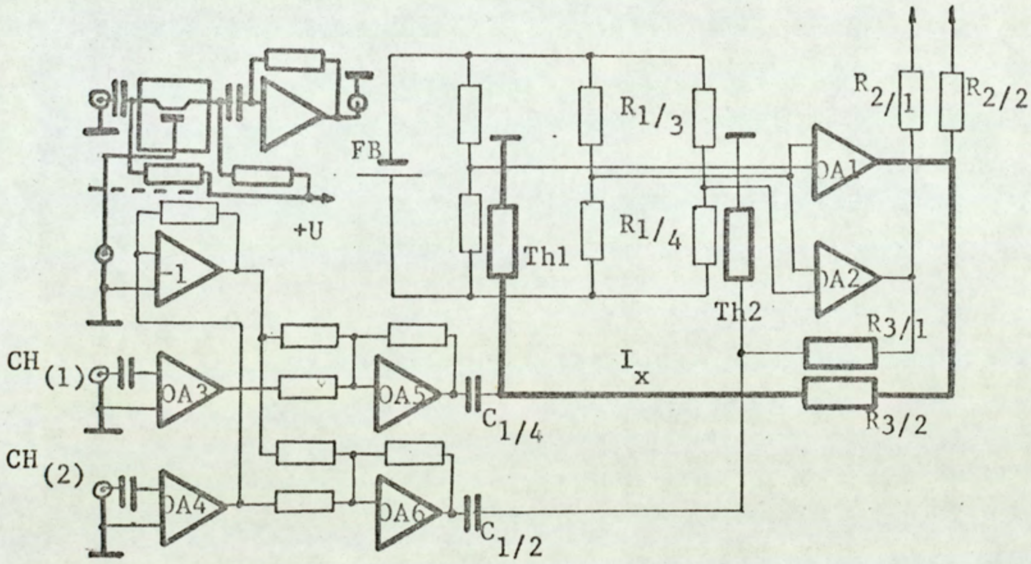


Fig.6.4a.

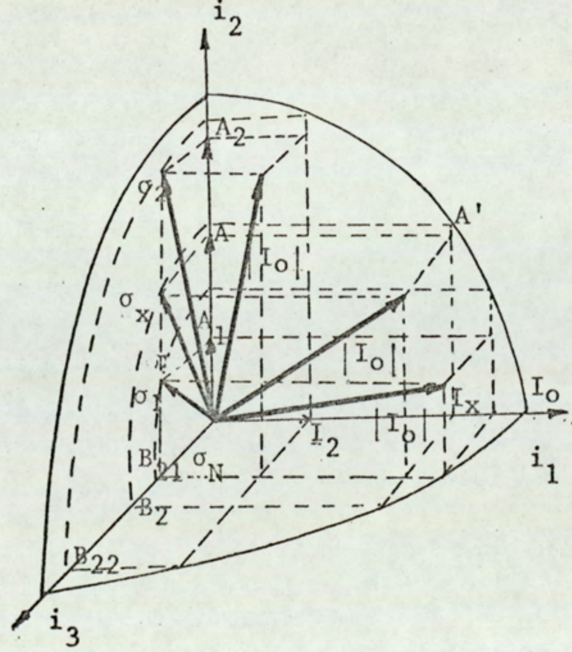


Fig.6.4b.

-1, and the others being two adders, marked OA5 and OA6. In principle this arrangement does not differ from the correlator used by Arthur. The aim was to improve the method by using sampling techniques which enabled $\overline{[x(t) + y(t+\tau)]^2}$ and $\overline{[x(t) - y(t+\tau)]^2}$ 6.3.5 to be measured simultaneously by switching a common amplifier channel several hundred times per second instead of perhaps once in from ten to twenty seconds manually as was done by Arthur and Hathaway. The sampling method has the further advantage that it was possible to check the zero balance by sampling the voltage of the common arm, $R_{1/3}$ and $R_{1/4}$ of the double bridge.

When x(t) is applied to Channel 1 input, there will result a current due to the signal content and the noise content of the output x(t). For the present discussion it is immaterial whether n(t) is an additive noise or is partially or completely dependent on s(t). The reference signal $y(t+\tau)$ is applied to Channel 2 input. In the thermistor heater Th1 there will be a current $i_r(t)$ and in the heater Th2 an inverted current of the same value, $-i_r(t)$.

Let us now use a similar three dimensional diagram, Fig.6.4b to that used previously to visualise better the relationships existing between the various currents involved.

If the reference signals in the two thermistor heaters had not been applied there would have been the same situation as in the previous case, from which we would find

$$\sigma_x^2 = 2I_o.\Delta I - (\Delta I)^2 \qquad 6.3.6$$

or in accordance with the notation used in Fig. 6.4 b

$$\sigma_x^2 = 2I_o(I_o - I_x) + (I_o - I_x)^2 \qquad 6.3.7$$

The total dissipation in each of the bridges would be I_o^2 if we take the thermistor heater resistances to be normalised, i.e. Th1 = Th2 = 1.

When the coherent reference signal current is added and after the bridges have reached a balance we have

$$I_2^2 + \overline{|s(t) + n(t) + i(t)|^2} = I_1^2 + \overline{|s(t) + n(t) - i(t)|^2}$$

$$\therefore I_2^2 + \overline{|S(t) + i(t)|^2} = I_1^2 + \overline{|S(t) - i(t)|^2} \qquad 6.3.8$$

$$\overline{4S(t).i(t)} = I_1^2 - I_2^2 \qquad 6.3.9$$

$$\text{or} \quad 4\psi_{xy}(0) = (I_1 - I_2)(I_1 + I_2) \qquad 6.3.10$$

By comparison with the approximate expression used by Arthur, Hathaway and Bozic

$$4\psi_{xy}(0) = 2I_o(\Delta I_2 - \Delta I_1) = 2I_o(I_1 - I_2) \qquad 6.3.11$$

it is obvious that the error in the expression they used is negligible only when

$$I_o >> \Delta I_1 \quad ; \quad I_o >> \Delta I_2 \qquad 6.3.12$$

$$\text{i.e. when} \quad I_1 + I_2 = 2I_o \qquad 6.3.13$$

Exact expressions could be written

$$4\psi_{xy}(0) = (I_o - \Delta I_1 + I_o - \Delta I_2)(I_1 - I_2)$$

$$= 2I_o(I_1 - I_2)\left(1 - \frac{\Delta I_1 + \Delta I_2}{2I_o}\right) \qquad 6.3.14$$

The relative percentage error is then seen to be

$$\varepsilon\,\% = \frac{\Delta I_1 + \Delta I_2}{2I_o} \cdot 100 \qquad 6.3.15$$

which decreases as the reference signal is reduced.

6.4 Mean power measurements on signals and noise using only a.c. feedback.

A simplified diagram of the arrangement to be discussed is shown in Fig.6.5. The essential feature is a double bridge arrangement, $R_{1/1}$, $R_{1/2}$, $R_{1/3}$, $R_{1/4}$, Rt1 and Rt2, with a supplementary comparison bridge $R_{3/1}$, $R_{3/2}$, $R_{3/3}$ and $R_{3/4}$. A comparator circuit, marked "Comp" is introduced as well as an electronically controlled attenuator, marked "var. att.", a compensating attenuator and a simple diode bridge rectifier with a mean current indicating micro-ammeter.

Two input circuits are provided, both with variable resistors for matching purposes if necessary. At Channel 1 input the unknown complicated voltage is applied. At Channel 2 a sinusoidal signal of constant amplitude and frequency is applied. Through the resistors R_3 a constant d.c. thermistor current is applied. When there is no signal applied, the d.c. currents through the thermistor heaters are adjusted so that both thermistors, Rt1 and Rt2 have equal resistances. For selected and initially matched thermistors, the d.c. currents through the two heaters should be almost the same when the double bridge is balanced. The auxiliary comparison bridge, formed of the resistances R_3 and its comparator circuit are so adjusted that the variable attenuator has a very high attenuation. The small leakage of comparison signal is compensated by applying the same amount of attenuated comparison signal to the non-inverting input of the same operational amplifier, OA6. When signal and noise is applied on Channel 1 input, the thermistor heater Th1 receives an increase in heating power, which unbalances the left hand portion of the double bridge. Consequently, the auxiliary bridge comparison circuit changes the bias on the F.E.T. in the variable attenuator and the comparison signal goes through OA6 to Th2 thereby increasing the dissipation in Th2.

When the balance has been restored there is almost the same dissipation in the two thermistor heaters, but both are higher than they were initially on account, not only of the signal and noise applied though Channel 1 input to Th1, but also on account of comparison signal applied through Channel 2 to Th2. For a comparison signal which is sinusoidal the mean effective value may be measured by the rectified current, i_c, in the instrument shown in Fig.6.5b.

The r.m.s. value is given by

$$I_c = I_{max}\sqrt{2}/2 = i_c\sqrt{2}/4 \qquad 6.4.1$$

The indication of the instrument can be directly calibrated using the relation

$$I_c \equiv 1.11\, i_c \qquad 6.4.2$$

If a calibrated vacuum tube voltmeter is used at the output of the amplifier no correction is necessary since such voltmeters are always calibrated to indicate the r.m.s. voltages assuming the input is sinusoidal. For measuring the input as a current it is necessary to know the transfer impedance of the amplifier, while to measure it as a voltage, it is necessary to know the voltage gain of the amplifier.

Fig.6.5a explains in more detail the relationships between the thermistor heater currents. At the commencement, a d.c. current in Th1 and Th2 is balanced to the value I_o, indicated by the vector OG. When the external signal is applied

$$o_x^2 + I_o^2 = I_1^2 \text{ (for normalised resistance Th1,} \qquad 6.4.3$$

and for the restored balance, the power in Th2 is

$$I_{ac}^2 + I_o^2 = I_1^2 \quad \text{when Th1 = Th2 = 1.} \quad \text{Therefore, numerically}$$

$$I_{ac} = \sigma_x \qquad 6.4.4$$

Here exist a linear relationship between I_{ac} and σ_x which removes the error due to the approximation which was necessary when d.c. feedback was

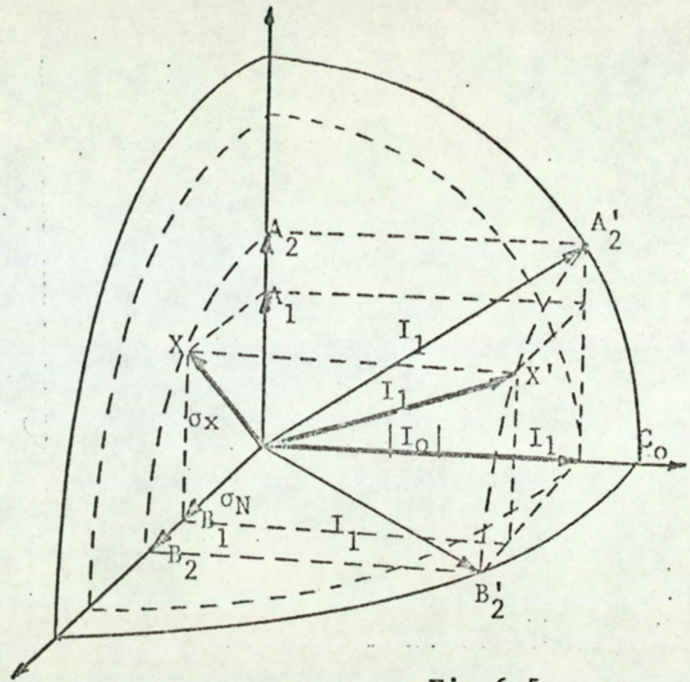


Fig.6.5a

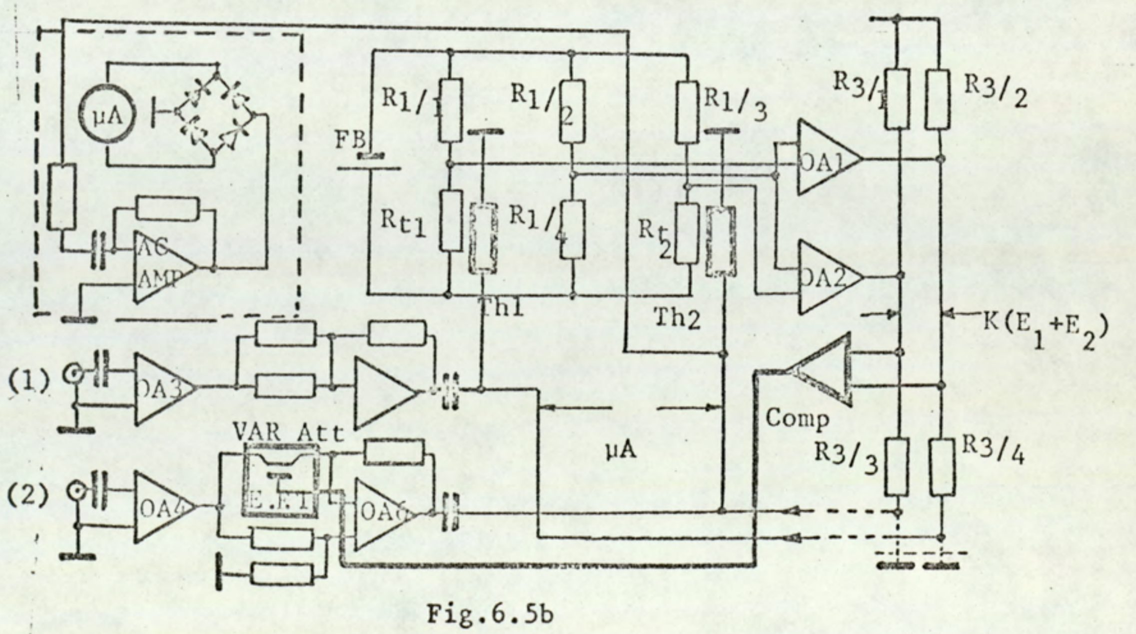


Fig.6.5b

considered in the last section.

The disadvantage of this arrangement is that the temperatures of the heaters and beads of the thermistors change, and if they are not exactly matched, an error will arise. (Appendix 5)

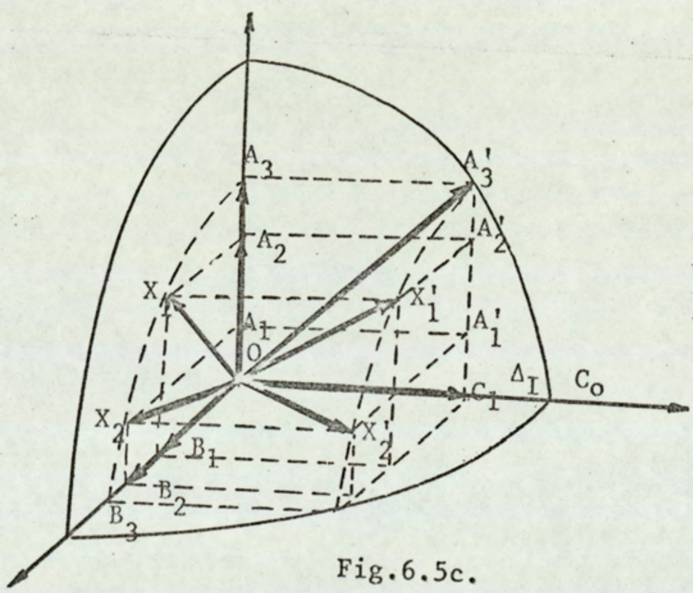


Fig.6.5c.

6.5 Mean power measurement using simultaneous d.c. and a.c. feedback.

In order to improve the method described in Section 6.4a.c. and d.c. feedback are combined, as shown in the diagram shown in Fig.6.5b This has been done simply by connecting the thermistor heaters Th1 and Th2 to the resistances $R_{3/3}$ and $R_{3/4}$ respectively. D.C. feedback here serves no direct measuring purpose, but to stabilise the temperatures of the thermistor beads. Measurement of unknown r.m.s. values is carried out by measuring the mean current or voltage due to a pure sinsoidal comparison current, or voltage.

In Fig.6.5c it is clear that on account of the increased total dissipation, the locus of points in the r.m. space will be X' for Th1 and A_2' for Th2. For a given mean power input, independently of the signal to noise ratio, the vector representing the operating point moves in the arc $A'_2 \rightarrow B'_2$.

An increase in the temperature of Th1 and Th2 will unbalance both arms Rt1 and Rt2 by the same amount in reference to the common comparison arm, $R_{1/3}$ and $R_{1/4}$. In order to restore the initial thermal conditions, both operational amplifiers OA1 and OA2 will take more current thereby decreasing the d.c. current in Th1 and Th2 from I_o to $(I_o - \Delta I)$. The thermistor heaters will then attain their initial temperatures. Using Fig. 6.5 it will be seen that the power relationships may be written as

$$I_o^2 = (I_o - \Delta I)^2 + \sigma_x^2, \text{ for Th1, and} \qquad 6.5.1$$

$$I_o^2 = (I_o - \Delta I)^2 + I_c^2, \text{ for Th2, and}$$

$$I_c = \underline{OA_2} \text{ in Fig.6.5c.} \qquad 6.5.2$$

By inspection we find

$$I_c = \sigma_x = (2I_o.\Delta I - (\Delta I^2)^{\frac{1}{2}} \qquad 6.5.3$$

which shows that σ_x could be estimated also by measuing ΔI as was described in Section 6.2, but with the inherent drift error when d.c. feedback only is used and the non-linear dependence of σ_x^2 on ΔI.

The corresponding constant mean power locus for combined a.c. and d.c. feedback is on the arc $A'_3 - B'_3$ which is confined to the spherical surface having $(P_{tot})^{\frac{1}{2}} = I_o = OC_o$ in Fig.6.5c when the thermistor resistances Th1 and Th2 are normalised to unity.

6.6 Measuring the r.m.s. signal value by the use of correlation.

For the measurement of signals by the use of correlation it is necessary to have a sample or reference signal of the same frequency and in the same phase as the transmitted signal, and of constant amplitude. It is evident that these conditions cannot be met if the signal is to be used for sending information, since the signal or a replica of it is supposed to be available at the receiver, in a form which has not been transmitted through the system under investigation. In these circumstances, however, the use of correlation serves as a guide as to the best results which can be obtained on transmitting signals through the system, and the best way of detecting it. The coherence between the reference signal and the received signal can be used to detect sinusoidal or discontinuous components of any periodic or other function masked by noise.

When a signal is transmitted through a system or medium which not only noise of the well known shot and thermal types, but also fluctuating parameters, it is distorted by the fluctuating parameters both in amplitude and phase. The condition that the reference signal must be in phase with the received signals which is to be measured is, in fact, never completely fulfilled. When the phase of the received signal is a slowly varying function of time, some locking systems to be described later can be devised which enable a very high degree of coherence to be obtained.

If the phase has random shifts, i.e. jitter, which are very fast and random, then coherence may be obtainable only over the central part of the band of frequencies transmitted.

The following considerations treat any component of the received signal and noise mixture which is coherent with the reference signal as being signal, and the remainder as noise. That is, no distinction will be made between noise which is inherent in the transmitting system and which is added to the signal from noise which has been made evident by the application of the signal or which has been produced by conversion of some of the signal power into noise once it has been received. The way in which the system modifies the signal can be determined only from a study of the effects of various modulating and detecting methods in modifying the signal to noise ratio finally detected by using correlation.

To discuss the principles of coherent detection and measurement consider at first only two arms of the double bridge as shown in Fig. 6.6a As was done previously, consider the thermistor heater resistances to be normalised so that Th1 = Th2 = 1. For calculations of actual powers it is necessary to use the actual resistance values, which have been set at 100 ohms.

On account of the initial d.c. current define the initial power dissipation in Th1 and Th2 by

$$P_{dc} = Th(1 \text{ or } 2).I_o^2 = I_o^2 \text{ when } Th1 = Th2 = 1. \qquad 6.5.1$$

It is permissible to have different powers from P_{dc} in either of the thermistor heaters so long as the total power is less than the maximum permitted dissipation in the heaters, the change in power being due to the effect of additional voltages and currents caused to flow when the bridge is subjected to received signal, noise and feedback.

I_r is represented by OA_1 along the axis for $s^{\frac{1}{2}}$, where s is the signal power present,

I_1 is the new d.c. current value in each thermistor heater when the same value of I_r, the reference signal is applied to both heaters simultaneously.

Then $I_o^2 = I_1^2 + I_r^2$ 6.5.2

and this equation defines an operating locus represented by the points

A_1^{I}, A_1^{II}, A_1^{III} and A_1^{IV} (See p. 122)

Denote the applied signal current by s(t) and the noise current by n(t). Apply (s(t) + n(t)) to Th1 and invert them so as to apply -(s(t) + n(t)) to Th2, omitting the time variable,

$$\overline{(I_{r1} + s + n)^2} + I_2^2 = I_o^2 \quad \text{and} \qquad 6.5.3$$

$$(I_{r2} - s - n)^2 + I_2^2 = I_o^2 \qquad 6.5.4$$

I_r has changed to two new values

$$I_{r1} = \sqrt{\overline{i_{r1}^2}} \quad ; \quad I_{r2} = \sqrt{\overline{i_{r2}^2}} \qquad 6.5.5$$

which are seen in Fig.5b to be represented by the vectors OA_4 and OA_3. Further manipulation leads to

$$\overline{(i_{r1} + s)^2} = \overline{(i_{r2} - s)^2} \qquad 6.5.6$$

or

$$\sqrt{\overline{s^2}} = \sigma_s = (I_{r2} - I_{r1})/2 \qquad 6.5.7$$

This is a linear relationship between the effective signal amplitude and the difference between two currents.

Measuring sinusoidal signals, and writing

$$\bar{\bar{s}} = s \; \frac{\overline{\bar{i}_{r1} - \bar{i}_{r2}}}{2} \qquad 6.5.8$$

since there is a linear relationship

$$\overline{\overline{s^2}} = (\pi/2^{\frac{1}{2}}). \; \bar{\bar{s}} \qquad 6.5.9$$

where $\bar{\bar{s}}$ defines the positive average value of the short term averages, $\bar{s}$, introduced by Arthur. The same argument leading to the measurement of rectified mean currents as a measure of r.m.s. or effective values applies here as before.

The last expression is true for a stationary random process when the time constant of the averaging circuit is longer than the inverse of the lowest frequency component of its power spectrum. For non-stationary random processes, low frequency components are usually dominant, and it is not possible to make time constants long enough to enable measurements of averages in a finite or reasonable time. In fact, it may be inferred, or even was demonstrated by Bull in developing a theory of the flicker effect, that the mean square voltage or current fluctuation increases proportionally to the time constant of the circuits, and that the flicker effect is an effect brought about by the fluctuations in the characteristics or parameters of the system. Consequently it is not possible to improve the stability of average values merely by increasing the time constants.

For stationary signals buried in noise it is quite irrelevant in the measurements just described whether the signal and noise is inverted and keep the same phase for the reference signal, or whether the reference signal is inverted to hold the received signal and noise in the same phase. For achieving a high precision in measurement it is probably better to invert the reference signal, since this will usually be at a single frequency and more likely to have a more stable phase and amplitude that has the signal and noise. Consequently, the inversion operation can be better carried out and tested for accuracy and stability on the reference signal than on the noise. Furthermore, inversion of the reference signal leads to at least partial suppression of the signal, resulting in the possibility that measurements of very low flicker effect noise in the vicinity of and due to the signal itself could be made.

If we apply two sinusoidal signals of equal magnitude and the same frequency they will cause equal d.c. current changes through both the thermistor heaters, irrespective of any relative phase differences between them. This opens up other possibilities.

The frequency of the reference signal must be equal to the mean frequency of the incoming signal which is to be measured. Since the communication system we shall describe may be capable of causing noticeable phase shifts in the received signal, in the measurements to be described, therefore, it will be necessary to distinguish two cases, one in which we are interested to measure the total effect of noise and phase shift on the received signal and the other in which we wish to separate out the effect of phase shifts and jitter in the transmission path. For the former the reference signal will either be a replica of the transmitted signal or a system having long time constant and automatic phase control. For the latter the reference signal will be made to track the incoming signal as quickly as possible by having phase control circuits having short time constants.

In either case the reference signal is applied to Channel 2 in Fig.6.6 i.e to the operational amplifier OA4, which has high input impedance and low output impedance. Switches S2 should be closed and resistance $R_{4/2}$ adjusted to meet matching conditions. Since OA4 has unity voltage gain, a reference signal of the same amplitude is applied to both the summing amplifiers OA7 and OA8, the phase of the output from OA7 is inverted, so that an inverted reference signal is applied to the thermistor heater Th1.

At the same time an attenuated reference signal is applied by the electronically controlled attenuator to the input of operational amplifier OA6. A signal of the same size is also applied to the non-inverting input through a manually adjustable attenuator.

At the input of OA6 there will be a hardly measureable amount of reference signal, and the indicating micro-ammeter, Inst, will show zero deflection. It is obvious that the accuracy of this method of measuring the coherence between signals depends critically on the common mode rejection of the amplifier OA6. This should be of the order of 60 db or higher. Since in this amplifier only a single frequency is concerned, it is necessary to obtain such accurate balance only at one frequency, and this condition is not difficult to fulfil. At both the input and the output of OA5 there will also be no discernible signal.

If we apply at Channel 1 a transmitted signal with the accompanying noise, it will be transferred via the amplifiers OA5 and OA7 and OA8 to the heaters Th1 and Th2 in phase. The first result will be that the double bridge and the comparison bridge will go out of balance, and this will be sensed by the comparator circuit, marked Comp. The bias on the FET in the electronically controlled attenuator, marked Var will be changed, and at the input of amplifier OA5 a reference signal, in opposite phase from that of the incoming signal, will appear. Its magnitude will increase until the unbalance caused by the incoming signal is corrected. This will happen when the reference signal from A_- is very nearly equal in magnitude to the portion of the incoming signal which is coherent with it.

There is then a very compressed signal, formed of the difference between the reference signal and the part of the signal which is coherent with it, together with noise. The noise consists partly of white noise filtered by the limited band of the system together with flicker type noise due to the transmitted signal and the d.c. excitation of the circuits. On account of the very strong feedback in the amplifier OA5 any noise generated in it could be neglected for a very wide range of levels of measured noise. This is discussed fully in App.14.

Since the regulated reference and the coherent part of the measured or incoming signal have very nearly equal magnitudes, rectified mean values and r.m.s. values, we can measure the mean value of the rectified, noiseless pure sinusoidal regulated reference signal with an instrument calibrated in r.m.s. voltages, thereby determining the coherent part of the incoming, masked signal.

At the output of OA5 where will appear the total noise and a very much reduced, and in many cases negligible amount of incoming signal.

This noise could be measured using the bridge with combined d.c. and a.c. feedback described above for measuring the mean power of (signal and noise, but in this case, the signal will usually be negligible. To eliminate the remaining, unsuppressed, part of the signal, a preliminary calibration of the equipment with a signal almost free from noise would be necessary.

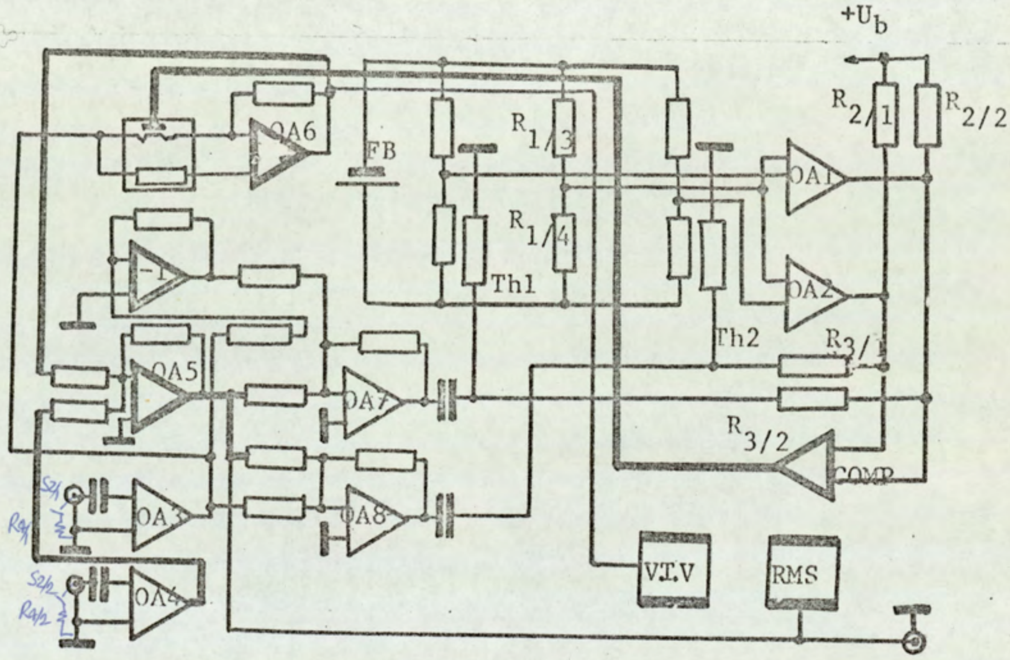


Fig.6.6a

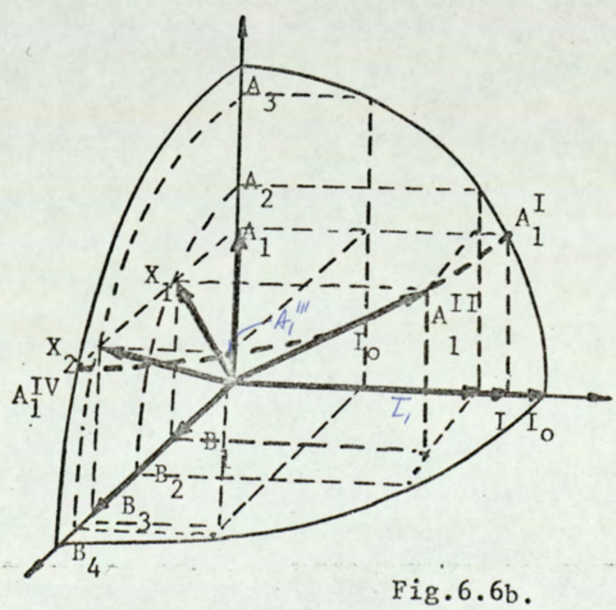


Fig.6.6b.

CHAPTER 7

Detection of Signal Fluctuation

7.1 Short Term Shifting Interval Correlator

This equipment was primarily developed in order to provide a means of determining the correlation between two voltages or currents over short periods of time, i.e. with short averaging times, for signals up to several hundres kc/s. The values of correlation coefficient so determined will be called the "short term correlation" factors. For a rapidly fluctuating signal these factors are fluctuating rapidly too, so that an analog continuous recording has to be provided.

The short term interval correlator consists of a buffer stage, which is an amplifier and an impedance transformer, a phase comparator, which enables locking to the incoming reference signal, and an RC-FET voltage controlled Wien Bridge oscillator and a shifting interval coherent detector.

The synchronism was not very good, when a replica of the transmitted 4Mc/s signal was used to synchronise local oscillator. This was due, as it was later experimentally found, to the lack of coherence in phase fluctuations of signals. It was observed that the oscillator locks easily to the received 200 kc/s reference signal in the case when the received 4 Mc/s carrier, after restoring the original phase and after amplitude fluctuation suppression (by using FAAGC and limiting) was used for second detection (i.e. mixing).

When intense fluctuations of signals were experienced, it was not at all possible to lock the local 200 kc/s oscillator on the received reference signal without a limiting stage for the 4 Mc/s reference signal, or rapid acting A.G.C. or both.

For simultaneous recording phase and magnitude fluctuations of the reference signal, two separate bridge multipliers, using 4 diodes each, were used for orthogonal coherent detection.

Before proceeding with the description of the equipment, let us explain the basic idea.

Mathematically the average value of a non-periodic function defined by

$$\overline{y(t)} = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} y(t)\, dt \qquad 7.1.1$$

cannot be measured since the definition implies that it is averaged over an infinite time. Since the function is non-periodic, there can be no certainty over a finite time that the function will not at some moment attain such a large value that the average value changes noticeably at that moment.

On the other hand, the average defined for a periodic function by the same equation will have a regularly decreasing variation as the time limits, T_1 and T_2, over which integration is carried out are varied so that the averaging is done over a regularly increasing period $(T_1 - T_2)$. Whenever the averaging time $(T_1 - T_2)$ is equal to an integral number of periods of the function the average has a definite value which is independent of the phase at which the time T_1 is taken.

For a non-periodic function which lasts for a limited time, the usual approach in mathematics is to define it as a truncated periodic function and find its average value for the duration. The time average values for a function of limited duration tends to zero as the time T tends to infinity for all well behaved functions.

For functions which last an unlimited time it is possible to find time averages in finite time intervals, but the averages are in general functions of time. A similar method could be applied to functions of limited duration by dividing the total duration of the function, T, into smaller observation intervals, ΔT, such that

$$N . \Delta T = T \qquad 7.1.2$$

By increasing N the observation intervals, ΔT will become smaller and smaller. If the process described by the time function is stationary, averaging over different time intervals will give almost the same result for any relatively small value of N. By increasing N towards high values, the time intervals ΔT will tend to become infinitessimal increments, and the average value taken over each element ΔT will approach the value of the function during that interval, i.e.

$$\lim_{(T_1 - T_2) \to dt} \left[\overline{y(t_1)} \right] = \lim_{(T_1 - T_2) \to dt} \left[\frac{1}{T_1 - T_2} \int_{T_1}^{T_2} y(t)\, dt \right]$$

$$= y(t_1) \qquad \text{where } (T_1 + T_2)/2 = t_1$$

If the time function, when transformed into the frequency domain, has a limited spectrum of frequencies, so that $f_{max} \leqq B$, then in order to find the function values $y(t_i)$ for a specified time t_i, without ambiguity, it is necessary to find its short term average in the small time interval

$$(t_i - \frac{\Delta T}{2} - (t_i + \frac{\Delta T}{2}) \quad \text{where} \quad \Delta T = 1/2B \qquad 7.1.4$$

Thus,

$$y(t_i) = \overline{\overline{y}}(t_i) = \frac{1}{2B} \int y(t)\, dt \qquad 7.1.5$$

This equation being valid whether the time function is deterministic in the full sense or taken from a set of sample functions which describe a band limited random function.

7.2 The detection of signals by the use of short term correlation.

Suppose that $y(t)$ is a signal and that we are interested in the determination of successive values of $y(t_i)$, $i = 0, 1, 2, \ldots N$. We can do this by considering the results of using an ideal averaging filter having a response defined by (Fig. 7.1)

$$\begin{aligned} h(t_i) &= 1 \quad \text{for the period from } (t_i - \Delta T) \text{ to } t_i \\ &= 0 \quad \text{otherwise outside the } i^{th} \text{ interval } T. \end{aligned} \qquad 7.2.1$$

The filter time response to a set of impulses $y(t_i)$ will be a staircase function. For a continuous function $y(t)$ at the input, the output will be continuous function also. Let us denote the output time function by $z(t)$, defined by the Green or Duhamel superposition or convolution integral

$$z(t) = \frac{1}{\Delta T} \int_{t-\Delta T}^{t} h(t-\lambda).y(\lambda)d\lambda \qquad 7.2.2$$

where λ is a dummy variable of integration. Thus the averaging filter performs an averaging operation for the immediately past period ΔT. As will be seen later, this averaging filter is useful either in detecting small signals masked in white noise or to find the successive values of say $y_1(t)$, which may be for example the amplitude of an unmodulated carrier which is being transmitted through a physical system which has time varying parameters.

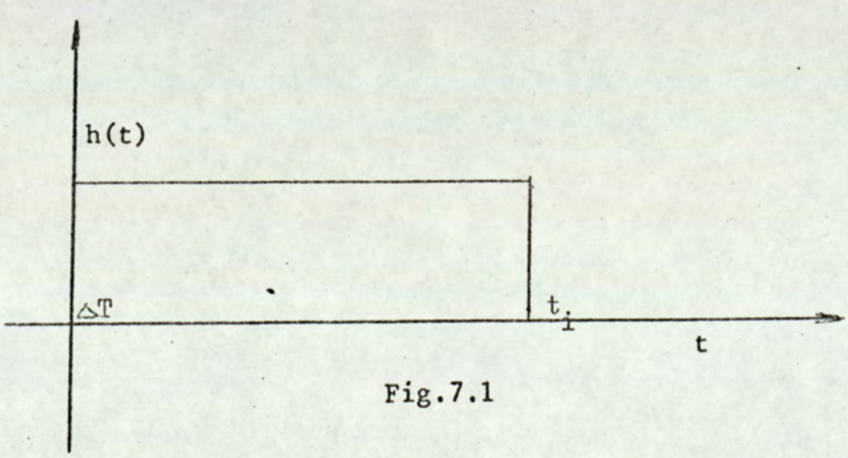


Fig.7.1

Description of the short term detector

The sinusoidal output from the 200 kc/s locked oscillator was applied to a Schmidt trigger circuit, and the resulting rectangular output pulses differentiated, so that suitable pulses for triggering were obtained.

These pulses were used to trigger a chain of three flip-flops, the outputs of which were so combined through a diode matrix, that eight gates could be operated in succession and in strong synchronism with either received or transmitted signal. Each gate was open for an interval of half of the signal cycle, thus enabling half cycles of the fluctuating signal to be separated and addressed to one of the eight "hold" circuits. Each hold circuit comprises a low loss mica capacitor, which was charged through a high impedance circuit. Therefore, at the end of the half cycle the voltage across its terminals was proportional to the area under particular half cycle portion of the test signal. During each half cycle, one of the eight capacitors is completely discharged, so that the new charge gives the information on the next corresponding half cycle signal intensity. Locking the lockal 200 kc/s oscillator to the transmitted 200 kc/s signal, and adjusting them to be orthogonal, the same equipment was also used as the phase fluctuation detector.

In the first case, when amplitude fluctuation was of interest, the total charge on "hold" capacitors is proportional to the area of the chopped and rectified received signal for that interval. Using a summing operation al amplifier, an output is obtained, which after some smoothing gives a measure of the short term average of signal amplitudes, i.e. of their fluctuations. All incoherent signals and additive noise are to some extent suppressed. This suppression is evidently more effective for unwanted signals and noise components whose frequencies are somewhat separated from the test signal frequency.

In the second case, when a phase fluctuation was of interest, the total charge on "hold" capacitors varied proportionally to the mean of the phase difference between the transmitted and received signals, being less sensitive to the fast phase jitter due to additive noise.

Due to the noticeable delay which the hold capacitors introduce, this system was not found to be suitable for either rapid acting A.G.C. or rapid acting A.P.C. applications, as it causes instability, even for moderate loop gains. However, for measurements and recordings of fluctuations, being less sensitive to additive perturbations of any kind, it was found to be very useful.

In the next few pages the properties and mechanism of the shifting interval correlator will be described in more detail. It should be noticed that coherent detection improves the signal to noise ratio, suppressing additive noises, but in contrast to ordinary envelope detection it is very sensitive to phase fluctuations.

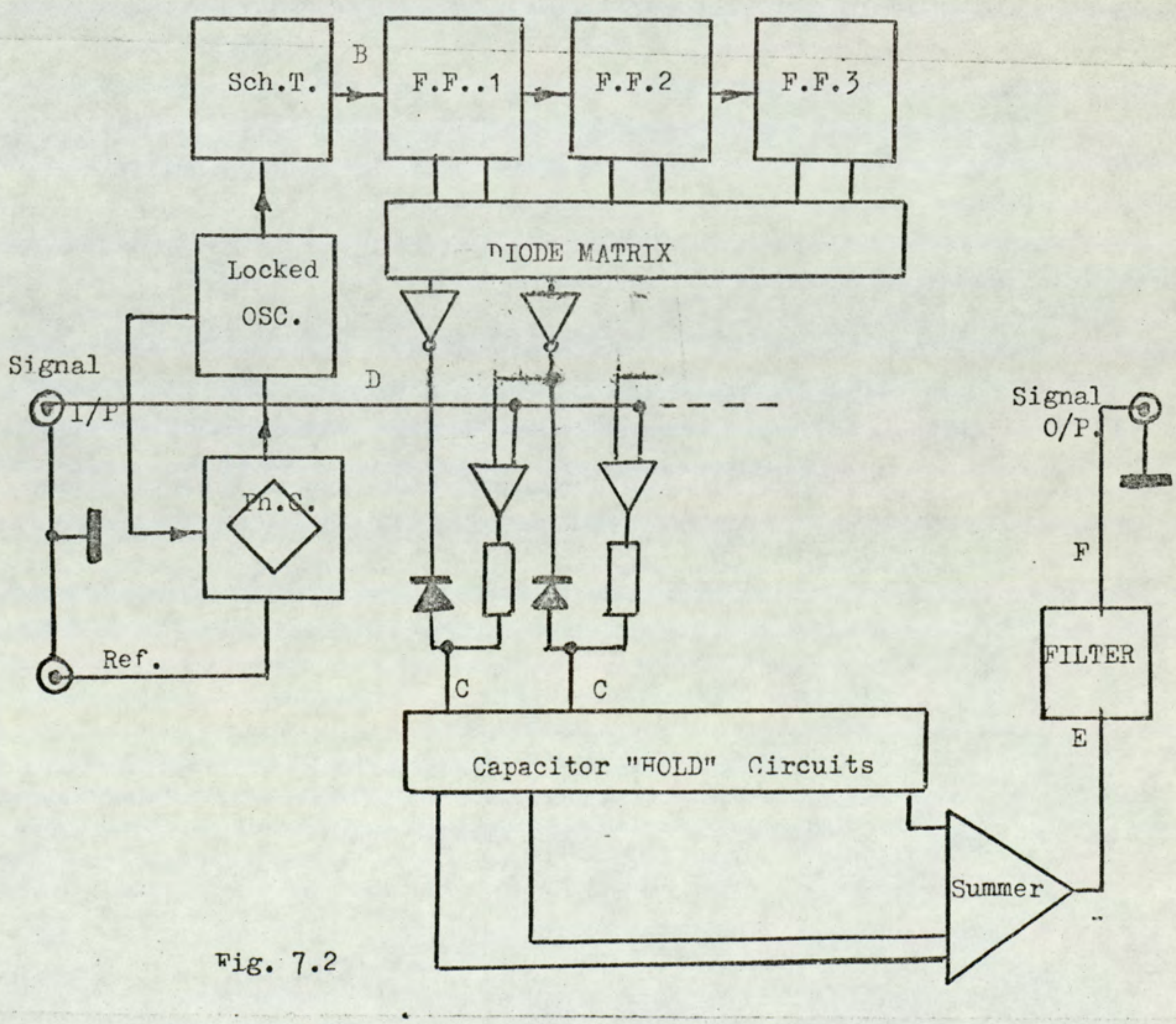


Fig. 7.2

SHIFT INTERVAL CORRELATOR WAVESHAPES.

Integrator outputs when no signal and noise is present

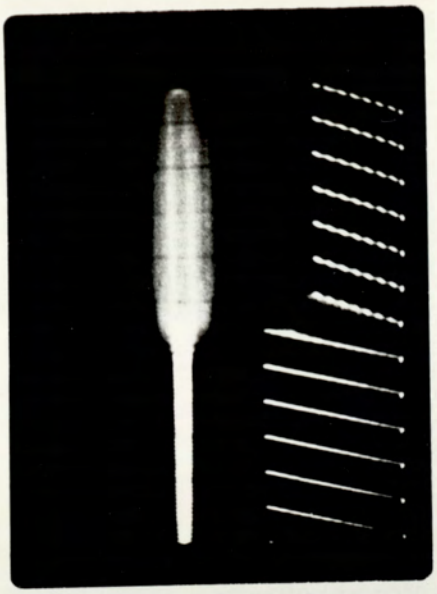
Demonstration of disturbance influence on integrator action

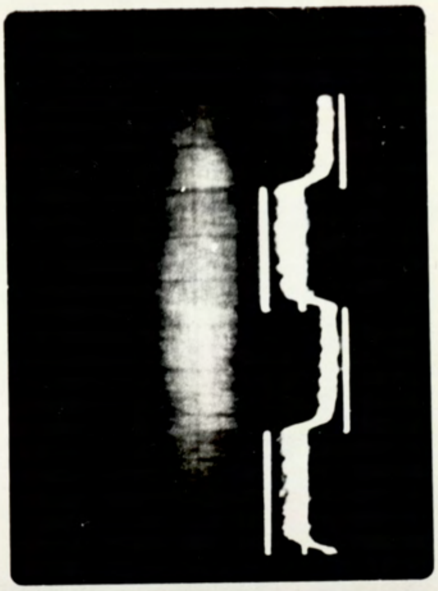
Detection of signal amplitude fluctuation (one integrator only)

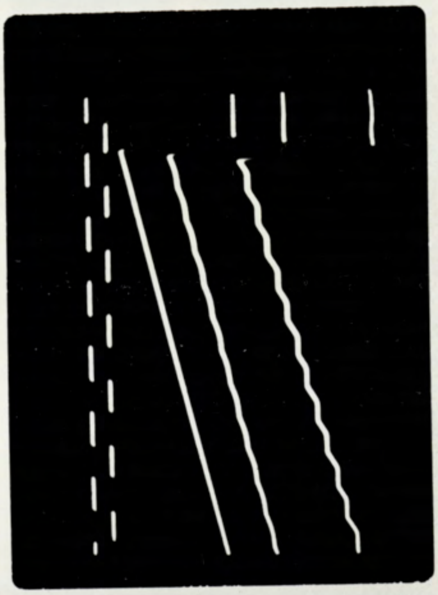
Detection of an amplitude modulated signal(one integrator output only)

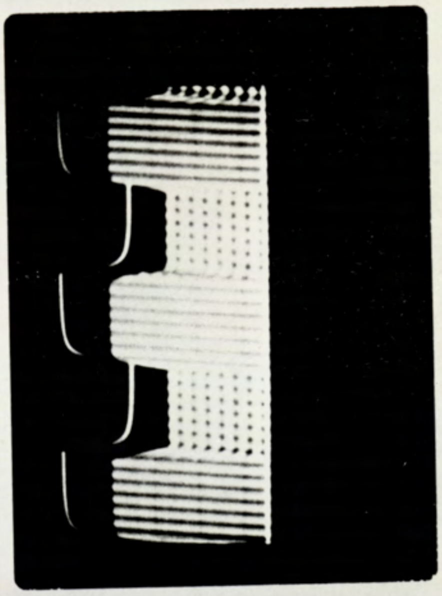
Detection of an amplitude modulated signal(all integrators)

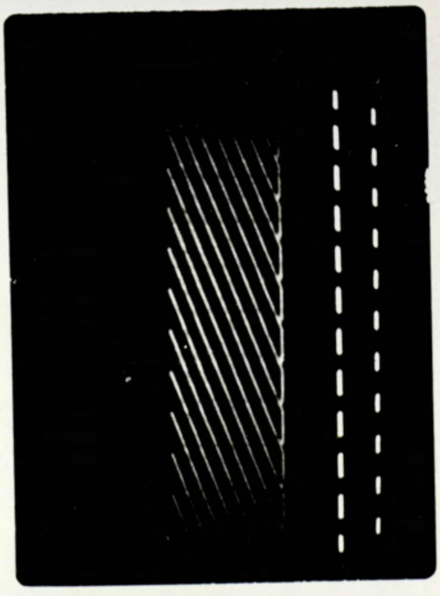
Detection of an amplitude modulatef signal burried in aditive noise.

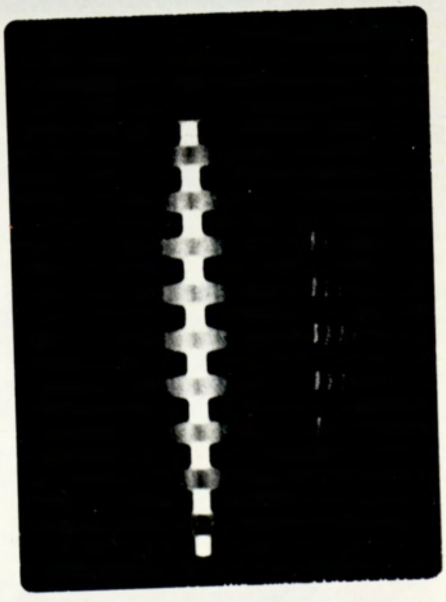












7.3 Short term coherence by synchronous sampling.

Let us divide the incoming signal up into a number of separate portions each of which starts when the carrier signal crosses the time axis, and also each portion has a width

$$\frac{T_s}{2} = \frac{1}{2f_s} = \frac{1}{2f_c} \qquad 7.3.1$$

where f_c = the carrier frequency (as above), and f_s = the frequency of the occurrence of the portions of the signal, i.e. the frequency of sampling.

Thus dividing incoming signal into a set of identical positive and negative carrier half-cycles of identical duration but not necessarily of the same amplitude, during a time T there will occur

$$N = 2Tf_s = 2Tf_c \qquad 7.3.2$$

of such portions. It is possible to ensure that such portions can be obtained by locking the sampling mechanism to the carrier.

In order to detect the information, which is conveyed by the variations in the amplitude modulation, it is necessary to use only either the positive or the negative half-cycles or to invert either set and add the un-inverted portions to them.

On the other hand, the mean value of a modulated carrier during half-cycle starting at the time t_i

$$\bar{s}_i = \bar{s}_{ci} = \frac{2}{T}\int_{t_i - \frac{T_s}{4}}^{t_i + \frac{T_s}{4}} \sqrt{2}\, s_c \left[1 + 2 \sum_{n=1}^{m} a_n \cos(n\omega_n t_i + \theta_n)\right] . \cos\omega_c t \, dt$$

$$= \frac{2\sqrt{2}}{T_s \pi} . s_c \left[1 + 2 \sum_{n=1}^{m} a_n \cos(n\omega_n t_i + \theta_n)\right] \qquad 7.3.3$$

where $\bar{s}_i$ is the short time average of the signal.

This shows that the output is proportional to the information signal amplitude at time t_i.

Taking the average of several demodulated signal carrier averages the short term information signal average can be written

$$\bar{s} = \sum_{i=1}^{N} \bar{s}_{ci} = \sum_{i=\mathrm{odd}} \overline{(+)\,S}_{ci} + \sum_{i=\mathrm{even}} \overline{(-)\,S}_{cn} \qquad 7.3.4$$

where $\overline{(+)s}_{ci}$ are the averages of the modulated carrier amplitudes for positive half cycles, while

$\overline{(-)s}_{ci}$ are the average values of the half cycles of the modulated carrier for the negative half cycles.

A sequence of half cycles carrier averages may be expressed as a set of values written in a matrix form,

$$S(t) = \{s_1, s_2, s_3, \ldots\ s_N\}^T \qquad 7.3.5$$

where the column matrix is represented as the transpose of a row matrix. The shifting or running short term averages of the information signal may then be written as the product of two matrices,

$$\bar{S}(t) = \frac{1}{K} \begin{array}{c} \uparrow \\ K \\ \downarrow \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \left\{ \begin{array}{cccccccc} 1 & 0 & 0 & & & & & \\ 1 & 1 & 0 & & & & & \\ 1 & 1 & 1 & & & & & \\ 0 & 1 & 1 & & & & & \\ 0 & 0 & 1 & 1 & 0 & 0 & & \\ & & & 1 & 1 & 0 & & \\ & & & 1 & 1 & 1 & 0 & 0 \\ & & & 0 & 1 & 1 & 1 & 0 \\ & & & 0 & 0 & 1 & 1 & 1 \end{array} \right\} \{s_i\} \qquad 7.3.6$$

where the first matrix of $[N \times (N + K)]$ order represents integration of K half cycles of the modulated carrier.

If the signal arrives with an additive white noise which is sampled and averaged in intervals 0 to $T_s/2$ and is denoted by the quantities n_i, then the sequence can be represented in matrix form by

$$N(t) = \{n_i\} = \{n_1, n_2, \ldots\ldots\ n_N\}^T \qquad 7.3.7$$

The noise samples n_i, being the results of integrating over finite intervals of time will have a mean value which will be much smaller than the standard deviation. The short term averages of the noise samples will tend to even smaller values, so that the signal to noise ratio will be improved proportionally to the square root of the number of samples taken, as will be seen from Section equation.

The number of shifting samples to be averaged cannot be increased without limit if we wish to detect the most rapid changes in the information signal, since the shifting or running average will smooth out the rates of change of signal modulation. Thus for information signals having a maximum frequency

$$f_{max} = B \quad \text{with} \quad T = 1/2B \qquad 7.3.8$$

and the carrier frequency f_c, the maximum value for K is, if no information is to be lost in the formation of the shifting averages,

$$K_{max} = T/T_s = f_c/f_{max} \qquad 7.3.9$$

Thus, having a carrier frequency of 4M Hz, and f_{max} = 250 k Hz,

$$K_{max} = 4 \times 10^6/0.25 \times 10^6 = 16 \qquad 7.3.10$$

The expected improvement in noise to signal ratio will then be four times in amplitude, i.e. 6 db. This, together with the 3 db obtained by the use of coherent detection will give a total improvement of 9 db.

The same method can be used to detect the random fluctuations of an incoming signal which has become fluctuating on account of fluctuation in the parameters of the transmission system of medium. This is one of the main objects of the experiments which have been carried out for this thesis.

The power density frequency spectrum has already been shown (Bull and Bozic, 1967) to have its maximum values close to the signal frequency, and under the conditions used in their experiments, was shown to fall off at a rate of about 6 db per octave of the difference frequency, $\Delta f = |(f_s - f_{meas})|$, where

f_s is the signal frequency and

f_{meas} is the frequency at which noise measurements are made.

In Bull and Bozic's experiments the frequency analyser used had a bandwidth of about 4 kHz and so it was not possible to make noise measurements at frequencies closer than about 15 kHz to the signal. For a deterministic signal of 200 kHZ it is therefore possible only to take about 12 samples in such an investigation if it were repeated using a sampling method.

Since the short term coherence measurement improves the signal to noise ratio for signals buried in additive noise and helps to detect noise due to fluctuations in the parameters of the system, it has been used extensively in this research,

(1) to measure the mean signal level of the fluctuating signal, and

(2) in connection with a fast acting automatic gain control, A.G.C., and automatic frequency control, A.F.C. to extract the remaining fluctuation in order to be recorded and analysed.

7.4 Broad-band coherent detection

For the coherent detection of a signal consisting of a constant amplitude of a sinusoidal voltage it is necessary, as has been already seen in Section 2.4 to have a reference signal of the correct phase and frequency.

If S is the r.m.s value of the signal of constant amplitude then the rectified short term average is

$$\bar{S} = \frac{2\sqrt{2}}{\pi} . S_{eff} = 0.9 \, S_{eff} \qquad 7.4.1$$

By taking 2BT samples we can improve the signal to noise ratio, S/N, very greatly. It must be borne in mind that this signal does not require any bandwidth for its transmission, so that the observation time, T, can be chosen arbitrarily long. (Only one bit of information) It cannot transmit information. Amplitude modulated signals have a frequency spectrum spread out over a band up to ±B, where B corresponds to the highest modulation frequency.

When white noise accompanies amplitude modulated signal, the input to the coherent detector may be written

$$e(t) = E_c \left[1 + m(t)\right] \cos(\omega_c t + \phi_c) + r(t)\cos(\omega_c t + \phi_r(t)) \qquad 7.4.2$$

This signal is multiplied by the reference signal

$$E_{ref} = E_o \cos(\omega_c t + \phi_o) \qquad 7.4.3$$

The second term in 7.4.2 having a randomly varying amplitude and phase is shown by Rice (1948) to be equivalent to white noise.

The maximum value of the detected baseband signal occurs for $\phi_c = \phi_o$ and is

$$e_{BS} = \overline{E_c E_o m(t) \cos^2 \omega_c t} = \frac{E_c E_o}{2} m(t) \qquad 7.4.4$$

The baseband output due to noise is accordingly

$$e_{BN} = \tfrac{1}{2} r(t) E_o \cos(\phi_r(t) - \phi_o) \qquad 7.4.5$$

so that the mean square noise output is

$$\overline{(e_{BN})^2} = \frac{E_o^2}{4} \overline{r^2(t)} \cdot \overline{\cos^2(\phi_r(t) - \phi_o)} = \frac{E_o^2}{8} \overline{r^2(t)} \qquad 7.4.6$$

since all values of $\phi_r(t)$ are equally likely, and so

$$\overline{\cos^2(\phi_r(t) - \phi_o)} = \tfrac{1}{2} \qquad 7.4.7$$

The output signal power of the wideband coherent detector is then

$$\left(\frac{S}{N}\right)_{out} = \frac{\left(\frac{E_o E_c}{2}\right)^2 \overline{m^2(t)}}{\overline{r^2(t)} \cdot E_o^2/8} = 2E_c^{\,2} \frac{\overline{m^2(t)}}{\overline{r^2(t)}} \qquad 7.4.8$$

At the input, if we do not take into account the carrier power, we may put the signal to noise ratio as

$$\left(\frac{S}{N}\right)_{in} = \frac{E_c (\overline{m^2(t)}/2 + \overline{m^2(t)}/2)}{\overline{r^2(t)}/2} = E_c^2 \frac{\overline{m^2(t)}}{\overline{r^2(t)}} \qquad 7.4.9$$

so that the improvement in the signal to noise ratio for broadband coherent detection is 3db, independently of the signal to noise ratio. In other words, there is no threshold effect.

7.5 Improving the signal to noise ratio by coherent detection.

From the Appendices 2 - 4 it has been seen that by keeping the same time interval T for making observations and by reducing the number of different distinguishable signals one can increase the certainty of detection. It is also clear that by increasing the observation time, T, or the number of observations, B, within limits specified and discussed briefly at Appendix 4, the certainty of detection is increased.

Let us assume that in a time interval from 0 - T there is a constant d.c. signal of amplitude S buried in noise n(t) with a flat power spectrum in the bandwidth B and having an r.m.s. value $N^{\frac{1}{2}}$.

The incoming signal could be represented by the function

$$f(t) = n(t) + S \qquad 7.5.1$$

which by the sampling theorem is characterised by 2BT sample values, specified by

$$\begin{aligned} f(i) &= f(i/2B) \\ n(i) &= n(i/2B) \qquad \text{where } i = 1, 2, \ldots.. 2BT \qquad 7.5.2 \\ s(i) &= \text{constant} \end{aligned}$$

The various samples f(i) are independent and have a Gaussian distribution with variance N and a mean value S. If there is no interdependence of the signal and the noise introduced by the correlation detection process we may put the mean value when the signal is absent as

$$\mu_o = 0 \qquad 7.5.3$$

and with the signal present as

$$\mu_S = S \qquad 7.5.4$$

The variange $\sigma_N{}^2$ is the same in both cases. We shall use the ratio

$$\frac{\mu_S - \mu_o}{\sigma_N} \qquad 7.5.5$$

as a measure of the effectiveness with which the detection process is operating.

The variance is given by

$$\sigma_n^{\,2} = \frac{\sum_1^{2BT}(s+n_i).\sum_1^{2BT}(s+n_j) - s^2}{(2BT)^2}$$

$$= \frac{\sum_1^{2BT}\sum_1^{2BT}(s^2 + sn_i + sn_j + n_i n_j - s^2)}{(2BT)^2} \qquad 7.5.6$$

$$\frac{\sum_1^{2BT}(n_i)^2}{(2BT)^2} = \frac{N^2}{(2BT)^2} \quad \text{or} \qquad 7.5.7$$

$$\sigma_n = N/(2BT)^{\frac{1}{2}}$$

from which we get

$$\frac{\mu_s - \mu_o}{\sigma_n} = (2BT)^{\frac{1}{2}}.\frac{S}{N} \qquad 7.5.8$$

or, expressed in words, the effect of taking 2BT samples is to improve the signal to noise ratio by a factor $(2BT)^{\frac{1}{2}}$. It must be noted that this is strictly valid for additive noise and a sinusoidal signal.

CHAPTER 8.

Experimental Results.

The first experiments were taken in order to measure signal to noise ratio of a received signal and noise, when a deterministic signal was transmitted, as well as to determine the power density spectrum of noise. For this purpose the equipment described in Chapter 6 and in Chapter 7, in connection with Wayne-Kerr Spectrum Analyzer (LF) was utilised. As the time spent for one data point was of the order of 30 sec. this method was abandoned and fluctuations were recorded for subsequent digital analysis. Anyway, a few power density spectra were obtained by this analog method, which have clearly shown that the slope of the spectra depend not only on the flame flow and fuel characteristics, which could be set constant and automatically controlled but also on the type of antennas used and also especially, on the method of signal detection. It was obvious that all spectra obtained were of the $(1/\Delta f)^{\alpha}$ shape, where the α coefficients were always higher than 1. Typically, all the spectra had slopes of about 10 to 12 db/octave for horn antennae, and 12 to 15 db/octave for cylindrical resonators.

Coherent signal detection gave 6 to 8 db more noise power and steeper spectra at high frequency end of the spectrum. At the time of the initial experiments it was not appreciated that the signal is converted to noise partially on account of magnitude fluctuations, and partially on account of phase uncertainty and fluctuation. In order to remove difficulties of interpretation on account of these differences in behaviour of the different methods of detection, some of these experiments were repeated when digital methods were introduced. As an example, Fig.8.1 is obtained when coherent detection was done and Fig.8.2 when an ordinary envelope detection was made.

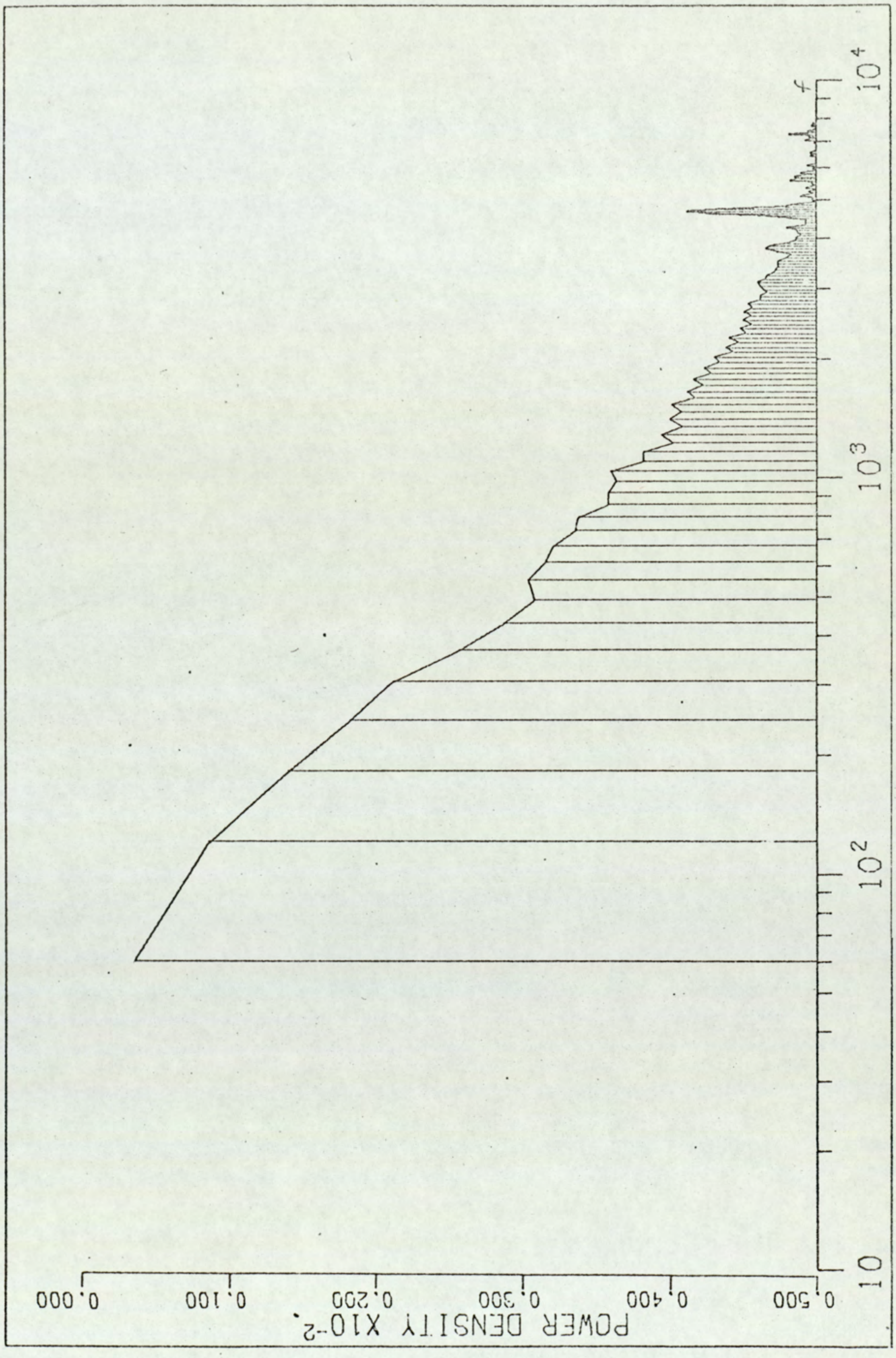

POWER DENSITY X10^-2
0.000
0.100
0.200
0.300
0.400
0.500
10
10^2
10^3
10^4
f

Fig.8.1

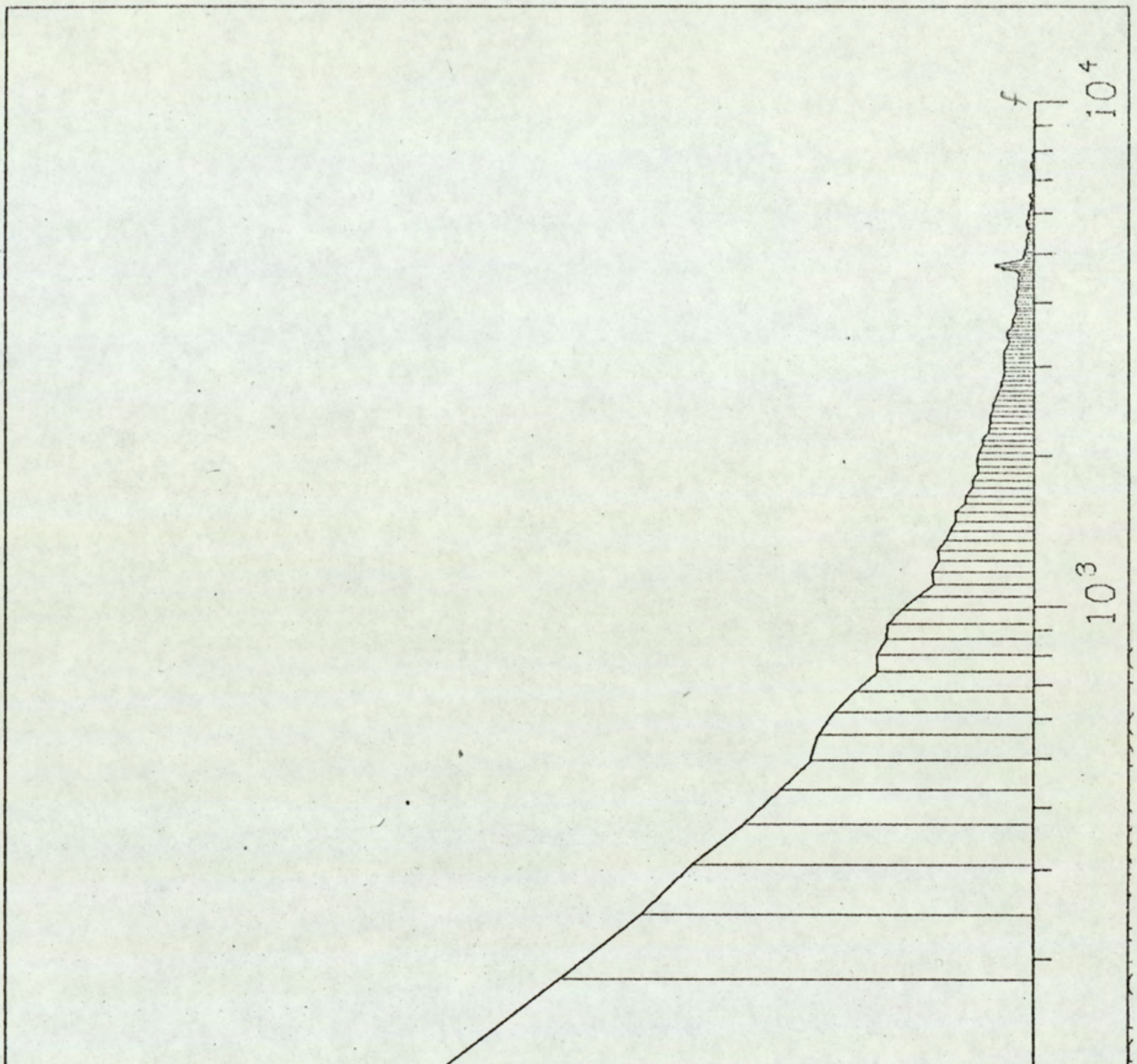

POWER DENSITY X10-2
0.000
0.100
0.200
0.300
0.400
0.500
10
10^2
10^3
10^4
f

Fig.8.2

Having this in mind, therefore, all the following experiments were conducted with the local oscillator locked either to the transmitted reference, when a phase fluctuation was of interest, or to the received reference, when the magnitude fluctuation was studied. It should be emphasised that the phase tracking must be fast, i.e. almost instantaneous, which could be achieved only by using for mixing the received reference instead of the local oscillator signal.

8.1 Autocovariance Function Estimation

On Fig.8.3 three experimentally obtained autocovariance functions are represented, obtained in 62.5msec, 125msec, and 250msec respectively. It must be emphasised that for some other particular interval of time different functions could be obtained, especially for observation times of less than 100msecs. For this particular case, when flame was turbulent but in other respects stable, extending the observation time did not improve or change the shape of the function obtained. Compare, for example, the autocoherence function on Fig.8.3c, obtained in 250msec, with the one obtained in 1 sec represented on Fig.8.4.

On the other hand, the differences were more obvious when the corresponding power density spectra were compared. The Fig.8.5 and Fig.8.6 illustrate this more clearly. Both power density spectra on Fig.8.5a and Fig.8.5b were obtained from records of 62.5msec duration, i.e. each from 1000 data points. The power spectra on Fig.8.6a and Fig.8.6b are from the same experiment, but for the observation time of 250 and 1000msec respectively.

It was found out that by averaging results from 4 - 6 portions of 1024 data points (Δt = 62.5μsec) reasonably smooth power density spectra could be obtained. Autocovariance functions and power density spectra were computed for 128 discrete points which corresponded to τ_{max} = 8msec (Fig.8.4) and f_{max} = 4kc/s (Fig.8.6) respectively.

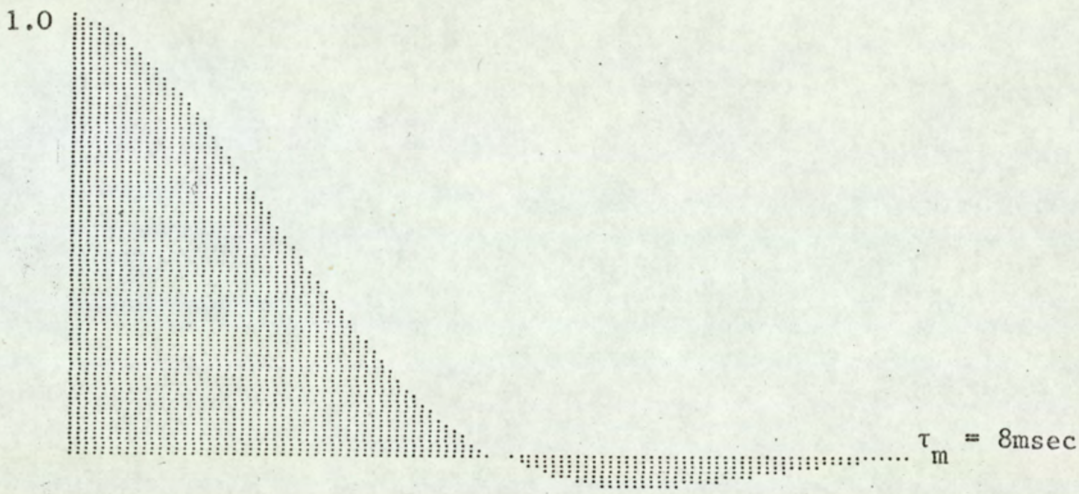


Fig.8.3a

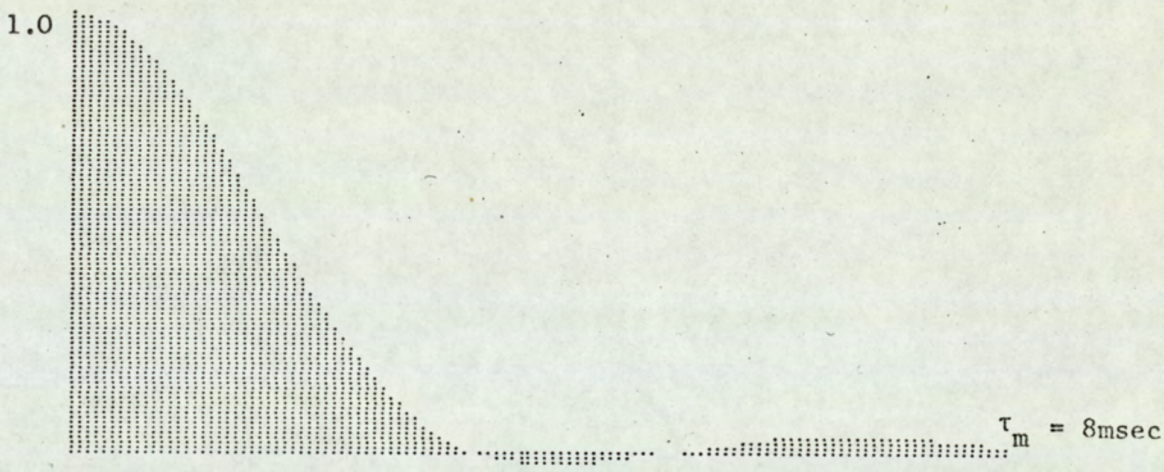


Fig.8.3b

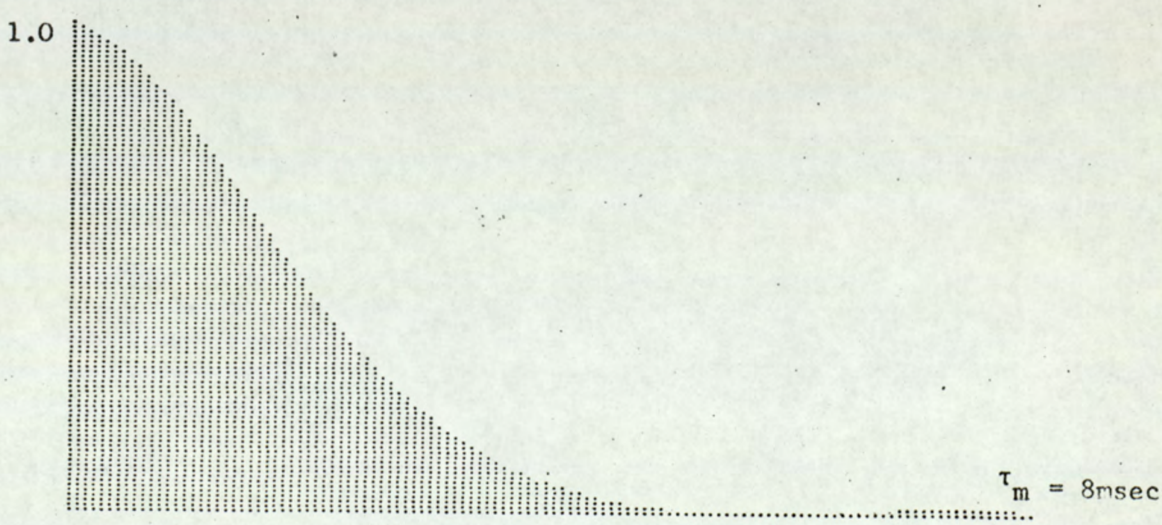


Fig.8.3c

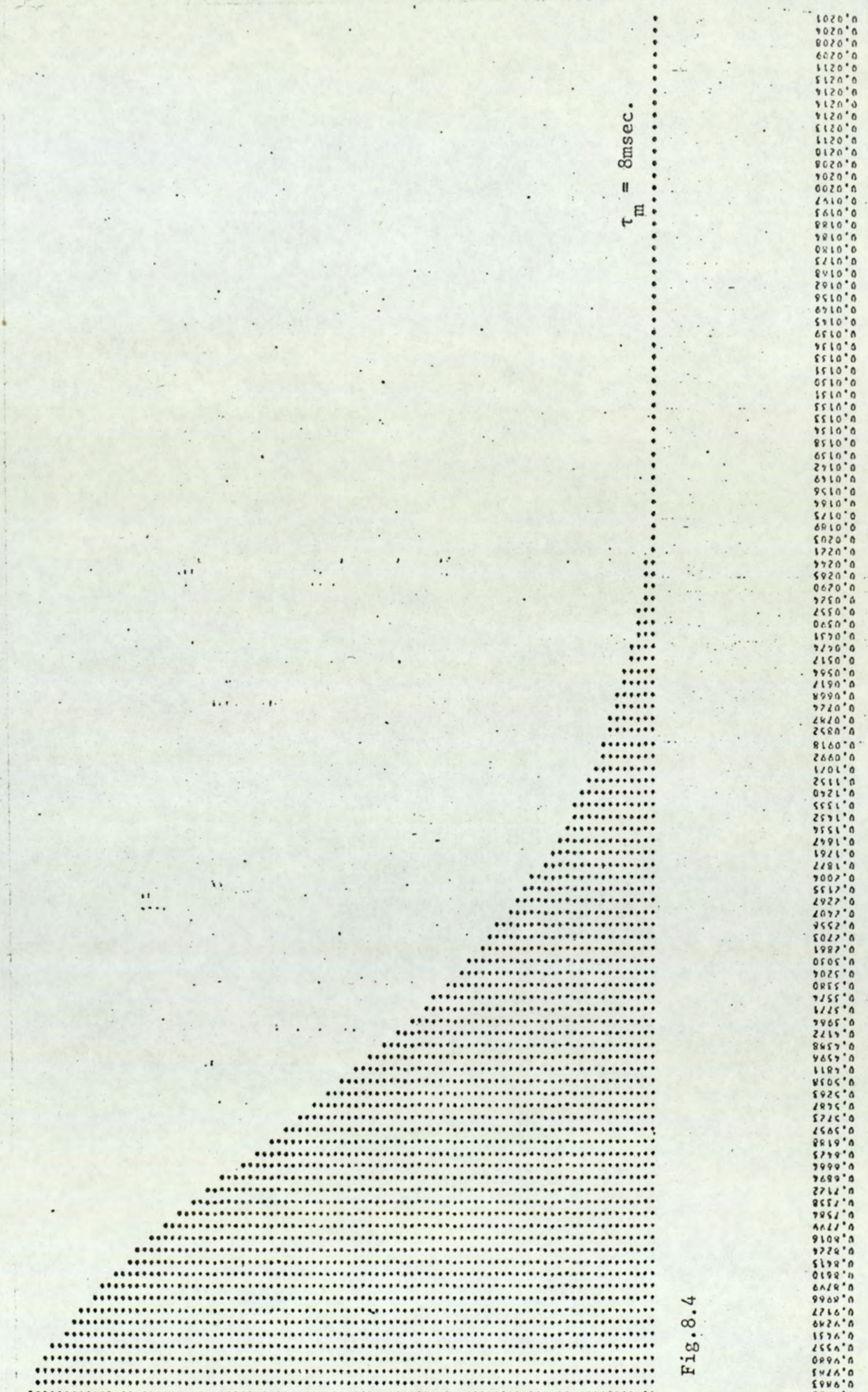


Fig.8.4

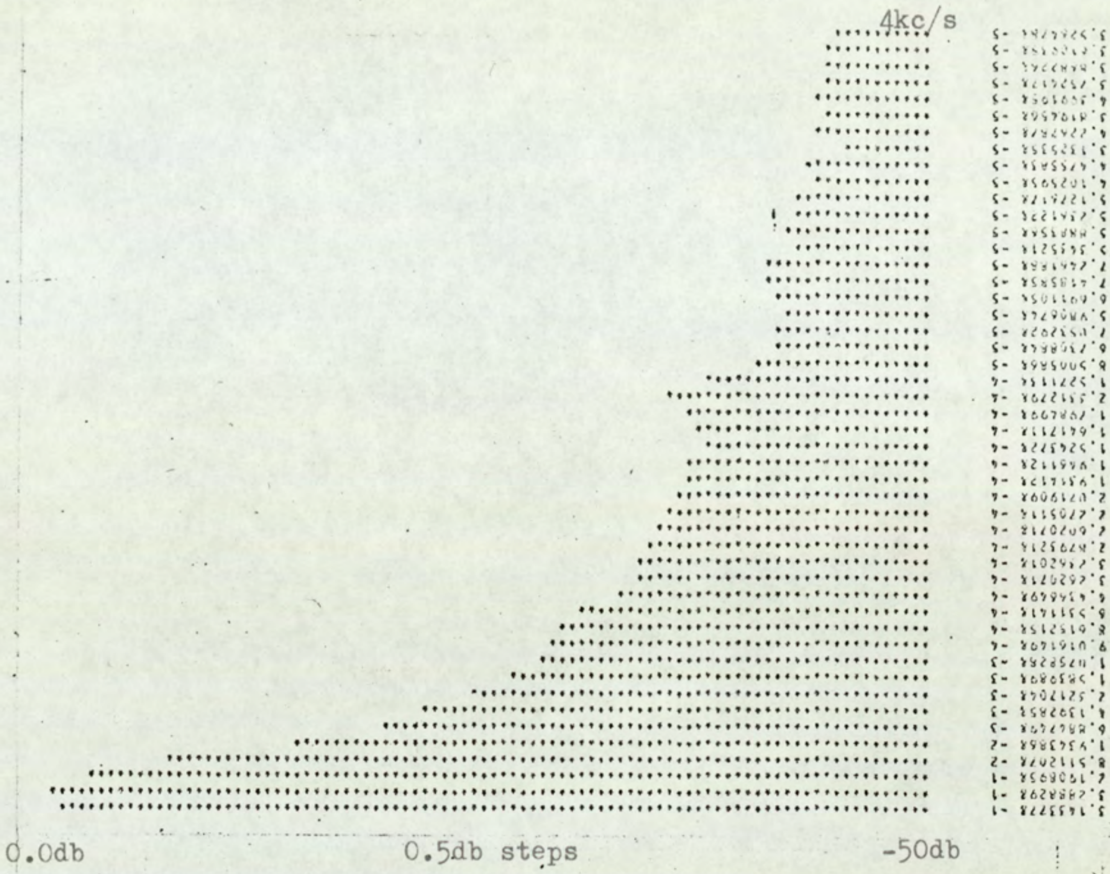


Fig.8.5b

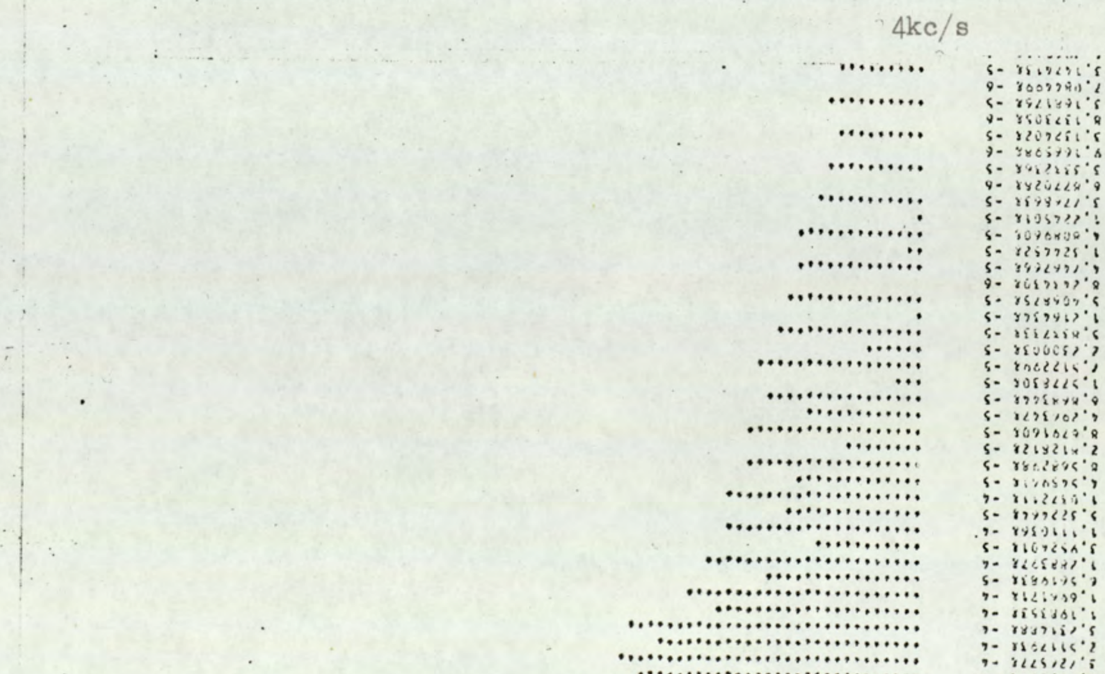


Fig.8.5a

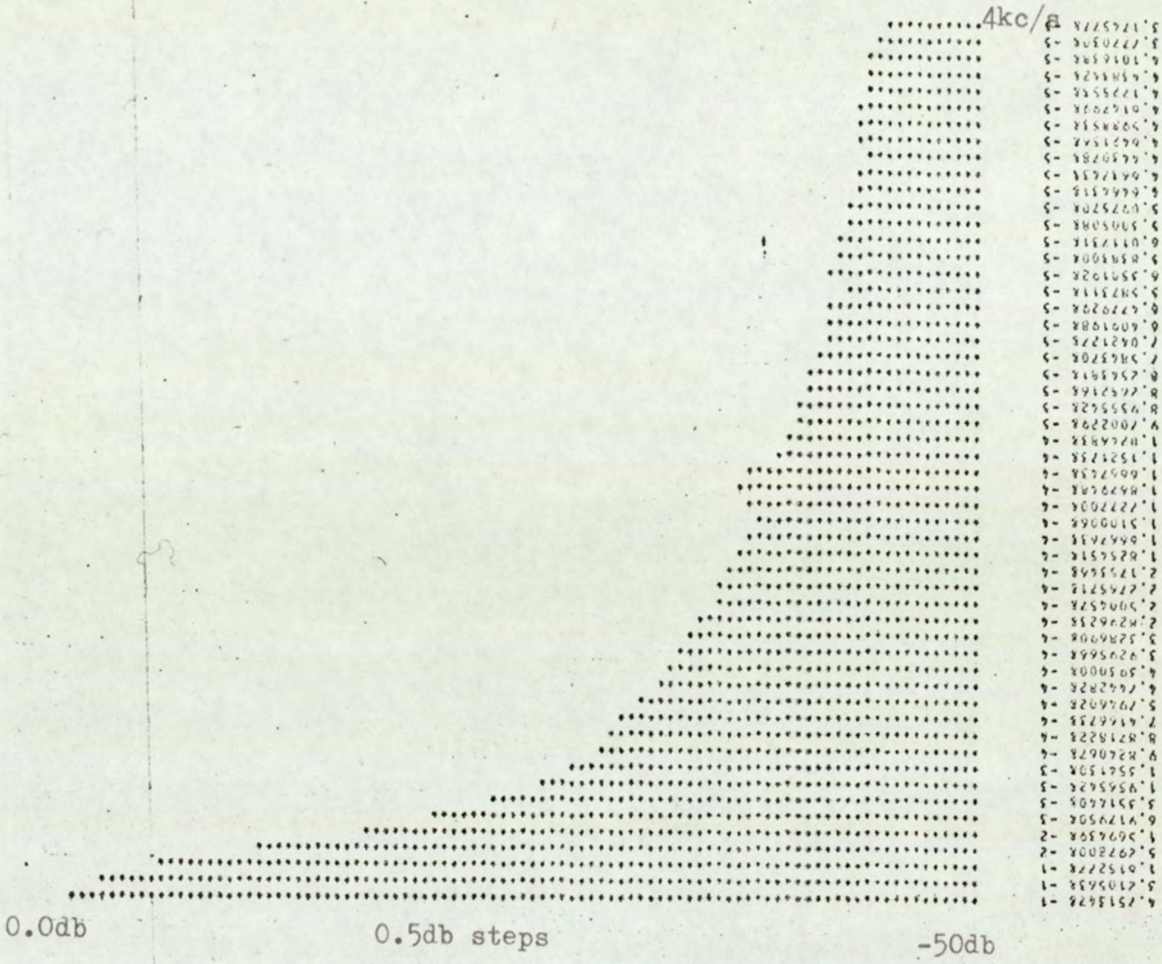


Fig.8.6b

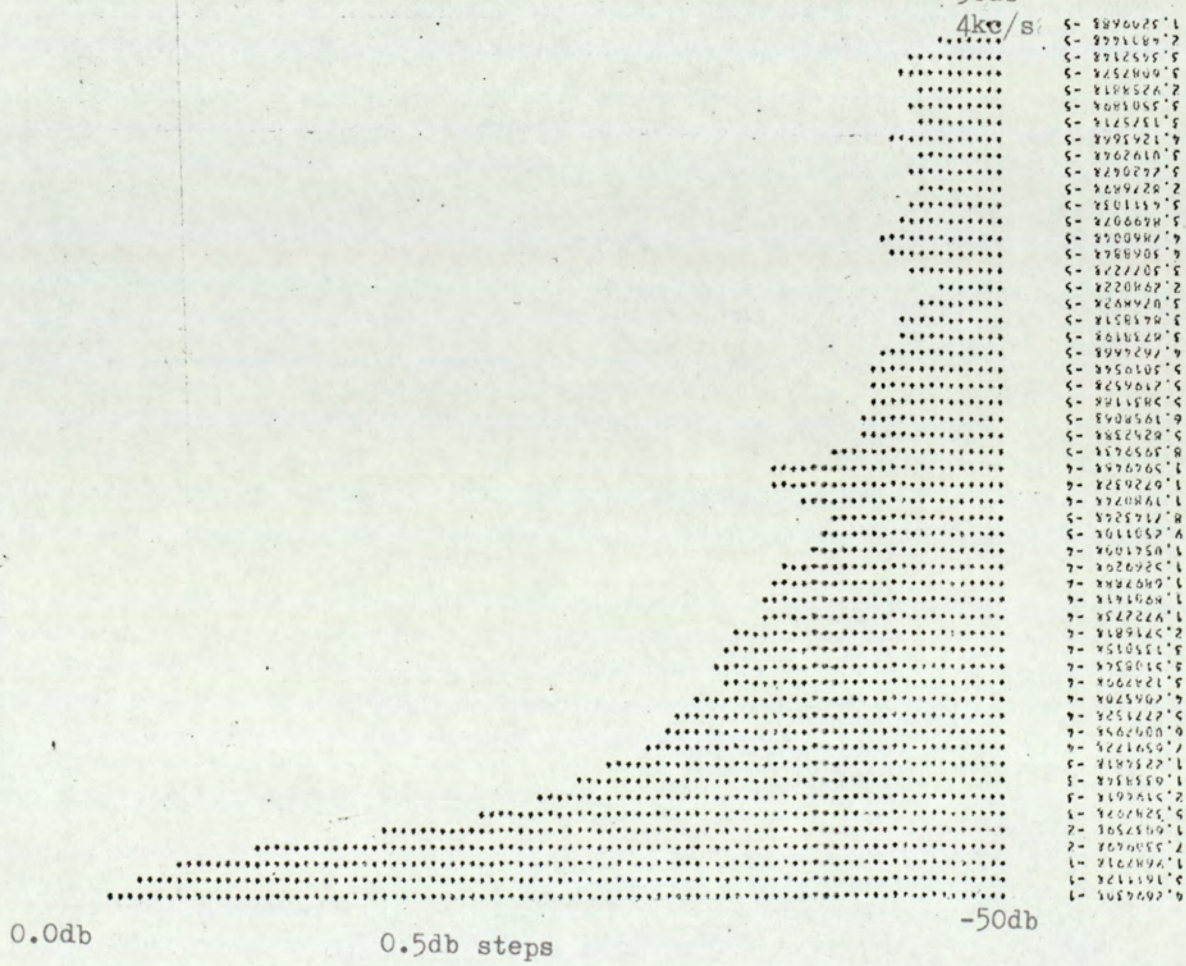


Fig.8.6a

8.2 Instantaneous transmission of two signals through the same physical system.

A stochastic signal, generated by a Markovian (message) process can be represented in the frequency domain by its real and imaginary components. Depending on the type of modulation used, these components are spread symmetrically or unsymmetrically about the carrier.

The simplest case is when there are only two frequency components, which could be obtained by sending an SSB with unsuppressed carrier (Fig.8.7a). The frequency difference could be easily adjusted by changing the modulation frequency. For frequency differences larger than several kc/s, the second signal could be added in a separate stage.(Fig.5.7) The total received signal plus noise power is linearly increased by increasing any of the signal levels. It is of specific interest that the second signal too will be spread in the frequency band, so that instead of a discrete spectral line, a power spectrum similar to the spectrum of the first signal is obtained. (Fig.8.7b). This could be intuitively expected, as there is no operational discrimination of the system on any signal component present. All signal components are convoluted by the same or similar Fourier Transform of the system correlation function. The next step in the fluctuation analysis is to find out to which extent these fluctuations are correlated, i.e. to establish the complex coherence measure, and the corresponding coherence function. If two signal components are too close in frequency, the corresponding output spectra will be interleaved and their separation almost impossible. But if two signals were mutually orthogonal, it was expected that their power spectra should be also orthogonal.

It will be shown later that although signals and their spectra were orthogonal, signal fluctuations were to a high degree correlated. (pp 155-160)

When a signal component is added at the receiver end, i.e. without being transmitted through the system with fluctuating parameters (Fig.8.7d) a corresponding single spectral line is obtained. Due to the artificial

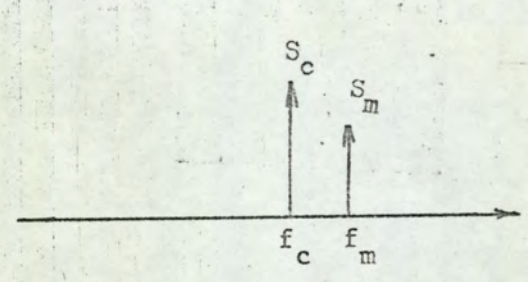


Fig.8.7a.

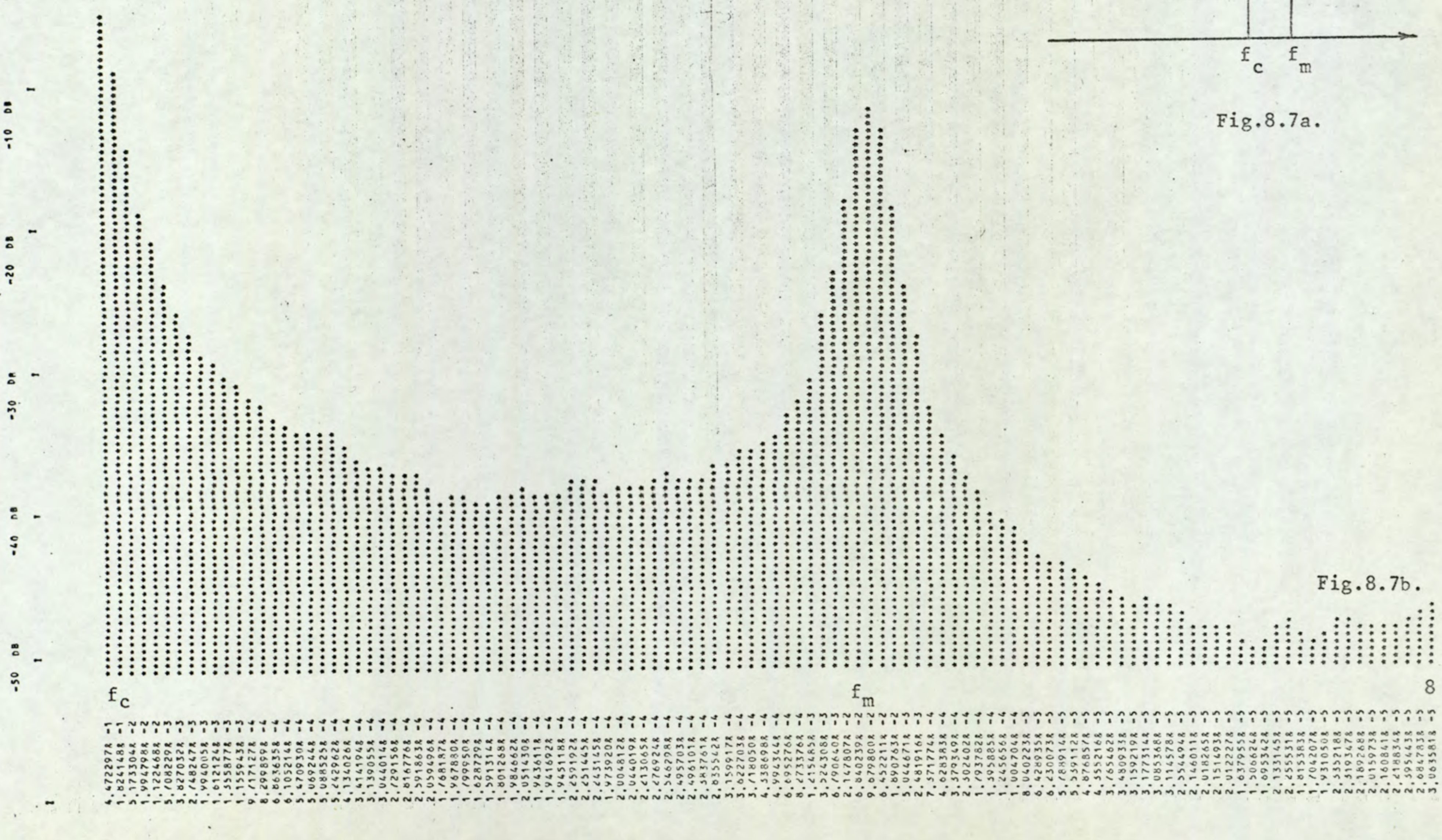


Fig.8.7b.

smoothing of spectra (Fig.8.7c) included in the program SHORTBITS (Procedure CORRFUN) this line is smeared.

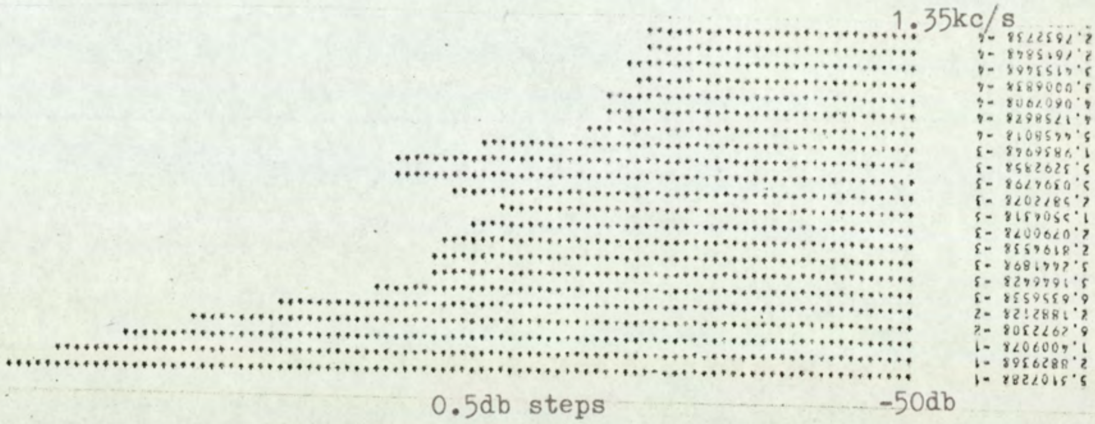


Fig.8.7c.

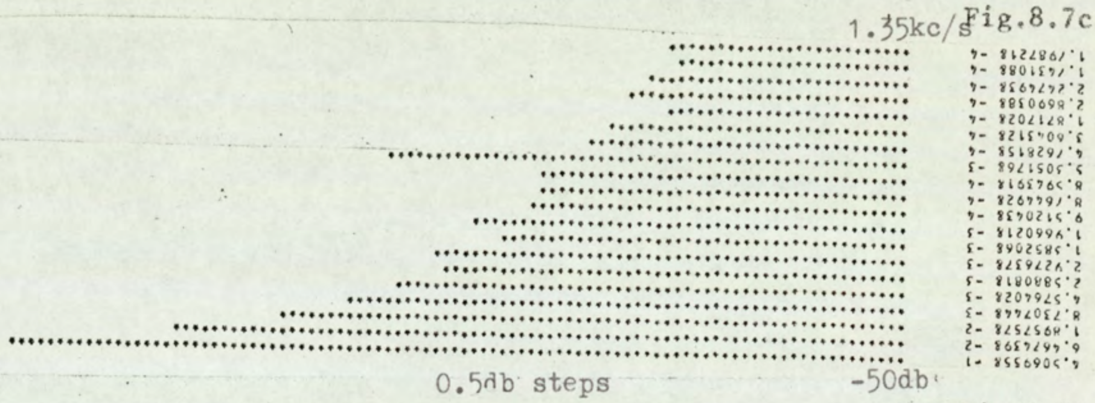


Fig.8.7d.

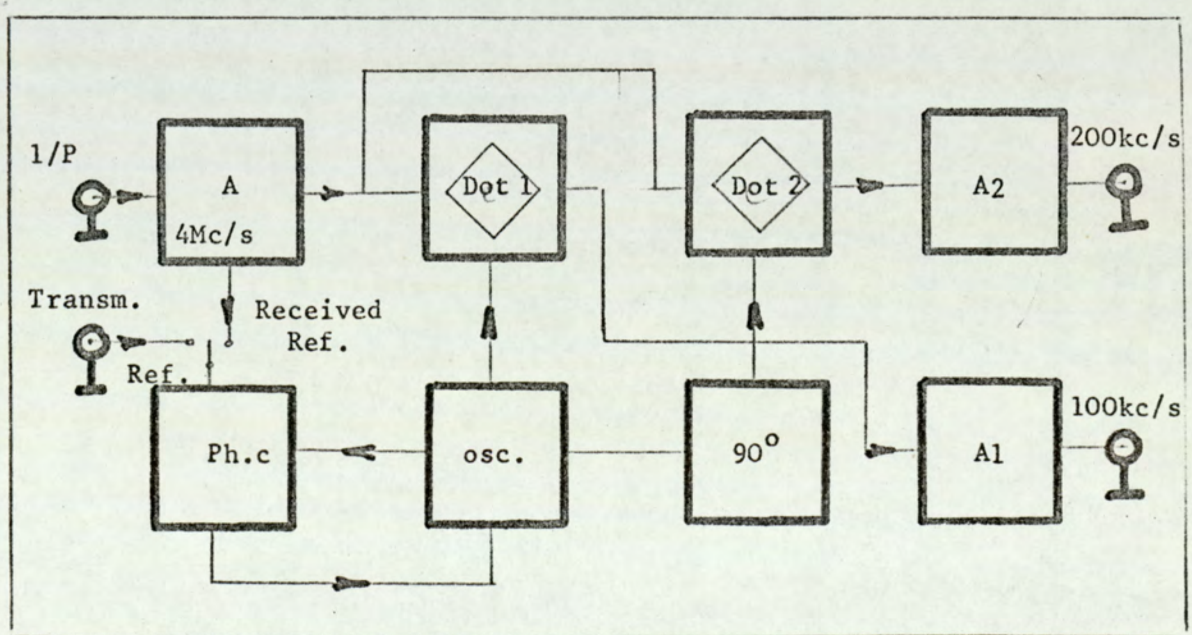


Fig.8.8

8.3 Coherence of fluctuations of two orthogonal signals.

For this experiment a 4Mc/s subcarrier was amplitude modulated by 200 kc/s signal, and this subcarrier rotated by 90 degrees. The rotated carrier was again AM modulated by the second 100 kc/s signal. Such a complex signal was used to modulate 9.35 Gc/s microwave carrier and then passed through the TM 040 resonator with an unstable and turbulent flame inside it. At the receiver end, after the first mixing, an IF signal and two orthogonal sidebands 100 kc/s apart were amplified by 40dB IF (Fig.8.8) amplifier. A phase-lock local 4 Mc/s oscillator was used for synchronous detection of both signals. For this purpose a 90^{o} phase rotation was made by strongly coupled resonant 4 Mc/s circuits. A 4 Mc/s rotated local oscillator signal was amplified by a wideband buffer amplifier and then applied to the balanced diode detector. At the output of amplifiers A1 and A2 100 kc/s and 200 kc/s fluctuating signals are obtained. After demodulation, signal amplitude fluctuations are recorded and analysed (see Data Acquisition and Reduction Sec.5.1)

Using procedure Coherence, autospectra as well as cross spectrum were computed. Both autospectra and the real part of the obtained cross spectrum were identical and therefore coherence function had a constant value ($\gamma^2(\omega) = 1.0$) independent of frequency. (See Fig.8.16a)

This experiment was repeated several times, and although the autospectra and cross spectrum varied slightly from one experiment to the other, the coherence function, and consequently cumulative coherence measure, was invariantly equal to 1.0 independently on the duration of a particular experiment (0.1 sec to 4.0 sec).

The same experiments were repeated, but the 4 Mc/s transmitted reference was substituted with the 4 Mc/s received reference. As the mean frequency in both cases was exactly the same, and the dominant filter time constant was approximately 10 m.sec. no difference in experimental results was expected which proved to be correct.

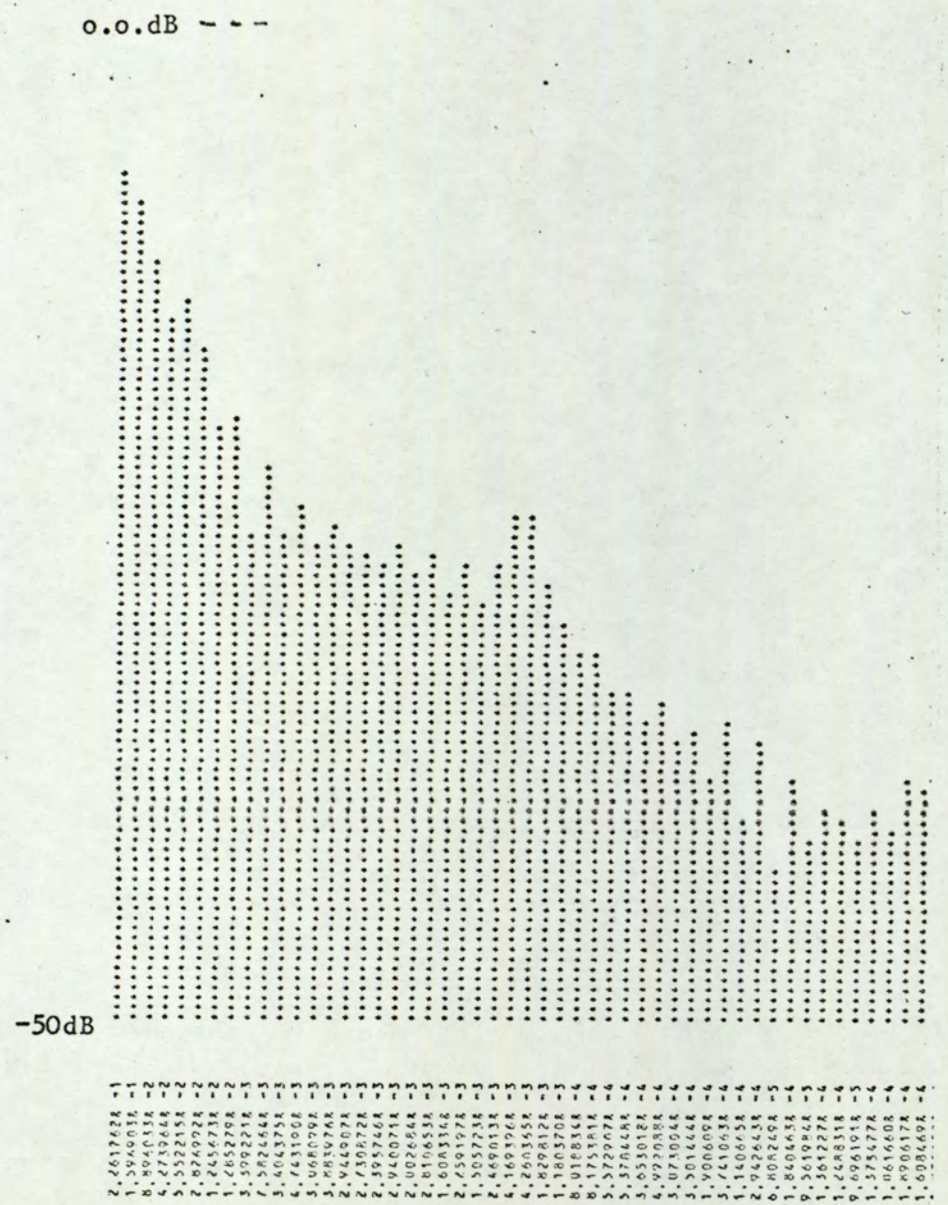


Fig.8.9a

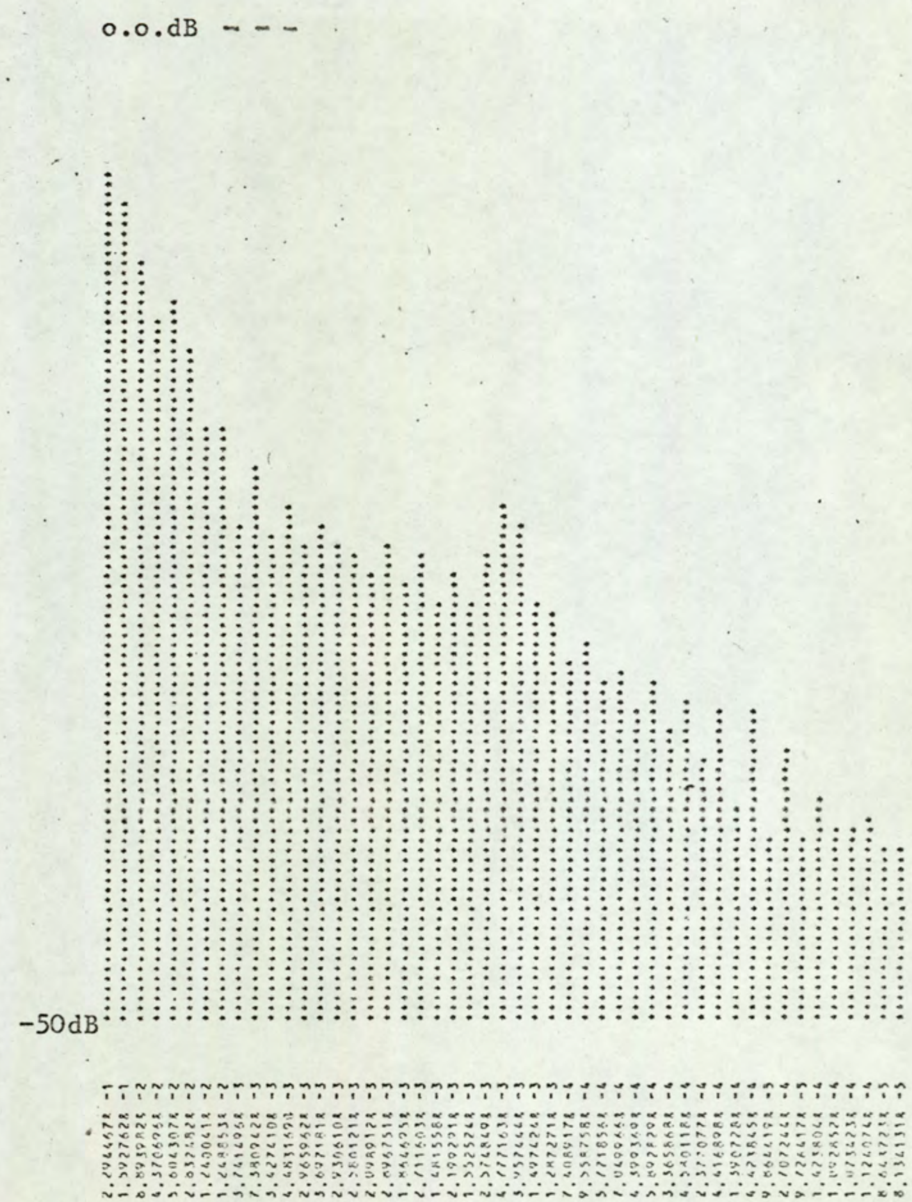


Fig.8.9b

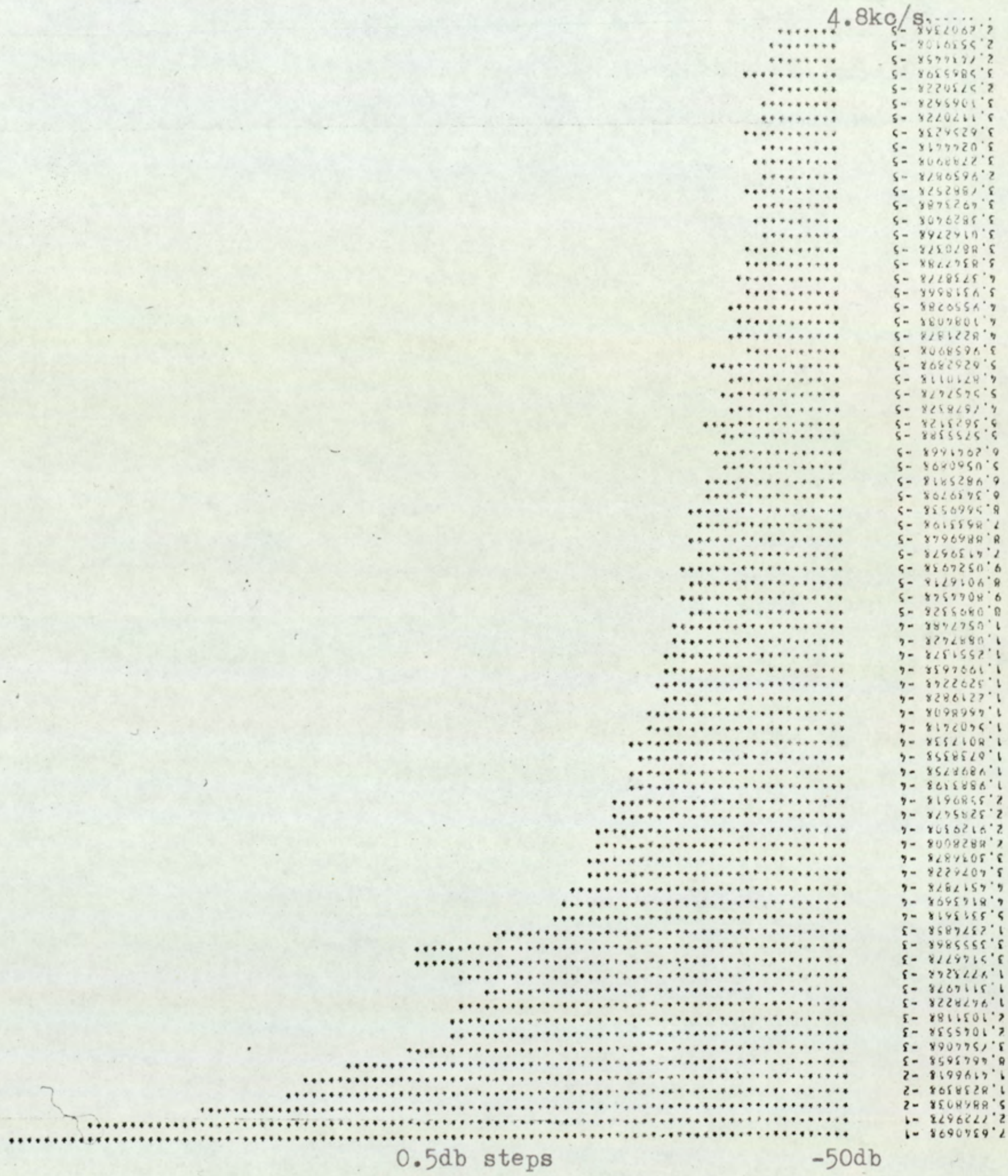


Fig.8.9c. Autospectrum of a fluctuating signal. Fluctuation was detected by short-term correlator(Chapter 7) locked to the transmitted reference signal.

Then the time constant was decreased to 0.1msec, which affected the R.M.S. value of the measured 200 kc/s noise fluctuation by some 6 dB. The corresponding coherence function was unaltered, but both auto spectra and cross spectrum were obviously less steep at the low frequency end of the spectrum (Fig.8.9a and 8.9b) than before(Fig.8.9c).

Coherence of amplitude and phase of signal fluctuations

The previous experiment has shown that the amplitude signal fluctuations (100 kc/s and 200 kc/s signals) depend to some extent on the phase fluctuations of the 4 Mc/s signal. In order to establish coherence between these fluctuations the following experiment was made. An unmodulated 4 Mc/s signal was transmitted, using microwave arrangement, through the same flame as before. (Sec.5.3)

After IF amplification the signal was applied to a synchronous detector. The 4 Mc/s IF signal from the second amplifier output was applied through a limiter stage (Fig.8.10) simultaneously to a balanced diode detector which was used as a phase comparator stage, for signal phase fluctuation detection. A transmitted 4 Mc/s reference signal was applied to the buffer amplifier stage with a 0° and 90° reference rotation. Both fluctuations were recorded and analysed.

There was a slight difference in power density spectra of amplitude and phase fluctuations especially noticeable in higher power density frequency components (Figs.8.12 & 8.13). A typical complex coherence measure function is given in Figs.8.14 & 8.15. It can be seen that the real part of this function is slightly affected in the higher frequency end (N.B. the log scale). The imaginary part rarely exceeds a few percent and is higher at the high frequency end.

The low frequency components, due to $1/\Delta f^{\alpha}$ spectrum shape, have added more weight to the cumulative coherence measure,which did not differ very much from its maximum value of 1.0. For example, in three successively made experiments, of 0.25 sec. duration each, cumulative coherence measure values obtained were 0.99, 0.99, 0.995.

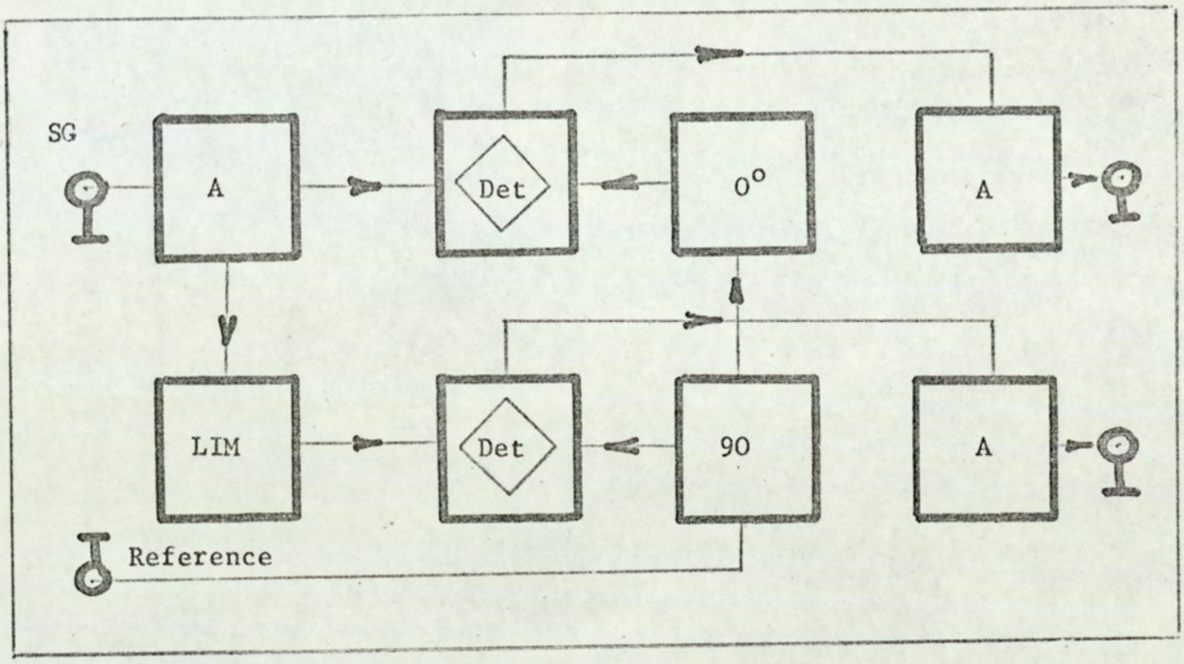


Fig.8.10

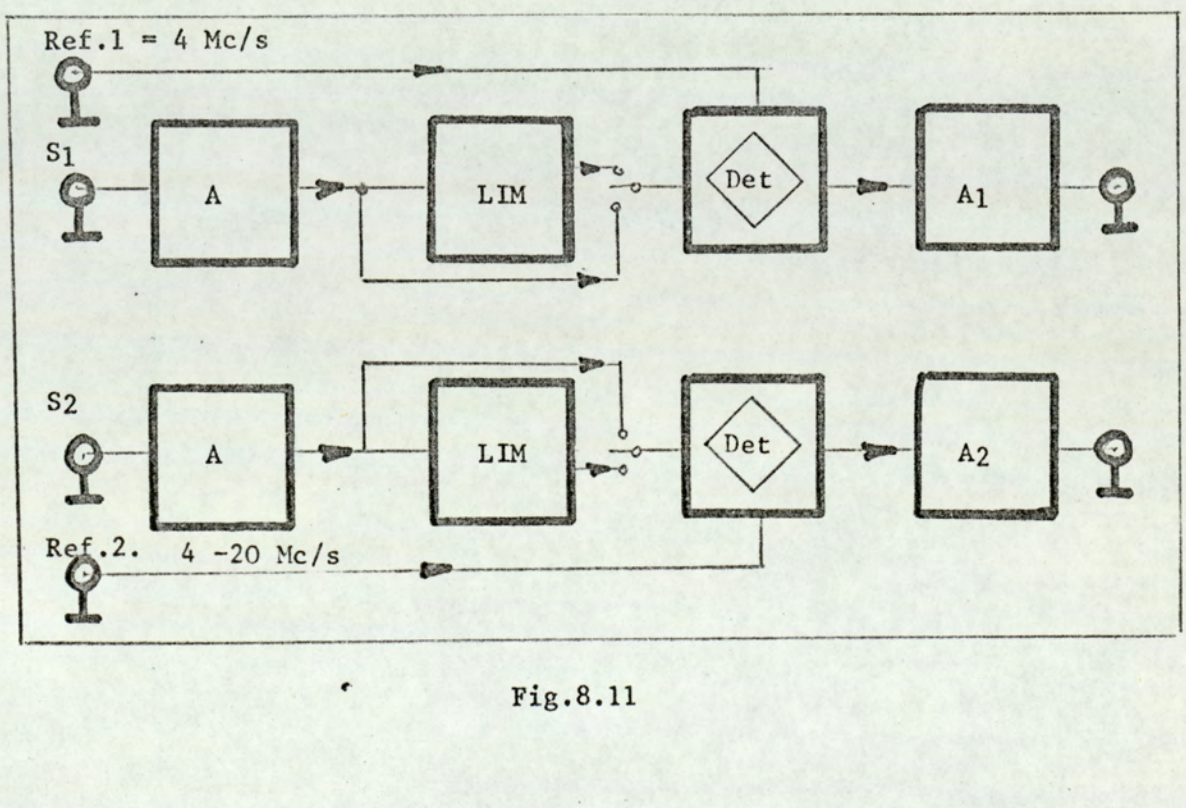


Fig.8.11

Coherence of the fluctuations of two signals

Previous experiments have shown that fluctuations of two orthogonal signals are highly correlated. Some departure from the maximum value of the cumulative coherence factor was attributed partially to imperfections in orthogonality of the reference signal as received and partially to the presence of additive noise and other interference picked up.

In order to investigate signal coherence when signal components are further apart in frequency, two deterministic signals S_1 and S_2 were transmitted and received. One signal, S_1, was again at 200 kc/s, on a subcarrier of 4 Mc/s, and the other, S_2, was 100 kc/s, on another subcarrier frequency which could be adjusted to 4 Mc/s, 8 Mc/s, 12 Mc/s, 16 Mc/s and 20 Mc/s, resulting in a maximum frequency difference of 16 Mc/s.

The receiving arrangement (Fig.8.11) consisted of two almost identical low gain (20dB) wide band amplifiers, each of which was followed by a limiter stage and a synchronous diode bridge detector. At the output of the detector stage 200 kc/s and 100 kc/s signals were available. Limiter stages were used only for phase coherence measurements between the two signals, when both subcarriers were not modulated at all by deterministic signals.

When a single path transmission was simulated, the cumulative coherence factor did not change very much even when two signals subcarriers were 16 Mc/s apart. From the maximum value of 0.995 obtained for $\Delta f = 0.1$ Mc/s it has fallen down to approximately 0.97 - 0.98 at 8 Mc/s and 0.95 $\div$ 0.96 at 16 Mc/s (Fig.8.19-SP)

Such a relatively small drop in cumulative coherence factors was obviously the result of a negligible delay difference occurring in the transmission path.

In order to make two signal fluctuations less coherent, an arrangement simulating two path transmission was introduced, which consisted of two resonators with very sumilar, almost identical transmission characteristics.

Fig.8.12.

Fig.8.13.

Samples of simultaneous autospectra of two fluctuating signals for a particular time interval are given on Figs. 8.12 and 8.13. It is quite obvious that both spectra are of the same shape whatever the carrier frequency. When samples were taken in a longer time interval, there was not an easily distinguishable difference in their autospectra. The real(Fig.8.14) and the immaginary(Fig. 8.15) cross spectra for two signals 100 kc/s appart are also obtained(T=0.1s) The waveguide path difference was approximately 5m, which introduced a time delay difference of the order of 0.1μsec. The short term coherence (ΔT = 0.1 sec) was obtained first (Fig.8.16) from which it was found that coherence at some frequencies in a short time interval is accidentally high but at the average is very low. Note that here as before the vertical scale is logarithmic, and then the base values are -50dB in reference to the maximum coherence value of 1.0 (0dB).

Much smoother coherence functions were obtained for an observation time of 1 sec. (Fig.8.17).

The cumulative coherence factor Γ versus frequency diagram is given on Fig.8.19-DP.1.

For double path transmission, even signals of very nearly the same frequency (Δf = 0.1 Mc/s) were evidently less coherent. To demonstrate this fact, two coherence functions are displayed (Figs.8.18). From these representations it is clear that there exist larger phase fluctuation discrepancy (Fig.8.18b) than in the case of magnitude fluctuation coherence (Fig.8.18a). The cumulative factors do not differ appreciably in these two cases as the discrepancy is at the higher frequencies of the fluctuation spectra.

A second set of double path transmission experiments was carried out for a much smaller waveguide path difference corresponding to approximately 10 nsec. The cumulative coherence factor is for this case also represented on Fig.8.19 and marked DP-2.

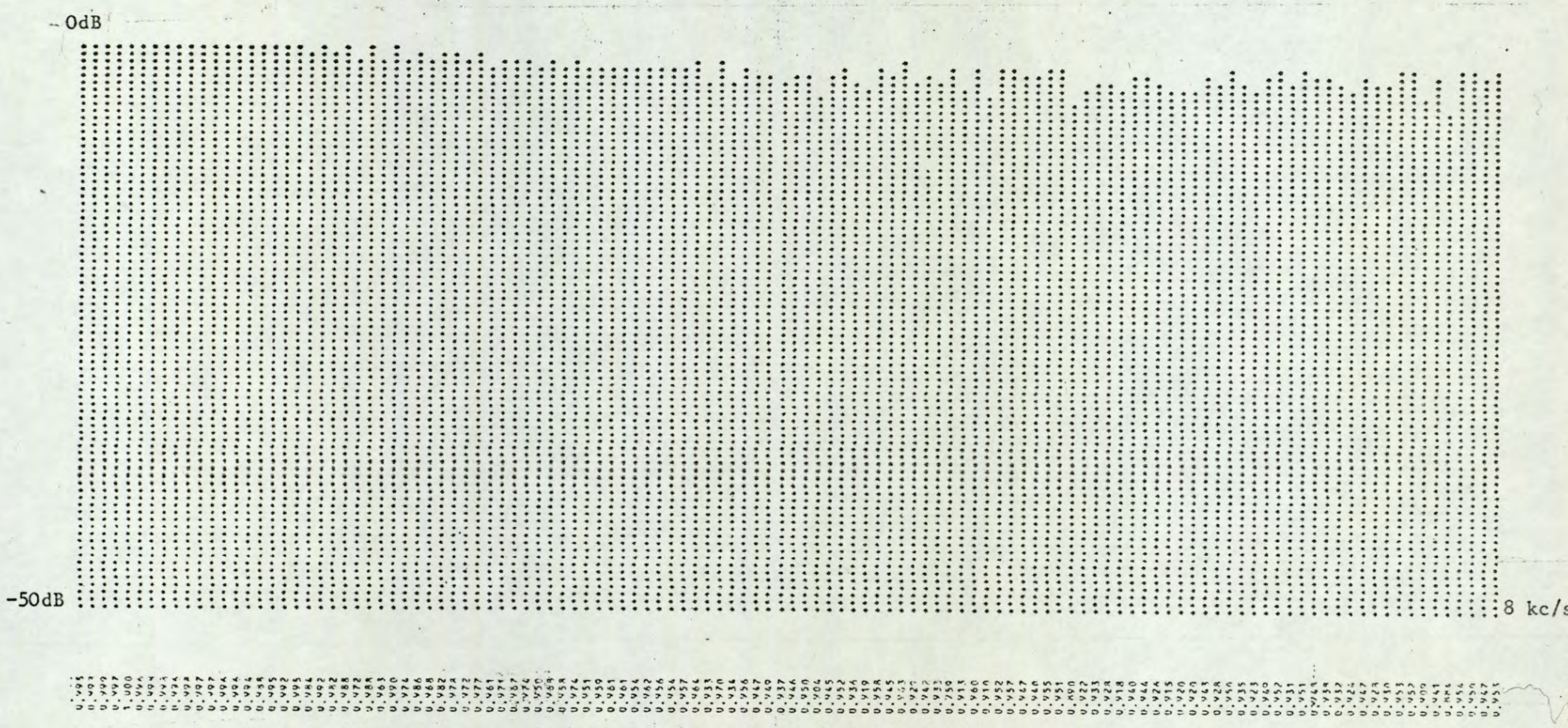


Fig.8.14

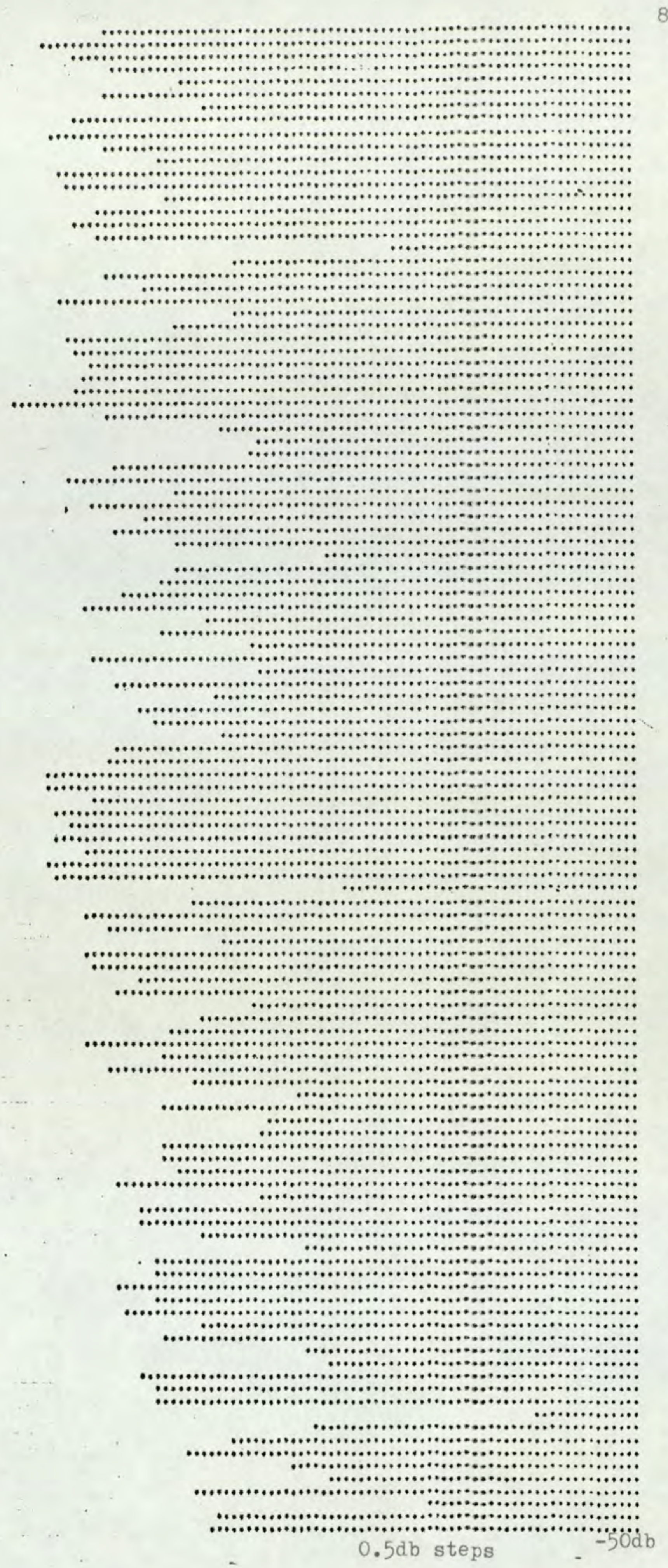
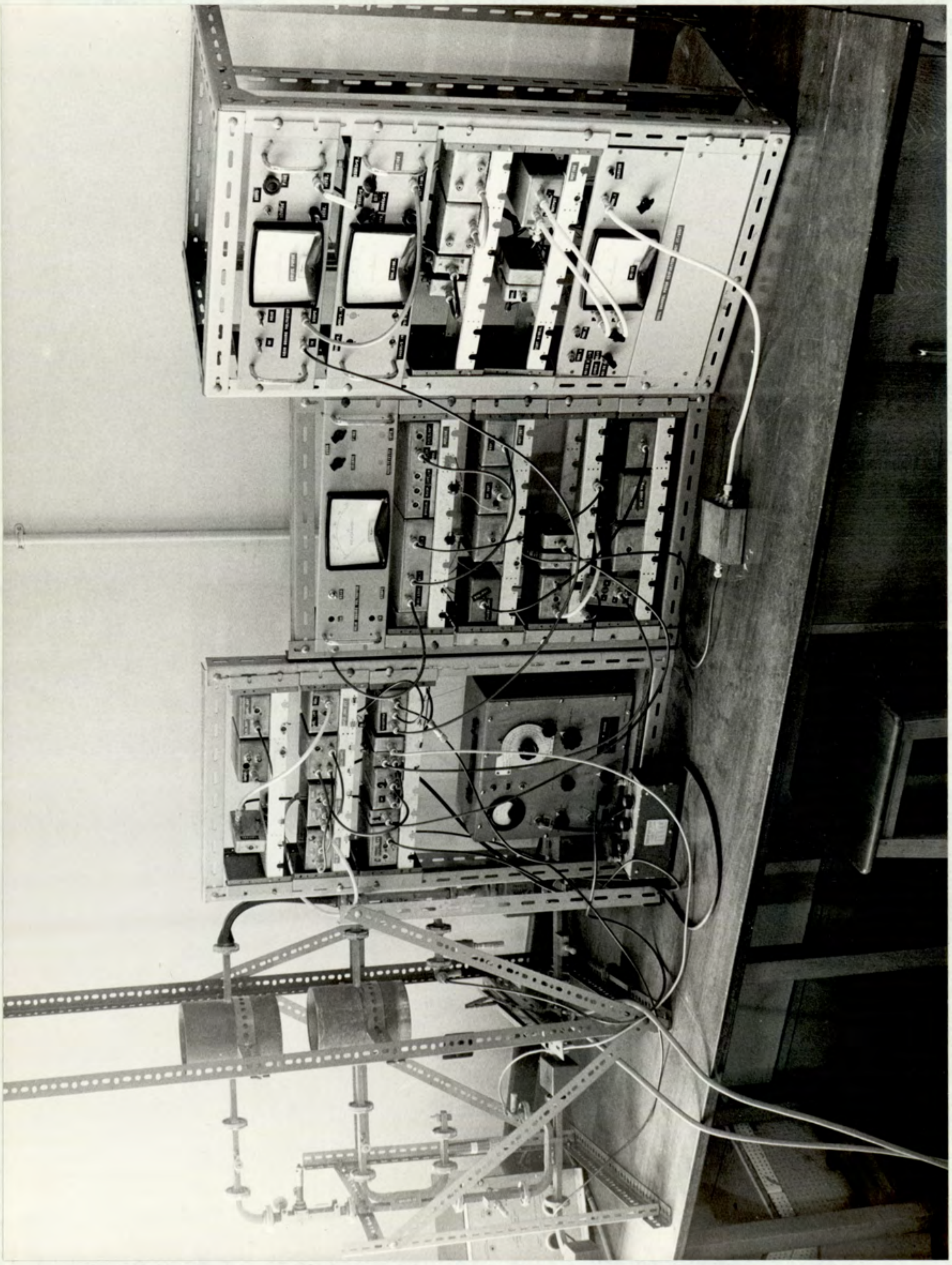


Fig.8.15



On Fig.8A (below) it is represented experimental arangement utilized when double path ,10 nsec. differential delay was simulated. For such small path difference only one flame plum is necessary.

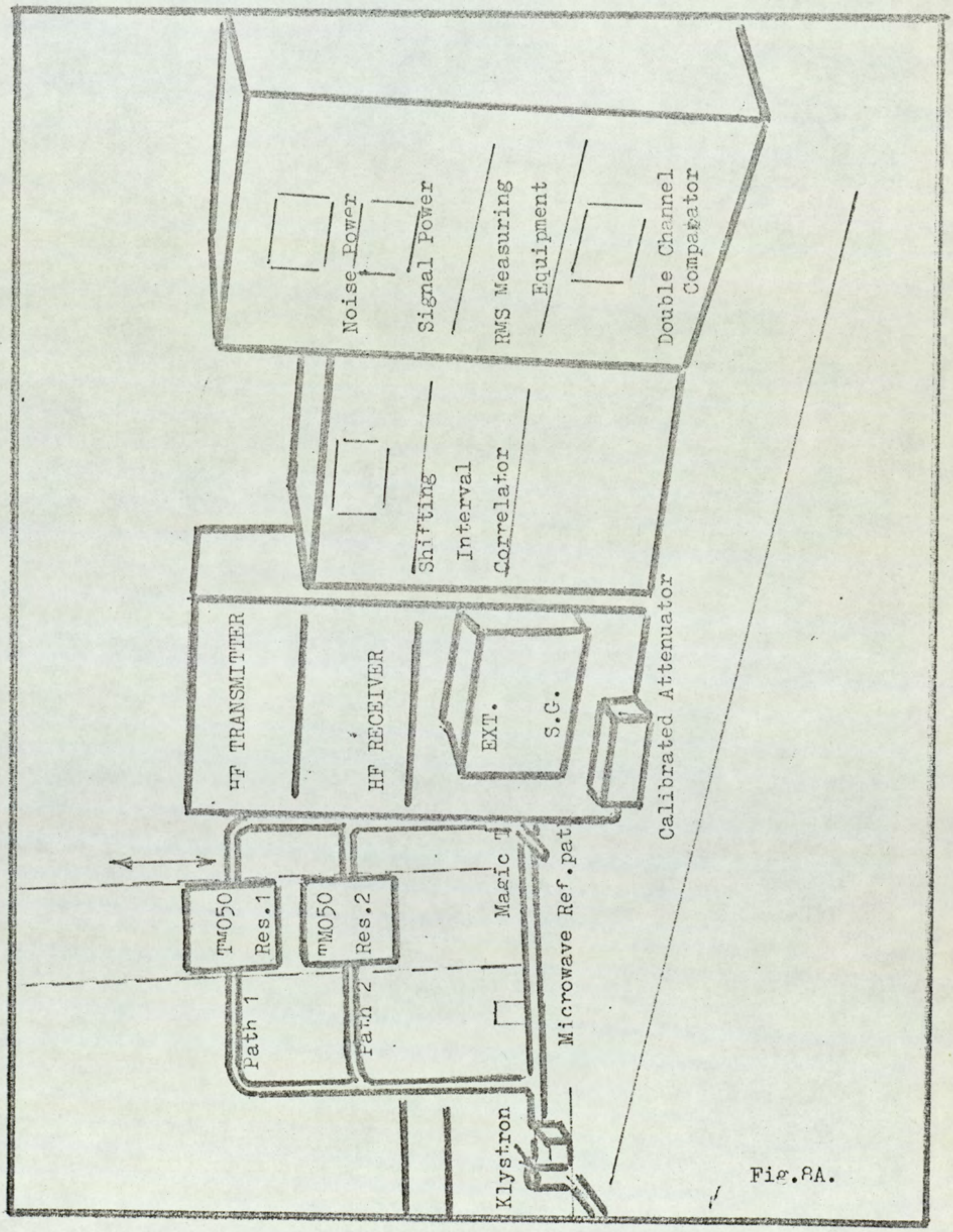
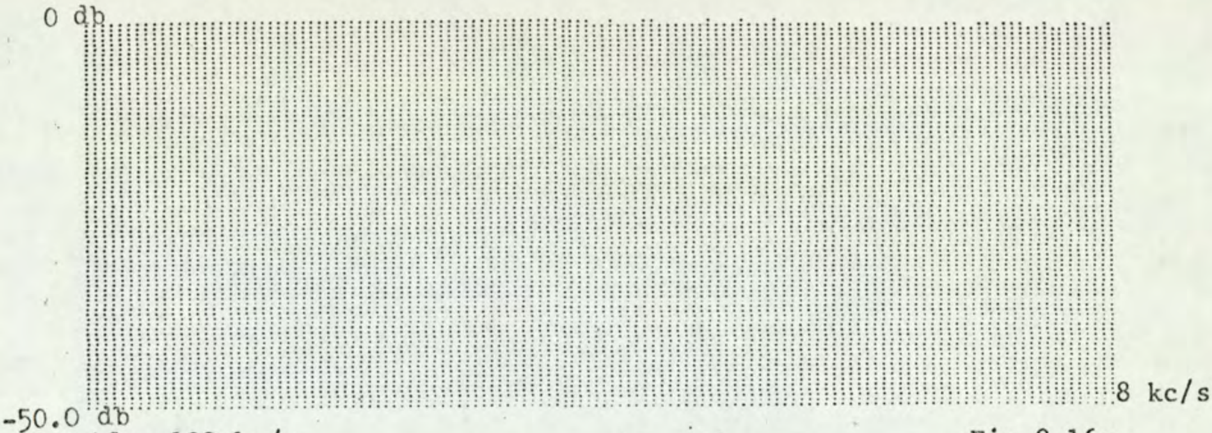
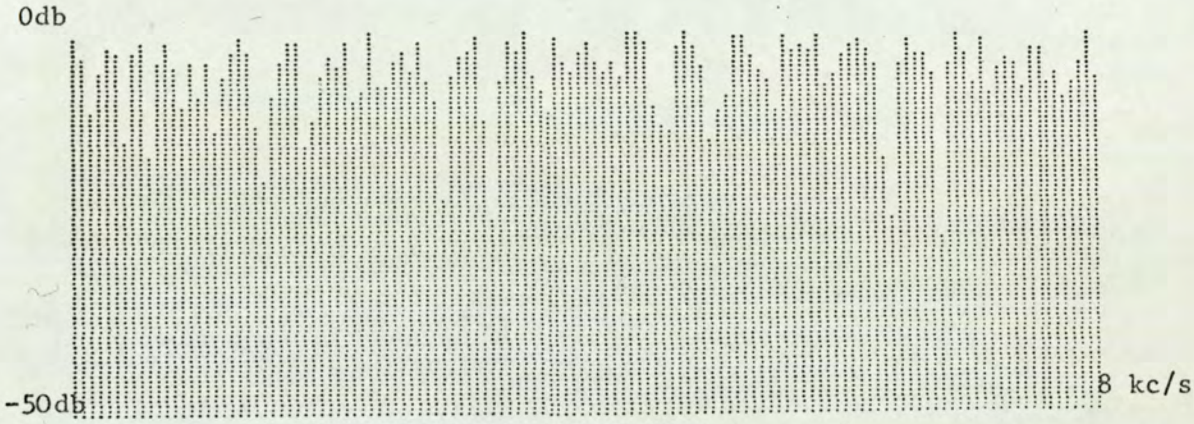


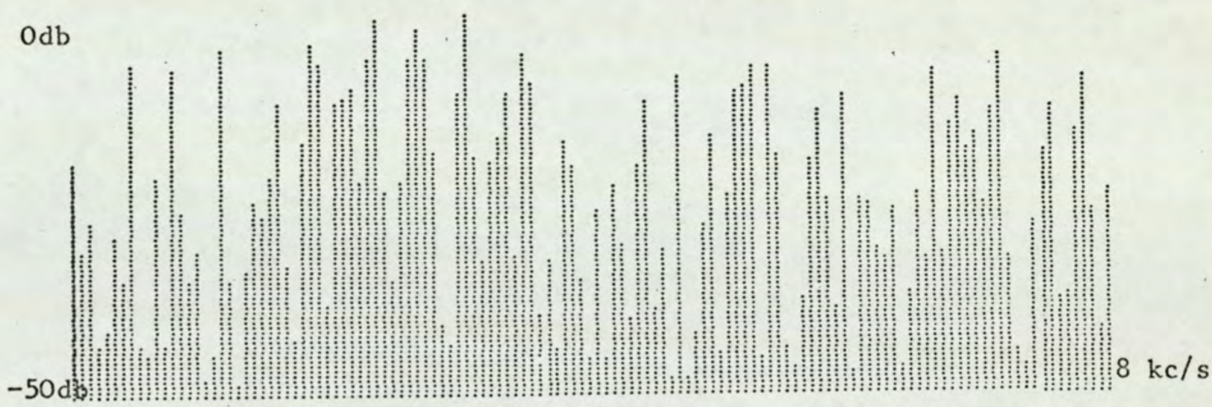
Fig.8A.



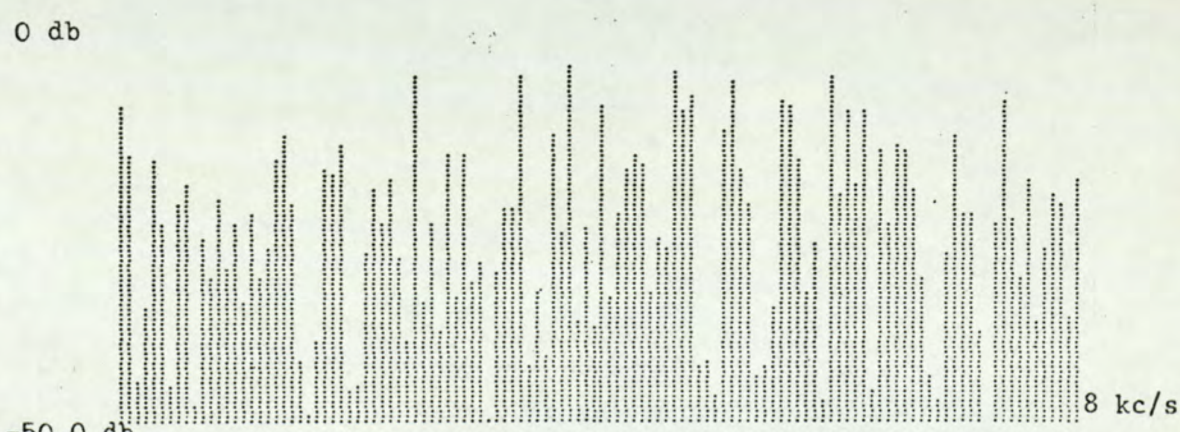
Δf = 100 kc/s   Fig.8.16a



Δf = 4 Mc/s   Fig.8.16b



Δf = 12 Mc/s   Fig.8.16c



Δf = 16 Mc/s   Fig.8.16d

Fig.8.16

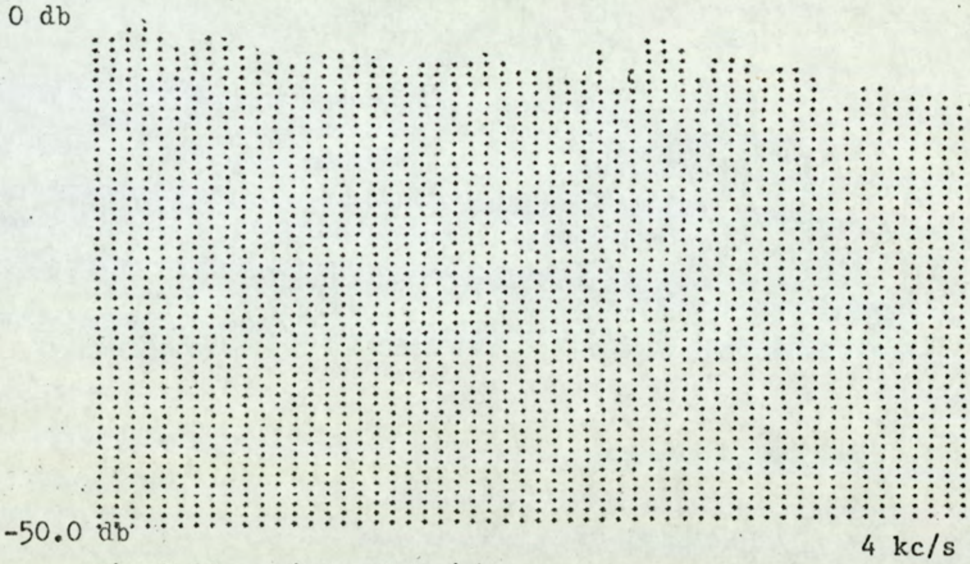


Fig.8.17a (Δf = 8 Mc/s)

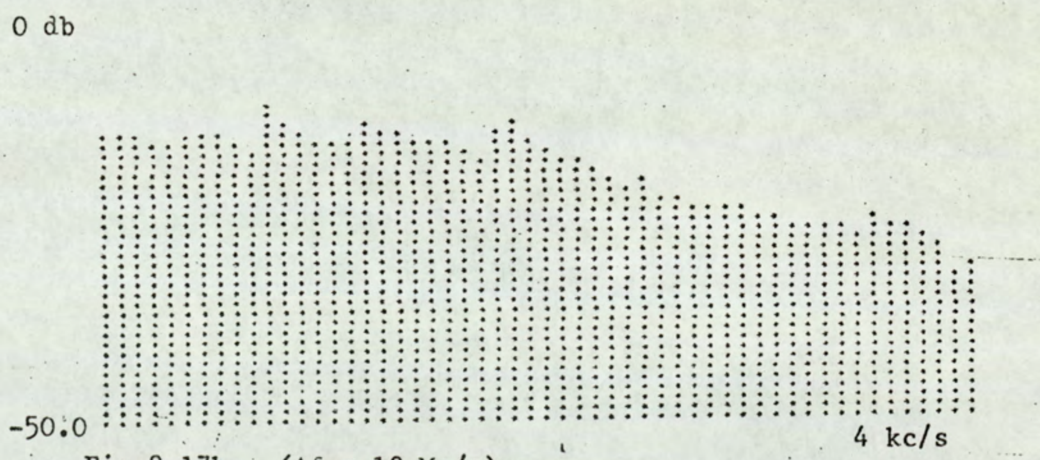


Fig.8.17b (Δf = 12 Mc/s)

Fig.8.17

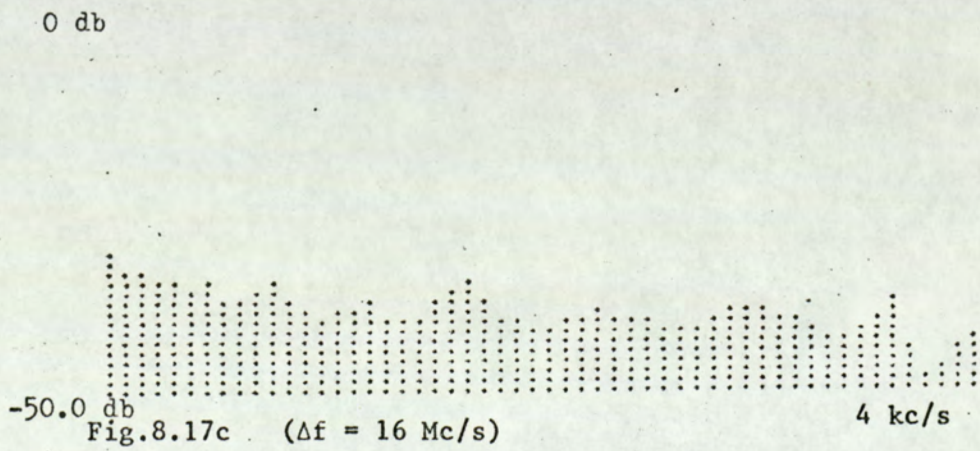


Fig.8.17c (Δf = 16 Mc/s)

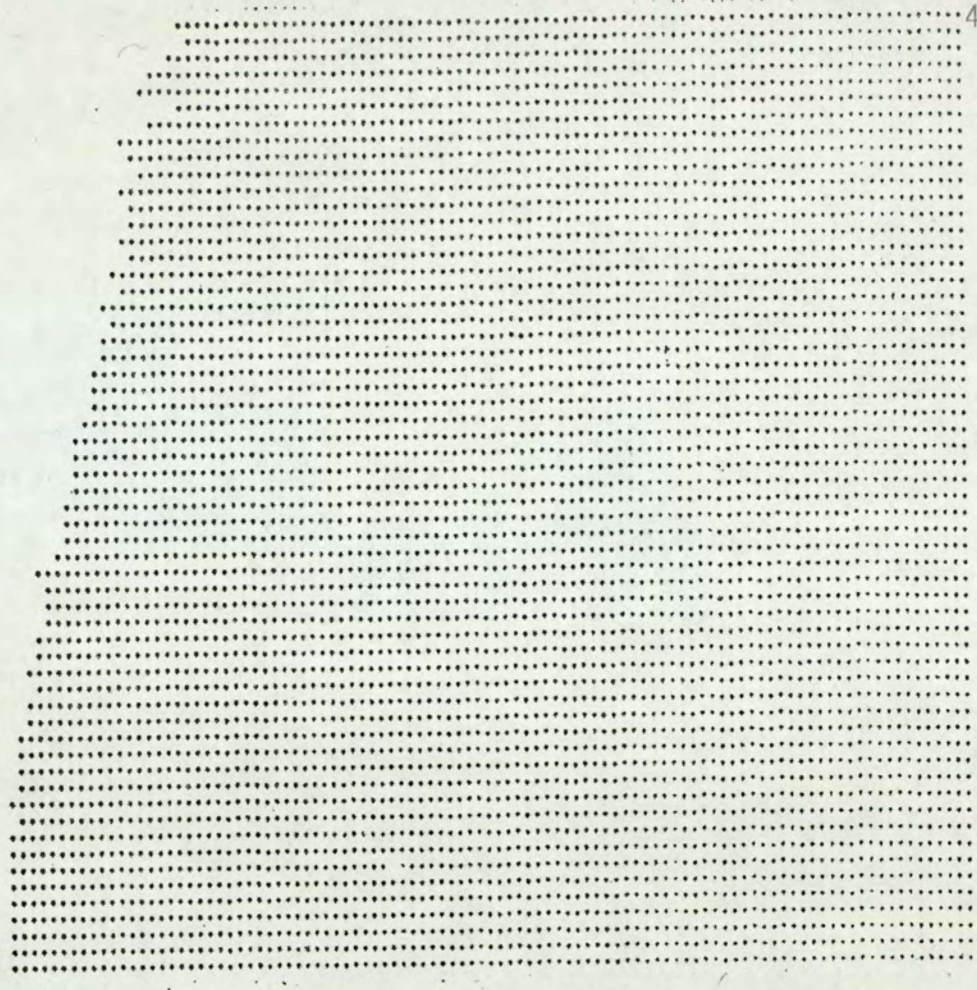


Fig.8.18b

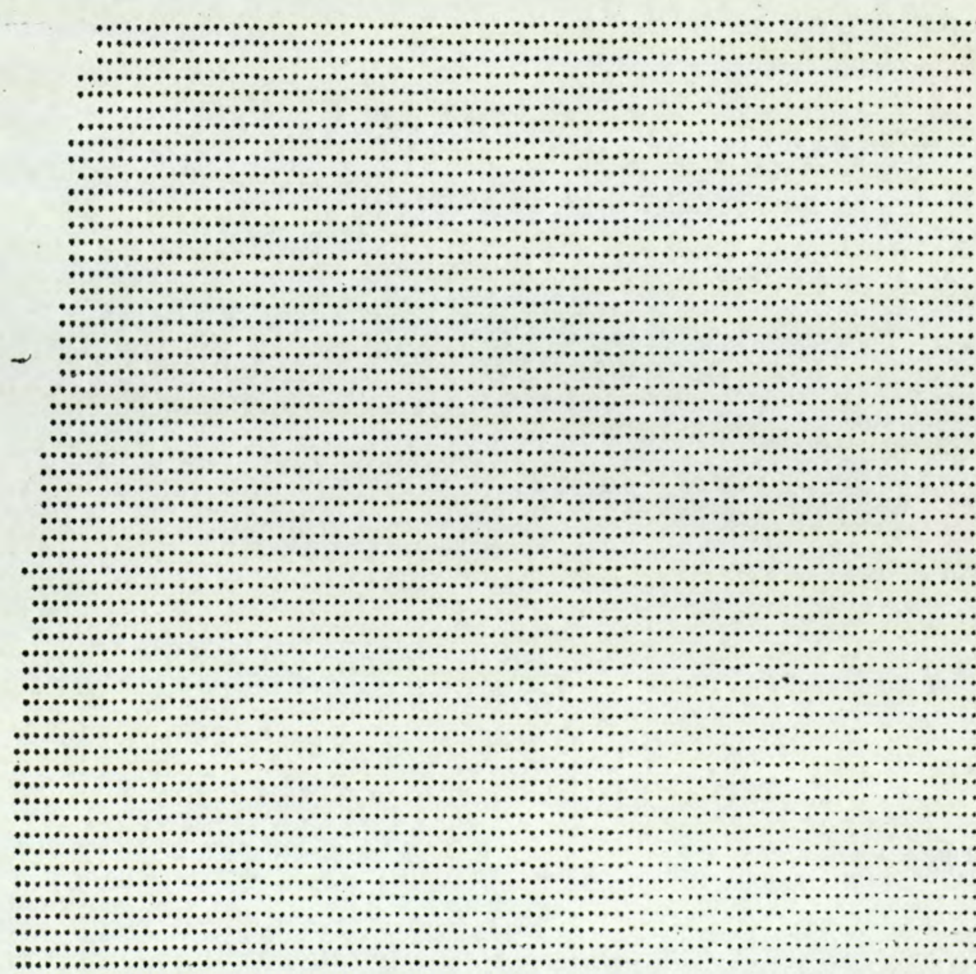


Fig.8.18a

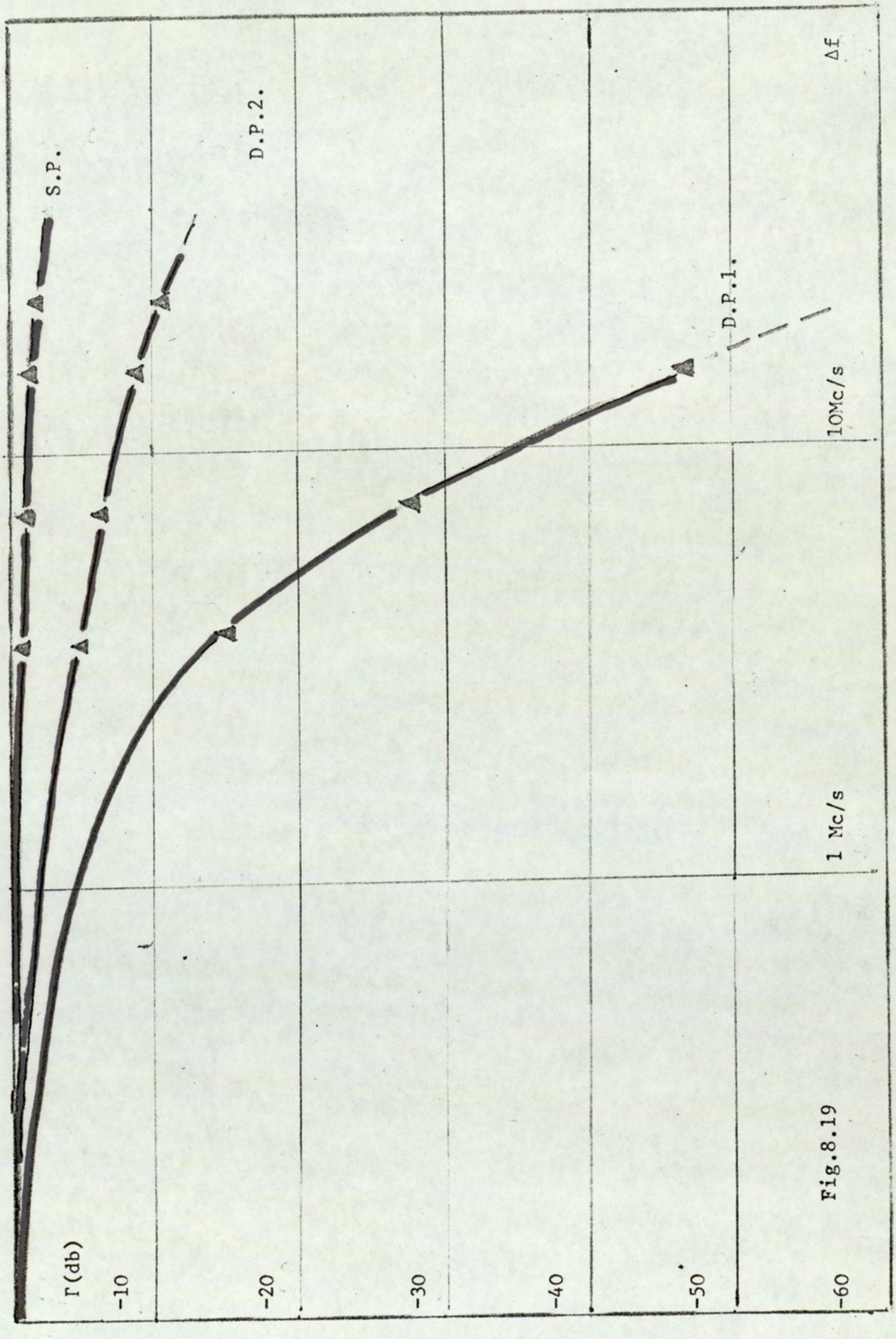


Fig.8.19

CHAPTER 9

9.1 Optimum Receiver - The WIENER FILTER

On a given perturbed signal x(t), it is desired to operate at the receiver in such a way, as to obtain as close as possible the original signal s(t).

As it was already shown, the noise might be either independent of the signal power, i.e. pure additive noise, or dependent on signal power, i.e. what we have called multiplicative noise. According to the kind of mechanism which generates it, the random process can be statistically stationary or nonstationary.

Additive Noise - Stationary case

The received signal x(t) is the sum of true signal s(t) and additive noise n(t). This is the simplest and most studied case. (Wiener, 1949). Consider the time invariant receiving filter following the detection stage shown in Fig. 9.1.

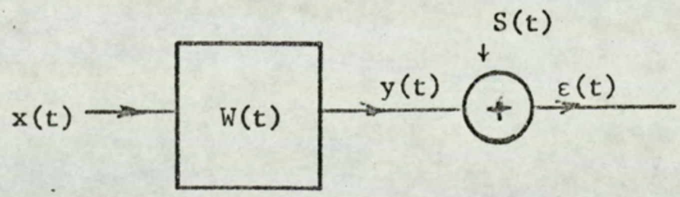


Fig.9.1

where x(t) = s(t) + n(t). Note that x(t) is the signal obtained after the necessary detection, but contaminated by noise.

The receiving filter is assumed to have an impulse response w(t). If the desired transmitted signal before the process of modulation were present at the receiver, there could be found a signal error ε(t), as the difference between the desired and the actual output, that is

$$\varepsilon(t) = s(t) - y(t) \qquad 9.1.1$$

The performance criterion of the quality of the total system is the mean squared error $\overline{\varepsilon^2(t)}$, i.e.

$$\overline{\varepsilon(t)^2} = \overline{[s(t) - y(t)]^2} \qquad 9.1.2$$

The filter output can be expressed in terms of the input x(t) and the impulse response w(t). It is assumed also that the system is noiseless, and the total additive noise is added in the transmitting channel. Starting from

$$y(t) = \int_0^t x(t - \tau) \, w(\tau) d\tau \qquad 9.1.3$$

and the minimum mean squared error criterion, it can be shown by the use of the calculus of variations (Esgolc, 1962) that

$$\int_0^t \psi_{xx}(\tau - \sigma) w_o(\sigma) d\sigma = \psi_{xs}(\tau) \qquad 9.1.4$$

which is the well known Wiener-Hopf integral equation. Transferring this equation in the complex frequency domain, and using spectrum factorization technique, the desired filter transfer function w(s) is obtained.

The optimum filter problem for additive stationary noise was solved by Kolmogoroff (1941) and Wiener (1949). A simplified derivation of linear least square filtering is done by Bode and Shannon (1950), which is summarized in Appendix 13. From the derived expression

$$W(j\omega) = \frac{P(\omega)}{P(\omega) + N(\omega)} \, e^{j\alpha(\omega)} \qquad 9.1.5$$

it is obvious that the optimum filter has the frequency response which depends on both the signal $P(\omega)$ and noise $N(\omega)$ power density spectra as well as on the delay coefficient $\alpha(\omega)$, which it should be remembered, is valid only if the noise is independent of the signal and is stationary.

9.2 Adaptive Receiver - The KALMAN FILTER

A new method of filtering nonstationary random processes has been described by Kalman and Bucy (1960,1961).

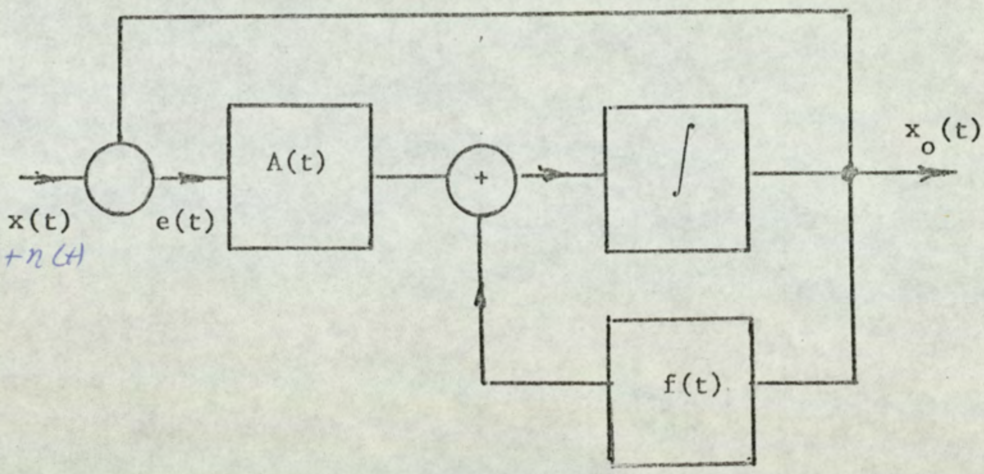


Fig.9.2

The optimum estimate (Fig.9.2) ($x_o(t)$) is given by

$$x_o(t) = \int_{t_0}^{t} \omega_o(t,\tau)Z(\tau)d\tau \qquad 9.2.1$$

where $\omega_o(t,\tau)$ is the weighting function of the optimum - adaptive time varying filter,

$$\text{and } Z(t) = x(t) + n(t) \qquad 9.2.2$$

Starting from the Wiener-Hopf equation for time varying systems

$$\int_{t_0}^{t} Wo(t, \tau)\psi_{2z}(\sigma,\tau)d\tau = \psi_x (t,\sigma) \qquad 9.2.3$$

It could be shown (Shear, 1965), that

$$\frac{dx_o}{dt} = \int_{t_0}^{t} \frac{\partial}{\partial t} |\omega_o(t,\tau)|Z(\tau)d\tau + \omega_o(t,\tau)Z(t) \qquad 9.2.4$$

Defining a time varying gain at (t)

$$A(t) = \omega_o(t,t) \qquad 9.2.5$$

and substituting in the last equation

$$\frac{dx_o}{dt} = f(t)x_o(t) - A(t)x_o(t) + A(t)Z(t) \qquad 9.2.6$$

By introducing the optimal error $e_o(t) = x(t) - x_o(t)$ the following differential equation may be written

$$\frac{de_o}{dt} = |f(t) - A(t)|e_o(t) + \xi(t) - A(t)n(t) \qquad 9.2.7$$

where it was assumed that the process can be assumed to be generated by a time varying system which behaves in accordance with the differential equation

$$\frac{dx}{dt} = f(t)x(t) + \xi(t) \qquad 9.2.8$$

The time derivative of the optimal error can be written as

$$\frac{de_o^{\,2}}{dt} = 2e_o \frac{de_o}{dt} \qquad 9.2.9$$

which is a function of time, (say p(t) = $\overline{e_o(t)^2}$)

Therefore

$$\frac{dp(t)}{dt} = 2|f(t) - A(t)|p(t) + 2\xi(t)\varepsilon_o(t) - 2A(t)n(t)e_o(t) \qquad 9.2.10$$

Denoting $\xi(t)e_o(t) = \frac{1}{2}\,Q(t)$, and

$$n(t)e_o(t) = \frac{1}{2}\,A(t)r(t) \qquad 9.2.11$$

The differential equation for p(t) can now be written in the form

$$\frac{dp}{dt} = 2|f(t) - A(t)|p(t) + Q(t)+A^2(t)r(t) \qquad 9.2.12$$

It has been shown by Kalman (1961) that

$$A(t) = \frac{p(t)}{r(t)} \qquad 9.2.13$$

and

$$\frac{dp}{dt} = 2f(t)p(t) - \frac{p^2(t)}{r(t)} + Q(t) \qquad 9.2.14$$

which is a form of differential equation known as the Riccati equation. This equation allows finding solution for p(t), i.e. $e_o^{\,2}(t)$, when the initial value $p(t_o)$ is given. (Piaggio, 1952)

9.3 Vector Representation of minimisation of $p^2(t)$.

The ensemble of outputs <Z(t)> can be represented as a vector in a multidimensional space. The ensemble of desired outputs <x(t)> can also be represented by another vector. The ensemble of errors <e(t)> is given by the vector difference of these two. The square of the magnitude of the error vector is equal to the sum of squares of the individual error components. The mean squared error is minimum when the end vector points of the actual and the derived signal are as close as possible. Suppose that it is possible to apply the actual output signal through an attenuating or amplifying system whose gain or attenuation can be regulated, so that the magnitude of the signal obtained can be adjusted at will. (Kalman 1960, Yaglom 1962)

Taking for the simplicity of representation a three dimensional space, which can be visualised, it becomes obvious that the error vector is a minimum when it becomes orthogonal to the actual output vector. Increasing the number of vector components does not change this property. For the well known Gaussian distributed, white spectrum additive noise, no further improvement is possible and the error is minimised.

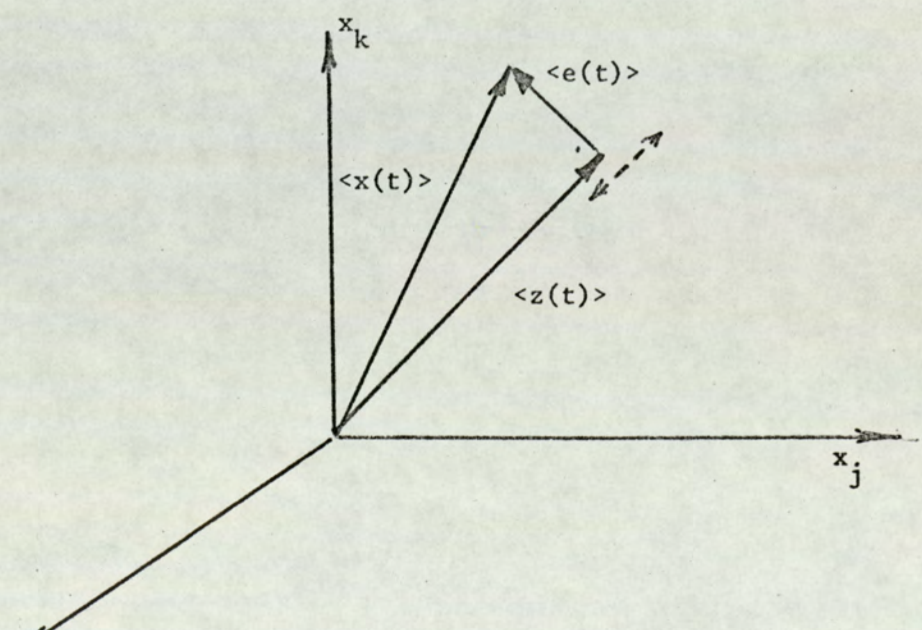


Fig.9.3.

9.4 Aston modification of Kalman Filtering.

When the signal is passed through a system with time varying parameters, the error vector depends on the signal intensity (Bull, 1961, Bull and Bozic, (1967). Reducing the signal intensity to zero makes the dependent portion of the noise power disappear too. Furthermore, the detected noise is not only due to the rapid magnitude perturbations of the signal, there is experimental evidence (Chapter 8) that the observed noise is more dependent on phase than magnitude fluctuations (Figs 8.1 & 8.2

Hence, for the multiplicative type of noise, by changing the gain of the amplifier, the length of the error vector will be reduced by suppressing the signal magnitude fluctuations.

Further reductions will be achieved if the phase error is corrected simultaneously, as will be demonstrated in this chapter.

It should be noted that the signal carrier phase fluctuations might be observed as being magnitude fluctuations, but not vice versa. Similarly, when a phase modulated signal is transmitted, through a system which in turn will add phase randomness, the received signal will be fluctuating in magnitude as well as phase.

The observed magnitude fluctuations, especially when a multipath signal propagation is possible, can be due to a great extent to the time dependent cancellation of the various signal vector components.

A simultaneous transmission of a deterministic reference signal, through the same physical system will help in suppressing both intensity and phase fluctuations. The rapid acting A.G.C. will keep the received reference to an almost constant value, reducing considerably fluctuations of the reference and the information signal components which are in the same frequency band.

Slowly fluctuating phase errors are usually of no importance, since it is always possible, and in fact is already done, to utilise a frequency and/or a phase locked local oscillator at the receiver. When the phase fluctuations are fast, such an oscillator will lock to the mean carrier frequency, or to any other desired frequency differing from the mean carrier frequency by a fixed amount.

Using the reference signal for mixing, instead of a locked local oscillator signal, will bring further improvement, since phase fluctuations of all signal components are coherent in a limited frequency band.

In the following few pages it will be described the system utilising simultaneously rapid acting phase and rapid acting gain control, which we will call 'ASTON FILTERING'.

$*S_o(t)$ received reference signal

$*\omega_c$ frequency of the received reference signal

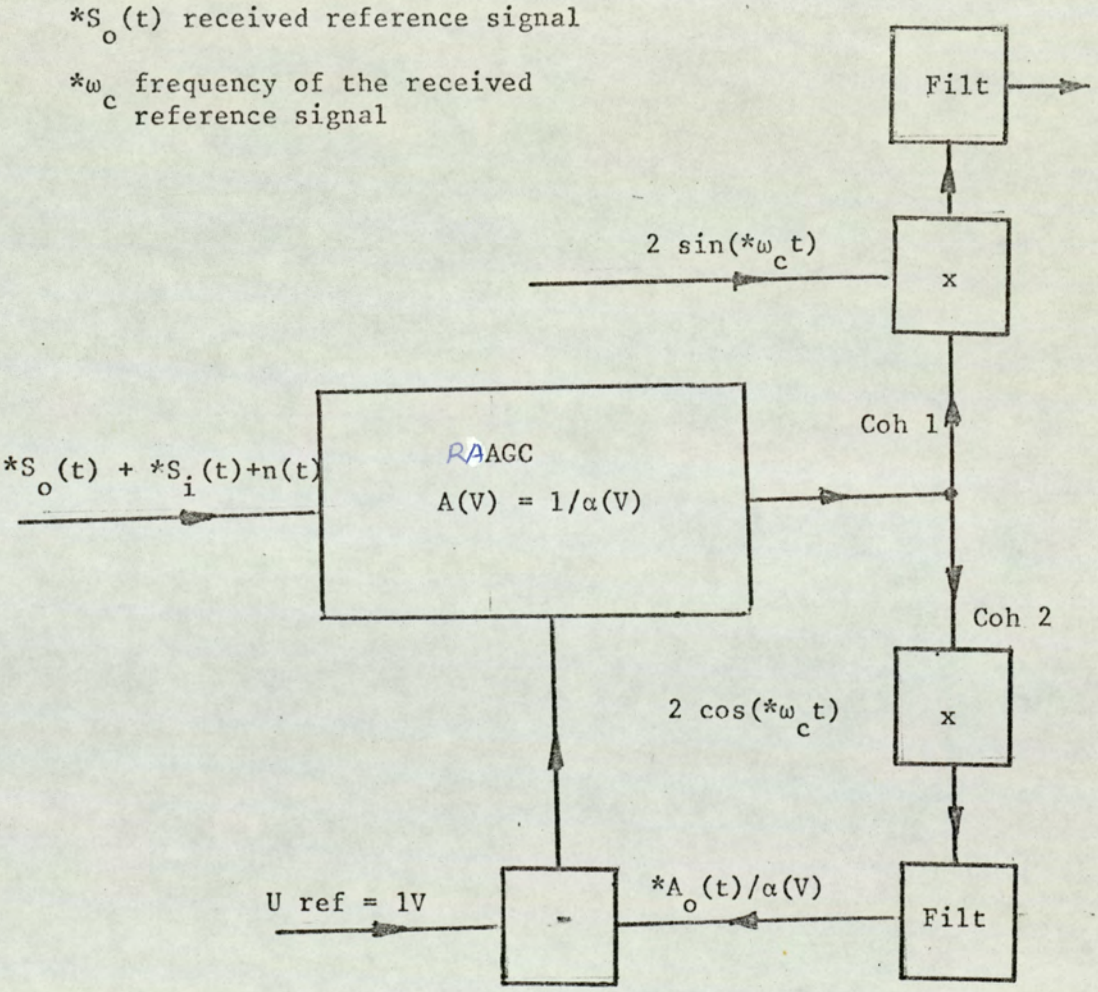


Fig.9.4.

9.5 R.A.A.G.C. Coherent band receiver - Aston Filtering.

A time truncated signal $m(t)$ defined in the interval $0 - T$ may be treated mathematically as a periodic signal and therefore treated as a Fourier series

$$m(t) = A_o + \sum_{n=-\infty}^{\infty} \cos \frac{2\pi n}{T} t + \sum_{n=-\infty}^{\infty} \sin \frac{2\pi n}{T} t \qquad 9.5.1$$

Let m(t) be the information to be transmitted through the transmission system which has fluctuating parameters. Since the information must be transmitted in a limited bandwidth, the frequency f_{max} defines n_{max}, the highest harmonic number, by

$$n_{max} = M = f_{max} \cdot \frac{T}{2} \qquad 9.5.2$$

so that $m(t)$ may be re-written as

$$m(t) = \sum_{n=-M}^{M} C_n \exp\left(jm\omega t + \theta_n\right), \text{ where } C_n = \sqrt{A_n^2 + B_n^2}, \qquad 9.5.3$$

$$\theta_n = \arctan B_n/A_n$$

For the double sideband suppressed carrier, DSBSC system the signal is given by

$$S_I(t) = \tfrac{1}{2} \sum_{n=-M}^{M} C_n \exp(jm\omega t) \left[\exp(j\omega_c t) + \exp(-j\omega_c t)\right] \qquad 9.5.4$$

where $\omega_c = 2\pi f_c =$ the angular frequency of the carrier. This expression consists of spectral pairs of the type

$$\left(C_n \cos(\omega_c + n\omega)t + \theta_n\right) \text{and} \left(C_{-n} \cos(\omega_c - n)t + \theta_{-n}\right) \qquad 9.5.5$$

Ideally both components of a spectral line are equal, and the pair is considered to be symmetrical, so that $\theta_n = \theta_{-n}$.

Suppose the carrier $S_o(t) = A_o \cos\omega_c t$. Then the AMDSB signal is

$$S_{AM}(t) = S_o(t) + S_I(t) = A_o\left(1 + m(t)\right) \cos \omega_c t \qquad 9.5.6$$

Let us consider only one spectral pair

$$^{(1)}S_{AM}(t) = A_o \left(1 + \frac{2C_1}{A_o} \cos(\omega t + \theta_1)\right) \cos \omega_{ct} \qquad 9.5.7$$

Changing the phase of the carrier by $\pi/2$ (or of the sidebands by $-\pi/2$).

$$^{(1)}S_{AM}(t) = A_o \sin\omega_c t + 2C_1 \cos(\omega t + \theta_1) \cos\omega_c t \qquad 9.5.8$$

which can be expressed in terms of a magnitude and an instantaneous phase by

$$^{(1)}A(t) = |S(t)| = A_o \sqrt{1 + 2 \frac{C_1}{A_o} - 2 \frac{C_1}{A_o} \cos(2\omega t + 2\theta_1)} \qquad 9.5.9$$

$$^{(1)}\psi(t) = \frac{C_1 \cos(\omega t + \theta_1)}{A_o} \qquad 9.5.10$$

It is evident that the signal is both amplitude and phase modulated.

When transmitted through a transmission channel whose parameters vary randomly with time, both the amplitude and phase will be altered. Let $^{(k)}\eta(t)$ be a sample function from an ensemble of random functions $\{\eta(t)\}$, which operates on the magnitude of the transmitted signal to give the received signal. Similary, let $^{(k)}\psi_r(t)$ define the random amount which must be added to the instantaneous phase of the signal. In general different spectral components will suffer different random modifications by the fluctuations in the parameters of the transmission system. Restrict the attention to the coherent portion of the spectrum, i.e. to the bandwidth, $B_c m$ over which the fluctuations in magnitude of the individual spectral components are correlated and in which all the phases are changed by the same amount on account of the fluctuations in the parameters of the transmission system.

In cases in which the bandwidth required for transmitting information is larger than that in which the amplitude and phase coherence persists, then the information carrying spectrum must be split into several portions over each of which coherence is maintained. After detecting and restoring each coherent channel separately they can all be re-combined to form the received information signal.

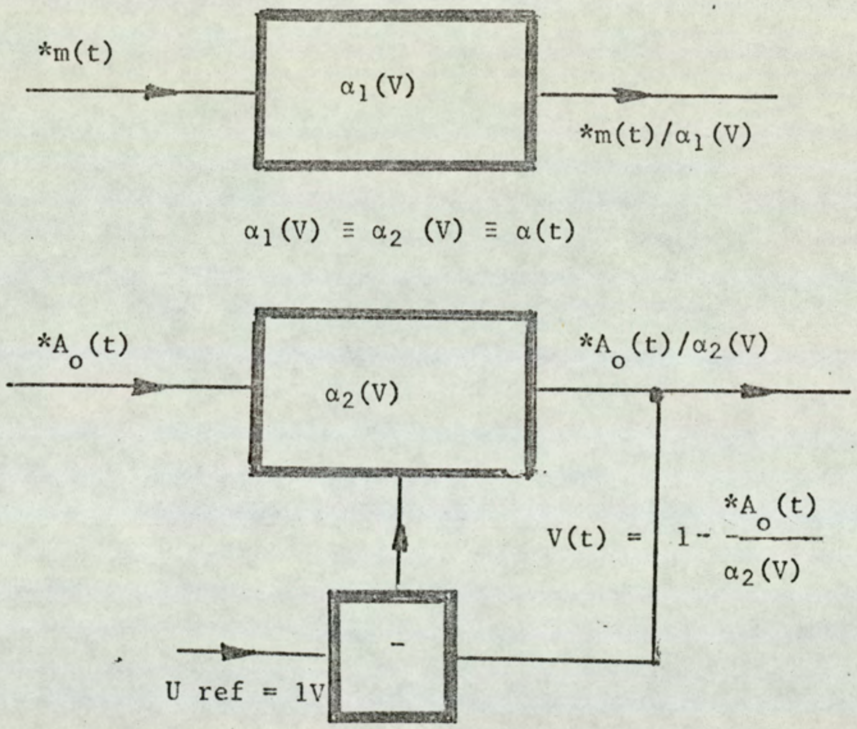


Fig.9.5.

9.6 Analysis of a method of detection using fast correlation with feedback.

Experiments have been carried out in which correlation is measured in a short period, i.e. the averaging associated with the estimation of correlation is carried out, not as is usual, over successive long periods of time but over a succession of short periods of time.

The description of the utilisation of this method will be assisted by the use of block diagrams showing how the action can be represented. It should be noticed that at the transmitter end of the system a relatively low frequency carrier is made to be orthogonal to the information spectrum sidebands, in the manner discussed above. In the experiments, the range of frequencies was in the range 4 MHz < F < 20 MHz, but these signals and their carrier were transmitted as amplitude modulation on a final carrier of about 10 GHz, i.e. in the 3 cm band of wavelengths.

After the first mixing stage in the receiver the I.F. signal is restored, which consists of a DSB spectrum and an I.F. carrier rotated through $\pi/2$ from its initial phase before transmission. It is well known that coupled tuned circuits at resonance change the phase by $\pi/2$. This is used as a convenient method of obtaining the necessary reference signals, $2 \cos \omega_c^* t$ and $2 \sin \omega_c^* t$ where ω_c is the instantaneous I.F. radian frequency at the receiving end of the system. On account of phase shifts in the transmission path which vary in a random manner the recovered instantantaneous reference signal frequency is not necessarily equal to that of the transmitted signal.

If $*(t)$ represent the carrier, with its random fluctuations due to the fluctuations of the transmission system, then

$$S_o^*(t) = {}^{(k)}\eta(t).A_o \sin \omega_c t + {}^{(k)}\psi_r(t) + \psi(t) \qquad 9.6.1$$

$$S_I^*(t) = {}^{(k)}\eta(t).m(t) \cos \omega_c t + {}^{(k)}\psi_r(t) + \psi(t) \qquad 9.6.2$$

are expressions for the amplitude of noise and signal after transmission and before reception provided all the frequencies considered are in the coherence bandwidth.

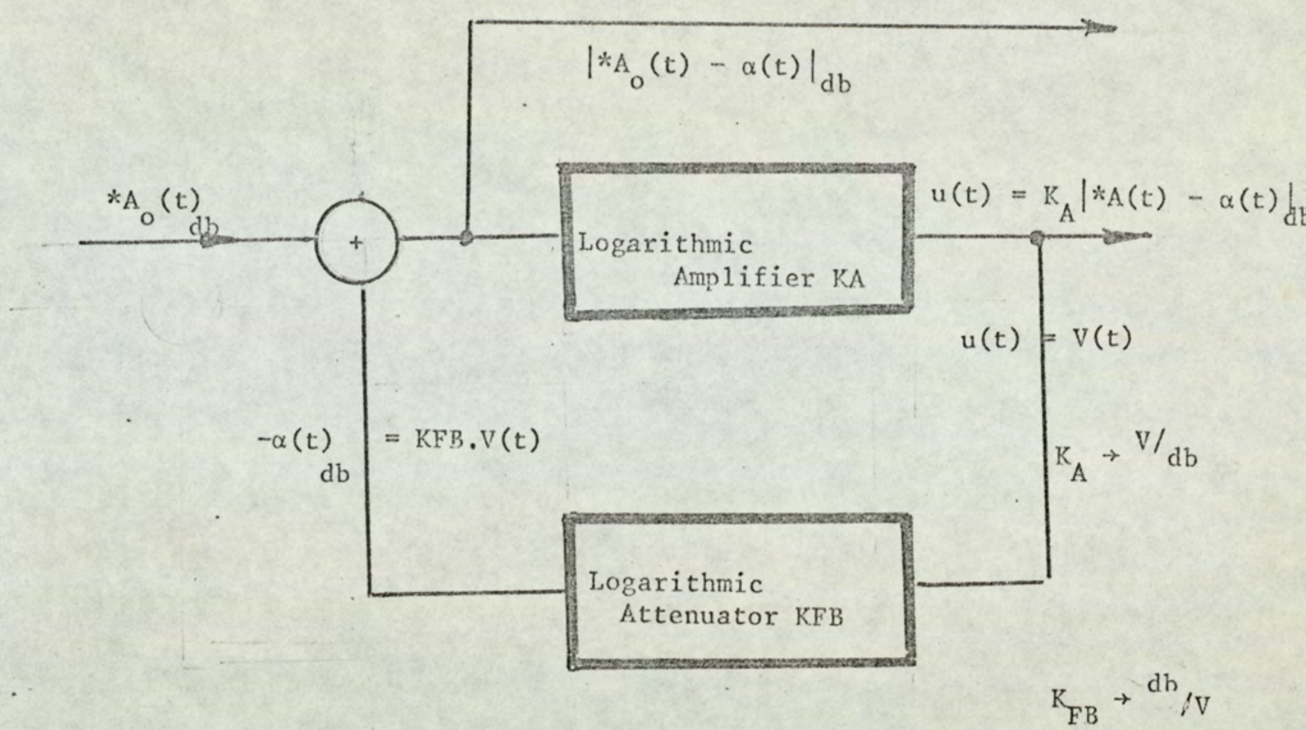


Fig.9.6(a)

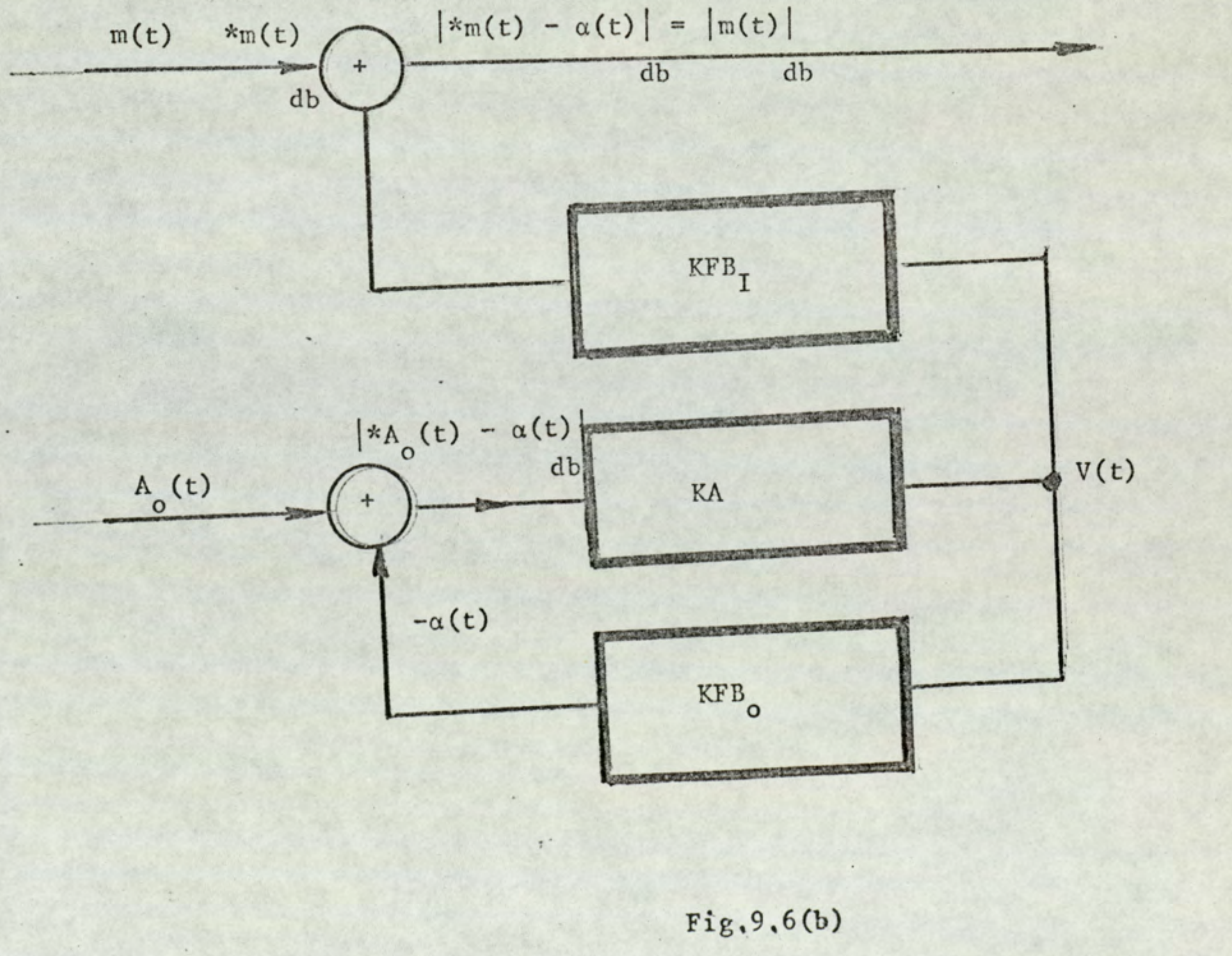


Fig.9.6(b)

In the block diagram, Fig. 9.4 AGC represents the wideband I.F. amplifier with fast automatic gain control. This has a gain $A(V)$ which is a function of V, the signal voltage.

When there is no L.F. carrier, this stage has a maximum gain, A_{max}. As the I.F. carrier amplitude increases, the amplification decreases. This can be expressed as an attenuation $\alpha(V) = 1/A(V)$, this attenuation being proportional to the carrier amplitude.

After I.F. amplification both components of the signals, $S_o^*(t)$ and $S_I^*(t)$ are separately detected by two coherent detectors, $C_{oh}O$ and $C_{oh}I$.

To simplify the notation, put

$$A_o^*(t) = A_o \, {}^{(k)}\eta(t) \qquad 9.6.3$$

$$m^*(t) = m(t) \, {}^{(k)}\eta(t)$$

$$\omega_c^* = \omega_c + {}^{(k)}\psi_r(t)$$

At the coherent detectors output, neglecting additive noise interference,

$$\frac{A_o^*(t)}{\alpha(V)} \quad 1 - \cos 2\,\omega_c^* t + 2\,\frac{\omega^*(t)}{\alpha(V)} \,.\, \cos \omega_c^* t . \sin \omega_c^* t \qquad 9.6.4$$

as the output from $C_{oh}(O)$. Similarly, from the detector $C_{oh}(I)$ the output is

$$\frac{m^*(t)}{\alpha(V)} \quad 1 + \cos 2\,\omega_c^* t + 2\,\frac{A_o^*(t)}{\alpha(V)} \,.\, \cos \omega_c^* t .\ \sin \omega_c^* t \qquad 9.6.5$$

At the corresponding filter outputs there exist only the lowest frequency terms, for all high frequency components are suppressed and become negligible.

It is almost obvious that since both signals are treated in the same way by amplifier RAAGC and the signals are orthogonal to one another, nothing would be changed if two separate amplifiers with identical gains at all times were used. One may therefore separate out the circuit shown in Fig.9.4 into the two parts shown in Fig.9.5.

As the coherence detectors and the filters eliminate high frequency components, further simplification is possible. Thus, at the input of the voltage regulated amplifier $m^*(t)$ and $A_o^*(t)$ appear separately.

Since the control voltage V is a function of time

$$V(t) = 1 - \frac{A_o^*(t)}{\alpha_2(V)} = 1 - \frac{{}^{(k)}\eta(t).A_o}{\alpha_2(V)} \qquad 9.6.6$$

where A_o is the constant amplitude of the I.F. carrier transmitted, and which serves as a reference signal for amplitude. ${}^{(k)}\eta(t)$ is a sample function from all ensembles of a random function, which is a Markovian process.

$\alpha_2(V)$ is a function of the regulating voltage $V(t)$, and this is itself a function of time. Let us define the voltage dependence

$$\begin{aligned}\alpha_2(V) &= \alpha_o K(V) \\ &= \alpha_o \exp_k 1 - \frac{A_o(t)}{\alpha(V)}\end{aligned} \qquad 9.6.7$$

The RAAGC is provided in order to change

$$\alpha(t) = \alpha(V(t)) \qquad 9.6.8$$

as nearly as possible in synchronism with $\eta^*(t)$, and make ${}^{(k)}\eta(t).A_o/\alpha(V(t))$ as close as possible to unity at all times. Thus, when the RAAGC operates in the way intended,

$$V(t) = 1 - \frac{A_o^*(t)}{\alpha(t)} \qquad 9.6.9$$

and is a very small quantity close to zero.

AGC operation is based on the existence of a high degree of non-linearity in the control of the amplifier. In order to use approximations and methods already adopted in control theory, let us use some simplifications similar to those used by Victor and Brockman (W.K.Victor and M. H. Brockman, 1960). The essentials are based in the transformation of an absolute signal magnitude, expressed in volts, into relative magnitudes with reference to unit voltage, represented here as 1V, corresponding to 0 db, in other words, introducing time functions expressed in db units. Note that relative voltages become dimensionless, as are amplification and attenuation.

Also, an exponential dependence on voltage becomes linear, so that logarithmic voltage converters may be substituted for linear devices, as shown in Fig. 9.6a.There a logarithmic amplifier and attenuator is substituted for a voltage to attenuation conversion in Fig.9.6b

At the output of the logarithmic amplifier

$$U(t) = K_A(A^*(t)_{db} - \alpha(t)_{db}) \qquad 9.6.10$$

where K_A is in volts per db, and $U(t)$ is in volts.

On the other hand,

$$- \alpha(t)_{db} = K_{FB}. V(t) \qquad 9.6.11$$

where K_{FB} x is in db per Volt.

Also put

$$U(t) = 20 K_A \log_{10} \frac{A_o^*(t)}{\alpha(t)} \qquad 9.6.12$$

since $A_o^*(t)/\alpha(t)$ is very nearly equal to unity and

$$V(t) = \frac{A_o^*(t)}{\alpha(V)} \qquad 9.6.13$$

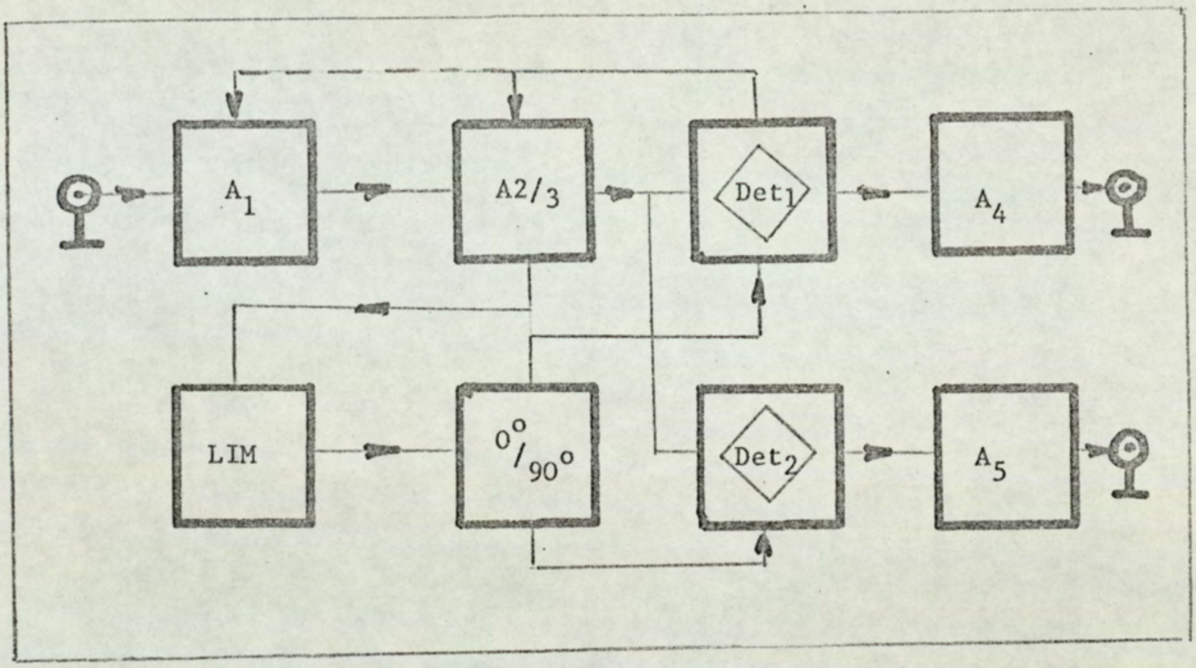

A1
A2/3
Det1
A4
LIM
0°/90°
Det2
A5

Fig.9.7

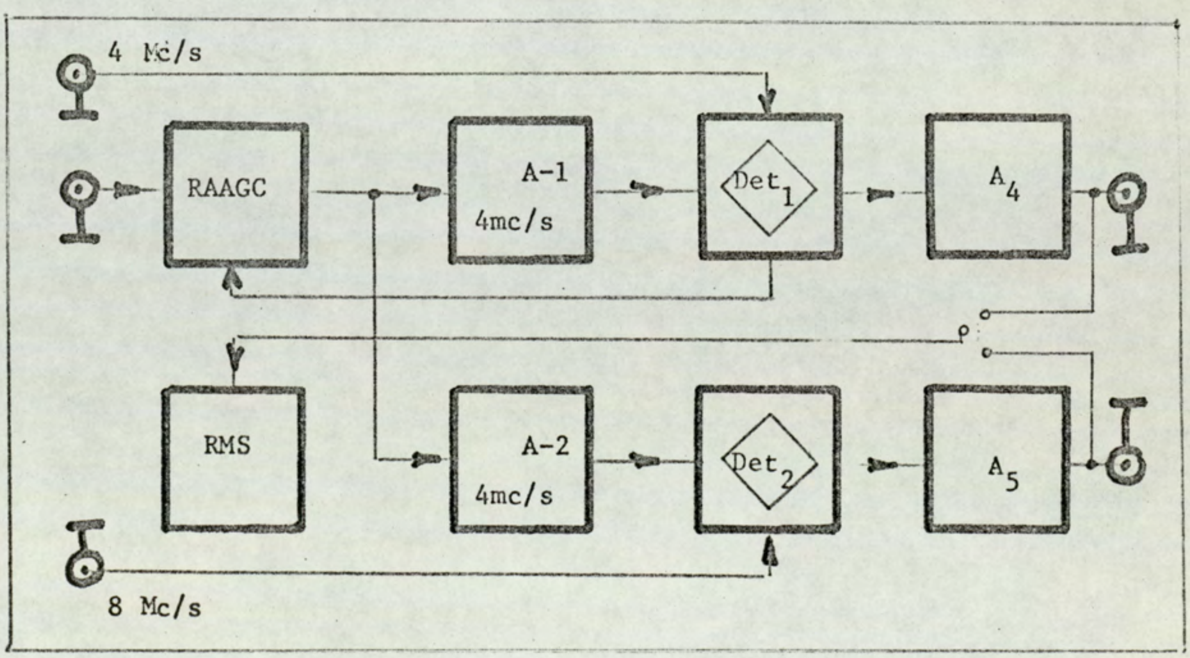

4 Mc/s
RAAGC
A-1
4mc/s
Det1
A4
RMS
A-2
4mc/s
Det2
A5
8 Mc/s

Fig.9.8

under which conditions

$$U(t) \equiv V(t) \qquad 9.6.14$$

and one may close the loop without altering the regulation.

Introducing this concept (Fig.9.6) note that K_{FB} is split into two identical paths to be in accordance with (Fig.9.5)

If A_o be taken as the reference level, i.e. $A_o = 1$, then recalling that

$$A_o^*(t)_{db} - (t)_{db} \equiv 0 \text{ db} \qquad 9.6.15$$

$$\alpha(t)_{db} = {}^{(k)}\eta(t)_{db} \qquad 9.6.16$$

and therefore the output from the upper voltage controlled attenuator

$$m^*(t)_{db} - \alpha(t)_{db} = m(t)_{db} \qquad 9.6.17$$

and the original information is retrieved completely when

$$\alpha(t)_{db} = {}^{(k)}\eta(t)_{db} \qquad 9.6.18$$

i.e. when the relative variation of the attenuation is exactly the same as the fluctuation of the signal intensity on account of the fluctuations in the parameters of the transmission system.

It was originally assumed that the low pass filter has a flat frequency response up to the highest frequency spectrum component of interest in the fluctuation, and that it has a very high attenuation for higher frequencies. The frequency characteristic of the AGC loop will be discussed in a later section.

The consideration of the influence of White additive noise has been completely omitted for two reasons,

1. All experiments were conducted with signal strengths which were well above any possible level of additive noise, and
2. Coherence detection inherently reduces the effect of white noise, even when wideband amplifiers are used.

9.7 Experiments using Rapid Acting Feedback in the Coherent Band.

Aston filtering, as it has been already shown is based on existence of coherence of fluctuations in magnitude as well as in phase of all signal components in a limited band of frequencies. Depending on the maximum delay differences encountered, this limited band in some cases may extend to several Mc/s, or to be only several kc/s or even less, when the volume of the propagation medium through which signals can travel from transmitter to receiver is very large and there exist several paths, i.e. there is a multipath transmission. Fortunately, the bigger delay differences are, the slower the fluctuations which can be expected, so that coherence bandwidth is usually wider than the band in which noise components due to signal fluctuations exist. Although, therefore, the reference signal and the information bearing signal, in general will be easy to separate, in the case of a very narrow coherence band, this could become a problem. It is therefore suggested, that the orthogonality of these signals be the basic form of transmission. On the other hand, when there is enough bandwidth available and these signals may be easily separated by means of band filtering, the reference signal and the information signal may be appreciably separated in frequency. The available bandwidth and the number of channels to be used will decide the method of modulation to be chosen.

Almost all experiments were carried out for an orthogonal reference and information signals. The test frequency, adopted for experiments, was 4 mc/s, where the rapid acting AGC and the rapid acting APC has been used.

The receiver consisted of a three-stage amplifier, A_1, A_2 and A_3 (Fig.9.7). The first stage is controlled by an ordinary AGC which brings the average signal level to be fairly constant and therefore enabled the RAAGC, A_2 and A_3 stages, to operate always in their optimum operating range. The gain of the AGC loop was also made to be manually regulateable

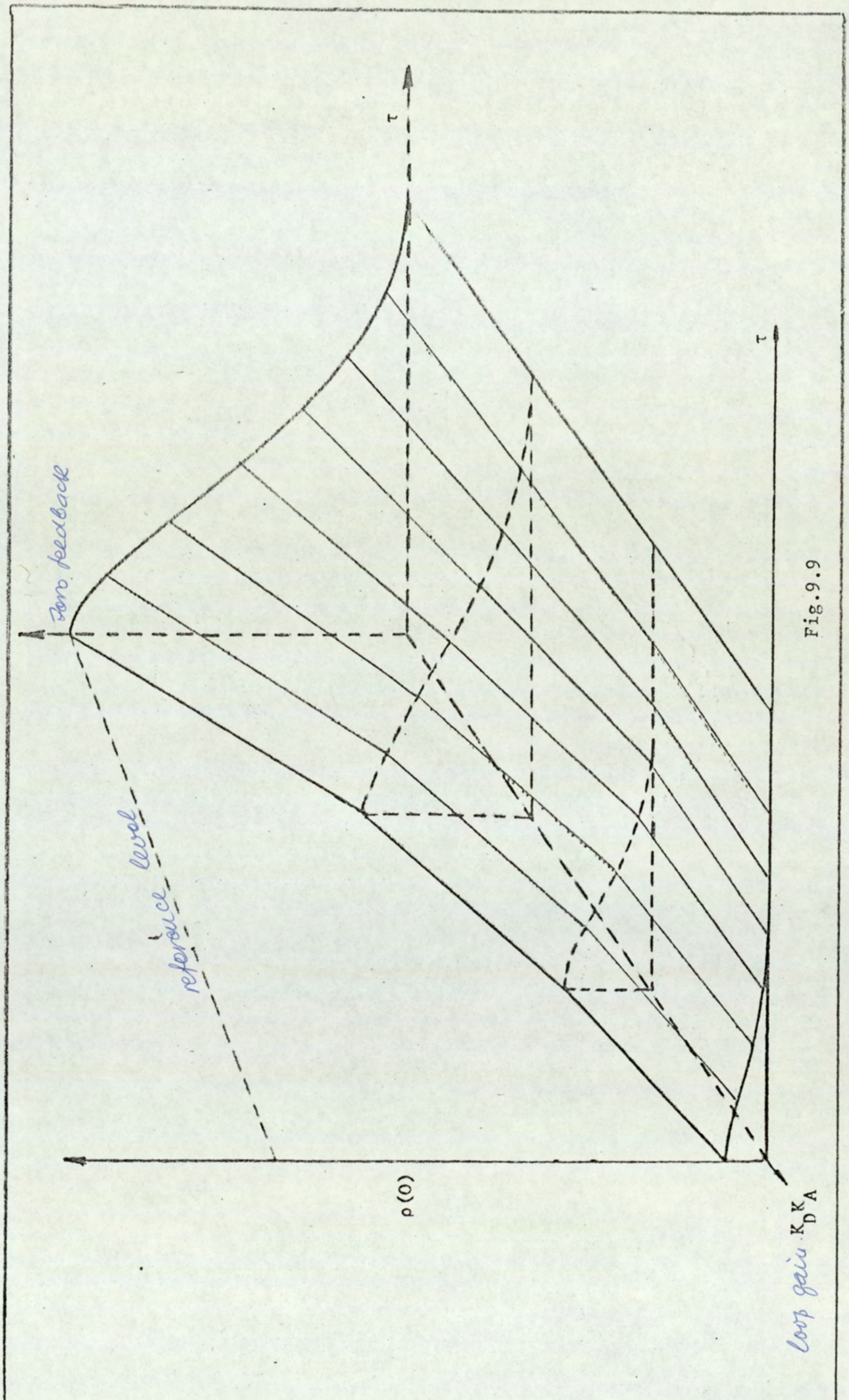


Fig.9.9

by changing the forward gain of the aplifier A_3, thus making possible different degrees of noise suppression.

The reference signal, after being separated, was first limited in amplitude and then applied to a circuit giving outputs consisting of two orthogonal signals, which could be used for synchronous detection. At the output of detector stage, Det.1, a fluctuating voltage becomes available, which after necessary filtering, was used for the slow and the rapid acting AGC loops. The same fluctuations were recorded for the purpose of short term and longer term statistical analysis.

It was found (Table 4), as it was expected, that the change of the see p 198 loop gain influences the intensity of the detected signal fluctuations (Fig.9.9). If the fluctuation power, at the beginning of the experiment has been taken for the reference level, then by changing the gain, the new noise power level has been suppressed several times, depending on the loop gain factors $K_A.K_D$.

The suppression of fluctuations improved the observed signal to noise ratio but the improvement depends, as will become clearer later, on the initial S/N ratio.

This might seem now to be obvious, but it was not appreciated fully at the beginning of these experiments.

The property of every good AGC loop is a linear relation of signal levels changes at the input and the output of the regulated amplifiers, when these levels are expressed in db's.

For a slow change, of say 40 db of the input signal level, 3-4 db of its output could be expected. For the most recent integrated circuits, for example SL 612, made for this purpose by Plessey Microelectronics, a regulating range of 70 db gives only 4db signal change at the output.

In any case, the net suppression of fluctuation is the difference between the input change in db and the output change expressed in db, giving a net 36 db suppression for the AGC amplifier, which was used in these experiments. For a less intense fluctuation say of 20db at the input, the output will vary only 2db, making a net 18db suppression of the fluctuation.

It should also be borne in mind that the observed fluctuations were not slow and deterministic, but fairly fast and random, i.e. their spectra were of the $(1/\Delta f)^{\alpha}$ shape. Therefore, the larger fluctuating components are more suppressed than the less intense at the high frequency end of the spectrum density distribution. (Figs. 9.11b and 9.12)

This situation becomes even more complicated due to the presence of necessary loop filters, which, although designed to cut-off at relatively high frequencies, influence to some extent even those high frequency components of fluctuations which should be suppressed.

The last two facts explain the change in the shapes of power density spectra (Fig.9.11) as well as the change in the autocovariance functions obtained (Fig.9.10).

In order to give some quantitative information about the change in shapes of the experimentally obtained power density spectra, Table 1 is provided.

Related and similar effects on the dependence of the power density spectrum of flicker noise due to the feedback ratio in an amplifier using feedback has also recently been reported in other investigations (Letter sent for publication, copy in Appendix 14).

As it has been shown in Chapter 4 of this thesis, a power density spectrum can be approximated by meromorphic functions, using cosine, alpha or beta coefficients computed from the experimental power density spectra data. (Program ANALYSIS)

In the course of this work it has been found, that on account of the accumulation of errors when fitting experimental curves, higher order meromorphic functions are not suitable, and that the best approximations, with errors less than 2db's are obtained, when a third or fourth order meromorphic function is used.

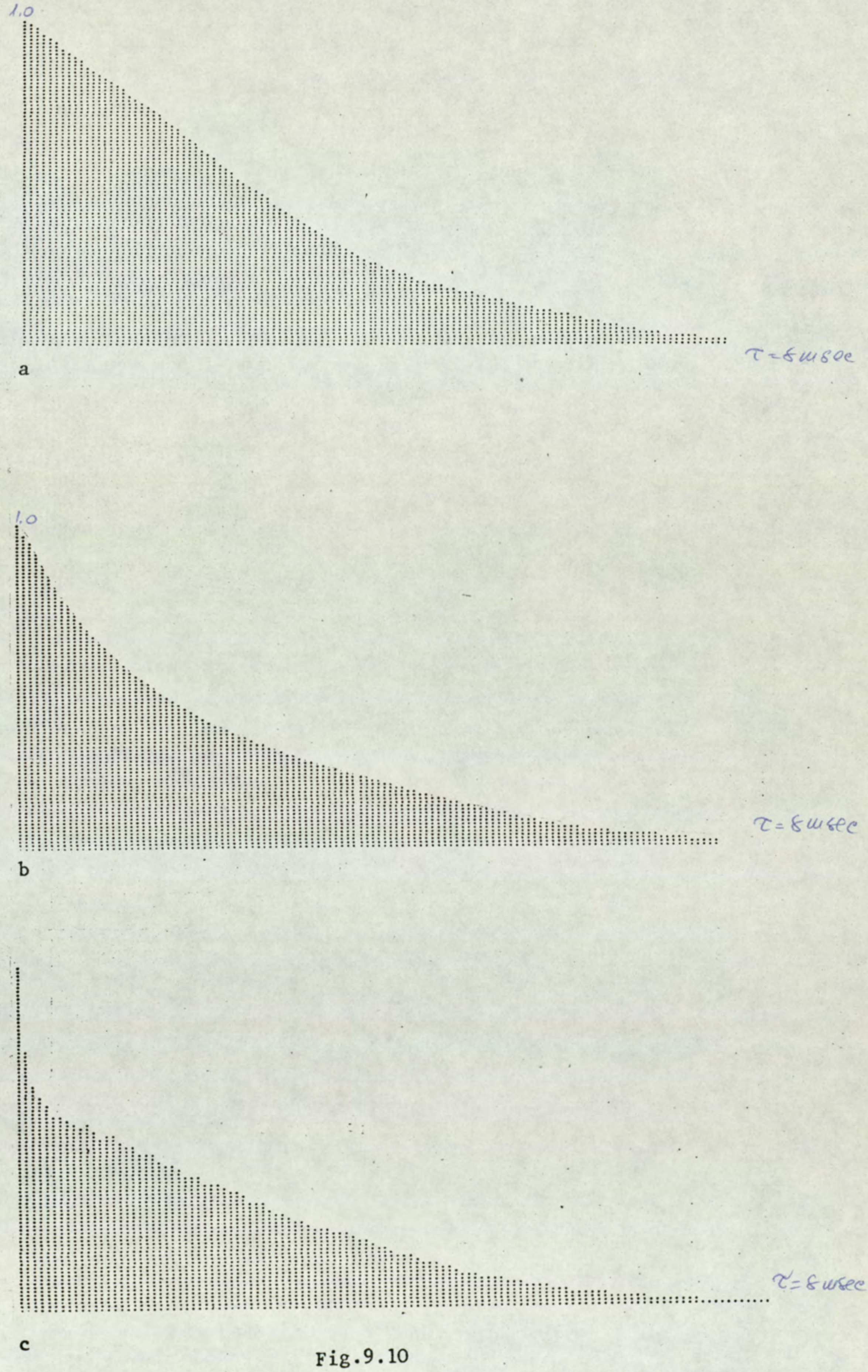
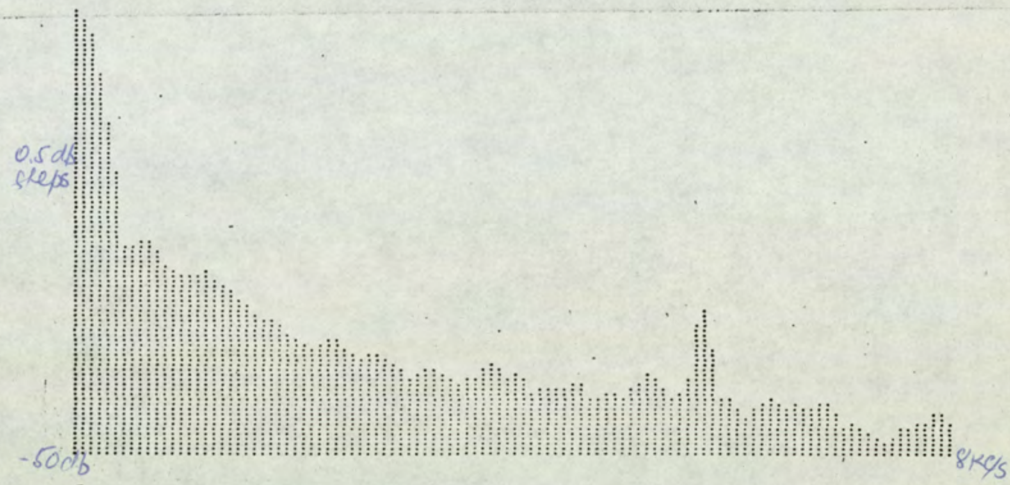


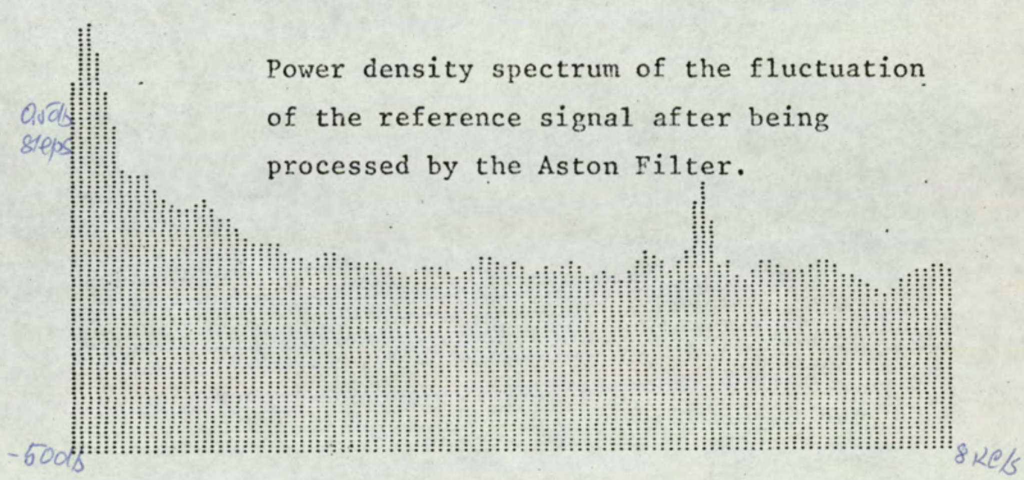
Fig.9.10

For comparison, fitting with Levy's method has been also undertaken, which is given in Table 3. (Program SYSTEM)



a

Power density spectrum of the fluctuation of the reference signal.



Power density spectrum of the fluctuation of the reference signal after being processed by the Aston Filter.

b

Fig.9.11

N.B. In both diagrams the total noise power is proportional to the signal power. The first diagram refers to the spectrum obtained when ASTON filtering was not applied. The second diagram refers to noise recorded at the same time, but with ASTON filtering and refers to a much reduced noise power.

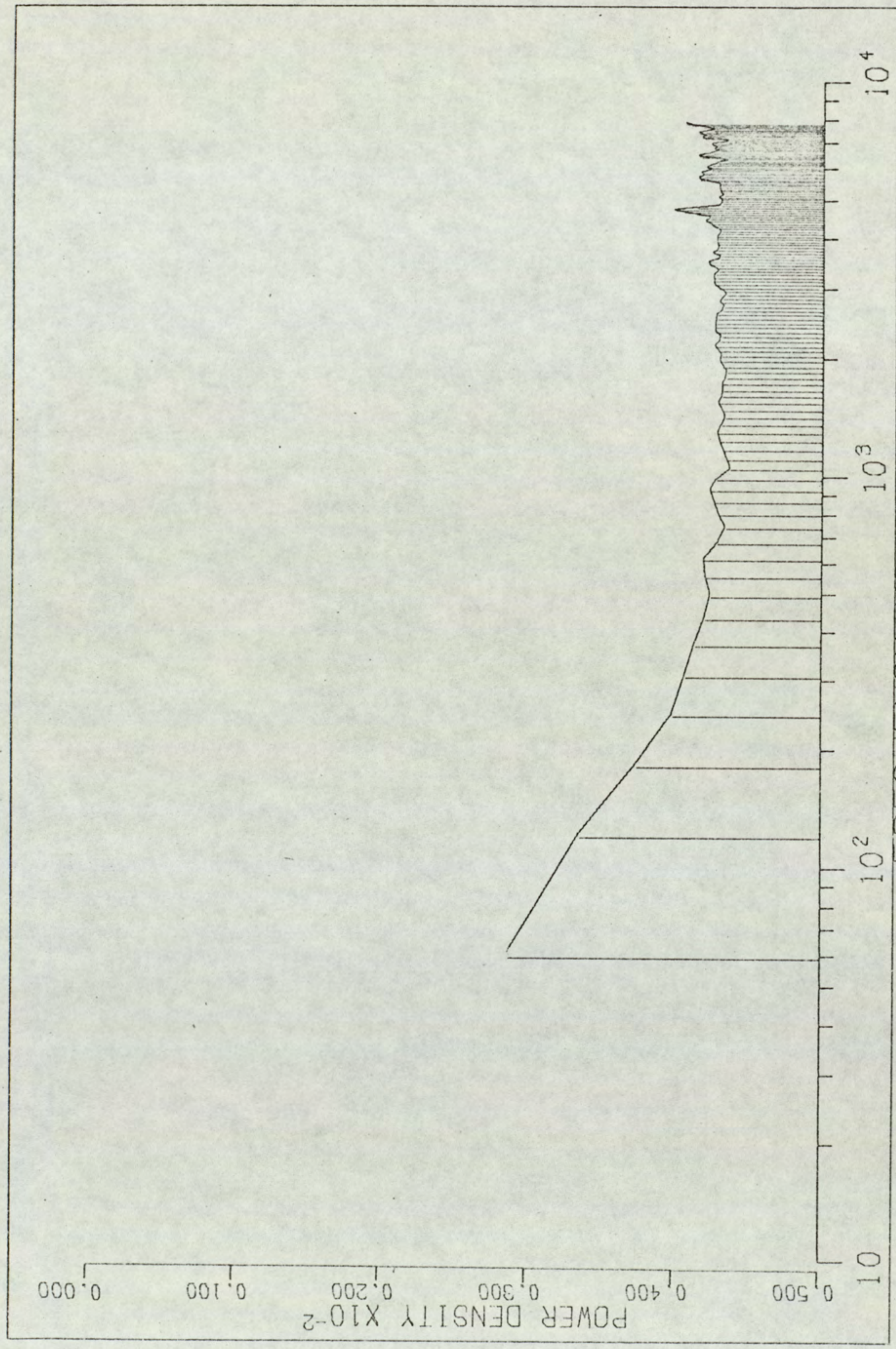

POWER DENSITY X10-2
0.000
0.100
0.200
0.300
0.400
0.500
FREQUENCY KC/S
10
10²
10³
10⁴

Fig.9.12a.

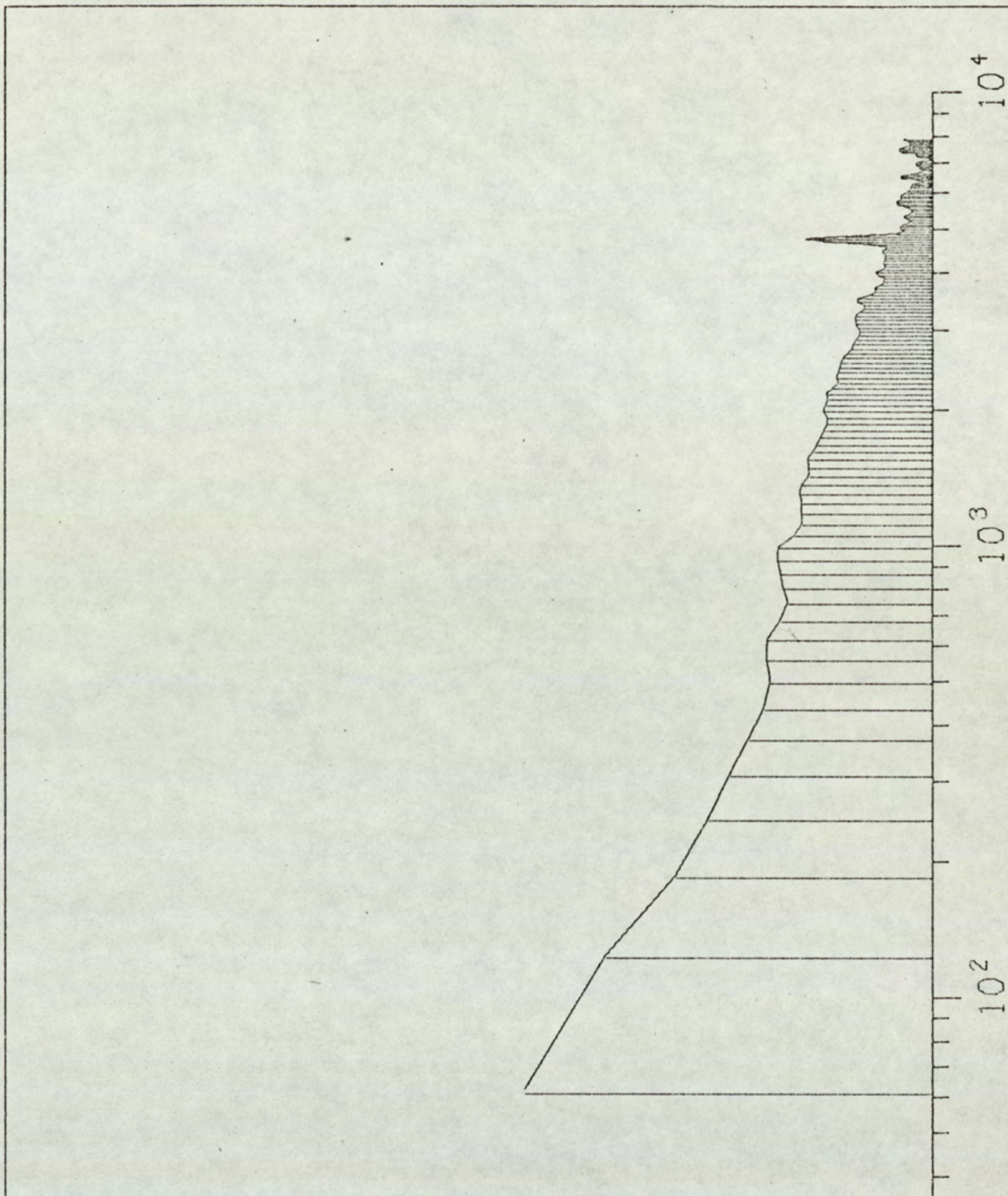

POWER DENSITY X10-2
0.000
0.100
0.200
0.300
0.400
0.500
FREQUENCY KC/S
10
10²
10³
10⁴

Fig.9.12b.

In order to keep the system stable when the amount of feedback is increased, the dominant (filter) time constant is changed which influences signal fluctuation power density spectrum. Compare power

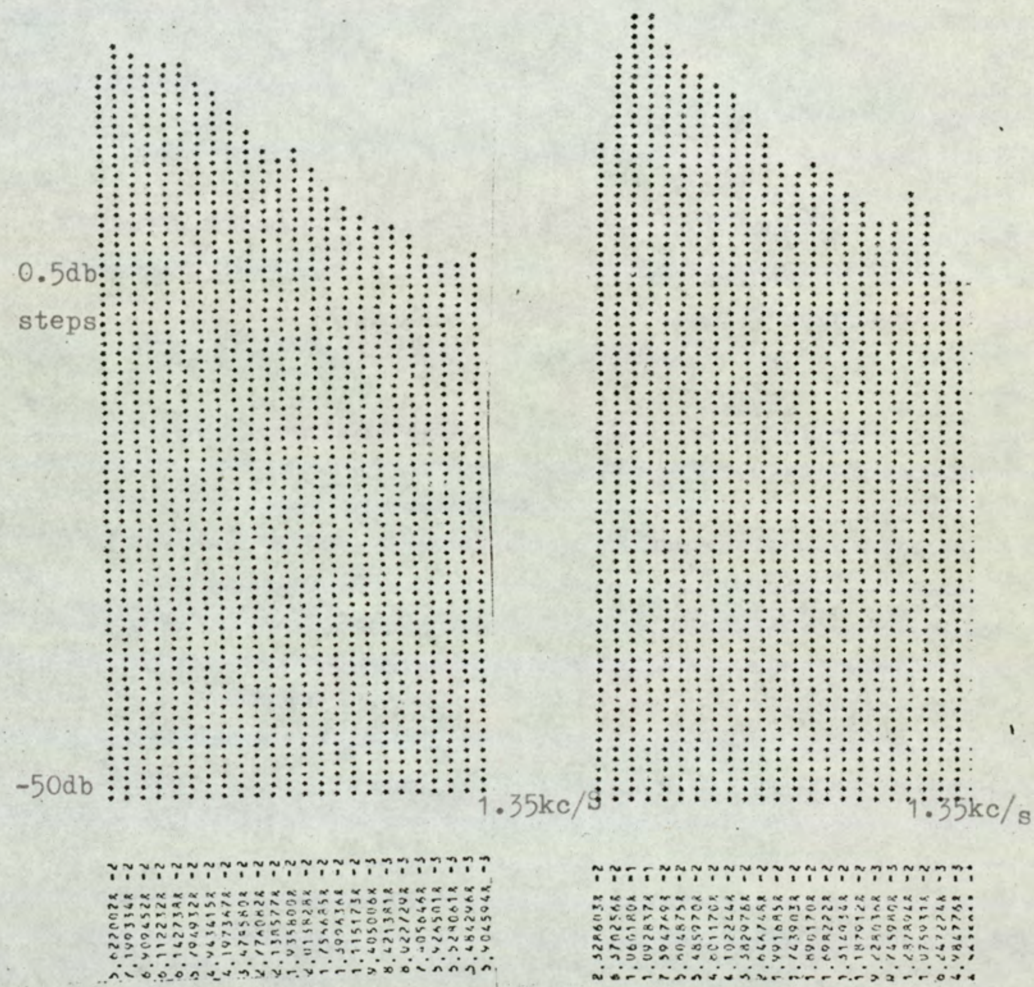


Fig.9.13

density spectrum for 26db feedback (Fig.9.12a) and 20db feedback (Fig.9.12b).

When a slower AGC is acting at the same time, the low frequency end of the spectrum is suppressed even more (Fig.9.13b.)

Table 1. Change of power density shape due to feedback

Order	cos. coef.	alpha coef	beta coef	F.B.
0	$2.7423 \cdot 10^{-1}$	$5.2653 \cdot 10^{-2}$	$9.2837 \cdot 10^{-1}$	0 db
1	$2.5950 \cdot 10^{-1}$	$-2.5970 \cdot 10^{-1}$	-2.6217	
2	$2.2157 \cdot 10^{-1}$	$4.4314 \cdot 10^{-1}$	2.0513	
3	$1.7306 \cdot 10^{-1}$	$6.9227 \cdot 10^{-1}$	$6.3225 \cdot 10^{-2}$	
0	$3.5865 \cdot 10^{-1}$	$1.2627 \cdot 10^{-1}$	1.0593	10 db
1	$3.1863 \cdot 10^{-1}$	$-1.3022 \cdot 10^{-1}$	-2.0116	
2	$2.3239 \cdot 10^{-1}$	$4.6478 \cdot 10^{-1}$	1.8397	
3	$1.4962 \cdot 10^{-1}$	$5.9847 \cdot 10^{-1}$	$1.2279 \cdot 10^{-1}$	
0	$5.7148 \cdot 10^{-1}$	$3.9650 \cdot 10^{-1}$	1.2294	20 db
1	$4.0811 \cdot 10^{-1}$	$1.8336 \cdot 10^{-1}$	$1.2391 \cdot 10^{-1}$	
2	$1.7498 \cdot 10^{-1}$	$3.4996 \cdot 10^{-1}$	1.5552	
3	$7.4916 \cdot 10^{-2}$	$2.9966 \cdot 10^{-1}$	$2.6344 \cdot 10^{-1}$	

Table 2. Power density spectra approximated by Levy's method.

$$P(\omega) = \frac{A + B\omega^2 + C\omega^4 + D\omega^6}{E + F\omega^2 + G\omega^4 + H\omega^6}$$

	I	II	III
A	1.0496	1.0000	$9.4731 \cdot 10^{-1}$
B	$-8.3942 \cdot 10^{-1}$	$-4.2793 \cdot 10^{-1}$	-1.7232
C	$2.2782 \cdot 10^{-2}$	$8.9145 \cdot 10^{-3}$	$1.6495 \cdot 10^{-1}$
D	$9.9922 \cdot 10^{-5}$	$-6.0663 \cdot 10^{-13}$	$-8.2427 \cdot 10^{-4}$
E	1.0000	1.0000	1.0000
F	$-1.3127 \cdot 10$	-5.6911	-2.7742
G	$7.6576 \cdot 10$	2.2612	$5.5257 \cdot 10.1$
H	$-1.1538 \cdot 10$	$-4.6918 \cdot 10^{-2}$	$-1.0581 \cdot 10^{-2}$

I Coherent detection

II Envelope detection

III Aston filtering

Table 3. Marcov Filter parameters estimation by Levy's method.

$$H_M(s) = \frac{A_o + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4}{b_o + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4}$$

	I	II	III
a_o	$9.2725 \cdot 10^{-1}$	$9.9339 \cdot 10^{-1}$	$9.4089 \cdot 10^{-1}$
a_1	$7.1394 \cdot 10^{-1}$	$6.6619 \cdot 10^{-2}$	$4.6466 \cdot 10^{-2}$
a_2	$-5.1703 \cdot 10^{-2}$	$-2.6129 \cdot 10^{-1}$	$-3.5109 \cdot 10^{-1}$
a_3	$7.8828 \cdot 10^{-5}$	$3.2278 \cdot 10^{-3}$	$2.8112 \cdot 10^{-3}$
a_4	$-2.2293 \cdot 10^{-4}$	$1.1896 \cdot 10^{-3}$	$3.7989 \cdot 10^{-3}$
b_o	1.0000	1.0000	1.0000
b_1	3.0669	1.3112	$3.7025 \cdot 10^{-1}$
b_2	$-1.5061 \cdot 10$	-7.0391	-2.0203
b_3	$-1.7961 \cdot 10$	$-8.0649 \cdot 10^{-1}$	$-7.5408 \cdot 10^{-2}$
b_4	$5.1521 \cdot 10^{-1}$	$7.7665 \cdot 10^{-1}$	$1.3973 \cdot 10^{-1}$

I app. gas nozzle speed $0.5\ m.sec^{-1}$
II " " " " $0.7\ m.sec^{-1}$
III " " " " $1.0\ m.sec^{-1}$

Suppression of noise in function of feedback

From the experimental results (Table 4) and previous theoretical consideration (Sec.9.6) it is evident that S/N ratio improvement is bigger for stronger feedback, ($KF \to 0$) but also depends on the initial N/S ratio. Obviously for $KF = 1$ (open loop) or $N/S \approx 0$ there is no improvement (Fig.9.11b).

On the other hand, for a chosen feedback(return) ratio, the ratio of absolute noise powers before and after ASTON Filtering is dependent solely on the power density distribution of fluctuation.

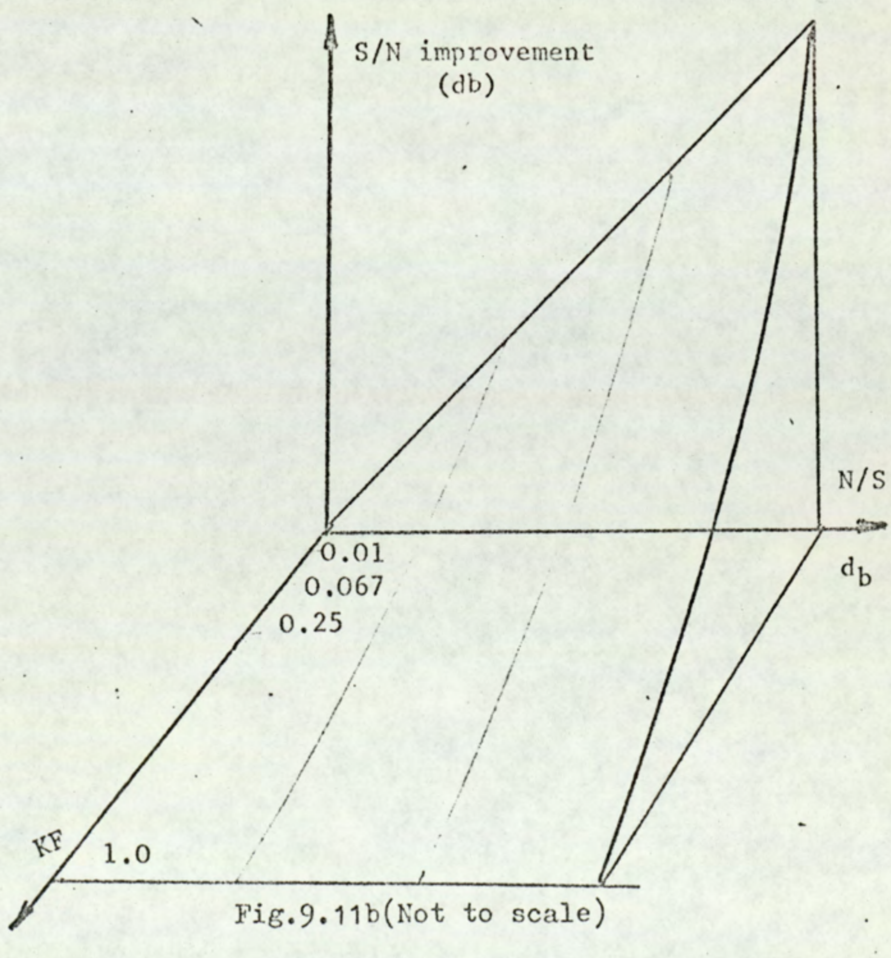


Fig.9.11b(Not to scale)

9.8 Effect of Aston Filtering on signal components outside the coherence band.

For the purpose of this investigation a reciving arrangement (Fig.9.8) was utilised. It consists of a wideband RAAGC amplifier and two selective amplifiers A_1 and A_2. The resonant frequencies of A_1 and A_2 has been set at 4 mc/s and 8 mc/s respectively.

Two signals with subcarrier frequencies 4 Mc/s and 8 Mc/s, were simultaneously transmitted and separately synchronously detected. The fluctuation of the signal with the 4 mc/s subcarrier was utilised for RAAGC.

The effect of slow acting AGC may be demonstrated by Fig. 9.13(p191) By increasing the loop bandwidth from 25c/s to 200c/s a drop in the lower frequency fluctuation component was observed with an increase of about the same amount in the vicinity of 200 c/s. It must be emphasised that this increase is only a relative one, as the total fluctuation power is in fact reduced.

The same effect of deterioration of higher frequency components of fluctuations is observed in the case of a RAAGC.

As it can be seen (Fig.9.14a) this fluctuation was suppressed by more than 20 db, while the other (Fig.9.14b) has not. The real part of the complex cross spectrum, obtained by correlating these two fluctuations, is shown on Fig.9.14c, and the imaginary parts on Fig.9.14d. Furthermore, noise power measurement revealed an increase in the noise power of approximately 3 db for the signal with the 8 c/s subcarrier.

The associated coherence function was also computed (Fig.9.15a) which has shown very high coherence at low frequency end, much higher than without feedback, as may be seen from Fig. 9.15a. and almost no coherence at the higher fluctuation frequencies. This experiment was repeated with the gas speed at the nozzles doubled, which made both power spectra less steep, but the coherence function shape did not change appreciably (Fig. 9.15b)

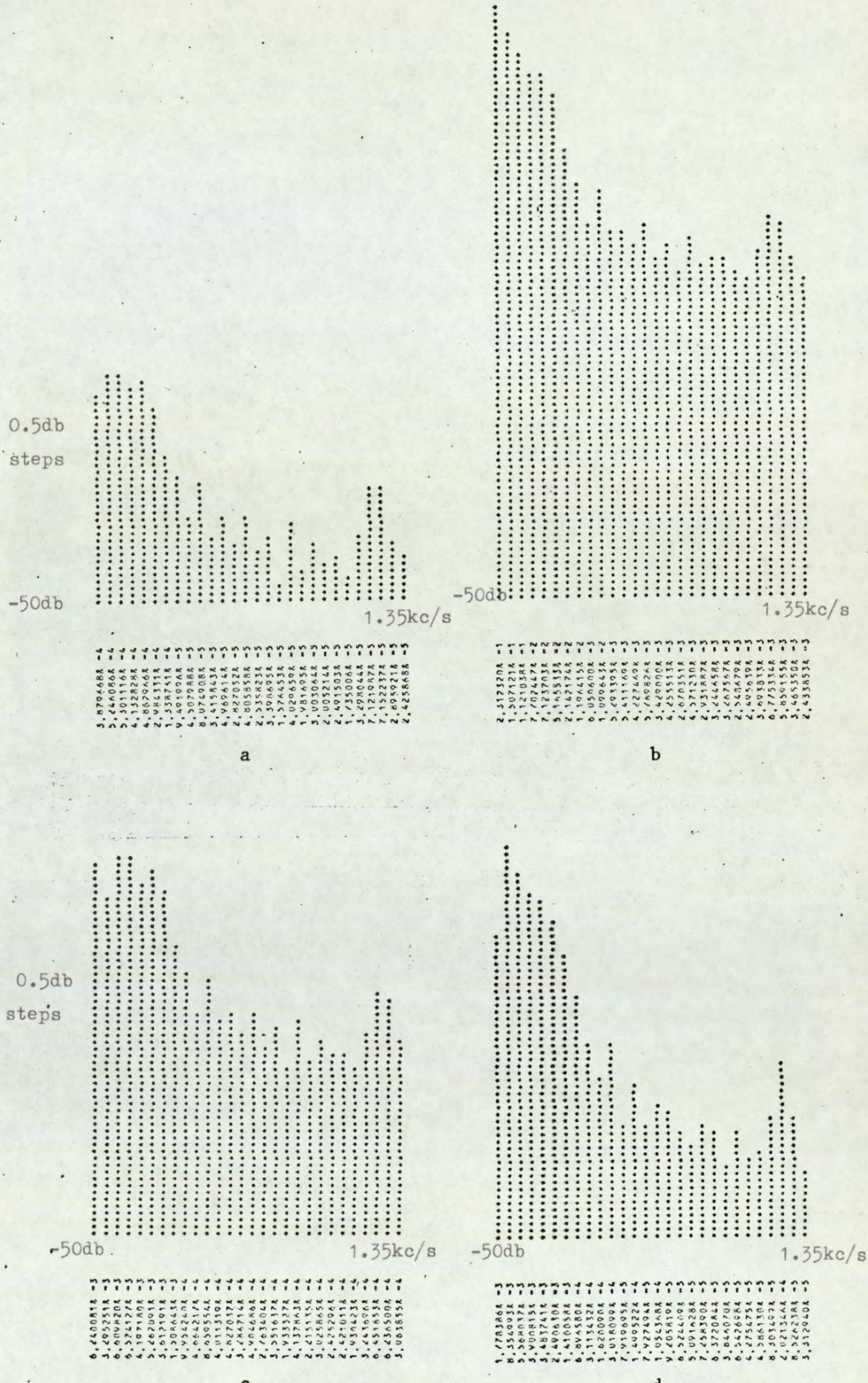

0.5db
steps
-50db
1.35kc/s
a
-50db
1.35kc/s
b
0.5db
steps
-50db
1.35kc/s
c
-50db
1.35kc/s
d

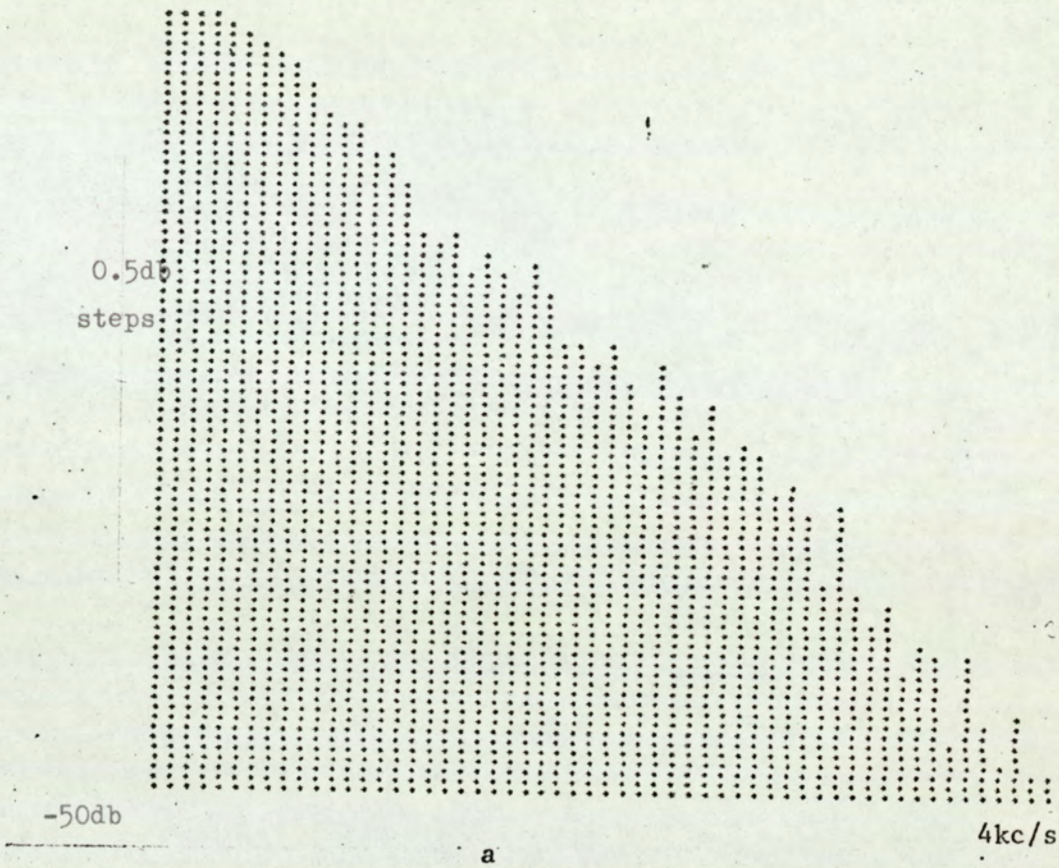
Fig.9.14

It can be concluded that by the use of the RAAGC and RAAPC in the coherence band a noise suppression can be obtained, which depends on the dimensionless factor $K_F = \frac{1}{1+K_{FB}.K_A}$ when the same RAAGC is utilised for correcting signal component fluctuations outside the coherence band, a deterioration in quality of communication is obtained making N/S noise ratio for approximately 3 db bigger.

Table 4.

K_F	III	II	I
0.25	9.9	11.1	12.4
0.0625	15.9	16.2	18.6
0.01	23.9	24.1	26.4

S/N ratio improvement in function of feedback factor K_F ,for different gas nozzle speeds(see p.194).



a

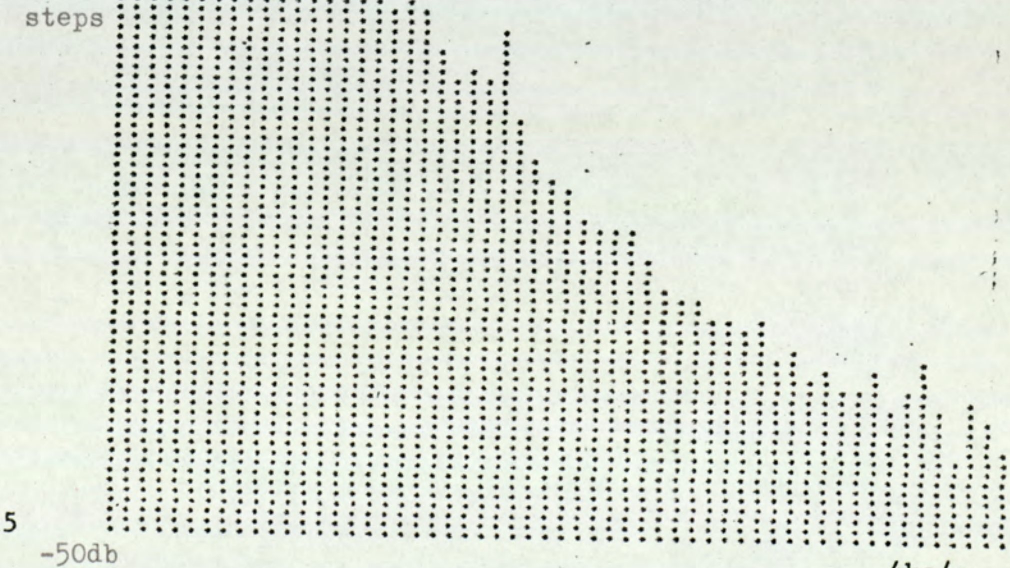


Fig.19.15

b

9.9 Note on System Stability

It has been shown by theoretical considerations and also experimentally (Victor and Brockman, 1960) that by expressing signal levels and the attenuation of the regulating amplifier in db, the AGC loop could be approximated by a linear feedback system. Thus a basically nonlinear system, whose stability could be tested rigorously* only by applying Liapunov approach (1966), may be approximated under certain assumptions by a linear system. By supposing that the level of the testing signal is much smaller than the D.C. reference in the loop, the linearized open loop function will be a good approximation of the loop transfer function. Larger testing signals would give bigger discrepancies. As the loop contains the necessary band pass filter, which attenuates higher frequency components more than D.C. and low frequency components, in order to get readable values at the output of the open loop system, it is necessary to change the testing signal level during measurements. Provided that the output of the test signal is kept constant the discrepancies will not be excessive.

It was found that the experimental and theoretical open loop magnitude and phase curves (Program NYQUIST) fit over a wide range, but not far enough, so that this approach could be used for stability testing provided Kochenburger (1950) criterion is applied. The best that can be done without recourse to methods used for nonlinear systems is to find the stability of the system when the testing signal level is low, and hence the linear feedback theory is applicable. The next step is to explore the trend of the system for higher signal levels. In an AGC loop, the most dominant time constant is in the loop filter. If the parameters of the filter and the gain bandwidth product of the forward gain are known, a very good approximation of the open loop transfer function may be obtained.

For ordinary AGC systems there is no discrepancy when this approach

(*Zubov, 1950)

is used, so the time constant of the filter is several orders higher than any other time constant in the loop. Consequently, the open loop transfer function is usually of the first and sometimes of the second order.

Unfortunately the simple integrating filter is obviously not suitable for a rapid acting feedback. Even with the proper first order filters precautions must be made so that the filter is wide enough, otherwise the high frequency components of the signal fluctuations are not suppressed and on account of the phase lag are even distorted.

The dominant time constant of the R.A.A.G.C. loop was chosen to be 100μsec. The two-stage regulating amplifier was designed with the gain bandwidth product corresponding to time constant of 10 and 5μsec respectively.

In order to compensate the obtained third order open loop transfer function, two zeros were introduced with corresponding time constants of 40μsec and 2μsec. respectively.

Therefore the obtained open loop transfer function ϕ_o is of the form:

$$\phi_o = \frac{(1 + 0.4S)\ (1 + 0.02S)}{(1 + S)\ (1 + 0.1S)\ (1 + 0.05S)}$$

where τ = 100μsec was used as time unit for normalising and scaling. Applying the root locus stability test (Program SCANNING) it was found that the system is stable even when gain bandwidth product of an amplifier was greatly reduced (Fig.9.14) and the corresponding time constant of 5 microsec increased to 25 (dashed root locus).

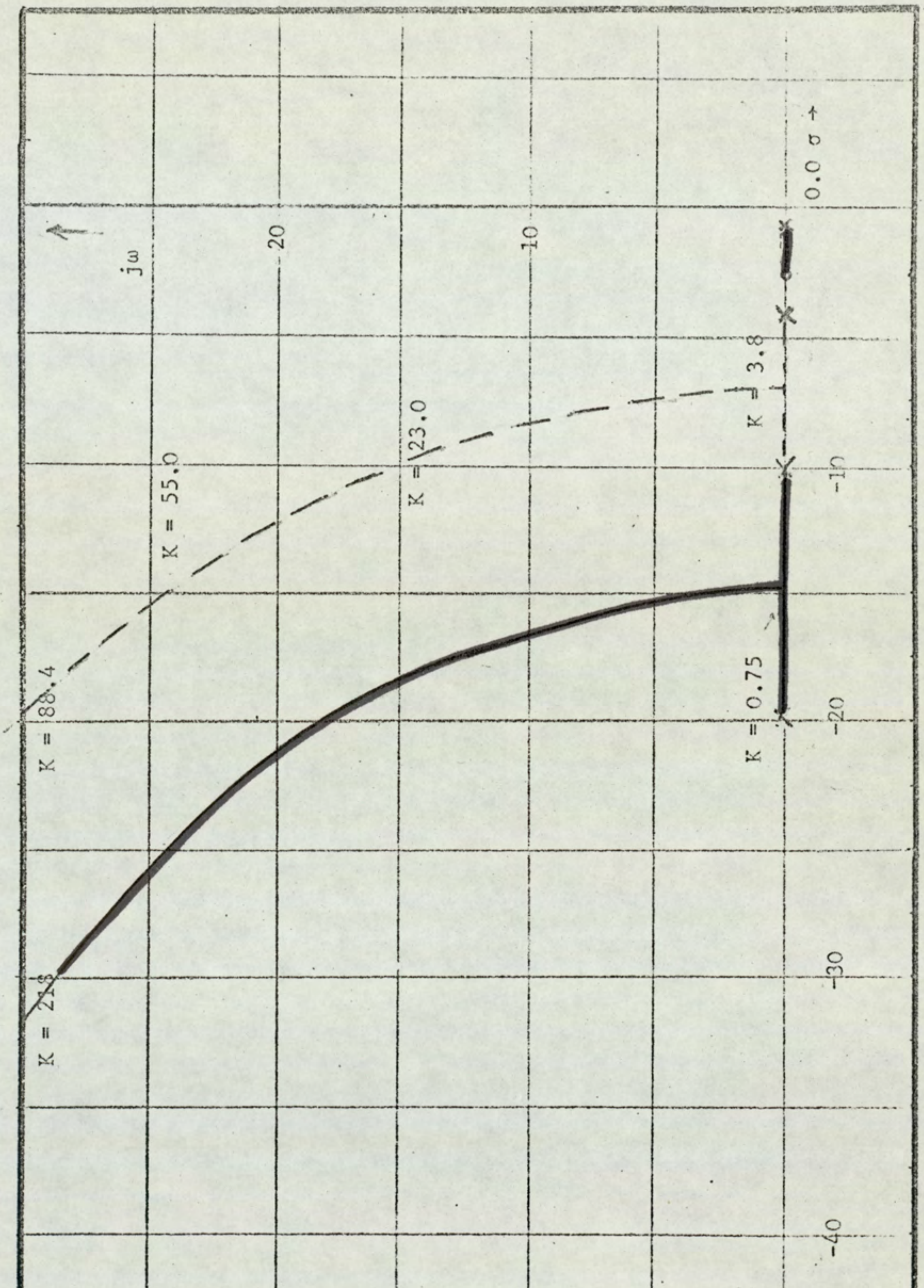


Fig.9.4.

CHAPTER 10

Conclusions and Recommendations

The variations in the parameters of the transmission path do not merely add noise to the received signals. In fact, if there is no signal carrier or signals, the noise due to the transmission path parameter fluctuation is not detectable. What takes place is that each frequency component of the signal is modulated both in amplitude and phase by the variations in the parameters of the transmission path, and these variations cannot be detected until there is a carrier present. The only noise present when no carrier is present is thermal and plasma radiation noise as well as noise due to the radiation resistance of the aerial and the receiver circuits and shot noise etc. in the amplifiers. These noises in communications to and from rockets are usually of far smaller power than the received signal. No economy of transmission energy will be obtained by transmitting a sufficiently small power to ensure that the received signal is so small that the gain of the receiver has to be sufficiently great to make the thermal and shot noise detectable, for then some of the signal power will be masked by these noises in a manner which cannot be corrected by the use of feedback or AGC and APC of any kind. The effect of them can be removed only by reducing the rate of signalling in some way.

The effect of the transmission path is due to vast numbers of atoms or ions in correlated movement in clouds, turbulence, etc. and it can therefore be treated as a macroscopic phenomenon amenable to treatment by electronic means as used in feedback control. Over a band of frequencies called the coherence bandwidth every component of the information signal and the reference signal are modulated in almost the same way, both in phase and amplitude. Over the coherence bandwidth the amplitude and the phase relation between the reference signal and the information signals is therefore preserved at the values pertaining when the signal was transmitted, even though the phase and amplitude relation between the transmitted signal and that received is varying on account of the action of the transmission path.

It has been found possible, by using the amplified reference signal which, by the rapid acting AGC action derived from it has been brought to a constant level, and by using a relatively wide-band amplifier for the reference signal, to preserve the phase changes in the reference signal unaltered. The reference signal can then be used in place of a local oscillator, and it was found that over the coherence bandwidth the effect of amplitude and phase errors in the received signal were greatly reduced.

When both the phase and amplitude defects arising in the transmission path have been minimized in this way it is permissible to reduce the transmitted power to such a level that the thermal and shot noise in the receiver are only just detectable when the signal has faded to its lowest values. No improvement in signal to noise ratio will be obtained by increasing the signal power, since the noise fluctuation power is proportional to the signal power. (Bozic, 1966). It is an economic consideration depending on the additive noise level to determine whether it is better to have more power available at the receiver and use a receiver of smaller gain or to transmit a smaller power and use a receiver of larger gain.

Outside the coherence bandwidth the action of the rapid acting AGC and APC will produce variations in the gain which have no relation to the noise generated outside the coherence band. Consequently, the noise will be increased if a bandwidth greater than the coherence band is permitted to pass through the receiver. To increase the bandwidth used for signalling the signal should be divided into frequency bands each with its own reference signal and each having a bandwidth equal to the coherence bandwidth.

The coherence bandwidth can be determined by using a signal consisting of two single frequencies, one of fixed frequency reference signal while the other is of variable frequency and measuring coherence function of the signal envelope fluctuations. It will be found that the cumulative coherence factor between the noise on the signal frequencies decreased as their frequencies are separated further and further.

In the example of the way the rapid acting AGC was utilised, a 4.0 MHz carrier is first modulated with the signal. The carrier is then removed and replaced with a reference signal locked to the original carrier in frequency, but shifted by 90^{o} in phase. The resultant signal and reference is modulated in amplitude moduclation on a carrier at 9,000 MHz.

On reception the signal and the reference signal are orthogonal to one another, and can be separated in a phase detector operated by the reference signal itself after it has been brought up to an appreciable amplitude and at the same time provided a rapidly acting AGC voltage whereby the received signal can be amplified to have the same phase and amplitude relation to the received reference signal as the original transmitted signal had to the transmitted reference signal. The signal can then be detected in a phase detector by passing the received reference signal through a tuned circuit which shifts the phase by 90^{o}. In the detector for the reference signal the signal sidebands will have no effect on the reference signal detector.

It is not necessary to adhere to amplitude modulation in applying the rapid acting AGC. The amplitude is brought to a constant value by combining rapid AGC and limiting processes, but the phase relation is still maintained, so that coherence detection of the signal can still be obtained by using the amplified version of the reference signal in place of the local oscillator.

The transmission of the signal in several neighbouring coherence bands each with its own reference signal is similar to multiplex telephony, except that the signals in neighbouring bands are related to one another and not as is usual in telephony, of a totally unrelated kind.

The effects arising from a multichannel transmission using a "comb" of reference signals, remain to be further investigated.

It was seen that by the use of phase shifts in the transmitted signal and reference signal it is possible to send the reference signal with no appreciable increase in signal power. It is not essential to send the reference signal in this form however, when the coherence bandwidth is large, and other means of separating the reference signal from the information signal at the receiver may be used.

The principles involved in the practical applications may then be summarised as follows:-

1. The coherent band of frequencies for the particular transmission path under consideration is determined.
2. This coherent band will not depend greatly on the actual r.f. carrier frequency, so that if necessary, several equal coherent signal bands lying side by side may be used at one time, with safety margin gaps.
3. At the transmitter a relationship is established between the information signal and the reference signal in each coherent band.
4. At the receiver the signal and the reference signal in each band are separated. The reference signal is then brought up to a constant level and in doing so, rapidly acting AGC voltages are produced which can be used to control signal amplifiers which bring the signals up to the same phase and amplitude relationship as they had on transmission.
5. The amplified reference signal is then used, after further treatment, such as, for example, a phase shift, or filtering as a local oscillator.
6. The methods outlined operate successfully since the transmission path is, from the electronic point of view, a macroscopic entity which can change only slowly in comparison with the rate at which electronic amplifiers can operate.

APPENDICES

APPENDIX 1

Sampling Theorem

Suppose that the complex signal a(t) which contains signal and noise, is known in the interval 0 to T to contain no power at frequencies above B Hertz. Assuming that the signal is periodic, that is, it is repeated in all time intervals $\{nT - (n + 1)T\}$, $-\infty < n < +\infty$, the signal may be expressed as a Fourier series with fundamental frequency $f_s = T^{-1}$. The fact that the function of time does not exist outside the basic period T limits the existence of the series coefficients physically to the same time interval (Lee, 1966). Thus the total number of harmonic frequencies which may be present is obtained by dividing the total bandwidth by the frequency of the basic harmonic or fundamental frequency f_s.

Each harmonic is defined by two constants, for example, the magnitude and phase, so that the total number of independent values such as Fourier coefficients, which can specify the samples taken from the function is

$$N = 2B/T^{-1} = 2BT \qquad \text{A1.1}$$

For finding N Fourier coefficients we require a knowledge of N independent amplitudes of the time function, that is, the time sampling should be carried out at time intervals

$$t = t_s = 1/f_s = 1/2B \qquad \text{A1.2}$$

More sampling values are not necessary, since they would not contain new and independent information. When white noise is present some redundancy might be useful, as will be shown in a later chapter.

Obviously, signals may be represented in a two dimensional space for each sample value for N (= 2BT) successive sampling times or in a multidimensional space as a N dimensional vector $\underline{S}$. Therefore a complex entity can be represented either in a simple space, i.e. on a plane, with many point values, or a vector in a complex N-dimensional space. In either case, since all values are, at least in initial considerations, independent of one another, the N co-ordinates must be orthogonal, i.e. at right angles to one another.

Suppose that a(t) exists for a finite time, i.e. it is a truncated signal which has the Fourier transform defined by

$$\underset{\sim}{a}(t) = \int_{-B}^{+B} A(j\omega)\exp(j\omega t)\, df \; ; \quad \omega = 2\pi f \; . \qquad \text{A1.3}$$

where $A(j\omega)$ defines the spectral density of $\underset{\sim}{a}(t)$. Considering, as before, that the band limited spectrum is due to one complete period in the frequency domain, then $A(j\omega)$ may be represented by the Fourier series

$$A(j\omega) = \sum_{n=-\infty}^{\infty} a_n . \exp(j\,\frac{\pi n f}{B}) \qquad \text{A1.4}$$

for the range $-B<f<B$

where

$$a_n = \frac{1}{2B} \int_{-B}^{+B} A(j\omega)\, \exp(-j\frac{2\pi n f}{2B}).df \qquad \text{A1.5}$$

A continuous signal can be approximated by interpolation between the sampling instants. The simplest method uses either the "staircase" or the "box-car" interpolation. The interpolation function is rectangular with a duration equal to the intervals between the sampling, that is, equal to the sampling time, $1/f_s$. Since this function is the response to an impulse of the network which is doing the interpolation, we can find its transform in the form

$$H(j\omega) = \int_{0}^{1/f_s} \exp(\,-j\omega).dt \qquad \text{A1.6}$$

$$= \frac{\sin(2\pi f/2f_s)}{\frac{2\pi f}{2f_s}} \; . \exp(-j.2\pi f/2f_s) \qquad \text{A1.7}$$

where the limits of intégration satisfy the interpolation process over the interval $t = 0$ to $t = 1/f_s$.

The $\dfrac{\sin 2\pi f/2f_s}{2\pi f/2f_s}$ filter response can be realised by a relatively simple "sample hold" circuit.

The staircase interpolation is piecewise continuous and gives rise to abrupt changes in the form of the reconstructed signal.

For the reconstructed signal to resemble $\underset{\sim}{a}(t)$ as closely as possible the interpolation function must have derivatives of the higher orders, which implies that a band limited interpolation is required. As all signals are band limited, and can be represented, as in equations

$$\underset{\sim}{a}(t) = \int_{-B}^{+B} \sum_{n=\infty}^{\infty} a_n \exp(j\frac{\pi n f}{B}) . \exp(j2\pi ft).df \qquad \text{A1.8}$$

$$= \sum_{n=\infty}^{\infty} a_n \int_{-B}^{+B} \exp\left[jf(2\pi t + \frac{n\pi}{B}).\right]df$$

$$= B \sum_{-BT}^{BT} a_n . \frac{\sin\left[2\pi B(t + n/2B)\right]}{2\pi B(t + n/2B)}$$

Since, from Shannon's sampling theorem (Shannon, 1949)

$$a_n = a(\frac{n}{2B}) ,$$

$$a(t) = \sum_{n=-\infty}^{\infty} a(\frac{n}{2B}) \frac{\sin\left[2\pi B(t + n/2B)\right]}{2\pi B(t + n/2B)} \qquad \text{A1.9}$$

so that the original function $a(t)$ may be reproduced by the sampling values

$$\sum_{n=-\infty}^{\infty} a(\frac{n}{2B}) \qquad \text{A1.10}$$

and the interpolation function

$$h(t + \frac{n}{f_s}) = \frac{\sin\left[2\pi B(t + n/2B)\right]}{2\pi B(t + n/2B)} \qquad \text{A1.11}$$

the last equation being the output response of an ideal low pass filter (having a cut-off frequency equal to B), to an input consisting of a unit impulse (Dirac delta function) occurring at time

$$t = \frac{n}{f_s} = \frac{n}{2B} \qquad \text{A1.12}$$

The form of the interpolation function does not change with time. For describing a signal in all necessary detail it is necessary to know the values of $a(n/2B)$ for $-\infty < n < +\infty$.

Therefore N points ($N = 2BT$) are sufficient for reconstructing a signal which is limited either in its time duration or in its frequency band. Mathematically it is not possible to consider the signal to be limited in both domains at the same time. However, the assumption that both exist gives a reasonably fair approximation for the reconstruction of a signal only if 2BT is large.

APPENDIX 2

Signals in the presence of white thermal band limited noise.

In the geometrical representation each signal point is surrounded by a small region of uncertainty due to additive noise. The perturbations of the different successive sample values are independent and have a Gaussian distribution of probabilities. The probability of a perturbation with co-ordinates

$$\{n_1,\ n_2,\ n_3,\ \ldots\ \ldots\ \ldots\ n_N\} \qquad \text{A2.1}$$

which together represent a small multidimensional radius around the origin, is the product of the individual probabilities of the various co-ordinates, namely

$$P(r) \ = \ \prod_{i=1}^{2BT} \frac{1}{2\pi.2BTN)^{\frac{1}{2}}} \ \exp(-(n_i)^2/2BTN) \qquad \text{A2.2}$$

where
$$H \ = \ \sigma_n^{\ 2} \ = \ \frac{1}{2BT} \sum_{1}^{2BT} (n_i)^2 \qquad \text{A2.3}$$

which gives
$$P(r) \ = \ \frac{1}{(2\pi.2BTN)^{TB}} \ \exp(- \sum_{1}^{2BT} (n_i)^2/2BT \qquad \text{A2.4}$$

Since it depends only on $2BTN \ = \ \sum_{1}^{2BT} (n_i)^2$, the probability of any given perturbation depends only on the distance from the original radius representing the signal, which indicates that in the signal space the uncertainty must have a spherical distribution. For a small number of dimensions, i.e. when 2BT is small, the limits of the uncertainty are not well defined. As the number of dimensions is increased to large values, the uncertainty tends to become the volume of a multidimensional hypersphere, which is very well defined by the surface of the hypersphere. This can be shown as follows.

The volume of a N dimensional hypersphere, $N \ = \ 2BT$, of radius R may be written

$$V(R) \ = \ M_R \ = \ C_\nu R^\nu$$

A shell of thickness s at the surface of the hypersphere has volume

$$V_s = V(R) - V(R - s)$$

$$= C_\nu (R^\nu - (R - s)^\nu)$$

$$= C_\nu R^\nu (1 - (1 - \frac{s}{R})^\nu) \qquad \text{A2.5}$$

Introducing the exponential function which by definition gives

$$\exp x = \lim_{n \to \infty} (1 - \frac{x}{n})^n \qquad \text{A2.6}$$

then

$$V_s = C_\nu R^\nu (1 - \exp(-s\nu/R)) \qquad \text{A2.7}$$

If $\nu = 2BT$ is large enough so that $s\nu >> R$, then V_s is practically the entire volume of hypersphere. In other words, for any reasonable, i.e. not too thin, shell, the volume of the shell is practically equal to the entire volume of the hypersphere. The volume of the hypersphere can be determined by calculating the constant C. Consider the integral

$$I = \int_{-\infty}^{+\infty} \exp(-\sum_1^{2BT} (n_i)^2/2BTN) \qquad \text{A2.8}$$

Putting $x_i^2 = (n_i^2/2BTN = (n_i)^2/\nu N$ A2.9

$$I = \int_{-\infty}^{+\infty} \exp|-(x_1^2 + x_2^2 + \dots +x_i^2 + \dots + x^2)|dx_1.dx_2 \dots .dx_\nu$$

$$= \int_{-\infty}^{+\infty} \exp(-x^2)dx = \pi^{\nu/2} \qquad \text{A2.10}$$

The same integral may be written in polar form, with

$r^{(\nu-1)}s_\nu$ as the surface area of a ν-dimensional sphere,

$$I = \int_0^\infty e^{-r^2} r^{(\nu-1)} s_\nu dr = \tfrac{1}{2} s_\nu \int_0^\infty e^{-t} t^{(\nu-2)} dt$$

$$= \tfrac{1}{2}{}_\nu (\frac{\nu}{2} - 1)! \qquad \text{A2.11}$$

Equating the two values for S we get

$$s_\nu = 2\pi^{\nu/2}/(\frac{\nu}{2} - 1)! \qquad \text{A2.12}$$

and the volume of the sphere is given as

$$V(R) = C_\nu R^\nu = \int_o^R S_\nu R^{\nu-1}\,dR = \pi^{\nu/2} R^\nu / \tfrac{\nu}{2}\,!$$

$$= \pi^{BT} R^{2BT} / \Gamma(BT+1) = C_{BT} R^{2BT} \qquad \text{A2.13}$$

$$\text{where} \quad (\nu/2)! = (BT)! = \Gamma(BT+1) \qquad \text{A2.14}$$

The gamma function may be evaluated either by using the Stirling approximation or tables which are available.

The probability distribution of r_N (= $\left[\sum_1^{2BT} (n_i^{\,2})\right]^{\frac{1}{2}}$ is the probability distribution of the r.m.s values of the noise, $r_N = \sigma_N(2BT)^{\frac{1}{2}}$. This can be found by integrating the joint probability density function over the spherical shell between r_N and $(r_N + dr_N)$ (W.W.Harman, 1963) to obtain

$$p(r_N)dr_N = 2BT.C_{BT}(2\pi N)^{-BT} r_N^{(2BT-1)} \exp(-r_N^{\,2}/2N) \qquad \text{A2.15}$$

For a large number of dimensions, this density function is very sharply peaked in the vicinity of

$$(2BTN)^{\frac{1}{2}} = \sigma_N(2BT)^{\frac{1}{2}} \qquad \text{A2.16}$$

which shows that all the highly probable noise voltages lie near the surface of the hypersphere of radius $(2BTN)^{\frac{1}{2}}$.

APPENDIX 3

The maximum number of distinct signals in the presence of noise.

For a signal of mean power P masked by an additive white noise power N, it is known exactly what point defines the signal in the signal space of 2BT dimensions. The point is probably inside and near the centre of a hypersphere of radius

$$r_n = (2BTN)^{\frac{1}{2}} \qquad \text{A3.1}$$

the volume of which is given by

$$V_n = C(BT).r_n^{\ 2BT} = C(BT).((2BTN)^{\frac{1}{2}})^{2BT} \qquad \text{A3.2}$$

The observed signal points due to interference from noise lie in a hypersphere of radius

$$r_{(s+n)} = (2BT(P+N))^{\frac{1}{2}} \qquad \text{A3.3}$$

and the volume of the corresponding hypersphere is

$$V_{(s+n)} = C(BT)((2BT(P+N))^{\frac{1}{2}})^{2BT} \qquad \text{A3.4}$$

The maximum number of non-overlapping signals is defined by the ratio

$$\frac{V_{(s+n)}}{V_n} = M_{max} = ((P+N)/N)^{BT} \qquad \text{A3.5}$$

from equations 3.2 and 3.4

If from $\log_2 M_{max}$ the maximum number of bits of information which can be transmitted in time T in the presence of noise, namely

$$\log_2 M_{max} = BT\ \log_2((P+N)/N) = BT\ \log_2(1+\frac{P}{N}) \qquad \text{A3.6}$$

which depends not only on the ratio of the signal power to the noise power but also on the product BT. By increasing the time of observation for a constant bandwidth we can detect signals buried in very intense additive noise.

In telecommunications a grouping of physical media and engineering devices for modulating, transmitting, propagating receiving and demodulating signals is know as a channel. The channel capacity is the amount of information which can be transmitted and interpreted without ambiguity after detection in unit time. Therefore, by dividing equation by T the channel capacity is

$$C = \frac{1}{T} \log_2 M_{max} = B \log_2 \frac{P+N}{N} = B \log_2 \left(1 + \frac{P}{N}\right) \qquad \text{A3.7}$$

It is seen that the channel capacity is proportional to the bandwidth passed and the ratio of the signal and noise powers after detection or demodulation at the output.

Noise might be added at any point in the system from the generation of the signal to the final output. As the signal to noise ratio deteriorates, the rate of transmission of information decreases. To increase the rate of transmission in the presence of additive noise is much more economically obtained by increasing the bandwidth and spreading the signal in a wider spectrum of frequencies rather than merely increasing the signal power transmitted.

For additive noise such as, for example, for thermal and shot noise, it is found that the total noise is proportional to the bandwidth, B. The signal is transmitted adequately and with good freedom from uncertainty due to noise if the ratio P/N is noticeably greater than or at least equal to unity, since $\log_2 (1 + \frac{P}{N})$ has its greatest rate of rise with P/N when P/N is in the range from unity to 2, and thereafter increases more and more slowly as P/N increases.

APPENDIX 4

Increasing the certainty of detecting the signal.

Let us denote by M(T) the number of distinguishable different signals elements from which an actual signal can be chosen or constructed and sent through the transmission channel.

If the channel capacity is C, in order to ensure the highest probability that the signal can be detected it is necessary that

$$M(T) < M_{max} = 2^{CT} \qquad A4.1$$

$$\text{or, } M(T) = \xi 2^{CT} \quad \text{where} \quad \xi < 1$$

$$= \xi \left(\frac{P+N}{N}\right)^{BT}$$

$$= \xi \left(((P+N)/N)^{\frac{1}{2}}\right)^{2BT} \qquad A4.2$$

The received noisy signals arising from the M(T) transmitted signals will each have an average power (P + N).
All received signals will be distributed over the surface of the hyper-sphere already discussed above, having a radius

$$r_{(S+N)} = (2BT(P+N))^{\frac{1}{2}} \qquad A4.3$$

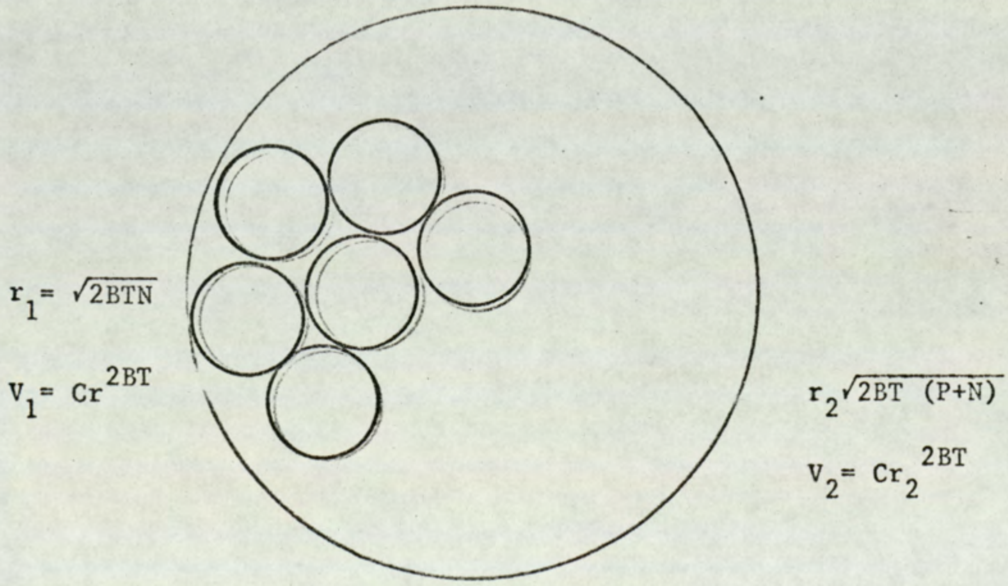


Fig. A.4.

In the two dimensional representation of the hypersphere as shown in Fig.   the signal transmitted is represented by the vector OB, and the observed or received signal at the output by the vector OA. The error of observation due to noise being

$$r_N = (2BTN)^{\frac{1}{2}} = \sigma_N(2BT)^{\frac{1}{2}} \qquad \text{A4.5}$$

The volume in which any reasonably possible transmitted signal is located is less than the volume of a sphere of radius h which was calculated by simple geometry.
From the equation

$$(2BTP)^{\frac{1}{2}}(2BTN)^{\frac{1}{2}} = h(2BT(P+N))^{\frac{1}{2}} \qquad \text{A4.6}$$

h is obtained

$$h = (2BT\,\frac{PN}{P+N})^{\frac{1}{2}} \qquad \text{A4.7}$$

Since the signals are distributed at random, the probability that any particular transmitted signal other than that represented by the vector OB in the diagram would be chosen is less than the ratio of the volumes of the volumes of spheres of radius h and one of radius $(2BTP)^{\frac{1}{2}}$. Altogether there are $\{M(T) - 1\}$ possible signals other than that represented by the vector OB. The total probability that any other signal than that represented by the vector OB was transmitted is then

$$\begin{aligned} P(\xi) &= \{M(T) - 1\}\,(\frac{N}{P+N})^{BT} \\ &= \left[\xi(\frac{P+N}{N})^{BT} - 1\right]\cdot\,(\frac{N}{P+N})^{BT} \\ &= \xi - (\frac{N}{P+N})^{BT} \\ &= \xi, \end{aligned} \qquad \text{A4.8}$$

since $N/(P + N) < 1$ and $BT >> 1$.

If M(T) is made much less than M_{max}, then $\xi << 1$ and the signal point or radius such as that represented by OB in the diagram can be located almost with certainty.

APPENDIX 5

Analysis of the thermistor double bridge.
The influence of small differences between the thermistors.

It has been shown experimentally that to a good approximation the law relating the resistance R of a thermistor bead to the absolute temperature $T^{o}K$ is

$$R = A. \exp(B/T) \qquad \text{A5.1}$$

where A is a constructional constant and

B is a constant determined by the bead construction.

The temperature coefficient of resistance at temperature $\theta^{o}C$ is by definition

$$\alpha_{\theta} = \frac{1}{R}. \frac{dR}{d\theta} \qquad \text{A5.2}$$

It follows that the temperature coefficient, is, from equations (1) and (2)

$$\alpha_{\theta} = -B/T^{2} \qquad \text{A5.3}$$

Clearly, two identical beads having the same value of B and at the same temperature must have equal temperature coefficients. Owing to inherent constructional differences it is inevitable that any pair, even if most carefully selected, will differ both in bead circuit resistance and in their heater circuit resistance. In an investigation into the possibility of using indirectly heated thermistors in a precise a.c./d.c. converter, F.C. Widds has shown that the bead temperature varies almost exactly linearly with the power dissipation up to a power of about 12 mW, or,

$$P = KT \qquad \text{A5.4}$$

The value 12 mW was adopted in the equipment constructed here as a maximum value for the total dissipation in each thermistor, i.e. the total $(P_o + P_B)$, of the dissipations in the heater P_o and the bead, P_B. The dissipation coefficient, K, expressed in mWK^{-1}, is therefore assumed to be constant.

The actual operating point of a bridge may be found graphically as the intersection of the characteristic curve of the thermistor,

$$U_B = f(I_B) \quad \text{for} \quad I_h = 10\text{mA} \qquad \text{A5.5}$$

and the operating line defined by

$$U_{supply} - U_B = R_s IB \qquad \text{A5.6}$$

where U_B is the voltage across the bead,

I_B is the bead current,

$U_{supply} = 2U =$ the bridge supply voltage

R_s is the value of a resistance in series with the bead,

I_h is the total heater circuit current.

The total dissipation, $P_T = P_H + P_B$, i.e. the sum of the dissipations in the bead and the heater can be minimised by introducing a sampling servo system. A sampling servo system has been used in the equipment constructed for this investigation.

In order to make P_B as small as possible and at the same time, in order to obtain a high sensitivity, to use a large value of bead voltage, U_B, the sampling time of the bridge, Δt, was chosen to be only 0.5% of the sampling time interval, t_s. A value of 7V was selected for U_B, so that

$$P_B = \frac{U_B^2}{R_B} \cdot \frac{\Delta t}{t_s} = \frac{49}{2.1} \cdot 10^{-3} \cdot \frac{1}{200} \qquad \text{A5.7}$$

$$= 0.125 \text{ mW}$$

where R_b is the bead resistance in the bridge shown in the diagram, R_T, when the total power dissipation in the thermistor, P_T, is set at 10 mW.

The two thermistor beads in the balanced bridge will have slightly different operating temperatures owing to the small but unavoidable differences in bead and thermistor heater dissipations. Denote this small difference by ΔP and the temperature difference due to ΔP by $\Delta\theta$, then

$$\Delta\theta = \Delta P/K \qquad \text{A5.8}$$

where K is the dissipation coefficient defined above in the paragraph following equation (4). The difference between the temperature coefficients of the two thermistors is then given by

$$\Delta\alpha = \frac{2\Delta\theta}{T} \cdot \alpha\theta \qquad \text{A5.9}$$

an equation which can be derived by differentiating equation (3) and changing differentials to finite differences.

If for some reason, for example a change in the ambient temperature, the temperature of the entire bridge is changed by an amount $\Delta\theta_a$ the bridge will go out of balance unless the temperature coefficients of the two thermistors are identical. In order to restore the balance, the bead temperature of the beat with the smaller temperature coefficient must be changed by applying a correcting temperature change $\Delta\theta_{corr}$. Then omitting the suffix θ on α, we get

$$\Delta\theta_{corr} = \Delta\theta_a \frac{\Delta\alpha}{\alpha - \Delta\alpha} \qquad \text{A5.10}$$

or, to a close approximation

$$\Delta\theta_{corr} = \Delta\theta_a \frac{\Delta\alpha}{\alpha} \qquad \text{A5.11}$$

or, by substituting for $\Delta\alpha$ from equation

$$\Delta\theta_{corr} = \Delta\theta_a \cdot \frac{2\Delta\theta}{T} \qquad \text{A5.12}$$

Since the power dissipation in the heater is

$$P = R_h . I_h^2 \qquad \text{A5.13}$$

it follows that a small change in heater current of amount ΔI_{corr} introduces a change in the dissipation of

$$\Delta P_{corr} = 2I_h . \Delta I_{corr} . R_h \qquad \text{A5.14}$$

so that

$$\Delta\theta_{corr} = \frac{\Delta P_{corr}}{K} = \frac{2I_h.\Delta I_{corr}.R_h}{K} \qquad A5.15$$

Using equation ( ) we obtain the dependence of the change on temperature of the bridge when a signal is applied, ΔP in equation ( ), in terms of the change in ambient temperature and the balance correction current in the form

$$\Delta\theta = \frac{2I_h.\Delta I_{corr}.R_h}{K} \cdot \frac{T}{2\delta\theta_a} \qquad A5.16$$

The temperature coefficient of the bridge may conveniently be defined as the fractional change in heater circuit current required to restore balance for 1^oC (or 1K) change in ambient temperature,

$$\alpha_{BR} = \frac{\Delta I_{corr}}{I_h} \cdot \frac{1}{\Delta\theta_a} \qquad (^oC^{-1}) \qquad A5.17$$

This temperature coefficient obviously depends on the differences in the thermistor characteristics. For an ideal case, $\Delta\alpha$; $\Delta\theta$; ΔI_{corr} and α_{BR} are all zero since all the components are exactly matched. By carefully selecting and balancing the component characteristics, a value of α_{BR} of the order 10^{-4} per degree Celsius can be obtained.

The temperature change due to changing the applied power can be expressed by using equations (16) and (17) as a function of ΔI_{corr}. Widdis has given a similar analysis, but for the same change, ΔI, in the thermistor beads. For carefully selected thermistors in which the difference between the values of the dissipation factors K was about 2%. α_{BR} was found to be 0.45×10^{-4} and $\Delta\theta$ to be 1.1^oC.

It is seen that the unbalance of the temperatures brought about by the change in ambient temperature $\Delta\theta_a$, namely $\Delta\theta$, is not at all important provided the fractional change in heater current, $\Delta I_{corr}/I_h$, brought about by a change in ambient temperature $\Delta\theta_a$ in order to restore the balance, is negligibly small.

If both d.c. and a.c. feedback systems are present, the total change in the heater power is partly due to the a.c. reference signal and partly due to the d.c. power change. In that case, ΔI_{corr} is constituted not only by a d.c. current change, but also by an equivalent change in r.m.s. values of the total d.c. and a.c. feedback currents.

Improvement in the bridge sensitivity could be achieved either by increasing the differential amplifier gain in each loop, or by using higher bridge supply voltages. If a very high d.c. gain is used it may increase the unwanted drift in d.c. current readings on the indicating instrument. High supply voltages would cause extra heater power dissipation, thereby reducing the ratio of controllable heater power dissipation. Furthermore, the thermistor bead resistance is a highly exponential function of the temperature, and this could introduce new unwanted errors. The solution adopted in the present investigation was to use the sampling servo system which enabled high bridge supply voltages to be used without increasing the total dissipation noticeably.

APPENDIX 6

Perturbing sample in Resonant cavity.

Consider an unperturbed, air filled cavity of resonant frequency ω_{01} and volume V, at each point of which are defined the fields $\underset{\sim}{E}_o$ and $\underset{\sim}{H}_o$. The permittivity and the permeability are respectively ε_o and μ_o.

Let a material of volume ΔV characterised by complex permittivity $\underset{\sim}{\varepsilon}$ and complex permeability $\underset{\sim}{\mu}$ be introduced into the cavity. The resonant frequency shifts now to ω_{02}. The fields at each point of the composite structure are denoted by $\underset{\sim}{E}$ and $\underset{\sim}{H}$.

At any point within the unperturbed cavity, Maxwell's curl equations may be written as

$$\underset{\sim}{\nabla} \times \underset{\sim}{H}_o = j\omega_{01}\varepsilon_o\underset{\sim}{E}_o \qquad \text{A6.1}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E}_o = j\omega_{01}\mu_o\underset{\sim}{H}_o \qquad \text{A6.2}$$

These are also valid for the complex conjugated $\underset{\sim}{E}_o{}^*$ and $\underset{\sim}{H}_o{}^*$

$$\underset{\sim}{\nabla} \times \underset{\sim}{H}_o = -j\omega_{01}\varepsilon_o\underset{\sim}{E}_o{}^* \qquad \text{A6.3}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E}_o = j\omega_{01}\mu_o\underset{\sim}{H}_o{}^* \qquad \text{A6.4}$$

For the perturbed cavity

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = j\omega_{02}\varepsilon_o\underset{\sim}{E} \quad \text{outside } \Delta V \qquad \text{A6.5}$$

$$= j\omega_{02}\underset{\sim}{\varepsilon}\,\underset{\sim}{E} \quad \text{inside } \Delta V \qquad \text{A6.6}$$

$$\nabla \times E = -j\omega_{02}\mu_o\underset{\sim}{H} \quad \text{outside } \Delta V \qquad \text{A6.7}$$

$$= -j\omega_{02}\underset{\sim}{\mu}\,\underset{\sim}{H} \quad \text{inside } \Delta V \qquad \text{A6.8}$$

Multiplying A6.5 and A6.6 by $\underset{\sim}{E}_o{}^*$ and A6.7 and A6.8 by $H_o{}^*$

$$\underset{\sim}{E}_o{}^* (\underset{\sim}{\nabla} \times \underset{\sim}{H}) = j\omega_{02}\varepsilon_o\underset{\sim}{E}_o{}^*\underset{\sim}{E}$$

$$= j\omega_{02}\underset{\sim}{\varepsilon}\,\underset{\sim}{E}_o{}^*\underset{\sim}{E} \qquad \text{A6.9}$$

$$\underset{\sim}{H}_o{}^* (\underset{\sim}{\nabla} \times \underset{\sim}{H}) = j\omega_{02}\mu_o\underset{\sim}{H}_o{}^*\underset{\sim}{H}$$

$$= j\omega_{02}\underset{\sim}{\mu}\,\underset{\sim}{H}_o{}^*\underset{\sim}{H} \qquad \text{A6.10}$$

and the unperturbed cavity equations (3) and (4) by E and H respectively

$$\underset{\sim}{E}\,(\underset{\sim}{\nabla} \times \underset{\sim}{H}_o{}^*) = -j\omega_{01}\varepsilon_o \underset{\sim}{E}_o{}^* \underset{\sim}{E} \qquad \text{A6.11}$$

$$\underset{\sim}{H}\,(\underset{\sim}{\nabla} \times \underset{\sim}{E}_o{}^*) = j\omega_{01}\mu_o \underset{\sim}{H}_o{}^* \underset{\sim}{H} \qquad \text{A6.12}$$

Applying the vector identity

$$\underset{\sim}{\nabla}\,(\underset{\sim}{A} \times \underset{\sim}{B}) \equiv \underset{\sim}{B}\,(\underset{\sim}{\nabla} \times \underset{\sim}{A}) - \underset{\sim}{A}\,(\underset{\sim}{\nabla} \times \underset{\sim}{B}) \qquad \text{A6.13}$$

to our set of equations

$$\underset{\sim}{\nabla}\,(E_o{}^* \times H) + \underset{\sim}{\nabla}\,(\underset{\sim}{E} \times \underset{\sim}{H}_o{}^*) \equiv - \left|\underset{\sim}{H}_o{}^*(\underset{\sim}{\nabla} \times \underset{\sim}{E}) + H(\underset{\sim}{\nabla} \times \underset{\sim}{E}_o{}^*)\right| + \left|\underset{\sim}{E}_o{}^*(\underset{\sim}{\nabla} \times \underset{\sim}{H}) + \underset{\sim}{E}(\underset{\sim}{\nabla} \times \underset{\sim}{H}_o{}^*)\right|$$

which after substituting corresponding terms and integrating over the total volume V become

$$\int \underset{\sim}{E}_o{}^*(\underset{\sim}{\nabla} \times \underset{\sim}{H}) + \underset{\sim}{E}(\underset{\sim}{\nabla} \times \underset{\sim}{H}_o{}^*) - \underset{\sim}{H}_o{}^*(\underset{\sim}{\nabla} \times \underset{\sim}{E}) + \underset{\sim}{H}(\underset{\sim}{\nabla} \times \underset{\sim}{E}_o{}^*) \; dV$$

$$= \int_V \left[j(\omega_{02} - \omega_{01})\varepsilon_o \underset{\sim}{E}_o{}^*\underset{\sim}{E} + j(\omega_{02} - \omega_{01})\,\mu_o \underset{\sim}{H}_o{}^*\,\underset{\sim}{H} \right] dV$$

$$+ \int_{\nabla V} \left[j\omega_{02}(\underset{\sim}{\varepsilon} - \varepsilon_o)\underset{\sim}{E}_o{}^*\underset{\sim}{E} + j\omega_{02}(\underset{\sim}{\mu} - \mu_o)\,\underset{\sim}{H}_o{}^*\underset{\sim}{H} \right] dV \qquad \text{A6.14}$$

Introducing the Gauss theorem, the left hand side of the equation (14) can be written as

$$\int_\nabla \left[\underset{\sim}{\nabla}(\underset{\sim}{E}_o{}^* \times \underset{\sim}{H}) + \underset{\sim}{\nabla}(\underset{\sim}{E} \times \underset{\sim}{H}_o{}^*) \right] dV$$

$$= -\int_S (\underset{\sim}{E}_o{}^* \times \underset{\sim}{H} + \underset{\sim}{E} \times \underset{\sim}{H}_o)\,d\underset{\sim}{s} \qquad \text{A6.15}$$

the surface integral is zero, since $\underset{\sim}{E}_o{}^* \times \underset{\sim}{H}$ and $\underset{\sim}{E} \times \underset{\sim}{H}$ is normal to $d\underset{\sim}{s}$.
Consequently

$$0 = j(\omega_{02} - \omega_{01}) \int_V (\varepsilon_o \underset{\sim}{E}{}^* \underset{\sim}{E} + \mu_o \underset{\sim}{H}_o{}^* \underset{\sim}{H})\; dV$$

$$+ j\omega_{02} \int_{\nabla V} \left[\underset{\sim}{\varepsilon} - \varepsilon_o)\underset{\sim}{E}_o{}^*\underset{\sim}{E} + (\underset{\sim}{\mu} - \mu_o)\, \underset{\sim}{H}_o{}^*\, \underset{\sim}{H} \right] dV \qquad \text{A6.16}$$

from which, finally

$$\frac{\omega_{02} - \omega_{01}}{\omega_{02}} = \frac{\int_{\Delta V} \left[\underset{\sim}{\varepsilon} - \varepsilon_o)\underset{\sim}{E}_o{}^*\underset{\sim}{E} + (\underset{\sim}{\mu} - \mu_o)\underset{\sim}{H}_o{}^*\, \underset{\sim}{H} \right] dV}{\int_V (\varepsilon_o \underset{\sim}{E}{}^*\underset{\sim}{E} + \mu_o \underset{\sim}{H}_o{}^*\, \underset{\sim}{H})\; dV} \qquad \text{A6.17}$$

APPENDIX 7

Simplified expressions for a flame plasma in a cavity resonator.

As there have been no assumptions in deriving expression (17) App (6), it is an exact expression.

For the case of interest, the perturbation volume ΔV is small so that its effects are negligible outside ΔV. Consequently, we may put

$$\underset{\sim}{E} = \underset{\sim}{E}_o \quad \text{and} \quad \underset{\sim}{H} = H_o \quad \text{outside } \Delta V \qquad \text{A7.1}$$

and the denominator in (7) Appendix (8) is essentially equal to (J.L.Altman, 1963).

$$\int_V \left[\varepsilon_o |E_o|^2 + \mu_o |H_o|^2\right] dV = 4(V_E + U_H) = 4VTOT \qquad \text{A7.2}$$

i.e. equal to 4 times the sum of the average electric stored energy V_E and the average magnetic stored energy V_H.

In practice, the perturbing element will be introduced either in region of negligible magnetic or negligible electric field. As in the flame $\underset{\sim}{\mu} = \mu_o$, the second term of the numerator of (A6.17) vanishes, and the maximum shift of the resonant frequency $\Delta\omega = \omega_{02} - \omega_{01}$ will be for the volume ΔV which corresponds spatially to the maximum intensity of the electric field. The expression (A6.17) then simplifies to

$$\frac{\Delta\omega}{\omega_{02}} = - \frac{\int_{\Delta V} (\underset{\sim}{\varepsilon} - \varepsilon_o)|E_o|^2}{4VTOT} \qquad \text{A7.3}$$

In general the perturbing flame sample is a good dielectric with the complex permittivity

$$\varepsilon = \varepsilon_o \, (\varepsilon_r' - j\varepsilon_r'') \qquad \text{A7.4}$$

where ε_r' is the real part of the relative permittivity (dielectric constant) and ε_r'' is the imaginary part of the relative permittivity. As the dielectric losses are mostly due to the conductivity σ of the flame

$$\varepsilon_o \varepsilon_r = \frac{\sigma}{2\pi f} \qquad A7.5$$

where f is the operating frequency.

Furthermore the complex resonant frequency ω_{co} of a high Q_o cavity with loss may be written

$$\omega_{co} = \omega_o + j\,\frac{\omega_o}{2Q_o} \qquad A7.6$$

when these concepts are applied to both ω_{01} and ω_{02}

$$\frac{\omega_{02} + \frac{j\omega_{02}}{2Q_{02}} - \omega_{01} - \frac{j\omega_{01}}{2Q_{01}}}{\omega_{02} + j\,\frac{\omega_{02}}{2Q_{02}}} = \frac{\left[\varepsilon_o(\varepsilon_r' - j\varepsilon_\theta'') - \varepsilon_o\right] \int_{\Delta V} \underset{\sim}{E}_o{}^* \underset{\sim}{E} dV}{4V\ TOT} \qquad A7.7$$

From the real part of the last equation, assuming $j\omega/2Q$ is negligible to ω in the denominator

$$\frac{\omega_{02} - \omega_{01}}{\quad} = -\,\varepsilon_o(\varepsilon_r' - 1)\ \frac{\int_{\Delta V} \underset{\sim}{E}_o{}^* \underset{\sim}{E} dV}{4V\ TOT} \qquad A7.8$$

whereas from the imaginary parts

$$\frac{1}{Q_{02}} - \frac{1}{Q_{01}} = \varepsilon_o\ \varepsilon_r''\ \frac{\int_{\Delta V} \underset{\sim}{E}_o{}^* \underset{\sim}{E} dV}{2Q\ TOT} \qquad A7.9$$

Thus, in principle, by measuring the shaft in resonant frequency and the change of Q, both ε_r' and ε_r'' (or σ) may be determined.

For a fixed position and constant size of the flame ΔV_o, not necessarily for a cylindrical resonator only, the ratio

$$\frac{\int_{\Delta Vo} \underset{\sim}{E}_o{}^* \underset{\sim}{E} dV}{2V\ TOT} = \frac{\int_{\Delta Vo} |E|^2 dV}{2V\ TOT} \qquad A7.10$$

is a constant which can be easily found experimentally by inserting a lossless specimen of the same shape and volume ΔV_o but of known dielectric constant ε_r' as function of frequency.

Chaffin (Chaffin,1968) has denoted this ratio as the effective filling factor P

$$P = \frac{\int_{\Delta V_o} |E|^2 dV}{2V_{TOT}} \qquad A7.11$$

so that expressions (8) and (9) may be rewritten as

$$2\,\frac{\omega_{02} - \omega_{01}}{\omega_{02}} = P\ \varepsilon_o\ (\varepsilon_r' - 1) \qquad A7.12$$

$$\frac{Q_{01} - Q_{02}}{Q_{01} \cdot Q_{02}} = P\ \varepsilon_o\ \varepsilon_r'' \qquad A7.13$$

By introducing (Drummond,1961)

$$\varepsilon_o\ (\varepsilon_r' - 1) = \frac{\omega_p^2}{\omega_o^2}$$

$$\varepsilon_o\ \varepsilon_r'' = \frac{\omega_p^2 \gamma_c}{\omega_o{}^3}$$

where ω_p is the plasma frequency

$$\omega_p = \left(\frac{N\ e^2}{\varepsilon_o m}\right)^{\frac{1}{2}}$$

N - number density of ions or electrons (part m^{-3})

e - electron charge 1.602. 10^{-19}C.

ε_o - permittivity of free space 8.854. 10-12 $Fm^{-1} = \frac{10^{-9}}{36\pi}\ Fm^{-1}$

m mass of an electron 9.109. 10^{-31} Kg.

and ν_ε - is the collision frequency of charged particles in a plasma.

By substituting these numerical values

$$\omega_p = 56.5\ \sqrt{N}\ \text{ rad/sec}$$

$$f_p = \frac{\omega_p}{2\pi} = 8.99.\ \sqrt{N}\ \text{ c/s}$$

From which a direct relationship between the resonant frequency shaft and number density of electrons could be derived

$$\Delta f \equiv 1.6.10^3.P.f_o.N$$

From the last expression it is obvious that high precision measurements of ionisation densities are possible only if the oscillator frequency is stable enough and means for measuring small frequency changes is available. Moreover, the Q_o of the resonator must be high, so that the adjustment on the resonant frequency is sharp enough.

APPENDIX 8

Dependence of complex dielectric constant on plasma frequency and collision frequency.

In order to simplify the approach some further assumptions are necessary:

- the equality of ion and electron number densities is not disturbed by the presence of microwave electric fields,
- the ions of gases are at least cca 1840 times as heavy as electrons and could be considered immobile,
- for fields that are not extremely strong, the electron motions are determined principally by the electric fields of the wave; the forces arising from the r.f. magnetic field can be neglected.

Taking into account the last assumption Lorentz force equation (the change of momentum) may be written

$$\frac{d(m\underset{\sim}{v})}{dt} = eE - m\underset{\sim}{v}\,\nu_{\epsilon} \qquad \text{A8.1}$$

where

$\bar{v}$ - is the average velocity of electrons in z direction

E_z - r.f. electric field (also exists only in z direction)

ν_{ϵ} mean frequency of inelastic collisions per electron

By further introduction of

$$E = |E_z|\, e^{j\omega_o} \qquad \text{A8.2}$$

$$\underset{\sim}{v} = |\bar{v}|\, e^{j\omega_o} \qquad \text{A8.3}$$

equation (8) may be rearranged to give the velocity in terms of the electric field as

$$\underset{\sim}{v} = \frac{e\underset{\sim}{E}}{m(v_e + j\omega_o)} \qquad \text{A8.4}$$

The electron density is undisturbed by the motions of the electrons since all paths lie in the transverse planes and are parallel with each other. The convection current density is

$$= Ne\underset{\sim}{v} = \frac{Ne^2E}{m(\nu_c + j\omega)} \qquad \text{A8.5}$$

Which we can substitute into the Maxwell curl equation

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = j\omega_o \varepsilon_o \underset{\sim}{E} + \frac{Ne^2\underset{\sim}{E}}{m(\nu_c + j\omega)}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = j\omega_o \varepsilon_o \left[1 - \frac{Ne^2}{\varepsilon_o m(\nu_c{}^2 + \omega^2)} + \frac{Ne^2\nu_c}{j\omega\varepsilon_o m(\nu_c + \omega^2)} \right]$$

$$= j\omega_o \varepsilon_o (\varepsilon_r' - j\varepsilon_r'') \underset{\sim}{E} \qquad \text{A8.6}$$

where the complex permittivity ε is

$$\varepsilon = \varepsilon_o \ (\varepsilon_r' - j\varepsilon_r'') \qquad \text{A8.7}$$

which was introduced earlier in the Appendix (7)

By comparison of the real and imaginary parts respectively

$$\varepsilon_r' = \varepsilon_o \left[1 - \frac{Ne^2}{\varepsilon_o m(\nu_c{}^2 + \omega^2)} \right] \qquad \text{A8.8}$$

$$\varepsilon_r'' = \frac{Ne^2\nu_c}{\omega\varepsilon_o m(\nu_c{}^2 + \omega^2)} \qquad \text{A8.9}$$

The imaginary part of the complex relative permittivity vanishes as the collision frequency ν_c goes to zero, then

$$\varepsilon = \varepsilon_o \varepsilon_r' = \varepsilon_o \left(1 - \frac{Ne^2}{\varepsilon_o m} \cdot \frac{1}{\omega^2} \right)$$

$$= \varepsilon_o \left[1 - \frac{\omega_p}{\omega} \right] \qquad \text{A8.10}$$

where ω_p is known as the plasma frequency,

$$\omega p = \frac{Ne^2}{\varepsilon_o m_e} \qquad \text{A8.11}$$

which was introduced already in Appendix (7)

Now ε_r'' may be also written in terms of ωp

$$\varepsilon_r'' = \frac{\omega_p{}^2 . \nu_c}{(\nu_c{}^2 + \omega_o{}^2)\omega_o} \qquad \text{A8.12}$$

Usually $\nu_c{}^2 << \omega_o{}^2$, so that approximately

$$\varepsilon_r'' = \frac{\omega_p{}^2}{\omega_o{}^3} . \nu_c \qquad \text{A8.13}$$

If, instead of the complex permittivity

$$\varepsilon = \varepsilon_o (\varepsilon_o - j\varepsilon_r) \qquad \text{A8.14}$$

we introduce complex conductivity

$$\sigma_c = \sigma + j\omega\varepsilon_o\varepsilon_r' \qquad \text{A8.15}$$

in Maxwell's H curl equation

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = (j\omega\varepsilon_o\varepsilon_r' + \sigma) \underset{\sim}{E} \qquad \text{A.8.16}$$

by comparison with

$$\sigma = \omega_o\varepsilon_o\varepsilon_r'' \qquad \text{A8.17}$$

so that r.f. conductivity may be expressed as

$$\sigma \equiv \frac{\omega_p{}^2}{\omega_o{}^2} . \nu_c \, \varepsilon_o = \frac{Ne^2}{m} \nu_c \qquad \text{A8.18}$$

from which it is obvious that $\sigma \propto Ne\nu_c$, i.e. r.f. conductivity is proportional to the product of electron density and collision frequency. Substituting numerical values for e and m

$$\sigma = 2.76.10^{-8} . \nu_c Ne \qquad \text{A8.19}$$

Ne, the number electron density was found to be proportional to the resonant frequency shift (Appendix 7, equation 8).

$$N_e \equiv 625.10^{-4} \; \frac{1}{P} \cdot \frac{\Delta f}{f_o} \qquad \text{A8.20}$$

Using (Appendix 7, Equation 13)

$$\frac{Q_{01} - Q_{02}}{Q_{01}\, Q_{02}} = P\; \varepsilon_o \varepsilon_r'' \qquad \text{A8.21}$$

and (Appendix 7, equation 9), we have

$$\sigma = \frac{\omega_o}{P} \; \left(\frac{1}{Q_{02}} - \frac{1}{Q_{01}}\right) \qquad \text{A8.22}$$

After combining (20) (21) (22) we finally get

$$V_c = \frac{1}{2.76}\, 10^8 \cdot \frac{\sigma}{N}$$

$$\equiv 3.63 \;.\; 10^{-5} \; \frac{f_o^2}{\Delta f} \cdot \; \left(\frac{1}{Q_{02}} - \frac{1}{Q_{01}}\right) \qquad \text{A8.23}$$

what is the relationship between collision frequency and measuring quantities Δf, Q_{01} and Q_{02}.

It should be noted that for finding out the collision frequency it is not necessary to know the effective fillings factor of the resonator.

As in practice we are more often interested in the plasma frequency measurements as well as in the r.f. conductivity, the effective filling factor of the resonator must be found at the very beginning of the measurements.

APPENDIX 9

The effective filling factor of the cylindrical open ended cavity.

Starting from the general expression (Appendix 7, equation 2), with the following assumptions:

- the relative permeability throughout the whole cavity is constant i.e. $\underset{\sim}{\mu} = \mu_o$.
- the energy stored in the cavity is equal to the sum of the average of the electric stored energy and the average of the magnetic stored energy. Taking these to be equal to each other (in resonance), total energy stored could be taken as two times the average electric stored energy, i.e.

$$\int_V \left[\varepsilon_o \underset{\sim}{E}^* \underset{\sim}{E} + \mu_o \underset{\sim}{H}_o{}^* \underset{\sim}{H} \right] dv = 2 \int_V |E^2| dv \qquad \text{A9.1}$$

we obtain

$$\frac{\omega_{02} - \omega_{01}}{\omega_{02}} = \frac{\int_{\Delta V} (\underset{\sim}{\varepsilon} - \varepsilon_o) |E^2| dv}{2 \int_V \varepsilon_o |E^2| dv} \qquad \text{A9.2}$$

For the ease of calculation let us substitute the slowly special varying dielectric constant in the flame, by the constant dielectric constant in an idealised flame of equivalent radius C. The electric field in the cylindrical cavity, with open ends follows the Bessel function distribution of the zero order (Adler, 1949), so that we may write

$$\frac{\omega_{01} - \omega_{02}}{\omega_{02}} = \frac{(\underset{\sim}{\varepsilon} - \varepsilon_o) \int J_o{}^2(R\rho)\rho d\rho}{2 \int J_o{}^2(K\rho)\rho d\rho} \qquad \text{A9.3}$$

where a is the radius of the cylinder;b is the rad. of the plasma

ρ is the variable of integration taken to be zero in the centre of the cavity.

J_o - Bessel function of the zero order (for TM-ONO cylindrical cavities)

$$\varepsilon_r - \varepsilon_o = \varepsilon_o\ (\varepsilon_r' - j\varepsilon_r'' - 1) \qquad \text{A9.4}$$

For the purpose of deriving effective filling factor ρ it is sufficient to take only real parts of the relative dielectric constant

$$\frac{\omega_{01} - \omega_{02}}{\omega_{02}} = \frac{\varepsilon_o(\varepsilon_r'-1) \int_o^b J_o{}^2(k\rho)\rho d\rho}{2\int_o^a J_o{}^2(k\rho)\rho d\rho} \qquad \text{A9.5}$$

By comparison to (Appendix 7)

$$\rho_o = \frac{\int_{\Delta V} |E^2| \ dv}{2V\ TOT} = \frac{\int_o^b J_o{}^2(k\rho)\rho d\rho}{\int_o^a J_o{}^2(k\rho)\rho d\rho} \qquad \text{A9.6}$$

$$= \left(\frac{b}{a}\right) \frac{J_o{}^2(kb) + J_1^2(kb)}{J_o{}^2(ka) + J_1^2(ka)} \qquad \text{A9.7}$$

where J_o is the Bessel function of the zero order

J_1 is the Bessel function of the first order

k is the wave number

a is the radius of the cylindrical cavity

b is the equivalent radius of the idealised flame with constant spacial relative dielectric constant.

Note that for resonance $J_o(ka) = o$ so that the effective filling factor

$$\rho_o = \left(\frac{b}{a}\right) \frac{J_d{}^2(kb) + J_1{}^2(kb)}{J_1{}^2(ka)} \qquad \text{A9.8}$$

Further simplification can be done by assuming cylindrical flame of very small radius, so that $J_1{}^2(kb) \rightarrow o$.

APPENDIX 10

Change of transmission coefficient on perturbed resonator Q factor.

Transmission coefficient of the equivalent circuit of a resonator is defined as the ratio of the load power P_L to the available power from generator Pavail. If the output impedance of generator is $Zg = Rg + jXg$, matching would be achieved for the load impedance Z_{Lm} which is the complex conjugate of Zg.

Then,

$$\text{Pavail} = \frac{E^2}{2(Rg + R_L)} = \frac{E^2}{4Rg} \qquad \text{A10.1}$$

Load power for the circuit in Fig.A10.1 is frequency dependent

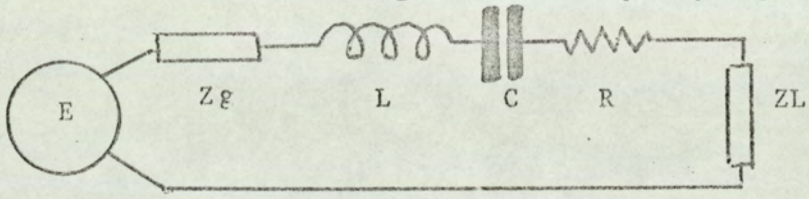


Fig.A10.1

$$P_L = Z_L \cdot I^2(\omega) \qquad \text{A10.2}$$

$$I_{(\omega)} = \frac{E}{Zg + Z_L + Zc} \qquad \text{A10.3}$$

$$= \frac{E}{Rg + R_L + R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) + j(Xg + X_L)}$$

where $(X_g + X_L)$ is total reactive component of the generator and the load, which generally do not compensate each other. Being either capacitive or inductive they slightly modify the cavity reactance.

For resonant coupling

$$X_g = X_L = o \qquad \text{A10.4}$$

and $I_{(\omega)}$ becomes

$$1_{(\omega)} = \frac{E}{R + R_g + R_L + j\omega L\left(1 - \frac{1}{\omega LC}\right)}$$

$$= \frac{E}{1 + \frac{Rg}{R} + \frac{R_L}{R} + jQ_{u1}\left(\frac{\omega}{\omega_{01}} - \frac{\omega_{01}}{\omega}\right)} \qquad \text{A10.5}$$

where $\omega_{01} = (LC)^{-2}$, i.e. resonant frequency of unloaded unperturbed cavity, and $Q_{u1} = \frac{\omega_{01}\ L}{R}$

i.e. unperturbed unloaded cavity Q factor. Transmission coefficient $T_{(\omega)}$ defined before becomes

$$T_{(\omega)} = \frac{4R_g}{E^2} \quad \frac{E^2}{1 + \frac{Rg}{R} + \frac{R_L}{R} + jQ_{u1}\left(\frac{\omega}{\omega_{01}} - \frac{\omega_{01}}{\omega}\right)} R_L \qquad \text{A10.6}$$

Magnitude $T_{(\omega)}$ is then

$$|T_{(\omega)}| = \frac{4\ R_g.R_L}{\left(1 + \frac{R_g}{R} + \frac{R_L}{R}\right)^2 + Q_{u1}^{\ 2}\left(\frac{\omega}{\omega_{01}} - \frac{\omega_{01}}{\omega}\right)^2} \qquad \text{A10.7}$$

what in resonance simplifies to:

$$|T_{(\omega_{01})}| = \frac{4R_g.R_L}{\left(1 + \frac{R_g}{R} + \frac{R_L}{R}\right)^2} \qquad \text{A10.8}$$

Inserting sample in the resonator resonant frequency shifts to ω_{02} with the simultaneous change in R.

$$|T_{(\omega_{02})}| = \frac{4R_gR_L}{\left(1 + \frac{R_g}{(R+\Delta R)} + \frac{R_L}{(R+\Delta R)}\right)^2} \qquad \text{A10.9}$$

If we express both transmission coefficients in decibels, the change in transition coefficient is their difference.

$$\text{Loss}_{(dB)} = \Delta t_{(dB)} = t_{1(dB)} - t_{2(dB)} = 10_{\log} \frac{|T_{(\omega_{02})}|}{|T_{(\omega_{01})}|}$$

$$= 10_{\log}\ t_{(2/1)} \qquad \text{A10.10}$$

Q factor of the unloaded unperturbed resonator was defined before as

$$Q_{ul} = \frac{\omega_{01} L}{R} \qquad \text{A10.11}$$

In fact as soon as the resonator is coupled Q decreases to

$$Q_{01} = \frac{\omega_{01} L}{R + R_g + R_L} \qquad \text{A10.12}$$

Similarly for perturbed cavity

$$Q_{02} = \frac{\omega_{02} L}{R + \Delta R + R_g R_L} \qquad \text{A10.13}$$

If the shift in resonant frequency is small, i.e.

$$\omega_{02} \equiv \omega_{01} = \omega_o \qquad \text{A10.14}$$

which also makes possible the assumption that R_g and R_L do not change

$$\frac{Q_{02}}{Q_{01}} = \frac{R + R_g + R_L}{R + \Delta R4 + R_g + R_L} = \frac{1 + \frac{R_g}{R} + \frac{R_L}{R}}{1 + \frac{R_g}{R + \Delta R} + \frac{R_L}{R + \Delta R}} \qquad \text{A10.15}$$

Furthermore, from A10.8, A10.9 and A10.10

$$\Delta t_{dB} = 10 \log \frac{1 + \frac{R_g}{R} + \frac{R_L}{R}}{1 + \frac{R_g}{R + \Delta R} + \frac{R_L}{R + \Delta R}} \qquad \text{A10.16}$$

What may be written

$$\Delta t_{dB} = 20 \log \frac{Q_{02}}{Q_{01}} \qquad \text{A10.17}$$

giving at the same time

$$\frac{|T(\omega_{01})|}{|T(\omega_{01})|} = \frac{Q_{02}}{Q_{01}} = t_{2/1} \qquad \text{A10.18}$$

In Appendix (8) we have derived expressions dependent on ratio $\frac{Q_{01}}{Q_{02}}$

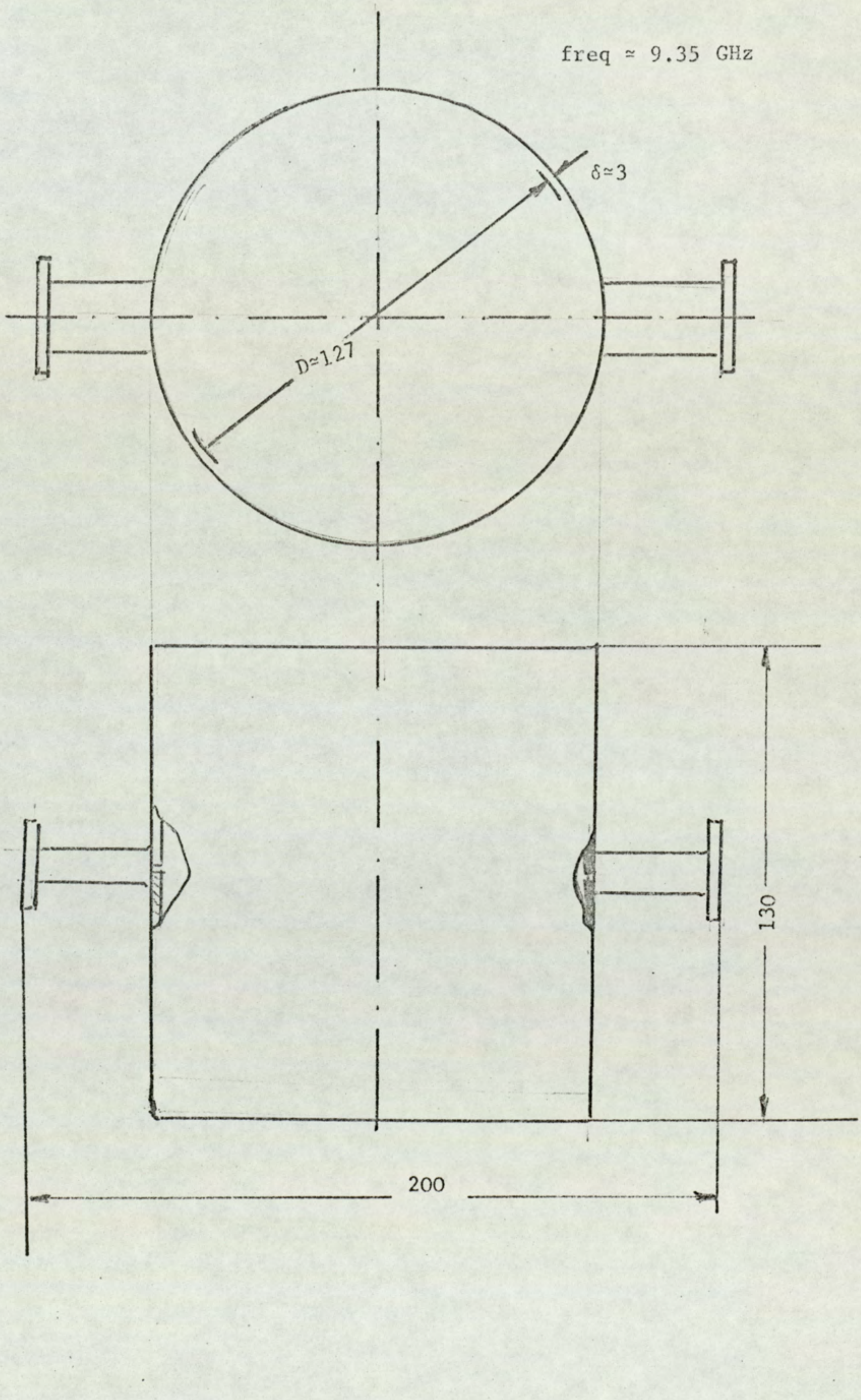
$$t_{2/1}^{-\frac{1}{2}} = \frac{Q_{01}}{Q_{02}} = \frac{|T(\omega_{02})|}{|T(\omega_{01})|} = 10^{\frac{\Delta t_{dB}}{10}}$$

$$= 10^{\frac{-\Delta t}{20}(dB)} = 10^{\frac{-L}{20}(dB)} \qquad A10.19$$

where Δt_{dB} can be measured directly.

APPENDIX 11

Open Ended Cylindrical Resonator TM-050.



System Identification

Determination of the system transfer function and the system impulse response.

Time invariant systems.

The method may be explained with the aid of the diagram (Fig.A12.1)

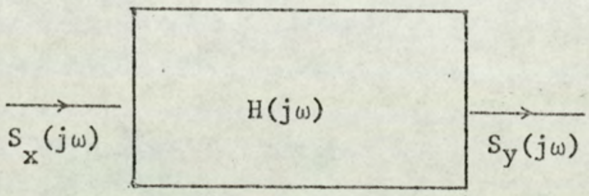


Fig.A12.1

where $S_y(j_\omega)$ and $S_x(j\omega)$ are the output and input linear spectra. If the input power spectrum is flat, i.e. if $G_{xx}(\omega)$ = constant

$$|H(j\omega)|^2 = G_{yy}(j\omega)/K \qquad \text{A12.1}$$

where

$G_{yy}(j\omega)$ is the output power spectrum

$H(j\omega)$ = system transfer function

It is clear that in order to find $|H(j\omega)|$ we must find the value of $G_{yy}(\omega)$ for any of the frequencies in the band of frequencies of interest. Notice that if at the input there is only one spectral line, the output of the system will be also only one line, i.e. the input multiplied by the value of the transfer function at that frequency.

The output response $y(t)$ is the inverse F.T. of $S_y(j\omega)$. The auto-correlation functions $\psi_{xx}(\tau)$, $\psi_{yy}(\tau)$ and the cross correlation function $\psi_{xy}(\tau)$ are defined by

$$\psi_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_o^T x(t + \tau).x(t).dt \qquad \text{A12.2}$$

$$\psi_{yy}(\tau) = \lim \frac{1}{T} \int_o^T y(t + \tau).y(t)\, dt \qquad \text{A12.3}$$

$$\psi_{xy}(\tau) = \lim \frac{1}{T}\int_o^T x(t+\tau).y(t)\,dt$$

$$= \lim \frac{1}{T}\int_o^T x(t+\tau)\int_o^t h(\lambda).x(t-\lambda).d\lambda.dt$$

A12.3

changing the order of integration

$$\psi_{xy}(\tau) = \int_o^t h(\lambda)\left\{\lim \frac{1}{T}\int_o^t x(t+\tau).x(t-\lambda)\,dt\right\} d\lambda$$

A12.4

If we take the Fourier transforms of both sides, since $\psi_{xy}(\tau)$ is on a symmetrical function,

$$G_{xy}(j\omega) \int_{-\infty}^{\infty}\psi_{xy}(\tau)\, e^{-j\omega\lambda} d\tau \qquad \text{A12.5}$$

Then, when we introduce

$$e^{-j\omega\tau} = e^{-j\omega\lambda}.e^{-j\omega(\tau-\lambda)} \qquad \text{A12.6}$$

we can now express the cross power density spectrum as

$$G_{xy}(j\omega) = \int_{-\infty}^{\infty}\int_o^t h\,(\lambda)\;\; e^{-j\omega\tau}\left\{\lim \frac{1}{T}\int_0^T x(t+\tau).x(t-\lambda)\,d\lambda\right\}d\tau$$

A12.7

which could also be written as

$$G_{xy}(j\omega) = \int_{-\infty}^{\infty} h(\lambda)\, e^{-j\omega\lambda}\left\{\int_o^t \psi_{xx}(\tau-\lambda)\, e^{-j\omega(\tau-\lambda)}\, d\tau\right\} d\lambda$$

A12.8

Finally, the cross power density spectrum,

$$G_{xy}(j\omega) = H(j\omega).\,G_{xx}(j\omega) \qquad \text{A12.9}$$

i.e. it can be found from the product of the auto-power spectrum and the transfer function.

The last expression shows that the transfer function of a stationary time function could be written as

$$H(j\omega) = G_{yx}(j\omega)/G_{xx}(j\omega) \qquad \text{A12.9}$$

but it should be noticed that $G_{xx}(j\omega)$, being the Fourier transform of an even function, has only a real part, $G_{xx}(\omega)$. Furthermore, as

$$G_{yx}(j\omega) = S_y(j\omega).\,S_x(-j\omega) \text{ and } G_{xx}(j\omega) = S_x(j\omega).S_x(-j\omega)$$

A12.10

we find also

$$H(j\omega) = S_y(j\omega)/S_x(j\omega) \qquad \text{A12.11}$$

$$\psi_{yx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\eta).h(\xi).d\eta.d\xi \left\{ \int_{0}^{t} x(t + \tau - \eta).x(t - \xi)dt \right\} \qquad A12.12$$

As the expression in the $\left\{ \right\}$ brackets is the autocorrelation function of the input,

$$\psi_{yx}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\eta).h(\xi)\ d\eta.d\xi.\psi_{xx}(\tau + \xi - \eta) \qquad A12.13$$

Taking the Fourier transforms of both sides

$$G_{yy}(\omega) = \int_{-\infty}^{\infty} \psi_{yy}(\tau)\ e^{-j\omega\tau} d\tau$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} e^{-j\omega\tau} h(\eta).h(\xi)\ \psi_{xx}(\tau + \xi - \eta)\ d\eta$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} e^{-j\omega(\tau + \xi - \eta)}\ e^{j\omega\xi}\ e^{-j\omega\eta} h(\ ).h(\ ).$$
$$\psi_{xx}(\tau - \xi - \eta).d\eta$$

$$= \int_{-\infty}^{\infty} h(\eta) e^{-j\omega\eta} d\eta \int_{-\infty}^{\infty} h(\xi).e^{j\omega\xi} d\xi \left[\ \cdot\ \right] \qquad A12.14$$

$$\text{where } \left[\ \cdot\ \right] = \left[\int_{-\infty}^{\infty} \psi_{xx}(\tau + \xi - \eta)\ e^{-j\omega(\tau + \xi - \eta)} d\tau \right] \qquad A12.15$$

so that finally

$$G_{yy}(\omega) = H(j\omega).\ H(-j\omega).G_{xx}(\omega) = |H(j\omega)|^2\ G_{xx}(\omega). \qquad A12.16$$

The last expression proves that the spectral density $G_{yy}(\omega)$ of the output of a linear system can be found by multiplying the square of the modulus of the transfer function by the input spectral density $G_{xx}(\omega)$.

APPENDIX 13

Wiener - Kolmogoroff Filter

The mean squared error, which is the figure of merit (performance index) of the optimum filtering for the stationary additive noise, can be written

$$\overline{\varepsilon^2(t)} = \overline{\left[s(t) - \int_0^t x(t-\tau)w(\tau)d\tau\right]^2} \qquad \text{A13.1}$$

Expanding the squared sum

$$\overline{\varepsilon^2(t)} = \overline{s^2(t)} - \overline{2s(t)} \overline{\int_0^t x(t-\tau)w(\tau)d\tau}$$

$$\overline{+ \int_0^t x(t+\tau)w(\tau)d\tau \int_0^t x(t-\sigma)w(\sigma)d\sigma} \qquad \text{A13.3}$$

Interchanging the operations of integration and averaging

$$\overline{\varepsilon_2(t)} = \psi_{ss}(o) - \int_0^t \psi_{sx}(\tau)w(\ )d\tau +$$

$$\int_0^t\int_0^t \psi_{xx}(\tau-\sigma)w(\tau)w(\sigma)d\tau d\sigma \qquad \text{A13.3}$$

where

$$\psi_{xs}(\tau) = \overline{x(t-\tau)s(t)} \qquad \text{A13.4}$$

$$\psi_{xx}(\tau-\sigma) = \overline{x(t-\tau)x(t-\sigma)} \qquad \text{A13.5}$$

i.e. $\psi_{xs}(\tau)$ and $\psi_{xx}(\tau-\sigma)$ are the cross correlation and auto correlation functions of noisy signals respectively.

The problem is to choose the impulse response of the system $w_o(t)$ which causes the mean squared error to be minimum. This means that if the input $x(t)$ is applied to a linear system with the impulse response $w_o(t)$, the mean squared error between the actual output $y(t)$ and the desired output $s(t)$ is less than would be the case if the same input were applied to a filter with any other impulse response $w_x(t)$ (Fig.A13.1)

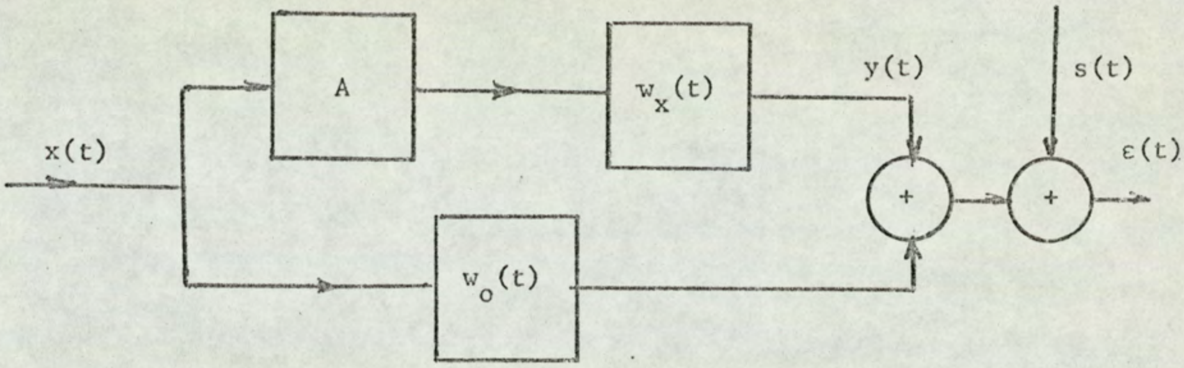


Fig.A13.1

Consider the combination of an actual filter $w(t)$ and an optimum filter $w_o(t)$, which acts as a single system, with the impulse response:

$$w(t) = A.w_x(t) + w_o(t) \tag{A13.6}$$

A is for the moment an arbitrary gain which can be varied continuously over a range of positive and negative values. As $w_o(t)$ is an optimum impulse response, the minimum mean squared error, must obviously be for $A = 0$.

Defining the change in the mean squared error

$$\Delta\varepsilon(A) = \overline{\varepsilon^2(t)} - \overline{\varepsilon_o^2(t)} \tag{A13.7}$$

where $\varepsilon(t)$ corresponds to the combined $w(t)$

$\varepsilon_o(t)$ corresponds to $w_o(t)$ alone

Substituting corresponding expressions, $\Delta\varepsilon(A)$ becomes

$$\begin{aligned}\Delta\varepsilon(A) = & \int_o^t\int_o^t \psi_{xx}(\tau-\sigma)\left[w_o(\tau)+Aw_x(\tau)\right]\left[w_o(\sigma)+Aw_x(\sigma)\right]d\sigma d\tau \\ & - \int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_o(\tau)w_o(\sigma)d\tau d\sigma \\ & - 2\int_o^t \psi_{xs}(\tau)\left[w_o(\tau) + Aw_x(\tau)\right]d\tau \\ & + 2\int_o^t \psi_{xs}(\tau)w_o(\tau)d\tau\end{aligned} \tag{A13.8}$$

Note that terms $\psi_{ss}(o)$ are cancelled.

Expanding the separate terms and cancelling when possible

$$\Delta\varepsilon(A) = 2A \int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_o(\tau)w_o(\sigma)d\tau d\sigma$$

$$+A^2 \int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_x(\tau)w_x(\sigma)d\tau d\sigma$$

$$-2A \int_o^t \psi_{xx}(\tau)w_x(\tau)d\tau \qquad \text{A13.9}$$

The minimum error is obtained when

$$\frac{d[\Delta\varepsilon(A)]}{dA} = 0 \quad \text{and} \quad \frac{d^2[\Delta\varepsilon(A)]}{dA^2} > 0 \qquad \text{A13.10}$$

Differentiating $\Delta\varepsilon(A)$

$$\frac{d[\Delta\varepsilon(A)]}{\phi\nabla} = 2A \left[\int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_x(\tau)w_x(\sigma)d\tau d\sigma\right.$$

$$\left. + 2\left[\int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_o(\tau)w_x(\sigma)d\tau d\sigma - \int_o^t \psi_{xs}(\tau)w_x(\tau)d\tau\right]\right. \qquad \text{A13.11}$$

The requirement $\frac{d|\Delta\varepsilon(A)}{dA}$ will be satisfied if and only if

$$\int_o^t\int_o^t \psi_{xx}(\tau-\sigma)w_o(\sigma)w_x(\tau)d\tau d\sigma - \int_o^t \psi_{xs}(\tau)w_x(\tau)d\tau = o \qquad \text{A13.12}$$

The last expression can be written in the form

$$\int_o^t \left[\int_o^t \psi_{xx}(\tau-\sigma)w_o(\sigma)d\sigma - \psi_{xs}(\tau)\right] w_x(\tau)d\tau = o \qquad \text{A13.13}$$

As $w_x(\tau) \neq o$, the quantity in the square beackets is zero for all τ greater or equal to zero.

Therefore

$$\int_o^t \psi_{xx}(\tau-\sigma)w_o(\sigma)d\sigma = \psi_{xs}(\tau) \qquad \text{A13.14}$$

which is a very well known expression used as the starting point in the system of identification by cross correlation. This is an integral equation (as the unknown function $w_o(t)$ appears under an integral sign) known as the Wiener Hopf equation

$$\text{At} \quad A = o; \quad \frac{d^2[\Delta\varepsilon(A)]}{dt^2} = 2 \cdot \int_o^t \int_o^t \psi_{xx}(\tau-\sigma) w_x(\tau) w_x(\sigma) d\tau d\sigma \qquad \text{A13.15}$$

An interpretation of the right side of A13.15 can easily be obtained by considering the system with impulse reponse $w_x(t)$ with the input $x(t)$ applied to it.

It is a special case of the expression for the mean squared error with the desired output $s(t)$ taken to be identical to zero and the impulse response to be $w_x(t)$. The expression is simply double the mean squared value of $y_x(t)$ (Fig.A13.2) and must therefore be a positive quantity. This means that the second derivative is positive, and hence that the condition represented by the Wiener Hopf equation (A13.15) gives a minimum of the mean squared error.

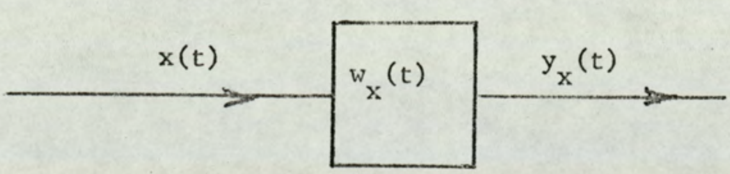


Fig.A13.2

In the frequency domain, the filter is defined with the transfer function

$$w(j\omega) = e^{A(\omega)} e^{j\phi(\omega)} \quad \text{as} \quad A(\omega) = \ell n|w(\omega)|$$

$$= |w(\omega)|\, e^{j\phi(\omega)} = C_{(\omega)}\, e^{j\phi(\omega)} \qquad \text{A13.16}$$

The requirement of physical realisability leads to the well know loss-phase relations. For a given gain

$A(\omega) = \ell n|w(\omega)|$ there is a minimum phase characteristic which is related to $A(\omega)$ by the following expression (Bode, 1945)

$$\phi(\omega_o) = \frac{2.\omega_o}{\pi} \int_o^\infty \frac{A(\omega) - A(\omega_o)}{\omega^2 - \omega_o^2}\, d\omega \qquad \text{A13.17}$$

A rational function $|\gamma(\omega)|^2 = \gamma(\omega).\gamma(\overset{*}{\omega})$ may be expressed as the ratio of two polynomials

$$\frac{N(\omega)}{D(\omega)} = k2 \frac{(\omega - \alpha_1)\ (\omega - \alpha_1^*)\ (\omega - \alpha_2)\ (\omega - \alpha_2^*)\ \ldots}{(\omega - \beta_1)\ (\omega - \beta_1^*)\ (\omega - \beta_2)\ (\omega - \beta_2^*)} \qquad \text{A13.18}$$

where * denotes complex conjugates. The minimum phase network has the transfer function

$$\gamma(\omega) = k \frac{(\omega - \alpha_1)\ (\omega - \alpha_2)\ \ldots .}{(\omega - \beta_1)\ (\omega - \beta_2)\ \ldots .} \qquad \text{A13.19}$$

Consider the components of the signal and noise at a particular frequency $f1 = \frac{\omega_1}{2\pi}$. It is assumed that signal and noise are independent and thus completely incoherent at all frequencies. Then at frequency f1, there will be a contribution to the error due to noise, equal to

$$N(\omega_1)\ .\ |\gamma(\omega_1)|^2 \qquad \text{A13.20}$$

where $N(\omega_1)$ is the average noise power at f1

Let $P(\omega_1)$ denote the signal power at the same frequency.

There is also a contribution to the error due to the phase change $|\alpha(\omega_1)|$ of signal passing through the system. Hence, there is a power error

$$E(\omega_1) = |\gamma(\omega_1) - e^{j\alpha(\omega_1)}|^2 \, P(\omega_1) + |\gamma(\omega_1)|^2 \, N(\omega_1) \qquad \text{A13.21}$$

and the total mean square error

$$E = \iint_{-\infty}^{\infty} \left[C^2_{(\omega)} + 1 - 2C_{(\omega)} \cos(\alpha(\omega) - \phi_{(\omega)} \right] P_{(\omega)} + C^2_{(\omega)} N_{(\omega)} \quad d\omega \qquad \text{A13.22}$$

For $\phi_{(\omega)} = \alpha_{\ \omega} \equiv \alpha\omega$, after some algebra

$$E_{opt} = \iint_{-\infty}^{\infty} \left[C^2 \sqrt{P+N} - \frac{P}{\sqrt{P+N}} \right] + \frac{PN}{P+N} \quad d\omega \qquad \text{A13.23}$$

The square bracketed term is the square of the real number, and therefore positive or zero. To minimise E, $C_{(\omega)}$ should be chosen to be

$$C_{(\omega)} = |\gamma_{(\omega)}| = \frac{P(\omega)}{P_{(\omega)} + N_{(\omega)}} \quad \text{and, hence} \qquad \text{A13.24}$$

$$W(j\omega) = \frac{P(\omega)}{P(\omega) + N(\omega)} \, e^{j\alpha\omega} = \frac{1}{1 + \frac{N(\omega)}{P(\omega)}} \, e^{j\alpha\omega} \qquad \text{A13.25}$$

The optimum Wiener-Kolmogoroff filter has a dependence on the signal to noise ratio frequency characteristic.

APPENDIX 14

The effect of feedback on the fluctuations of amplifiers.

Measurements were undertaken some time ago to determine in what way the fluctuations in gain of an amplifier mask the signals available in the mixture of signal and noise obtained at the output. The amplifier under test was, in the first experiments, followed by a valve amplifier, while the signal in the output was detected by measuring the correlation between the output and the input signals. No feedback was used in the valve amplifier following the amplifier under test. Using this method, Hathaway[1] found that when the signal output was much greater than the noise output, there is a constant ratio between the signal output amplitude and the input amplitude. Since the noise under these conditions is most often relatively small it was taken that this constant ratio indicated 100% correlation between input and output.

As the signal input was reduced, after the point at which the noise output becomes comparable with the signal, the correlation percentage decreased until, at signal inputs of the order of 10^{-6} volts, almost half the expected signal amplitude in the output had disappeared, i.e. only about 25% of the expected signal output power could be detected. It was established experimentally that this loss of signal power was not due to failure of the correlation method to detect all the signal available. It was also demonstrated experimentally that the total power output of noise and signal was exactly equal to that which would have been expected had all the signal power expected at the output been available. It was apparent therefore that some of the signal power was being converted into an exactly equal amount of noise power. This effect will be known as the conservation of power or energy between noise and signal, and is a newly observed phenomenon which has not hitherto been taken into account in fluctuation theory. Measurements of mean square voltages or powers will not normally detect this effect.

Similar results were obtained when either a valve or a transistor amplifier were in the first stage under test.

Later it was attempted to repeat these results. In this attempt Arthur[2] used a transistor amplifier with several stages in cascade to amplify the output of the first stage. The transistors were arranged in groups of three around each of which feedback was used to stabilise the gain and reduce the impedances of the circuits. This was done to reduce the effects of interference picked up from the mains and to reduce the physical size of the apparatus so that earth loops did not present so many opportunities for errors to be introduced. It was found that, even when a valve which had been tested by Hathaway was under test, it was not possible to observe for certain that any signal power was being converted into noise, even at signal strengths considerably smaller than those used by Hathaway. It was not at that time possible to reconstruct Hathaway's original apparatus to duplicate the original experiments exactly.

It was thought at the time that the feedback used in the second set of experiments could not have caused the discrepancy, since the conventional view is that feedback cannot improve the sensitivity of an amplifier. However, later related investigations led to the view that this might not be the case, especially since theory which has already been published[3,4] showed that the fluctuations in gain remain important even when the signal strength is large, and also become more important as the time constants of the circuits are increased. Consequently, the fluctuation phenomena become more akin to turbulent and other macroscopic random events rather than to atomic events due to the action of single electron charges. When the opportunity arose therefore it was decided to make exploratory experiments which might indicate whether or not feedback could influence the fluctuations of an amplifier.

A three-stage transistor amplifier was constructed in which, by means of a six position switch, the gain and the feedback system could be changed. In positions 1 to 3 the feedback was around the last two stages. In positions 4 to 6 the feedback was around all three stages. In positions 1 and 4 the gain was set to 60 db by adjusting the resistance values in the feedback paths. Similarly, in positions 2 and 5 the gain was set to 50 db and in positions 3 and 6 to 40 db. In general terms it will be seen that the feedback increased in amount in moving through the switch positions from 1 to 6. In positions 1 to 3 the low frequency cut-off was at about 500 Hz, while in positions 4 to 6 it was about 50 Hz, the difference arising from the presence of by-pass condensers across bias resistors.

When the low frequency noise in the range from 10 Hz to 200 Hz was measured, with no signals applies, it was found that in the first position there is a noise like the flicker effect, but falling off markedly less rapidly than as 1/f. The rate of fall-off of the noise increased in positions 2 and 3 to a value of about 6 db per octave of f, i.e., a rate much greater than as 1/f.

In positions 4 to 6 the noise was traced only to about 100 Hz, since the rate of fall-off was much greater, reaching a value of about 20 db per octave over the range from 10 Hz to 40 Hz in position 6, where the feedback is largest. It is evident that the low frequency noise in the absence of signals is considerably reduced by the use of feedback.

An excess noise due to signals, similar to the $1/\Delta f$ noise reported by Bozic[5] and Bull and Bozic[6] was also observed. In particular, for a signal at 14 kHz, the rate of fall-off of the noise was, in position 1 about 8 db per octave of Δf, i.e., much faster than as $1\Delta f$. In position 4, i.e., with the same gain and the greater feedback over a wider frequency range, the noise falls off even more rapidly, and is at all values of Δf, at least two or three db's lower than in position 1.

Tests were also made on an integrated circuit amplier, in which the gain could be adjusted by varying the input resistance, so that gains of 60 db, 40 db and 20 db could be selected. The gain of the amplifier is flat down to very low frequencies. It was found that as the gain is decreased, i.e. as the feedback is increased, the rate of fall-off of the noise with Δf decreases from 20 db per octave of Δf when the gain is 60 db to 40 db per octave when the gain is 20 db.

It was not possible in the time available to measure the sensitivity of the amplifiers. The indications are, however, that for both the $1/f$ and the $1/\Delta f$ types of fluctuation, which are caused by fluctuation in the parameters of the amplifier, the fluctuations can be brought under considerable and significant control in the same way as are the macroscopic variations of the amplifier gain in feedback control systems.

Theory of the kind referred to in Reference 3 and 4 show that two kinds of fluctuation are developed in an amplifier on account of the fluctuations in the gain. One has a total power which is proportional to the square of the signal amplitude, i.e. to the signal input power or the expected power output. The other is proportional to the signal amplitude. The former fluctuation is that reported in References 5 and 6. The latter is that which would be expected on conventional theories of the fluctuations, since the numbers of electron charges brought into consideration for each sign event is proportional to the signal amplitude. For what are usually regarded as "normal" statistical situations, the mean square deviations of such numbers is proportional to the number. It can be shown, however, that this type of fluctuation cannot, for small enough a signal, meet the requirement that the signal and noise powers are conserved at the output. It is in fact the type of fluctuation which was shown by Hathaway[1] to be cause of the irreversible loss of signal power when the signal is small.

It is also seen now that there is a change, not only in the noise power, but also in the noise spectrum when signals are applied, and that the signal to noise ratio reaches a constant value as the signals are increased, and does not increase without limit as is generally supposed.

On account of the constant signal to noise ratio for large signals, every stage of the amplifier without feedback, from the first to the last, contributed significantly to the noise at the output, and caused a corresponding loss of signal power. In the amplifier used by Arthur, in which feedback was used, the fluctuations in gain, and therefore the loss of signal power at all stages of the amplifier was considerably reduced. This is probably the best general explanation of the large discrepancy between the results obtained by Hathaway and Arthur.

The effects described are not of simple kind, and they have considerable interconnections and ramifications. They are however of technical importance and will have an influence on the development of the theory of fluctuations and the detection of signals in noise.

A longer paper, which it is hoped will eventually be published, giving more complete experimental details and theoretical discussions is in preparation.

July 1971. C.S.Bull, D.E.G.Hathaway, F.Arthur, S.M. [illegible] and A.K.C.Lee.

References:-

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APPENDIX 15

Phase lock

A phase lock loop is a servo system comprising (Fig.A15.1)

Phase comparator (Ph.C)

Voltage controlled oscillator (VCF)

Low Pass Filter (W(s))

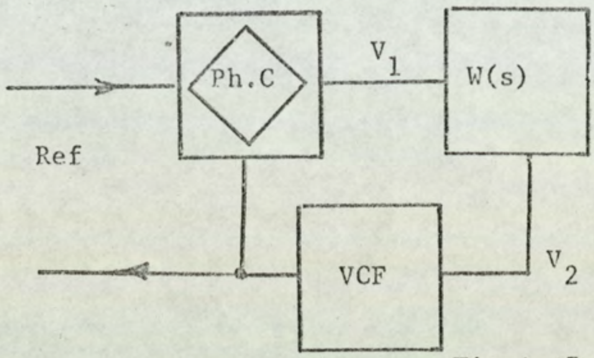


Fig.A.15.1.

The phase sensitive detector compares the phase of a periodic input signal $A\sin(\omega t+\phi)$ against the phase of the phase of the controlled frequency oscillator (VCF). The output of PhC is proportional to the cosine of the phase difference. As two signals are kept almost orthogonal, the output voltage can be approximately written

$$V_1 = K_1 \Delta\phi = K_1(\phi_1 - \phi_2) \text{ where } K_1 = \frac{2A}{\pi} \qquad 15.1$$

usually PhC output is dependent also on the amplitude of the smaller of compared signals. In order to make the voltage dependent solely on the phase difference, the smaller signal must be of a constant amplitude. The output voltage V_1 is filtered and then used as the controlled oscillator voltage V_2. As the transfer function of the simple first order filter is

$$W_{(s)} = \frac{1}{1 + ST_1} \qquad 15.2$$

where T_1 us the Re time constant.

The transfer function of the controlled oscillator is given by

$$F_{(s)} = \frac{K2}{S(1 + ST_2)} \qquad 15.3$$

where K_2 is the gain given in rad.V.sec^{-1}.

T_2 is the inertial (or the time constant) of the oscillator frequency change due to the controlled voltage change

Therefore ϕ_2 can be expressed

$$\phi_2(s) = V_2\ F(s) = \frac{K_2V_2}{S(1 + ST_2)} \qquad 15.4$$

Substituting

$$V_2 = V_1\ W_{(s)} \qquad 15.5$$

$$\phi_2(s) = \frac{K_2V_1}{S(1 + ST_1)(1 + ST_2)} \qquad 15.6$$

As $T_2 << T_1$, the approximate expression for $\phi_2(s)$

$$\phi_2(s) = \frac{K_2V_1}{S(1 + ST_1)} \qquad 15.7$$

From 15.1

$$\phi_1(s) = \frac{V_1}{K_1} + \phi_2(s) = \frac{V_1}{K_1} + \frac{K_2V_1}{S(1 + ST_1)} \qquad 15.8$$

and the loop transfer function

$$H(s) = \frac{\phi_2(s)}{\phi_1(s)} = \frac{1}{1+S/\left[K_1K_2.W(s)\right]} \qquad 15.9$$

$$= \frac{K_1.K_2/T_1}{S^2 + S/T_1 + K_1.K_2/T_1} = \frac{K}{S^2 + 2\xi\omega_n + \omega_n{}^2} \qquad 15.10$$

By inspection, the natural frequency $\omega_n = \sqrt{\frac{K}{T_1}} \quad \sqrt{\frac{K_1K_2}{T_1}}$ and the damping factor 15.11

$$\xi = \frac{1}{2}\sqrt{\frac{1}{K_1.K_2T_1}} = \frac{1}{2}\sqrt{\frac{1}{K.T}} \qquad 15.12$$

It is obvious that the loop becomes less stable as the product K.T is increased.

If the amplitude of the smaller signal (A) is not kept constant (by combined age and limiting process) the loop gain will change with the variation of A. This will make the loop tend to unlock at low signal levels.

Double Channel RMS Voltmeter

The basic principles of opperation was already considered in Chapter 6. and what follows is the description of the associated sampled-data system. The mean currents through T1 and T2 depend on the widths of NAND3 and NAND2 pulses respectively(Fig16.1 and Fig.16.8). Suppose that a signal and/or noise is applied on I/P2. In order to keep the bridge in balance the NAND4 pulse becomes narrower,decreasing the mean (DC) current through T2(Th2 on Fig.6.5b).

At the same time ,note that R10 and R11 correspond to R3/3 and R3/4 on Fig.6.5b.As the bridge is slightly unbalanced,which is sensed by COMP circuit

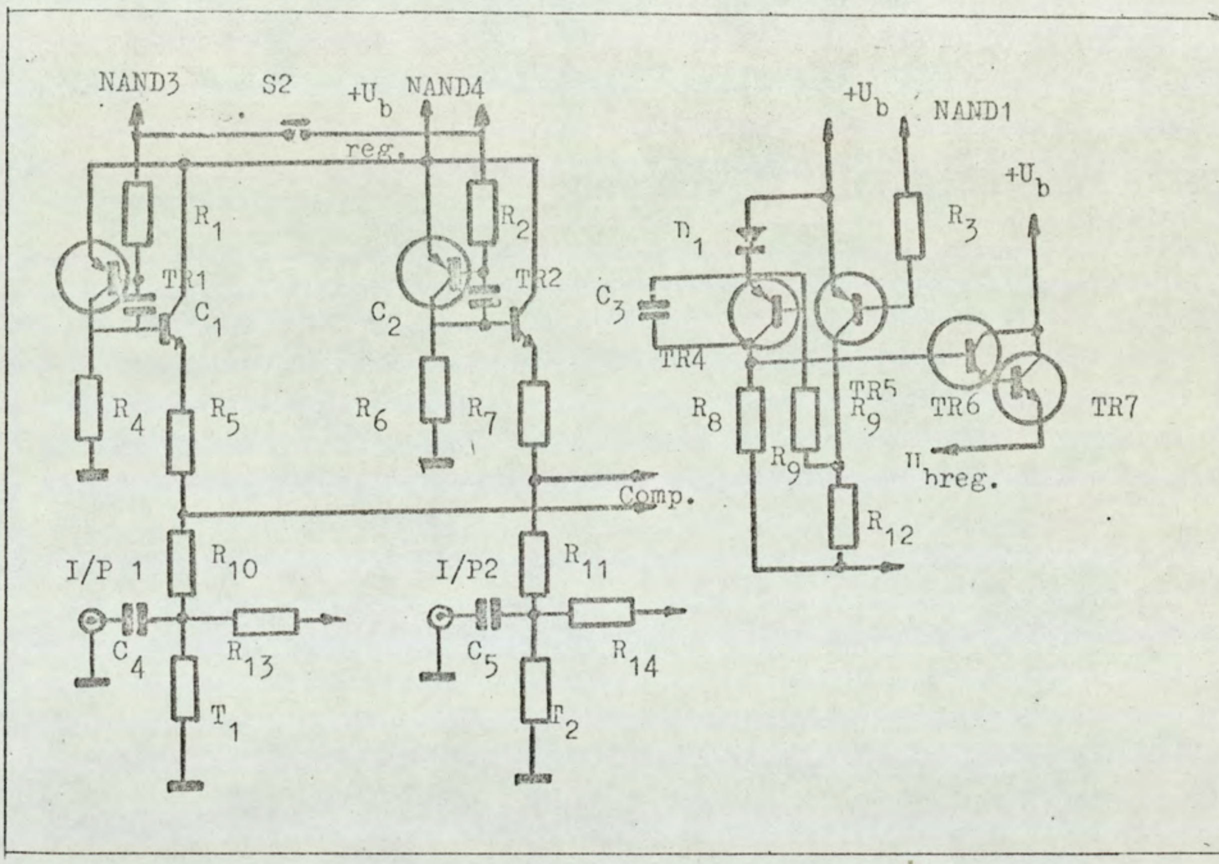


Fig. 16.1

(Fig6.5b),the intensity of the comparisson signal will be increased.This is due to the change of the attenuation (Var.Att.Fig6.5b) caused by a new gate voltage setting.When the bridge balance is restored,the comparisson signal and the measured signal and/or noise have(within 0.5%) the same RMS value .

All the necessary basic pulses for the opperation of the sampled data measuring servo system are generated by the master multivibrator(MMV-Fig.16.6). and the accompanied flip-flop deviders(FF2,FF3,FF4,FF1, and FF5-Fig.16.6) which are in the ripple carry mode opperation. Using NOR1,NOR2 and a bistable multivibrator(BS2) the remaining basic waveshapes(Fig.16.2) are obtained.

Short sampling pulses for sensing bridge balance(R1/1-R1/4 and thermistor beads Rt1 and Rt2-Fig.6.5b) are obtained utilizing AND2 AND3 and AND4 circuits(Fig.16.3),from the already described basic pulses(Fig.16.4).

The inverted output from AND2 with 1/8th of the MMV frequency, samples the reference branch(R1/2 -R1/4-Fig6.5b) which when in balance (i.e. when a negligible drift error voltage is present) produces(Fig. 16.7) a verry narrow NAND1 pulse.This pulse is inverted(TR5,Fig.16.1), integrated(TR4), and the corresponding mean current amplified(TR6/TR7 Fig.16.1).Therfore,whatever the source of a drift in the bridge balance by changing U_{breg} (Fig.16.1) this drift will be automatically reduced to a negligible value.

Similarly any change in R1/1-Rt1 and/or R1/3-Rt2 branch balance is corrected by corresponding changes in NAND3 and/or NAND4 pulse actions(Figs 16.7 and 16.8)

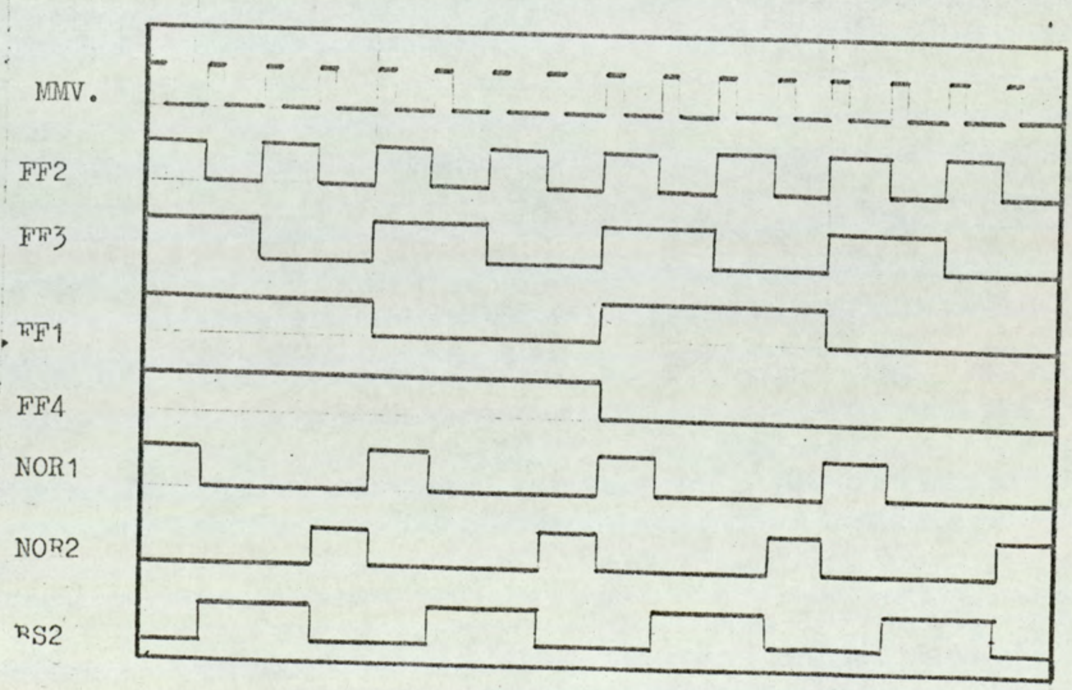
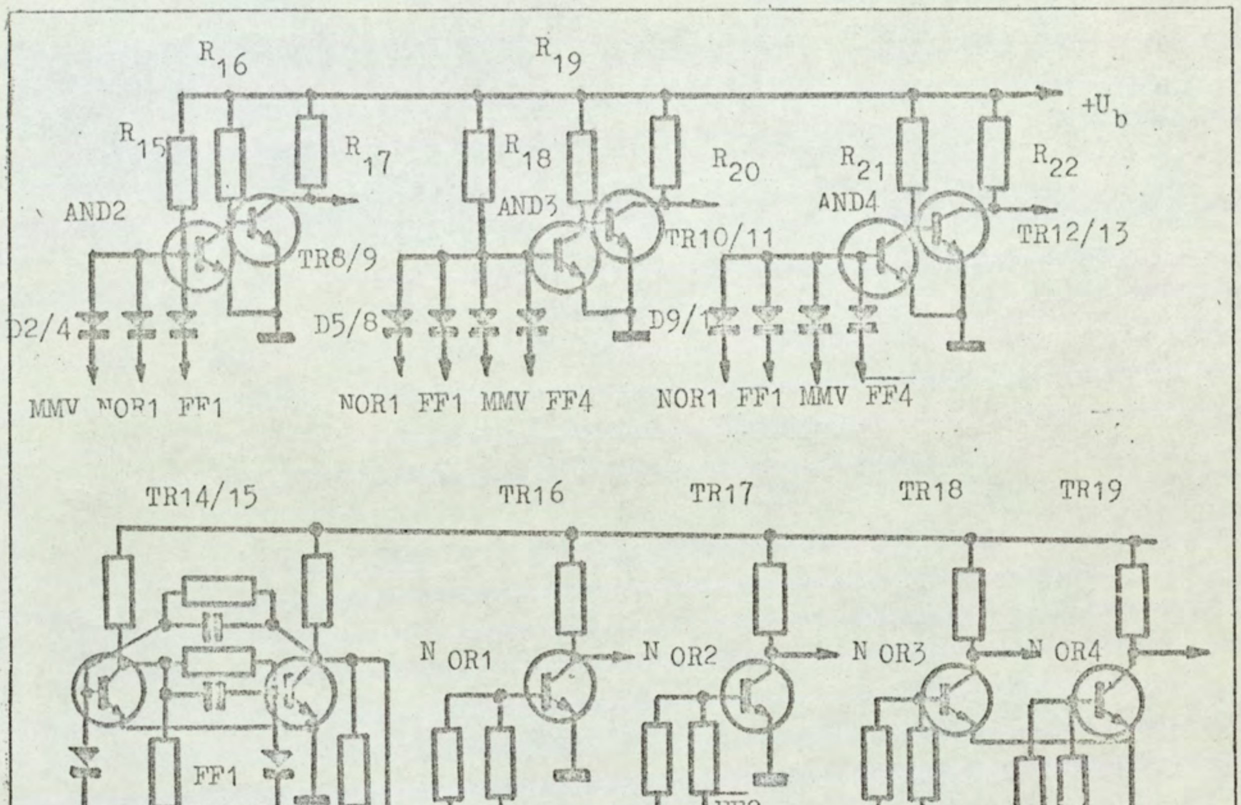


Fig.16.2


R16
R19
+Ub
R15
R17
R18
R20
R21
R22
AND2
AND3
AND4
TR8/9
TR10/11
TR12/13
D2/4
D5/8
D9/1
MMV NOR1 FF1
NOR1 FF1 MMV FF4
NOR1 FF1 MMV FF4
TR14/15
TR16
TR17
TR18
TR19
NOR1
NOR2
NOR3
NOR4
FF1
FF2
From FF3
FF2
From FF3 To FF4
AND3
AND4
S
MMV
FF2
FF3
TR20/21
TR22/23
TR24/25
Synchr.

16.3

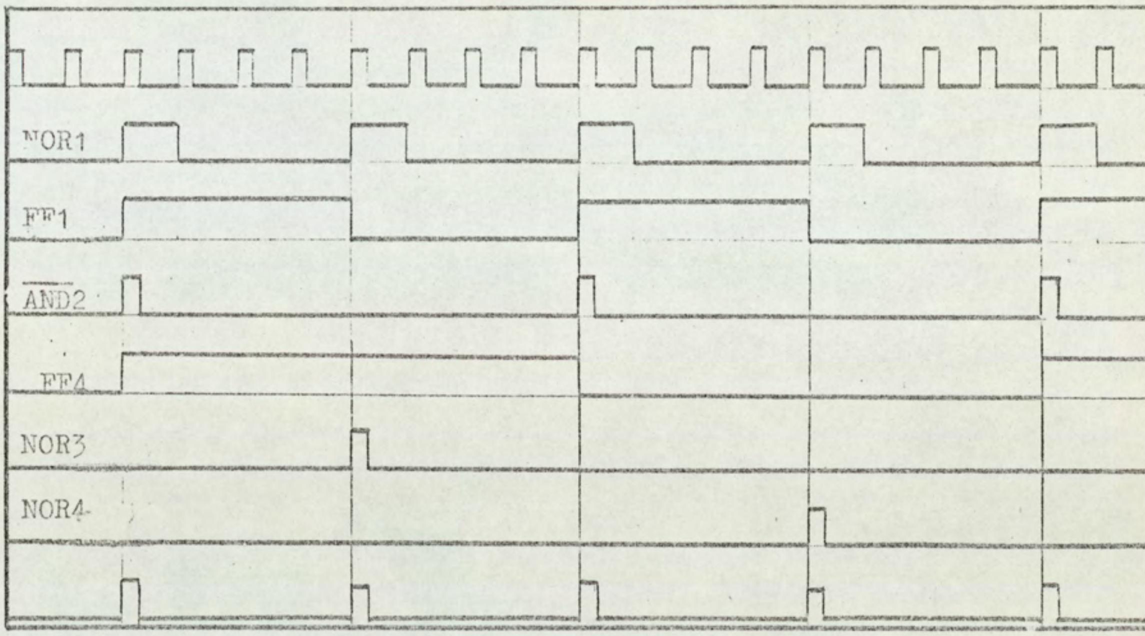


Fig.16.4

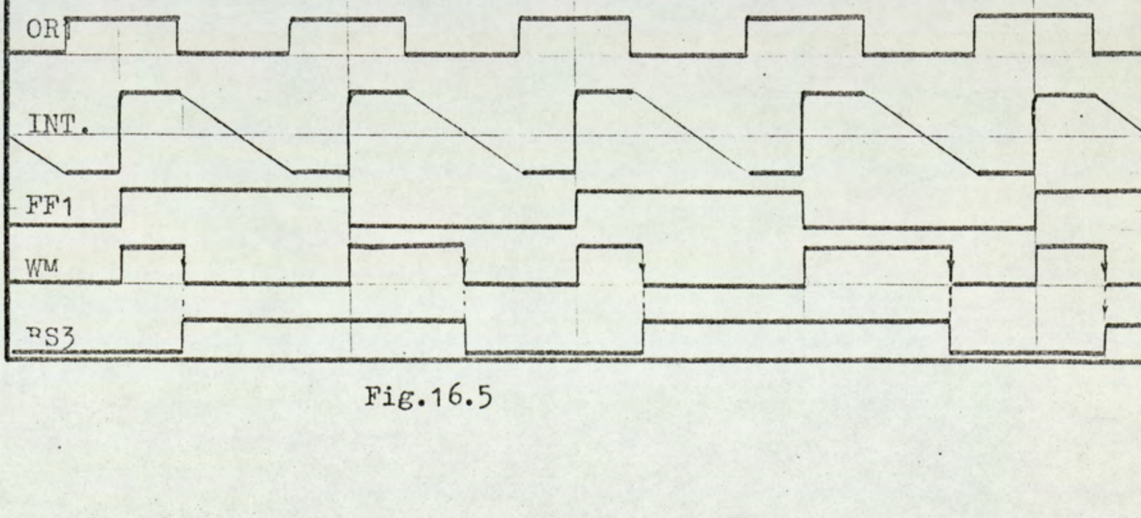
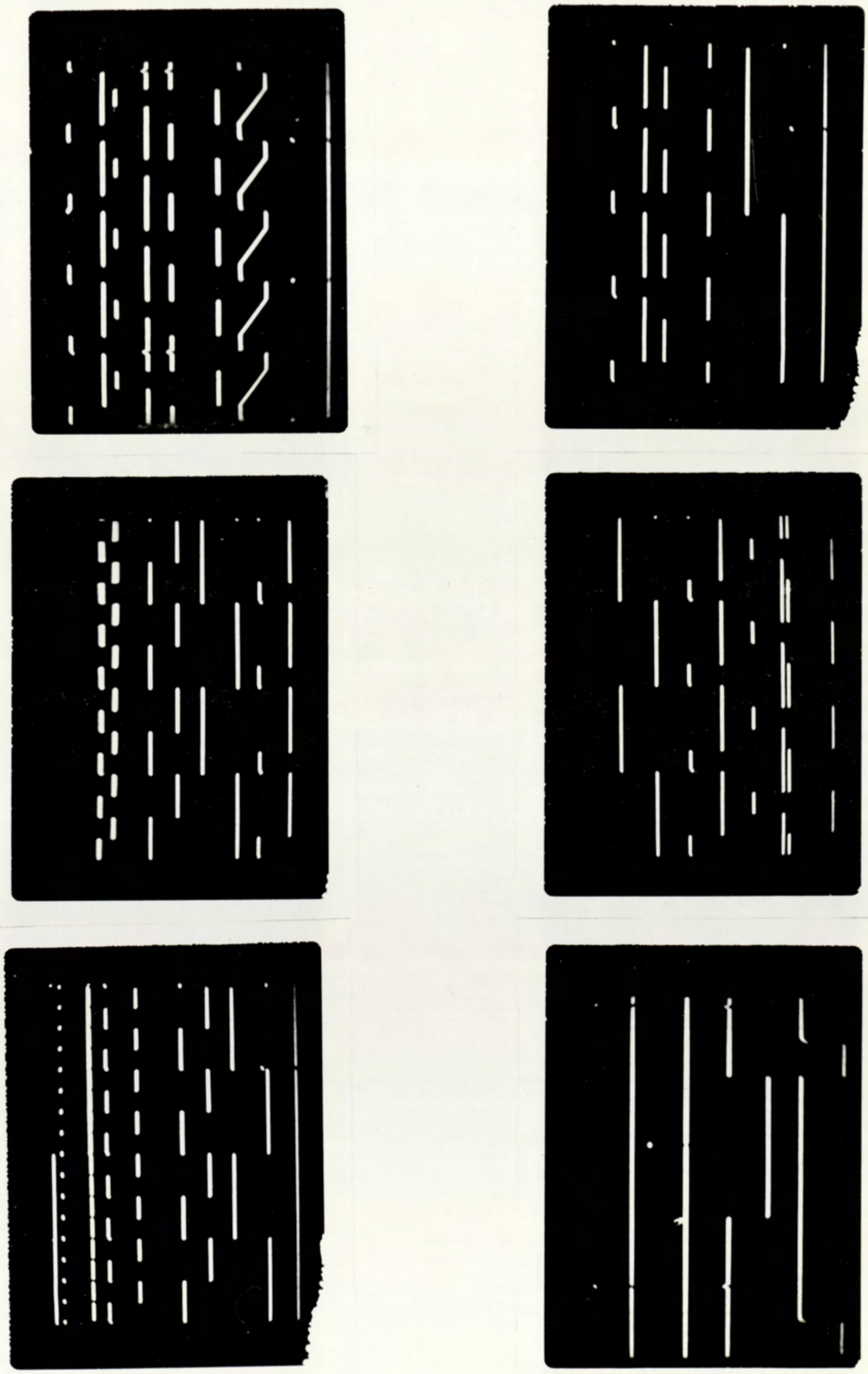
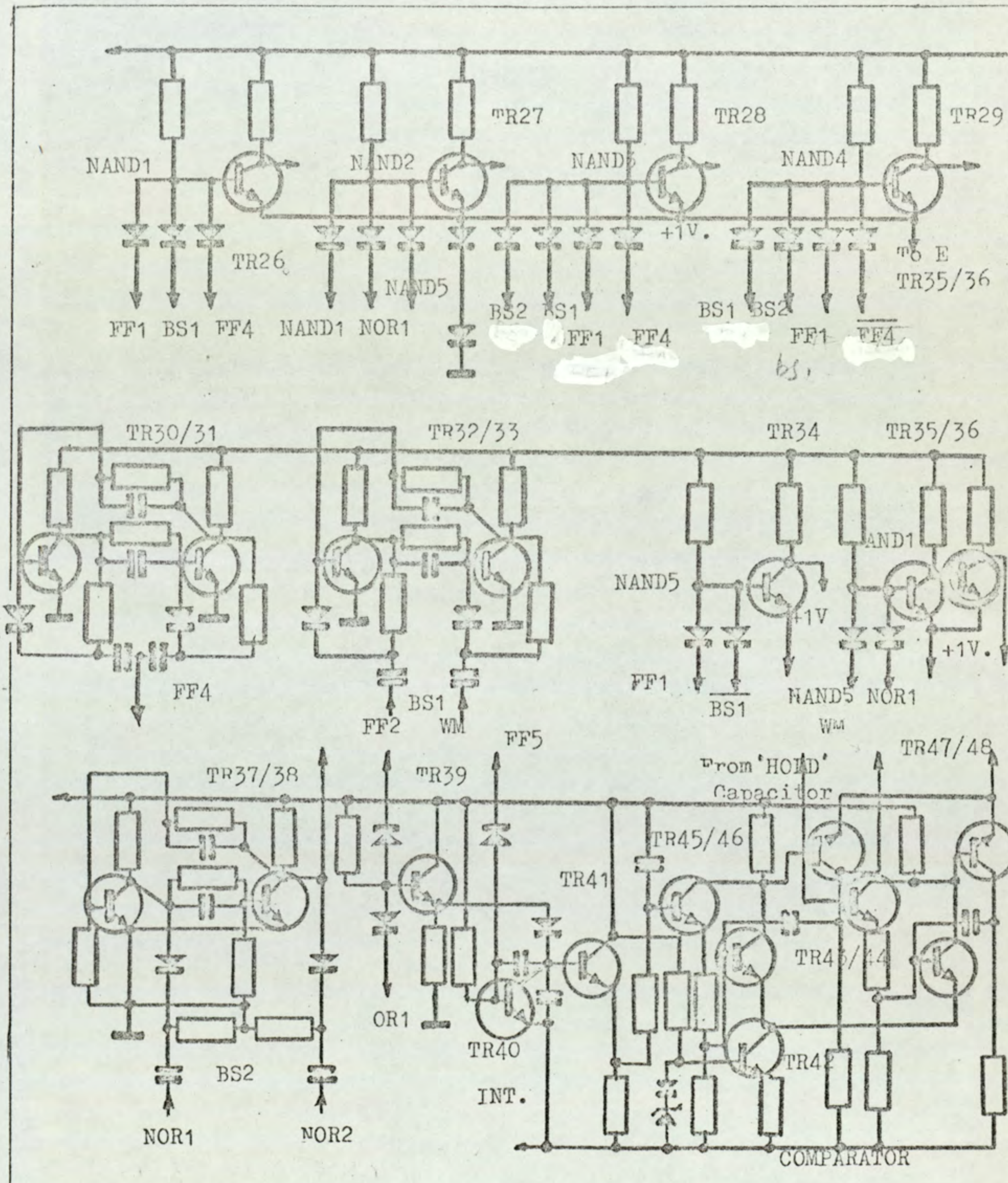


Fig.16.5



The main part in height(HOLD-Fig.16.5) to width conversion (WM-Fig.16.5) of error pulses is done by the comparator circuit (TR41-TR48-Fig.16.5).On thr base of TR44,the error voltage from the 'HOLD' capacitor is applied.Simultaneously,on the base of TR41 a modified ramp voltage(Fig.16.5) from the integrator circ-



cuit(TR40) is applied.Obviously(Fig.16.5) the output WM(TR45) will be a width modulated pulse.

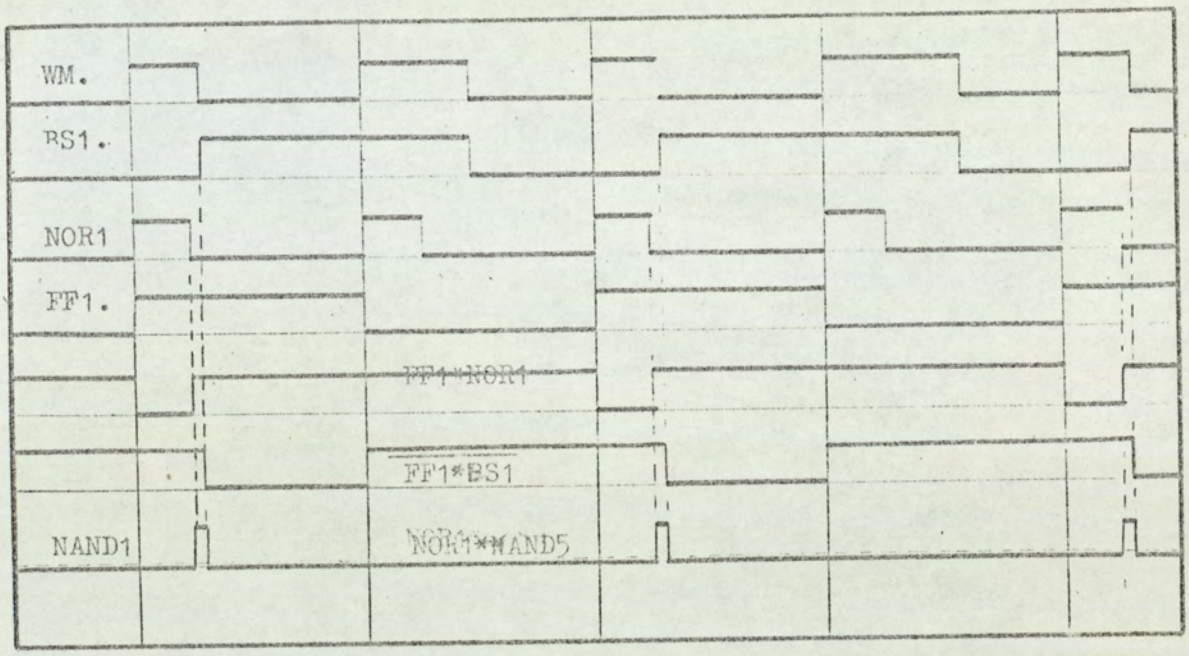


Fig.16.7

After necessary sifting,sorting and shaping(Fig.16.7 and Fig.16.8) this results in the already described servo balance and separate coherent signal and noise measurements.

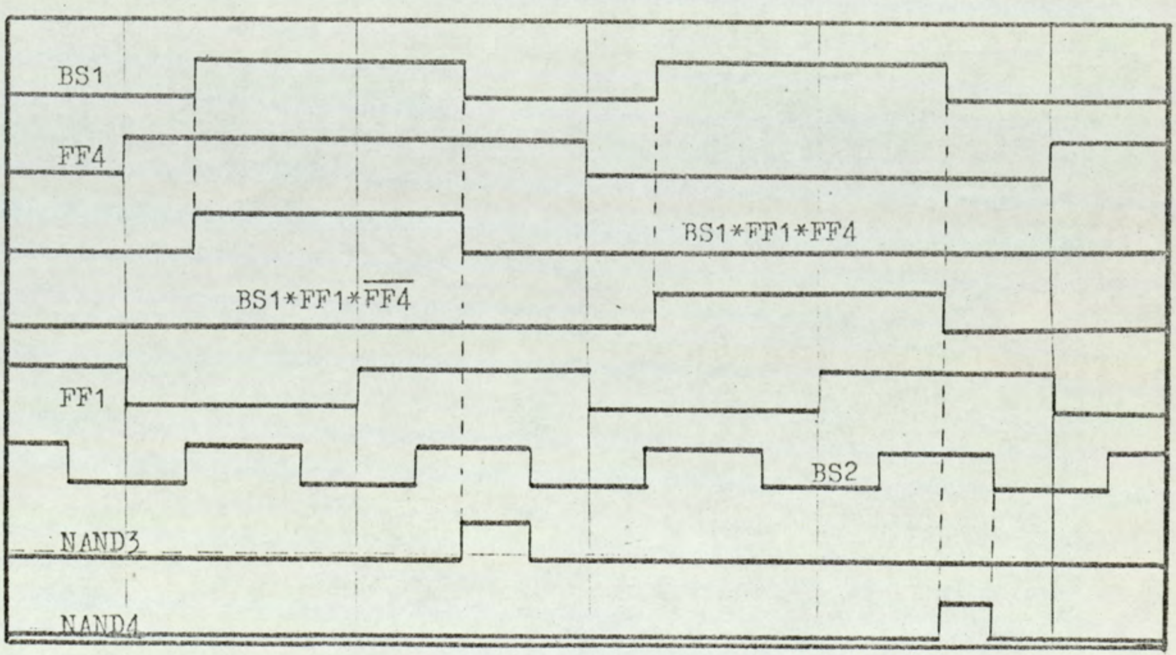
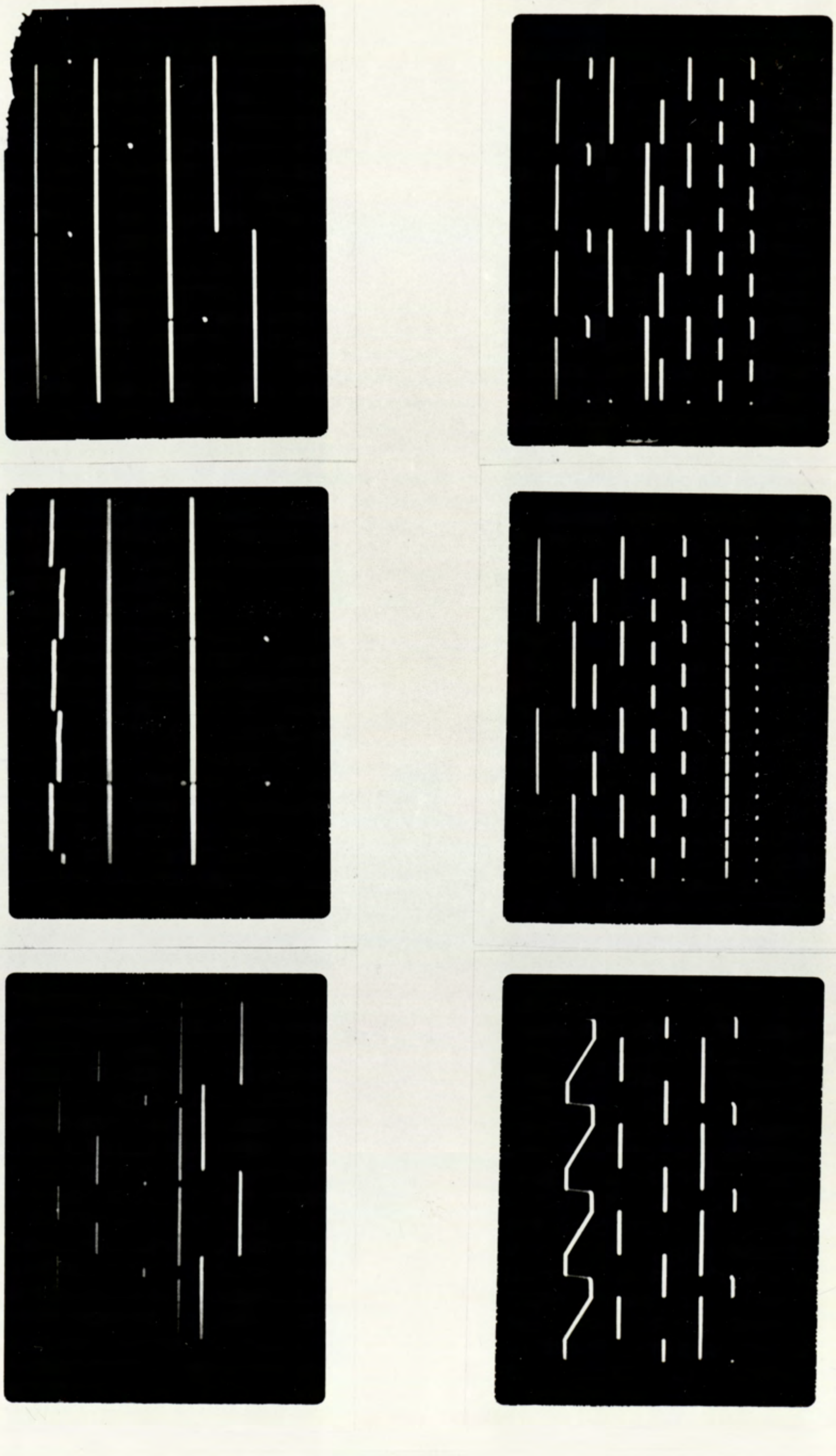


Fig.16.8.



Computer Programs and Procedures

```
'BEGIN'

 'PROCEDURE' FLICKER(NTOT,TSEC,FLAT, TCNT, AVER, VAR,MNT,X0);
 'VALUE' AVER, VAR, NTOT;
 'INTEGER' NTOT, MNT, X0;
 'REAL' TSEC, TCNT, AVER, VAR;
 'BOOLEAN' FLAT;

  'COMMENT' ****************************************
FLICKER GENERATES MARKOVIAN NUMBERS ,WHERE
     NTOT  IS THE TOTAL NUMBER OF GENERATED NUMBERS IN
             SEQUENCE,
     TSEC  IS THE SEQUENCE DURATION TIME,
     FLAT  DEFINES EITHER UNIFORM (FLAT ) OR GAUSSIAN
           DESTRIBUTION,
     TCNT IS THE SIMULATION SYSTEM TIME CONSTANT,
     AVER  IS THE DESIRED AVERAGE VALUE,
     VAR  IS THE DESIRED VARIANCE,
     MNT  IS THE MAXIMUM EXPONENT VALUE OF 2 TO SATISFY
              THE LARGEST PERMISSIBLE INTEGER VALUE,
     X0  IS AN ARBITRARY INTEGER IN THE RANGE -2↑MNT TO
             +2↑MNT DEVIDED BY  5. TO CHANGE X0  WILL
              PRODUCE DIFFERENT SEQUENCES************;


  'BEGIN'

    'PROCEDURE' RANDOM(MEAN,SIGMA,K,GAUSS,X0,MOD);
     'VALUE' MEAN,SIGMA,MOD;
     'REAL' MEAN,SIGMA,GAUSS;
     'INTEGER' K, X0,MOD;

        'BEGIN'  'INTEGER' I, L, M, N;
        'REAL' SUM, A, B, RND, X;

 'COMMENT' *****************************************
 THE PROCEDURE RANDOM, WHICH IS A SUBROUTINE, GENERATES
 PSEUDO-RANDOM  NUMBERS  WITH THE DESIRED  STANDARD DE-
VIATION AND MEAN VALUE.  IT HAS BEEN SHOWN BY COVEYOU
 (1960), THAT FOR MULTIPLICATIVE CONGRENTIAL METHODS OF
 GENERATING PSEUDO-RANDOM NUMBERS, THE CORRELATION COEF.
 BETWEEN SUCCESIVE NUMBERS IS APPROXIMATELY RECIPROCAL
 OF THE MULTIPLYING FACTOR F.  AS THE RECIPROCAL IN THIS
 CASE  IS   ONLY   0.2,   THIS PROCEDURE IS NOT QUITE
 SATISTACTORILLY FOR GENERATING MARCOVIAN SEQUENCES OF
 ZERO'TH ORDER ***************************************;

        'BEGIN'            SUM:=0.0;
     X:=X0; N:=MOD'/'2; M:=N'/'2; L:=M'/'2;
        A:=-1.0;   B:=1.0;
     'FOR' I:=1 'STEP' 1 'UNTIL' K 'DO'  'BEGIN'
            X:=5*X;
        'IF' X 'GE' N 'THEN' X:=X-N;
        'IF' X 'GE' M 'THEN' X:=X-M;
        'IF' X 'GE' L 'THEN' X:=X-L;
          RND:=X/L*(B-A) + A;
     SUM:=SUM +  RND;
        'END';
```

```
  GAUSS:=MEAN + SIGMA*SUM*SQRT(3.0/K);
 X0:=X;
 'END';

'COMMENT' ****************************************
GROM GREENBERGER CONDITION (1962), WHICH STATES THAT
 FACTOR F SHOULD BE CONSIDERABLY LESS THAN THE SQUARE
ROOT OF THE MODULUS, IT FOLLOWS THAT THE MULTIPLICATIVE
FACTOR OF 25 OR EVEN 125 COULD BE USED, WITH THE AP-
PROPRIATE CHANGE IN THE PROCEDURE RANDOM.*************;

 'END';
 'END' PROC;

'INTEGER' NPSEC, NTCNT, NCORR, I, J, L, M, N, MOD, K;
'REAL' DELTT, FMIN, FMAX, TAUCR, PI, CTOFF, STOCH, GAUSS,
 MEAN, SIGMA;

'BEGIN'  PI:=4.0*ARCTAN(1.0);
 NPSEC:=ENTIER(NTOT/TSEC);   FMAX:= NPSEC/2;   NEWLINE(1);
  WRITETEXT('('MAXIMUM%GENERATED%FREQUENCY')'); SPACE(2);
 PRINT(FMAX,0,6);  WRITETEXT('('CPERSEC')');  NEWLINE(1);
 DELTT:=1/NPSEC;
 WRITETEXT('('SAMPLING%INTERVAL')');  SPACE(12);
PRINT(DELTT,0,6);      WRITETEXT('('SEC')');      L:=0;

 LAB:L:=L+1;    NCORR:=128*L;
 TAUCR:=NCORR*DELTT;
 'IF'  TCNT 'GT' TAUCR/2 'THEN' 'GOTO' LAB;
 NEWLINE(1);        WRITETEXT('('MAXIMUM%DELAY')');
SPACE(16);  PRINT(TAUCR,0,6);  WRITETEXT('('SECONDS')');
 'IF' TCNT  'LT' DELTT 'THEN' TCNT:=DELTT;
 NTCNT:=TCNT/DELTT;
 NEWLINE(1);  WRITETEXT('('TIME%CONSTANT')');  SPACE(16);
PRINT(NTCNT,0,6);   WRITETEXT('('NUMBER%OF%%DELAY%UNITS')');
 CTOFF:=NPSEC/(2*NTCNT);              NEWLINE(1);
 WRITETEXT('('CUTOFF%FREQUENCY')');  SPACE(13);
PRINT(CTOFF,0,6);  WRITETEXT('('CYCLES%PER%SECONDS')');
 FMIN:=FMAX/NCORRR;     N:=ENTIER(NCORR/NTCNT);
 FMAX:=0.0;  M:=NCORR - N + 1;
 'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO'
 FMAX:=FMAX + 1/I;  FMAX:=NCORR/(N + FMAX);
NEWLINE(1);   WRITETEXT('('MULTIPLICATION%FACTOR')');
SPACE(8);    PRINT(FMAX,0,6);

 'IF' FLAT 'THEN' 'BEGIN'  MEAN:=AVER;  SIGMA:=SQRT(VAR);
 K:=12   'END' 'ELSE'  'BEGIN' MEAN:=0.0;  SIGMA:=1.0;
   'IF' NTCNT 'GE' 16 'THEN' K:=1;
   'IF' NTCNT 'LT' 16 'THEN' K:=16 -NTCNT   'END';
  MOD:=2↑(MNT-1);

 NEWLINE(1);  WRITETEXT('('MODULUS')');   PRINT(MOD,0,6);

 CTOFF:=1/(1 + 1/NTCNT);  TAUCR:=1/(1 + NTCNT);
 STOCH:=0.0;    GAUSS:=0.0;    VAR:=SQRT(3*VAR);

 NEWLINE(1);   WRITETEXT('('PROCEDURE%FLICKER')');

 SELECT OUTPUT(4);
```

```
WRITETEXT('('DOCZTAPE')');   NEWLINE(1);   RUNOUT;

 'FOR' I:=1 'STEP' 1 'UNTIL' NTOT'/'4 'DO' 'BEGIN'
  NEWLINE(1);     SPACE(2);
 'FOR' J:=1 'STEP' 1 'UNTIL' 4 'DO' 'BEGIN'
     RANDOM(MEAN,SIGMA,K,GAUSS,X0,MOD);

 'COMMENT' ******************************************
  RANDOM IS A SUBROUTINE TO GENERATE A RANDOM NUMBER.RE-
 PEATED CALL OF THIS PROCEDURE GENERATES A SEQUENCE OF IN-
 DEPENDENT NUMBERS WHICH ARE GAUSSIAN DISTRIBUTED FOR  K
  GT 10 OR UNIFORM (FLAT) DISTRIBUTED FOR K=1,
     MOD  IS THE LARGEST INTEGER NUMBER USED IN THE SUB-
             ROUTINE (SE R.W. HAMMING: NUMERICAL METHODS
             FOR SCIENTISTS AND ENGINEERS**************;
     STOCH:=STOCH*CTOFF + GAUSS*TAUCR;

 'COMMENT' *********************************************
     STOCH IS A MARKOVIAN NUMBER WHICH IS PARTIALLY  DE-
             PENDENT ON PREVIOUS STOCH VALUES.  IN ORDER
             TO CORRECT ITS VARIANCE IT MUST BE MULTIPLIED
             BY THE COMPUTED FACTOR FMAX.*****************;

      GAUSS:=STOCH*FMAX*VAR + AVER;
      PRINT(GAUSS,1,4);     'END';  'END';
 'END' BLOCK;     'END' FLICKERPROC;

    'BEGIN' 'INTEGER' NTOT,MNT,X0,I; 'REAL' TSEC,TCNT,AVER,VAR;
            'BOOLEAN' FLAT;
            NTOT:=5120;  TSEC:=1.024;  MNT:=23;  X0:=865432;
            TCNT:=0.04;      AVER:=0.0;  VAR:=1.0;


 'COMMENT'   ******************************************
     THESE PARAMETERS CAN BE ADJUSTED TO SUIT PARTICULAR
             CASE.***************************************;


  FLICKER(NTOT,TSEC,'FALSE',TCNT,AVER,VAR,MNT,X0);

  WRITETEXT('('****')');     NEWLINE(2);     RUNOUT;

'COMMENT' *****************************************
  RANDOMNESS OF NUMBERS IN A SEQUENCE COULD BE CHECKED BY
  PERFORMING THE POKER TEST (KENDAL ET. AL. 1938).********;

  NEWLINE(1);
  WRITETEXT('('COMPUTERZGENERATEDZMARKOVIANZNUMBERS')');
 'END' DATAINPUTING;     'END' PROGRM;
**          *
```

```
'BEGIN'
'COMMENT' ************************************************
   THIS PROGRAM GENERATES A SEQUENCE OF NUMBERS WITH
   MARCOVIAN PROPERTIES OF A HIGHER ORDER.  OPOSITE TO THE
   PROCEDUTE FLICKER WHICH IS ENTIRELY IN THE TIME DOMAIN,
   THE SUBROUTINE PATTERN USED IN THIS PROGRAM IS MADE IN
   THE FREQUENCY DOMAIN.  PROCEDURE PATTERN IS (CHAPTER 2)
   THEORETICALLY BASED ON THE FACT THAT A WHITE NOISE
   EXICTATION OF A SYSTEM RESULTS IN AN OUTPUT WHOSE
   POWER DENSITY SPECTRUM HAS IDENTIAL FREQUENCY DE-
   PENDENCE AS THE SQUARE OF ITS TRANSFER FUNCTION.  THE
   DESIRED CORRELATION FUNCTION IS TRANSFORMED TO THE
   FREQUENCY DOMAIN AND CORRESPONDING POWER DENSITY
   SPECTRUM IS OBTAINED.  THIS ONE IS THEN   DECOMPOSED.
    TO THE REAL AND IMAGINARY PART OF THE  ASSOCIATED
  TRANSFER FUNCTION.  (PROCEDURE BODE)
  IT IS ALSO POSSIBLE TO INPUT  THE  DESIRED POWER DENSITY
 SPECTRUM, OR THE REAL AND IMAGINARY PARTS OF  THE MARKO-
 VIAN FILTER TRANSFER FUNCTION WHICH INFLUENCE IS TO BE
 SIMULATED.

   A MARCOVIAN SEQUENCE OF THE ZERO'TH ORDER IS GENERATED
   AND TRANSFORMED TO THE FREQUENCY DOMAIN.  THE REAL AND
   THE IMAGINARY PARTS ARE THEN MULTIPLYED BY BOTH THE
   REAL AND IMAGINARY PARTS OF THE TRANSFER FUNCTION, AND
   THEN RETRANSFORMED TO THE TIME DOMAIN. ******************;

    'PROCEDURE' REAL2(A,B,M);
    'VALUE' M;  'INTEGER' M;  'ARRAY' A,B;
    'BEGIN' 'INTEGER' J,N;  N:=2↑M;
    TRAN(A,B,N,'TRUE');
    'FOR' J:=N-1 'STEP' -1 'UNTIL' 0 'DO' B[J]:=-B[J];
    FTNB(A,B,N,M,N);
    'FOR' J:=N-1 'STEP' -1 'UNTIL' 0 'DO' 'BEGIN'
    A[J]:=0.5*A[J];  B[J]:=-0.5*B[J];  'END';
    REORD(A,B,N,M,N,'TRUE');
  'COMMENT' ************************************************
   PROCEDURE REAL2 COMPUTES THE FINITE INVERS FOURIER TRANS-
   FORM OF 2↑M+1 DATA POINTS. THE ARRAYS A[0:N] AND B[0:N] ,
  WHERE N=2↑M, INITIALLY CONTAIN THE FOURIER COSINE AND SINE
   TERMS RESPECTIVELY. NOTE THAT A[0] IS THE MEAN VALUE AND
   B[0]=0.0. *********************************************;
    'END' PROC;
    'COMMENT' ***********************************************
    ALL INTEGER,REAL AND BOOLEAN VARIABLES USED IN THIS PRO-
    GRAM SHOULD PROPERLY BE DECLARED,(SEE PROGRAMS SHORTBITS
    AND COHERENCE). THE LISTING GIVEN HERE IS A SHORTENED VER-
    SION OF THE PROGRAM GENERATE,IN ORDER NOT TO  REPEAT PRO-
    CEDURES USED IN OTHER PROGRAMS FOR THIS RESEARCH*********;
```

```
'PROCEDURE' FTNB(A,B,N,M,KS);
'VALUE' N,M,KS;  'INTEGER' N,M,KS;
'ARRAY' A,B;          'BEGIN'
  'INTEGER' K0,K1,K2,K3,SPAN,J,JJ,K,KB,KN,MM,MK;
  'REAL' RAD,C1,C2,C3,S1,S2,S3,CK,SK,SQ,
  A0,A1,A2,A3,B0,B1,B2,B3;
'INTEGER' 'ARRAY' C[0:M];
SQ:=0.70710678119;  SK:=0.38268343237;  CK:=0.92387953251;
  C[M]:=KS;  MM:=(M'/'2)*2;  KN:=0;
  'FOR' K:=(M-1) 'STEP' (-1) 'UNTIL' 0 'DO'
C[K]:=C[K+1]'/'2;  RAD:=6.2831853072/(C[0]*KS);  MK:=M-5;
 L:   KB:=KN;  KN:=KN + KS;
  'IF' MM 'NE'  M 'THEN'  'BEGIN'
  K2:=KN;    K0:=C[MM] + KB;
 L2: K2:=K2 - 1;  K0:=K0 - 1;
  A0:=A[K2];  B0:=B[K2];
  A[K2]:=A[K0] - A0;  A[K0]:=A[K0] + A0;
  B[K2]:=B[K0] - B0;  B[K0]:=B[K0] + B0;
  'IF' K0 > KB 'THEN' 'GOTO' L2   'END';
  C1:=1.0;  S1:=0.0; JJ:=0;  K:=MM-2;  J:=3;
  'IF' K 'GE' 0 'THEN' 'GOTO' L4 'ELSE' 'GOTO' L6;
 L3:  'IF' C[J] 'LE' JJ 'THEN'  'BEGIN'
  JJ:=JJ - C[J];  J:=J - 1;
  'IF' C[J] 'LE' JJ 'THEN'  'BEGIN'
  JJ:=JJ - C[J];  J:=J - 1;  K:=K + 2;
  'GOTO'  L3  'END'     'END';
  JJ:=C[J] + JJ;  J:=3;
 L4:  SPAN:=C[K];
  'IF' JJ 'NE' 0  'THEN'  'BEGIN'
  C2:=JJ*SPAN*RAD;  C1:=COS(C2);  S1:=SIN(C2);
 L5:  C2:=C1↑2 - S1↑2;  S2:=2.0*C1*S1;
  C3:=C2*C1 - S2*S1;  S3:=C2*S1 + S2*C1 'END';
  'FOR' K0:=KB+SPAN-1 'STEP' (-1) 'UNTIL' KB 'DO'
  'BEGIN'  K1:=K0 + SPAN;  K2:=K1 + SPAN;  K3:=K2 + SPAN;
  A0:=A[K0];  B0:=B[K0];  'IF' S1 = 0 'THEN' 'BEGIN'
  A1:=A[K1];  B1:=B[K1];  A2:=A[K2];  B2:=B[K2];
  A3:=A[K3];  B3:=B[K3]   'END'  'ELSE'  'BEGIN'
  A1:=A[K1]*C1 - B[K1]*S1;
  B1:=A[K1]*S1 + B[K1]*C1;
  A2:=A[K2]*C2 - B[K2]*S2;
  B2:=A[K2]*S2 + B[K2]*C2;
  A3:=A[K3]*C3 - B[K3]*S3;
  B3:=A[K3]*S3 + B[K3]*C3  'END';
  A[K0]:=A0 + A2 + A1 + A3;
  B[K0]:=B0 + B2 + B1 + B3;
  A[K1]:=A0 + A2 - A1 - A3;
  B[K1]:=B0 + B2 - B1 - B3;
  A[K2]:=A0 - A2 - B1 + B3;
  B[K2]:=B0 - B2 + A1 - A3;
  A[K3]:=A0 - A2 + B1 - B3;
  B[K3]:=B0 - B2 - A1 + A3   'END';
 'IF' K > 0 'THEN' 'BEGIN' K:=K - 2; 'GOTO' L4 'END';
  KB:=K3 + SPAN;
  'IF' KB < KN 'THEN' 'BEGIN'
  'IF' J = 0 'THEN' 'BEGIN' K:=2;  J:=MK;
  'GOTO' L3 'END';  J:=J - 1;  C2:=C1;
  'IF' J = 1 'THEN'  'BEGIN'
  C1:=C1*CK + S1*SK;  S1:=S1*CK - C2*SK 'END'
  'ELSE' 'BEGIN' C1:=(C1 - S1)*SQ;
  S1:=(C2 + S1)*SQ 'END';   'GOTO' L5 'END';
 L6:  'IF' KN < N 'THEN' 'GOTO' L 'END' PROC;
```

***

```
'PROCEDURE' BODE(A,B,N);
'VALUE' N;  'INTEGER' N;  'ARRAY' A,B;
'BEGIN' 'INTEGER' I,J,K;  'REAL' PI,Q,SUM;
PI:=4.0*ARCTAN(1.0);  SUM:=0.0;
'COMMENT' ******************************************
PROCEDURE BODE COMPUTES PHASE OF A MINIMUM PHASE NETWORK
WHEN MAGNITUDE IS KNOWN.MAGNITUDE IS SAMPLED FROM F=0 TO
N IN EQUALLY SPACED FREQUENCY INTERVALS,WHERE  DELTAF IS
NORMALIZED TO 1.0. AT THE OUTPUT OF THE PROCEDURE ARRAY
A[0:N] CONTAINS MAGNITUDE VALUES AND B[0:N] THE CORRESP-
ONDING PHASE VALUES EXPRESSED IN RAFIANS.TO FIND THE REAL
AND IMAGINARY PARTS PROCEDURE RANDI(SEE PROGRAM SYSTEM )
IS NECESSARY. IF DIGITAL FILTERING IS NOT DONE WITH A
MINIMUM PHASE NETWORK THIS PROCEDURE MAY BE OMITTED.***;

A[N]:=B[N]:=B[0]:=0.0;  PAPERTHROW;
'FOR' I:=0 'STEP' 1 'UNTIL' N-1 'DO'  'BEGIN'
'IF' A[I] 'LT' 10.0↑(-6) 'THEN' A[I]:=10.0↑(-6);
A[I]:=ABS(A[I]);     A[I]:=LN(A[I])/2.0;     'END';
'FOR' I:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
Q:=I*I;  SUM:=SUM +(A[1]-A[I])/Q; 'END' ILOOP;
B[1]:=2.0*(SUM-A[1])/PI;
'FOR' J:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
SUM:=0.0;  K:=J-1;
'FOR' I:=0 'STEP' 1 'UNTIL' K 'DO' 'BEGIN'
Q:=(I*I/J - J);  SUM:=SUM + (A[I]-A[J])/Q;  'END';
K:=J+1;
'FOR' I:=K 'STEP' 1 'UNTIL' N  'DO' 'BEGIN'
Q:=(I*I/J - J);  SUM:=SUM +(A[I]-A[J])/Q; 'END';
B[J]:=2.0*SUM/PI;
NEWLINE(1); PRINT(A[I],0,6);
PRINT(A[J],0,6);  PRINT(B[J],0,6);  'END' JLOOP;
'END' PROC;

'PROCEDURE' PATTERN(A,B,C,D,M);
'VALUE' M; 'INTEGER' M;  'ARRAY' A,B,C,D;
'BEGIN' 'INTEGER' J,N;  'REAL' P,Q;
N:=2↑M;  REAL1(A,B,M);
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
P:=A[J]*C[J] - B[J]*D[J];
B[J]:=A[J]*D[J] + B[J]*C[J];  A[J]:=P;  'END';
REAL2(A,B,M);

'COMMENT' ***********************************************
FOR PROCEDURE PATTERN IT IS NECESSARY TO (SEE PROGRAMS
COHERENCE AND SHORTBITS) DECLARE  REAL1 AND REAL2 PRO-
CEDURES INCLUDING THEIR SUBROUTINES.********************;

SPECTR:=REIGH:='FALSE';
NG:=READ;  M:=READ;  K:=READ;  L:=2↑M;
'IF' K 'EQ' NG 'THEN' SPECTR:='TRUE';
K:=(NG'/'2)*2;     N:=L;
'IF' K 'EQ' NG 'THEN' REIGH:='TRUE';
SELECT INPUT(3);  WRITETEXT('('GEN')');
'BEGIN'  'ARRAY'  A[0:N],B[0:N],C[0:N],D[0:N];
'INTEGER' X0;     'REAL' GAUSS;
'COMMENT' ***********************************************
 PROCEDURE RANDOM (SEE PROGRAM FLICKER) MUST BE DECLARED.
 ********************************************************;
```

```
'IF' SPECTR'THEN' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
B[L-I]:=A[I]:=C[I];  'END';
REAL2(A,B,M);  BODE(C,D,L);  RANDI(C,D,E,F,PI)
'END' 'ELSE' 'BEGIN'
REAL1(C,D,M); BODE(C,D,L);  RANDI(C,D,E,F,PI)
'END';
'COMMENT' ********************************************
A,B,C,D,E,F ARE ARRAYS WHERE CORRELATIN FUNCTION, POWER
DENSITY SPECTRUM OR REAL AND IMAGINARY PARTS OF THE
MARKOVIAN FILTER ARE STORED.CARE SHOULD BE TAKEN THAT
FINALLY  SPECTR = TRUE AND THAT THE REAL AND IMAGINARY
COMPONENTS OF THE CORRESPONDING FILTER ARE STORED IN
ARRAYS A  AND  D.**************************************;

NEWLINE(1);    RUNOUT;    X0:=READ;

'FOR' I:=1 'STEP' 1 'UNTIL' NG 'DO' 'BEGIN'
RUNOUT;
'FOR' J:=0 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
RANDOM(0.0,1.0,12,GAUSS,X0);
A[J]:=GAUSS;
RANDOM(0.0,1.0,12,GAUSS,X0);
B[J]:=GAUSS;                          'END';
'IF' REIGH 'THEN' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
RANDOM(0.0,1.0,12,GAUSS,X0);
A[J]:=SQRT(A[J]*A[J] + GAUSS*GAUSS);
RANDOM(0.0,1.0,12,GAUSS,X0);
B[J]:=SQRT(B[J]*B[J] + GAUSS*GAUSS);
'END'    'END';
'COMMENT' ********************************************
REIGH IS A BOOLEAN  WHICH IS TRUE FOR A  RAYLEIGH DISTR-
IBUTION.**********************************************;

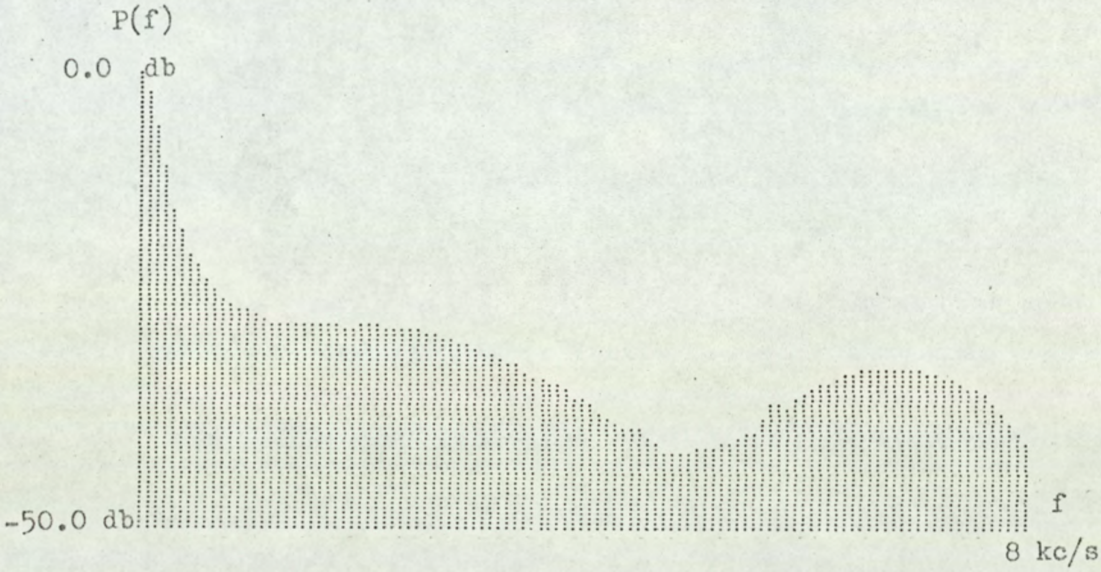

PATTERN(A,B,C,D,M);

'COMMENT'*********************************************
DATA SHOULD BE STORED ON DISK OR MAGNETIC TAPE.IF DATA
ARE NECESSARY ON A PAPER TAPE THEN  PROCEED AS FOOLOWS***;


SELECT OUTPUT(4);  RUNOUT;
'FOR' J:=0 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
N:=N+1;  'IF' N 'GT' NG  'THEN' 'BEGIN'
NEWLINE(1);    N:=0    'END';
PRINT(A[J],0,4);                          'END';
'FOR' J:=1 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
N:=N+1;  'IF' N 'GT' NG  'THEN' 'BEGIN'
NEWLINE(1);    N:=0    'END';
'COMMENT' NG IS THE NUMBER OF DATA TO BE PRINTED IN A ROW;
'END';

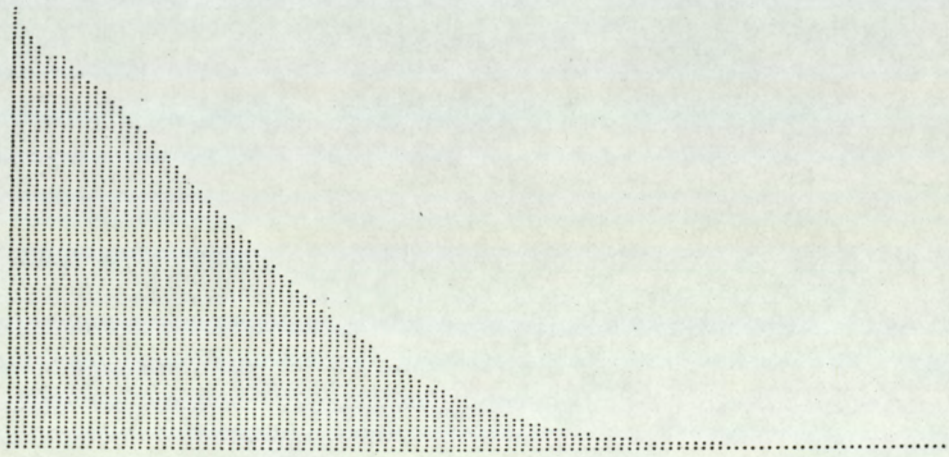
'END' NGLOOP;    'END' BLOCK;    'END' PRGRM;
```

****



A sample of desired power density spectrum used in the computer program GENERATE

Autocovariance function obtained from a sequence of numbers generated by digital computer , using program GENERATE and the above sample of the desired power density spectrum.



Programs PROBAB&PROBABPLOT

The noise is an actual communication system which takes many forms but most of the theoretical work done has been based on the assumption that the noise is the very common and relatively tractable Gaussian noise. The values taken on can be described by single-variable, bi-variable or multivariable distributions, each describing the values which may be taken on at several instants of time, and each distribution is Gaussian.

The special importance of the Gaussian distribution is that it is the form said to be taken when the distribution is the result of the addition of effects due to many independent randomly varying phenomena, each of which may not itself be Gaussian. The variance and the mean values of each process is regarded as being small compared with the total effect which is being studied.

Since electrical effects often result from the superposition of the effects of many discrete events, e.g. the passage of electrons in a circuit, the Gaussian (normal) distribution has very often been observed.

A Gaussian distribution due to a single effect is characterised completely by two parameters, the first moment or its mean value

$$\bar{x}(t) = \int x.\, p(x)\, dx$$

and the second moment or variance

$$\sigma_x^2 = \int (x - \bar{x}(t))^2 p(x).\, dx$$

where $p(x)$ is the probability that x will be observed within an interval between x and $x + dx$.

The square root of the variance of x is called the standard deviation of x, and is indicated by σ_x.

The probability density function $p(x)$ for the single variable can be written

$$p(x\ \bar{x},\ \sigma_x) = \frac{1}{\sqrt{2\pi}\ \sigma_x} \exp\{-(x - \bar{x})^2/2\sigma_x^2\}$$

For a random process it is often useful to know the higher, k^{th}, moment about the mean, or the k^{th} central moment, since $p(x\ \bar{x},\ \sigma_x)$ for the normal distribution is symmetrical about the mean value, the central moments for k odd are all zero. By repeated integration by parts the central moments when k is even are given by

$$M_k\ \{(x - \bar{x})\} = \frac{(x - \bar{x})^k}{\sqrt{2\pi}\ \sigma} . \exp\{-(x - \bar{x})^{k/}2\sigma\}\ dx$$

$$= 1.3.5\ \ldots\ldots\ (k - 1)\sigma^k$$

The left hand side of this equation is a symbolical expression representing the k^{th} central moment, and can in principle, for a Gaussian distribution, as shown by this expression be related to the second moment.

The first and second moments can be related to readily understood physical properties of the process. For example, in an electrical process, the first is a measure of the mean current due to the flow of electrons, while the second is proportional to the power dissipated in a circuit by a noise voltage or fluctuating current, regardless of the type of distribution, i.e. whether or not it be symmetrical or Gaussian. Although, for the Gaussian distribution the higher moments can be calculated in terms of the second moment, in general the higher moments than the second have no readily expressible physical meaning similar to those of the first and second moments.

In general, the first and second moments are not sufficient for a complete statistical description of a random process it it is not Gaussian, but they are useful and the most readily available or measurable data for describing a random process.

PROGRAMS PROBAB AND PROBABPLOT

```
'BEGIN'
'COMMENT' **********************************************
THESE PROGRAMS ARE MADE IN ORDER TO FIND OUT THE
PROBABILITY DENSITY FUNCTIONS OF A RANDOM SEQUENCE OF
NUMBERS.  THE PROGRAM CONTAINS A SUBROUTINE NORMAL
WHICH FINDS THE MEAN VALUE AND THE STANDARD DEVIATION.
N  REAL DATA POINTS ARE STORED IN AN ARRAY X[1:N].
PROCEDURE NORMAL COMPUTES THE STANDARD DEVIATION AND
THE AVERAGE VALUE OF THE SEQUENCE OF  N  NUMBERS.  THE
SEQUENCE IS NORMALISED BY DEDUCTING THE AVERAGE VALUE
AND THEN DIVIDING ALL THE NUMBERS IN THE SEQUENCE BY
THE STANDARD DEVIATION ******************************;


'INTEGER' I, J, JJ, K, L, M, N, H, BOX1, BOX2, BOX3, BOX4, BOX5,
NPSEC;
'REAL' P, Q, XPREV, XPRIM, XSEC, ALPHA, BETA, DELTA,ZZ;


    'REAL' 'PROCEDURE'  NORMAL(F,N,OUT);
      'VALUE' N;  'INTEGER' N;  'ARRAY' F;
      'BOOLEAN'  OUT;
      'BEGIN'  'REAL' AVER, SUM, SIGMA; 'INTEGER' I,J,K,M;

      AVER:=0.0;  SUM:=0.0;
     'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
      AVER:=AVER + F[I];             'END';

      AVER:=AVER/N;      NEWLINE(1);

     'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
      F[I]:=F[I]-AVER;
      SUM:=SUM + F[I]↑2;             'END';

      SUM:=SQRT(SUM);
      SIGMA:=SUM/SQRT(N);   NEWLINE(1);
    'FOR' I:= 1 'STEP' 1 'UNTIL' N 'DO'
    F[I]:=F[I]/SIGMA;
      'IF' OUT 'THEN'
      'BEGIN'
  J:=K:=0;
  SELECT OUTPUT(4);
  WRITETEXT('('END%OF%NORMALIZED%DATA')');
RUNOUT;
      'FOR' I:=1 'STEP' 1 'UNTIL'  N 'DO'
  'BEGIN'
  J:=J + 1;
  'IF' J = 4 'THEN'  'BEGIN'
J:=0;    K:=K + 1;
  NEWLINE(1)    'END';
  'IF' K = 64 'THEN'   'BEGIN'
  NEWLINE(1);    RUNOUT;    K:=0
  'END';
      PRINT(F[I],1,3);
  'END';
      PRINT(SIGMA,0,6);    NEWLINE(1);
      PRINT(AVER,0,6);    NEWLINE(1);
       'END';
 WRITETEXT('('****')');    NEWLINE(2);
 WRITETEXT('('NORMALIZED%DATA')');
 RUNOUT;                                   'END' PROC;
```

```
'COMMENT' ****************************************

N  IS THE NUMBER OF DATA POINTS IN THE SEQUENCE TO BE
ANALYSED.
M  DEFINES THE NUMBER OF STATES FOR COMPUTING
DISTRIBUTION.
L  IS THE NUMBER OF STEPS FOR TRANSITION PROBABILITY
COMPUTATION.
K  IS THE INTEGER NUMBER WHICH DEFINES THE INITIAL
STATE OF THE RANDOM VARIABLES FOR WHICH TRANSITION
PROBABILITIES ARE COMPUTED.
AT THE END OF THE PROGRAM PROBAB ARRAY PRDENS  [-H,H]
CONTAINS NUMERICAL VALUES OF THE DISCRETE FIRST
PROBABILITY DENSITY DISTRIBUTION. ********************;


 'BEGIN'
 N:=READ;  M:=READ;  L:=READ;  H:=M'/'2;  K:=READ;
 AGAIN:
      ZZ:=0.0;
    'BEGIN'
 'REAL' 'ARRAY' X[1:N], PRDNS[-H:H], TOTPR[-H:H], TRPR[-H:H,-H:H],
 Y[-H:H], ANGLE[-36:36], MARCO[1:L,-H:H], VELPR[-H:H], ACCPR[-H:H];
 'INTEGER' 'ARRAY' MEM[1:L];
    'BEGIN'

 'FOR' I:=1 'STEP' 1 'UNTIL' L 'DO'  'BEGIN'
 'FOR' J:=-H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
 MARCO[I,J]:=0.0;
                              'END'; 'END';
 'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO'  'BEGIN'
 PRDNS[I]:=0.0;  TOTPR[I]:=0.0;
 VELPR[I]:=0.0;  ACCPR[I]:=0.0;
 'FOR' J:= (-H) 'STEP' 1 'UNTIL' H 'DO'  'BEGIN'
 TRPR[I,J]:=0.0;
 'END';  'END';


      'COMMENT' ****************************************
      ARRAY  TOTPR  [-H:H]  CONTAINS  M+1  NUMBERS OF
      CUMULATIVE PROBABILITY, I.E. IN THE TOTPR [I] THE
      CUMULATIVE PROBABILITY FROM -H TO I IS STORED.  ARRAY
      TRPR [-H:H,-H:H] CONTAINS NUMERICAL VALUES OF THE
      DISCRETE SECOND PROBABILITY DENSITY DISTRIBUTION.
      ARRAY MARCO [1:L,-H:H] CONTAINS TRANSITION PROBABIL-
     ITIES FROM THE INITIAL  K -TH STATE FOR  L STEPS.*****;


       SELECT INPUT(3);.
 WRITETEXT('('DOC%TAPE')');
 NEWLINE(1);    RUNOUT;
 LAB:
     'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
 'BEGIN'   X[J]:=READ;
 'END';
 NORMAL(X,N,'FALSE');

 P:=H/3.0;

 'FOR' I:= 1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
 X[I]:=P*X[I] + 0.5;  'END';
```

```
XPREV:=X[L-2] - X[L-3];
XPRIM:= X[L-1] - X[L-2];

'FOR' I:= L 'STEP' 1 'UNTIL' N 'DO'  'BEGIN'
BOX1:=ENTIER(X[I-1]);      BOX2:=ENTIER(X[I]);
XPRIM:= X[I] - X[I-1];
XSEC:=XPRIM - XPREV;
XPREV:=XPRIM;
      BOX4:=ENTIER(20.0*XSEC + 0.5);
      BOX3:=ENTIER(20.0*XPRIM + 0.5);

      'IF' ABS(BOX1) > H 'THEN'
      'BEGIN'  'IF'  BOX1 > 0 'THEN'  BOX1:=H
               'ELSE'  BOX1:=-H 'END';
      'IF' ABS(BOX2) > H 'THEN'
      'BEGIN'  'IF'  BOX2 > 0 'THEN'  BOX2:=H
               'ELSE'  BOX2:=-H 'END';
      'IF' ABS(BOX3) > H 'THEN'
      'BEGIN'  'IF'  BOX3 > 0 'THEN'  BOX3:=H
               'ELSE'  BOX3:=-H 'END';
      'IF' ABS(BOX4) > H 'THEN'
      'BEGIN'  'IF'  BOX4 > 0 'THEN'  BOX4:=H
               'ELSE'  BOX4:=-H 'END';
PRDNS[BOX1]:=PRDNS[BOX1] + 1;
TRPR[BOX1,BOX2]:=TRPR[BOX1,BOX2] + 1;
VELPR[BOX3]:=VELPR[BOX3] + 1;
ACCPR[BOX4]:=ACCPR[BOX4] + 1;
MEM[1]:=ENTIER(X[I-L+1]);
'IF' MEM[1] = K  'THEN'  'BEGIN'   MARCO[1,K]:=MARCO[1,K] + 1;
'FOR' JJ:=2 'STEP' 1 'UNTIL' L 'DO'  'BEGIN'
MEM[JJ]:=ENTIER(X[I-L + JJ]);
'IF' ABS(MEM[JJ]) > H 'THEN'
    'BEGIN'  'IF'  MEM[JJ] > 0 'THEN'  MEM[JJ]:=H
                                'ELSE'  MEM[JJ]:=-H   'END';

MARCO[JJ,(MEM[JJ])]:=MARCO[JJ,(MEM[JJ])] + 1;
'END'  'END';

'END';
ZZ:=ZZ + 1.0;

NEWLINE(1);  SPACE(2);
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
PRDNS[I]:=PRDNS[I]/(N-L+1);
'END';
'BEGIN'  TOTPR[-H]:=PRDNS[-H];

'FOR' J:=(-H+1) 'STEP' 1 'UNTIL' H 'DO'  'BEGIN'
TOTPR[J]:=TOTPR[J-1] + PRDNS[J];
    'END'; 'END';

     'COMMENT'*********************************************
     SOME FURTHER INFORMATION ON THE DISTRIBUTION OF A
     RANDOM VARIABLE IS OBTAINED FROM THE THIRD AND THE
     FOURTH MOMENTS.  IF P(X) IS SYMMETRICAL ABOUT ITS MEAN
     VALUE ALL CENTRAL MOMENTS OF ODD ORDER MUST VANISH.
     THIS IS TRUE FOR AN IDEAL GAUSSIAN DISTRIBUTION BUT NOT
     NECESSARILY FOR AN ACTUAL AND ARBITRARY ONE, (CH.3).
    THE ASSYMETRY MAY BE COMPARED BY THE USE OF THE THIRD
     CENTRAL MOMENT M3.  TO MAKE IT NORMALISED IT SHOULD BE
     DEVIDED BY SIGMA MULTIPLIED BY 3. AS SIGMA IS ALREADY
     MADE TO BE 1.0 (PROCEDURE NORMAL) THE SKEW COEFFICIENT
     CORRESPONDS NUMERICALLY TO THE THIRD CENTRAL MOMENT.*****;
```

```
NEWLINE(1);  SPACE(2);
RUNOUT;
PRINT(ZZ,4,0);
XPRIM:=XSEC:=0.0;    P:=1.0/P;
'FOR' I:=-H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
BETA:=I*P;  ALPHA:=BETA*BETA*BETA;
BETA:=ALPHA*BETA;
XPRIM:=XPRIM + PRDNS[I]*ALPHA;
XSEC:=XSEC + PRDNS[I]*BETA;  'END';

'COMMENT' ***************************************
THE FOURTH CENTRAL MOMENT  OF THE GAUSSIAN DISTRIBUTION
IS EQUAL TO 3 TIMES  SIGMA **4.
THE NONDIMENSIONAL QUANTITY M4/SIGMA↑4 IS FOR A GAUSSIAN
DISTRIBUTION EQUAL TO 3.  THEREFORE BY INTRODUCING THE
COEFFICIENT OF EXCESS WHERE EXCESS  =  3 - M4/SIGMA **4
WHICH FOR A GAUSSIAN DISTRIBUTION IS OBVIOUSLY ZERO,
BUT FOR OTHER DISTRIBUTIONS HAVING THE SAME VARIANCES
AS GAUSSIAN WILL INDICATE WHETHER THE DISTRIBUTION HAS
A FLATTER (EXCESS < 0) OR SHARPER (EXCESS > 0) PEAK
THAN THE GAUSSIAN (BECKMAN 1960) **********************;

WRITETEXT('('SKEW%COEFFICIENT')');
PRINT(XPRIM,0,6);
WRITETEXT('('EXCESS%COEFFICIENT')');
SPACE(4);  PRINT(XSEC,0,6);
XPRIM:=XSEC:=0.0;
NEWLINE(1);

WRITETEXT('('FLICKER%AMPLIFIER%NOISE')');
SELECT OUTPUT(4);
WRITETEXT('('DOC%TAPE')');
      NEWLINE(1);
RUNOUT;


   NEWLINE(1);     SPACE(2);
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'


K:=(ENTIER(I/3))*3;   P:=K;   Q:=I;
'IF' P = Q 'THEN' NEWLINE(1);   SPACE(2);
PRINT(TOTPR[I],0,6);
'END';
NEWLINE(1);  SPACE(2);
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'

K:=(ENTIER(I/3))*3;   P:=K;   Q:=I;
'IF' P = Q 'THEN' NEWLINE(1);   SPACE(2);
PRINT(PRDNS[I],0,6);
'END';
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
     ZZ:=0.0;
NEWLINE(1);
'FOR' J:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
'IF' J = 0 'THEN' NEWLINE(1);    SPACE(1);
PRINT(TRPR[I,J],3,0);

       :=ZZ + TRPR[I,J];
    'END';
```

```
NEWLINE(1);  SPACE(2);
PRINT(ZZ,4,0);
    'END';
'FOR' I:=1 'STEP' 1 'UNTIL' L 'DO'  'BEGIN'
    ZZ:=0.0;
NEWLINE(1);
'FOR' J:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
'IF' J = 0 'THEN' NEWLINE(1);    SPACE(1);
PRINT(MARCO[I,J],2,0);
    ZZ:=ZZ + MARCO[I,J];
  'END';

NEWLINE(1);  SPACE(2);
PRINT(ZZ,4,0);
    'END';
    ZZ:=0.0;
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
'IF' I = 0 'THEN' NEWLINE(1);    SPACE(1);
PRINT(VELPR[I],2,0);
    ZZ:=ZZ + VELPR[I];
'END';
NEWLINE(1);  SPACE(2);
PRINT(ZZ,4,0);
    ZZ:=0.0;
NEWLINE(1);
'FOR' I:= (-H) 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
'IF' I = 0 'THEN' NEWLINE(1);    SPACE(1);
PRINT(ACCPR[I],2,0);
    ZZ:=ZZ + ACCPR[I];
'END';
NEWLINE(1);  SPACE(2);
PRINT(ZZ,4,0);
    NEWLINE(2);
RUNOUT;
   'END' BLOCK;  'END' ARRAY;
                                   'END' READ;  'END' PROGRAM;


  'BEGIN'
'INTEGER' I,J,L,M,N,H,K;  'REAL' P,Q,ZZ,DP,DQ,DN;
H:=READ;  L:=READ; M:=READ;
N:=(2*H + 1); DN:=2.0*H/8.0;  DP:=4.0/L;     DQ:=2.5/L;
PRINT(L,0,6);  PRINT(DP,0,6);  PRINT(DQ,0,6);

 'COMMENT'*******************************************
 PROGRAM PROBABPLOT  PLOTS THE PROBABILITY COMPUTED
BY PROBAB.THE OUTPUT TAPE FROM PROBAB IS THE INPUT
TAPE FOR PROBABPLOT. PROGRAMS ARE WRITTEN FOR PAPER
TAPES BUT FILING IS ALSO POSSIBLE ON MAG.TAPE.IT IS
 SEPARATED FROM THE PROBAB IN ORDER NOT TO EXHAUST THE
 STORAGE CAPACITY OF THE COMPUTER WHEN PROBAB IS USED.
 APPLYING THE OVERLAY TECHNIQUE THESE TWO PROGRAMS COULD
 BE MERGED ****************************************;

 PRINT(H,0,6);  PRINT(N,0,6);  PRINT(DN,0,6);
```

```
'BEGIN'


'ARRAY' X[-H:H], Y[-H:H], YY[-H:H,1:N], YX[-H:H];

'PROCEDURE' STARP(M,Z,FACTOR);
'VALUE' Z;  'ARRAY' Z;  'INTEGER' M, FACTOR;
'BEGIN'  'INTEGER' I, J, MT, K;  K:=(17'/'M) + 1;


'COMMENT' ******************************************
 STARP IS A SIMPLE PROCEDURE FOR PLOTTING PROBABILITIES(I.
E. PROBABILITY DENSITY) STORED IN A DUMMY ARRAY Z. 2*M ,THE
 MAXIMUM NUMBER OF DISCRETE VALUES  OF CORRESPONDING FUNC
TION  IS VIRTUALLY  LIMMITED BY THE SIZE OF THE COMPUTER
 STORE, BUT FOR PRACTICAL REASONS SHOULD BE LESS OR EQUAL
 TO 100.*******************************************;


'FOR' I:= -M 'STEP' 1 'UNTIL' M 'DO'  'BEGIN'
NEWLINE(K);  PRINT(Z[I],1,4);  Z[I]:=FACTOR*Z[I];
MT:=ENTIER(Z[I]);  SPACE(10);
'IF' MT 'GE' 100 'THEN' MT:=100;
'FOR' J:=1 'STEP' 1 'UNTIL' MT 'DO'
PRINTCH(26);  'END';     'END' PROC;


'COMMENT' ***************************************
PROBAB OUTPUT TAPE IS THE INPUT TAPE FOR PROBABPLOT.**;


SELECT INPUT(3);
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Q:=Y[I]:=READ;  P:=X[I]:=I/DN;  Y[I]:=Y[I]*5.0;
'IF' ABS(Y[I]) 'GT' 5.0 'THEN' Y[I]:=5.0;
'END';                PAPERTHROW;

WRITETEXT('('CUMULATIVE%PROBABILITY')');  STARP(H,Y,M);

'BEGIN'  OPENPLOT;  P:=-19.0;    Q:=8.0;


    'COMMENT' *******************************************
    FOR ALL USED GRAPH PLOTTER PROCEDURES SEE THE CORRESPONDING
    ICL 1900 COMPUTER SERIES  MANUAL.ALL LENGHTS AND COORDINATES
    ARE SPECIFIED IN INCHES. *********************************;


HGPLOT(P,Q,0,4);  HGPLINE (X,Y,N,+1);
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=READ;  P:=X[I]:=I/DN;  Y[I]:=2*M*Y[I];
'IF' ABS(Y[I]) 'GT' 5.0 'THEN' Y[I]:=5.0; Q:=Y[I];
```

```
HGPDASHLN(P,0.0,P,Q,0.0);          'END';

HGPLINE (X,Y,N,+1);  PAPERTHROW;     STARP(H,Y,M);
STRARR(X,21,'('NORMALIZED%VARIABLE%X')');
STRARR(YY,17,'('TOTAL%PROBABILITY')');
HGPAXISV(0.0,0.0,YY,17,5.0,90.0,0.0,0.1,0.5,2);
HGPAXISV(-4.0,0.0,X,21,8.0,0.0,-5.0,1.0,0.8,2);
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPDASHLN(-6.0,-1.5,-5.0,-1.5,0.0);
HGPDASHLN(4.0,-1.5,5.0,-1.5,0.0);

HGPLOT(0.0,0.0,3,0);  P:=0.0;    Q:=8.5;  HGPLOT(P,Q,0,4);
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'

'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
P:=X[I]:=I/DN;    YY[I,J]:=READ;  'END';
ZZ:=READ;  'IF' ZZ 'LE' 1 'THEN' ZZ:=1;
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
YY[I,J]:=(YY[I,J]/ZZ)*5.0;
'IF'  YY[I,J] 'GE' 5.0 'THEN'  YY[I,J]:=5.0;
YX[I]:=YY[I,J];                        'END';
HGPLINE(X,YX,N,+1);                    'END';


HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPRECT(-7.0,-1.5,17.0,12.0,0.0,3);


STRARR(X,21,'('NORMALIZED%VARIABLE%X')');
HGPAXISV(-4.0,0.0,X,21,8.0,0.0,-5.0,1.0,0.8,2);

HGPLOT(0.0,0.0,3,0);  P:=14.0;    Q:=-8.5;
HGPLOT(P,Q,0,4);
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
K:=J - (N+1)'/'2;
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
P:=X[I]:=I/DN;  YX[I]:=YY[I,J]*Y[K];  'END';
HGPLINE(X,YY,N,+1);     'END';
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
STRARR(X,21,'('NORMALIZED%VARIABLE%X')');
STRARR(Y,23,'('SEC%PROBABILITY%DENSITY')');
HGPAXISV(-4.0,0.0,X,21,8.0,0.0,-5.0,1.0,0.8,2);
HGPAXISV(0.0,0.0,Y,23,5.0,90.0,0.0,0.1,0.5,2);
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPDASHLN(-6.0,-1.5,-5.0,-1.5,0.0);
HGPDASHLN(4.0,-1.5,5.0,-1.5,0.0);

HGPLOT(0.0,0.0,3,0);  P:=2.0;   Q:=6.0;
HGPLOT(P,Q,0,4);
STRARR(X,21,'('NORMALIZED%VARIABLE%X')');
HGPAXISV(-2.0,0.0,X,21,4.0,0.0,-5.0,1.0,0.4,2);
STRARR(Y,23,'('SEC%PROBAB%DENSITY')');
HGPAXISV(0.0,0.0,Y,23,2.0,90.0,0.0,0.25,0.5,1);
'FOR' J:=1 'STEP' 1 'UNTIL' L 'DO' 'BEGIN'
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=READ;  'END';  ZZ:=READ;
'IF' ZZ 'LE' 1 'THEN' ZZ:=1;
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
X[I]:=I/(2.0*DN);  Y[I]:=(Y[I]/ZZ)*2.0;
'IF' ABS(Y[I]) 'GT' 2.0 'THEN' Y[I]:=2.0;  'END';
```

```
'IF' J 'NE' 1 'THEN'  HGPLINE(X,Y,N,+1)
'ELSE'     HGPDASHLN(1.0,0.0,1.0,2.0,0.0);
HGPLOT(0.0,0.0,3,0);  P:=-DP;    Q:=DQ;
HGPLOT(P,Q,0,4);                       'END';

'COMMENT'*******************************************
THE NEXT SECTION (DOWN TO CLOSEPLOT) SHOULD BE OMMITED WHEN
VELOCITY AND ACCELERATION PROBABILITIES ARE NOT OUTPUTED FROM
THE PROGRAM PROBAB.WHEN OVERLAY TECHNIQUE IS USED THESE DATA
ARE STORED IN  VELPR[-H:H] AND ACCPR[-H:H]. *****************;

HGPLOT(0.0,0.0,3,0);  P:=2.0;     Q:=0.0;
HGPLOT(P,Q,0,4);
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPRECT(-7.0,-1.5,17.0,12.0,0.0,3);

HGPLOT(0.0,0.0,3,0);  P:=0.0;     Q:=8.5;
HGPLOT(P,Q,0,4);
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=READ;  'END';  ZZ:=READ;
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=(Y[I]/ZZ)*5.0;
'IF' ABS(Y[I]) 'GT' 5.0 'THEN' Y[I]:=5.0;
Q:=YX[I]:=Y[I];    P:=X[I]:=I/DN;
HGPDASHLN(P,0.0,P,Q,0.0);              'END';
HGPLINE (X,Y,N,+1);
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPRECT(-7.0,-1.5,8.4,12.0,0.0,3);
STRARR(X,19,'('NORMALIZED%VELOCITY')');
STRARR(Y,23,'('VELOCITY%PROBAB%DENSITY')');
HGPAXISV(-4.0,0.0,X,19,8.0,0.0,-5.0,1.0,0.8,2);
HGPAXISV(0.0,0.0,Y,23,5.0,90.0,0.0,0.1,0.5,2);
PAPERTHROW;                     STARP(H,YX,M);

HGPLOT(0.0,0.0,3,0);  P:=-14;     Q:=0.0;
HGPLOT(P,Q,0,4);
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=READ;     'END';  ZZ:=READ;
'IF' ZZ 'LE' 1 'THEN' ZZ:=1;
'FOR' I:= -H 'STEP' 1 'UNTIL' H 'DO' 'BEGIN'
Y[I]:=(Y[I]/ZZ)*5.0;      P:=X[I]:=I/DN;
'IF' ABS(Y[I]) 'GT' 5.0 'THEN' Y[I]:=5.0;
HGPDASHLN(P,0.0,P,Q,0.0);            'END';
HGPLINE (X,Y,N,+1);
HGPRECT(-4.0,-0.5,6.0,8.0,0.0,3);
HGPRECT(-7.0,-1.5,8.4,12.0,0.0,3);
STRARR(X,23,'('NORMALIZED%ACCELERATION')');
STRARR(Y,23,'('ACCELRTN%PROBAB%DENSITY')');
HGPAXISV(-4.0,0.0,X,23 ,8.0,0.0,5.0,1.0,0.8,2);
HGPAXISV(0.0,0.0,Y,23,5.0,90.0,0.0,0.1,0.5,2);

CLOSEPLOT;


PAPERTHROW;    STARP(H,YX,M);  'END';

'END';  'END' BLOCK;  'END' PRGRM;
****
```

Program SHORTBITS

Digital Computation of the Normalised Autocovariance Function.

Analogue methods of evaluation of correlation functions are inherently very slow and tedious. The averaging process must be carried out for all delays of interest, which usually requires an appreciable time.

Even for systems in which the statistical properties do not change quickly, i.e. they are quasi stationary processes, this could lead to serious errors.

In general, assuming that the fast digital computer is available, the easiest and fastest way to get the autocovariance function reliably is to use a sampling technique and numerical methods of analysis.

Sampling should be done in accordance with the Shannon sampling theorem, i.e. at the Nyquist frequency rate.

Sampling and averaging a finite number, say N, of the sample products

$$\rho_{xx}(m.\Delta\tau = \frac{1}{N} \sum_{I=1}^{N} X(I.\Delta t).X(I\Delta t - m\Delta\tau)$$

This should be computed for as many values of τ, i.e. for $m = 1, 2, 3, \ldots\ldots M$, as we need, bearing in mind that the maximum delay $\tau_{max} = M.\Delta\tau$ cannot be longer than about 5% of the record duration, which is equal to

$N.\Delta t = N.\Delta\tau = T_1 - T_2$

so that $M.\Delta\tau \leqq \frac{1}{20}(N.\Delta\tau)$ or, $M \leqq N/20$

In using the autocovariance function it is assumed that the mean value of the function is zero (or the mean value has been subtracted).

For the normalised autocovariance function or correlation function, the value of the variance must be found using

$$\sigma^2 = \frac{1}{N} \sum_{I=1}^{N} (X_I - \overline{X_{(N)}})^2$$

where

$$\overline{X_{(N)}} = \frac{1}{N} \sum_{I=1}^{N} X_I$$ = the average value over N observations being utilised.

The normalised autocovariance function can then be expressed as

$$\chi(M\ ) = \frac{\sum_{1}^{N} \chi(I) - \frac{1}{N}\sum_{J=1}^{N} \chi(J) \qquad \chi(K) - \frac{1}{N}\sum_{J=1}^{N} \chi(J)}{\sum_{L=1}^{N} \chi(L) \ - \sum_{J=1}^{N} \chi(J)^{\,2}}$$

where $\Delta t \ = \ I\Delta t - \tau(I,K) \ = \ I\Delta t - M\Delta\tau$

Since generally, at least for the case of computation, $\Delta\tau = \Delta t$, i.e. the sampling interval and the unit of the delay variable are taken to be the same, we may write

$$K = I - M$$

Instead of normalising the autocovariance function it is more convenient to normalise the recorded time function by subtracting the mean value and dividing each sampled value by the precomputed standard deviation, i.e. by the r.m.s. value. Thus

$$\chi\ (M\Delta\tau) \ = \ \sum_{I}^{N} \xi(I).\xi(K) \ = \ \sum_{I}^{N} \xi(I).\xi(I - M)$$

where $\xi(I)$ represent the sampled value of the normalised sample function of the random variable.

Changing the values of M from $0 < M < M_{max}$, the autocovariance function is obtained. (See Appendix : Computer programme for auto-correlation function estimation).

The Fourier transform of any truncated and sampled time function, i.e. observed over a finite time integral results in discrete spectrum lines, which makes it equivalent to a Fourier series strictly mathematically defined only for periodic function.

That means that a truncated function has a spectrum defined only in the same time interval as the truncated time function. The highest component in the spectrum with the highest frequency which one can compute without ambiguity is equal to the Nyquist sampling frequency,

$$f_{max} = 1/2\Delta t$$

where t is the sampling interval.

The lowest frequency is inversely proportional to the total time duration of the record, or, in the case of the power density spectrum, inversely proportional to the maximum time delay utilised for its computation,

$$f_{min} = 1/2/\tau_{max}|$$

$$\text{where } \tau_{max} = M.\Delta\tau = M\Delta t$$

and M is a integer defined for the computation of the correlation function. It is obvious that

$$f_{max}/f_{min} = M$$

as was postulated earlier in order to fulfil the condition $M \lesssim N/20$.

$$\text{Thus } f_{min} = 20\ f_{max}/N = 10/T_{rec}$$

$$\text{where } T_{rec} = N_{\Delta}t\text{, i.e. the total record duration in seconds.}$$

In order to attain a high upper frequency limit, say several kilocycles per second, and starting from the relation

$$f_{max} = 10.M/T_{rec}$$

we see that M must also be of the same order, i.e. M must have values ranging up to several thousand or even higher. Very often it is not possible to sample the recorded time function fast enough to obtain the component of the spectrum with the highest frequency. In such a case the tape speed, both for recording and replaying, should be changed accordingly. The maximum sampling rate of the programme used was determined by the PDP9 computer and ADC converter, and was 1,000 cycles per second.

```
'BEGIN' 'INTEGER' I,J,M,MN,N,NG,IN,UT;
'REAL' SIGMA,AVER,RATIU,MEAN,VAR,FF;
'BUULEAN' HAMMING,UUT;   HAMMING:='FALSE';
  SELECT INPUT(3);
'COMMENT' DATA ARE IN THE FOLLOWING ORDER:
DAY, MUNTH, YEAR UF SAMPLING, TAPE COLOUR,
TAPE NUMBER, NUMBER UF 2↑10 DATA GROUPS,
EXPERIMENTAL SIGNAL TU NUISE RATIU,
FLUCTUATIUN NURMALIZING RATIU,
SIGNAL LEVEL IN VULTS,SMUUTHING.
FUR FAST FUURIER TRANSFURM  NUMBER UF DATA
IN A GRUUP IS READ BEFURE THE PRUGRAM STARTS.
*************************************************;

I:=READ;  J:=READ;  M:=READ;
MN:=READ;  N:=READ;  NG:=READ;
RATIU:=READ;  NEWLINE(2);
WRITETEXT('('DATE%UF%SAMPLING')');
PRINT(I,3,0);  PRINT(J,3,0);  PRINT(M,5,0);
SPACE(12);    PRINT(N,3,0);
'IF' MN 'EQ' 1 'THEN'
WRITETEXT('('GREEN%TAPE%NU%')');
'IF' MN 'EQ' 2 'THEN'
WRITETEXT('('PINK%TAPE%NU%')');
'IF' MN 'EQ' 3 'THEN'
WRITETEXT('('BLUE%TAPE%NU%')');
'IF' MN 'EQ' 4 'THEN'
WRITETEXT('('WHITE%TAPE%NU%')');
'IF' MN 'EQ' 5 'THEN'
WRITETEXT('('MAGNETIC%TAPE')');
SPACE(1);  PRINT(N,3,0);
NEWLINE(2);
WRITETEXT('('NUMBER%UF%2↑10%DATA%GRUUPS')');
SPACE(2);  PRINT(NG,2,0);
NEWLINE(2);
WRITETEXT('('EXPERIMENTAL%SIGNAL%TU%NUISE%RATIU')');
SPACE(2);  PRINT(RATIU,0,6);
SPACE(1);  WRITETEXT('('CURRESPUNDING%TU')');
AVER:=LN(10.0);  SIGMA:=20.0*LN(RATIU)/AVER;
SPACE(2);  PRINT(SIGMA,2,3);
WRITETEXT('('DB')');
SIGMA:=READ;  RATIU:=RATIU/SIGMA;  NEWLINE(2);
WRITETEXT('('NURMALIZED%FLUCTUATIUN%RATIU')');
SPACE(8);  PRINT(SIGMA,0,6);
SIGMA:=20.0*LN(SIGMA)/AVER;
SPACE(1);  WRITETEXT('('CURRESPUNDING%TU')');
SPACE(2);  PRINT(SIGMA,2,3);
WRITETEXT('('DB')');
NEWLINE(2);  WRITETEXT('('UUTPUT%SIGNAL%LEVEL')');
SIGMA:=READ;  RATIU:=RATIU*SIGMA;
SPACE(2);  PRINT(SIGMA,2,4);
SPACE(2);  WRITETEXT('('VULTS')');
SIGMA:=SIGMA/SQRT(0.075);
SPACE(14); WRITETEXT('('CURRESPUNDING%TU')');
SPACE(1);  PRINT(SIGMA,3,3);  WRITETEXT('('DBM')');
NEWLINE(2);     RATIU:=1.0/RATIU;
WRITETEXT('('CUMPUTATIUNAL%RATIU')');
PRINT(RATIU,0,6);  MEAN:=VAR:=0.0;
```

```
M:=READ;  'IF' M 'EQ' 1 'THEN'
HAMMING:='TRUE';  FF:=READ;
'IF'  FF 'NE' 0 'THEN' 'BEGIN'
M:=READ;  N:=2↑M;  MN:=N 'END' 'ELSE'
'BEGIN'  NEWLINE(1);
 M:=7;  MN:=2↑M;  N:=1024  'END';
 'BEGIN'  'ARRAY' X[1:N], Y[0:MN], Z[0:MN],
            SPECT[0:MN], CORF[0:MN];
 'COMMENT' ****************************************************
 PROCEDURE NORM1 (NORM=TRUE) NORMALIZES N REAL DATA CONTAINED IN
 ARRAY X[1:N]. WHEN OUT IS TRUE ,A PAPER TAPE IS OUTPUTED FOR
 SOME FURTHER USE. WHEN NORM IS FALSE ,SIGMA AND AVER OF X[1:N]
 ARE  COMPUTED BUT ACTUAL INPUT DATA ARE  NOT NORMALIZED.*******;
 'PROCEDURE' NORM1(X,N,SIGMA,AVER,OUT,NORM);
 'VALUE' N,OUT,NORM;  'INTEGER' N; 'ARRAY' X;
 'REAL' SIGMA,AVER; 'BOOLEAN' OUT,NORM;
 'BEGIN' 'INTEGER' I,J,JJ;  'REAL' SUM,DUM;
 AVER:=0;  SUM:=0; J:=0;  NEWLINE(1);
 'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
 X[I]:=READ;  AVER:=AVER+X[I];  'END';
 AVER:=AVER/N;
 'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
 DUM:=X[I]-AVER;  SUM:=SUM+DUM*DUM;  'END';
 SIGMA:=SQRT(SUM/N);
 'IF' NORM 'THEN' 'BEGIN'
 'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
 X[I]:=(X[I]-AVER)/SIGMA;   J:=N 'END';

 'IF' OUT 'THEN' 'BEGIN'  J:=0;  JJ:=0;
 SELECT OUTPUT(4);    RUNOUT;
 'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
 J:=J+1;  PRINT(X[I],2,4);
 'IF' J = 4 'THEN' 'BEGIN' NEWLINE(1);
 JJ:=JJ+1;  J:=0  'END';
 'IF' JJ= 64 'THEN' 'BEGIN' RUNOUT; JJ:=0 'END';
 'END' ILOOP;  J:=N 'END'; 'END' PROC;

 'PROCEDURE' CORRFUN(N,MN,X,Y,OUT,HAMMING);
 'VALUE' N,MN,X;  'BOOLEAN' OUT,HAMMING;
 'INTEGER' N, MN;  'ARRAY' X, Y;
 'BEGIN' 'INTEGER' I,J;  'REAL' T,PI;
'COMMENT' ****************************************************
 PROCEDURE CORRFUN COMPUTES THE AUTOCOVARIANCE FUNCTION OF X[1:N].
 WHEN  HAMMING IS TRUE ,SMOOTHING OF THE CORRESPONDING SPECTRUM
 IS DONE. CORRESPONDING AUTOCOVARIANCE FUNCTION  IS MODIFIED
 AND REPRESENTS MATHEMATICALLY DEFINED  APPARENT  COVARIANCE
 FUNCTION OF A TIME TRUNCATED REAL FUNCTION. IF THE TIME LIMITS
 OF INTEGRATION ARE SET TO BE EQUAL TO THE BEGINING AND THE END
 OF THE TIME INTERVAL  OF THE EXISTENCE OF THE REAL FUNCTION
 AND THERFORE NO SMOOTHING IS NECESSARY HAMMING SHOULD BE FALSE.
 FOR OUT = TRUE,DISCRETE VALUES OF THE HALF OF THE COMPUTED
 AUTOCOVARIANCE FUNCTION ARE OUTPUTED ON THE PAPER TAPE FOR SOME
 FURTHER USE,I.E. PLOTTING,FILING ETC. *************************;
 PI:=4.0*ARCTAN(1.0);
 'FOR' I:=0 'STEP' 1 'UNTIL' MN 'DO'  Y[I]:=0.0;
 'FOR' I:=1 'STEP' 1 'UNTIL' (N-MN) 'DO' 'BEGIN'
 'FOR' J:=0 'STEP' 1 'UNTIL' MN 'DO' 'BEGIN'
 Y[J]:=Y[J] + X[I]*X[I+J];  'END'; 'END';
 'IF'  HAMMING  'THEN'  'BEGIN'
 'FOR' I:=1 'STEP' 1 'UNTIL' MN 'DO' 'BEGIN'
 T:=I/MN;  T:=COS(PI*T)*0.46;
 Y[I]:=Y[I]*(0.54+T)/Y[0];          'END'
```

```
'END' 'ELSE' 'BEGIN'
'FOR' I:=1 'STEP' 1 'UNTIL' MN 'DO'
Y[I]:=Y[I]/Y[0];    J:=0  'END';
Y[0]:=1.0;  'IF' OUT 'THEN' 'BEGIN'
SELECT OUTPUT(4);      RUNOUT;
'FOR' I:=0 'STEP' 1 'UNTIL' MN 'DO' 'BEGIN'
NEWLINE(1);  PRINT(Y[I],0,6);  'END';  I:=0 'END';
'END' PROC;

'PROCEDURE' STARX(F,N,FACTOR,SPECTR,FF);
'VALUE' F, N, FACTOR, SPECTR;
'INTEGER' N, FACTOR; 'ARRAY' F;
'BOOLEAN' SPECTR;   'REAL' FF;
'BEGIN'
 'COMMENT' ***************************************************
PROCEDURE STARX DISPLAYS EITHER THE AUTOCOVARRIANCE FUNCTION
(SPECTR=FALSE)    OR THE POWER DENSITY SPECTRA (SPECTR=TRUE)
USING THE LINEPRINTER OUTPUT.    WHEN SPECTRA ARE DISPLAYED
ORDINATE VALUES ARE IN DBS IN REFERENCE TO THE  TOTAL NORMAL-
IZED SPECTRAL POWER,MADE TO BE EQUAL TO 1.0(SIGMA=1.0).********;
'INTEGER' I, J, NP;  'REAL' L1, FD;
SELECT OUTPUT(0);  PAPERTHROW;  FF:=0.0;
'IF' SPECTR 'THEN' 'BEGIN'
L1:=LN(10.0);  SPACE(13);  NP:=-50;
'FOR' J:=1 'STEP' 1 'UNTIL' 6 'DO' 'BEGIN'
PRINT(NP,2,0);    NP:=NP + 10;
WRITETEXT('('DB')');  SPACE(13);  'END';
NEWLINE(1);  SPACE(19);
'FOR' J:=1 'STEP' 1 'UNTIL' 6 'DO' 'BEGIN'
WRITETEXT('('I')');  SPACE(19);  'END';
'END' SCALE;    NEWLINE(4);
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'IF' SPECTR 'THEN' 'BEGIN'
FD:=FACTOR*F[I];  FF:=FF+FD;  FD:=ABS(FD);
NEWLINE(1);
PRINT(F[I],0,6);    SPACE(4);
'IF'  FD  'LT'  0.00001  'THEN'
FD:=0.00001;
FD:=20.0*LN(FD)/L1;
NP:=ENTIER(FD);  NP:=100 + NP;
'IF' NP 'GT' 100 'THEN' 'BEGIN'
WRITETEXT('('POWER%DENSITY%')');
'GOTO'  AGAIN 'END';

'FOR' J:=1 'STEP' 1 'UNTIL' NP 'DO'
PRINTCH(26);    AGAIN:  NP:=0
'END' 'ELSE' 'BEGIN'
NEWLINE(1);    PRINT(F[I],1,3);
FD:=FACTOR*F[I];
NP:=ENTIER(FD);
'IF' NP 'LT' 0 'THEN' 'BEGIN'
NP:=ABS(NP);
'IF' NP 'GT' 19 'THEN' NP:=20;
NP:=21 - NP;  SPACE(NP);
NP:=21 - NP  'END' 'ELSE' SPACE(20);
'IF' NP 'GT' 100 'THEN' NP:=100;
'FOR' J:=1 'STEP' 1 'UNTIL' NP 'DO'
PRINTCH(26);    NP:=0
```

Digital Fourier Transforms.

Basically, there are two methods for computing the power density spectrum of a time varying signal. The first method was just described as

$$G_{xx} = \text{F.T.}\ \{\chi_{xx}\}$$

The other approach is based on the computation of the linear spectrum of a truncated portion of the total available time varying function, and then by squaring it, obtaining the power density spectrum valid in this small time interval.

If we average, say 100 of these spectra, we will obtain an estimate of the power density spectrum which will be identical to that pertaining to the previous estimates, provided both were calculated for the same total time interval. (Fig.

The second method is extensively used by the so-called "on-line" computation of the power spectra using the Fast Fourier Transform.

The Fast Fourier Transform.

For computing large transforms Colley and Tukey have shown that very large savings in time can be achieved by using their algorithm. When the total number of sampled units is N chosen to be a power of 2, the computing time for their algorithm is proportional to $N\log_2 N$ instead of N^2. This transform is equally applicable to direct spectrum analysis as well as for the method using computation through the use of the correlation function.

The transformation of a time series to an amplitude spectrum, or, in other words, to a set of coefficients for a Fourier Series, has been used for the past two decades on digital computers. Given a real sequence of a sample function, whether it be deterministic or random

$$\{{}^{k}\tilde{x}\} = \{x_0, x_1, \ldots\ x_j, \ \ldots\ x_n\}$$

the Fourier cosine and sine coefficients, A_k and B_k, can be written

$$A_k = \frac{2}{n} \sum_{j=0}^{n-1} x_j \cos(2\pi jk/N) \quad \text{for} \quad k = 0,1, \ldots N/2$$

$$B_k = \frac{2}{N} \sum_{j=0}^{N-1} x_j \sin(2\pi jk/N) \quad \text{for} \quad k = 0,1, \ldots (N/2 - 1)$$

The inverse relationship can also be easily obtained and are well known. (Hamming, R.W.).

These series are known as Discrete Fourier Transforms, D.F.T., and they have the spectra resolution

$$\Delta f = 1/T$$

where T is the time duration of the truncated portion of the observed or recorded time function.

In the exponential form

$$S_x(M\Delta f) = \frac{1}{N} \sum_{j=0}^{N-1} \chi(j\Delta t).\exp(-i.2\pi jk/N)$$

and inversely,

$$x_j = \sum_{k=0}^{N-1} d_k \exp(i.2\pi jk/N)$$

where $i = \sqrt{-1}$ and d_k and χ_j are complex vectors of N dimensions.

In Matrix notation

$$\{\underset{\sim}{\chi}\} = \{\underset{\sim}{T}\}\ \{\underset{\sim}{d}\}$$

where $\{\underset{\sim}{T}\}$ is an $N \times N$ matrix of complex exponential terms

$$t_{jk} = \exp(j.2\pi jk/N)$$

The inverse transform can be written similarly,

$$\{\underset{\sim}{d}\} = \frac{1}{N}\ \{\underset{\sim}{T^*}\}\ \{\underset{\sim}{x}\}$$

where $\{\underset{\sim}{T^*}\}$ is the complex conjugate of $\{\underset{\sim}{T}\}$, i.e.

$$t_{jk} = \exp(-j.2\pi jk/N)$$

The matices $\{\underset{\sim}{T}\}$ and $\{\underset{\sim}{T^*}\}$ have the relation

$$\{\underset{\sim}{T}\}\ \{\underset{\sim}{T^*}\} \quad = \quad N\ \{\underset{\sim}{I}\}$$

where $\{\underset{\sim}{I}\}$ is the N x N diagonal unity matrix.

It is obvious that the inverse of $\{\underset{\sim}{T}\}$ is

$$\{\underset{\sim}{T}\} \ = \ N\ \{\underset{\sim}{T^*}\}$$

The ordinary transform calculation uses direct computation by matrix multiplication. The Cooley and Tukey fast transform algorithm decomposes the matrix $\{\underset{\sim}{T}\}$ into the product of M elementary matrices, i.e. transformations followed by a permutation matrix $\{\underset{\sim}{P}\}$ of the results. It can be shown (Goertzel, G.)

$$\{\underset{\sim}{T}\} \ = \ \{\underset{\sim}{P}\}\ \ \{\underset{\sim}{S}\} = \{\underset{\sim}{S}\}\ \ \{\underset{\sim}{P}\}$$

$\{\underset{\sim}{P}\}$ and $\{\underset{\sim}{S}\}$ represent permutation and transformations respectively.

$\{\underset{\sim}{S}\}$ mat be decomposed into elementary transformations and $\{\underset{\sim}{P}\}$ into several step elementary permutations.

Permutation consists of changing the sequence from the normal binary order into a reverse binary order. Let us represent an indes j as

$$j \ = \ j_{m-1}2^{m-1} \ + \ j_{m-2}2^{m-2} \ + \dots \ + \ j_0$$

the associated inverse binary order

$$r \ = \ j_o 2^{m-1} \ + \ j_1 2^{m-2} \ + \dots\dots \ + j_{m-1}$$

(Singleton, R.) Thus indices from 0 to 15 are permuted as follows (by considering their digital binary representation):

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0	8	4	12	2	10	6	14	1	9	5	13	3	11	7	15

Note that 1111 represents 15, and remains unchanged as does 1001 representing 9, 0110 representing 6 and 0000 representing 0. Elementary transformations consist of simple multiplications and groupings of terms. Thus for $N = 8 = 2^3$

$$
\begin{array}{lll}
\alpha_0 & \alpha_0 + \alpha_4 & (\alpha_0 + \alpha_4) + (\alpha_2 + \alpha_6) \\
\alpha_1 & \alpha_1 + \alpha_5 & (\alpha_1 + \alpha_5) + (\alpha_3 + \alpha_7) \\
\alpha_2 & \alpha_2 + \alpha_6 & (\alpha_0 + \alpha_4) - (\alpha_2 + \alpha_6) \\
\alpha_3 & \alpha_3 + \alpha_7 & (\alpha_1 + \alpha_5) - (\alpha_3 + \alpha_7) \\
\alpha_4 & \alpha_0 + \alpha_4 & (\alpha_0 - \alpha_4) + C^2 (\alpha_2 - \alpha_6) \\
\alpha_5 & \alpha_1 - \alpha_5 & (\alpha_1 - \alpha_5) + C^2 (\alpha_3 - \alpha_7) \\
\alpha_6 & \alpha_2 - \alpha_6 & (\alpha_0 - \alpha_4) - C^2 (\alpha_2 - \alpha_6) \\
\alpha_7 & \alpha_3 - \alpha_7 & (\alpha_1 - \alpha_6) - C^2 (\alpha_3 - \alpha_7) \\
 & S_1 & S_2
\end{array}
$$

where $C^L = \exp(i.2\pi L/N)$ with $L = jk$, for example $C^2 = \exp(i.\pi/2)$. For $N = 8$ we need one more transformation, which will result in a new sequence of values representing χ_j in the reverse binary order.

$$
\begin{array}{l}
[(\alpha_0 + \alpha_4) + (\alpha_2 + \alpha_6)] + [(\alpha_1 + \alpha_5) + (\alpha_3 + \alpha_7)] = \chi_0 \\
[(\alpha_0 + \alpha_4) + (\alpha_2 + \alpha_6)] - [(\alpha_1 + \alpha_5) + (\alpha_3 + \alpha_7)] = \chi_4 \\
[(\alpha_0 + \alpha_4) - (\alpha_2 + \alpha_6)] + C^2 [(\alpha_1 + \alpha_5) - (\alpha_3 + \alpha_7)] = \chi_2 \\
[(\alpha_0 + \alpha_4) - (\alpha_2 + \alpha_6)] - C^2 [(\alpha_1 + \alpha_5) - (\alpha_3 + \alpha_7)] = \chi_6 \\
[(\alpha_0 - \alpha_4) + C^2(\alpha_2 - \alpha_6)] - C [(\alpha_1 - \alpha_5) + C^2(\alpha_3 - \alpha_7)] = \chi_1 \\
[(\alpha_0 - \alpha_4) + C^2(\alpha_2 - \alpha_6)] - C [(\alpha_1 - \alpha_5) + C^2(\alpha_3 - \alpha_7)] = \chi_5 \\
[(\alpha_0 - \alpha_4) - C^2(\alpha_2 - \alpha_6)] + C^3 [(\alpha_1 - \alpha_5) - C^2(\alpha_3 - \alpha_7)] = \chi_3 \\
[(\alpha_0 - \alpha_4) - C^2(\alpha_2 - \alpha_6)] - C^3 [(\alpha_1 - \alpha_5) - C^2(\alpha_3 - \alpha_7)] = \chi_7
\end{array}
$$

Results are obtained in reverse binary order, and the proper sequence is obtained after reordering positions according to the indices 000 001 010 011 100 101 110 111 is obtained from 000 100 010 110 001 101 011 111.

A computer programme based on the FFT by Singleton and originated by Cooley and Tukey(1967).

```
'END'     'END' LOOP;  NEWLINE(1);
   'IF' SPECTR 'THEN'
   WRITETEXT('('CUMULATIVE%POWER%IS%')');
   PRINT(FF,0,6);        'END' PROC;

   'PROCEDURE'  REORD(A,B,N,M,KS,REEL);
   'VALUE' N,M,KS,REEL;
   'INTEGER' N,M,KS;
   'BOOLEAN' REEL;  'ARRAY' A,B;
   'BEGIN'
   'INTEGER' I,J,JJ,K,KK,KB,K2,KU,LIM,P;
   'REAL' TEMP;  'INTEGER'  'ARRAY'  C,LST[0:M];

   'COMMENT' ****************************************
  PROCEDURE REORDER PERMUTES DATA FROM NORMAL TO REVERSE
  BINARY ORDER OR VICE VERSA.  FOR A SINGLE VARIABLE
  TRANSFORM  N = KS = 2**M.  FOR SHORT BITS " REEL " IS
  TRUE, MAKING THEN  A[2*J+1] AND  B[2*J] ARE EXCHANGED
  FOR ALL J'S UP TO (N-2)/2.  THEN ADJACENT POINTS OF
  ENTRIES IN  A  AND  B  ARE PERMUTED TO A REVERSE
  BINARY ORDER.  AFTER REORDERING THE EVEN INDEXED
  ENTRIES ARE IN  A  AND THE ODD INDEXED ENTRIES ARE IN
  B, EACH IN REVERSE  BINARY ORDER ********************;
   C[M]:=KS;
   'FOR' K:=M 'STEP' -1 'UNTIL' 1 'DO'
   C[K - 1]:=C[K]'/'2;
   P:=J:=M - 1;  I:=KB:=0;
   'IF' REEL 'THEN'
   'BEGIN'
   KU:=N - 2;
   'FOR' K:=0 'STEP' 2 'UNTIL' KU 'DO'
   'BEGIN'
   TEMP:=A[K+1];  A[K+1]:=B[K];
   B[K]:=TEMP  'END'
   'END'  'ELSE' M:=M - 1;
   LIM:=(M+2)'/'2;
   'IF' P 'LE' 0 'THEN'  'GOTO' L4;
   L:  KU:=K2:=C[J] + KB;
   JJ:=C[M-J];  KK:=KB + JJ;
   L2:  K:=KK + JJ;
   L3:  TEMP:=A[KK];  A[KK]:=A[K2];
   A[K2]:=TEMP;  TEMP:=B[KK];
   B[KK]:=B[K2];  B[K2]:=TEMP;
   KK:=KK + 1;  K2:=K2 + 1;
   'IF'  KK 'LT' K 'THEN'  'GOTO' L3;
   KK:=KK + JJ;  K2:=K2 + JJ;
   'IF'  KK 'LT' KU 'THEN'  'GOTO' L2;
   'IF' J 'GT' LIM  'THEN'  'BEGIN'
   J:=J - 1;  I:=I + 1;
   LST[I]:=J;  'GOTO' L  'END';
   KB:=K2;
   'IF' I 'GT' 0 'THEN'  'BEGIN'
   J:=LST[I];  I:=I - 1;'GOTO' L 'END';
   'IF'  KB 'LT' N 'THEN'  'BEGIN'
   J:=P;     'GOTO' L 'END';
   L4:     'END'  PROC;
```

```
'PROCEDURE' INVRS(A,B,N,M,KS);
'VALUE' N,M,KS;
'INTEGER' N,M,KS;  'ARRAY' A,B;
'BEGIN'
'INTEGER' K0,K1,K2,K3,K,SPAN;
'REAL' A0,A1,A2,A3,B0,B1,B2,B3,
RAD,DC,DS,C1,C2,C3,S1,S2,S3;
RAD:=4.0*ARCTAN(1.0);  N:=N-1;
'COMMENT' ****************************************************
PROCEDURE INVRS COMPUTES THE FFT FOR ONE VARIABLE OF DIMENSION
N=2↑M, WHERE N IS THE NUMBER OF DATA POINTS.ARRAYS A[0:N-1] AND
B[0:N-1] ORIGINALLY CONTAIN THE REAL AND IMAGINARY COMPONENTS OF
THE DATA IN NORMAL ORDER.ON EXIT,THE REAL AND IMAGINARY  COMPON-
ENTS OF THE RESULTING FOURIER COFFICIENTS ARE IN THE  REVERSE
BINARY  ORDER(SEE SINGLTON,ALGORITHMS 338&345).*****************;
K0:=0;     SPAN:=KS;

'BEGIN'
LA:      K2:=K0 + SPAN;

A0:=A[K2];  B0:=B[K2];
A[K2]:=A[K0] - A0;  A[K0]:=A[K0] + A0;
B[K2]:=B[K0] - B0;  B[K0]:=B[K0] + B0;
K0:=K2 + SPAN;
'IF' K0 < N 'THEN' 'GOTO' LA;
K0:=K0 - N;
'IF' K0 'NE' SPAN 'THEN' 'GOTO' LA;
SPAN:=SPAN + SPAN;  RAD:=0.5*RAD 'END';

'FOR' M:=(M-2) 'WHILE' M 'GE' 0 'DO'
'BEGIN'
C1:=1.0;  S1:=0.0;  K0:=0;  RAD:=0.25*RAD;
DC:=2.0*SIN(RAD)↑2;  DS:=SIN(RAD + RAD);
K:=KS;
LB:
K1:=K0 + SPAN;
K2:=K1 + SPAN;
K3:=K2 + SPAN;
A0:=A[K0];  B0:=B[K0];
'IF' S1 = 0 'THEN'
'BEGIN'
A2:=A[K1];  B2:=B[K1];
A1:=A[K2];  B1:=B[K2];
A3:=A[K3];  B3:=B[K3]

'END' 'ELSE' 'BEGIN'

A2:=A[K1]*C2 - B[K1]*S2;
B2:=A[K1]*S2 + B[K1]*C2;
A1:=A[K2]*C1 - B[K2]*S1;
B1:=A[K2]*S1 + B[K2]*C1;
A3:=A[K3]*C3 - B[K3]*S3;
B3:=A[K3]*S3 + B[K3]*C3
'END';
A[K0]:=A0 + A2 + A1 + A3;
B[K0]:=B0 + B2 + B1 + B3;
A[K1]:=A0 - A2 - B1 + B3;
B[K1]:=B0 - B2 + A1 - A3;
A[K2]:=A0 + A2 - A1 - A3;
```

```
B[K2]:=B0 + B2 - B1 - B3;
A[K3]:=A0 - A2 + B1 - B3;
B[K3]:=B0 - B2 - A1 + A3;
K0:=K3 + SPAN;
'IF' K0 < N 'THEN' 'GOTO' LB;
K0:=K0 - N;
'IF' K0 'NE' K 'THEN' 'GOTO' LB;
'IF' K0 'NE' SPAN 'THEN'
          'BEGIN'
C2:=C1 - (DC*C1 + DS*S1);
S1:=(DS*C1 - DC*S1) + S1;
C1:=C2;

C2:=C1↑2 - S1↑2;  S2:=2.0*C1*S1;
C3:=C2*C1 - S2*S1;  S3:=C2*S1 + S2*C1;
K:=K + KS;  'GOTO' LB 'END';
SPAN:=4*SPAN
'END'      'END' PROC;


'PROCEDURE' TRAN(A,B,N,EV);
'VALUE' N,EV;  'INTEGER' N;
'BOOLEAN' EV;  'ARRAY' A,B;
'BEGIN'  'INTEGER' K,NK,NH;
'REAL' AA,AB,BA,BB,RE,IM,CK,SK,DC,DS,R;
NH:=N'/'2;  R:=4.0*ARCTAN(1.0)/N;
DS:=SIN(R);  R:=(2.0*SIN(0.5*R))↑2;
'COMMENT' ******************************************
 THIS PROCEDURE UNSCRAMBLES THE SINGLE VARIATE COMPLEX
TRANSFORM OF THE  N  EVEN NUMBERED AND  N  ODD NUMBERED
ELEMENTS OF A REAL SEQUENCE OF LENGTH 2*N,  WHERE THE
EVEN NUMBERED ELEMENTS WERE ORIGINALLY IN  A  AND THE
ODD NUMBERED ELEMENTS IN  B.  THEN IT COMBINES TWO
TRANSFORMS TO GIVE THE FOURIER COSINE (A) AND SINE  (B)
COEFFICIENTS ***************************************;

DC:=0.5*R;  CK:=1.0;  SK:=0.0;
'IF' EV 'THEN' 'BEGIN' CK:=1.0;  DC:=-DC  'END'
'ELSE' 'BEGIN'  A[N]:=A[0];  B[N]:=B[0]  'END';
'COMMENT' *******************************************
FOR THE INVERSE TRANFORM, I.E. FOR USE IN THE REAL 2
PROCEDURE THE PROCESS IS REVERSED AND THE SET OF
FOURIER COSINE AND SINE COEFFICIENTS ARE MADE READY FOR
EVALUATION OF THE CORRESPONDING FOURIER SERIES BY MEANS
OF THE INVERSE COMPLEX TRANSFORM.  FOR THE INVERSE
TRANSFORM  EV= 'TRUE'. *******************************;
'FOR' K:=0 'STEP' 1 'UNTIL' NH 'DO' 'BEGIN'
NK:=N-K;  AA:=A[K] + A[NK];  AB:=A[K] - A[NK];
BA:=B[K] + B[NK];  BB:=B[K] - B[NK];
RE:=CK*BA + SK*AB;  IM:=SK*BA - CK*AB;
B[NK]:=IM - BB;  B[K]:=IM + BB;
A[NK]:=AA - RE;  A[K]:=AA + RE;
R:=CK - (DC*CK + DS*SK);
SK:=(DS*CK - DC*SK) + SK;
CK:=1.5 - 0.5*(R↑2 + SK↑2);
SK:=CK*SK;  CK:=CK*R  'END'  'END' TRANPROC;
```

```
'PROCEDURE' REAL1(A,B,M,OUT);
'VALUE' M;    'INTEGER' M;  'ARRAY'  A, B;
'BOOLEAN' OUT;
'BEGIN'
'INTEGER' J, JJ, N;    N:=2↑M;

'COMMENT' ****************************************
PROCEDURE REAL1 COMPUTES THE FINITE FOURIER TRANSFORM
OF 2↑M+1 REAL DATA POINTS.  THE ARRAYS A[0:N] AND
B[0:N], WHERE N=2↑M, INITIALLY CONTAINS THE FIRST  N
DATA POINTS IN A[0:N-1] AND THE REMAINING  N  DATA
POINTS ARE STORED IN B[0:N-1].  ON COMPLITION OF THE
TRANSFORM THE ARRAYS  A  AND  B  CONTAIN THE FOURIER
COSINE AND SINE TERMS RESPECTIVELY.
SUBROUTINES REORD, INVRS, AND TRAN DESCRIBED IN
OTHER PROGRAMS ARE ALSO USED IN THIS PROCEDURE AND
MUST BE PROPERLY DECLARED. **************************;
REORD(A,B,N,M,N,'TRUE');
INVRS(A,B,N,M,1);
TRAN(A,B,N,'FALSE');
JJ:=0;
NEWLINE(1);  SPACE(1);  PRINT(A[0],0,6);
'IF' OUT 'THEN' 'BEGIN'
SELECT OUTPUT(4);  NEWLINE(1);  JJ:=1   'END';
NEWLINE(1);  SPACE(1);
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO'
'BEGIN' JJ:=JJ + 1;
A[J]:=A[J]/(2.0*N);     B[J]:=B[J]/(2.0*N);
'IF' JJ 'EQ' 4 'THEN'  'BEGIN'
NEWLINE(1);  SPACE(1);  JJ:=0 'END';
PRINT(A[J],0,6)  'END';
'END' PROC;
     'PROCEDURE' FTNB(A,B,N,M,KS);
     'VALUE' N,M,KS;  'INTEGER' N,M,KS;  'ARRAY' A,B;
     'BEGIN'
       'INTEGER' K0,K1,K2,K3,SPAN,J,JJ,K,KB,KN,MM,MK;
       'REAL' RAD,C1,C2,C3,S1,S2,S3,CK,SK,SQ,
       A0,A1,A2,A3,B0,B1,B2,B3;
       'INTEGER' 'ARRAY' C[0:M];
SQ:=0.70710678119;  SK:=0.38268343237;  CK:=0.92387953251;
'COMMENT' ****************************************************
THIS PROCEDURE (FTNB) COMPUTES THE FAST  FFT FOR ONE VARIABLE OF
DIMENSION N=2↑M.ARRAYS A[0:N-1] AND B[0:N-1] ORIGINALLY CONTAIN
THE REAL AND IMAGINARY COMPONENTS OF  DATA WITH THE INDICES
OF EACH VARIABLE IN  INVERSE BINARY ORDER.ON  COMPLETION OF THE
TRANSFORM THE REAL AND IMAGINARY COMPONENTS  OF THE RESULTING
FOURIER COEFFICIENTS ARE IN  A AND B IN NORMAL ORDER(SINGLTON,
ALGORITHM  338,GENTLMAN AND SANDE,1966). **********************;

       C[M]:=KS;  MM:=(M'/'2)*2;  KN:=0;
      'FOR' K:=(M-1) 'STEP' (-1) 'UNTIL' 0 'DO'
       C[K]:=C[K+1]'/'2;
       RAD:=6.2831853072/(C[0]*KS);  MK:=M-5;
    L:   KB:=KN;  KN:=KN + KS;
       'IF' MM 'NE'  M 'THEN'  'BEGIN'
       K2:=KN;   K0:=C[MM] + KB;
```

```
L2: K2:=K2 - 1;  K0:=K0 - 1;
   A0:=A[K2];  B0:=B[K2];
  A[K2]:=A[K0] - A0;  A[K0]:=A[K0] + A0;
   B[K2]:=B[K0] - B0;  B[K0]:=B[K0] + B0;
   'IF' K0 > KB 'THEN' 'GOTO' L2       'END';
   C1:=1.0;  S1:=0.0;
   JJ:=0;  K:=MM-2;  J:=3;
   'IF' K 'GE' 0 'THEN' 'GOTO' L4 'ELSE' 'GOTO' L6;
L3:  'IF' C[J] 'LE' JJ 'THEN'  'BEGIN'
   JJ:=JJ - C[J];  J:=J - 1;
   'IF' C[J] 'LE' JJ 'THEN'  'BEGIN'
   JJ:=JJ - C[J];  J:=J - 1;  K:=K + 2;
   'GOTO'  L3  'END'                         'END';
   JJ:=C[J] + JJ;  J:=3;
L4:  SPAN:=C[K];
   'IF' JJ 'NE' 0  'THEN'  'BEGIN'
   C2:=JJ*SPAN*RAD;  C1:=COS(C2);  S1:=SIN(C2);
L5:  C2:=C1↑2 - S1↑2;  S2:=2.0*C1*S1;
   C3:=C2*C1 - S2*S1;  S3:=C2*S1 + S2*C1
        'END';
   'FOR' K0:=KB+SPAN-1 'STEP' (-1) 'UNTIL' KK 'DO' 'BEGIN'
  K1:=K0 + SPAN;  K2:=K1 + SPAN;  K3:=K2 + SPAN;
   A0:=A[K0];  B0:=B[K0];
   'IF' S1 = 0 'THEN' 'BEGIN'
   A1:=A[K1];  B1:=B[K1];
    A2:=A[K2];  B2:=B[K2];
   A3:=A[K3];  B3:=B[K3]
          'END'  'ELSE'  'BEGIN'
   A1:=A[K1]*C1 - B[K1]*S1;
   B1:=A[K1]*S1 + B[K1]*C1;
   A2:=A[K2]*C2 - B[K2222]]*S2;
   B2:=A[K2]*S2 + B[K2]*C2;
   A3:=A[K3]*C3 - B[K3]*S3;
   B3:=A[K3]*S3 + B[K3]*C3
                 'END';
   A[K0]:=A0 + A2 + A1 + A3;
   B[K0]:=B0 + B2 + B1 + B3;
   A[K1]:=A0 + A2 - A1 - A3;
   B[K1]:=B0 + B2 - B1 - B3;
   A[K2]:=A0 - A2 - B1 + B3;
   B[K2]:=B0 - B2 + A1 - A3;
   A[K3]:=A0 - A2 + B1 - B3;
   B[K3]:=B0 - B2 - A1 + A3
   'END';
  'IF' K > 0 'THEN' 'BEGIN' K:=K - 2; 'GOTO' L4 'END';
   KB:=K3 + SPAN;
   'IF' KB < KN 'THEN' 'BEGIN'
   'IF' J = 0 'THEN' 'BEGIN' K:=2;  J:=MK; 'GOTO' L3 'END';
   J:=J - 1;  C2:=C1;
   'IF' J = 1 'THEN'  'BEGIN'
   C1:=C1*CK + S1*SK;  S1:=S1*CK - C2*SK 'END'
   'ELSE' 'BEGIN' C1:=(C1 - S1)*S0;  S1:=(C2 + S1)*S0 'END';
   'GOTO' L5 'END';
L6:  'IF' KN < N 'THEN' 'GOTO' L 'END' PROC;
```

```
 'PROCEDURE' REAL2(A,B,M);
'VALUE' M;  'INTEGER' M;  'ARRAY' A,B;
'BEGIN' 'INTEGER' J,N; N:=2↑M;
'COMMENT'*******************************************
 THIS PROCEDURE IS DESCRIBED IN PROGRAM GENERATE.*******;
 TRAN(A,B,N,'TRUE');
 'FOR' J:=N-1 'STEP' - 1 'UNTIL' 0 'DO' B[J]:=-B[J];
 FTNB(A,B,N,M,N);
 'FOR' J:=N-1 'STEP' -1 'UNTIL' 0 'DO' 'BEGIN'
 A[J]:=0.5*A[J];  B[J]:=-0.5*B[J];  'END';
 REORD(A,B,N,M,N,'TRUE');
 'END' TRANPROC;

 SELECT INPUT(IN)

 'FOR' I:=0 'STEP' 1 'UNTIL' MN 'DO' 'BEGIN'
 CORF[I]:=0.0;  SPECT[I]:=0.0;    'END';
 RATIO:=READ;
'IF' OUT 'THEN' 'BEGIN'
 SELECT OUTPUT(OT);
 WRITETEXT('('DOC%TAPE')');  NEWLINE(1);
   RUNOUT 'END';


 'FOR' J:=1 'STEP' 1 'UNTIL' NG 'DO' 'BEGIN'
'IF' FF 'NE' 0 'THEN' 'BEGIN'
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' A[I]:=READ;
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' B[I]:=READ;
'IF' FF 'EQ' 1 'THEN' 'BEGIN'
 STARX(A,N,1.0,'TRUE',VAR);
 STARX(B,N,1.0,'TRUE',VAR);
 REAL2(A,B,M);
 STARX(A,N,100,'FALSE',FF);
 STARX(B,N,100,'FALSE',FF)
 'END' 'ELSE' 'BEGIN'
 STARX(A,N,100,'FALSE',FF);
 STARX(B,N,100,'FALSE',FF);
 REAL1(A,B,N,'TRUE');
 STARX(A,N,1.0,'TRUE',VAR);
 STARX(B,N,1.0,'TRUE',VAR)
'END' 'END' 'ELSE' 'BEGIN'

 NORM1(X,N,SIGMA,AVER,'FALSE','TRUE');

 SIGMA:=SIGMA*SIGMA;
 MEAN:=MEAN + AVER;    VAR:=VAR + SIGMA;

 CORRFUN(N,MN,X,Y,'TRUE',HANNING);
 SELECT OUTPUT(0);
 NEWLINE(1);  WRITETEXT('('AVERAGE%VALUE')');
 SPACE(2);  PRINT(AVER,0,6);
 SPACE(4);  WRITETEXT('('STANDARD%DEVIATION')');
 SPACE(2);  PRINT(SIGMA,0,6);
```

PROGRAMS COHERENCE AND SPECTRPLOT

```
'PROCEDURE' CONVUL(Z,A,B,C,D,M,XX,YY,SCALE);
'VALUE' M,SCALE;    'INTEGER' M;
'REAL' SCALE;   'ARRAY' Z, A, B, C, D, XX, YY;
'BEGIN'
'INTEGER' J,KK,KS,N;
'REAL' AA,AB,BA,BB,IM;
N:=2↑M;
FTNB(A,B,N,M,N);
'FOR' KS:=0 'STEP' 1 'UNTIL' N-1 'DO' 'BEGIN'
C[N]:=0.0;     D[N]:=0.0;
C[KS]:=A[KS]/N;    D[KS]:=B[KS]/N;   'END';
REORD(C,D,N,M,N,'FALSE');
KK:=N'/'2;

'FOR'  J:=1 'STEP' 1 'UNTIL' KK 'DO'  'BEGIN'
IM:=C[N-J] -C[J];
C[J]:=C[J] + C[N-J];
C[N-J]:=IM;
IM:=D[J] - D[N-J];
D[J]:=D[J] + D[N-J];
D[N-J]:=C[N-J];
C[N-J]:=IM;
XX[J]:=XX[J] + C[J]*C[J] + C[N-J]*C[N-J];
YY[J]:=YY[J] + D[J]*D[J] + D[N-J]*D[N-J];
                                        'END';
XX[0]:=XX[0] + 4.0*C[0]*C[N];
YY[0]:=YY[0] + 4.0*D[0]*D[N];

J:=1;
C[0]:=4*(A[0]*B[0]);
L:  KK:=J;  KS:=J:=J + J;
L2: KS:=KS - 1;
AA:=A[KK] + A[KS];  AB:=A[KK] - A[KS];
BA:=B[KK] + B[KS];  BB:=B[KK] - B[KS];
IM:=BA*BB + AA*AB;
AA:=AA*BA - AB*BB;
C[KK]:=AA - IM;  C[KS]:=AA + IM;
KK:=KK + 1;
'IF' KK 'LT' KS 'THEN' 'GOTO' L2;
'IF' J 'LT' N   'THEN' 'GOTO' L ;
KK:=N'/'2;  KS:=KK - 1;  SCALE:=SCALE/(8*N);
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'
D[J]:=C[J + KK];
INVRS(C,D,KK,M-1,1);
TRAN(C,D,KK,'FALSE');
C[0]:=SCALE*C[0];  C[KK]:=SCALE*C[KK];
Z[0]:=Z[0] + C[0];
Z[KK]:=Z[KK] + C[KK];
'FOR' J:=1 'STEP' 1 'UNTIL' KS 'DO'
'BEGIN'
C[N-J]:=SCALE*(C[J] - D[J]);
C[J]:=SCALE*(C[J] + D[J]);
Z[N-J]:=Z[N-J] + C[N-J];
Z[J]:=Z[J] + C[J];
'END'  'END' PROC;
```

```
'COMMENT' ************************************************
COHERENCE CONSISTS  OF THE PROCEDURE CONVOL,WHICH IS
ALSO A MODIFICATION AND EXPANSION OF SINGLETON'S
CONVOLUTION(1968). IT COMTAINS THE SAME PREVIOUSLY
DESCRIBED PROCEDURES FTNB,INVRS,TRAN WHICH SHOULD BE
DECLARED AND BUILT INTO THIS PROGRAM. CONVOL COMPUTES
THE CONVOLUTION OF TWO REAL VECTORS OF DIMENSION N=2↑M.
THE TWO VECTORS A AND B ARE FIRST TRANSFORMED WITH A
SINGLE COMPLEX FOURIER TRANSFORM OF DIMENSION N. THE
COMPLEX PRODUCY IS THEN FORMED,LEAVING THE RESULT IN
REVERSE BINARY ORDER. DATA ARE TRANSFERED FROM A AND B
INTO C AND D,REORDERED AND UNSCRUMBLED. AUTO POWER SPECTRA
ARE OBTAINED AS ARRAY C[0:N'/'2-1] AND C[N'/'2:N-1].
REAL CROSS SPECTRUM IS IN THE FIRST PART OF ARRAY A AND
IMAGINARY PART IN THE SECOND PART. COMPLEX COHEEENCE
FUNCTION IS STORED IN THE ARRAY XX[0:N'/'2] AND THE CROSS
CORRELATION FUNCTION IN  Z[0:N]************************;

 'PROCEDURE' SAMPLE(X,Y,MN,AUTO);
 'VALUE' MN;  'INTEGER' MN;
 'ARRAY' X,Y;  'BOOLEAN' AUTO;
 'BEGIN'  'INTEGER' I;
 'IF' AUTO 'THEN' 'BEGIN'
 'FOR' I:=0 'STEP' 1 'UNTIL' MN-2 'DO' 'BEGIN'
 X[I]:=Y[I]:=READ;  'END';
 WRITETEXT('('AUTOCORRELATION')');  I:=0 'END' 'ELSE' 'BEGIN'
 'FOR' I:=0 'STEP' 1 'UNTIL' MN-2 'DO' 'BEGIN'
 X[I]:=READ;    Y[I]:=READ;    'END';
 X[MN]:=Y[MN]:=0.0;
 WRITETEXT('('CROSSCORRELATION')');
 Y[MN-1]:=X[MN-1]:=(Y[0]+X[0])/2.0 'END'; 'END' PROC;

 'PROCEDURE' NORM2(X,Y,MN);
 'VALUE' MN;  'INTEGER' MN;  'ARRAY' X,Y;
 'BEGIN' 'INTEGER' I;  'REAL' SU1,SU2,SI1,SI2;
 SU1:=SU2:=SI1:=SI2:=0.0;
 'FOR' I:=0 'STEP' 1 'UNTIL' MN-1 'DO' 'BEGIN'
 SU1:=SU1 + X[I];  SU2:=SU2 + Y[I]; 'END';
 SU1:=SU1/MN;  SU2:=SU2/MN;
 'FOR' I:=0 'STEP' 1 'UNTIL' MN-1 'DO' 'BEGIN'
 X[I]:=X[I] - SU1;  Y[I]:=Y[I] - SU2;
 SI1:=SI1 + X[I]*X[I];  SI2:=SI2 + Y[I]*Y[I]; 'END';
 SI1:=SQRT(SI1/MN);  SI2:=SQRT(SI2/MN);
 'FOR' I:=0 'STEP' 1 'UNTIL' MN-1 'DO' 'BEGIN'
 X[I]:=X[I]/SI1;  Y[I]:=Y[I]/SI2;.'END';
 PRINT(SU1,0,6);  PRINT(SU2,0,6);
 PRINT(SI1,0,6);  PRINT(SI2,0,6);  'END' PROC;
 'COMMENT' ********************************************
N DATA FROM EACH FUNCTION ARE SAMPLED AND NORMALIZED.
SEVERAL TRANSFERS AND DATA REORDERINGS ARE DONE IN ORDER
TO PREPARE FOR THE FINAL TRANSFORM.THEN THE DATA ARE STO-
RED IN AVAILABLE ARRAYS.CROSS-CORRELATION FUNCTION IS
STORED IN ARRAY Z, THE LINEAR AUTOSPECTRA IN C(THE FIRST
IN THE FIRST HALF AND SECOND IN THE SECOND HALF).COMPLEX
COHERENCE FUNCTION IN ARRAY B(THE REAL AND IMAGINARY PARTS
IN THE FIRST AND SECOND PART RESP.).THE MEAN SPECTRA PRO-
DUCT,WHICH HAS ONLY TEMPORARELY COMPUTATIONAL USE IS STO-
RED IN ARRAY D.AT THE AND OF THE PROCEDURE,AVERAGED COHER-
ENCE FUNCTION IS STORED IN XX[0:N'/'2].ALL RESULTS ARE DI-
SPLAYED BY THE LINE PRINTER USING ALREADY DESCRIBED PRO-
CEDURE STARX(P.277),WHICH SHOULD BE DECLARED AND BUILT IN
THIS PROGRAM ******************************************;
```

```
'INTEGER' J,JJ,KS,M,N,NG;
'REAL' SCALE;     'BOOLEAN' AUTO;
AUTO:='FALSE';
NG:=READ;   M:=READ;   N:=READ;
'IF' N 'EQ' 1 'THEN' AUTO:='TRUE';
N:=2↑M;      KS:=N'/'2;
'BEGIN'
'REAL' 'ARRAY' A[0:N], B[0:N], C[0:N],
D[0:N], Z[0:N], XX[0:KS], YY[0:KS], ZZ[0:N];

'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO' 'BEGIN'
Z[J]:=0.0;  Z[N-J]:=0.0;
XX[J]:=0.0; YY[J]:=0.0;  'END';
SELECT INPUT(3);
'FOR'  J:=0 'STEP' 1 'UNTIL'  N-2 'DO'  ZZ[J]:=READ;
SCALE:=1.0/N;

'FOR' JJ:=1 'STEP' 1 'UNTIL' NG 'DO' 'BEGIN'

NEWLINE(1);   SAMPLE(A,ZZ,N,AUTO);

'FOR'  J:=0 'STEP' 1 'UNTIL'  N-1 'DO'  B[J]:=ZZ[J];

NORM2(A,B,N);   CONVOL(Z,A,B,C,D,M,XX,YY,SCALE);

'END' NGLOOP;

'FOR' KS:=0 'STEP' 1 'UNTIL' N 'DO'  'BEGIN'
Z[KS]:=Z[KS]/NG;        'END';
KS:=N'/'2;  NG:=2*NG;

NEWLINE(1);        JJ:=N'/'2;
'FOR' J:=0 'STEP' 1 'UNTIL' JJ 'DO' 'BEGIN'
XX[J]:=XX[J]/NG;  YY[J]:=YY[J]/NG;
'END';
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'  'BEGIN'
C[J]:=XX[J];  C[KS+J]:=YY[J];
XX[J]:=Z[J];  YY[J]:=Z[KS+J];  'END';
XX[KS]:=YY[KS]:=0.0;        M:=M-1;
REAL1(XX,YY,M);
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'  'BEGIN'
A[J]:=XX[J];  A[JJ+J]:=YY[J];
'END';
XX[0]:=2.0*C[0];  YY[0]:=2.0*C[KS];
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'  'BEGIN'
XX[J]:=C[J]*C[KS+J];
D[J]:=D[KS+J]:=SQRT(XX[J]);
'IF' D[J] 'LT' 0.000000001 'THEN'  B[J]:=0.0
'ELSE' 'BEGIN' B[J]:=A[J]/D[J];
B[KS+J]:=A[KS+J]/D[KS+J] 'END';
YY[J]:=A[J]*A[J] + A[JJ+J]*A[JJ+J];
'IF'  XX[J]  'LT'  1.0& -16 'THEN'
XX[J]:=0.0 'ELSE' XX[J]:=YY[J]/XX[J];
'END';
```

```
N:=N-1;
STARX(A,N,80,'TRUE');
WRITETEXT('('CROSS%SPECTRUM')');
STARX(B,N,80,'TRUE');
WRITETEXT('('COMPLEX%COHERENCE%MEASURE')');
STARX(C,N,80,'TRUE');
WRITETEXT('('AUTO%SPECTRA')');
STARX(XX,128,80,'TRUE');
WRITETEXT('('COHERENCE%FUNCTION')');
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'  'BEGIN'
XX[J]:=SQRT(XX[J]);
                                    'END';

STARX(XX,128,80,'FALSE');
WRITETEXT('('COHERENCE%FUNCTION')');
STARX(Z,N,80,'FALSE');
WRITETEXT('('CROSS%CORRELATION%FUNCTION')');
STARX(D,N,80,'TRUE');
WRITETEXT('('MEAN%SPECTRA%PRODUCT')');

NEWLINE(1);  SPACE(4);
WRITETEXT('('CROSS%CORRELATION')');
SPACE(2);
WRITETEXT('('LINEAR%AUTO%SPECTRA')');
SPACE(2);
WRITETEXT('('CROSS%POWER%SPECTRUM')');
SPACE(1);
WRITETEXT('('COMPLEX%COHERENCE%MEASURE')');
SPACE(2);
WRITETEXT('('LINEAR%SPECTRA%PRODUCT')');
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
NEWLINE(1);  SPACE(4);  PRINT(Z[J],0,6);
SPACE(8);  PRINT(C[J],0,6);
SPACE(8);  PRINT(A[J],0,6);
SPACE(8);  PRINT(B[J],0,6);
SPACE(8);  PRINT(D[J],0,6);
'END';       NEWLINE(2);
'FOR' J:=0 'STEP' 1 'UNTIL' KS 'DO'
PRINT(XX[J],0,6);

SELECT OUTPUT(4);  WRITETEXT('('DOC%TAPE')');
NEWLINE(1);  RUNOUT;

'FOR' J:=0 'STEP' 1 'UNTIL' JJ 'DO' 'BEGIN'
NEWLINE(1);  PRINT(C[J],0,4);  PRINT(C[JJ+J],0,4);
PRINT(A[J],0,4);  PRINT(A[JJ+J],0,4);  PRINT(B[J],0,4);
PRINT(B[JJ+J],0,4);   PRINT(XX[J],0,4);  'END';
WRITETEXT('('****')'); NEWLINE(1); RUNOUT;
WWWRITETEXT('('DOC%TAPE')');  NEWLINE(1); RUNOUT;
NEWLINE(1);
'FOR'  J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
PRINT(Z[J],0,6); NEWLINE(1); 'END';
PRINT(B[0],0,6); PRINT(B[N],0,6);
'END';    'END' PROGRM;
```

CROSS CORRELATION	LINEAR AUTO SPECTRA	CROSS POWER SPECTRUM	COMPLEX COHERENCE MEASURE	LINEAR SPECTRA PRODUCT
4,486887& -2	0.000000& 0	6.240081& -3	0.000000& 0	0,000000& 0
4,483787& -2	3.879668& -4	3.295251& -3	3.479103& -1	9,471555& -3
4,354633& -2	5.246086& -4	6.609820& -3	7.433577& -1	8,891843& -3
4,165951& -2	5.301116& -4	6.574127& -3	8.527944& -1	7,708924& -3
3,957479& -2	4.132828& -4	4.197180& -3	7.439086& -1	5,642064& -3
3,778113& -2	4.862966& -4	5.262091& -3	8.940357& -1	5,885773& -3
3,614324& -2	2.984331& -4	3.612191& -3	9.200038& -1	3,926278& -3
3,570990& -2	1.338761& -4	1.508643& -3	8.908562& -1	1,693475& -3
3,616199& -2	9.491996& -5	9.054240& -4	9.340893& -1	1,065663& -3
3,581813& -2	4.507988& -5	4.664217& -4	8.930754& -1	5,222646& -4
3,495735& -2	8.074908& -5	8.659554& -4	9.189690& -1	9,423120& -4
3,332312& -2	3.415863& -5	4.011319& -4	9.433461& -1	4,252224& -4
3,115889& -2	4.969614& -5	4.827037& -4	9.308372& -1	5,185694& -4
2,869417& -2	2.827052& -5	3.286714& -4	9.507249& -1	3,457061& -4
2,624740& -2	4.805858& -5	4.904686& -4	9.314175& -1	5,265830& -4
2,413006& -2	2.535823& -5	2.763404& -4	9.371329& -1	2,948786& -4
2,234057& -2	3.398695& -5	3.551637& -4	9.338918& -1	3,803050& -4
2,112105& -2	1.576453& -5	1.933327& -4	9.422812& -1	2,051751& -4
2,014743& -2	4.029659& -5	4.137863& -4	9.641074& -1	4,291910& -4
1,905039& -2	1.081695& -5	2.215135& -4	9.189845& -1	2,410416& -4
1,773318& -2	3.003064& -5	3.025865& -4	9.693549& -1	3,121524& -4
1,603231& -2	2.005214& -5	2.425296& -4	9.474845& -1	2,559721& -4
1,412869& -2	2.491503& -5	2.421011& -4	9.607458& -1	2,519929& -4
1,218687& -2	1.735006& -5	1.930423& -4	9.277076& -1	2,080853& -4
1,041510& -2	3.298044& -5	3.243696& -4	9.293370& -1	3,490333& -4
9,192600& -3	7.121987& -5	6.451853& -4	9.287921& -1	6,946498& -4
8,318240& -3	7.152237& -5	6.236500& -4	9.633907& -1	6,473489& -4
7,831284& -3	2.895921& -5	3.063855& -4	9.671667& -1	3,167867& -4
7,575199& -3	2.425868& -5	2.267701& -4	9.332648& -1	2,429857& -4
7,328169& -3	8.459375& -6	8.464305& -5	9.114171& -1	9,286972& -5
7,057974& -3	2.445591& -5	2.100833& -4	9.306401& -1	2,257407& -4
6,573439& -3	6.403834& -6	6.931801& -5	9.197210& -1	7,536852& -5
5,881100& -3	1.338268& -5	9.280455& -5	9.134371& -1	1,015993& -4
5,269275& -3	4.120057& -6	3.549318& -5	8.324821& -1	4,263537& -5
4,727859& -3	1.246756& -5	8.317663& -5	9.103594& -1	9,136680& -5
4,526403& -3	4.523707& -6	3.466401& -5	7.828871& -1	4,427716& -5
4,644227& -3	8.492011& -6	5.627607& -5	9.053890& -1	6,215679& -5
4,996769& -3	2.885084& -6	2.275678& -5	7.462726& -1	3,049392& -5
5,579571& -3	9.079075& -6	5.713793& -5	8.920239& -1	6,405426& -5
6,157611& -3	2.716409& -6	2.113976& -5	7.703864& -1	2,744047& -5
6,614755& -3	8.860574& -6	5.116847& -5	8.608513& -1	5,943939& -5
6,775243& -3	1.771932& -6	1.149669& -5	7.258607& -1	1,583870& -5
6,682323& -3	6.181885& -6	3.350339& -5	8.237839& -1	4,067012& -5
6,492575& -3	2.483033& -6	1.781628& -5	8.070229& -1	2,207655& -5
6,225963& -3	5.497245& -6	2.421165& -5	7.209203& -1	3,358436& -5
6,147474& -3	1.841594& -6	1.596084& -5	7.926531& -1	2,013598& -5
6,222655& -3	7.189346& -6	2.938711& -5	7.385372& -1	3,979097& -5
6,485305& -3	1.812954& -6	1.537281& -5	7.413629& -1	2,073587& -5
6,889850& -3	8.231641& -6	3.660578& -5	8.146326& -1	4,493532& -5
7,273275& -3	1.421685& -6	1.486350& -5	7.258239& -1	2,047811& -5
7,550606& -3	1.004520& -5	5.340886& -5	8.943821& -1	5,971593& -5
7,574547& -3	1.575601& -6	1.307084& -5	6.393494& -1	2,044397& -5
7,300253& -3	7.845109& -6	4.971221& -5	9.083265& -1	5,472945& -5
6,867843& -3	2.026679& -6	1.222078& -5	5.616958& -1	2,175693& -5
6,329254& -3	8.594016& -6	5.039501& -5	9.089824& -1	5,544113& -5
5,868526& -3	1.841007& -6	1.157913& -5	6.240501& -1	1,855481& -5
5,626171& -3	9.669859& -6	5.725056& -5	9.354695& -1	6,119981& -5
5,463559& -3	1.959951& -6	1.302415& -5	6.751755& -1	1,929002& -5
5,441824& -3	9.089288& -6	5.237816& -5	9.283994& -1	5,641771& -5
5,383297& -3	2.159558& -6	1.380166& -5	7.488860& -1	1,842958& -5
5,202820& -3	8.680652& -6	4.954866& -5	9.172833& -1	5,401675& -5
4,876219& -3	1.774199& -6	9.955965& -6	6.370894& -1	1,562727& -5
4,400710& -3	8.993186& -6	4.116972& -5	8.838956& -1	4,657758& -5
3,869975& -3	1.415022& -6	7.769013& -6	6.540381& -1	1,187853& -5
3,503090& -3	7.588040& -6	3.384632& -5	8.655217& -1	3,910511& -5
3,227383& -3	1.823210& -6	1.008836& -5	6.934229& -1	1,454864& -5
3,204170& -3	7.014404& -6	3.062714& -5	8.807272& -1	3,477484& -5
3,311132& -3	1.641848& -6	7.000427& -6	5.207803& -1	1,344219& -5
3,532199& -3	7.078656& -6	2.939898& -5	8.720509& -1	3,371245& -5
3,784835& -3	1.146786& -6	5.039368& -6	5.095054& -1	9,890706& -6
4,013491& -3	6.138506& -6	2.469301& -5	7.984150& -1	3,092754& -5
4,195577& -3	1.399023& -6	5.880554& -6	5.456096& -1	1,077795& -5
4,340468& -3	5.039558& -6	1.788058& -5	7.262879& -1	2,461914& -5
4,399076& -3	1.368424& -6	7.353042& -6	6.408864& -1	1,147324& -5
4,436011& -3	4.872283& -6	1.226765& -5	5.565261& -1	2,204325& -5
4,442319& -3	1.299675& -6	8.712717& -6	6.816246& -1	1,278228& -5
4,626678& -3	4.057256& -6	6.974579& -6	3.763371& -1	1,853280& -5
4,713675& -3	1.578855& -6	1.254936& -5	7.855231& -1	1,597580& -5
4,912331& -3	4.725487& -6	4.737939& -6	2.437077& -1	1,944107& -5
5,099381& -3	1.068372& -6	9.056056& -6	6.716263& -1	1,348377& -5
5,336212& -3	4.172383& -6	6.672339& -6	3.308211& -1	2,016903& -5
5,593480& -3	1.191147& -6	9.450023& -6	6.158231& -1	1,534535& -5
5,880601& -3	4.835337& -6	7.021570& -6	3.297958& -1	2,129066& -5
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6,234839& -3	4.534611& -6	1.345515& -5	5.562456& -1	2,418922& -5
6,314154& -3	6.574301& -7	6.048952& -6	6.137534& -1	9,855672& -6
6,400777& -3	4.779534& -6	1.372477& -5	5.744421& -1	2,389235& -5
6,430419& -3	7.290362& -7	3.943545& -6	3.857672& -1	1,022260& -5
6,460764& -3	4.560173& -6	1.682107& -5	6.815747& -1	2,467972& -5
6,505247& -3	5.178625& -7	2.771463& -6	3.621323& -1	7,653178& -6
6,640206& -3	6.445303& -6	1.643186& -5	6.207408& -1	2,647136& -5
6,699938& -3	8.933725& -7	6.507166& -6	6.736739& -1	9,792819& -6
6,785743& -3	6.694271& -6	1.823750& -5	7.175273& -1	2,541716& -5
6,781037& -3	8.553006& -7	5.418618& -6	6.014325& -1	9,009520& -6
6,787060& -3	6.243283& -6	1.655237& -5	7.233043& -1	2,288438& -5
6,737624& -3	6.366535& -7	5.428127& -6	6.992008& -1	7,763331& -6
6,672962& -3	6.725515& -6	1.576631& -5	6.943391& -1	2,270693& -5
6,556738& -3	6.834449& -7	3.924242& -6	5.559107& -1	7,059122& -6
6,494316& -3	6.894105& -6	1.489305& -5	7.219353& -1	2,062934& -5
6,489640& -3	7.717401& -7	3.184954& -6	4.416513& -1	7,211467& -6
6,452625& -3	8.241344& -6	1.976671& -5	7.221517& -1	2,737196& -5
6,427966& -3	3.801961& -7	3.239248& -6	6.566806& -1	4,932761& -6
6,379752& -3	1.327147& -5	1.086216& -5	4.268950& -1	2,544458& -5
6,113614& -3	2.748552& -6	7.408160& -6	4.707708& -1	1,592741& -5
5,722436& -3	4.377478& -6	1.478274& -5	7.439231& -1	1,987133& -5
5,306060& -3	1.793165& -6	7.301442& -6	7.128079& -1	1,024321& -5
4,892213& -3	6.278362& -6	1.030410& -5	5.636413& -1	1,828131& -5
4,604064& -3	1.267971& -6	6.802037& -6	6.808404& -1	9,990648& -6
4,357257& -3	5.198533& -6	1.203247& -5	6.573707& -1	1,830393& -5
4,255233& -3	1.131032& -6	7.414500& -6	7.247021& -1	1,023116& -5
4,125396& -3	4.391950& -6	4.351885& -6	2.738398& -1	1,589208& -5
4,002884& -3	1.278784& -6	8.734059& -6	7.070523& -1	1,235278& -5
3,771191& -3	4.292585& -6	3.406454& -6	2.087396& -1	1,675031& -5
3,417180& -3	1.063015& -6	7.985833& -6	6.876870& -1	1,161260& -5
2,933038& -3	4.688619& -6	5.166674& -6	2.912843& -1	1,773756& -5
2,404349& -3	1.226029& -6	8.623371& -6	6.837046& -1	1,261271& -5
1,926906& -3	5.402492& -6	4.727324& -6	2.624032& -1	1,801550& -5
1,554662& -3	8.891560& -7	5.445883& -6	5.557150& -1	9,799776& -6
1,357195& -3	5.623260& -6	7.123522& -6	3.555533& -1	2,003503& -5
1,305633& -3	5.262793& -7	3.553446& -6	4.890264& -1	7,266368& -6
1,316910& -3	5.424059& -6	1.000763& -5	4.837779& -1	2,068642& -5
1,200376& -3	6.303907& -7	4.559843& -6	5.995759& -1	7,605113& -6
1,289418& -3	6.171436& -6	9.382866& -6	4.705164& -1	1,994163& -5
1,530592& -3	8.468495& -7	1.947712& -6	2.456671& -1	7,928258& -6
1,700194& -3	6.132820& -6	1.128908& -5	5.619943& -1	2,008753& -5
1,852037& -3	1.022125& -6	6.475755& -6	6.925104& -1	9,639935& -6
2,060782& -3	5.215089& -6	6.691635& -6	4.163244& -1	1,607313& -5
3,082102& -3	1.332711& -6	-4.175877&-11	-3.670335& -6	1,137737& -5
3,778914& -3	0.000000& 0	1.778386& -3	0.000000& 0	0,000000& 0

2,942392& -3	2,312320& -1	8,354993& -3	8,821142& -1	9,471555& -3
2,279959& -3	1,507121& -1	5,568037& -3	6,261960& -1	8,891843& -3
1,827183& -3	1,121038& -1	3,900815& -3	5,060129& -1	7,708924& -3
1,651321& -3	7,702447& -2	3,483741& -3	6,174585& -1	5,642064& -3
1,526816& -3	7,123703& -2	2,490630& -3	4,231610& -1	5,885773& -3
1,020804& -3	5,165533& -2	1,410058& -3	3,591336& -1	3,926278& -3
8,185884& -4	2,142174& -2	6,896580& -4	4,072442& -1	1,693475& -3
1,283493& -3	1,196415& -2	3,113432& -4	2,921593& -1	1,065663& -3
9,867275& -4	6,050600& -3	1,620090& -4	3,102048& -1	5,222646& -4
6,979589& -4	1,099643& -2	3,018099& -4	3,202866& -1	9,423120& -4
5,389673& -4	5,293395& -3	7,919695& -5	1,862483& -1	4,252224& -4
4,593841& -4	5,411169& -3	1,499522& -4	2,891652& -1	5,185694& -4
5,798250& -4	4,227440& -3	7,927494& -5	2,293131& -1	3,457061& -4
8,688760& -4	5,769826& -3	1,054368& -4	2,002283& -1	5,265830& -4
1,231192& -3	3,429000& -3	9,205819& -5	3,121902& -1	2,948786& -4
1,225877& -3	4,255513& -3	6,524409& -5	1,715573& -1	3,803050& -4
1,565336& -3	2,670351& -3	5,091628& -5	2,481601& -1	2,051751& -4
1,780946& -3	4,571230& -3	7,258390& -5	1,691179& -1	4,291910& -4
1,821600& -3	2,931887& -3	6,372084& -5	2,643562& -1	2,410416& -4
1,751298& -3	3,244658& -3	3,866070& -5	1,232113& -1	3,121524& -4
1,773348& -3	3,267567& -3	6,542608& -5	2,556020& -1	2,559721& -4
1,789166& -3	2,548479& -3	4,275475& -5	1,696665& -1	2,519929& -4
2,006394& -3	2,495639& -3	4,586130& -5	2,203966& -1	2,080853& -4
2,381678& -3	3,693833& -3	8,303499& -5	2,378999& -1	3,490333& -4
2,822499& -3	4,775334& -3	2,021346& -4	2,909878& -1	6,946498& -4
3,165521& -3	5,859155& -3	8,526238& -5	1,317101& -1	6,473489& -4
3,294575& -3	3,465350& -3	3,312940& -5	1,045795& -1	3,167867& -4
3,249420& -3	2,433853& -3	4,580427& -5	1,885060& -1	2,429857& -4
3,058869& -3	1,019553& -3	1,430347& -5	1,540165& -1	9,286972& -5
2,800043& -3	2,083703& -3	5,669973& -5	2,511720& -1	2,257407& -4
2,561301& -3	8,870333& -4	9,637048& -6	1,278657& -1	7,536852& -5
2,220243& -3	7,713263& -4	2,284271& -5	2,248314& -1	1,015993& -4
2,033828& -3	4,412014& -4	9,642429& -6	2,261603& -1	4,263537& -5
1,808117& -3	6,695690& -4	1,243712& -5	1,361229& -1	9,136680& -5
1,624548& -3	4,333761& -4	7,756529& -6	1,751813& -1	4,427716& -5
1,397926& -3	4,549530& -4	8,701483& -6	1,399925& -1	6,215679& -5
1,067479& -3	3,223059& -4	6,693972& -6	2,195182& -1	3,049392& -5
6,447225& -4	4,519126& -4	8,296648& -6	1,295253& -1	6,405426& -5
1,650993& -4	2,771966& -4	2,326714& -6	8,479134& -2	2,744047& -5
-3,735916& -4	3,987373& -4	9,258845& -6	1,557695& -1	5,943939& -5
-8,057652& -4	1,415769& -4	-7,002608& -7	-4,421200& -2	1,583870& -5
-1,283039& -3	2,675655& -4	3,902809& -6	9,596255& -2	4,067012& -5
-1,604732& -3	1,962818& -4	-1,217761& -6	-5,516086& -2	2,207655& -5
-1,895903& -3	2,051772& -4	3,575113& -7	1,064517& -2	3,358436& -5
-2,152421& -3	2,201666& -4	4,201296& -7	2,086463& -2	2,013598& -5
-2,434268& -3	2,202316& -4	-3,771288& -6	-9,477748& -2	3,979097& -5
-2,828209& -3	2,371680& -4	2,999030& -6	1,446300& -1	2,073587& -5
-3,265639& -3	2,452953& -4	-1,041786& -5	-2,318412& -1	4,493532& -5
-3,739654& -3	2,049691& -4	2,621079& -7	1,279942& -2	2,047811& -5
-4,293983& -3	3,549945& -4	-1,151370& -5	-1,928079& -1	5,971593& -5
-4,819186& -3	2,652676& -4	7,894914& -7	3,861733& -2	2,044397& -5
-5,350075& -3	3,818064& -4	-1,000352& -5	-1,827813& -1	5,472945& -5
-5,794779& -3	2,335664& -4	-5,799018& -7	-2,665336& -2	2,175693& -5
-6,083352& -3	3,576580& -4	-6,202580& -6	-1,118769& -1	5,544113& -5
-6,322463& -3	1,870070& -4	-9,254263& -7	-4,987527& -2	1,855481& -5
-6,459935& -3	3,873290& -4	-5,360819& -6	-8,759536& -2	6,119981& -5
-6,642825& -3	1,898541& -4	-2,218710& -6	-1,150185& -1	1,929002& -5
-6,888962& -3	3,501878& -4	-3,421939& -7	-6,065363& -3	5,641771& -5
-7,296195& -3	1,572773& -4	-2,014712& -6	-1,093195& -1	1,842958& -5
-7,854814& -3	3,361279& -4	4,380782& -6	8,110043& -2	5,401675& -5
-8,404678& -3	1,376460& -4	-1,910113& -6	-1,222295& -1	1,562727& -5
-9,009080& -3	2,412349& -4	3,262684& -7	7,004838& -3	4,657758& -5
-9,435807& -3	9,971546& -5	-1,954077& -7	-1,645049& -2	1,187853& -5
-9,509089& -3	2,015289& -4	2,384729& -6	6,098255& -2	3,910511& -5
-9,392414& -3	1,160936& -4	3,355395& -6	2,306329& -1	1,454864& -5
-8,996989& -3	1,724009& -4	1,403107& -6	4,034834& -2	3,477484& -5
-8,569633& -3	1,100543& -4	1,759036& -6	1,308593& -1	1,344219& -5
-8,231514& -3	1,605573& -4	5,798319& -6	1,719934& -1	3,371245& -5
-8,085103& -3	8,530451& -5	3,084628& -6	3,118714& -1	9,890706& -6
-8,244010& -3	1,558217& -4	4,740792& -6	1,532871& -1	3,092754& -5
-8,407639& -3	8,303238& -5	3,679561& -6	3,413970& -1	1,077795& -5
-8,596347& -3	1,202689& -4	9,110426& -6	3,700546& -1	2,461914& -5
-8,523612& -3	9,619470& -5	3,233598& -6	2,818383& -1	1,147324& -5
-8,205373& -3	9,972840& -5	7,011650& -6	3,180860& -1	2,204325& -5
-7,715096& -3	1,257135& -4	2,234423& -6	1,748063& -1	1,278228& -5
-7,038668& -3	8,465440& -5	4,212596& -6	2,273049& -1	1,853280& -5
-6,529615& -3	1,616526& -4	-3,516513& -7	-2,201150& -2	1,597580& -5
-6,343407& -3	7,098230& -5	4,570668& -7	2,351037& -2	1,944107& -5
-6,453996& -3	1,701767& -4	3,537686& -6	2,623662& -1	1,348377& -5
-6,969273& -3	9,749577& -5	-6,574798& -6	-3,259849& -1	2,016903& -5
-7,531464& -3	1,976917& -4	4,311936& -6	2,809930& -1	1,534535& -5
-8,197093& -3	9,374572& -5	-1,160372& -5	-5,450148& -1	2,129066& -5
-8,658675& -3	1,792512& -4	4,779825& -6	3,160882& -1	1,512181& -5
-8,872747& -3	1,290338& -4	-1,049658& -5	-4,339365& -1	2,418922& -5
-8,890797& -3	1,477484& -4	1,888472& -6	1,916127& -1	9,855672& -6
-8,777425& -3	1,194351& -4	-1,068190& -5	-4,470844& -1	2,389235& -5
-8,837347& -3	1,633422& -4	3,829710& -7	3,746316& -2	1,022260& -5
-9,217044& -3	1,335670& -4	-9,287800& -6	-3,763333& -1	2,467972& -5
-9,938935& -3	1,131017& -4	-4,206613& -7	-5,496557& -2	7,653178& -6
-1,108377& -2	1,087200& -4	-1,026295& -5	-3,877001& -1	2,647136& -5
-1,239603& -2	1,073453& -4	-2,429886& -7	-2,481294& -2	9,792819& -6
-1,372578& -2	9,650521& -5	-5,834515& -6	-2,295502& -1	2,541716& -5
-1,485222& -2	9,490400& -5	8,769353& -7	9,733430& -2	9,009520& -6
-1,557568& -2	8,388132& -5	-5,421306& -6	-2,368998& -1	2,288438& -5
-1,601842& -2	9,466579& -5	-9,430018& -7	-1,214687& -1	7,763331& -6
-1,617149& -2	7,666396& -5	-5,507642& -6	-2,425534& -1	2,270693& -5
-1,631316& -2	7,291181& -5	1,002350& -6	1,419936& -1	7,059122& -6
-1,665824& -2	6,172951& -5	-6,080515& -6	-2,947508& -1	2,062934& -5
-1,723031& -2	6,738701& -5	-5,749404& -7	-7,972710& -2	7,211467& -6
-1,808244& -2	9,091044& -5	-7,881035& -6	-2,879236& -1	2,737196& -5
-1,892327& -2	6,399888& -5	-1,094842& -7	-4,044067& -2	4,932761& -6
-1,972575& -2	4,878335& -5	8,422849& -6	3,310272& -1	2,544458& -5
-2,020406& -2	9,229675& -5	9,111522& -6	5,720655& -1	1,592741& -5
-2,009075& -2	9,020489& -5	5,588583& -6	2,812385& -1	1,987133& -5
-1,948454& -2	5,851295& -5	3,739467& -6	3,650679& -1	1,024321& -5
-1,829402& -2	5,323142& -5	4,917319& -6	2,689808& -1	1,828131& -5
-1,694800& -2	7,871873& -5	1,729734& -6	1,731353& -1	9,990648& -6
-1,567808& -2	6,444778& -5	3,857850& -6	2,107662& -1	1,830393& -5
-1,455113& -2	9,254850& -5	1,876415& -6	1,834031& -1	1,023110& -5
-1,375334& -2	5,750483& -5	6,052770& -7	3,808670& -2	1,589208& -5
-1,289806& -2	1,193251& -4	1,777519& -6	1,438963& -1	1,235278& -5
-1,188747& -2	6,536224& -5	4,245219& -8	2,534412& -3	1,675031& -5
-1,035996& -2	1,268585& -4	2,474425& -6	2,130810& -1	1,161260& -5
-8,056443& -3	6,710317& -5	-3,425010& -6	-1,930936& -1	1,773756& -5
-5,185781& -3	1,297527& -4	4,113238& -6	3,261184& -1	1,261271& -5
-1,777435& -3	6,007563& -5	-3,877400& -6	-2,152258& -1	1,801550& -5
1,760514& -3	1,080076& -4	2,166254& -6	2,210514& -1	9,799776& -6
5,150999& -3	7,138253& -5	-3,361523& -6	-1,677822& -1	2,003503& -5
8,082560& -3	1,003272& -4	5,909305& -7	8,256264& -2	7,266368& -6
1,023780& -2	7,889443& -5	-1,898970& -6	-9,179790& -2	2,068642& -5
1,236689& -2	9,174904& -5	-7,731309& -7	-1,016594& -1	7,605113& -6
1,486007& -2	6,443698& -5	-3,353334& -6	-1,681574& -1	1,994163& -5
1,819090& -2	7,422405& -5	1,947083& -7	2,455877& -2	7,928258& -6
2,252761& -2	6,579501& -5	-1,470647& -6	-7,321191& -2	2,008753& -5
2,768051& -2	9,091682& -5	9,110796& -8	9,451097& -3	9,639935& -6
3,348758& -2	4,953806& -5	2,455170& -6	1,527500& -1	1,607313& -5
3,959670& -2	9,712880& -5	-4,169803& -6	-3,664996& -1	1,137737& -5
0,000000& 0	0,000000& 0	0,000000& 0	0,000000& 0	0,000000& 0

PROGRAM SPECTRPLOT

```
'BEGIN'
'COMMENT' ****************************************
RESULTS EITHER FROM SHORTBITS OR COHERENCE CAN BE ALSO DESPLAY-
ED USING STANDARD ICL 1900 GRAPH PLOTTER PROCEDURES WHICH ARE
COMBINED IN PROGRAM SPECTRPLOT.NUMBER OF INDIVIDUAL DIAGRAMS
TO BE PLOTTED MAY BE VARIED AT WILL.DIAGRAMS HAVE THE SIZE OF
A3  OR A4 SHEETS.TOTAL NUMBER OF SHEETS CAN BE VARIED FROM 1
TO 18,WHICH IS DEFINED BY PARAMETER NG.BOOLEAN VARIABLE ANAFOU IS
INTRODUCED IN ORDER TO SWITCH PROGRAM FROM PLOTTING SHORTBITS TO
COHERENCE PLOTTING.WHEN ONLY ONE BIG GRAPH IS DESIRED THAN
SIX IS FALSE.TWO OR THREE GROUPS OF SIX A4 SHEETS OF POWER
DENSITY SPECTRA IS POSSIBLE TO PLOT AT A TIME.BY PROPER CHANGE
OF  THE PARAMETER K MORE THAN ONE FUNCTION CAN BE PLOTTED ON
THE SAME A4 SHEET. *****************************************;

'INTEGER' I,J,K,L,M,N,NG;  'BOOLEAN' SIX, ANAFOU;

SIX:='FALSE';  ANAFOU:='FALSE';
M:=READ;  L:=READ;  NG:=READ;  K:=READ;
'IF' K 'EQ' 1 'THEN' SIX:='TRUE';        K:=READ;
'IF' K 'EQ' 1 'THEN' ANAFOU:='TRUE';     K:=READ;

'BEGIN'  'REAL' P,Q,PI,C,XDUM,YDUM,F;
'REAL' 'ARRAY' X[1:M],Y[1:M],U[1:M],A[1:M],B[1:M],
               CV[1:L],  CW[1:L],
               T[1:M],Z[1:M],V[1:L],W[1:L],S[1:M];
            N:=0;     SELECT INPUT(3);

F:=READ;  'IF' F 'EQ' 0.0 'THEN' 'BEGIN'
'FOR' I:=1 'STEP' 1 'UNTIL' 6 'DO' F:=READ;
'FOR' I:=1 'STEP' 1 'UNTIL' M-1 'DO' 'BEGIN'
Y[I]:=READ;  U[I]:=READ;  A[I]:=READ;
B[I]:=READ;  T[I]:=READ;  Z[I]:=READ;
S[I]:=READ;                         'END';
Y[M]:=U[M]:=A[M]:=B[M]:=T[M]:=Z[M]:=S[M]:=0.00001;
F:=0.5              'END' 'ELSE' 'BEGIN'
'FOR' I:=2 'STEP' 1 'UNTIL' M 'DO'  Y[I]:=READ;
N:=0     'END';     Y[1]:=F;

OPENPLOT;  L:=0;     C:=LN(10.0);
STRARR(V,13,'('POWER%DENSITY')');
STRARR(W,14,'('FREQUENCY%KC/S')');
STRARR(CV,20,'('CORRELATION%FUNCTION')');
STRARR(CW,10,'('TIME%DELAY')');

'IF' ANAFOU 'THEN'  'BEGIN'
WRITETEXT('('ANAFOU')');  'END'  'ELSE'  'BEGIN'
WRITETEXT('('FAST%FOURIER%TRANSFORM')');  'END';

'IF' SIX 'THEN' 'GOTO' SMALL;  P:=-3.0;     Q:=3.0;
'IF' NG 'EQ' 1 'THEN' SIX:='TRUE';  BIG:
```

```
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1);
N:=N+1;  NEWLINE(1);  HGPLOT (P,Q,0,4);  C:=LN(10.0);
'FOR' I:=1 'STEP' 1 'UNTIL' M  'DO' 'BEGIN'
X[I]:=4.0*I;  Y[I]:=ABS(Y[I]);
PRINT(X[I],0,6);     PRINT(Y[I],0,6);
'IF' Y[I] 'LE' 0.00001   'THEN'  Y[I]:=0.00001;
X[I]:=8.0*LN(I)/C  + 6.4;  Y[I]:=5.0*(LN(Y[I])/C);
PRINT(X[I],0,6);    PRINT(Y[I],0,6);
'IF' Y[I] 'GT'  0.5 'THEN' Y[I]:=0.5;  'END';
HGPLINE (X,Y,128,1);   Q:=0.0;   P:=0.0;
'FOR' I:=1 'STEP' 1 'UNTIL' 5 'DO' 'BEGIN'
HGPDASHLN (0.0,Q,24.0,Q,0.0);

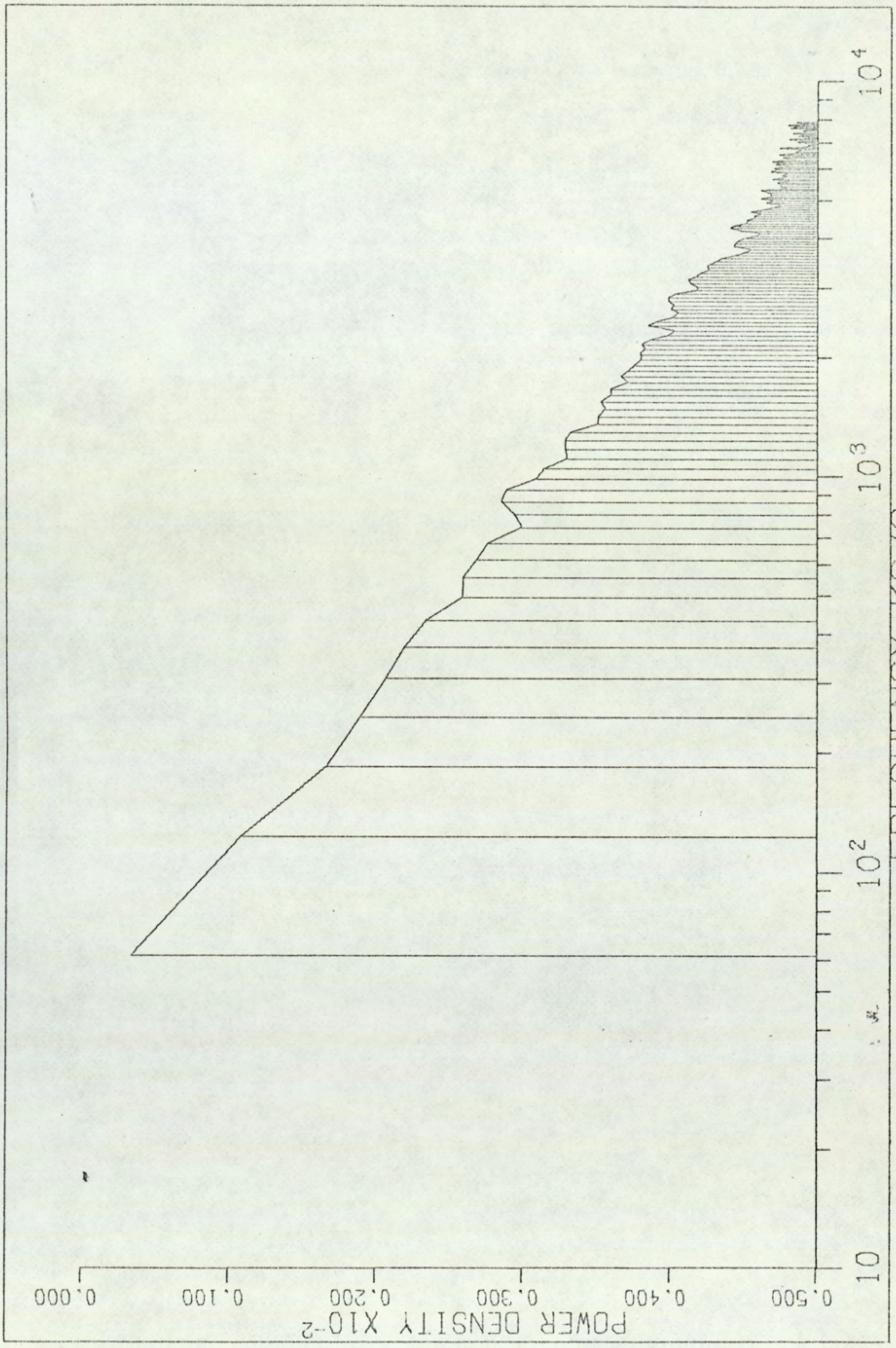
Q:=Q-5.0;       'END';

'FOR' I:=1 'STEP' 1 'UNTIL' M  'DO' 'BEGIN'
P:=X[I];    Q:=Y[I];
HGPDASHLN (P,-25.0,P,Q,0.0);       'END';
HGPRECT (-1.0,-26.0,28.0,26.0,0.0,3;
HGPAXISV (0.0,-25.0,V,13,25.0,90.0,50.0,-10,5.0,3);
HGPAXISV (24.0,-25.0,V,13,25.0,90.0,50.0,-10.0,5.0,3);
HGPLOGAXIS (0.0,-25.0,W,-14,24,0.0,1,4);
HGPLOT(0.0,0.0,3,0);
'IF' N 'LT' NG 'THEN' SIX:='FALSE' 'ELSE' SIX:='TRUE';

NEWLINE(1);  WRITETEXT('('BIG%DIAGRAM%IS%DONE')');
'IF' SIX 'THEN' 'GOTO' FIN 'ELSE'
'BEGIN'  L:=0;  P:=-42;  Q:=0.0;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1);
HGPLOT(P,Q,0,4);       'GOTO'  AGAIN  'END';

SMALL:      P:=-16.0;    Q:=3.0;      AGAIN:
'IF' N 'GE' NG 'THEN' 'GOTO' FIN;
PI:=8.0/3.0;       NEWLINE(1);
'FOR' J:=1 'STEP' 1 'UNTIL' K 'DO'  'BEGIN'      N:=N+1;
'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
'IF' ANAFOU 'OR' (N'/'2)*2 'EQ' N 'THEN' 'BEGIN'
X[I]:=4.0*I;  X[I]:=PI*LN(X[I])/C + 0.5;
'END' 'ELSE' X[I]:=I/16.0;    'END' LOOP;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1);
HGPLOT(P,Q,0,4);
'IF' ANAFOU 'AND' NG 'EQ' 7 'THEN' 'BEGIN'
'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
'IF' N 'EQ' 2 'THEN' Y[I]:=U[I];
'IF' N 'EQ' 3 'THEN' Y[I]:=A[I];
'IF' N 'EQ' 4 'THEN' Y[I]:=B[I];
'IF' N 'EQ' 5 'THEN' Y[I]:=T[I];
'IF' N 'EQ' 6 'THEN' Y[I]:=Z[I];
'IF' N 'EQ' 7 'THEN' Y[I]:=S[I];
'END' 'END' 'ELSE' 'BEGIN'
```

```
'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
Y[I]:=READ;
'IF' ANAFOU 'OR' (N'/'2)*2 'EQ' N 'THEN' 'BEGIN'
Y[I]:=ABS(Y[I]);
'IF' Y[I] 'LE' 0.00001 'THEN'  Y[I]:=0.00001;
Y[I]:=LN(Y[I])/C;
'IF' ABS(Y[I]) 'GT' 5.0 'THEN'  Y[I]:=SIGN(Y[I])*5.0;
'IF' Y[I] 'GT' 2.0 'THEN' Y[I]:=2.0
'END' 'ELSE' Y[I]:=4.0*Y[I] - 4.0;
PRINT(Y[I],0,6);              'END';
'END' CONDITION;           NEWLINE(1);
HGPLINE(X,Y,M,1);
'IF' ANAFOU 'OR' (N'/'2)*2 'EQ' N
'AND' K 'EQ' 1 'THEN' 'BEGIN'
'FOR' I:=1 'STEP' 1 'UNTIL' M  'DO' 'BEGIN'
P:=X[I];     Q:=Y[I];
HGPDASHLN(P,-5.0,P,Q,0.0);      'END';
'END' DASHING;
'IF' ANAFOU 'OR' (N'/'2)*2 'EQ' N 'THEN' 'BEGIN'
HGPAXISV(0.0,-5.0,V,13,5.0,90.0,50.0,-10.0,1.0,2);
HGPLOGAXIS(0.0,-5.0,W,-14,8.0,0.0,1,4)
'END' 'ELSE' 'BEGIN'
HGPAXISV(0.0,-5.0,CW,10,8.0,0.0,0.0,16,0.8,2);
HGPAXISV(0.0,-5.0,CV,20,5.0,90.0,-0.2,0.2,1.0,2)
'END' AXISPLOT 'END' KLOOP;
HGPRECT(-0.5,-5.5,6.0,9.0,0.0,3);  L:=L + 1;
'IF' L = 1 'THEN' 'BEGIN'
HGPLOT(0.0,0.0,3,0);  K:=1;   P:=0.0;  Q:=8.5;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1); 'END';
'IF' L = 2 'THEN' 'BEGIN'
HGPRECT(-2.5,-6.5,17.0,12.0,0.0,3);
HGPLOT(0.0,0.0,3,0);  K:=1;  P:=14.0;  Q:=-8.5;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1); 'END';
'IF' L = 3 'THEN' 'BEGIN'
HGPLOT(0.0,0.0,3,0);  K:=1;  P:=0.0;  Q:=8.5;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1); 'END';
'IF' L = 4 'THEN' 'BEGIN'
HGPRECT(-2.5,-6.5,17.0,12.0,0.0,3);
HGPLOT(0.0,0.0,3,0);  K:=1;  P:=0.0;  Q:=9.0;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1); 'END';
'IF' L = 5 'THEN' 'BEGIN'
HGPRECT(-2.5,-6.5,8.5,12.0,0.0,3);
HGPLOT(0.0,0.0,3,0);  K:=1;  P:=-14.0; Q:=0.0;
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1); 'END';
'IF' L 'LT' 6 'THEN' 'GOTO' AGAIN;
HGPRECT(-2.5,-6.5,8.5,12.0,0.0,3);
'IF' NG 'GE' 8 'AND' NG 'GT' N 'THEN' 'BEGIN'
HGPLOT(0.0,0.0,3,0);
P:=-27.0;     Q:=-18.0;  'GOTO' BIG 'END';
FIN:
NEWLINE(1);  PRINT(P,0,6);  SPACE(4);  PRINT(Q,0,6);
WRITETEXT('('ORIGIN%IS%SHIFTED')');  NEWLINE(1);
CLOSEPLOT;      'END';
****                                      'END';
```



Linear Spectrum of a computer generated sequence of numbers with Markovian properties. Sequencen was generated by program GENERATE,analyzed by program COHERENCE(AUTO=TRUE) and desplayed by program SPECTR-PLOT

PROGRAM ANALYSIS

```
'BEGIN'
 'INTEGER' I, II, J, JJ, L, M, N, DUM, K, MT;
 'REAL'  PI, THETA, ANGLE, DANGLE;
 'BOOLEAN' ODD;
 'COMMENT'
 FROM THE EXPERIMENTAL POWER DENSITY  DATA POINTS,COEFFICIENTS
 OF THE CORRESPONDING MEROMORPHIC FUNCTION IS OBTAINED,BASED
 ON THE WIENER LEE METHOD AND CHEBYSHEV POLYNOMIALS APPROXIMATION;
 PI:=4.0*ARCTAN(1.0);     N:=128;
 M:=19;   K:=18;
 PAPERTHROW;

     'BEGIN'

 'REAL' 'ARRAY' C[0:K],TEMPO[0:(K+1)],TEMP2[0:(K+1)],
 TEMP1[0:(K+1)],TEMP3[0:(K+1)],ALPHA[0:K],BETA[0:K],
FREQ[0:N], PSPECTR[0:N], FI[1:M], SPECTR[1:M], YAR[0:M,0:M];

 'PROCEDURE' BINCOEF(C,N,V);
 'INTEGER' V,N;  'ARRAY' C;
 'BEGIN' 'INTEGER' I,J;
 'ARRAY' T1[0:N],T2[0:N];
 'COMMENT'
 THIS PROCEDURE COMPUTES  BINARY COEFFICIENTS OF  AN N ORDER
 POLYNOMIAL,BASED ON THE PASCAL TRIANGLE METHOD.THIS PROCEDURE

 IS USED AS A SUBROUTINE FOR  ANOTHER PROCEDURE(PREPARE);
 'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' C[J]:=1.0;
 'IF' N 'LE' 1 'THEN' 'GOTO' BFIN;
 T1[0]:=T1[1]:=1.0;
 'FOR' I:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
 'FOR' J:=1 'STEP' 1 'UNTIL' I-1 'DO'
 T2[J]:=T1[J-1] + T1[J];  T2[I]:=1.0;
 'FOR' J:=1 'STEP' 1 'UNTIL' I 'DO' T1[J]:=T2[J]; 'END';
 'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' C[J]:=T2[J];
 BFIN: 'IF' V 'LT' 0 'THEN' 'BEGIN' I:=(N'/'2)*2;
 'IF' I 'NE' N 'THEN' I:=N 'ELSE' I:=N-1;
 'FOR' J:=1 'STEP' 2 'UNTIL' I 'DO' C[J]:=-C[J];
 V:=-V  'END';  'END' PROC;

 'PROCEDURE' ACOEF(A,N,B);
 'INTEGER' N;  'ARRAY' A,B;

 'BEGIN' 'ARRAY' YAR[0:N,0:N];
 FORMAR(YAR,N);  TRANSF(A,YAR,B,N);
 'END' PROC;

 'PROCEDURE' FORMAR(YAR,N);
 'INTEGER' N;  'ARRAY' YAR;
 'BEGIN' 'INTEGER' I,J,K;
```

```
'INTEGER' N;  'ARRAY' A,YAR,B;
'BEGIN' 'INTEGER' I,J;
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
B[I]:=0.0;  'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO'
B[I]:=B[I] + YAR[I,J]*A[J];  'END' ILOOP; 'END' PROC;

'PROCEDURE' BCOEF(A,N,B);
'INTEGER' N;  'ARRAY' A,B;
'BEGIN' 'ARRAY' YAR[0:N,0:N];
PREPAR(YAR,N);  TRANSF(A,YAR,B,N);  'END' PROC;

'PROCEDURE' COSCOEFF(N,M,FI,SPECTR,COEFF,EREUR,MT,PI);
'VALUE' N, M, EREUR;
'INTEGER' N, M, MT;
'REAL' EREUR,PI;
'ARRAY' FI, SPECTR,COEFF;
'BEGIN'
'INTEGER' I, J, JJ;
'REAL' SUM,SUM1,SUM2,SUM3,ALPHA,BETA,GAMMA,
DELTA,ETA,U,V,PLNOM,MEAN,ERR,EST,FMAX,W;
'ARRAY' A[0:(M+1)], B[0:M];
'COMMENT'
THIS PROCEDURE (HUNTER ,1967) APPROXIMATES A FUNCTION BY COSINE
SERIES OF PERIOD 2*PI,FROM VALUES SPECIFIED NOT NECESSARILLY
EQUALLY SPACED IN THE RANGE  0-2*PI.THE INPUT PARAMETERS ARE
N  NUMBER OF FUNCTION VALUES GIVEN
M  ORDER OF FUNCTION VALUES GIVEN
SPECTR  KNOWN DISCRETE VALUES OF POWER DENSITY SPECTRUM AT
     FREQUENCIES FI.
FI  NINETEEN FREQUENCIES,WHICH ARE CHOSEN TO BE ALMOST EQUALLY
     SPACED IN THE ANGLE DOMAIN(SEE P. 53)
NORMALLY MT=M, BUT PROVISION IS INCLUDED TO CALCULATE FEWER
     HARMONICS IF ROUNDING ERROR BEGIN TO ACCUNULATE(CLENSHAW)
ERREUR IS THE PERMISSIBLE ERROR IN FITTING BY ORTHOGONAL
     POLYNOMIALS(WATT)
COEFF  ARE COSINE COEFFICIENTS COMPUTED TO FIT KNOWN SPECTRUM
     TRANSFERRED TO THE ANGLE DOMAIN;
MT:=0;
A[M+1]:=ALPHA:=EST:=0.0;
BETA:=SUM3:=MEAN:=1.0;
'FOR' I:=0 'STEP' 1 'UNTIL' M 'DO'
COEFF[I]:=0.0;
'FOR' I:=0 'STEP' 1 'UNTIL' M 'DO'
'BEGIN'
B[I]:=A[I]:=0.0;
'END';
B[0]:=-1.0;
AGAIN: ETA:=0.0;
'FOR' I:=0 'STEP' 1 'UNTIL' MT 'DO'
'BEGIN'
'IF' I = 0 'THEN'
'BEGIN'
DELTA:=A[1] + A[I+1] - BETA*B[I] - ALPHA*A[I];
   'GOTO'  SKIP    'END';
DELTA:=B[I-1] + A[I+1] - BETA*B[I]- ALPHA*A[I];
```

```
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO'
YAR[I,J]:=0.0;  'END' ILOOP;

I:=1;  J:=-1;  K:=1;
L13:    J:=J + 1;
'IF' J'/'2  'EQ' J 'THEN' 'BEGIN'
YAR[0,J]:=1.0*I;  I:=-I
'END' 'ELSE' 'BEGIN'
YAR[1,J]:=K*J;  K:=-K  'END';
'IF' J 'LT' N 'THEN' 'GOTO' L13;
YAR[0,0]:=1.0;

'FOR' I:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'FOR' J:=I 'STEP' 1 'UNTIL' N 'DO'
YAR[I,J]:=2.0*YAR[I-1,J-1]-YAR[I,J-2]; 'END';
'END' PROC;

'PROCEDURE' POLMULT(A,B,C,M1,M2);
'INTEGER' M1,M2;  'ARRAY' A,B,C;
'BEGIN' 'INTEGER' I,J,M12,IJ;  M12:=M1+M2;
'COMMENT'
THIS PROCEDURE IS A SUBROUTINE FOR POLYNOMIAL MULTIPLICATION
USED  FOR ANOTHER SUBROUTINE(PREPARE);
'FOR' I:=0 'STEP' 1 'UNTIL' M12 'DO' C[I]:=0.0;
'FOR' I:=0 'STEP' 1 'UNTIL' M1 'DO' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' M2 'DO' 'BEGIN'
IJ:=I+J;  C[IJ]:=C[IJ] + A[I]*B[J];
'END' ILOOP;  'END' JLOOP;  'END' PROC;

'PROCEDURE' PREPAR(YAR,N);
'INTEGER' N;  'ARRAY' YAR;
'BEGIN' 'INTEGER' I,J;  'ARRAY' A[0:N],B[0:N],C[0:N];
'COMMENT'
THIS PROCEDURE (SEE P. 52) GENERATES TERMS IN  MATRIX YAR WHICH
IS USED TO TRANSFORM ALPHA COEFFICIENTS(HYPERGEOMETRIC FUNCTION)
TO BETA COEFFICIENTS(MEROMORPHIC FUNCTION);
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
A[I]:=B[I]:=C[I]:=0.0; 'END';
'IF' J 'EQ' N 'THEN' A[0]:=1.0 'ELSE' 'BEGIN'
BINCOEF(A,N-J,1)  'END';
'IF' J 'EQ' 0 'THEN'  B[0]:=1.0 'ELSE'
'BEGIN'  BINCOEF(B,J,-1) 'END';
POLMULT(A,B,C,N-J,J);
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' YAR[I,J]:=C[I];
'END' JLOOP;    'END' PROC;

'PROCEDURE' TRANSF(A,YAR,B,N);
```

```
SKIP:  B[I]:=A[I];
A[I]:=DELTA;   DELTA:=ABS(DELTA);
'IF' DELTA > ETA 'THEN' ETA:=DELTA
            'END';
SUM1:=SUM2:=DELTA:=ERR:=0.0;
'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
GAMMA:=2*COS(FI[I]);
'IF' MT = 0 'THEN'
'BEGIN'
PLNOM:=0.5;  'GOTO' EVALUATE
            'END';
U:=V:=0.0;
'FOR' J:=MT 'STEP' (-1) 'UNTIL' 1 'DO'
'BEGIN'
W:=GAMMA*U - V + A[J];
V:=U;  U:=W;
            'END';
SUM:=1.0;
PLNOM:=0.5*(U*GAMMA + A[0]) - V;
EVALUATE:
W:='IF' FI[I] 'EQ' 0 'OR' FI[I] 'EQ' PI
'THEN' 0.5 'ELSE'  1.0;
DELTA:=DELTA + W*PLNOM*SPECTR[I];
'IF' MT > 0 'THEN' ERR:=ERR + W*PLNOM*MEAN*SUM;
PLNOM:=W*PLNOM↑2;
SUM1:=SUM1 + GAMMA*PLNOM;
SUM2:=SUM2 + PLNOM
            'END';
'IF' EST > EREUR 'THEN' 'GOTO' FIN;
EST:=EST + ETA*ABS(ERR)/SUM2;
ALPHA:= SUM1/SUM2;  BETA:=SUM2/SUM3;
DELTA:=DELTA/SUM2;  SUM3:=SUM2;
'FOR' I:=0 'STEP' 1 'UNTIL' MT 'DO'
COEFF[I]:=COEFF[I] + DELTA*A[I];
MT:=MT + 1;

'IF' MT 'LE' M 'THEN' 'GOTO' AGAIN;
FIN: MT:=MT - 1;
            'END';

II:=0;          WORK:
M:=19;   K:=18;    JJ:=0;
PI:=4.0*ARCTAN(1.0);     N:=128;

II:=II + 1;  NEWLINE(1);
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
THETA:=J;  PSPECTR[J]:=  READ;                         'END';

ALPHA[0]:=PSPECTR[0];   BETA[0]:=0.0;
'FOR' J:=1,2,3,4,5,6,7,8,10,12,14,17,21,27,37,56,114 'DO'
'BEGIN'
JJ:=JJ + 1;
```

```
BETA[JJ]:=2.0*ARCTAN(J/10);
ALPHA[JJ]:=PSPECTR[J];
'END';

BETA[K]:=PI;
ALPHA[K]:=PSPECTR[N];

'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO'
'BEGIN'
FI[J]:=BETA[J-1];
SPECTR[J]:=ALPHA[J-1];
'END';

K:=18;

COSCOEFF(M,K,FI,SPECTR,C,0.00001,MT,PI);

'FOR' I:=0 'STEP' 1 'UNTIL' 18 'DO' 'BEGIN'
TEMP1[I]:=TEMP2[I]:=TEMP3[I]:=0.0;
TEMPO[I]:=ALPHA[I]:=BETA[I]:=0.0;   'END';

ACOEF(C,K,ALPHA);

BCOEF(ALPHA,K,BETA);
JJ:=0;

NEWLINE(1);   SPACE(2);
WRITETEXT('('SPECTRUM%ANALYSIS%NO')');
SPACE(2);  PRINT(II,2,0);     NEWLINE(1);
NEWLINE(1);  SPACE(2);
WRITETEXT('('POWER%DENSITY%SPECTRUM')');
NEWLINE(1);
WRITETEXT('('ORDER')');
SPACE(12);
    WRITETEXT('('SPECTRUM%VALUE')');
SPACE(2);
WRITETEXT('('COSINE%COEFFICIENTS')');
SPACE(4);
WRITETEXT('('ALPHA')');
SPACE(12);
WRITETEXT('('BETA')');
'FOR' I:=0 'STEP' 1 'UNTIL' M-1 'DO'
'BEGIN'
NEWLINE(1);
PRINT(I,2,0);   SPACE(9);
PRINT(PSPECTR[I],0,6);
SPACE(4);
'IF'  I 'GT' K 'THEN' 'GOTO'  OVER;
PRINTCH(26);
SPACE(2);     PRINT(C[I],0,6);
SPACE(2);  PRINT(ALPHA[I],0,6);
SPACE(2);  PRINT(BETA[I],0,6);
OVER:  'END';
'IF' II 'LT' 6 'THEN' 'GOTO' WORK;

'END';
NEWLINE(10);
'END';
```

****

```
NEWLINE(1);   SPACE(2);
WRITETEXT('('SPECTRUM%ANALYSIS')');
NEWLINE(1);  SPACE(2);
WRITETEXT('('POWER%DENSITY%SPECTRUM')');
NEWLINE(1);
WRITETEXT('('ORDER')');
SPACE(12);
    WRITETEXT('('SPECTRUM%VALUE')');
SPACE(2);
WRITETEXT('('COSINE%COEFFICIENTS')');
SPACE(4);
WRITETEXT('('ALPHA')');
SPACE(12);
WRITETEXT('('BETA')');
'FOR' I:=0 'STEP' 1 'UNTIL' M-1 'DO'
'BEGIN'
NEWLINE(1);
PRINT(I,2,0);   SPACE(9);
PRINT(PSPECTR[I],0,6);
SPACE(4);
'IF'  I 'GT' K 'THEN' 'GOTO'  OVER;
PRINTCH(26);
SPACE(2);    PRINT(C[I],0,6);
SPACE(2);  PRINT(ALPHA[I],0,6);
SPACE(2);  PRINT(BETA[I],0,6);
OVER:  'END';
'IF' II 'LT' 4 'THEN' 'GOTO' WORK;

'END';
NEWLINE(10);


'END';
****
```

PROGRAM NYQUIST

```
'BEGIN'
'PROCEDURE' SIMPLE(A,B,C,M,N,II,WMIN,WMAX,SCALE,OUT);
'VALUE' A,B,C,M,N;  'INTEGER' M,N,II,OUT;
'REAL' WMIN,WMAX,SCALE;
'ARRAY' A,B,C;
'BEGIN'
'INTEGER' I,J,JJ;
'REAL'  RATIO,PI,WMID,XM,XN,YM,YN,VAL,W,MN,MM,LOGC;
'ARRAY' AM[1:M], AD[1:N];
I:=0;     RATIO:=WMIN/WMAX;
'IF' RATIO 'LT' 0.00000001 'THEN'  'BEGIN'
RATIO:=0.00000001;  WMID:=WMAX/10000.0;
WMIN:=WMID/10000.0;
'GOTO'  SKIP    'END';
RATIO:=WMAX/WMIN;      PI:=SQRT(RATIO);
WMID:=SQRT(WMAX*WMIN);
WMAX:=PI*WMID;  WMIN:=WMID/PI;
SKIP:
SPACE(8);
WRITETEXT('('LOWEST%RADIAN%FREQUENCY')');
SPACE(8);
WRITETEXT('('CENTER%RADIAN%FREQUENCY')');
SPACE(8);
WRITETEXT('('MAXIMUM%RADIAN*FREQUENCY')');
NEWLINE(1);     SPACE(16);
PRINT(WMIN,0,6); SPACE(16);
PRINT(WMID,0,6);  SPACE(16);
PRINT(WMAX,0,6);  SPACE(16);
SCALE:=1.0/WMID;    WMID:=1.0;
WMIN:=WMIN*SCALE;  WMAX:=WMAX*SCALE;
SCALE:=1.0/SCALE;
NEWLINE(1);     SPACE(16);
PRINT(WMIN,0,6); SPACE(16);
PRINT(WMID,0,6);  SPACE(16);
PRINT(WMAX,0,6);
WRITETEXT('('NORMALIZED')');
PI:=4.0*ARCTAN(1.0);
PI:=1.0/PI;
W:=WMIN;     II:=1;
NEWLINE(6);  SPACE(6); WRITETEXT('('FREQ')');
SPACE(9);
WRITETEXT('('REAL%PART')');
SPACE(4);
WRITETEXT('('IMAGINARY%PART')');
SPACE(4);
WRITETEXT('('MAGNITUDE')');
SPACE(4);
WRITETEXT('('RADIAN%PHASE')');
SPACE(4);
WRITETEXT('('MAGNITUDE%IN%DB')');
SPACE(4);
WRITETEXT('('PHASE%IN%DEGREES')');
LOGC:=20.0/LN(10.0);
LAB:
```

```
'IF' II 'GT' RATIO 'THEN' 'GOTO' FIN;

'IF' OUT 'EQ' 0 'THEN' SELECT OUTPUT(0);
'ELSE' 'BEGIN' SELECT OUTPUT(4);
WRITETEXT('('DOCZTAPE')'); NEWLINE(1);
RUNOUT; OUT:=4.0 'END';

'FOR' VAL:=1.41,2,3,4,5,6,8,10 'DO' 'BEGIN'
I:=I + 1;
'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
AM[J]:=C[J] - A[J]*W*W; 'END';
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
AD[J]:=C[M+J] - A[M+J]*W*W; 'END';
XM:=AM[1]; YM:=B[1]*W; MM:=XM;
'FOR' J:=1 'STEP' 1 'UNTIL' M-1 'DO' 'BEGIN'
XM:=(MM*AM[J+1] - YM*W*B[J+1]);
YM:=(YM*AM[J+1] + MM*W*B[J+1]);
MM:=XM;                        'END';
XN:=AD[1]; YN:=B[M+1]*W; MN:=XN;
'FOR' J:=1 'STEP' 1 'UNTIL' N-1 'DO' 'BEGIN'
XN:=(MN*AD[J+1] - YN*W*B[M+J+1]);
YN:=(YN*AD[J+1] + MN*W*B[M+J+1]);
MN:=XN;                        'END';
MM:=XM*XN + YM*YN; MN:=XN*XN + YN*YN;
WMID:=MM/MN;            MM:=YM*XN - YN*XM;
YM:=MM/MN;              XM:=WMID;
XN:=SQRT(XM*XM + YM*YM);
YN:=ARCTAN(YM/XM);
MM:=LOGC*LN(XN); MN:=180.0*YN*PI;
NEWLINE(1);     SPACE(1);        PRINT(W,0,6);
PRINT(XM,0,6);     SPACE(1);
PRINT(YM,0,6);     SPACE(1);
PRINT(XN,0,6);     SPACE(1);
PRINT(YN,0,6);     SPACE(1);
PRINT(MM,0,6);     SPACE(1);
PRINT(MN,0,6);     SPACE(1);

W:=VAL*WMIN;
'END' VALLOOP;
II:=II*10;   WMIN:=W;
'IF' OUT 'EQ' 1 'OR' OUT 'EQ' 3 'THEN' 'BEGIN'
NEWLINE(1); WRITETEXT('('REFFZRADZFREQ')');
SPACE(4); PRINT(WMIN,0,6);       OUT:=4.0 'END';
'IF' WMIN 'LT' WMAX 'THEN' 'GOTO' LAB;
FIN: NEWLINE(4);
PRINT(II,3,0); SPACE(4); PRINT(RATIO,2,1);
PRINT(SCALE,0,6);    'END' PROCSIMPL;
```

```
'INTEGER' M,N,NM,I,J,II;
'REAL' WMIN,WMAX,SCALE;
M:=READ;     N:=READ;     J:=0;     OUT:=READ;
NM:=N + M;     II:=0;
'BEGIN'
'ARRAY' A[1:NM],B[1:NM],C[1:NM];
NEWLINE(1);     SPACE(10);
WRITETEXT('('NOMINATORZCOEFFICIENTS')');
'FOR' I:=1 'STEP' 1 'UNTIL' NM 'DO'     'BEGIN'
NEWLINE(1);     SPACE(2);
II:=II + 1;     PRINT(II,2,0);     SPACE(4);
'IF' II = M+1 'THEN' 'BEGIN'
WRITETEXT('('DENOMINATORZCOEFFICIENTS')');
II:=-N;
NEWLINE(1);     SPACE(2);
PRINT(II,2,0);     SPACE(4);
                                          'END';


A[I]:=READ;   PRINT(A[I],0,6);   SPACE(4);
B[I]:=READ;   PRINT(B[I],0,6);   SPACE(4);
C[I]:=READ;   PRINT(C[I],0,6);   SPACE(4);
                                     'END';
WMIN:=READ;     WMAX:=READ;
NEWLINE(1);     SPACE(2);
SIMPLE(A,B,C,M,N,II,WMIN,WMAX,SCALE,OUT);
NEWLINE(1);     SPACE(2);
SPACE(1);     PRINT(WMIN,0,6);
SPACE(1);     PRINT(WMAX,0,6);
'END';
SPACE(4);
WRITETEXT('('NYQUISTZANDZBODEZCOMPUTATION')');

'END' PROGE;
****
```

PROGRAM SCANNING

```
'BEGIN'

'COMMENT' ****************************************************
THIS PROGRAM COMPUTES THE ROOT LOCUS OF A KNOWN OPEN  LOOP
TRANSFER FUNCTION.TRANSFER FUNCTION SHOULD BE EXPRESSED AS THE
PRODUCT OF QUADRATICS IN ITS NOMINATOR AND DENOMINATOR.
BEFORE READING  THIS DATA  L AND M MUST BE SPECIFIED,I.E.
THE NUMBER OF QUADRATICS IN THE DENOMINATOR AS WELL AS IN THE
NOMINATOR.THE COMPLEX PLANE IS THEN SCANNED FROM XPOZ TO XEND
(WHICH SHOULD BE SPECIFIED AS POSITIVE) IN STEPS OF DX.IN THE
Y DIRECTION(IMGINARY) JMAX DEFINES THE NUMBER OF STEPS TAKEN
IN INCREMENTS OF 0.4 OF FREQUENCY UNITS.FOR EACH  POINT ON THE
THE ROOT LOCUS GAIN VALUE IS COMPUTED AND OUTPUTED AFTER COR-
RESPONDING COORDINATE VALUES. *********************************;

'INTEGER' JN,M,L,JMAX,ZX,ML,JK;
'REAL' X,Z,TOL;     'BOOLEAN' POZ;

'PROCEDURE' OLTF(A,B,C,X,Y,RF,IF,DEN,IX,L,M);
'VALUE' M,L;  'REAL' X;  'INTEGER' M,L,IX;
'ARRAY' A,B,C,RF,IF,DEN,Y;
'BEGIN'  'INTEGER' I,J;
'REAL' P,R,Z,W,QZ,W1,W2,Z1,Z2,D;
'FOR' I:=0,1 'DO' 'BEGIN'
W:=1.0;  Z:=0.0;
'FOR' J:=1+I*M 'STEP' 1 'UNTIL' M+I*L 'DO' 'BEGIN'
D:=A[J] + B[J]*X;     P:=W;
QZ:=D + (X*X - Y[IX]*Y[IX])*C[J];
R:=2.0*X*Y[IX]*C[J] + Y[IX]*B[J];
W:=W*QZ - Z*R;  Z:=Z*QZ + P*R;  'END';
'IF' I 'EQ' 0 'THEN' 'BEGIN'
W1:=W;  Z1:=Z  'END' 'ELSE' 'BEGIN'
W2:=W;  Z2:=Z  'END';  'END' ILOOP;
DEN[IX]:=W2*W2 + Z2*Z2;
RF[IX]:=W1*W2 + Z1*Z2;
IF[IX]:=Z1*W2 - W1*Z2;   'END' PROC;

'COMMENT' *****************************************************
PROCEDURE OLTF COMPUTES THE REAL AND IMAGINARY  PARTS OF THE
OPEN LOOP TRANSFER FUNCTION. *********************************;

'PROCEDURE' ROOTS(C,B,A,X,Y);
'REAL' C,B,A,X,Y; 'BEGIN' 'REAL' DISC,DEN;
'IF' C 'NE' 0 'THEN' 'BEGIN'
DEN:=1/(2*C);   DISC:=B*B - 4*A*C;
'IF' DISC 'NE' 0 'THEN' 'BEGIN'
'IF' DISC 'GT' 0 'THEN' 'BEGIN'
DISC:=SQRT(DISC); X:=-B/DEN + DISC/DEN; Y:=0
'END' 'ELSE' 'BEGIN' X:=-B/DEN;  Y:=SQRT(-DISC);
Y:=Y/DEN  'END' 'END' 'END' 'ELSE' 'BEGIN'
X:=-A/B;  Y:=0 'END';  'END' PROC;
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'COMMENT' *********************************************
PROGRAM STARTS BY READING DATA **************************;
   SELECT INPUT(3);
 NEWLINE(1);  WRITETEXT('('INPUT%DATA')');

 M:=READ;  L:=READ;  JMAX:=READ;  TOL:=READ;
 NEWLINE(1);  WRITETEXT('('M=')');  PRINT(M,2,0);
 SPACE(4);  WRITETEXT('('L=')');  PRINT(L,2,0);
 SPACE(4);  WRITETEXT('('TOL=')');  PRINT(TOL,0,6);
 ML:=M + L;  JK:=JMAX + 1;  ZX:=0;

 'BEGIN' 'INTEGER' J,K,IX;  'REAL' QX,ZZ,XPOZ,DX,XEND,
        CJ,BJ,AJ;
 'ARRAY' A[1:ML],B[1:ML], C[1:ML],V[1:2],
         Y[1:2],REA[1:2],IMA[1:2],ADEN[1:2];

 XPOZ:=READ;  DX:=READ;  XEND:=READ;
 SPACE(4);  WRITETEXT('('XEND=')'); PRINT(XEND,0,3);

 NEWLINE(3);  WRITETEXT('('DENOMINATOR%COEFFICIENTS')');
 NEWLINE(1);
 'FOR' J:=1 'STEP' 1 'UNTIL' ML 'DO' 'BEGIN'
 A[J]:=READ;  B[J]:=READ;  C[J]:=READ; JK:=READ;
 AJ:=A[J];  BJ:=B[J];  CJ:=C[J];
'IF' JK 'LT' 0 'THEN' 'BEGIN'
 AJ:=AJ/CJ;  BJ:=BJ/CJ;  Z:=BJ;  X:=AJ;

 A[J]:=AJ*AJ + BJ*BJ;  CJ:=C[J]:=1;  B[J]:=2*AJ
'END' 'ELSE' ROOTS(CJ,BJ,AJ,X,Z);

 'COMMENT'*******************************************
 QUADRATIC TERMS ARE OF THE FORM (A+B*S+C*S↑2)
 OR EXPRESSED IN PARTITIONED FORM, I.E AS THE PRODUCT
 (C*S+A+J*B)*(C*S+A-J*B),WHEN BOTH TERMS ARE COUNTED
 AS ONE QUADRATIC TERM.FOR THE LATTER CASE JK < 0.
 THERFORE BEFORE VALUES OD A,B, AND C ARE USED IN THE
 PROCEDURE OLTF ,THEY MUST BE RECALCULATED INTO PROPER
 QUADRATIC COEFFICIENTS********************************;

 'IF' J 'EQ' M+1 'THEN' 'BEGIN'
NEWLINE(1); WRITETEXT('('DENOMINATOR%COEFFICIENTS')');
 NEWLINE(3);  WRITETEXT('('NOMINATOR%COEFFICIENTS')');
 NEWLINE(1)  'END';
 NEWLINE(1);  PRINT(A[J],0,6);  PRINT(B[J],0,6);
 PRINT(C[J],0,6);       SPACE(20);
WRITETEXT('('REAL%ROOT')');  PRINT(X,0,6);
SPACE(8);  WRITETEXT('('IMAGINARY%ROOT')');
PRINT(Z,0,6);        'END' READ;

 PAPERTHROW;

 'FOR' X:=XPOZ 'STEP' -DX 'UNTIL' -XEND 'DO' 'BEGIN'
 ZX:=ZX + 1;  V[1]:=Y[1]:=0.0;  ZZ:=-1.0;
 OLTF(A,B,C,X,Y,REA,IMA,ADEN,1,L,M);
 'IF' ADEN[1] 'LT' 10↑(-5) 'AND' REA[1] 'LT' 0.0
 'THEN' 'BEGIN' ZZ:=10↑5;
 NEWLINE(1);  PRINT(ZX,3,0);  PRINT(X,2,1);
 PRINT(Y[1],2,3);  PRINT(ZZ,0,6);  'END';
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'IF' ADEN[1] 'GE' 10↑(-5)  'AND'  REA[1] 'LE' 0.0
'THEN' 'BEGIN'  ZZ:=-REA[1]/ADEN[1];
NEWLINE(1);  PRINT(ZX,3,0);  PRINT(X,2,1);
PRINT(Y[1],2,3);  PRINT(ZZ,0,6);  'END';
V[2]:=Y[1]:=Y[1] + 2.0*TOL;

'FOR' JK:=1 'STEP' 1 'UNTIL' JMAX 'DO' 'BEGIN'
'IF'  V[2] 'NE' V[1] 'THEN' 'BEGIN'
Y[1]:=V[1] + 1.0*TOL;
OLTF(A,B,C,X,Y,REA,IMA,ADEN,1,L,M);
IX:=0 'END';
V[2]:=Y[2] :=0.40 + V[1];
OLTF(A,B,C,X,Y,REA,IMA,ADEN,2,L,M);

'IF' IMA[1] 'EQ' 0.0 'THEN' 'BEGIN'
ZZ:=-REA[1]/ADEN[1];
NEWLINE(1);  PRINT(ZX,3,0);  PRINT(X,2,1);
PRINT(Y[1],2,3);  PRINT(ZZ,0,6);  'END'
'ELSE'   ZZ:=IMA[2]/IMA[1];
'IF' IMA[1] 'GT' 0.0 'THEN' POZ:='TRUE' 'ELSE' POZ:='FALSE';

'IF' ZZ 'LT' 0.0 'THEN' 'BEGIN'
'FOR' V[1]:=(Y[1] + Y[2])/2.0  'WHILE'
ABS(Y[1] - Y[2]) 'GT' TOL 'DO' 'BEGIN'

OLTF(A,B,C,X,V,REA,IMA,ADEN,1,L,M);
'IF' IMA[1] 'GT' 0.0 'THEN' 'BEGIN'
'IF' POZ 'THEN' Y[1]:=V[1]  'ELSE'
Y[2]:=V[1]  'END' 'ELSE' 'BEGIN'
'IF' POZ 'THEN' Y[2]:=V[1]  'ELSE'
Y[1]:=V[1]   'END'  'END';

'IF' ADEN[1] 'LT' 10↑(-5) 'AND' REA[1] 'LT' 0.0
'THEN' 'BEGIN' ZZ:=10↑5;
NEWLINE(1);    SPACE(40);
PRINT(ZX,3,0);  PRINT(X,2,2);
SPACE(20);
PRINT(Y[1],2,3);  PRINT(ZZ,0,6);  'END';

'IF' ADEN[1] 'GE' 10↑(-5)  'AND'  REA[1] 'LE' 0.0
'THEN' 'BEGIN'  ZZ:=-REA[1]/ADEN[1];
NEWLINE(1);    SPACE(40);
PRINT(ZX,3,0);  PRINT(X,2,2);
PRINT(Y[1],2,3);  PRINT(ZZ,0,6);  'END'

'END' 'ELSE' 'BEGIN'

Y[1]:=V[1]:=Y[2];     REA[1]:=REA[2];
IMA[1]:=IMA[2];    ADEN[1]:=ADEN[2]
'END'   'END' JK LOOP;

'END' XLOOP;

NEWLINE(1);  WRITETEXT('('ROOT%LOCUS%BY%SCANNING')');

'END' BLOCK;  'END' PRGRM;
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Program SYSTEM

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'BEGIN' 'INTEGER' M,FL,MI,I,J,N,D,II;
'BOOLEAN' CPS;  'REAL' ZZ;

'PROCEDURE' BODE(AM,B,N);
'VALUE' N;  'INTEGER' N;  'ARRAY' AM,B;
'BEGIN' 'INTEGER' I,J,K,NN;  'REAL' PI,Q,SUM;
PI:=4.0*ARCTAN(1.0);  SUM:=0.0;
NN:=2*N;  'BEGIN'  'ARRAY' A[0:NN];
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
A[I]:=ABS(AM[I]);
A[I]:=LN(A[I]);     'END';
AM[N]:=ABS(AM[N]);
'FOR' I:=N+1 'STEP' 1 'UNTIL' NN 'DO' 'BEGIN'
A[I]:=AM[N]/(I-N);  A[I]:=LN(A[I]);  'END';
B[0]:=0.0;     SUM:=0.0;
'FOR' I:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
Q:=I*I;  SUM:=SUM+(A[1]-A[I])/Q; 'END';
B[1]:=2.0*(A[1]-A[0]-SUM)/PI;
'FOR' J:=2 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
SUM:=0.0;  K:=J-1;
'FOR' I:=0 'STEP' 1 'UNTIL' K 'DO' 'BEGIN'
Q:=(I*I/J - J);  SUM:=SUM + (A[I]-A[J])/Q; 'END';
K:=J+1;
'FOR' I:=K 'STEP' 1 'UNTIL' NN 'DO' 'BEGIN'
Q:=(I*I/J - J);  SUM:=SUM +(A[I]-A[J])/Q; 'END';
B[J]:=2.0*SUM/PI;     'END' JLOOP;
'END' BLOCK;  'END' PROC;


'VALUE' N;  'INTEGER' N;  'ARRAY' A,B;
'BEGIN' 'INTEGER' I;  'ARRAY' C[0:N];
'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'PROCEDURE' RANDI(A,B,N);
B[I]:=0.0;
C[I]:=A[I]*SIN(B[I]);  A[I]:=A[I]*COS(B[I]);
B[I]:=C[I];  'END' ILOOP; 'END' PROC;

'PROCEDURE' TRANSFIT(X,Y,CF,C,FF,K,M);
'VALUE' K,M,X,Y,FF;  'INTEGER' K,M;
'ARRAY'  X,Y,FF,CF,C;
'BEGIN' 'INTEGER' I,J,JK,JT,L,N;
'REAL' SUMW,SUMX,SUMY,SUMZ;
SUMX:=C[M+1]:=0.0;     N:=2*M;    PAPERTHROW;
'BEGIN'  'ARRAY' FS[1:K],TEMP1[1:N],TEMP2[1:N],TEMP3[1:N];

'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,J]:=0.0; 'END'; C[I]:=0.0; 'END';
CF[0,0]:=K+1;  WRITETEXT('('PROCEDURE%TRANSFIT')');
NEWLINE(1);    C[0]:=X[0];
'FOR'  JK:=1 'STEP' 1 'UNTIL'  K 'DO' 'BEGIN'
C[0]:=C[0] + X[JK];  FS[JK]:=FF[JK]  'END' KLOOP;
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
SUMW:=SUMX:=SUMY:=SUMZ:=0.0;  JT:=(J'/'2)*2;
'FOR' JK:=1 'STEP' 1 'UNTIL' K 'DO' 'BEGIN'
'IF' JT 'EQ' J 'THEN' 'BEGIN'
SUMX:=SUMX + FS[JK]*X[JK];
SUMW:=SUMW + FS[JK]
'END' 'ELSE' 'BEGIN'
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'FOR' I:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
'FOR' J:=0 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,J]:=0.0; 'END'; C[I]:=0.0; 'END';
CF[0,0]:=K+1;   WRITETEXT('('PROCEDURE%TRANSFIT')');
NEWLINE(1);     C[0]:=X[0];
'FOR'  JK:=1 'STEP' 1 'UNTIL'  K 'DO' 'BEGIN'
C[0]:=C[0] + X[JK];  FS[JK]:=FF[JK]  'END' KLOOP;
'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
SUMW:=SUMX:=SUMY:=SUMZ:=0.0;  JT:=(J'/'2)*2;
'FOR' JK:=1 'STEP' 1 'UNTIL' K 'DO' 'BEGIN'
'IF' JT 'EQ' J 'THEN' 'BEGIN'
SUMX:=SUMX + FS[JK]*X[JK];
SUMW:=SUMW + FS[JK]
'END' 'ELSE' 'BEGIN'
SUMZ:=SUMZ+FS[JK]*(X[JK]*X[JK]+Y[JK]*Y[JK])*FF[JK];
SUMY:=SUMY + FS[JK]*Y[JK]  'END';
FS[JK]:=FS[JK]*FF[JK];          'END'  JKLOOP;
TEMP1[J]:=SUMW;     TEMP3[J]:=SUMZ;
TEMP2[J]:=SUMX + SUMY;     'END' JLOOP;
NEWLINE(1);  WRITETEXT('('OMEGA%MATRIX')');
'FOR' I:=0 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
NEWLINE(1);
'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO'    'BEGIN'
CF[I,J]:=TEMP1[I+J];     PRINT(CF[I,J],0,6);  'END';
'END';  WRITETEXT('('OMEGA%MATRIX%END')');
'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO'
CF[I,0]:=TEMP1[I];        NEWLINE(1);
'FOR' I:=0 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
L:=-1;       NEWLINE(1);
'FOR' J:=0 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
'IF' CF[I,J] 'NE' 0 'THEN' 'BEGIN'
L:=-L; CF[I,J]:=L*CF[I,J]  'END';
PRINT(CF[I,J],0,5);  'END' JLOOP; 'END' ILOOP;
NEWLINE(1);  WRITETEXT('('LOWER%RIGHT%MATRIX')');

JK:=0;
'FOR' I:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,M]:=CF[M,I]:=TEMP2[I];      'END';

'FOR' I:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[0,I]:=TEMP2[I-M];        NEWLINE(1);
'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
CF[I,J-1]:=CF[I-M-1,M+J]:=TEMP2[J+JK];
CF[I,M+J]:=TEMP3[J+JK];  PRINT(CF[I,M+J],0,6);
'END' JLOOP;     JK:=JK+1;     'END' ILOOP;
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L:=0;  NEWLINE(2);  WRITETEXT('('LOWER%LEFT%MATRIX')');
'FOR' I:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
L:=L+1;  JK:=1;    NEWLINE(1);
'IF' (L'/'2)*2 'NE' L 'THEN''BEGIN' JK:=-1;
'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
CF[I,J]:=JK*CF[I,J];      PRINT(CF[I,J],0,6);
'IF' (J'/'2)*2 'EQ' J 'THEN' JK:=-JK; 'END';
JK:=0 'END' 'ELSE' 'BEGIN' JK:=1;
'FOR' J:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
CF[I,J]:=JK*CF[I,J];      PRINT(CF[I,J],0,6);
'IF' (J'/'2)*2 'NE' J 'THEN' JK:=-JK; 'END' JLOOP;
PRINT(I,2,0)  'END' CONDITION; 'END' ILOOP;
L:=0;  NEWLINE(1);  WRITETEXT('('UPPER%RIGHT%MATRIX')');
'FOR' I:=0 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
L:=L+1;  JK:=1;  NEWLINE(1);

'IF' (L'/'2)*2 'EQ' L 'THEN'  'BEGIN' JK:=-1;
'FOR' J:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,J]:=JK*CF[I,J];      PRINT(CF[I,J],0,6);
'IF' (J'/'2)*2 'NE' J 'THEN' JK:=-JK;
'END'  JLOOP; JK:=0
'END' 'ELSE' 'BEGIN' JK:=1;
'FOR' J:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,J]:=JK*CF[I,J];      PRINT(CF[I,J],0,6);
'IF' (J'/'2)*2 'EQ' J 'THEN' JK:=-JK;
'END' JLOOP;
JK:=0 'END' CONDITION; 'END' ILOOP;
NEWLINE(1);  WRITETEXT('('UPPER%RIGHT%MATRIX%END')');

'FOR' I:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
NEWLINE(1);
'IF' (I'/'2)*2 'EQ' I 'THEN' JK:=-1 'ELSE' JK:=1;
'FOR' J:=M+1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
CF[I,J]:=JK*CF[I,J];      PRINT(CF[I,J],0,6);
'IF' CF[I,J] 'EQ' 0 'THEN' JK:=-JK;
'END' JLOOP;   'END' ILOOP;
'FOR' I:=1 'STEP' 1 'UNTIL' M 'DO' C[I]:=TEMP2[I];
'FOR' I:=M+2 'STEP' 1 'UNTIL' N 'DO'
C[I]:=TEMP3[I-M-1];
NEWLINE(1);  WRITETEXT('('PROCEDURE%TRANSFIT%IS%DONE')');
'FOR' I:=0 'STEP' 1 'UNTIL' MT 'DO' 'BEGIN' NEWLINE(1);
'FOR' J:=0 'STEP'  1 'UNTIL' MT 'DO'  PRINT(CF[I,J],0,2);
PRINT(C[I],0,3);         'END' ILOOP;
'END' BLOCK;              'END' PROC;
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```
'PROCEDURE' PIVOT(A,Y,N);
'VALUE' N;  'INTEGER' N; 'ARRAY' A,Y;
'BEGIN' 'ARRAY' B[1:N,1:N+1];
         'INTEGER' M,I,J,K;  'REAL' PIV,TEMP;
        M:=N+1;        PAPERTHROW;
        'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'
        'BEGIN'  B[I,M]:=Y[I];  NEWLINE(1);
        'FOR' J:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
         B[I,J]:=A[I,J];    PRINT(B[I,J],0,3);
'END';
        SPACE(2);
        PRINT(B[I,M],0,4);          'END';
        'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'                             0
        'BEGIN'      PIV:=B[I,I];
        'FOR' J:=(I+1)  'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
        'IF'  ABS(PIV) < ABS(B[J,I]) 'THEN'
        'BEGIN' PIV:=B[J,I];
        'FOR' K:=1 'STEP' 1 'UNTIL' M 'DO' 'BEGIN'
        TEMP:=B[I,K]; B[I,K]:=B[J,K];  B[J,K]:=TEMP;
             'END' PIV:=B[I,I];  'END';
'END';
        'FOR' K:=M 'STEP' (-1) 'UNTIL' I 'DO'
         B[I,K]:=B[I,K]/B[I,I];
        'FOR' J:=(I+1) 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
        'FOR' K:=M 'STEP' (-1) 'UNTIL' I 'DO'
         B[J,K]:=B[J,K]-B[I,K]*B[J,I];    'END';
          'END';

         Y[N]:=B[N,M];
        'FOR' I:=(N-1) 'STEP' (-1) 'UNTIL' 1 'DO'
        'BEGIN' Y[I]:=B[I,M];
        'FOR' K:=N 'STEP' (-1) 'UNTIL' (I+1) 'DO'
    'BEGIN'
         Y[I]:=Y[I]-B[I,K]*Y[K];
'END' KLOOP;    'END' ILOOP;  NEWLINE(1);
WRITETEXT('('COMPUTED%VALUES')');
NEWLINE(4);  SPACE(6);
        'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO' 'BEGIN'
WRITETEXT('('X')');  PRINT(I,1,0);  SPACE(8);  'END';
NEWLINE(1);  SPACE(4);
        'FOR' I:=1 'STEP' 1 'UNTIL' N 'DO'  PRINT(Y[I],0,4);
'END' PROC;
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'PROCEDURE' RESPONSE(B,M,FL,X,Y,F,CPS);
'INTEGER'  M,FL;  'ARRAY' B,X,Y,F;
'BOOLEAN' CPS;
'BEGIN' 'INTEGER' I,MH,K,MT;
'REAL' FF,AN,BN,AD,BD,PI;
MH:=M'/'2;  MT:=2*M + 1;     'BEGIN'
'ARRAY' B1[1:MH],B2[1:MH],B3[1:MH],B4[1:MH];
FF:=B[M+1];    WRITETEXT('('FREQUENCY%RESPONSE')');
'FOR' I:=0 'STEP' 1 'UNTIL' MT 'DO' 'BEGIN'
B[I]:=B[I]/FF;                              'END';
PI:=4.0*ARCTAN(1.0);
'IF' CPS 'THEN' 'BEGIN'
'FOR' I:=0 'STEP' 1 'UNTIL' FL 'DO'
F[I]:=2.0*PI*F[I];  I:=0  'END';
AN:=B[0];  AD:=1.0;  BN:=BD:=0.0;  FF:=1.0;
'FOR' K:=1 'STEP' 1 'UNTIL' MH 'DO' 'BEGIN'
I:=2*K;  B2[K]:=B[I-1];     B1[K]:=B[I];
         B4[K]:=B[M+I];  B3[K]:=B[M+1+I];  'END';
K:=-1;  X[0]:=B[0];  Y[0]:=0.0;
'FOR' I:=1 'STEP' 1 'UNTIL' MH 'DO' 'BEGIN'
B1[I]:=K*B1[I];     B3[I]:=K*B3[I];  K:=-K;
B2[I]:=K*B2[I];     B4[I]:=B4[I]*K;     'END';
NEWLINE(1);  WRITETEXT('('DOUBLE')');  FF:=1.0;

'FOR' I:=1 'STEP' 1 'UNTIL'  FL 'DO' 'BEGIN'
'FOR' K:=1 'STEP' 1 'UNTIL' MH 'DO' 'BEGIN'
FF:=FF*F[I];
BN:=BN + B2[K]*FF;     BD:=BD + B4[K]*FF;
FF:=FF*F[I];
AN:=AN + B1[K]*FF;     AD:=AD + B3[K]*FF;      'END' KLOOP;
PI:=AD*AD + BD*BD; FF:=1.0;
X[I]:=(AN*AD + BN*BD)/PI;
Y[I]:=(AN*BD - AD*BN)/PI;   Y[I]:= -Y[I];
NEWLINE(1);  PRINT(I,2,0); SPACE(4);
PRINT(F[I],0,6);  SPACE(4);
AN:=SQRT(Y[I]*Y[I] + X[I]*X[I]);
AD:=ARCTAN(Y[I]/X[I]);
PRINT(X[I],0,6);  PRINT(Y[I],0,6);
PRINT(AN,0,6);     PRINT(AD,0,6);
AN:=B[0];  AD:=1.0;  BN:=BD:=0.0;   'END' ILOOP;
NEWLINE(1);  WRITETEXT('('CHECKING')');
'END' BLOCK;                'END' PROC;
```

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