Mach Edges: A critical test of the nonlinear 3rd derivative model for edge detection

EPSRC

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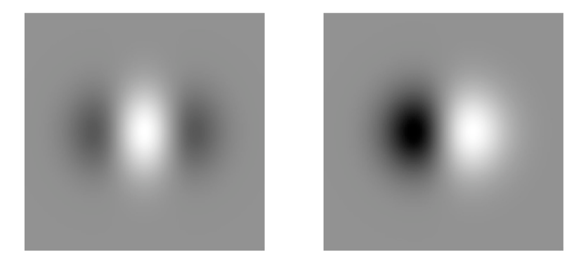
20th April 2007

1. Introduction to edge detection

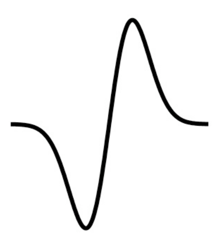
Edges are key points of information in visual scenes. But, how are they extracted from the eye's neural output?

It is widely accepted that the retinal image is filtered by even- and odd-symmetric spatial operators of various scales, early in the visual pathway [1].

One view of these operators is that their role is to compute the spatial gradients and higher derivatives of the image. For the odd-symmetric operator shown here, this process can be illustrated as follows….

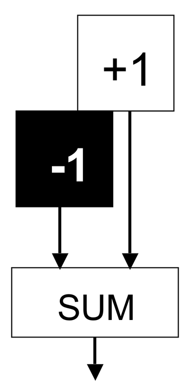


Odd-symmetric operator profile



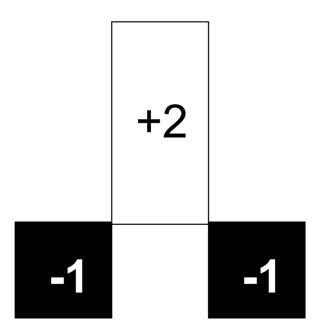
A horizontal section through the odd-symmetric profile shown above produces the profile shown here (left). This can be simplified to two adjacent regions of positive and negative response. If the output of these two regions is summed, this is equivalent to obtaining the difference in luminance between these two regions.

If this operator is 'swept along' (convolved with) a 1-D image so that it makes this comparison between every point and its neighbour then the output at every point is proportional to the rate of change of luminance, i.e. its 1st derivative.



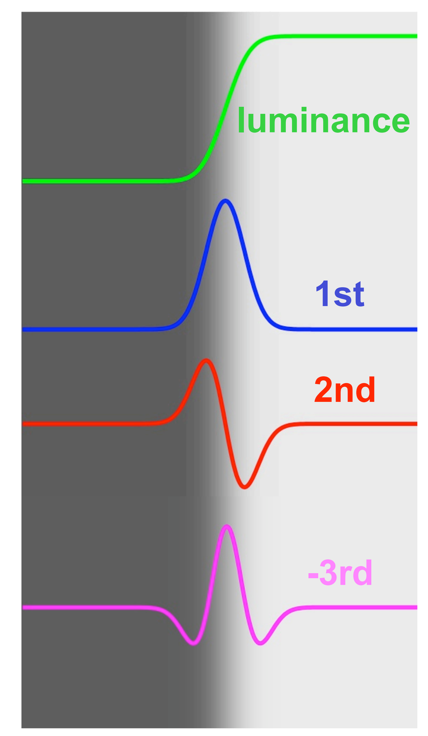
A similar argument applies to obtain operators that compute the 2nd, 3rd or any higher derivative, but with different weights applied to different regions.

Simplified 2nd derivative operator



One important class of models supposes that edges correspond to the steepest parts of the luminance profile, implying that they can be found as peaks and troughs in the response of a gradient (first derivative) filter [2], or as zero-crossings (ZCs) in the second derivative [3].

A blurred edge and its 1st 3 derivatives

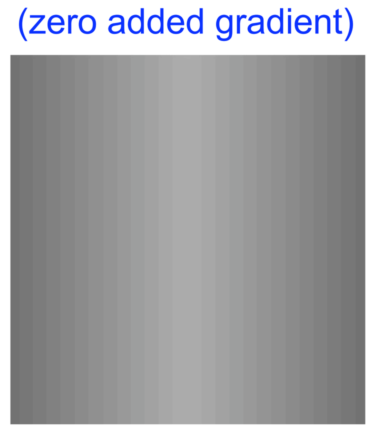


However, our previous feature-marking experiments found that peaks in the (inverted) 3rd derivative can signify edges where there are no 1st derivative peaks nor 2nd derivative zero-crossings [4]. These results on 'Mach edges' (the edges of Mach bands) were nicely predicted by a new <u>non-linear</u> model based on 3rd derivative filtering (Georgeson et al, 2007, in press). The model uses 2 stages of rectification to remove spurious edges (the troughs shown here).

2. Rationale and stimuli

As a critical test of the model, we add a linear luminance ramp to the blurred triangle waves used previously. This ramp has no effect on the second or higher derivatives, but the nonlinear 3rd derivative model predicts: (1) For ramp gradients of less than ±1 (below, 2 left plots) the gradient (1st derivative) profile has both positive and negative parts - 2 edges are predicted. (2) For steeper added ramp gradients (below, 3 right plots) the gradient (1st derivative) does not change sign. This causes one 3rd derivative peak to be blocked by a rectifier in the model - one edge predicted.

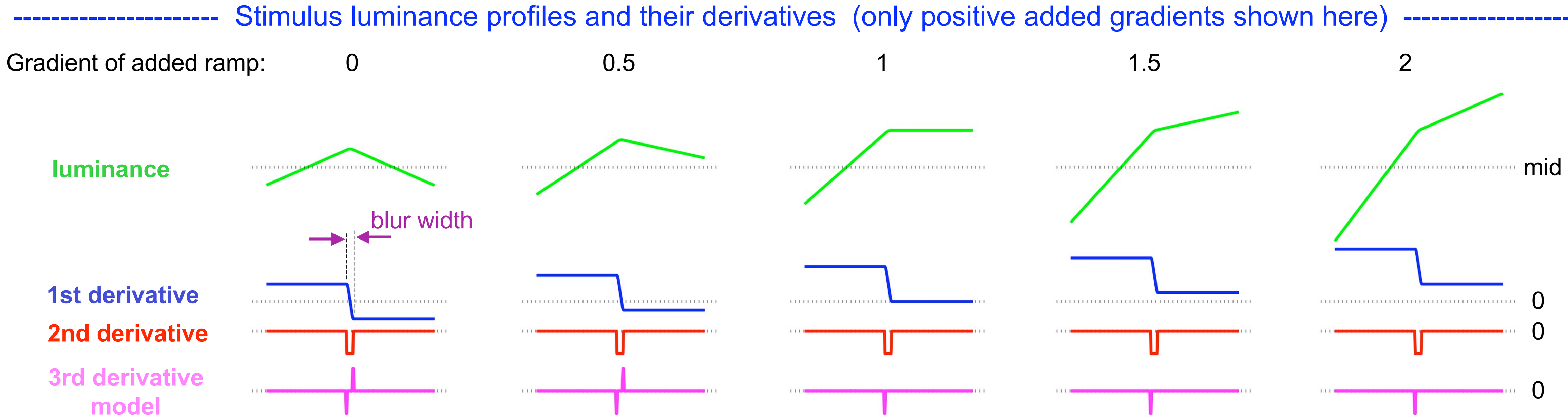
Experimental image (zero added gradient)



<u>Stimulus generation</u>
- Basic stimulus was one period of a triangle-wave (green, below, left) blurred by a rectangular function 2 or 8 pixels wide. The gradients of its rise and fall are defined as +1,-1.
- Linear luminance ramp was added to form the stimulus luminance profile. Its gradient was -2, -1, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, or 2.
- Image size was 256 by 256 pixels (4.27 degrees), surrounded by a full screen of mid-grey.
- Waveforms were also inverted to obtain opposite phase images.
- 2 blur widths x 9 ramp gradients x 2 phases = 36 conditions.

Question? WILL SUBJECTS SEE 1 OR 2 MACH EDGES?

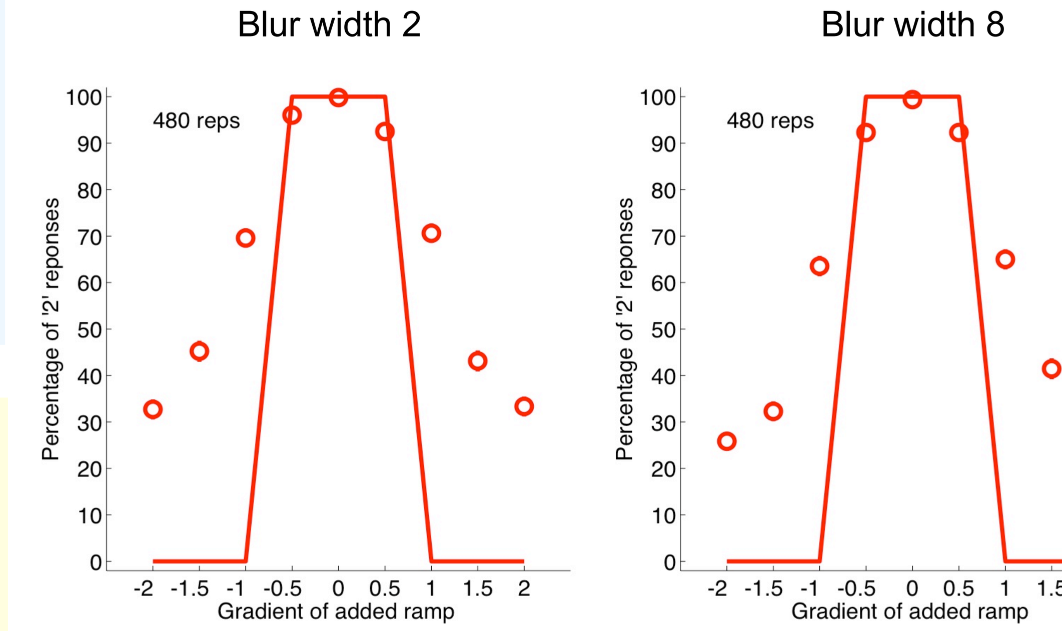
---------------------- Stimulus luminance profiles and their derivatives (only positive added gradients shown here) ----------------------



3. Experiment 1: Yes-No paradigm

<u>Procedure</u>
- Images were presented once for 0.3s.
- Task: indicate whether 1 or 2 edges were seen at the centre of the image.
- Interval between presentations was at least 1s.
- 1 session had 10 repetitions of the 18 conditions at one blur width, in random order, and took about 5 minutes.
- 4 subjects each performed 6 sessions for blur width 8 pixels followed by 6 sessions for blur width 2 pixels.

Data (symbols) and model predictions (line)



<u>Results</u>
- Data broadly similar for all 4 subjects & both phases; averages shown here.
- Error bars calculated from the standard deviation in binomial sampling:

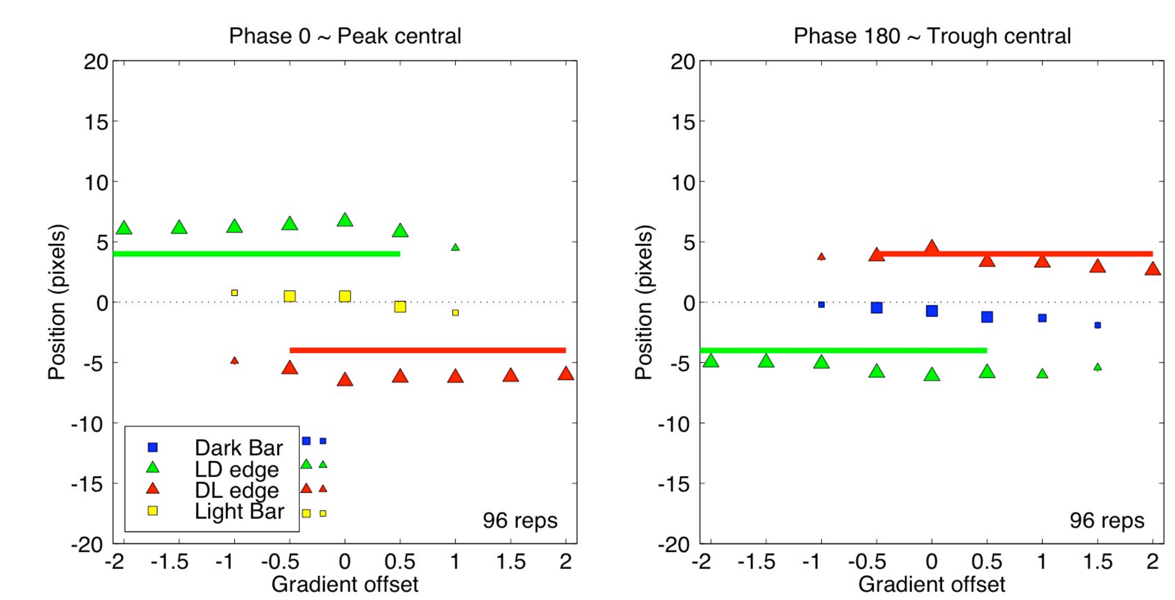
$$\sqrt{p.\frac{1-p}{n}}$$

where p is the probability of response and n is the number of trials (480 here).
- Results were almost identical for the two blur widths.
- Two edges were seen reliably with added gradients up to ±0.5. This matched model predictions closely (solid line).
- With steeper added ramps, reports of two edges fell away as the model predicts, but more gradually than predicted.

4. Experiment 2: Feature-marking paradigm

<u>Procedure</u>
- Images were flashed repeatedly (0.3s on, 0.6s off).
- Task: mark the position and polarity of all edges and bars seen
- The marker comprised two black dots, each 1 x 3 pixels, vertically arranged, each 32 pixels (0.6 deg) from image midline
- Subjects were instructed to fixate midway between the dots
- 1 blur width (8 pixels) x 9 ramp gradients x 2 phases = 18 conditions.
- 1 session consisted of 6 repetitions of each of the 18 conditions, in randomised order, and took about 30 minutes.
- 4 subjects each performed 4 sessions.

Data (symbols) and model predictions (lines)



<u>Main Results</u>
- Data were broadly similar for all 4 subjects; averages are shown here.
- The task was reliable - Error bars are ±1se and are plotted behind symbols.
- All subjects marked a central bar flanked by 2 edges for gradients up to ±0.5
- For gradients beyond ±0.5, bars and one edge were marked less frequently - smaller symbols here.
- Position and polarity of marked edges was well predicted by the 3rd derivative model. Absolute position was better for phase 180 than phase 0.

<u>Secondary Result</u>
- The light bars appeared wider than the dark bars.

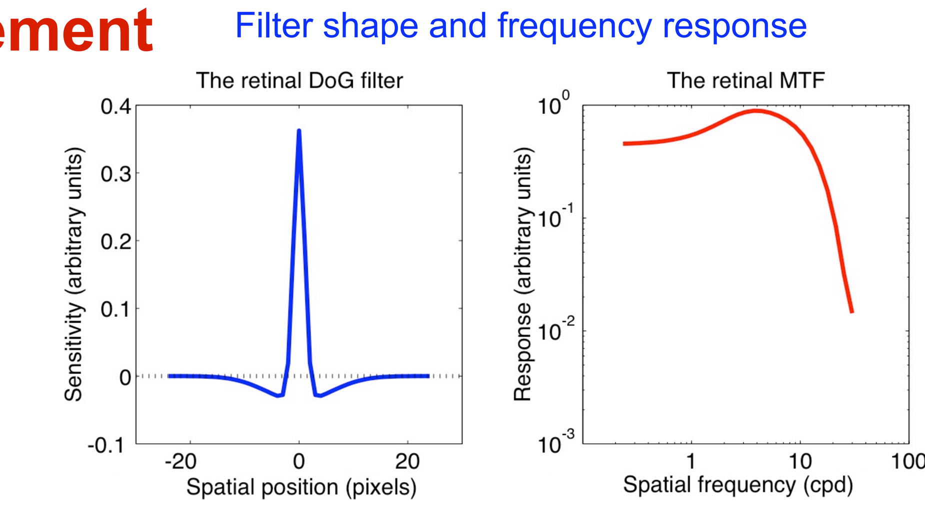
This may be an example of the Helmholtz irradiation effect [5].

5.Conclusions and model refinement

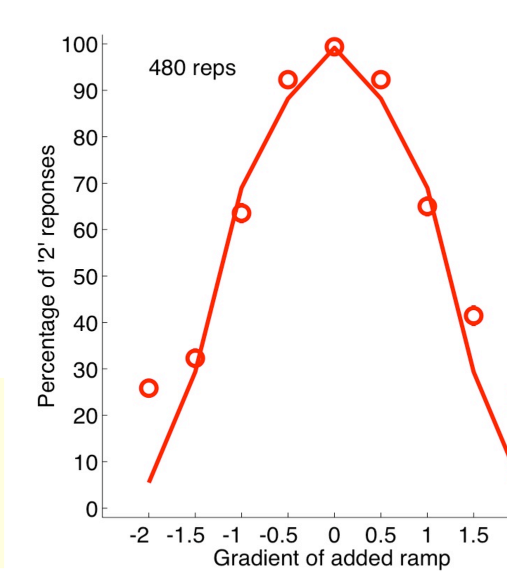
<u>Conclusion</u>

Two Mach edges were reliably seen in a triangle-wave, but one disappeared when a steep linear ramp was added. This pattern of results was nicely predicted by the nonlinear 3rd derivative model. But the transition from 2 to 1 edge was less rapid than the model predicted (Box 3). We suggest that mild, bandpass filtering (e.g. by the retina) can explain this.

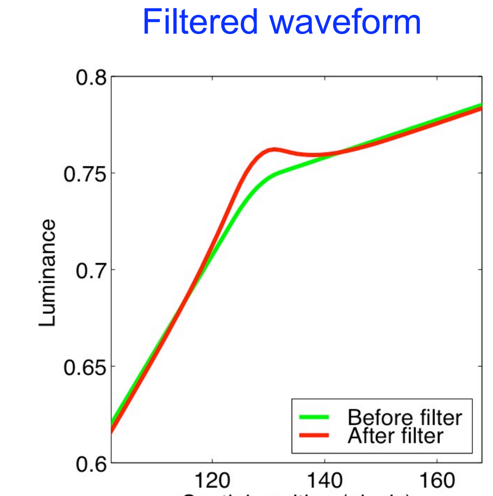
Filter shape and frequency response



Data (Box 3, blur 8) with refined predictions



Filtered waveform



<u>Model refinement - Add a retina</u>

An additional pre-filter was added to the model. It has a Difference-of-Gaussians receptive field profile, based on P cells in central vision [6]. RF shape and spatial frequency response are shown above.

Its effect is to accentuate edges as shown here (left). The filter modifies this waveform so that it now has a small central negative gradient, which allows both 3rd derivative edges to pass through the rectifiers in the model.

The revised predictions, which also assume a noisy decision process, match the data well (right).

<u>References:</u>

1. Bruce et al, 2003, *Visual Perception: physiology, psychology and ecology*. 4th Hove: Psychology Press.
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3. Marr, 1980, *Vision*. San Francisco: Freeman and Co.
4. Wallis & Georgeson, 2007, Perception **36** (2), 314-315.
5. Mather & Morgan, 1985, *Vision Research*, 26 (6), 1107-1015.
6. Croner & Kaplan, 1995, *Vision Research*, 35, 7-24.