

# Volatility Transmission between Stock and Bond Markets

by

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## Abstract

A two-factor no-arbitrage model is used to provide a theoretical link between stock and bond market volatility. While this model suggests that short-term interest rate volatility may, at least in part, drive both stock and bond market volatility, the empirical evidence suggests that past bond market volatility affects both markets and feeds back into short term yield volatility. The empirical modelling goes on to examine the (time-varying) correlation structure between volatility in the stock and bond markets and finds that the sign of this correlation has reversed over the last twenty years. This has important implications for portfolio selection in financial markets.

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## 1 Introduction

The world-wide downturn in equity prices in October 1987 focussed academic and practitioner attention on to the international transmission of financial market volatility. It was clear at that time that shocks were being transmitted around the global trading system. Evidence of an international volatility contagion effect was documented by King and Wadhvani (1990), who found that the correlation between market movements in different countries and general levels of volatility were positively related. Understanding the nature of linkages between financial markets, whether intra- or international, is fundamental to establishing the limits of diversification, to security pricing, and to successful asset allocation. While there is a large literature examining the international transmission of equity market volatility, and a growing literature examining the international transmission of bond market volatility, there are relatively few intra-national studies, and then usually within one asset class. By contrast, this study aims to explore the intra-national transmission of volatility between short-term risk-free yields, long-term bond yields and equity returns in the UK.

During the period immediately following the 1987 equity market crash, the flow of investment funds out of the equity market and into the gilt-edged market was substantial. The Stock Exchange (1988) reported that gilt-edged market average customer turnover reached a record £3,114 million per day during November 1987, following the record average customer turnover of £1,342 million per day during October 1987 in the equity market. In fact, during the second quarter of 1987, gilt-edged market turnover had declined. It was not until 1993, that turnover in either of the markets reached the levels observed during 1987. Indeed, during the intervening period, total average daily turnover values in each of the markets have been around one half of the levels experienced in the immediate post-crash period, see Stock Exchange (1994).

Over the period October 1 to November 30, 1987, prices in the equity market fell at an annualised rate of nearly 600 percent, while prices in the gilt-edged market rose at an annualised

rate of nearly 40 percent. These observations and the Exchange's report on turnover activity suggested a clear link between the behaviour of the two markets at that time. The more recent Asian crisis in global financial markets during the late Nineties had a similar impact in the the gilt-edged market; Steeley and Ahmad (2002) document the empirical effects of the gilt-edged market becoming a safe-haven for international capital during this period. While these are both "headline" market events, the long term nature of any relationships between the behaviour of prices and returns in the two markets over a longer time span has not received the same attention. In particular, there has been no systematic documentation of the relationship between return volatility in the two markets. It is the aim of this study to examine the nature of the dynamic relationships between equity and bond price movements both in theory and practice in the UK, with particular reference to the time series behaviour of the processes capturing the volatility in each of the two markets.

A number of studies have examined the interdependence of equity market volatility, typically using the framework of generalised autoregressive conditional heteroscedasticity (GARCH) time series models, for example, Hamao et al. (1990) and Koutmos and Booth (1995). Hamao et al. (1990) discovered that shocks to the volatility of financial market returns in one country could influence both the conditional volatility and the conditional mean of the returns in another country, while Koutmos and Booth observed asymmetric volatility relations between the financial markets of the USA, the UK and Japan, where the influence of negative shocks was different in both scale and direction to positive shocks. This kind of volatility asymmetry has become known as the "leverage effect" (Black (1976) and Christie (1982)), since an increase in a firm's debt to equity ratio will lead to both an increase in the risk and required return on equity that, *ceteris paribus*, will reduce the value of equity. Studies by Bekaert and Wu (2000) and Brailsford and Faff (1993) are representative of the global nature of this empirical phenomenon.

The GARCH modelling framework has also been applied to analysing volatility spillovers between equity portfolios for a single country, sorted by market capitalisation. Studies by Conrad et al. (1991) and Kroner and Ng (1998) for the US equity market, and Chelley-Steeley and Steeley (1996) for the UK equity market have found a further form of asymmetry in the transmission of volatility. While past shocks to the volatility of large firm portfolios appeared to influence the volatility of small firm portfolios, the reverse was not found to be the case. Alli et al. (1994) have applied the same technique to examine volatility spillovers between different sectors of the US oil industry.

In parallel with this analysis, an increasing number of studies have examined changes in the correlation among worldwide equity markets. Both Longin and Solnik (1995) and Chelley-Steeley and Steeley (1999) document increases in correlations among European countries' equity markets since the 1970s. Goetzmann, Li and Rouwenhorst (2001) found that international equity correlations change dramatically through time, with peaks in the late 19th Century, the Great Depression, and the late 20th Century.

By contrast to studies of global equity markets, analyses of the interdependence of international bond markets are relatively few in number. Ilmanen (1995) used a linear regression model to forecast the excess returns of long-term international bonds. The excess returns were found to be highly correlated indicating considerable integration among international bond markets. Clare and Lekkos (2000) used a VAR model to measure the interaction between US, UK and German bond markets, and found that transnational factors were more important during times of instability. Driessen et al (2003) analyze the bond markets of US, Japan and Germany using a principal components analysis.

Bond markets, however, have been the setting for some of the key developments in GARCH methods, such as the ARCH-M model (Engle et al, 1987) and the Factor-ARCH model (Engle et al, 1990), and ARCH methods were also used to examine the properties of certain theoretical models of the yield curve, (see Steeley, 1990 and Chan et al, 1992). The application

of these methods to the study of bond market integration has, however, been more recent (see Laopodis, 2002, Christiansen, 2004 and Skintzi and Refenes, 2005). While Laopodis (2002) and Christiansen (2004) assume constant correlation structures, Skintzi and Refenes (2005) model a time varying (parametric) correlation structure among bond market volatilities, using a model previously applied to foreign exchange by Darbar and Deb (2002). In this study, I also use a GARCH modelling framework to examine the interdependence between stock, bond and interest rate volatility. The model will include volatility spillovers and asymmetries and a time-varying (non-parametric) correlation structure, similar to that used by Berben and Jansen (2002) to study international equity market integration.

The remaining sections of the paper are as follows. Section 2 works through a no-arbitrage model that provides a natural link between the volatility of stocks, bonds and short term interest rates. Section 3 describes the GARCH modelling framework that will be employed. In Section 4, summary statistics for the data are reported, along with an analysis of the estimated coefficients of the GARCH models. Section 5 contains the conclusions of the study.

## **2 A model of volatility integration**

To underpin the empirical analysis, I first explore a model of the theoretical relation between the volatility of short term interest rates, long term interest rates (on default-free debt instruments), and equity. This model is based on models developed by Merton (1974) and Shimko et al (1993), which study the relation between the default-risk premia of corporate bonds and the stochastic behaviour of firm value, and uses the no-arbitrage framework of Black and Scholes (1973). Thus, the model views equity holders as owning a call option on the asset value of the firm that has an exercise price equal to the face value of the firm's (risky) zero-coupon debt.

Consider then a simple firm that issues (zero-coupon) bonds with a face value of  $D$  and maturity  $T$  secured on the assets of the firm,  $V$ . We assume that investors agree on the following (geometric Brownian Motion) stochastic process describing the evolution of firm value,  $V(t)$ ,

$$dV = \mu V dt + \sigma V dw \quad (1)$$

where  $dV = \lim_{\Delta t \rightarrow 0} (V(t + \Delta t) - V(t))$ , where  $\mu$  and  $\sigma$  are the instantaneous mean and standard deviation, respectively, of the proportional change in firm value,  $dV/V$ , and where  $dw$  are increments to a standard Wiener process (a random process whose values are independently and identically normally distributed with mean zero and variance equal to  $t$ ).

As the equity of the firm is a call option on the value of the firm, we can apply Ito's Lemma to equation (1) to find the instantaneous standard deviation of the rate of return on equity ("equity volatility"). Specifically,

$$\text{Std. Dev.} \left( \frac{dS}{S} \right) \equiv \sigma_s = \sigma \frac{V}{S} \frac{\partial S}{\partial V} \quad (2)$$

where  $S$  is the market value of the firm's equity.

Under an assumption of constant short-term interest rates, the Black-Scholes (1973) formulae for the price and delta of a (European) call option can be used to make substitutions for the value of equity,  $S$ , and for  $\partial S/\partial V$  in equation (2), and so

$$\sigma_s = \frac{VN(d_1)}{VN(d_1) - D e^{-r_f T} N(d_2)} \sigma \quad (3)$$

where  $D e^{-r_f T}$  is the present (risk-free) value of the face value of the firm's debt,  $N(x)$  is the cumulative normal probability of the unit normal variate,  $x$ ,  $d_1 = (\ln(V/D) + (r_f + 0.5\sigma^2)T)/\sigma\sqrt{T}$  and  $d_2 = d_1 - \sigma\sqrt{T}$ .

Equation (3) can be rearranged as

$$\sigma_s = \frac{1}{1 - (D/V)e^{-r_f T} [N(d_2)/N(d_1)]} \sigma \quad (4)$$

and since  $(D/V) \leq 1$ ,  $e^{r^*T} < 1$ , and  $N(d_2) \leq N(d_1)$ , the variance of return on equity must be greater than the variance of the proportional change in firm value. Moreover, as reductions in firm value reduce the ratio  $N(d_2)/N(d_1)$  proportionately less than the ratio  $(D/V)$  increases, equity return volatility will rise. Since, also  $N(d_1) = \partial S / \partial V > 0$ , reductions in firm value must also reduce the value of equity. Taken together, the risk and return on levered equity will be negatively correlated, that is, exhibit the "leverage effect".

In order to integrate interest rate volatility and stock market volatility, it is necessary to replace the constant interest rate assumption in the Black-Scholes (1973) and Merton (1974) models, with a stochastic process for the short term interest rate. Shimko et al (1993) suggest incorporating the framework of Vasicek (1977) and assuming that the instantaneous short interest rate,  $r$ , follows an Ornstein-Uhlenbeck process, that is,

$$dr = \kappa(r^* - r)dt + \sigma_r dz \quad (5)$$

where  $dr$  is the instantaneous change in output, that is,  $dr = \lim_{\Delta t \rightarrow 0} (r(t + \Delta t) - r(t))$ , and where  $dz$  are the increments of another standard Weiner process, which need not be uncorrelated with  $dw$ .<sup>1</sup> Short term interest rates are assumed, therefore, to be drawn towards a long term mean value,  $r^*$ , at a rate  $\kappa$ , while the instantaneous standard deviation of the interest rate ("interest rate volatility") is given by  $\sigma_r$ . Equation (5) is intuitively reasonable for countries where short term interest rates are adjusted in response to perceived changes in a longer term policy target.

Vasicek (1977) shows that equation (5), Ito's Lemma and no-arbitrage imply that long term default-free (zero-coupon) bond prices,  $P_T$ ,

$$P_T = \exp(b_T(R_\infty - r) - R_\infty T - 0.25b_T^2\sigma_r^2/\kappa) \quad (6)$$

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<sup>1</sup> In earlier drafts of this paper, the two Weiner processes were assumed to be independent.

where  $b_T = (1 - \exp(-\kappa T))/\kappa$ , and where  $R_\infty$  is a constant that depends on the parameters of equation (5) and the constant market price of interest rate risk.<sup>2</sup> From the correspondence between bond prices and yields,  $R_T = -\ln(P_T)/T$ , long term yields are given by

$$R_T = -(b_T(R_\infty - r) - R_\infty T - 0.25b_T^2\sigma_r^2/\kappa)/T \quad (7)$$

Thus, the instantaneous variance of long term yields ("bond market volatility") is given by

$$\text{Var}(R_T) = \left(\frac{b_T}{T}\right)^2 \sigma_r^2 \quad (8)$$

and since  $b_T < T \quad \forall \kappa > 0$ , long term yields will exhibit lower variance than short term interest rates. A graph of the function  $b_T/T$  is given in Figure 1, and indicates that long term yields will show less volatility than short term yields.

Returning to equity volatility, it is now assumed that stock returns depend on current firm value, short term interest rates and time, so we can use the multi-dimensional version of Ito' Lemma to relate the processes for  $r$ ,  $V$  and  $S$ , that is,

$$dS = \frac{\partial S}{\partial r} dr + \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial t} dt + \frac{1}{2} \frac{\partial^2 S}{\partial r^2} \sigma_r^2 dt + \frac{1}{2} \frac{\partial^2 S}{\partial V^2} \sigma^2 dt + \frac{\partial^2 S}{\partial r \partial V} \rho \sigma \sigma_r dt \quad (9)$$

where the (constant) correlation between the Wiener processes is given by  $dwdz = \rho dt$ .

On dividing both sides of (9) by  $S$  and calculating the variance of the resulting equity returns,  $dS/S$ , one obtains

$$\sigma_S^2 = \text{Var}\left(\frac{\frac{\partial S}{\partial r} \sigma_r}{S} dz + \frac{\frac{\partial S}{\partial V} \sigma V}{S} dw\right) = \left(\frac{\left(\frac{\partial S}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial S}{\partial V}\right)^2 \sigma^2 V^2 + 2 \frac{\partial S}{\partial r} \frac{\partial S}{\partial V} \sigma_r \sigma V \rho}{S^2}\right) dt \quad (10)$$

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<sup>2</sup> See Vasicek (1977), p.181.

Under stochastic interest rates, the value of equity, which is given by the Black Scholes (1973) formula in the case of constant interest rates, becomes instead

$$S = VN(d_1^*) + D^*N(d_2^*) \quad (11)$$

where, now,  $d_1 = (\ln(V/D^*) + 0.5T^*)/\sqrt{T^*}$  and  $d_2 = d_1 - \sqrt{T^*}$ , where  $T^*$ , which is the variance of  $D$  over its time to maturity, is a function of  $\sigma$ ,  $\sigma_r$ ,  $\rho$ ,  $\kappa$  and  $T$ , and  $D^*$  is the present value of  $D$  evaluated under the stochastic (varying) short term interest rate.<sup>3</sup>

From (11), it can be established that  $\partial S/\partial V = N(d_1^*)$  and  $\partial S/\partial r = b_T D^* N(d_2^*)$  and so, the equivalent expression to (3), with stochastic interest rates, is

$$\sigma_S = \frac{\left( [b_T D^* N(d_2^*)]^2 \sigma_r^2 + N(d_1^*)^2 \sigma^2 V^2 + 2b_T D^* N(d_2^*) N(d_1^*) \sigma_r \sigma V \rho \right)^{\frac{1}{2}}}{VN(d_1^*) + D^* N(d_2^*)} \quad (12)$$

where, now, the combined impact of changes to the volatility of both interest rates and firm value on equity volatility is considerably more complex.

Thus, while this theoretical framework provides a sound basis for examining the links between interest rate, bond market and equity market volatility, establishing the characteristics for a particular case necessarily becomes an empirical problem. Moreover, the empirical framework that is described below permits further generalizations to (i) time varying correlations between the two sources of risk, and (ii) richer dynamics of the relationships between the three volatility measures.

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<sup>3</sup> See Shimko et al (1993) for the detail.

### 3 Modelling Volatility Linkages

The generalised autoregressive conditional heteroscedasticity (GARCH) family of statistical processes (Engle, 1982 and Bollerslev, 1986) is used to model the variance processes of the returns in the three markets.<sup>4</sup> Specifically, the basic model is

$$R_{i,t} = \alpha_{i,1} + \alpha_{i,2}Tue_t + \alpha_{i,3}Wed_t + \alpha_{i,4}Thur_t + \alpha_{i,5}Fri_t + \sum_{j=1}^3 \beta_{i,j}R_{j,t-1} + \delta_{i,1}C87_t + \delta_{i,2}C98_t + \varepsilon_{i,t} \quad (13)$$

where  $\varepsilon_{i,t} | \Omega_{t-1} \sim N(0, h_{i,t})$ , and

$$h_{i,t} = m_i + b_i \varepsilon_{i,t-1}^2 + c_i h_{i,t-1} + g_{i,t} D_{i,t} \varepsilon_{i,t-1}^2 \quad (14)$$

where  $R_{i,t}$  is daily the return from a particular market in week  $t$ ,  $m_i > 0$ ,  $b_i, c_i \geq 0$ ,  $b_i + c_i < 1$ , and  $\Omega_{t-1}$  is the set of all available information at time  $t - 1$ . The constant and the dummy variables  $Tue_t$ ,  $Wed_t$ ,  $Thur_t$ , and  $Fri_t$  are used to pick up any day-of-the-week effects in either of the markets. Although early studies by Board and Sutcliffe (1988) and Choy and O'Hanlon (1989) documented evidence in the UK equity market that the average returns on particular week days are significantly different from each other, studies using more recent data, for example, Steeley (2001) and Ahmad (2004) have suggested otherwise. The variables  $C87_t$ ,  $C98_t$  control for the effects of the 1987 stock market crash and the 1998 stock market falls. Specifically,  $C87_t = 1$  between 19 October 1987 and 30 October 1987 and takes the value zero otherwise, and  $C98_t = 1$  between the FTSE100 index market peak of 6179 on 20 July 1998 and the trough of 4648 on 5 October 1998 and takes the value zero otherwise.

The form of the variance equation in equation (14) is known as a GARCH(1,1) specification, since the conditional variance is a function of its past values and past squared residuals in only the immediate past period, that is, at lag 1. In this model, the coefficient  $b$

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<sup>4</sup> See, also, the survey paper by Bollerslev et al (1992).

measures the tendency of the conditional variance to cluster, while the coefficient  $c$  (in combination with  $b$ ) measures the degree of persistence in the conditional variance process. The coefficient  $g$  captures the "leverage" effect; the dummy variable  $D_{i,t}$  takes the value of one when  $\varepsilon_{i,t-1} < 0$  and zero otherwise. Specifically, this allows for the possibility of a negative relationship between returns and volatility, and implies bad news shocks will have a greater impact on volatility than good news shocks. The leverage effect has been documented in many equity markets including in the UK, for example, Chelley-Steeley and Steeley (1996), and so is included in the specification here.<sup>5</sup>

To capture volatility transmission effects within the GARCH framework, it is possible to augment the conditional variance equation of the GARCH model with terms representing volatility shocks in another market. Thus, and also recognizing the possibility of asymmetries in the variance specification, equation (14) becomes

$$h_{i,t} = m_i + b_i \varepsilon_{i,t-1}^2 + c_i h_{i,t-1} + g_i D_{i,t} \varepsilon_{i,t-1}^2 + \sum_{j \neq i} k_{i,j} \varepsilon_{j,t-1}^2 + \sum_{j \neq i} l_{i,j} D_{j,t} \varepsilon_{j,t-1}^2 \quad (15)$$

where the coefficients  $k_{i,j}$  measure the impact of shocks to the volatility in market  $j$  on the conditional volatility in market  $i$ . The coefficients  $l_{i,j}$  determine whether this relationship is asymmetric such that negative shocks in market  $j$  have a bigger impact on the conditional volatility in market  $i$  than do positive shocks. Other terms are as previously defined.

The estimation of equation (15) for a given a market can be accomplished in one of two ways. In the first method, the volatility shock series from the other markets,  $\varepsilon_{j,t-1}$ , are estimated using equation (14). These then enter equation (15) as exogenous variables. While it would be possible to thereafter engage in an iterative process to update the estimated volatility series, a more convenient method to improve the efficiency of the estimation is to estimate equation (15) for each market simultaneously, using a multivariate extension of the GARCH framework.

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<sup>5</sup> The leverage effect was first introduced into a GARCH framework by Nelson (1991). The specification used here was suggested by Glosten et al (1993) and is recommended against many alternatives by Engle and Ng (1993). Although the term "leverage effect" is not applicable to bond and money market volatilities, this does not preclude the possibility that returns and volatility in these markets are negatively correlated also. Hence, this asymmetry captured by the Glosten et al (1993) model is applied to each of the three markets analyzed here.

In this multivariate frameworkcase, it is also necessary to model the covariances between the volatility processes. Perhaps the simplest possible specification is the constant conditional correlation model proposed by Bollerslev (1987), whereby

$$h_{ij,t} = \rho_{ij} \sqrt{h_{i,t} h_{j,t}} \quad \forall i \neq j \quad (16)$$

As an aim of this study is to examine how (and indeed whether) the correlation between the three markets evolves through time, a more general specification for the correlation process is required.

Within a multivariate setting it is possible, in principle, for each conditional variance or covariance term to depend on all the lagged variance and covariance terms, which would generate around 50 parameters within even the most basic GARCH(1,1) specification. Although a number of "intermediate" specifications have been examined in the literature, see for example, Chowdhury, Kroner and Sultan (1996) and the survey in Kroner and Ng (1998), these can still involve the estimation of large numbers of parameters. Instead, in this paper, an alternative approach to introducing time varying correlation was adopted. This is a straightforward generalisation of the constant conditional correlation assumption to one of there being two correlation regimes that the markets move between during the sample period. Intuitively, this is like generalizing a constant to a weighted average, where the weights change throughout the sample.

While the simplest of linear weights could be used, Berben and Jansen (2002) have recently explored weights based on the logistic function

$$G(t; \gamma_{i,j}, \psi_{i,j}) = \frac{1}{1 + \exp(-\gamma_{i,j}(t - \psi_{i,j}))} \quad (17)$$

such that the conditional correlation process becomes

$$h_{i,j,t} = ((1 - G(t, \gamma_{i,j}, \psi_{i,j}))\rho_{0,i,j} + G(t, \gamma_{i,j}, \psi_{i,j})\rho_{1,i,j})\sqrt{h_{i,t}h_{j,t}} \quad \forall i \neq j \quad (18)$$

and there is now a smooth transition between the correlation regimes,  $\rho_{0,i,j}$  and  $\rho_{1,i,j}$ .<sup>6</sup>

This smooth-transition correlation GARCH (STC-GARCH) model can capture a wide range of patterns of change. If  $\rho_{0,i,j}$  and  $\rho_{1,i,j}$  differ, correlations move monotonically upward or downward. The change between the regimes is more gradual for smaller values of  $\gamma_{i,j}$  and more sudden for larger values of this parameter. The parameter  $\psi_{i,j}$  represents the "mid-point" of the transition between regimes. For estimated values of  $\psi_{i,j}$  around the middle of the sample period, the transition function would appear, for example, as a straight line for  $\gamma_{i,j} = 1$ , and S-shape for  $\gamma_{i,j} = 5$  and a step function for  $\gamma_{i,j} = 100$ . The constant correlation model are the special cases that  $\rho_{0,i,j} = \rho_{1,i,j}$ ,  $\gamma_{i,j} = 0$  or both. Hence, this model provides a simple framework to test for time-varying correlation between markets.

## 4 Data and Results

Daily closing observations on the FTSE-100 share price index (FT100), to represent stock returns, the index of prices of long term (more than 15 years to maturity) Government Stocks (FTLG), representing the return on long term bonds, and the index of prices of short term (less than 5 years to maturity) government stocks (FTSG), to represent short-term risk-free yields, were obtained for the twenty-year period June 1984 - June 2004, providing some 5050 observations for each series. Returns series are calculated as log differences in the respective price index.<sup>7</sup>

Table 1 contains summary statistics for the returns series in each of the markets. The average growth in the long term gilt-edged market over the sample period was positive and

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<sup>6</sup>In the parametric approach of Skintzi and Refenes (2005), the correlation process is specified as a logistic transformation of a GARCH(1,1)-style relation for the conditional covariance between markets.

<sup>7</sup>The data were obtained from Datastream (codes FTSE100, FTBGSHT, FTBGLNG). Short term bond returns were preferred to returns on money market instruments or short term interest rate measures due to the stronger influence of monetary policy discreteness and in some cases the inter-bank credit risk.

represents an annualised rate of about 2.44 percent, while the short term gilt index showed an average annualised growth rate of 0.36 percent. The average growth rate on the FTSE 100 share index over the sample is an annualised rate of 7.40 percent. The equity return series has the greatest standard deviation, and the estimates reflect the well understood differences in risk between the three markets. The distribution of the long term gilt returns series is more symmetric and less leptokurtic than the equity returns series, while the short term gilt series also displays high kurtosis. These findings strongly accord with previous studies, such as Poon and Taylor (1992) for the UK equity market, and Steeley (1992) for the gilt-edged market.

In all of the three series, there is evidence of significant first order autocorrelation, but the autocorrelation function shows a fairly rapid decay in all cases. The cross serial correlations indicate that past movements in short term interest rates affect both the long term gilt-edged market and the stock market, and with higher significance that past movements in long term yields affect current movements in short term yields. This latter result confirms well understood theories regarding the forward-looking behaviour of long term bond yields.

A returns series that has either a changing conditional or unconditional variance will exhibit high levels of autocorrelation among its squared and absolute returns. Autocorrelation of this type suggests that large absolute returns are more likely to be followed by large absolute returns than by small absolute returns. This is known as variance clustering and was first identified in US equity returns by Fama (1965). There is strong evidence of variance clustering in the equity returns and weaker, though still significant, evidence in the long term and short term gilt returns.

Maximum likelihood estimates of the parameters in equations (13) and (14) were obtained using the Berndt, Hall, Hall and Hausman (1974) algorithm, and are reported on the left side of Table 2. First, there is clear evidence that the returns in all the three markets are different across the days of the week. The coefficients that capture cross serial correlation in the returns series confirm the picture established in Table 1. The 1987 equity market crash had a strongly significant positive effect on gilt yields, as well as the expected negative effect on equity returns,

while the 1998 market downturn affected only long term gilts (positively) and equities (negatively). The parameters of the variance processes indicate that the long term yield volatility series is the most persistent, although all are highly persistent. The leverage effect is present in the equity market, and also in long term bond yields. By contrast, there is an inverse leverage effect in short term yield volatility, whereby negative shocks tend to reduce future volatility.

The lower panel on the left side of Table 2 provides estimates of the contemporaneous and serial cross correlations of the  $\varepsilon_{i,t}^2$  from this model. There appears to be significant correlation among the squared returns, indicating a clear link between volatility in the three markets. Furthermore, past long term yield volatility appears to influence current volatility in all three markets, and past short term yield volatility also influences current equity volatility. The right side of Table 2 reports the results of re-estimating the GARCH models, with equation (15) substituting for equation (14), and using the residuals from equation (14) as additional variables in (15). Again the influence of past long yield volatility is clear, with it affecting both the sign and magnitude of current short rate volatility. By contrast, and contrary to the theoretical model that has the short term interest rate as the driving force, volatility shocks to the short term interest rate do not seem to spillover into the other markets. The estimated values of the other parameters in this revised models are very similar to those obtained from the model without the inclusion of the volatility spillover terms.

In Table 3, the estimated coefficients are reported for estimating the GARCH models for all three markets together, using an multivariate extension of the GARCH modelling approach that also specifies the correlation structure between the three volatility processes. On the left side of the table, the correlation is assumed to be fixed throughout the sample period while on the right side of the table, the correlation is allowed to vary according to a smooth transition between two regimes. Since, the parameter estimates for variables that are common to the models in Table 2 are very similar to those obtained there, and retain the same interpretations, this discussion will focus on the estimated correlation structures. For the smooth transition model, the correlation between short and long term yield volatilities appears remarkably stable changing

only from around 0.75 to 0.60 through the sample. By contrast, the correlations of each of these volatilities and equity market volatility have reversed in sign during the sample period. This suggests that whereas stock and bond volatility used to move together, they now tend to move in opposite directions. This transition seems to have taken place around the time of the aftermath of the Asian crisis and the beginnings of the technology stock bubble, when equity markets have appeared relatively more volatile while interest rates, whose level has been falling, have appeared more stable. These effects are clearly shown in Figure 2 that displays the transition processes for the three correlation pairs and maps this onto the path of the equity index over the sample period. This analysis clearly demonstrates the importance of modelling correlation structures using time varying specifications.

## **5 Conclusions**

In this study, a theoretical model was used to provide the basis for examining the links between the volatility of short term yield, long term bond yields and stock returns. The empirical analysis used a GARCH framework that permitted richer structures than could be analyzed using the theoretical model. In particular, the impact of dynamic spillovers and time-varying correlations among the volatility processes could be examined. The time-varying correlations used a non-parametric smooth transition process that allowed the correlation between market shocks to evolve across the sample period.

Using data for the UK stock and bond markets, it was found that the correlation between short term yield shocks and long term bond yield shocks was relatively stable during the sample period, while the correlation between each of these markets and the equity market reversed sign. This clearly has important implications regarding the increased hedging potential of the bond market market in recent years, as the correlations among market shocks are now strongly significantly negative. It also makes apparent the importance of permitting correlation structures

to evolve within empirical specifications. While this paper has considered only one country, it could easily be applied to other countries, and across countries, where modelling time varying correlation structures is also likely to be a key factor. Such applications are left for future research.

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**Table 1: Summary Statistics of Returns**

|       | N.Obs. | $\bar{x} \times 10^3$ | $s(x) \times 10^3$ | Skew.  | Kurt.  | Min.   | Q1     | Med.  | Q3    | Max.  |
|-------|--------|-----------------------|--------------------|--------|--------|--------|--------|-------|-------|-------|
| FTSG  | 5050   | 0.0142                | 1.9840             | -1.239 | 14.921 | -0.020 | -0.001 | 0.000 | 0.001 | 0.021 |
| FTLG  | 5050   | 0.0957                | 5.7784             | -0.167 | 2.9512 | -0.039 | -0.003 | 0.000 | 0.003 | 0.037 |
| FT100 | 5050   | 0.2838                | 10.617             | -0.746 | 10.142 | -0.130 | -0.005 | 0.001 | 0.006 | 0.076 |

Autocorrelations of Returns at Lag

|       | 1     | 2      | 3      | 4      | 5      | 6      | 7      | 8     | 9      | 10     |
|-------|-------|--------|--------|--------|--------|--------|--------|-------|--------|--------|
| FTSG  | 0.063 | 0.016  | 0.008  | -0.022 | 0.048  | -0.025 | 0.006  | 0.019 | -0.008 | 0.052  |
| FTLG  | 0.053 | -0.012 | -0.018 | -0.013 | 0.011  | 0.019  | -0.010 | 0.037 | -0.004 | 0.014  |
| FT100 | 0.035 | -0.026 | -0.049 | 0.043  | -0.012 | 0.034  | 0.010  | 0.046 | 0.027  | -0.009 |

Autocorrelations of Squared Returns at Lag

|       | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| FTSG  | 0.051 | 0.058 | 0.052 | 0.014 | 0.012 | 0.008 | 0.015 | 0.013 | 0.004 | 0.040 |
| FTLG  | 0.086 | 0.057 | 0.085 | 0.070 | 0.077 | 0.087 | 0.048 | 0.056 | 0.034 | 0.074 |
| FT100 | 0.507 | 0.289 | 0.178 | 0.177 | 0.146 | 0.110 | 0.090 | 0.128 | 0.097 | 0.112 |

Cross Serial Correlations of Returns at Lag

|            | -1     | 0     | 1      |
|------------|--------|-------|--------|
| FTSG-FTLG  | -0.001 | 0.658 | 0.084  |
| FTSG-FT100 | 0.033  | 0.125 | -0.001 |
| FTLG-FT100 | 0.033  | 0.103 | -0.002 |

## Notes:

The returns are calculated from daily observations on the FTA Government Stocks (<5 years) Index (FTSG), the FTA Government Stocks (>15 years) Index (FTLG), and the FTSE 100 Share Index (FT100), between July 7, 1984 and July 6, 2004.  $\bar{x}$ ,  $s(x)$ , Skew and Kurtosis are the sample mean, standard deviation, skewness and kurtosis of the returns series. Min., Max., and Med., are the two extreme and the central values of the sample distribution, and Q1 and Q3 capture the inter-quartile range. The cross autocorrelation at lag  $\tau$  is the correlation coefficient between the return of the first named series in period  $t$  and the return of the second named series in period  $t - \tau$ . For normally distributed returns, the (5 percent) critical value for the autocorrelation coefficients is 2.81 percent.

**Table 2: Univariate GARCH Models**

This table contains the estimated coefficients from the model

$$R_{i,t} = \alpha_{i,1} + \alpha_{i,2}Tue_t + \alpha_{i,3}Wed_t + \alpha_{i,4}Thur_t + \alpha_{i,5}Fri_t + \sum_{j=1}^3 \beta_{i,j}R_{j,t-1} + \delta_{i,1}C87_t + \delta_{i,2}C98_t + \epsilon_{i,t}$$

$$\epsilon_{i,t} | \Omega_{t-1} \sim \mathcal{N}(0, h_{i,t}) \quad h_{i,t} = m_i + b_i \epsilon_{i,t-1}^2 + c_i h_{i,t-1} + g_i D_{i,t} \epsilon_{i,t-1}^2 + \sum_{j \neq i} k_{i,j} \epsilon_{j,t-1}^2 + \sum_{j \neq i} l_{i,j} D_{j,t} \epsilon_{j,t-1}^2$$

where  $R_{i,t}$  are the returns on the FTSG, FTLG or FT100 indices as defined in Table 1. All the estimated parameters are denoted by a caret, and \*\*\*, \*\*, \* indicate statistical significance at the 1%, 5% and 10% levels, respectively. The variables *Tue*, *Wed*, *Thur*, and *Fri* are day of the week dummy variables. The dummy variable,  $D_{i,t}$ , takes the value of one if  $\epsilon_{i,t-1} < 0$  and zero otherwise. The variables *C87*, *C98*, control for the effects of the 1987 stock market crash and the 1998 stock market falls. Log-L is the value of the log likelihood function of the model, and  $Q(10)$  is the Box-Ljung test for autocorrelation in the residuals to lag 10. On the right of the table, the coefficient  $\hat{k}_{i,j}$  measures the impact of past volatility surprises to series  $j$  on the conditional variance of series  $i$ , while the coefficient  $\hat{l}_{i,j}$  indicates whether this volatility transmission effect is asymmetric. These coefficients are set to zero on the left side of the table where, instead, the cross correlations of the estimated squared residuals,  $\epsilon_{i,t}^2$ , are reported.

|   | FTSG       | FTLG       | FT100                          | FTSG            | FTLG       | FT100      |
|---|------------|------------|--------------------------------|-----------------|------------|------------|
| $\hat{\alpha}_1 \times 10^3$            | -0.440 *** | -0.768 *** | -0.592 **                      | -0.420 ***      | -0.745 *** | -0.592 **  |
| $\hat{\alpha}_2 \times 10^3$            | 0.437 ***  | 0.839 ***  | 1.037 ***                      | 0.405 ***       | 0.725 ***  | 1.051 ***  |
| $\hat{\alpha}_3 \times 10^3$            | 0.352 ***  | 0.894 ***  | 1.140 ***                      | 0.361 ***       | 0.858 ***  | 1.147 ***  |
| $\hat{\alpha}_4 \times 10^3$            | 0.302 ***  | 0.678 ***  | 0.820 **                       | 0.299 ***       | 0.575 ***  | 0.824 **   |
| $\hat{\alpha}_5 \times 10^3$            | 0.943 ***  | 1.560 ***  | 1.437 ***                      | 0.959 ***       | 1.644 ***  | 1.436 ***  |
| $\hat{\beta}_1$                         | 0.023      | -0.163 *** | 0.128                          | 0.040 *         | -0.174 *** | 0.134 *    |
| $\hat{\beta}_2$                         | 0.028 ***  | 0.097 ***  | 0.045                          | 0.022 ***       | 0.092 ***  | 0.043      |
| $\hat{\beta}_3$                         | -0.001     | 0.013 *    | 0.024 *                        | -0.001          | 0.012      | 0.025 *    |
| $\hat{\delta}_1 \times 10^3$            | 2.233 ***  | 7.263 ***  | -61.80 ***                     | 2.189 ***       | 8.509 ***  | -59.50 *** |
| $\hat{\delta}_2 \times 10^3$            | 0.518 *    | 2.204 ***  | -3.622 ***                     | 0.252           | 2.053 ***  | -3.684 *** |
| $\hat{m} \times 10^6$                   | 1.107 ***  | 0.725 ***  | 2.229 ***                      | 0.703 ***       | 0.739 ***  | 2.107 ***  |
| $\hat{b}$                               | 0.625 ***  | 0.926 ***  | 0.894 ***                      | 0.681 ***       | 0.918      | 0.895      |
| $\hat{c}$                               | 0.130 ***  | 0.041 ***  | 0.045 ***                      | 0.090 ***       | 0.044 ***  | 0.046 ***  |
| $\hat{g}$                               | -0.077 *** | 0.021 ***  | 0.076 ***                      | -0.078 ***      | 0.028 ***  | 0.072 ***  |
| Cross correlations of squared residuals |            |            | Volatility spillovers included |                 |            |            |
|   | <i>t</i>   |            |                                |                 |            |            |
| <i>t</i> - 1                            | 0.043 ***  | -0.009     | 0.034 ***                      | $\hat{k}_{i,1}$ | -0.040 *   | -0.091     |
|   | 0.066 ***  | 0.078 ***  | 0.078 ***                      | $\hat{k}_{i,2}$ | 0.007 ***  | 0.005      |
|   | 0.007      | 0.017      | 0.269 ***                      | $\hat{k}_{i,3}$ | <0.001 *   | 0.001      |
| <i>t</i>                                | 1.000      |            |                                | $\hat{l}_{i,1}$ |            | 0.119      |
|   | 0.421 ***  | 1.000      |                                | $\hat{l}_{i,2}$ | 0.006 ***  | 0.001      |
|   | 0.081 ***  | 0.135 ***  | 1.000                          | $\hat{l}_{i,3}$ | -0.001 *** | <0.001     |
| Log-L                                   | 29096      | 23734      | 21171                          |                 | 29150      | 23736      |
| Q(10)                                   | 11.955     | 9.277      | 10.679                         |                 | 12.392     | 9.926      |

**Table 3: Multivariate GARCH Models**

This table contains the key estimated coefficients from the model

$$R_{i,t} = \alpha_{i,1} + \alpha_{i,2}Tue_t + \alpha_{i,3}Wed_t + \alpha_{i,4}Thur_t + \alpha_{i,5}Fri_t + \sum_{j=1}^3 \beta_{i,j}R_{j,t-1} + \delta_{i,1}C87_t + \delta_{i,2}C98_t + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} | \Omega_{t-1} \sim N(0, h_{i,t}) \quad h_{i,t} = m_i + b_i \varepsilon_{i,t-1}^2 + c_i h_{i,t-1} + g_i D_{i,t} \varepsilon_{i,t-1}^2 + \sum_{j \neq i} k_{i,j} \varepsilon_{j,t-1}^2 + \sum_{j \neq i} l_{i,j} D_{j,t} \varepsilon_{j,t-1}^2$$

$$h_{i,j,t} = ((1 - G(t, \gamma_{i,j}, \psi_{i,j})) \rho_{0,i,j} + G(t, \gamma_{i,j}, \psi_{i,j}) \rho_{1,i,j}) \sqrt{h_{i,t} h_{j,t}} \quad \forall i \neq j \quad G(t; \gamma_{i,j}, \psi_{i,j}) = \frac{1}{1 + \exp(-\gamma_{i,j}(t - \psi_{i,j}))}$$

where  $R_{i,t}$  are the returns on the FTSG, FTLG or FT100 indices as defined in Table 1. On the right side of the table, the coefficients  $\rho_{0,i,j}$  and  $\rho_{1,i,j}$  measure the correlations in the two regimes, while the parameters  $\hat{\gamma}_{i,j}$  and  $\hat{\psi}_{i,j}$  measure the "rate" and "mid-point" (as a proportion of the length of the sample) of a smooth transition process between the two correlation regimes. The date below  $\hat{\psi}_{i,j}$  converts this "mid-point" back to a date. On the left side of the table,  $\rho_{0,i,j} = \rho_{1,i,j} = \rho_{i,j}$  is imposed. Other terms are as described in Table 2.

|                              | FTSG                  | FTLG       | FT100      | FTSG                           | FTLG       | FT100      |
|------------------------------|-----------------------|------------|------------|--------------------------------|------------|------------|
| $\hat{\alpha}_1 \times 10^3$ | -0.443 ***            | -0.668 *** | -0.558 **  | -0.454 ***                     | -0.710 *** | -0.557 **  |
| $\hat{\alpha}_2 \times 10^3$ | 0.458 ***             | 0.761 ***  | 1.001 ***  | 0.448 ***                      | 0.714 ***  | 0.880 ***  |
| $\hat{\alpha}_3 \times 10^3$ | 0.372 ***             | 0.859 ***  | 1.077 ***  | 0.377 ***                      | 0.836 ***  | 1.146 ***  |
| $\hat{\alpha}_4 \times 10^3$ | 0.323 ***             | 0.497 **   | 0.677 *    | 0.351 ***                      | 0.525 ***  | 0.594 *    |
| $\hat{\alpha}_5 \times 10^3$ | 0.927 ***             | 1.497 ***  | 1.373 ***  | 0.925 ***                      | 1.496 ***  | 1.345 ***  |
| $\hat{\beta}_1$              | 0.037 *               | -0.145 *** | 0.125      | 0.027                          | -0.124 *** | 0.146 *    |
| $\hat{\beta}_2$              | 0.024 ***             | 0.090 ***  | 0.043      | 0.025 ***                      | 0.083 ***  | 0.042      |
| $\hat{\beta}_3$              | -0.001                | 0.010      | 0.025      | -0.001                         | 0.004      | 0.022      |
| $\hat{\delta}_1 \times 10^3$ | 2.550 ***             | 8.007 ***  | -55.80 *** | 2.358 ***                      | 6.008 ***  | -57.44 *** |
| $\hat{\delta}_2 \times 10^3$ | 0.331                 | 1.755 ***  | -2.424 *   | 0.357                          | 2.042 ***  | -2.507 *   |
| $\hat{m} \times 10^{61}$     | 0.736 ***             | 0.809 ***  | 0.002 ***  | 0.840 ***                      | 0.820 ***  | 1.840 ***  |
| $\hat{b}_1$                  | 0.678 ***             | 0.928 ***  | 0.903 ***  | 0.648 ***                      | 0.930 ***  | 0.906 ***  |
| $\hat{c}_1$                  | 0.088 ***             | 0.042 ***  | 0.048 ***  | 0.114 ***                      | 0.042 ***  | 0.047 ***  |
| $\hat{g}_1$                  | -0.073 ***            | 0.006      | 0.068 ***  | -0.101 ***                     | 0.006      | 0.058 ***  |
| $\hat{k}_{i,1}$              |                       | -0.039 *** | -0.120     |                                |            |            |
| $\hat{k}_{i,2}$              | 0.006 ***             |            | 0.006      | 0.006 ***                      | -0.016     | -0.121     |
| $\hat{k}_{i,3}$              | <0.001 *              | 0.001      |            | <0.001                         | <0.001     | 0.002      |
| $\hat{l}_{i,1}$              |                       | 0.029      | 0.125      |                                |            |            |
| $\hat{l}_{i,2}$              | 0.006 ***             |            | 0.001      | 0.007 ***                      | -0.022     | 0.122      |
| $\hat{l}_{i,3}$              | -0.001 ***            | <0.001     |            | -0.001 ***                     | <0.001     | <-0.001    |
|                              | Constant correlations |            |            | Smooth transition correlations |            |            |
|                              | FTSG-FTLG             | FTLG-FT100 | FT100-FTSG | FTSG-FTLG                      | FTLG-FT100 | FT100-FTSG |
| $\hat{\rho}_{i,j}$           | 0.657 ***             | 0.211 ***  | 0.187 ***  | $\hat{\rho}_{0,i,j}$           | 0.747 ***  | 0.401 ***  |
|                              |                       |            |            | $\hat{\rho}_{1,i,j}$           | 0.619 ***  | -0.300 *** |
|                              |                       |            |            | $\hat{\gamma}_{i,j}$           | 12.79 ***  | 4.487 ***  |
|                              |                       |            |            | $\hat{\psi}_{i,j}$             | 0.292 ***  | 0.730 ***  |
|                              |                       |            |            | (dd/mm/yy)                     | (09/05/90) | (10/02/99) |
|                              |                       |            |            |                                | (22/05/98) |            |
| LogL                         | 75468                 |            |            | 75713                          |            |            |
| Q(10)                        | 12.350                | 9.341      | 10.267     | 12.335                         | 9.798      | 10.872     |

**Figure 1: Long Yield Volatility Multiplier**

This figure shows the function

$$\frac{b_T}{T} = \frac{(1 - \exp(-\kappa T))}{\kappa T}$$

which determines the relationship between the volatility of short interest rates and long term bond yields, when short interest rates follow the process  $dr = \kappa(r^* - r)dt + \sigma dz$ , equation (5).

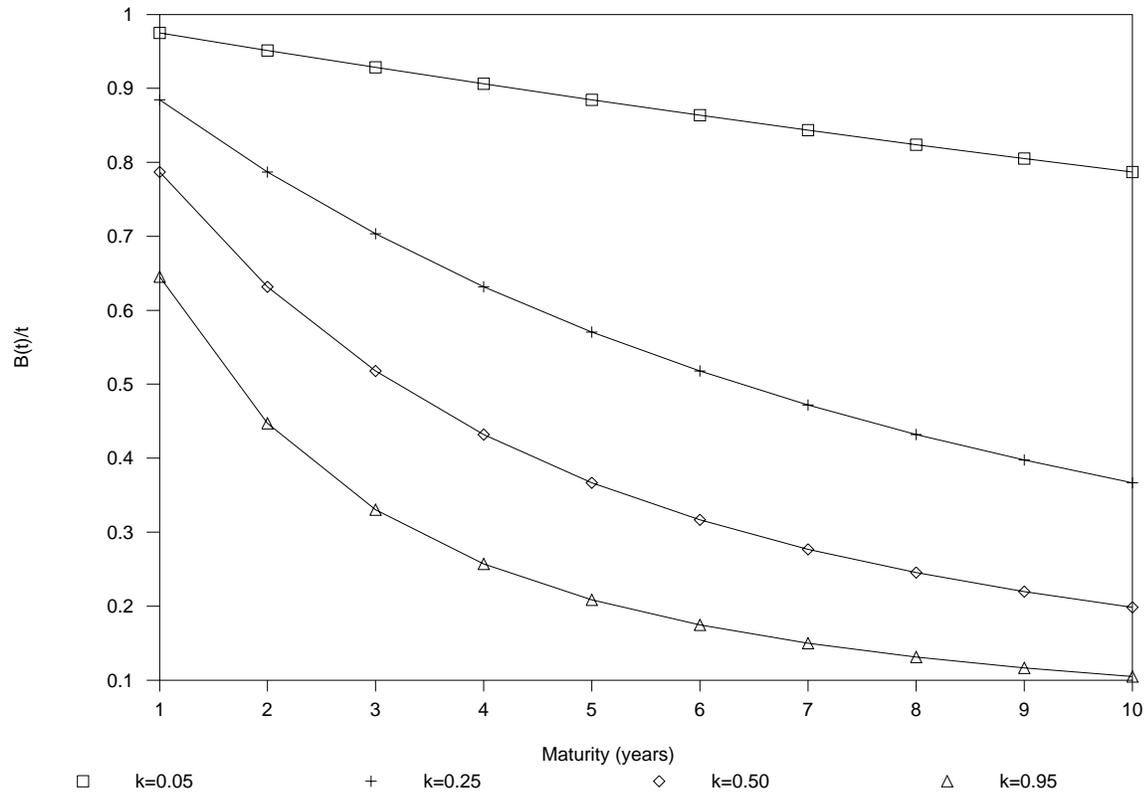


Figure 2: Smooth Transition Functions for Correlation Coefficients

This figure shows the estimated smooth transition functions for the correlations coefficients between the three markets, that is

$$\rho_{i,j,t} = ((1 - G(t, \gamma_{i,j}, \psi_{i,j}))\rho_{0,i,j} + G(t, \gamma_{i,j}, \psi_{i,j})\rho_{1,i,j}) \quad \forall i \neq j \quad G(t; \gamma_{i,j}, \psi_{i,j}) = \frac{1}{1 + \exp(-\gamma_{i,j}(t - \psi_{i,j}))}$$

where the coefficients  $\rho_{0,i,j}$  and  $\rho_{1,i,j}$  measure the correlations in the two regimes. The parameters  $\hat{\gamma}_{i,j}$  and  $\hat{\psi}_{i,j}$  measure the "rate" and "mid-point" (as a proportion of the length of the sample) of the smooth transition between the two correlation regimes.

