ACOUSTICS OF SMALL ROOMS INTENDED FOR USE AS STUDIOS

A Thesis submitted in accordance
with the requirements of the
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To my wife Salwa and son Mohamed
in gratitude for their patience
and encouragement throughout the
work.

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SUMMARY

The aims of the work reported in this thesis are:

(1) To investigate the application of a Wave Analysis Approach in the study of the acoustics of small rooms where the conditions for a Geometrical Statistical Analysis are not valid.

Physical parameters, describing room model reponse to acoustic excitation, are derived and their values for proposed design models characterized by different boundary conditions for a small room are theoretically predicted.

(2) To establish methods of experimentally verifying the predicted values of those parameters from complex response measurements in steady-state conditions, by applications of resonance testing techniques in the analysis of measured results. These results are compared with predicted values and the variance that may occur between them is explained.

(3) To probe the feasibility of implementation of the theoretical analysis and experimental techniques to practical conditions in the design and testing of broadcasting and recording studies for good acoustical performance.

A survey of the subjective requirements in a studio, as viewed by the user, is carried out.

An endeavour to relate the subjective requirements to the physical parameters already established, is made. The success of this will help to establish a design criteria for small studios solely based on wave acoustics.

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CHAPTER ONE

INTRODUCTION

1.1) OBJECTIVES OF THESIS:-

The objectives of this thesis are to endeavour to reconcile the often conflicting demands of the listener and performer, the theories and techniques of the acoustician and the design considerations of the architect in the field of room acoustics with a view to providing a means of designing good small auditoria.

The problem has largely been overcome in the case of large auditoria as the geometrical analysis renders some physical acoustical properties of sound fields in enclosures which can be related to subjective criteria similar to the faculties of human perception. The same physical properties can be interpreted in terms of design parameters upon which the architectural design can be based.

In the case of small regularly-shaped rooms, the problem is, to the best of the author's knowledge, unsolved.

The wave analysis approach can be used to give a physical description of the sound field in terms of the acoustic properties of the bounding surfaces of the room in terms of some measurable parameters. The task of giving subjective values to those parameters, and of expressing them in terms of architectural design data, is a field still not fully explored.

1.2) SCOPE OF RESEARCH.

The scope of the research here, is confined to considering small rooms, with simple geometrical shapes with small absorption. The bounding surfaces are assumed to have uniform, but different coverage of absorbing materials.

These are the conditions likely to prevail in broadcasting and recording studios; for besides if they give great simplification to the complexity of the wave theory, the extension of the analysis to the more complicated case of non-uniform coverage can be achieved. In practice, the simple condition is the more likely to meet the architects' search for aesthetic qualities.

1.3) METHOD OF APPROACH.

A modal wave analysis is adopted in the study of the acoustics of small rooms. The parameters derived are used in evaluating the response of the room to acoustic excitation. Similarly, indices for describing the absorbing qualities of the wall materials are sought.

Theoretical predictions of the values of these parameters in terms of the materials absorbing indices are computed and their verification by experimental measurements carried out.

1.4) FORM OF WORK.

The theoretical work is based on the study of steady-state conditions of acoustic excitation. This is in contrast to transient-state conditions associated with geometrical theories. The former are considered here to be more closely linked with the hearing qualities of studios.

The experimental work includes complex response measurements of the room as the study of phase as well as amplitude of sound signal yields more accurate values for the steady-state parameters.

The advent of modern technology makes digital computers an invaluable tool in overcoming the complexity of the mathematical analysis involved and the ranges of experimental data required.

1.5) PRESENTATION OF WORK.

The work in this thesis is presented in form of chapters, each considering a special aspect of the problem.

Chapter One is an introductory summary of the whole thesis. The objectives of the research are presented and the form of work discussed.

Chapter Two gives a brief account of the background to the subject. A survey of the historical progress of acoustics is made and the diversity of the subject outlined.

Chapter Three is an account of the wave modal theory as applied to rectangular rooms with different surfaces having uniform coverage of absorbing materials.

The response of the room to acoustic excitation and the associated parameters are derived in terms of the acoustic impedance of the materials of the room surfaces.

The statistics of the wave theory are applied in classifying the different types of modes of vibrations according to their common properties.

Chapter Four describes the theories of experimental techniques. These fall into two categories:

Those dealing with measurement of the specific acoustic impedance of abosrbing materials, and those applied for measurements of modal parameters (natural frequencies, damping constants and modal shapes).

Chapter Five covers the methods of numerical analysis applied in the derivations of theoretical predictions and data analysis of experimental measurements, by the use of digital computers.

Chapter Six deals with the experimental work. The experimental measurements and instrumentation set-ups are described. The results are presented, and analysed in comparison with predicted values.

Chapter Seven probes the feasibility of application of the work to practical conditions in the design of recording and broadcasting studios. A survey of the subjective requirements in a studio, as viewed by the user, is carried out. An endeavour to relate these requirements to the physical parameters already established in the thesis, is made. The success of this will help to establish a design criteria for studios solely based on wave acoustics.

Chapter Eight concludes on what has been achieved and gives proposals for future work.

CHAPTER TWO

BACKGROUND TO THE SUBJECT.

2.1) INTRODUCTION:

Man has strived, since the dawn of history, to satisfy his basic living needs - shelter from the severity of nature being a predominent one. Shelter, by definition an enclosed space, gave rise to complications associated with the distribution of sound in an enclosure with its consequent detrimental or otherwise beneficial effects on the intelligibility of speech and the quality of music.

With the progress of human knowledge in science, a better understanding of the phenomena of sound was achieved, through theoretical and experimental studies.

The exploration of the behaviour of sound in enclosures closely followed the pace of development in the study of the science of sound, better known as acoustics.

By the turn of the 19th Century, architectural acoustics emerged as an established science, free from the mystical myth which it has been associated with for so long. And with the advances in science and technology, its interrelations with other fields of acoustics like physical acoustics, electro-acoustics and psycho-acoustics gave impetus to the rise of a variety of theories and approaches for investigation into its diverse aspects. These theories and approaches were aimed at explaining the nature of the problems encountered, establishing criteria for judgement and suggesting solutions.

Statistical, geometrical and wave theories were introduced, their successful applications always depending on the relevant conditions for which they are appropriate.

The aim of this study is to seek solutions for a particular aspect of architectural acoustics; that of the design of small studios in the light of the prevailing room acoustics theories.

That a specific approach, relying on the wave modal theory has been adopted will, it is hoped, be justified through the pages of this study.

The intention here, is to give a brief summary of the development in acoustics from a theoretical and experimental point of view in general, and in architectural acoustics in particular, that may help to shed some light on our present day approach to the subject.

A critique of the existing room acoustics theories and an appraisal of associated parameters, for judging room performance, are made.

An account of the design requirements of studios, as reviewed in the literature, is given.

2.2) HISTORICAL REVIEW OF THE SUBJECT.

2.2.1) THEORETICAL ACOUSTICS:-

In early history, the study of sound was closely linked to music, an art practiced by old civilations as long ago as 4000 B.C. The Greek mathematician Pythagoras, about 2500 years ago, originated the study of musical intervals and ratios which became the basis of fantastic philosophical speculations, and so for another 2000 years, sound was mainly involved in a semi-mystical arithmetic of music[1].

Two hundred years later, Aristotle proposed an explanation of the mode of propagation of sound waves in air based on the air movement, an idea which prevailed until the time of Galileo, sixteen hundred years later. His hypothosis that a sharp sound was transmitted more quickly than a low tone, was only disproved by Pierre Gassendi[1] (1592-1655).

The theoretical aspects of sound began with Newton's derivation of an expression for the velocity of sound in terms of the elasticity and density of the medium of propagation. Some 70 years later Lagrange (1736-1813) pointed out the effect of temperature on elasticity. Pierre Laplace (1816) modified Newton's equation on the basis of Lagrange's argument and the velocity then calculated from the corrected formula was in agreement with the standard experimental value[1]. The theoretical study of sound and wave motion received a great stimulus as the result of the development of mathematical calculus by Liebniz (1646-1716) and by Newton, and was applied by the 18th Century mathematicians, like Lagrange, Euler, Poisson and D'Alembert, to many physical, including acoustical problems.

The two men mainly responsible for the establishment of sound as an exact science were Helmholtz and Lord Rayleigh (1842-1929)[7].

Helmholtz's contribution was the summation of tones and the effect of relative intensity of the harmonics on the timbre of sound.

Lord Rayleigh's "Theory of Sound" as considered later by W.H.Eccles[2] as the most comprehensive work on acoustics that has till then been published.

Fourier[7] (1768-1830) applied a theorem to the solution of problems in heat, and it was G.Ohms (1789-1854) who indicated its application to acoustic problems.

The concept of acoustic impedance was advanced by Webster in 1914[2].

The development of wave acoustics was pioneered by P.M.Morse in the early 1940's. He helped to give a clear insight into the problems of room acoustics.

2.2.2) EXPERIMENTAL ACOUSTICS:-

The period of experiments[2] started with Galileo (1564-1642) who laid the foundations of experimental science, certainly of experimental acoustics. He used the pendulum as a demonstration instrument. Mersenne (1588-1648) used it to time the speed of sound. Gassendi (1592-1655) repeated and extended the experiments of Mersenne.

In 1830 Savart used a rotating wheel against which a reed was pressed, originally invented by Robert Hooke (1681), for determination of the frequency of a sound.

Ghladni, in 1802 , determined the wave patterns of vibrating bodies by means of sand figures.

In 1858, Leconte discovered the sensitive flame, which provided a crude means of determining the intensity of sound waves.

In 1882, Lord Rayleigh invented the first practical precision instrument, the Rayleigh disk, for the measurement of the strength of a sound wave[2].

The use of electronic equipment in acoustic measurements only came in in 1908, when G.W.Pierce used a telephone receiver in a set-up that amounted to the first sound level meter.

Arnold and Crandall of the American Telephone and Telegraph and Western Electric Laboratory developed, in 1917, the thermophone - an electro-thermo-acoustical transducer, which served as an absolute standard of sound pressure. Wente, of the same group, developed an electrostatic microphone with a high diaphragm impedance, wide frequency range and good stability, and the ability to be calibrated with the aid of a thermophone.

It was the second world war that brought an unpredicted activity in applied acoustics in the United States and Great Britain, and the development of suitable sound sources for acoustical experiments was among the main achievements.

With the advance of computer technology in the last decade, the prospects for its utilization in acoustic research opened new horizons in the field of experimental acoustics.

2.2.3) ARCHITECTURAL ACOUSTICS:-

Vitruvius[1], the Roman architect, wrote about 20 B.C. a treatise on the acoustics of theatres, and his statements on the necessary conditions for good acoustics remain remarkably accurate.

He pointed out how interference, echos and reverberation - the re-inforcement factor as he called it - occur in auditoria. Wallace Sabine[3] in his 'Collected Papers on Acoustics' mentions the work of Vitruvious and quotes his proposal to use resonant vases to strengthen the voice of the actors.

During the Renaissance, the subject of architectural acoustics was ignored. This period saw the creation of Gothic Cathedrals with very long reverberation times, thus leading to the chanted rituals since normal speech was unintelligible. On the other hand, thick carpeting and hangings in large palaces lead to 'dead' situations, and chamber music was specially written for such conditions.

The design of rooms and halls for public speech and music had, until the turn of the 19th Century, no scientific basis. In fact the problem involved had not even been clearly defined.

That it consists, for public speech at least, in avoiding a long reverberation time was mentioned by Dr.D.B.Reid[6] in the British Association Report in 1835.

Extensive quantitative study of the subject has to wait until the 19th Century, when among others, W.C.Sabine did his pioneer work on the practice of acoustic design, when he was consulted about the Fogg Art Museum of Harvard University[3],

The theory of absorption by porous materials has been discussed by Rayleigh, but it was Sabine who introduced measuring techniques for determination of the absorption.

The relation $RT = \frac{KV}{A}$, expressing the reverberation time in terms of the room volume and the total absorption of its bounding surfaces, was first deduced by him and it was he who adopted the definition for the reverberation time as the time taken for the energy density to fall to the minimum audible value from an initial value of a million times greater i.e. a range of 60 dB[4].

Sabine, in his derivation of the above expression for the reverberation time, assumed that the sound field was diffuse - i.e. that the energy density was constant throughout room and that at any point the energy flow was equally distributed in all directions.

Sabine's empirical formula exhibits limitations when applied to 'live' rooms and it was Eyring[6] (1930) who prompted to bring modification to it. His result was:

$$RT = \frac{KV}{-S \; \ell n(1-\bar{a})}$$

where $\bar{a}$ is the average absorption coefficient of all the surfaces.

He assumed the acoustic behaviour of the room to be determined by the average absorption coefficient and considered a wave front travelling a fixed distance (the mean free path) between successive reflections at which it lost the same fraction of its incident energy.

A further modification of Eyring's formula was suggested by Millington[6] who considered the room to be characterised by the behaviour of a specific sound ray suffering successive reflections at surfaces whose absorption coefficients varied. The term of his formula is:

$$RT = - \frac{KV}{\Sigma S_i \, \ell n(1-\alpha_i)}$$

These formulae were the basis for what is termed the classical statistical and geometrical theory which treats the sound in an enclosure as a field through every point of which are passing simultaneously, a large number of waves reflected from bounding surfaces in a manner similar to light rays.

An alternative approach[5] - that of the wave theory - is based on the fact that the air space contained within an enclosure acts as a vibratory system which is excited by a sound signal.

The wave theory[5] of room acoustics was developed by

P.M.Morse and R.Bolt in the 1940's from one dimensional experiment first suggested by Rayleigh in 1878.

As early as 1855, Thomas Young[4] has called attention to the phenomena "that the air in an enclosure has a large number of possible modes of vibration corresponding to the partial tones it would emit if vibrating freely."

Knudsen[22] first showed experimentally that the reverberant sound has the characteristic frequencies of normal modes of vibration of the room and not necessarily the frequency of the sound source.

Wente[22] studied the steady state response of a room as a function of the frequency of the source and pointed out that the sharp response peaks occurred at the resonance frequencies of the room model.

Several other theoretical and experimental workers[8,9,10] have studied other aspects of wave acoustics.

With the application of the statistical approach, extended to wave acoustics by Schroeder[11], many of the properties of frequency and space responses in rooms can be studied.

2.3) SEARCH FOR CRITERIA.

The reverberation time, as a parameter of judging the acoustical qualities of rooms, has been used extensively in the design of enclosures. Beranek[2] questions whether such criteria ARE strictly applicable to modern auditoriums and studios because of the differences in shape between the rooms Sabine studied and rooms of modern designing. He enlists other quantities which can be measured and used as indices for judgement for the acoustic properties of enclosures. Among these are the fluctuations of sound during decay - the variations of sound pressure with position of source and receiver - the variations with frequency of the source - the ratio of direct to reverberent sound and modification of wave shape during transmission from one point to another. He concludes that each gives some useful information about the performance of the enclosure.

In the course of search for physical measurable indices of the acoustic quality of a room, in addition to the reverberation time, a number of studies have been made of the fluctuations occurring in the sound pressure level when the frequency of a simple harmonic source is varied or when the position of the source or the receiver is varied.

The theory of frequency irregularity originally given by Bolt and Roop[12], has been revised and extended by Schroeder[12].

Bolt,Doak and Westervelt[13] comment: "that it is well known that the reverberation time is not a completely adequate index of the acoustical quality of a room; implicit in the concept of the reverberation time is the assumption that the room reaches a steady state diffuse condition. In practice, most sounds of speech and music can b e classified as pulsed wave trains whose amplitudes and frequency components fluctuate rapidly so that the room seldom reaches steady state."

E.Meyer and H.Kuttruff[13] point out that the reverberation time, which is based on the statistical room acoustics, involves only the volume of the room and its sound absorption but not its shape, By contrast, the number of echoes,the sequence in time and intensity are exclusively questions of geometrical room acoustics, that is to say they depend decisively on the dimensions and shape of the room and the position of sound source and the observer.

The preceding discussions pin-point but a few of the limitations associated with the statistical geometrical approach to room acoustics. This is attributed to the fact that this theory idealizes the physical processes in the enclosure and leaves aside any consideration of the wave nature of sound. This is an over simplification of the conditions that prevail - a limitation only outweighed by the simplicity of its application and the fact that geometrical acoustics is as Knusden and Harris[16] conclude, under appropriate conditions, an approximation to wave acoustics.

On the other hand, the wave theory, based on the application of the methods of wave mechanics to architectural acoustics, has been the subject of wide disagreement on the practicality of its application.

Kinsler and Frey[15] comment that in spite of the greater complexity of the mathematical techniques involved, the application of the wave analysis to room acoustics has markedly broadened our knowledge of the behaviour of sounds in enclosures. They refer to how it lends itself to a consideration of such important factors as the shape of the enclosure, the effect of different distributions of absorbing surfaces, the behaviour of sounds in dead rooms and the effect of the position of the source or the receiver.

Knudsen and Harris[16] emphasize that consideration should be given to principles of physical acoustics in determining the location of absorptive materials and studio dimensions. They stress that physical acoustics is seen to provide a logical and practical guide to the problems of studio design, but nevertheless conclude that due to the complexity of the approach that it is not yet feasible to present a complete set of practical design formulae based on physical acoustics.

According to H.Kuttruff[17], the wave theory of room acoustics can only be carried out exactly in highly idealized cases, with reasonable effort. Its methods cannot be applied to types of rooms with which we are concerned in our daily life, such as concert halls, theatres, because they are irregular in shape, largely because of the furniture, the boundary conditions cannot be formulated in a satisfactory way and so the theory can only yield approximate or qualitative results. For this reason the immediate practical application of the wave theory to room acoustics is very limited.

P.M.Morse[18], by contrast, states that the fact that his analysis of standing waves has depended to some extent on the choice of rectangular rooms is not a serious limitation, for most rooms approximate a rectangular form.

V.S.Mankovsky[5] limits the practical value of the wave theory to problems connected with the influence of absorption distribution, the character of the sound field in a steady-state condition and the character of the decay of normal modes of vibration.

Knudsen and Harris[16] are more optimistic when they conclude that: "it is almost certain that in the years to come physical acoustics will make further important contributions to practical designing in architecture."

2.4) STUDIOS DESIGN REQUIREMENTS.

In the design of all rooms for speech and music, the basic requirements are:[16]

(1) to preserve the original qualities of a music tone such that the relative magnitudes of all harmonics remain unaltered as they are transmitted;

(2) to ensure the intelligibility of speech.

These conditions, which are applicable to all rooms used for speech and music may be achieved by satisfying the followng requirements:

(1) exclusion of intruding noise by high degree of insulation;

(2) optimum reverberation time over a wide frequency range;

(3) optimum diffusion of sound with a favourable ratio of direct to reflected sound to all auditors;

(4) provision for reinforcing the sound to adequate level in all parts of the room.

In the case of studios, the freedom from objectionable room resonance is an equally important requirement.

Knudsen and Harris[16] refer to the special nature of the problem of studios as a consequence of the difference between monoaural and binaural hearing. They emphasize that the microphone pick-up is subject to about the same peculiarities as is monaural hearing and so a studio should have about the same acoustical properties as would be required for the most favourable conditions for listening with one ear.

J.Borenius[19] outlines the most important criteria in talk studios, to be the clearness and intelligibility of reproduction i.e. to eliminate the acoustical effects of the place of performance "so that the listener gets the impression that the performance is in his own home". This could be attained by heavily damping the talk-studio, but this results in creating a "dead" unpleasant effect.

Coloration, due to predominance of some modes of vibration causes a 'hollow' tone in the reproduction.

The same author comments that the measurement of the reverberation time in small rooms does not reveal these acoustical defects. Moreover, measurement of the reverberation time at low frequencies, in small rooms, is a very problematic task. The conception of diffusion and its measurement is suggested instead. Polar pattern plots prepared with a microphone pick-up and compared with similar plots from an anechoic and diffuse rooms are used to establish a diffusity index for the room under study.

J.J.Geluk[20] suggests some other quantities, beside the reverberation time, in studio acoustics, like definition (distinctness in time) and directional diffusity and outlines methods for measuring them.

From the preceding discussion the characteristic features in the design of small studios seem to be influenced by the following considerations:

(1) the dimensional proportions of the studio and the modal set-up associated with them;

(2) the low frequency dominance and the subjective acoustical defects associated to it e.g. coloration;

(3) the need for other acoustic parameters beside the reverberation time as an index for the performance of the studio;

(4) the assumption of simple geometrical shape is more appropriate than in the large auditoria of theatres.

The wave theory seems to be the one most likely to lend itself to explaining these conditions and suggesting solutions to the problem of the design of small studios.

CHAPTER THREE

THE WAVE MODAL THEORY

3.1) INTRODUCTION:

In small rooms of regular geometrical shapes, with fairly hard walls, and for comparatively low frequency sound signals, the sound field is not diffuse and a wave acoustics approach must be applied in the study of the room performance.

From the wave acoustic point of view[15,18], the air in a room is treated as an assemblage of resonators, having different allowed modes of vibration, each with its own characteristic frequency of damped free vibration.

When a sound source is started, a steady-state vibration having the frequency of the source is set up, together with a transient free vibration composed of the various normal modes of vibration in the room.

The steady-state vibration may be considered to be made up of a large number of standing waves whose amplitudes depend on the frequency of the source, the impedance of the standing wave with respect to this frequency and the position of the source in the room.

The transient vibration will have the form necessary to satisfy the initial conditions in the room when the source is started and will therefore also be made of many standing waves, but each normal mode of the transient vibration will vibrate with its own natural frequency.

If the source source is operated continuously, each component frequency of the transient dies out exponentially at its own particular rate leaving only the steady-state vibration.

When the source is shut off, the steady-state waves, having the frequency of the impressed sound almost immediately disappear, and the acoustic energy in the room is considered to reside in these standing waves, largely in those having natural frequencies near that of the source.

As these waves damp out exponentially according to their individual free-vibration properties, they often interfere with one another and thus produce beat notes.

The resulting decaying amplitudes of these free vibrations constitute the phenomenon of reverberation.

The characteristic frequencies of vibration of the standing waves in a room depend primarily on the shape and size of the room; whereas their rates of damping depend chiefly on the boundary conditions.

Therefore, three aspects of the problem present themselves for study:

(1) The nature of the reaction between the sound wave and its boundary conditions i.e. the walls of the room.

(2) The nature of the steady-state response of the room to a source of sound.

(3) The nature of the transient response, in particular reverberation.

It is most appropriate to commence the study by considering the simplest possible boundary conditions, that of perfectly rigid walls and no damping in deriving expressions for the characteristic frequencies. The effect of the acoustic conditions at the walls on the damping of the vibrations may then be considered as a perturbation of these simple conditions.

3.2) THE SOUND FIELD IN RECTANGULAR ROOMS WITH RIGID BOUNDARIES:-

3.2.1) THE WAVE EQUATION.

During the passage of acoustic waves, there is no rotational motion of the particles, and so the velocity vector of a particle is an irrotational vector which may be represented as the gradient of some scalar potential function ϕ [21]

$$\text{i.e. } V = \nabla\phi \left(\text{i.e. } u = \frac{\partial\phi}{\partial x},\ v = \frac{\partial\phi}{\partial y},\ w = \frac{\partial\phi}{\partial z} \text{ in component terms}\right)$$

where $\phi(x,y,z,t)$ is the velocity potential. The propagation of the acoustical waves is governed by the wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi \qquad (3.1)$$

(c: speed of sound; ∇^2: the Laplacian operator)

which represents the propagation of the velocity potential (ϕ). Similar wave equations, in which other acoustical variables, such as the pressure serving as the dependent variable, can be obtained. The wave equation can be represented in any system of coordinates which allows its separation. It is fortunate[22] that the most usual room shape, the rectangular paralleopiped, is also the easiest to calculate.

The wave equation expressed in terms of the pressure in cartesian coordinates is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \qquad (3.2)$$

For a rectangular room whose sides have the lengths: L_x, L_y, L_z, the general expression for a plane wave is:

$$p = Ae^{-j(\omega t - k_x x - k_y y - k_z z)} \qquad (3.3)$$

If this equation is to satisfy the general wave equation, the constants k_x, k_y and k_z must satisfy

$$k = \frac{\omega}{c} = (k_x^2 + k_y^2 + k_z^2)^{\frac{1}{2}} \qquad (3.4)$$

as can be shown by direct substitution of equation (3.3) into equation (3.2).

If we replace any one, any two or all the three of the negative signs in equation (3.3) by positive signs, we obtain seven additional expressions similar to equation (3.3), all of which have identical values of k_x, k_y and k_z. This array of eight expressions represents the family of plane waves generated by the original wave as successive reflections occur at the six bounding surfaces of the room.

3.2.2) THE BOUNDARY CONDITIONS.

The origin of the coordinate system is chosen at the corner of the room Fig.(3.1) and the room extends from

$x = 0$ to $x = L_x$ in the x-direction

$y = 0$ to $y = L_y$ in the y-direction

$z = 0$ to $z = L_z$ in the z-direction

Since the room surfaces are assumed to be rigid, the velocity of the air particles near any surface must be parallel to that surface i.e. the normal component of the particle velocity must vanish. This is equivalent to:

$u = 0$ for $x = 0$ and $x = L_x$

$v = 0$ for $y = 0$ and $y = L_y$

$w = 0$ for $z = 0$ and $z = L_z$

where u,v and w are the components of the particle velocity vector in the x,y and z directions respectively.

From the equation of motion[21], the pressure is:

$$p = -\rho\frac{\partial\phi}{\partial t} = -\ j\omega\rho\phi \tag{3.5}$$

(where ρ is the density of the medium).

The component particle velocity is, by definition

$$u = \frac{\partial\phi}{\partial n} = \ jk_n\phi \tag{3.6}$$

where n denotes a direction normal to the surface.

The particle velocity in the x-direction can be obtained from equations (3.5) and (3.6) as:

$$u = -\ \frac{1}{j\omega\rho}\ \frac{\partial p}{\partial x} \tag{3.7}$$

with similar expressions for v in the y-direction and w in the z-direction.

3.2.3) CHARACTERISTIC VALUES AND CHARACTERISTIC FUNCTIONS:

Application of the boundary conditions at $x = 0$, $y = 0$ and $z = 0$ to the respective equations for particle velocities obtained from the array of the eight expressions represented by equation (3.3), results in the standing wave equation:

$$p = P(\cos k_x x \cos k_y y \cos k_z z)e^{-j\omega t} \tag{3.8}$$

Upon substitution of equation (3.8) into equation (3.7) the particle velocity in the x-direction becomes:

$$u = -\frac{k_x P}{j\omega\rho}(\sin k_x x \cos k_y y \cos k_z z)e^{-j\omega t} \tag{3.9}$$

which is zero for $x = 0$.

If the additional condition of $u = 0$ at $x = L_x$ is applied to equation (3.9), then:

$$\sin k_x L_x = 0 \quad , \text{ or:}$$

$$k_x = \frac{n_x \pi}{L_x} \qquad (n_x = 0,1,2,3 \ldots) \tag{3.10}$$

with similar expressions for k_y and k_z.

On substitution of these allowed values for k_x, k_y and k_z in equation (3.8) we obtain:

$$p_n(x,y,z,t) = P \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) e^{-j\omega t} \tag{3.11}$$

as a generalexpression for the characteristic function describing the pressure in any of the standing waves (as denoted by the subscript: n) inside a rigid-walled rectangular enclosure.

The component characteristic values k_x, k_y and k_z in this expression are limited to the values given by equation (3.10). This limitation, in turn, restricts the characteristic frequencies corresponding to the allowed normal modes of vibration as obtained upon substitution in equation (3.4) to:

$$f_n = \frac{\omega_n}{2\pi} = \frac{c}{2}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]^{\frac{1}{2}} \tag{3.12}$$

A particular normal mode of vibration corresponding to any set of values of n_x, n_y and n_z can be produced by starting a wave in the direction given by any combination of the direction cosines $\pm \frac{k_x}{k}$, $\pm \frac{k_y}{k}$ and $\pm \frac{k_z}{k}$ and letting it be reflected from the various room surfaces until it becomes a standing wave.

A knowledge of the characteristic frequencies of a room is essential to a complete understanding of its acoustic properties, for the room will act as a resonator and respond strongly to those impressed sounds having frequencies in the immediate vicinity of any of its characteristic frequencies.

3.2.4) FREQUENCY DISTRIBUTION OF NORMAL MODES:-

Equation (3.12) suggests[16] that $\bar{f}$ may be considered as a vector in frequency space having components $\frac{n_x c}{2L_x}$, $\frac{n_y c}{2L_y}$ and $\frac{n_z c}{2L_z}$.

The direction of the vector gives the direction of the wave producing the standing wave and the length of the vector the frequency.

A normal mode of vibration may therefore be represented by a point in this frequency space whose x-component is an integral number of unit lengths $\frac{c}{2L_x}$, its y-component is an integer times $\frac{c}{2L_y}$ etc. The length of the line joining this point to the origin is a measure of the characteristic frequency of this particular normal mode and the direction of this line is the direction of the wave that can be used to generate the standing wave.

A few of the characteristic points are shown in Fig.(3.2) and it can be seen that they correspond to the intersections of a rectangular lattice having f_x, f_y and f_z spacings equal to $\frac{c}{2L_x}$, $\frac{c}{2L_y}$ and $\frac{c}{2L_z}$ respectively.

This method of representing the normal modes of vibration by a lattice of characteristic points in frequency space is extremely useful in computing the number and types of normal modes having frequencies within a given frequency range.

Consider each lattice point as occupying a rectangular block in frequency space of dimensions $\frac{c}{2L_x}$, $\frac{c}{2L_y}$, $\frac{c}{2L_z}$ and volume = $\frac{c^3}{8L_xL_yL_z} = \frac{c^3}{8V}$.

Since all the normal modes of characteristic frequency f and below are included amoung the points contained within the octant($\frac{1}{8}$ of sphere), between the positive f_x,f_y and f_z axes and a spherical surface of radius f, the number of points can be obtained by dividing $\frac{\pi f^3}{6}$,the volume of the octant, by $\frac{c^3}{8V}$.

Accordingly, the number of normal modes of frequency below f is approximately:

$$N \simeq \frac{4\pi V}{3c^3}.f^3 \tag{3.13}$$

Actually the points fill all of the first octant in frequency space, which means that the space occupied is more than the first octant, since the parallelopipeds extend $\frac{c}{4L_y}$ beyond the (x-z) plane, and so on. The correct total volume occupied by points with frequency less than f is therefore:

$$\frac{\pi.f^3}{6} + \left(\frac{c}{4L_x} + \frac{c}{4L_y} + \frac{c}{4L_z}\right)\frac{\pi}{4} f^2 + \left(\frac{c^2}{16L_xL_y} + \frac{c^2}{16L_xL_z} + \frac{c^2}{16L_yL_z}\right) f + \ldots$$

and on dividing by the volume occupied by one point $\left(= \frac{c^3}{8V}\right)$ we obtain the number of modes of frequency below f:

$$N(f) = \frac{4\pi V}{3c^3} f^3 + \frac{\pi S}{4c^2} f^2 + \frac{L}{8c}.f + O(f) \tag{3.14}$$

when:-

$$V = L_xL_yL_z \quad , \quad S = 2(L_xL_y+L_xL_z+L_yL_z)$$

$$L = 4(L_x+L_y+L_z)$$

and O(f) is an irregular step function where magnitude is of the order of unity.

3.2.5) CLASSIFICATION OF WAVES IN A ROOM.

Various types of waves can occur in a rectangular room. Those for which none of the n's are zero are termed oblique waves, those for which one n is zero are tangential waves, and those for which two n's are zero are axial waves.

Axial waves are made up of two travelling waves propagated parallel to an axis and striking only two walls.

Tangential waves are built up of four travelling waves, reflecting from four walls and moving parallel to two walls.

Oblique waves are built up of eight travelling waves reflecting from all six walls.

Therefore the standing waves can be separated into three categories or seven classes:

Axial waves (for which two n's are zero):

x-axial waves parallel to the x-axis ($n_y, n_z = 0$)

y-axial waves parallel to the y-axis ($n_x, n_z = 0$)

z-axial waves parallel to the z-axis ($n_x, n_y = 0$)

Tangential waves (for which one n is zero)

y,z-tangential waves, parallel to the y,z-plane ($n_x = 0$)

x,z-tagential waves, parallel to the x,z-plane ($n_y = 0$)

x,y-tangential waves, parallel to the x,y-plane ($n_z = 0$)

Oblique waves (for which no n is zero)

Each of these three types of waves has different properties; even to the first approximation waves of different classes have different decay rates and accordingly different reverberation times.

There is also a difference in energy content, the total sound energy in the room for one of the waves being[18]

$$\omega_n = \frac{1}{2} \iiint \left[\rho U^2 + \frac{P^2}{\rho c^2} \right] dxdydz = \frac{V\epsilon}{2\rho c^2} \qquad (3.15)$$

WHERE V IS THE ROOM VOLUME

where the factor ϵ has the value $^1/_8$ for oblique waves, $\frac{1}{4}$ for tangential waves and $\frac{1}{2}$ for axial waves.

Consequently, it is important to count the number of the standing waves of a given class having frequency less than f.

3.2.6) AVERAGE FORMULAS FOR NUMBERS OF ALLOWED FREQUENCIES:

The representation in a lattic system is also useful here in computing the average number of standing waves of a given class having frequencies less than f.

Consider each lattice point to occupy a rectangular block $\frac{c}{2L_x} \cdot \frac{c}{2L_y} \cdot \frac{c}{2L_z} = \frac{c^3}{8V}$ in frequency space with the actual lattice point at the centre of the block. Then the average number of points is obtained by dividing the volume of frequency space considered by the volume $\frac{c^3}{8V}$ of each block.

The average number of x-axial waves equals the number of blocks in a rod of cross-sectional area $\frac{c^2}{4L_yL_z}$ and length f (Fig.3.3)

$$\therefore\ N_{ax-x} = \frac{c^2}{4L_yL_z} . f \Big/ \frac{c^3}{8V} = \frac{2.f.L_x}{c}$$

and the average number of all axial waves with frequency less than f:

$$N_{ax} = \frac{2.f.(L_x+L_y+L_z)}{c}$$

since $L_x+L_y+L_z = \frac{L}{4}$ (L being the sum of all the length of all the edges of the room)

$$N_{ax} = \frac{f.L}{2c} \qquad (3.16)$$

The average number of y,z-tangential waves is the number of blocks in a quarter of a disk of thickness $\frac{c}{2L_x}$ and radius f minus a correction for the axial waves already counted (Fig.3.4)

$\therefore$ the average number of y,z-tangential waves:

$$N_{ta-yz} = \frac{\pi f^2}{c^2} . L_yL_z - \frac{f}{c}(L_y+L_z)$$

and the average number of all tangential waves:

$$N_{ta} = \frac{\pi . f^2 . A}{2c^2} - \frac{fL}{2c} \qquad (3.17)$$

where $A = 2(L_xL_y+L_xL_z+L_yL_z)$.

The volume occupied by lattice points for the oblique waves of frequency less than f is the volume of $\frac{1}{8}$ of a sphere of radius f minus the volume already counted for the other classes

$$N_{ob} = \left(\frac{4\pi f^3 V}{3c^3}\right) - \left(\frac{\pi f^2 A}{4c^2}\right) + \left(\frac{fL}{8c}\right) \tag{3.18}$$

and the total number of standing waves of all classes which have frequencies less than f is:

$$N = \left(\frac{4\pi f^3 V}{3c^3}\right) + \left(\frac{\pi f^2 A}{4c^2}\right) + \left(\frac{fL}{8c}\right) \tag{3.19}$$

3.2.7) <u>MODAL DENSITY</u>:

A still more useful quantity is the number of normal modes having frequencies in a band of width Δf, centred on f. This is obtained by differentiating equation (3.19) with respect to f.

The result is:

$$\Delta N = \left(\frac{4\pi V}{c^3} . f^2 + \frac{\pi A}{2c^2} . f + \frac{L}{8c}\right) \Delta f \tag{3.20}$$

and the average modal density (i.e. the number of characteristic frequencies in one Hz band centred on f) equals:

$$\frac{\Delta N}{\Delta f} = \frac{4\pi V}{c^3} f^2 + \frac{\pi A}{2c^2} . f + \frac{L}{8c} \tag{3.21}$$

Similar expressions can be obtained for each class of waves by similar differentiation of corresponding equations.

For example the modal density of the x-axial waves:

$$\frac{dN_{ax-x}}{df} = \frac{2L_x}{c} \tag{3.22}$$

The above discussion indicates that the average number of allowed frequencies in a band increases with the square of the frequency or the size of the enclosure.

From equation (3.21) the mean separation between characteristic frequncies can be calculated:

$$\overline{\Delta f} = \left[\left(\frac{4\pi V.f^3}{c^3}\right) + \left(\frac{\pi A.f}{2c^2}\right) + \left(\frac{L}{8c}\right) \right]^{-1} \qquad (3.18)$$

which decreases as the frequency or the volume is increased. As a result, the spacing of the characteristic frequencies at high frequencies is so close that specific resonances may be neglected and the room may instead be considered as capable of carrying all frequencies in this region with equal readiness.

Since the standing wave pattern corresponding to each characteristic frequency is generally associated with some particular set of direction cosines, any increase in ΔN indicates an increase in the randomness of direction of the associated waves.

3.2.8) DEGENERATE NORMAL MODES:

The response of a room is observed to become less uniform as its symmetry is increased. This results from the increase in the number of degenerate modes i.e. standing waves having different n's but the same characteristic frequency. The degeneracy becomes worse with the increase of symmetry, for example in cubical rooms where even six-fold or more degeneracies are not uncommon. The response then becomes very irregular as the characteristic frequencies 'clump together' leaving large intervals within which there is no characteristic frequency.

The ratios of room dimensions determine the distribution of the characteristic frequencies and degeneracies can be eliminated by non-integral ratios of three dimensions.

3.3) THE SOUND FIELD IN RECTANGULAR ROOMS WITH NON-RIGID BOUNDARIES.

3.3.1) THE BOUNDARY CONDITIONS:

In the case of fairly hard walls, a certain degree of damping, expressed by the introduction of an exponential factor ($e^{-\delta t}$) in the standing wave equation, is taken into account, but it is assumed that the wave shape is not changed.

If the walls are not completely rigid, the normal components of the particle velocity at the wall surfaces will have non-vanishing values. This will affect the shapes and natural frequencies of the standing waves and must, therefore, be taken into consideration.

The boundary conditions, assumed earlier, of the velocity components normal to the walls being equal to zero, must be replaced by the more general condition of the complex ratio of the pressure to the normal component of the particle velocity at the wall surface being equal to the acoustic impedance of the wall.

The impedance is considered(22) as a more fundamental quantity for expressing the absorbing characteristics of the wall material as it depends on the physical properties of the material and not on the incident sound wave. In most cases of practical interest(22), the impedance is nearly independent of the angle of incidence sound and depends only on the frequency of the sound wave. Thus it is assumed that each wall has uniform physical properties expressed by a single value of its acoustic impedance which may differ from wall to wall.

In the range of validity of these assumptions, the expressions for the acoustic variables (e.g. the pressure) expressed in cartesian coordinates, are separable and so the characteristic values of each coordinate depends only on the impedance of the pair of walls relative to that coordinate only and not on the impedances of the other two pairs of walls.

3.3.2) CHARACTERISTIC VALUES AND CHARACTERISTIC FUNCTIONS:

Assuming, as before, a harmonic law for the acoustic variables, the wave equation (3.1), expressed in terms of the pressure wave can be written as:

$$\nabla^2 p + k^2 p = 0 \qquad (3.19)$$

This equation will yield non-zero solutions fulfilling the new boundary conditions only for particular discrete values of k - known as the characteristic values and denoted by the symbol k_n. These are the cases of importance in architectural acoustics[14]. It is implicitly contained in the acoustic impedance since the impedance depends, in general, on the frequency. It also depends on the size of the room and the shape and absorptive properties of the walls[12].

The pressure distribution in a single standing wave is expressed by:

$$P(x,y,z,t) = \psi_n e^{-j\omega t} \qquad (3.20)$$

where:

$\omega = \frac{k}{c}$ is the angular driving frequency in forced vibrations, or

$\omega = \omega_n = \frac{k_n}{c}$ is the angular characteristic frequency in free vibrations

and:

ψ_n is the characteristic function of the standing wave, being the solution of the wave equation which satisfies the proper boundary conditions.

For the rigid-walled condition, the characteristic values - k_n - are real and the characteristic functions (ψ_n) are expressed from equation (3.8) as:

$$\psi_n = \cos k_x x \cos k_y y \cos k_z z \qquad (3.21)$$

where the component characteristic values k_x, k_y and k_z are given by equation (3.10).

If the walls are not rigid, the characteristic values are complex $\left(k_n = \frac{\omega_n + j\delta_n}{c}\right)$ and so are their co-characteristic components: $k_x = \frac{\omega_x + j\delta_x}{c}$ with similar expressions for k_y and k_z.

Also some phase constants should be included in the arguments of the cosine functions of equation (3.21), which, together with the new characteristic components will be determined by solving for the boundary conditions.

Equation (3.21) is, thus, equivalent to:

$$\psi_n = \cos(k_x x+\phi_x)\cos(k_y y+\phi_y)\cos(k_z z+\phi_z) \qquad (3.22)$$

where:

ϕ_x, ϕ_y and ϕ_z are some phase constants.

Equation (3.22) can be expressed in the hyperbolic term, which yields simpler mathematical notations by using the standard transformations of trigonometric functions in terms of exponential functions. Accordingly, equation (3.22) becomes:

$$\psi_n = \cosh(jk_x x+\phi_x)\cosh(jk_y y+\phi_y)\cosh(jk_z z+\phi_z) \qquad (3.23)$$

As it has been assumed that the three terms are separable, only the x-terms will be used in deriving the values of the complex characteristic value k_x in terms of the properties of the boundary conditions.

It follows:

$$P_x = \psi_x = \cosh(jk_x x+\phi_x) \qquad (3.24)$$

and from equation (3.7)

$$U = -\frac{1}{j\omega\rho}\frac{\partial P}{\partial x} \qquad (3.7)$$

therefore, from (3.24) and (3.7), the normal velocity:

$$U = -\frac{k_x}{\omega\rho}\sinh(jk_x x+\phi_x) \qquad (3.25)$$

with similar expressions for P_y, P_z, V and W.

The boundary conditions, which fix the values of the constants $k_x, k_y, k_z, \phi_x, \phi_y$ and ϕ_z are that the ratio of the pressure to normal velocity into the surface of each wall equals the impedance (Z_x) of the surface.

For the x-walls:

$$Z_x = \frac{P_x}{U} = -\frac{\omega\rho}{k_x}\coth(jk_x x+\phi_x) \qquad (3.26)$$

or, expressing the impedance in units of the acoustic impedance (ρc) of air for plane waves, the impedance is denoted by $\xi = \frac{Z}{\rho c} = R+jx$ and termed the specific acoustic impedance of the material.

Equation (3.26) becomes:

$$\xi_x = -\frac{k}{k_x}\coth(jk_x x+\phi_x) \qquad (3.27)$$

If the specific impedances of the x-wall (at x = 0) is denoted by ξ_{x1} and that of the x-wall (at $x = L_x$) by ξ_{x2}, then, at x = 0 (substituting in equation (3.27):

$$\xi_{x1} = -\frac{P_x}{U\rho c} = \frac{k}{k_x}\coth(\phi_x)$$

or:

$$\phi_x = \coth^{-1}\left(\frac{\xi_{x1}\cdot k_x}{k}\right) \qquad (3.28)$$

at $x = L_x$:

$$\xi_{x2} = \frac{P_x}{U\rho c} = -\frac{k}{k_x}\coth\left(jk_x L_x+\coth^{-1}\left(\frac{\xi_{x1}\cdot k_x}{k}\right)\right)$$

which is equivalent to:

$$-\frac{\xi_{x2}\cdot k_x}{k} = \coth\left(jk_x L_x+\coth^{-1}\left(\frac{\xi_{x1}\cdot k_x}{k}\right)\right)$$

or

$$jk_x L_x + \coth^{-1}\left(\frac{\xi_{x1}\cdot k_x}{k}\right) + \coth^{-1}\left(\frac{\xi_{x2}\cdot k_x}{k}\right) = 0 \qquad (3.29)$$

which is the basic equation for k_x.

Similar two other equations are obtained for k_y and

k_z by applying the boundary conditions at the y-walls and the z-walls:

$$jk_yL_y + \coth^{-1}\left(\frac{\xi_{y1}\cdot k_y}{k}\right) + \coth^{-1}\left(\frac{\xi_{y2}\cdot k_y}{k}\right) = 0$$

and

$$jk_zL_z + \coth^{-1}\left(\frac{\xi_{z1}\cdot k_z}{k}\right) + \coth^{-1}\left(\frac{\xi_{z2}\cdot k_z}{k}\right) = 0$$

As discussed earlier, the complex characteristic value k_n can be expressed in terms of its component values:

$$k_x = \frac{\omega_x + j\,\delta_x}{c}\ , \text{ where:}$$

ω_x is the characteristic frequency component, and

δ_x is the damping constant component.

Likewise, we can utilize the expression for the specific acoustic admittance of the wall,

$\beta = \frac{1}{\xi} = \frac{\rho c}{z}$ to express these basic equations in the standard form:

$$jk_xL_x + \coth^{-1}\left(\frac{k_x}{k}\cdot\frac{1}{\beta_{x1}}\right) + \coth^{-1}\left(\frac{k_x}{k}\cdot\frac{1}{\beta_{x2}}\right) = 0$$

$$jk_yL_y + \coth^{-1}\left(\frac{k_y}{k}\cdot\frac{1}{\beta_{y1}}\right) + \coth^{-1}\left(\frac{k_y}{k}\cdot\frac{1}{\beta_{y2}}\right) = 0 \qquad (3.30)$$

$$jk_zL_z + \coth^{-1}\left(\frac{k_z}{k}\cdot\frac{1}{\beta_{z1}}\right) + \coth^{-1}\left(\frac{k_z}{k}\cdot\frac{1}{\beta_{z2}}\right) = 0$$

Due to the periodicity of the hyperbolic cotangent[24], there is an infinity of roots of each of the three equations. For each of the equations, the lowest value corresponds to a wave with no pressure node perpendicular to the respective direction, and the n^{th} value to one with n pressure nodes. The integers n_x, n_y and n_z, therefore serve to label the particular standing wave under consideration.

Solutions are obtained in the form of power series manipulations[22,23,24] or graphically by conformal mapping[22]. A power series approach, based on one suggested by P.M.Morse[24], is adopted in this work. Different formulae are valid for different ranges of convergence, where the convergence limits are set by the relative magnitude of the quantity (R) to unity, where

$$R = \frac{\omega L.|\beta|}{\pi.c.n} \quad (\text{if } n > 0)$$

and

$$R = \frac{\omega.L.|\beta|}{\pi.c} \quad (\text{if } n = 0)$$

where

$|\beta|$ is the magnitude of the specific acoustic admittance of the wall

ω : is the angular frequency

L : is the corresponding room dimension

c : is the speed of sound

and n : is the integer labelling the standing wave

Let the above condition be denoted by (R_{x1}) for the x-wall (at x = 0), and (R_{x2}) for the x-wall (at $x = L_x$), with similar notations for the y-walls and z-walls.

The power series solutions, expressing the component characteristic values in terms of the walls admittances and the frequency parameter (η_x), where:

$$\eta_x = \frac{\omega L_x}{\pi c}$$ for the x quantities - for the different

ranges of convergent limits are:

(1) for R_{x1} and R_{x2} small compared to unity,

and for n = 0:

$$k_x = \frac{\pi}{L_x} \sum_{m=0} \left(\frac{j^{(6m+1)} \eta_x^{(2m+1)} (\beta_{x1}^3 + \beta_{x2}^3)^{2m} . \pi^{(2m-1)}}{((2m+1)!)^{2m} (\beta_{x1}+\beta_{x2})^{(4m-1)}} \right)^{\frac{1}{2}} \qquad (3.31)$$

and for n > 0:

(1) contd.

$$k_x = \frac{\pi}{L_x} \sum_{m=0} \frac{j^{m^2}}{n_x^{(2m-1)}} \cdot \left(\frac{\eta_x(\beta_{x1}+\beta_{x2})}{\pi}\right)^m \qquad (3.32)$$

(2) for R_{x1} small and R_{x2} large compared to unity:

$$k_x = \frac{\pi(n_x+\frac{1}{2})}{L_x} + \frac{\pi}{L_x} \sum_{m=1} \frac{j^{m^2}}{(\pi(n_x+\frac{1}{2}))^m} \left(\left(\frac{\eta_x \cdot \beta_{x1}}{(n_x+\frac{1}{2})}\right)^m - j^{2m}\left(\frac{(n_x+\frac{1}{2})}{\eta_x\beta_{x2}}\right)^m \right) \qquad (3.33)$$

(3) for R_{x1} and R_{x2} large compared to unity:

$$k_x = \frac{\pi}{L_x} \sum_{m=0} j^{m^2}(n_x+1) \cdot \left(\frac{\beta_{x1}+\beta_{x2}}{\pi\eta_x\beta_{x1}\beta_{x2}}\right)^m \qquad (3.34)$$

similar expressions are derived for the y and z quantities.

3.3.3) RESONANCE FREQUENCIES AND DAMPING CONSTANTS.

Solutions of the three basic transcendental equations (3.30) make it possible to compute the component characteristic values k_x, k_y, k_z in terms of the specific acoustic admittances β_x, β_y and β_z of the room surfaces.

Since these component characteristic values can be represented as:

$$k_x = \frac{\omega_x+j\delta_x}{c}, \quad k_y = \frac{\omega_y+j\delta_y}{c} \quad \text{and} \quad k_z = \frac{\omega_z+j\delta_z}{c}$$

and are related, according to equation (3.4) by:

$$k_n = (k_x^2+k_y^2+k_z^2)^{\frac{1}{2}}$$

we obtain

$$\left(\frac{\omega_n+j\delta_n}{c}\right)^2 = \left(\frac{\omega_x+j\delta_x}{c}\right)^2 + \left(\frac{\omega_y+j\delta_y}{c}\right)^2 + \left(\frac{\omega_z+j\delta_z}{c}\right)^2 \qquad (3.35)$$

and equating the real and imaginary parts:

$$\omega_n^2 - \delta_n^2 = (\omega_x^2 + \omega_y^2 + \omega_z^2) - (\delta_x^2 + \delta_y^2 + \delta_z^2) \tag{3.36}$$

and

$$\delta_n = \frac{\delta_x\omega_x}{\omega_n} + \frac{\delta_y\omega_y}{\omega_n} + \frac{\delta_z\omega_z}{\omega_n} \tag{3.37}$$

In most case of practical interest[22], the damping constant (δ_n) is much smaller than the resonance frequency ω_n, and the term (δ_n^2) in equation (3.0) can be ignored. It follows that:

$$\omega_n = [(\omega_n^2-\delta_x^2)+(\omega_y^2-\delta_y^2)+(\omega_z^2-\delta_z^2)]^{\frac{1}{2}} \tag{3.38}$$

and, equation (3.37) can be written as

$$\delta_n = \frac{1}{2\omega_n}\left[2\delta_x\omega_x+2\delta_y\omega_y+2\delta_z\omega_z\right] \tag{3.39}$$

The characteristic frequency (ω_n) and the damping constant (δ_n) are the quantities usually determined experimentally, rather than their components $\omega_x, \omega_y, \omega_z$ and $\delta_x, \delta_y, \delta_z$. Therefore, for their computation in terms of the boundary conditions it is more suitable to obtain values for the quantities $(\omega_x^2-\delta_x^2)$, $(\omega_y^2-\delta_y^2)$, $(\omega_z^2-\delta_z^2)$ and $(2\delta_x\omega_x)$, $(2\delta_y\omega_y)$, $(2\delta_z\omega_z)$. These can be obtained from the transformation of k_x^2, k_y^2 and k_z^2 in terms of the specific acoustic admittance of the respective pairs of walls:

$$k_x^2 = \left(\frac{\omega_x+j\delta_x}{c}\right)^2 = \left(\frac{\omega_x^2-\delta_x^2}{c}\right)^2 + \frac{j2\delta_x\omega_x}{c^2}$$

i.e. $c^2k_x^2 = (\omega_x^2-\delta_x^2)+j2\delta_x\omega_x$.

The quantities $(\omega_x^2-\delta_x^2)$ and $2\delta_x\omega_x$ are the real and imaginary parts of $(c^2k_x^2)$.

To the first approximation, these results can be expressed in terms of the real and imaginary parts of the specific acoustic admittance (β), the specific acoustic conductance being denoted by (γ) and the specific acoustic susceptance as (σ) so that $\beta(\omega) = \gamma+j\sigma$.

From the expressions (3.31) and for small values of η_{x1} and η_{x2} we obtain:

$$(\omega_x^2 - \delta_x^2) \simeq \begin{cases} -\frac{kc^2}{L_x}(\sigma_{x1}+\sigma_{x2}) & ; n = 0 \\ \left(\frac{m\pi c}{L_x}\right)^2 + \frac{2kc^2}{L_x}(\sigma_{x1}+\sigma_{x2}) & ; n > 0 \end{cases} \qquad (3.40)$$

and:

$$2\delta_x\omega_x \simeq \begin{cases} \frac{kc^2}{L_x}(\gamma_{x1}+\gamma_{x2}) & ; n = 0 \\ \frac{2kc^2}{L_x}(\gamma_{x1}+\gamma_{x2}) & ; n > 0 \end{cases} \qquad (3.41)$$

with similar expressions for the y and z-components.

3.3.4) CONCLUSION.

The expressions for the component characteristic values k_x, k_y and k_z obtained for the solutions of equation (3.30) and for the phase constants from equations (3.28) enable us to evaluate the characteristic function of the standing wave by substituting these values in equation (3.22).

Similarly the evaluation of the real and imaginary parts of k_x^2, k_y^2 and k_z^2 render the values of the resonance frequency and damping constant by inserting these values in equations (3.38) and (3.39) respectively.

With these quantities determined, the steady-state response of the room can be readily obtained as an expansion in normal modes of vibration.

3.4) THE STEADY-STATE RESPONSE OF THE ROOM

3.4.1) INTRODUCTION

So far in this chapter the characteristic values and functions have been considered, and methods have been derived for computing their values in terms of acoustic admittance of the bounding surfaces and the driving frequency of the impressed sound in the steady-state sound field for individual modes of vibration. A logical extension for our analysis is to apply these results to the study of the steady-state response of the room.

The sound field in the room is represented by the wave equation; this, expressed in terms of the velocity potential, is:

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \qquad (3.1)$$

Assuming a harmonic law for the pressure variation

$$\nabla^2 P_o + k^2 P_o = 0 \quad \text{where } k = \frac{\omega}{c} \qquad (3.19)$$

The wave equaton will only yield solutions satisfying the boundary conditions, discussed earlier, for particular discrete values of (k) which are the characteristic values k_n of the room.

Each characteristic value (k_n) is associated with a solution ($\psi_{n(x,y,z)}$) which is the characteristic function of the room in such a way that:

$$P = A\psi_n$$

As the characteristic function form a complete set of orthogonal set of functions[17,18], the steady-state distribution of sound can be expressed as a series (ψ_n) i.e.

$$p = P_n e^{-j\omega t} = \sum_n A_n \psi_n e^{-j\omega t} \qquad (3.42)$$

3.4.2) THE SOURCE FUNCTION:-

If it is assumed that the effect of the source and the room boundaries could be replaced by a distribution of simple sources such that the element of volume (dv') at (x',y',z') has an equivalent outflow of air (q*dv') at (x',y',z') cm³/sec, where (q) is the density function, and that any source of sound may be considered as an assembledge of simple sources, then:

$$Q_{(t)} = \iiint_{room} q\,dv'$$

$Q_{(t)}$ being the equivalent volume velocity of the source.

If the source is simple harmonic, then:

$$q_{(t)} = qe^{-j\omega t} \quad \text{and} \quad Q_{(t)} = Qe^{-j\omega t} \tag{3.43}$$

Assuming the orthogonality property(18), the source distribution can, likewise, be expanded in a series of (ψ_n) i.e.

$$q_{(x,y,z)}e^{-j\omega t} = \sum_n C_n\psi_n e^{-j\omega t} \tag{3.44}$$

where

$$C_n = \frac{1}{V\Lambda_n} \iiint_{room} q\psi_n dv$$

3.45

and

$$\iiint_{room} \psi_n\psi_m dv = \begin{cases} 0 & n \neq m \\ V\Lambda_n & n = m \end{cases}$$

Λ_n being the average of the integral over the room volume.

3.4.3) THE GREEN'S FUNCTION OF THE ROOM:-

The wave equation for the velocity potential in the presence of a source distribution of density q is:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -q_{(x,y,z)}e^{-j\omega t} \tag{3.46}$$

the steady-state presence is

$$p = -\rho\frac{\partial\phi}{\partial t} \quad ; \quad p = j\omega\rho\phi \tag{3.47}$$

Equation (3.46), expressed in terms of the pressure is:-

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho \frac{\partial q_{(t)}}{dt} = -j\omega\rho q_{(t)} \tag{3.48}$$

and substituting:

$$p = Pe^{-j\omega t} \text{ and } q_{(t)} = qe^{-j\omega t}$$

equation (3.48) reads

$$\nabla^2 P_o + k^2 P = -j\omega\rho q \tag{3.49}$$

and inserting $P_o = \sum_n A_n \psi_n$ and $q = \sum_n C_n \psi_n$ in equation (3.49)

$$\nabla^2 \sum_n A_n \psi_n + k^2 \sum_n A_n \psi_n = -j\omega\rho \Sigma C_n \psi_n \tag{3.50}$$

Since ψ_n are solutions of the equation:

$$\nabla^2 \psi_n + k_n \psi_n = 0 \tag{3.51}$$

therefore $\nabla^2 \psi_n = -k_n^2 \psi_n$

Using this relation and equations term by term in equation (3.50):

$$-k_n A_n \psi_n + k^2 A_n \psi_n = -j\omega\rho C_n \psi_n$$

i.e.

$$A_n = j\omega\rho \frac{C_n}{k^2 - k_n^2} \tag{3.52}$$

substituting for C_n:

$$A_n = \frac{j\omega\rho\, {}^1/V\Lambda_n \iiint_{room} q\, \psi_n dV}{k^2 - k_n^2} \tag{3.53}$$

If the source dimensions are small, compared to the wave length of the radiated sound, it approximates to a simple source and the integral $\iiint q\psi_n dV$ (in equation (3.53)) reduces to $Qo\psi_{n(x_o,y_o,z_o)}$ where:

Q_o is the equivalent magnitude of source strength,

$\psi(x_o,y_o,z_o)$ is the value of ψ_n at the source position.

Equation (3.53) then reads:

$$A_n = j\omega\rho \frac{{}^1/V\Lambda_n Q_o \psi_n(x_o,y_o,z_o)}{(k^2-k_n^2)} \tag{3.54}$$

By substituting for (A_n) in equation (3.42), the expression for the instantaneous pressure at the point (x,y,z) in a room excited by a simple source of strength (Q_o) and frequency (ω) is:

$$p = jQ_o\omega\rho \frac{\Sigma\, {}^1/V\Lambda_n \psi_n(x,y,z)\psi_n(x_o,y_o,z_o)}{(k^2 - k_n^2)} \quad e^{-j\omega t} \tag{3.55}$$

where: $k = \frac{\omega}{c}$

and $k_n = \frac{(\omega_n+j\delta_n)}{c}$; ω_n = the natural frequency of the mode and δ_n its damping constant.

If (δ_n) is small compared to ω_n the term δ_n^2 can be ignored and the expression in equation (3.55) reduces to:

$$P = \frac{jQ_o\omega\rho c^2}{V} \quad \frac{\Sigma\, {}^1/\Lambda_n \psi_n(x,y,z)\psi_n(x_o,y_o,z_o)}{(\omega^2-\omega_n^2 + 2j\omega_n\delta_n)} \quad e^{-j\omega t} \tag{3.56}$$

which is the Green's Function of the room.

3.4.4) NORMALIZING FACTORS:-

These are weighting factors applying to each mode of vibration and determined by consideration of the location of the source and receiver in the room and the steady-state pressure amplitude to which each mode is excited[23].

The normalization factors for the function are the mean value of (ψ_n^2) averaged over the room volume.

i.e.

$$V\Lambda_n = \iiint_{room} \psi_n^2 \, dV \tag{3.57}$$

If the walls of the room are rigid, the functions ψ_n are the products of the three cosines, and since the cosine function has a mean-squared value of one for an argument equal to zero and $\frac{1}{2}$ for argument different from zero, the constant Λ_n in the above equation can be computed as follows:

$$\Lambda_n = \Lambda_{nx} \Lambda_{ny} \Lambda_{nz} \tag{3.58}$$

where $\Lambda_{nx} = 1$ for $n_x = 0$

$\Lambda_{nx} = \frac{1}{2}$ for $n_x > 0$

and similarly for Λ_{ny} and Λ_{nz}.

If the walls have some admittance, the function (ψ_n) are hyperbolic functions with complex arguments and their normalizing factors are given by(22):

$$\Lambda_{nx} = \left[1 - \left(\frac{j\omega L_x}{3c}\right) \frac{(\beta_{x1}{}^3 + \beta_{x2}{}^3)}{(\beta_{x1} + \beta_{x2})^2} \right] \tag{3.59}$$

and

$$\Lambda_{nx} = \left[\frac{1}{2} + \left(\frac{j\omega L_x}{\pi c n_x^2}\right) \right] \tag{3.60}$$

for $n_x > 0$

and similarly for Λ_{ny} and Λ_{nz}

β_{x1} and β_{x2} being the admittance of the opposite walls of a pair in the room.

3.4.5) THE STRENGTH OF THE SOURCE:-

In order to derive an expression for the strength of the source (Q_o) in terms of some measurable quantity, e.g. the sound pressure at a fixed distance from the source in free field conditions, the following assumptions are made:-

(1) the source is simple periodic,

(2) the radial velocity of the surface of the source is equal

(2) contd.

to the particle velocity of sound in the surrounding medium.

Consider a pulsating sphere of radius (a) and radial velocity $U_{(t)}$. The rate of flow of air away from the surface of the sphere[25]:

$$Q_{(t)} = 4\pi a^2 U_{(t)} = 4\pi a^2 U_r \qquad (3.61)$$

where U_r is the radial component of the particle velocity.

The wave equation in spherical polar coordinates is:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (3.62)$$

A general solution for the pressure wave is:

$$p = \frac{P_o}{r}(r-ct) \qquad (3.63)$$

by Newton's equation:

$$\rho \frac{\partial U_r}{\partial t} = -\frac{\partial p}{\partial r}$$

The requirement at the surface of the sphere is:

$$\frac{1}{r}\frac{dP_o}{dr} - \frac{P_o}{r^2} = -\frac{\rho}{4\pi a^2}\frac{dQ}{dt} \qquad \text{at } r = a$$

If (a) is small compared to the wavelength of sound the assumption that the source is simple source holds and in this case $\left(\frac{P_o}{r}\right)$ is much larger than $\left(\frac{dP_o}{dr}\right)$, and therefore at r=a:

$$P_o \simeq \frac{\rho}{4\pi}\frac{dQ}{dt}$$

The pressure at a distance (r) from the centre of the source is therefore:

$$p = \frac{\rho dQ\,(t-{}^r/_c)}{4\pi r\, dt} \qquad (3.64)$$

The function (Q) gives the instantaneous value of the total flow of air away from the centre of the source. The pressure at a distance (r) is proportional to the rate of change of this flow a time $(^r/c)$ earlier.

If the simple source is periodic:

$$Q_{(t)} = Q_o e^{-j\omega t}$$

and the pressure:

$$p = \frac{j\omega\rho}{4\pi r} Q_o e^{-j(\omega t - kr)}$$

for $p = Pe^{-j\omega t}$

$$P = \frac{j\omega\rho}{4\pi r} Q_o e^{+jkr} \tag{3.65}$$

and the strength of the source is:

$$Q_o = -\frac{j4\pi rP}{\omega\rho} e^{+jkr} \tag{3.66}$$

The strength of the source, thus evaluated, can be utilized for the evaluation of (Q_o) in equation (3.56) to give the Green's function of the room.

3.5) <u>CONCLUSION</u>:

The foregoing analysis makes it possible to express the components of the characteristic values of a mode of vibration for a particular room in terms of the specific acoustic impedance of the bounding surfaces - a quantity considered[22] fairly unique in describing the physical properties of an absorbing treatment, since it is independent of the measuring conditions, the distribution of sound and its angle of incidence and is only a function of its frequency.

The relation between the component characteristic values and the specific acoustic admittance - the reciprocal of the specific impedance - is described by the transcendental equation (3.30).

Solutions for these equations are obtained, numerically, in the form of series of powers or, graphically, by conformal transformation, and many alternatives are reported in the literature [17,18,22,23,24,25].

In this work an approach proposed by Morse[24] is adopted for deriving solutions.

From the characteristic values the natural frequencies, the damping constants and the characteristic functions of the mode are computed. These, in turn, make it possible to evaluate the resonant response of the mode when excited by a sinusoidal signal in the steady-state condition.

The natural frequency, the damping constant and the resonant response (in amplitude and phase) are measurable quantities and can therefore be employed as parameters for judging the acoustic performance of the room when a particular absorbing surface treatment is applied to its boundaries.

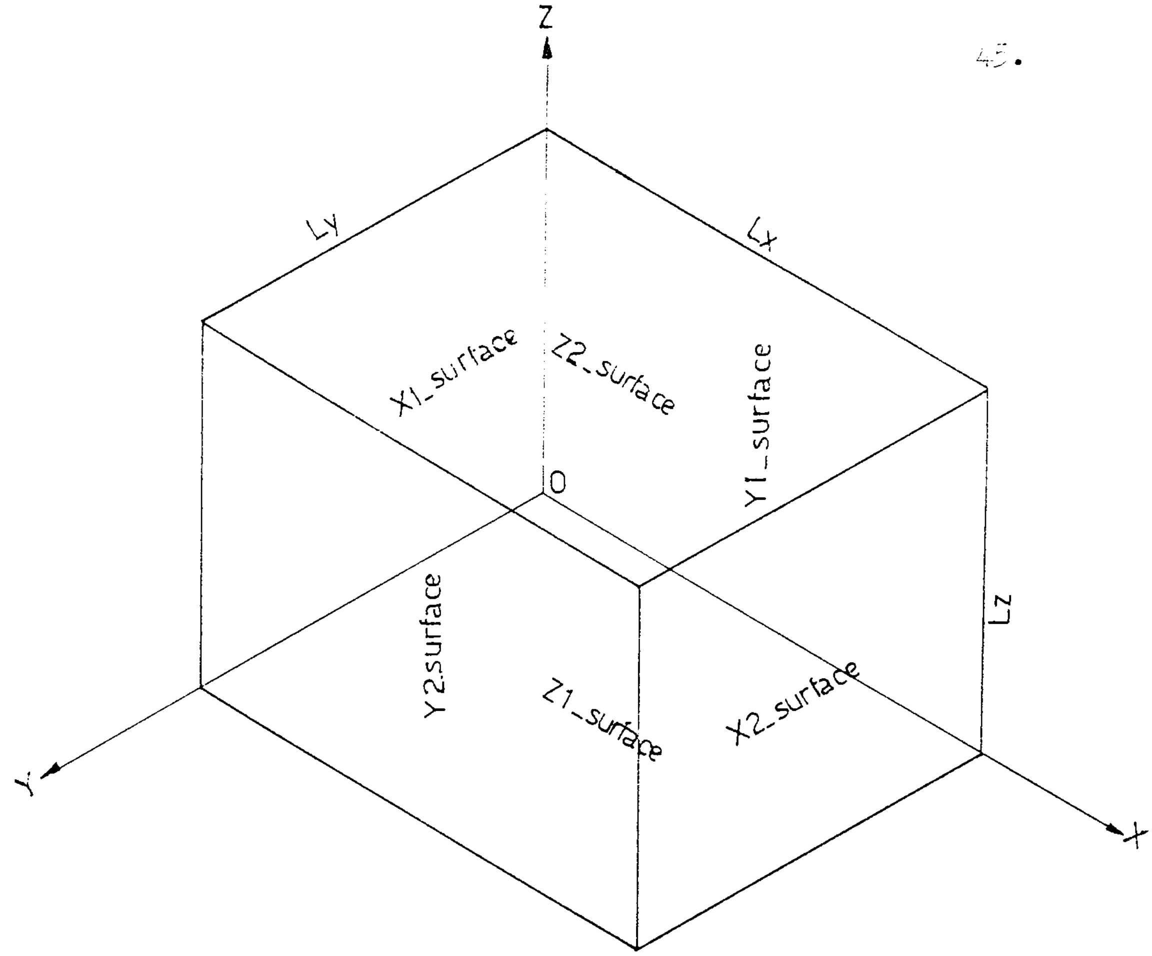
In this respect the calculated values can be compared with the measured values and used as design parameters in the same way the calculated and measured values of, for example, the reverberation time are used in transient-state conditions.

Similarly the shift in their values brought by introducing a different treatment can be used to evaluate the absorbing properties of these treatments.

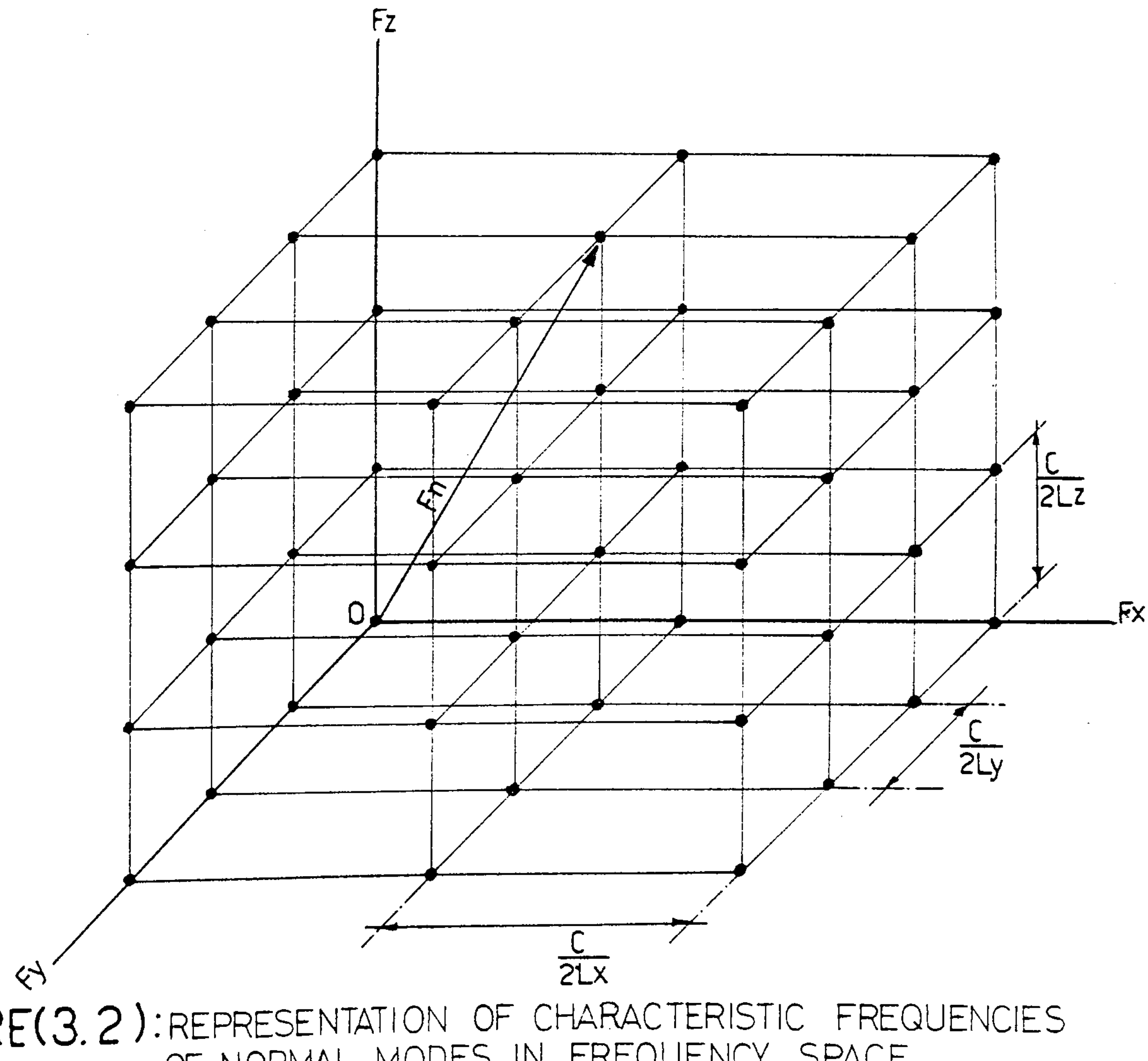
The characteristic functions describe the spatial distribution of the sound field in the presence of an excited mode, and the variation of its amplitude with distance governs the geometric structure of the shape of the mode.

Once again the calculated and measured values can be compared and used as a parameter for modes identification.

In the next chapter techniques will be derived which will make it possible to measure these parameters.



FIGURE(3.1): COORDINATE SYSTEM AND ROOM DIMENSIONS.



FIGURE(3.2): REPRESENTATION OF CHARACTERISTIC FREQUENCIES OF NORMAL MODES IN FREQUENCY SPACE.

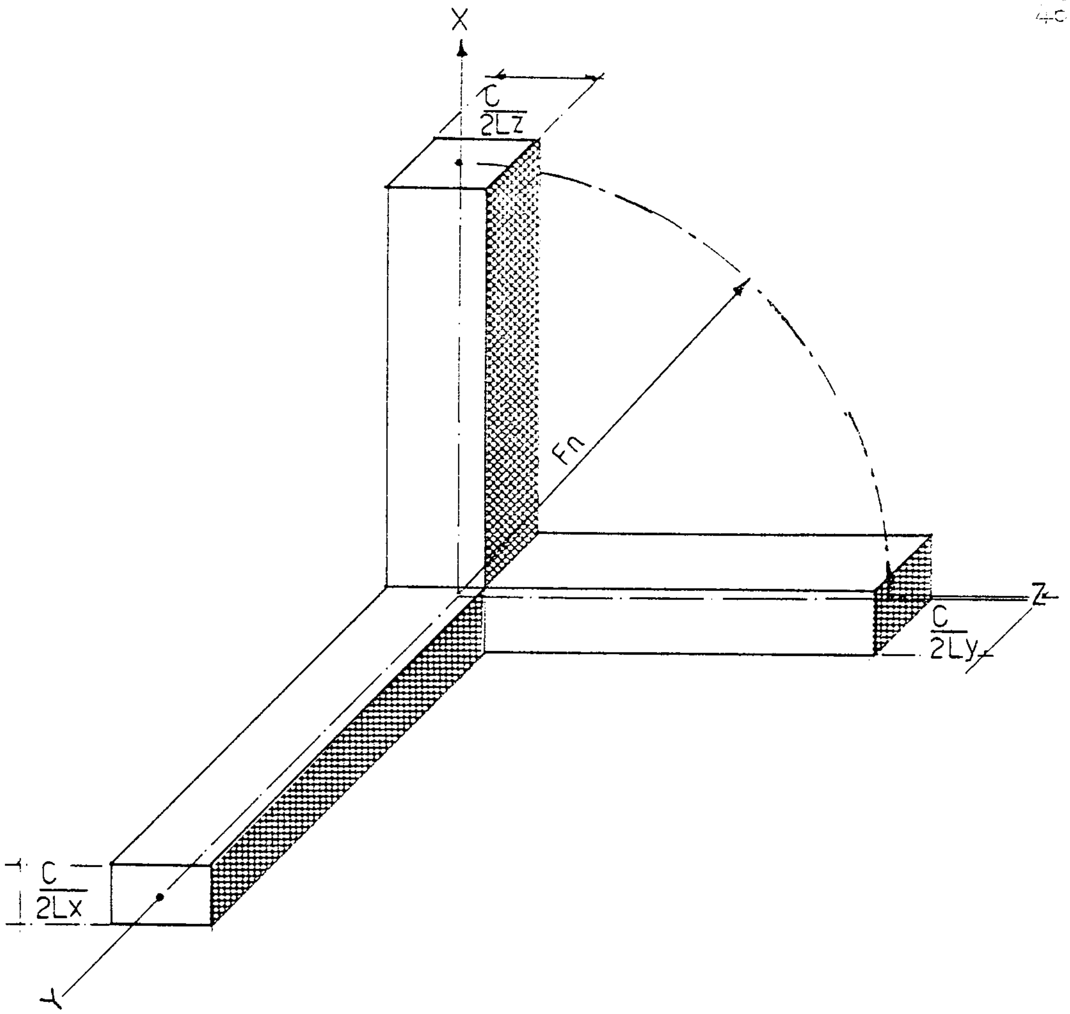


FIG.(3.3): REPRESENTATION OF AXIAL MODES IN FREQUENCY SPACE.

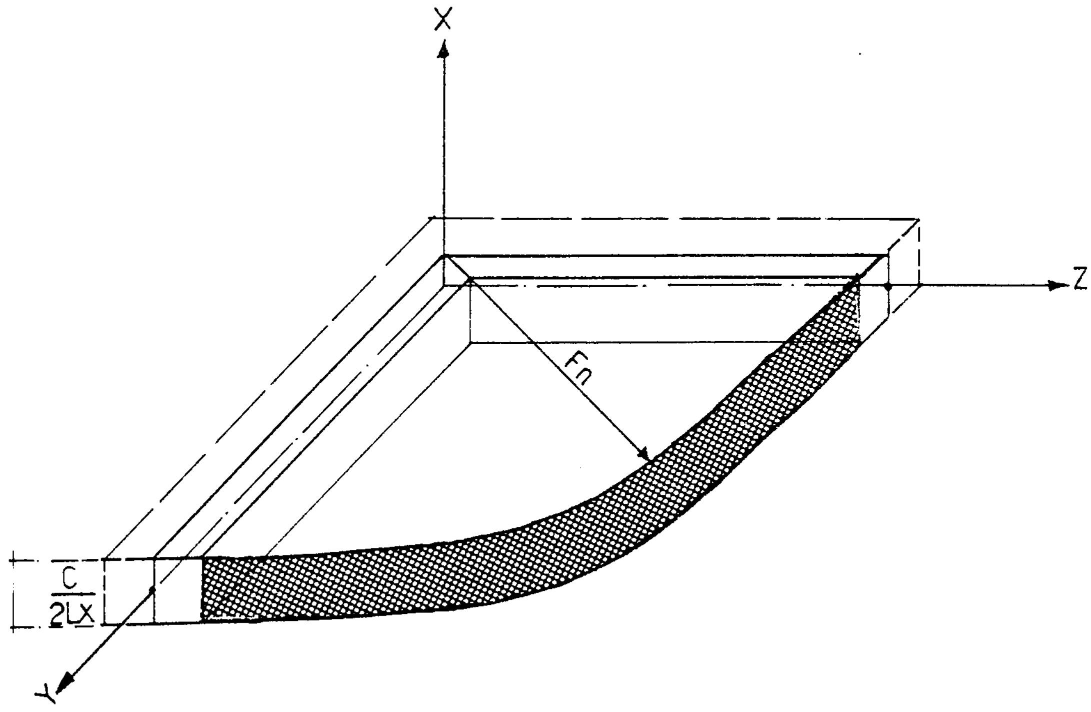


FIG.(3.4): REPRESENTATION OF TANGENTIAL MODES IN FREQUENCY SPACE.

CHAPTER FOUR

THEORIES OF EXPERIMENTAL TECHNIQUES

4.1) INTRODUCTION:

In applying the foregoing analysis, presented in Chapter Three, two problems arise; the first is to predict the acoustical behaviour of the room from a knowledge of the absorbing properties of the bounding surfaces - the acoustic impedance of the materials covering the walls - and the geometry of the room. The second is that of finding the impedance from measurements based on exploring the spatial distribution of sound fields in the presence of the absorbing material.

The steady-state acoustical behaviour of a room, when excited by a sinusoidal signal proves to be the combined effect of numerous systems(17). It is possible that the overall response of the room may be quite easily measured, but if the fundamental understanding of the problem is to be advanced, and if results from one particular experiment are to be reliably extrapolated to be a guide to the general problem, then a systematic study is required(26). This necessitates the ability of identifying the separate modes of vibration of the room and determining their characteristic properties.

The acoustic behaviour of the room, in the presence of an excited single mode is completely described by evaluating the natural frequency, the damping constant and the resonance response of the mode.

From measurements of the acoustic impedance of the materials used to form the bounding surfaces of the room, these parameters can be, theoretically, computed. Such impedance measurements are facilitated by use of the Standing Wave Tube and the Transmission Characteristic Methods. Both techniques will be described later in this chapter.

To verify the computations, methods are devised for measuring the modal parameters (i.e. the natural frequency, the damping constant and resonant response). This will provide a basis for comparisons of the predicted and measured values.

The methods are based on resonance testing techniques, originally applied in aeronautical engineering problems and adapted here to room acoustics measurements. Various techniques of resonance testing and their applications to acoustical measurements will be described, also, in this chapter.

The problem of finding the impedance of materials from exploring the spatial distribution of sound fields is investigated by two different techniques:-

1. The Standing Wave Tube Method, used with small sample of the material, and suitable for such materials whose acoustical performance is not dependent on the sample size.
2. The Transmission Characteristic Method used with large area of material and under actual mounting conditions, and suitable for the case when the surface material vibrates as a whole. In this case the tube measurements are not significant in representing the total impedance of the material since they are made on relatively small samples which allow little or no flexural vibration.

Also the second method can be employed for identification of the normal modes from the nodal surfaces of the modal shapes, displayed by the plots of spatial variations of resonance pressure amplitude with distance in the room.

4.2) DETERMINATION OF THE ACOUSTIC NORMAL IMPEDANCE OF SOUND ABSORBANTS

4.2.1) INTRODUCTION:

The Wave Modal Theory, describing the sound field in an enclosure, holds rigorously providing that the boundary conditions for the sound field are expressed in terms of the acoustic normal impedance of the boundary surfaces.

The normal impedance is defined as the ratio of pressure to normal air velocity at the surface of the material, and it is a quantity that depends on the physical properties of the material and not on the incident sound wave (except for variation with frequency). In certain cases, however, it changes with the angle of incidence of the sound wave.

In a modal sound field, different sanding waves impinge on the surfaces of the enclosure at different angles of incidence - defined by their directional cosines. The effect of the angle of incidence on the value of the normal impedance must therefore be considered.

In order to avoid complications in theory and practical applications, arising from the possible variation of the impedance with the angle of incidence, the choice of materials for surface treatment should be aimed at those exhibiting normal impedances which are independent of the angle of incidence.

Moreover, while the normal impedance is a quantity that can be determined accurately, measurements of the oblique incidence surface impedance are few and the techniques involved are not straightforward[27,28].

The choice of materials for surface treatment should thus be confined to those which are locally reactive in preference to materials of extended reaction, as for the former, the oblique incidence behaviour can adequately be determined by their normal impedance.

4.2.2) THEORY OF ACOUSTIC IMPEDANCE:

4.2.2.1) INTRODUCTION:

In order to investigate the effect of the angle of incidence on the normal impedance, the latter has to be expressed in terms of the physical properties of the material. The purpose is not to make a detailed analysis of the mechanism of sound absorption in the material, but rather to consider how the conditions for local reaction can be satisfied to obtain a normal impedance independent of the angle of incident of the sound wave.

4.2.2.2) LOCAL AND EXTENDED REACTION:

Different sorts of materials react differently to sound, depending on their structure.

Impervious materials yield slightly, and porous materials allow some air to penetrate below the surface. In any of these cases the reaction of the surface can be expressed in terms of a specific impedance, a complex ratio between pressure at the surface and the normal velocity. The normal velocity is due to a motion of the wall itself or else to a motion of air into the pores in the wall.

The impedance depends in general on the nature of the surface material, on the frequency of the wave and on its angle of incidence. The amount of dependence on the angle of incidence depends on how well wave motion can travel, in the surface material, parallel to the surface(18). If such motion is rapidly attenuated or is considerably slower than that of sound in air, then the material is locally reacting and the impedance is independent of the angle of incidence. On the other hand, if the wave motion in the surface is not attenuated and is faster than that in air, then the material is of extended reaction.

4.2.2.3) TYPES OF ABSORBING SURFACES:

The above distinction in the mechanism of absorption, as a function of the type of motion, leads to two different types of surface materials:

1. the panel type, where the normal velocity next to the wall is a motion of the panel as a whole, and the reaction of the wall to the pressure fluctuations is due to stiffness of the wall,
2. the porous type, where the normal velocity is due to the penetration of air into the pores of the material, and the reaction is due to the interaction of the penetrating air with the material of the pores.

In practical applications there are usually intermediate cases where the panel is backed by porous material or air space, or where the porous material also acts as a panel. These can be understood in the light of these two limiting cases[22].

Of the two, the porous type is more widely used in practice; besdies, its impedance can be easily determined by the Standing Wave Method. For these reasons, a detailed analysis for the derivation of its impedance will be presented in this work.

For the panel type, the work and conclusions of previous authors shall be adopted.

In the analysis, expressions of the impedance of oblique incidence will be derived in terms of the physical properties of the material and the angle of incidenceof sound waves. Similar expressions will be drawn for the normal impedance, in order to highlight the effect of the angle of incidence.

The conditions for local reaction will be considered with a view to proving that if the effect of the angle of incidence is negligible, then the normal impedance of a locally reacting material, as determined by the Standing Wave Method measurements, would be valid as a parameter for the study of modal fields in enclosures where waves impinge on walls at varying angles of incidence.

4.2.2.4) PORUS TYPE:

The theory of impedance of porous materials has been developed on the concept of a capillary pore medium, first suggested by Rayleigh[21], in which the sound energy dissipates into heat through viscous action. Accordingly, the impedance is derived as a function of macroscopic properties of the material (e.g. bulk modulus of elasticity, porosity, density etc.), and expressed in terms of two propagation parameters of the material (the propagation constant and the characteristic wave impedance).

An alternative theory is of a model of an array of parallel, elastic (or rigid) fibres in air[27]. This requires a knowledge of more basic aspects, related to the microstructure of the material (e.g. mean fibre radius, fibre concentration). Dissipation of energy inside the material is considered to take place on scattering at the fibre boundaries.

On the former concept, some theories have been developed which predict the impedance for isotropic materials[18,29,30] assuming a homogeneous medium. In such cases, the material is characterized by physical properties which are uniform, and the impedance is described by single propagation constants for the material.

Others[22,31] have allowed for anisotropy such that the principal axes run parallel and normal to the material surface, and bulk parameters being considered as vectors. The impedance is thus expressed in terms of different propagation parameters.

Following Pyett[31], the acoustic properties of a homogeneous isotropic medium may be expressed in terms of a characteristic wave impedance Z_W and a propagation constant γ, both can be determined from a knowledge of the apparent density ρ

and the effective bulk modulus of elasticity K. Z_W and γ are both functions of the frequency and may be complex quantities[28].

For a harmonic disturbance, the wave equation is obtained by combining the equation of motion:

$$\rho \frac{\partial u}{\partial t} = - \text{grad } p = - \frac{\partial p}{\partial x} \qquad (4.1)$$

and the equation of continuity (combined with the Gas Law)

$$- \frac{\partial p}{\partial t} = K \text{ div } u = K \frac{\partial u}{\partial x} \qquad (4.2)$$

resulting in:

$$\frac{\partial^2 p}{\partial x^2} = \frac{\rho}{K} \frac{\partial^2 p}{\partial t^2} \qquad (4.3)$$

and for a solution of the form:

$p = Pe^{j(\omega t - \gamma x)}$, then

$$\gamma^2 = - \frac{\rho}{K}\omega^2 \quad \text{i.e. } \gamma = j\omega\sqrt{\frac{\rho}{K}} \qquad (4.5)$$

by definition:

$$Z_W = \frac{p}{u}$$

then, inserting the value of u in equation (3.7) yields:

$$Z_W = \sqrt{K\rho} \qquad (4.6)$$

The propagation constant γ can be written as $\gamma = (\alpha + j\beta)$; α is the attenuation constant, β is the phase constant.

If there is no attenuation i.e. $\alpha = 0$, and K is real, the physical interpretation of $\sqrt{K/\rho}$ is C the velocity of propagation of sound in the medium. γ then reduces to $j\beta$, $\beta = \frac{\omega}{C}$, the wave number of the medium.

If there is internal damping, K is a complex quantity, defined as the compression modulus. The velocity of propagation C is then equal to $\text{Re}(\sqrt{K/\rho})$.

If the material is porous, the apparent density is a complex quantity defined as[28]

$$\rho = k\rho o/\bar{n} + \sigma/_{j\omega}$$

where:

k = the structural constant

h = porosity factor

σ = resistivity

the compression modulus is

$$K \simeq \frac{K_o}{h}$$

In an anisotropic medium, ρ and K will depend on the direction of u so, both could be expressed as vectors with components parallel and normal to the surface of the medium.

The equation of motion is then:

$$\frac{\partial(\rho_i u_i)}{\partial t} = - \frac{\partial p}{\partial x_i} \qquad (4.7)$$

and the equation of continuity:

$$\frac{\partial(K_i u_i)}{\partial t} = - \frac{\partial p}{\partial t} \qquad (4.8)$$

and:

$$\gamma_i = j\omega \sqrt{\rho_i / K_i} \qquad (4.9)$$

and

$$Z_{wi} = \sqrt{\rho_i K_i} \qquad (4.10)$$

The wave equation is then:

$$\frac{1}{\gamma_x^2} \frac{\partial^2 p}{\partial x^2} + \frac{1}{\gamma_y^2} \frac{\partial^2 p}{\partial y^2} + \frac{1}{\gamma_z^2} \frac{\partial^2 p}{\partial z^2} = p \qquad (4.11)$$

For a plane wave, incident at an angle θ to the normal (Fig.4.1), the plane of incidence making an angle ∂ with the y-axis, the sum of sound pressure for incident and reflected waves is

$$p_o = \left(P_o e^{ikx\cos\theta} + R_o P_o e^{+jkx\cos\theta}\right) e^{jk(y\cos\partial + z\sin\partial)\sin\theta} \qquad (4.12)$$

k = the wave number in free air.

The transmitted wave (Fig.4.2) may be written as:

$$p_t = \left(P_t e^{-qx} + R_t P_t e^{+qx}\right) e^{(ry+sz)}$$

From Equation (4.11):

$$\frac{q^2}{\gamma_x^2} + \frac{r^2}{\gamma_y^2} + \frac{s^2}{\gamma_z^2} = 1 \qquad (4.13)$$

The normal velocity in the transmitted wave is:

$$U_x = \frac{q}{\gamma_x Z_{wx}}\left(P_t e^{-qx} - R_t P_t\ e^{+qx}\right) e^{(ry+sz)} \qquad (4.14)$$

The continuity of pressure across the surface x = 0 for all values of y and z requires:

$$P_o(1+R_o) = P_t(1+R_t) \qquad (4.15)$$

$$r = jk\sin\theta\cos\partial \qquad (4.16)$$

$$s = jk\sin\theta\sin\partial \qquad (4.17)$$

at x = 0:

$$Z_o = \frac{P}{U_x} = \frac{\gamma_x Z_{wx}}{q} \cdot \frac{(1+R_t)}{(1-R_t)} \qquad (4.18)$$

at $x = \ell$

$$Z_\ell = \frac{\gamma_x Z_{wx}}{q} \cdot \frac{1+R_t e^{+2q\ell}}{1-R_t e^{+2q\ell}} \qquad (4.19)$$

setting:

$V = \dfrac{\gamma_x Z_{wx}}{q}$ in Equation (4.19)

the reflection factor of the transmitted wave:

$$R_t = \frac{(Z_\ell - V)}{(Z_\ell + V)} \cdot e^{-2q\ell} \qquad (4.20)$$

substituting R_t in Equation (4.18)

$$Z_o = V\ \frac{\left(1+\left(\frac{Z_\ell - V}{Z_\ell + V}\right) e^{-2q\ell}\right)}{\left(1-\left(\frac{Z_\ell - V}{Z_\ell + V}\right) e^{-2q\ell}\right)} \qquad (4.21)$$

putting:

$$\frac{Z_{\ell}-V}{Z_{\ell}+V} = e^{-2\psi} \tag{4.22}$$

Equation (4.21) becomes

$$Z_o = V.\ \frac{1+e^{-2(q\ell+\psi)}}{1-e^{-2(q\ell+\psi)}}$$

$$= \frac{\gamma_x Z_{wx}}{q} \operatorname{Coth}(q\ell+\psi) \tag{4.23}$$

where, from Equation (4.22):

$$\frac{Z_\ell}{V} = \operatorname{Coth}(\psi) \tag{4.24}$$

and, from Equation (4.13) and Equation (4.16) and (4.17)

$$q = \gamma_x \sqrt{1+k^2\sin^2\theta\left(\frac{\cos^2\partial}{\gamma_y^2} + \frac{\sin^2\partial}{\gamma_z^2}\right)} \tag{4.25}$$

If $\partial = 0$ or $\gamma_y = \gamma_z$,

the expression for q reduces to:

$$q = \gamma_x \left(1 + \frac{k^2\sin^2\theta}{\gamma_y^2}\right)^{\frac{1}{2}} \tag{4.26}$$

Thus, for an anisotropic material, the impedance at the surface at oblique incidence is, from Equation (4.23) and Equation (4.24):

$$Z_o = \frac{\gamma_x Z_{wx}}{q} \operatorname{Coth}\left(q\ell + \operatorname{Coth}^{-1} \frac{Z_\ell q}{\gamma_x Z_{wx}}\right) \tag{4.27}$$

For normal incidence, $\theta = 0$,

$$q = \gamma_x \tag{4.27A}$$

and

$$Z_o = Z_{wx} \operatorname{Coth}\left(\gamma_x \ell + \operatorname{Coth}^{-1} \frac{Z_\ell}{Z_{wx}}\right) \qquad (4.28($$

For an isotropic material,

$\gamma_x = \gamma_y$ and

$$q = \gamma\left(1 + \frac{k^2\sin^2\theta}{\gamma^2}\right)^{\frac{1}{2}} \tag{4.29}$$

where γ is given by Equation (4.5) and Z_W by Equation (4.6).

The above analysis expresses the impedance of a porous material in terms of:

γ = the propagation constant

Z_W = the characteristic wave impedance

Z_ℓ = the backing impedance.

Some of the special cases, encountered in practice will now be considered.

4.2.2.4.1) <u>MATERIAL WITH POROUS BACKING</u>:-

It is possible to calculate in principle the acoustic impedance of any combination of slabs of different characters and thicknesses by successive application of Equation (4.27): thus, the impedance Z_ℓ can be computed by:

$$Z_{(n-1)} = \frac{\gamma_{(n-1)}Z_{w(n-1)}}{q_{(n-1)}} \operatorname{Coth}\left(q_{(n-1)}\ell_{(n-1)} + \operatorname{Coth}\frac{Z_n \cdot q_{(n-1)}}{\gamma_{(n-1)}Z_{w(n-1)}}\right) \tag{4.30}$$

4.2.2.4.2) <u>MATERIAL WITH RIGID BACKING</u>:-

If the porous absorbent is backed by a rigid wall, as in normal practice, then:

$$Z_\ell = \infty$$

and Equation (4.21) reduces to:

$$Z_0 = \frac{\gamma_x Z_{wx}}{q} \cdot \frac{1+e^{-2q\ell}}{1-e^{+2q\ell}} = \frac{\gamma_x Z_{wx}}{q} \operatorname{Coth}(q\ \ell) \tag{4.31}$$

where q as per Equation (4.26) for oblique incidence, and as per Equation (4.27A) for normal incidence.

4.2.2.4.3) MATERIAL WITH AIR SPACE BACKING:-

If the porous material is backed by air space of distance equal to $\frac{1}{4}\lambda$ from a rigid wall, the impedance at the back of the material is $Z_\ell = 0$.

This can be shown from Equation (4.31) by setting

$\gamma_x = q$ (the case for normal incidence)

$\gamma_x = j\beta = \frac{j\omega}{c_0}$ (since there is no damping)

$\ell = \frac{\lambda}{4}$, the impedance at the back of the porous material

$$\therefore \quad Z_\ell = \frac{\gamma_x Z_{wx}}{q} \quad \text{Coth}\left(\frac{j\omega.\lambda}{C_o 4}\right) = Z_{wx} \quad \text{Coth}\left(\frac{j\pi}{2}\right) = 0$$

Substituting this in Equation (4.21)

$$Z_0 = \frac{\gamma_x Z_{wx}}{q}. \frac{1-e^{-2q\ell}}{1+e^{-2q\ell}} = \frac{\gamma_x Z_{wx}}{q} \tanh(q\ell) \tag{4.32}$$

4.2.2.4.4) CONDITIONS FOR LOCAL REACTION:

The dependence of impedance on the angle of incidence is portrayed by the term containing $(\sin\theta)$ in the expression for (q) given by Equation (4.26), which may be written as:

$$q = (\alpha_x + j\beta_x)\left(1 + \frac{k^2\sin^2\theta}{(\alpha_y + j\beta_y)^2}\right)^{\frac{1}{2}}$$

If the tangential constant of propagation is large, the value of $\frac{k^2\sin^2\theta}{(\alpha_y + j\beta_y)^2}$ is small, and $q \simeq \gamma_x$

which is identical with the case for normal incidence.

The quantity $\gamma_y = (\alpha_y + j\beta)$ must, thus, contain the two conditions necessary for local reaction; i.e.

1. the wave motion parallel to the surface, inside the material, being slower than that of sound in air,

2. the wave motion inside the material is rapidly attenuated.

The quantity y_y is equal to $j\omega\sqrt{\rho/K}$, which, as discussed earlier, may all be complex quantities.

The wave velocity C_t may also be complex if there is attenuation in the material, and as before, $c_t = \sqrt{\frac{K}{\rho}}$ [29].

Thus for a smaller value of c_t, the compression modulus and the porosity factor must be small, and the structural factor and resistivity must be large.

4.2.2.5) PANEL TYPE:

The other type of sound absorbent, normally encountered in practice, is that of an inpervious plane panel supported on a heavier structure, with distances between points of support of the same size or smaller than the wavelength of sound wave.

In this case, transverse waves cannot be propagated far along the surface without being stopped by the supporting structure, so that the panel is, on the average, a locally reacting surface[18]. Also,each portion of the panel enclosed between the supporting network acts as a stiff diaphram having an effective mass per unit area $M \simeq \rho\ell$, an effective stiffness constant K, and an effective resistance R.[22]

In this case the impedance of the panel surface is:

$$Z = Z_\ell - j\omega M + jK/\omega + R \qquad (4.33)$$

where:

Z_ℓ: the impedance of the material (or air space) behind the panel.

This impedance is independent of the angle of incidence.

4.2.2.6) COMPOSITE TYPES:-

If the porous material acts also as a panel, the normal velocity at the surface is the sum of the velocity of the material as a whole plus the velocity of air in the pores. The impedances of the two separate effects can be considered to be in parallel[22].

If the composite treatment consists of a stiff impervious panel mounted on a porous backing, the impedances of the two will act in series[22].

Impedances for other more complicated treatments can be built up in a similar manner.

4.2.2.7) CONCLUSION:

The cases considered provide a wide range of surface treatment for practical applications in room acoustics.

In the majority of cases, the acoustic impedance is independent of the angle of incidence.

This conclusion is in good agreement with the results obtained by many investigators on the subject of the dependence of impedance of the angle of incidence.

Beranek[32] states that "measurements performed in a model sound chamber show that the effect of variation of angle of incidence on the normal impedance is less than that produced by small changes in mounting conditions."

P.M.Morse[24] draws the conclusion that "the experiments indicate that the impedance depends only on the material and not on the incident wave, except for the variation with frequency."

Experimental measurements conducted by C.M.Harris[33] on celotex C-4 using the Transmission Characteristic Method indicate that the impedance of proous materials is not a function of the angle of incidence.

It therefore seems justifiable to consider the absorbing properties of a wall material to be adequately represented by a normal acoustic impedance, independent of the angle of incidence of the sound wave, but varies only with frequency.

The measurement of this normal impedance will be considered in the following pages.

4.2.3) MEASUREMENT OF NORMAL IMPEDANCE:

4.2.3.1) INTRODUCTION:

Several methods of measuring the acoustic normal impedance of sound absorbents have been developed. These can be broadly classified into two groups:

(1) The Standing Wave Tube methods

(2) The Transmission-Characteristic methods.

In the first group, use is made of a measuring tube - known as 'Kundts Tube' in order to create plane travelling waves. At one end the sound source is placed, whereas the tube is terminated by the sample under test. The sound pressure is measured as a function of the distance from the sample, of the frequency or the length of the tube[28].

The methods can also be classified[2] as surface, transmission line or comparison methods, corresponding to those which utilize data taken at the surface of the sample, at various points somewhat removed from the sample, or those which require a known standard acoustic impedance for comparison with the impedance to be determined.

Of the transmission line methods, the most widely used is a constant length method, first described by Taylor[2] for the determination of absorption coefficients, and offered by Wente and Bedell for measurement of impedance.

In the second group measurements are performed on a large area of the material under actual mounting conditions.

The method has been developed by C.M.Harris[33] on the mathematical analysis of wave theory in room acoustics as presented by P.M.Morse[18]. The assumption, underlining the method, is that the impedance of the bounding surfaces of the room can be determined from the acoustic properties of the room. These are the natural frequency of a mode of vibration, its decay constant and the spatial pressure distribution in the normal mode.

The spatial distribution property will be used in this thesis for impedance determination.

4.2.3.2) THE CONSTANT LENGTH INTERFEROMETER METHOD:

The method is based on the standing wave analysis and requires that a plane sound wave is generated in the tube and is partially reflected by the sample. As a result, a partially standing wave is formed in front of the sample. The sound field is probed by a microphone, and the ratio of the sound pressures at a loop and a node, and the position of a node relative to the surface of the sample are measured.

The tube should be long enough to permit the formation of, at least, one maximum and one minimum of the pressure distribution at the lowest frequency of interest.

The lateral dimension should be smaller than 0.5λ for the highest measuring frequency to ensure the formation of only plane waves[17]. This is based on the solution for the lowest cross-mode in a circular tube[34].

On the other hand, the cross-section of the tube must not be too small, since otherwise the wave attenuation due to friction on the wall surface would become too high.

4.2.3.2.1) THE STANDING WAVE ANALYSIS:-

The incident and reflected waves, forming the standing wave, have different amplitudes and phases.

The change in amplitude and phase are expressed by the pressure complex reflection factor:

$$R = |R|e^{j\Delta} \tag{4.34}$$

In the free air, assuming there is no damping, the propagation constant is:

$$\gamma = \frac{j\omega}{c}$$

the characteristic wave impedance is:

$$Z_w = \rho c$$

The sum of the pressures at a distance x from the sample is:

- omitting the time dependence factor $e^{j\omega t}$ -

$$p_{(x)} = p_i e^{-\frac{j\omega x}{c}} + Rp_i e^{+\frac{j\omega x}{c}} \tag{4.35}$$

The sum of the particle velocities is, similarly,

$$u_{(x)} = \frac{p_i}{\rho c} e^{-\frac{j\omega x}{c}} - Rp_i e^{+\frac{j\omega x}{c}} \tag{4.36}$$

At the surface of the sample (x=0), the normal impedance - from Equation (4.35) and Equation (4.36) is:

$$Z_{(o)} = \frac{p_{(o)}}{u_{(o)}} = \rho c . \frac{1+R}{1-R} \tag{4.37}$$

and from Equation (4.34)

$$Z_{(o)} = \rho c . \frac{1+|R|e^{j\Delta}}{1-|R|e^{j\Delta}} \tag{4.38}$$

This relation expresses the impedance of the absorbing surface as a function of the complex reflection factor.

From the tube measurements both the amplitude and phase angle can be determined.

4.2.3.2.2) DETERMINATION OF AMPLITUDE OF THE REFLECTION FACTOR:-

From Equation (4.34), Equation (4.35) takes the form:

$$p_{(x)} = p_i e^{-\frac{j\omega x}{c}} + |R| p_i e^{j(\frac{\omega x}{c} + \Delta)} \qquad (4.39)$$

The pressure, $p_{(x)}$, will be maximum for those values of (x) such that both terms are in phase, i.e. the difference of their arguments is zero:

i.e. $\left(\frac{\omega x}{c} + \Delta\right) - \left(-\frac{\omega x}{c}\right) = 0$

i.e. $\Delta = -\frac{2\omega x}{c}$

and $p_{max} = p_i e^{-\frac{j\omega x}{c}}(1+|R|)$

and its magnitude is:

$$|p_{max}| = p_i(1+|R|) \qquad (4.40)$$

Similarly the pressure, $p_{(x)}$, will be minimum for those values of (x) such that both terms in Equation (4.39) are in opposite phase i.e. the difference of their arguments equals: $\pi, 3\pi, ..$ i.e....$(2n+1)\pi$.

$$\left(-\frac{\omega x}{c}\right) - \left(\frac{\omega x}{c} + \Delta\right) = (2n+1)\pi$$

$$\Delta = -(2n+1)\pi - \frac{2\omega x}{c} \qquad (4.41)$$

and:

$$p_{min} = p_i\, e^{-\frac{j\omega x}{c}} \left(1 + |R| e^{-j(2n+1)\pi}\right)$$

$$\left[e^{-j-2n+1)\pi} = \cos(2n+1)\pi - j\sin(2n+1)\pi) = -1\right]$$

$$p_{min} = p_i e^{-\frac{j\omega x}{c}}(1 - |R|)$$

and its magnitude is

$$|p_{min}| = p_i(1-|R|) \qquad (4.42)$$

The standing wave ratio is defined as:

$$n = \frac{|p_{max}|}{|p_{min}|} = \frac{1+|R|}{1-|R|}$$

from which:

$$|R| = \frac{n-1}{n+1} \qquad (4.43)$$

and so the modulus of the reflection factor can be determined from measurement of the standing wave ratio.

Alternatively, if the scale of the measuring apparatus enables the energy absorption coefficient to be read off directly, as in B and K analyzers and amplifiers[35], then the modulus of the reflection factor $|R|$ can be obtained from the relation between the energy absorption coefficient α and the pressure reflection factor:

$$\alpha = 1-|R|^2 \qquad (4.44)$$

4.2.3.2.3) DETERMINATION OF THE PHASE ANGLE OF THE REFLECTION FACTOR:

The phase angle (Δ) of the complex reflection factor can be determined from measurement of the first minimum.

Setting $x = (-x_o)$ for the position of the first minimum, Equation (4.41) reads:

$$\Delta = \frac{2\omega x_o}{c} - \pi$$

i.e.

$$\Delta = \pi\left(\frac{4x_o}{\lambda} - 1\right) \qquad (4.45)$$

It turns out to be too inaccurate to compute (λ) from the frequency and sound velocity. More reliable values of (Δ) are obtained by measuring the positions of the two minima at ($-x_o$) and ($-x_1$) and eliminating (λ) in Equation (4.45).

If the first minimum occurs at ($-x_o$), then, from equation (4.41):

$$\frac{2\omega x_o}{c} - \Delta = \pi \qquad n = 0$$

– the second minimum occurs at ($-x_1$), then:

$$\frac{2\omega x_1}{c} - \Delta = 3\pi \qquad\qquad n = 1$$

subtracting:

$$\frac{2\omega}{c}(x_1 - x_0) = 2\pi$$

and $\lambda = 2(x_1 - x_0)$

substituting in Equation (4.45):

$$\Delta = \left(\frac{2x_0}{(x_1 - x_0)} - 1\right)\pi \tag{4.46}$$

So by measuring the standing wave ratio and the position of the first two minima relative to the surface of the sample, Equation (4.43) and Equation (4.46) completely define the complex reflection factor in amplitude and phase.

Equation (4.38) must then be used to find the normal impedance of the sound absorbent.

4.2.3.2.4) <u>APPRAISAL OF METHOD:-</u>

Impedance measurements by the constant Length Interferometer method are only meaningful for such materials whose acoustical behaviour is not dependent on the sample size i.e. vibration panels, large slit resonators, etc. The method is particularly suited to measurements of porous materials, resonance absorbents, light memebrane absorbents and composite constructions involving those types.

Even with these types the conditions for local reaction, discussed earlier, have to be satisfied if their impedance is to be used to describe the boundary conditions of an enclosure.

4.2.3.3) THE TRANSMISSION-CHARACTERISTIC METHOD:

The method is based on evaluating the specific acoustic impedance from steady-state measurements of the acoustic properties of the room as represented by the characteristic values (k_n) of the normal modes of vibration.

According to the wave theory analysis, discussed in Chapter Three, the characteristic values can be determined in terms of infinite series of the impedance that converge fairly rapidly (equation (3.31)). It has been suggested[36] that the impedance can be determined from a knowledge of the characteristic value if these series can be used conversely. This has been shown[36] to be possible for the case when the impedance is large and only then the first term of the series can be considered.

The characteristic values can be determined from measurements of the natural frequency and the damping constant or the pressure spatial distribution in the normal modes. Experimental measurements along these lines have been suggested by P.Morse[22] and R.Bolt and carried out by C.V.Harris[36] and others[37,38].

In this work, a method of exploring the spatial structure of modal sound fields in the room will be adopted for evaluating the characteristic values. However, its application will be restricted to the case of bare walls with large impedance in view of the simplified mathematical analysis involved, and since its extension to the study of soft walls is fraught with analytical errors.

The transmission characteristic method envisages a rectangular chamber in which measurements are performed on large samples of acoustic materials under actual mounting conditions.

The technique is based on measuring the variation with distance of the resonance pressure amplitude at the natural frequencies of the normal modes of vibration of the chamber. From these measurements the component characteristic values (k_x, k_y, k_z) can be determined and, using the series expansion, the impedance of the corresponding pair of walls can, in turn, be evaluated.

Another advantage of this method of exploring the spatial structure of sound fields is that it can be used in identification of modal shapes from the nodal pattern associated with each mode[39].

However, certain assumptions should be laid down for the technique to be plausible.

Firstly, it is assumed that some means of exciting the room in one of its pure modes could be sought. It is also assumed that the contribution to the measured response from off-resonant vibration, is negligible in the resonance condition of the mode under consideration. Both assumptions will be discussed later in this Chapter.

Secondly, it is assumed that the normal impedance of the wall is independent of the angle of incidence and therefore the component characteristic values (k_x, k_y, k_z) depend on the impedance of the corresponding pair of walls and not on the other two pairs of walls.

4.2.3.3.1) THE SPATIAL DISTRIBUTION ANALYSIS:

The complex pressure response of the room when a single mode is excited is given by equation (3.56):

$$P_{(x,y,z)} = \frac{j\omega\rho c^2 Q_o}{V\Lambda_n} \; \frac{\psi_{n(x_o,y_o,z_o)} \cdot \psi_{n(x,y,z)}}{(\omega^2-\omega_n^2)+2j\omega_n\delta_n} \qquad (3.56)$$

where

$$\psi_{n(x,y,z)} = \psi_x \cdot \psi_y \cdot \psi_z$$

with similar expression for $\psi_{n(x_o,y_o,z_o)}$

and:

$$\psi_x = \cosh(jk_x x + \phi_x)$$

with similar expressions for ψ_y and ψ_z.

If the sound source is placed at antinodes with respect to the three coordinates and the receiver on antinodes with respect to the y and z directions and moved only along the x-direction, and the driving angular frequency (ω) is adjusted to be equal to the natural frequency (ω_n), then the above expression reduces to:

$$P_{(x,y,z)} = A_n \cosh(jk_x x + \phi_x) \qquad (4.47)$$

where:

$$k_x = \frac{\omega_x + j\delta_x}{c}$$

ω_x being the component characteristic frequency and,

δ_x the component damping constant

and:

$$\phi_x = \coth^{-1}\left(\frac{k_x}{k\beta_{x1}}\right)$$

where

β_{x1} is the specific acoustic admittance of the x₁-wall

A_n a constant

If β_{x1} is small (as for a bare wall);

$$\phi_x \simeq \frac{1}{\coth\phi_x} = 0$$

and equation (4.47) simplifies to:

$$\begin{aligned} P_{(x,y,z)} &= A_n \cosh(jk_x x) \\ &= A_n \cosh\left(-\frac{\delta_x x}{c} + \frac{j\omega_x x}{c}\right) \\ &= A_n \cosh\left(\frac{\delta_x x}{c}\right) \cos\left(\frac{\omega_x x}{c}\right) - jA_n \sinh\left(\frac{\delta_x x}{c}\right) \sin\left(\frac{\omega_x x}{c}\right) \end{aligned}$$

and the square of the magnitude of $P_{(x,y,z)}$ is:

$$|P|^2 = |A_n|^2 \cosh^2\left(\frac{\delta_x x}{c}\right) \cos^2\left(\frac{\omega_x x}{c}\right) + |A_n|^2 \sinh\left(\frac{\delta_x x}{c}\right) \sin^2\left(\frac{\omega_x x}{c}\right) \qquad (4.48)$$

From the trigonometric and hyperbolic identities:

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

and

$$\cosh^2(\beta) - \sinh^2(\beta) = 1$$

equation (4.48) can be expressed as:

$$|P|^2 = |A_n|^2 \left(\cosh^2\left(\frac{\delta_x x}{c}\right) + \cos^2\left(\frac{\omega_x x}{c}\right) - 1\right) \qquad (4.49)$$

Also, since:

$$\cos(2\alpha) = 2\cos^2(\alpha) - 1$$

$$\cosh(2\beta) = 2\cosh^2(\beta) - 1$$

equation (4.49) can be written as

$$2|P|^2 = |A_n|^2 \cosh\left(\frac{2\delta_x x}{c}\right) + |A_n|^2 \cos\left(\frac{2\omega_x x}{c}\right)$$

The value of the constant $|A_n|$ can be determined from measurement of the pressure amplitude at $x = 0$

i.e. $|P_o|^2 = |A_n|^2$

i.e.

$$\frac{2|P|^2}{|P_o|^2} \quad \cosh\left(\frac{2\delta_x x}{c}\right) + \cos\left(\frac{2\omega_x x}{c}\right) \qquad (4.50)$$

Let the sound pressure level (in dB) corresponding to $|P|$ be SPL_x, and that corresponding to $|P_o|$ be SPL_o

then:

$$\left|\frac{P}{P_o}\right|^2 = 10^{(SPL_x - SPL_o)/10}$$

i.e.

$$\cosh\left(\frac{2\delta_x x}{c}\right) + \cos\left(\frac{2\omega_x x}{c}\right) = 2 \times 10^{(SPL_x - SPL_o)/10} \qquad (4.51)$$

From measurements of the sound pressure level at a minimum and another position in the room, along the x-direction, both the x-component characteristic frequency and damping constant can be determined.

(4.2.3.3.2) <u>DETERMINATION OF ω_x:</u>

The expression for the pressure amplitude, given by equation (4.51) assumes its minimum value when:

$$\cos\left(\frac{2\omega_x x}{c}\right) = -1$$

$$\text{i.e.} \quad \omega_x = \frac{\pi c}{2x_{min}} \qquad (4.52)$$

thus, from measurement of the first pressure minimum, the component characteristic frequency (ω_x) can be determined.

4.2.3.3.3) DETERMINATION OF δ_x:

At a point ($x_{(-20)}$) along the x-direction where the sound pressure level relative to that at the wall is (-20dB) lower, equation (4.51) reads:

$$.02 = \cosh\left(\frac{2\delta_x x_{(-20)}}{c}\right) + \cos\left(\frac{\pi . x_{(-20)}}{x_{min}}\right)$$

i.e.
$$\delta_x = \frac{c}{2x_{(-20)}}\left[0.02 - \cos\left(\frac{\pi . x_{(-20)}}{x_{min}}\right)\right] \quad (4.53)$$

4.2.3.3.4) DETERMINATION OF IMPEDANCE:

The real and imaginary parts of the square of the complex characteristic values (k_x^2) are:

$$\left(\frac{\omega_x^2 - \delta_x^2}{c^2}\right) \quad \text{and} \quad \frac{2\omega_x\delta_x}{c^2}$$

and can be evaluated from equations (4.52) and (4.53).

Using the first terms of the series expansions (3.40) and (3.41) conversely and assuming that the specific acoustic admittance of both x-walls is equal and small, we get for the real part (γ_x) and the imaginary part (σ_x) of the specific admittance:

$$\gamma_x = \frac{(2\omega_x\delta_x).L_x}{4kc^2} \quad (4.54)$$

and

$$\sigma_x = \frac{n_x^2\pi^2}{4kL_x} - \frac{(\omega_x^2 - \delta_x^2).L_x}{4kc^2} \quad (4.55)$$

From the specific admittance $\beta_x = (\gamma_x + j\sigma_x)$, the specific impedance $\xi_x = \frac{1}{\beta_x}$ can be determined.

4.2.3.3.5) APPRAISAL OF METHOD:

The advantage of the transmission-characteristic method is that the pressure amplitude plots are simple to produce experimentally, and their analysis for determining the impedance of fairly hard walls is a simple matter. It is also possible to determine the impedance of large samples of acoustical materials, and under actual mounting conditions.

The inherent limitation of the method, when applied to the study of soft materials, lies in the extent of analytical approximations involved in determining the component characteristic values from the measured amplitude and position of the sound pressure minima and the pressure at the wall, as the phase constant (ϕ) cannot, then be ignored.

If one of the pair of walls is soft and the other hard, their admittances are not additive and the series expansion of equation (4.40) cannot be used. Graphical solutions are reported in the literature(22) and should be used instead.

4.3) APPLICATION OF RESONANCE TESTING TECHNIQUES TO ROOM ACOUSTICS MEASUREMENTS.

4.3.1) INTRODUCTION:-

As pointed out, in the previous chapter, the resonance frequencies and damping constants are the quantities that can usually be determined experimentally, rather than the characteristic values of an isolated normal mode of vibration. It is, therefore, of prime importance to develop some techniques whereby the normal modes of a room can be identified and their associated resonance frequencies and damping constants measured, so as to employ them in evaluating the room response to acoustic excitation.

It is a common electrical technique to employ the width of the resonance curve as a measure of the effective dissipation of a resonant circuit, and its application in the acoustical case to determine the acoustical damping of a resonant mode of vibration was first suggested by F. Hunt(40) in 1939.

In aeronautical engineering, in particular, most of the techniques of resonance testing were developed and used to provide essential information for flutter calculations in aircraft structural vibrations(41).

Generally speaking, a resonance test may be carried out to determine the normal modes of the system, the associated natural frequencies and the damping.

The procedure is, basically, to excite the system and measure its reponse.

Various techniques for performing such measurements have been developed and may be distinguished by:

(1) the physical quantities which are measured

(2) the ways in which the experimental data are displayed (i.e. the graphs which are plotted)

(3) the ways in which the graphs are analysed

The relationship:

$$E = E_0 \, e^{-\delta t}$$

between the damping factor of a simple oscillator and the fractional loss of energy of free vibration, which expresses the effect of damping in terms of an exponential decay of the total energy, is assumed[18] to hold for any vibration, and can be used to study the damping of standing waves in rooms, since the air in a room can be considered as an assemblage of resonators - standing waves that can be set into vibration by a sound source.

In theory, the techniques of resonance testing can thus be applied in the study of room response to acoustic excitation. The intention, in this study, is to investigate such a possibility, experimentally.

4.3.2) RESONANCE TESTING TECHNIQUES:-

In the most basic form of resonance testing, a system is excited harmonically and the driving frequency adjusted until a resonant state is found.

The resonance frequency is assumed to be a natural frequency of the system, the resonant vibration assumed to take place accurately in the corresponding principal mode, which can be identified and measured, and the resonant magnification - or alternatively, the variation of response with driving frequency around the resonant condition - can be made to give a figure of the damping associated with that mode.

In particular, the system may possess close natural frequencies which will not permit violent vibrations to be excited separately in their corresponding principal modes, and the presence of heavy damping may obscure the detection of some of the modes.

The commonest method of resonance testing is to excite the system concerned and to measure the total forced vibration at enough points so as to be sure that all the modes will display their resonant characteristics in at least one of these responses. A response curve ot the total amplitude plotted against driving frequency is then constructed and the required information extracted from this curve.

This is known as the 'peak-amplitude' method

From a theoretical point of view, it is inadequate and may be misleading. Its main limitation lies in the fact that no account is taken of the changes in the phase lag of the response of the system behind the exciting force. Since this change in phase is most marked when the system is passing through resonance, information can be derived from the analysis of a simple phase-angle plot.

This fact has largely been overlooked(43), but subsequent analysis will show that phase-angle plots yield as much information as those of peak-amplitude.

This was recognized by Kennedy and Pancu(42) in 1947, who proposed an alternative and theoretically more attractive method, which was claimed to be superior(44) to the conventional peak-amplitude method of detection of natural frequencies and damping constants.

In the Kennedy and Pancu method, the amplitude and phase are measured, but instead of plotting two separate graphs, the amplitude and phase are plotted together on an Argand plane. This was claimed to be able to isolate modal frequencies and damping parameters of systems with relatively close natural frequencies by easily eliminating the contribution to over all vibration level of off-resonant motion. The method, as pointed out by Bishop and Gladwell(41), was based entirely on theoretical grounds, buttressed by numerical examples; nevertheless, it has found wide application in practice(44).

4.3.3) THEORIES OF RESONANCE TESTING AS APPLIED TO ROOM ACOUSTICS:-

The theoretical analysis for the techniques described above as applied to the study of vibration signals is widely reported in the literature(41,42,43).

In sound propgation, the acoustical variables are governed by the wave equation which for the pressure wave in the presence of a source distribution of density (q) is:

$$\nabla^2 P + k^2 P = - j\omega\rho q \qquad (3.49)$$

The solution has been discussed in Chapter Three and, for a single mode, is:

$$P_{(r,t)} = P_{(r)} \; e^{j\omega t} = Ae^{j(\omega t-\alpha)} = \frac{j\omega\rho Q_o}{V\Lambda_n} \cdot \frac{\psi_{n(r)} \cdot \psi_{n(r_o)}}{k^2-k_n^2} \cdot e^{j\omega t} \qquad (4.56)$$

where

$r = (x,y,z)$: the receiver's position

$r_o = (x_o,y_o,z_o)$: the source position

The transfer function of the system is obtained by dividing the response by some reference function ($e^{j\omega t}$) proportional and in phase with the input function, thus:

$$P_{(r)} = A_{(r)} e^{-j\alpha} = \frac{j\omega\rho Q_o}{V\Lambda_n} \cdot \frac{\psi_{n(r)} \cdot \psi_{n(r_o)}}{k^2-k_n^2} \qquad (4.57)$$

$P_{(r)}$: is the transfer function of the system, representing the complex pressure amplitude

$A_{(r)}$: the amplification (or gain) representing the magnitude of the pressure amplitude

α : the phase angle, the response lags behind a reference signal in phase with the input signal.

It will be necessary, experimentally, to keep the amplitude of the input signal constant so that the effect of variation in its magnitude will be eliminated. This is achieved by considering the receptance of the system under investigation.

By non-dimensionalizing the response in terms of the frequency and the source strength, the following expression results for the response per unit amplitude of the strength of the source, and independent of the driving frequency.

$$P = \frac{P_{(r)}}{\omega Q_o} = A_n e^{-j\alpha} = \frac{A_{(r)} e^{-j\alpha}}{\omega Q_o} = \frac{j\rho}{V\Lambda_n} \cdot \frac{\psi_{n(r)} \cdot \psi_{n(r_o)}}{k^2 - k_n^2} \qquad (4.58)$$

If δ_n is small compared to ω_n, Eq.(4.58) reduces to:

$$P = A_n e^{-j\alpha} = \frac{jc^2\rho}{V\Lambda_n} \cdot \frac{\psi_{n(r)} \cdot \psi_{n(r_o)}}{(\omega^2 - \omega_n^2) - 2j\omega_n\delta_n} \qquad (4.59)$$

If the position of the source and receiver are kept fixed then:

$$P = A_n e^{-j\alpha} = \frac{jR_n}{(\omega^2 - \omega_n^2) - 2j\omega_n\delta_n} \qquad (4.60)$$

where

$$R_n = \pm \frac{\rho c^2 \psi_{n(r)} \cdot \psi_{n(r_o)}}{V\Lambda_n} = \text{a constant} \qquad (4.61)$$

or:

$$P = (x_n + jy_n) = \frac{-2R_n\omega_n\delta_n}{(\omega^2 - \omega_n^2)^2 + 4\omega_n^2\delta_n^2} + j\frac{R_n(\omega^2 - \omega_n^2)}{(\omega^2 - \omega_n^2)^2 + 4\omega_n^2\delta_n^2} \qquad (4.62)$$

in rectangular coordinates.

x_n and y_n being the inphase and inquadratic components of the receptance.

And:

$$A_n = \frac{|R_n|}{[(\omega^2 - \omega_n^2)^2 + 4\omega_n^2\delta_n^2]^{\frac{1}{2}}} \qquad (4.63)$$

$$\alpha = \tan^{-1} - \frac{y_n}{x_n} = \tan^{-1} \frac{(\omega^2 - \omega_n^2)}{2\omega_n\delta_n} \qquad (4.64)$$

in polar coordinates.

Equation (4.62) is the theoretical basis for the Kennedy and Pancu Technique, while Equations (4.63) and (4.64) are the basis for the Peak-Amplitude and Phase-Angle Methods respectively.

The three techniques will now be discussed in more detail.

4.3.4) THE PEAK-AMPLITUDE METHOD:-

4.3.4.1) INTRODUCTION:-

In this technique, the system is excited harmonically and the response amplitude at a particular point measured over a range of frequency. The amplitude of the response depends, not only on the dynamic characteristics of the system investigated, but on the amplitude as well as the location and driving frequency of the exciting force. In order to eliminate the effect of variation of the magnitude of the exciting force, the receptance of the system is considered instead.

The source and receiver are both located at antinodes for the mode under investigation, so that the maximum response is attained.

4.3.4.2) DISPLAY OF RESULTS:-

The absolute value of the receptance is plotted against the driving frequency (FIG.4.2). It can be seen that the curve consists of a number of peaks, each in the vicinity of a natural frequency associated with a particular normal mode of vibration.

4.3.4.3) DETERMINATION OF NATURAL FREQUENCIES:-

From Equation (4.63), the absolute value of the receptance is given by

$$A_n = \frac{|R_n|}{[(\omega^2-\omega_n{}^2)^2+4\omega_n{}^2\partial_n{}^2]^{\frac{1}{2}}} \tag{4.63}$$

the maximum value of A_n occurs when:

$$\omega^2 \simeq \omega_n{}^2 \quad \text{i.e. } \omega \simeq \omega_n$$

This defines the natural frequency of the mode as the value of the frequency at which the peak of the receptance is attained.

Analytically, some approximations are made in the above definition:

Firstly, the peaks do not occur exactly at the natural frequency but at frequencies displaced one way or another from them.

Secondly, the receptance is made up from contributions from each of the excited modes in the immediate vicinity of the mode under consideration, and these will in turn tend to displace the peaks.

With heavy damping and closely spaced natural frequencies, an experimental difficulty is exhibited in that some peaks may be missed altogether from the plot due to the fact that off-resonant contribution from other modes may be comparable in magnitude to the vibration in the resonant mode; this will tend to flatten the curve at these frequencies.

The method, therefore, tends to be unreliable for modes which are heavily damped or with close natural frequencies, but may still provide a rough guide to their resonance properties.

4.3.4.4) <u>DETERMINATION OF DAMPING CONSTANTS:-</u>

If it can be assumed that each peak represents reponse of only one mode, then the damping constant may be calculated from the sharpness of the peak.

Under this assumption, the receptance in the mode is represented by:

$$A = \frac{|R_n|}{[(\omega^2-\omega_n^2)^2 + 4\omega_n^2\delta_n^2]^{\frac{1}{2}}} \tag{4.63}$$

the maximum value of A occurs at $\omega \simeq \omega_n$:

$$A_{peak} = \frac{|R_n|}{2\omega_n\delta_n} \tag{4.65}$$

Consider the particular value of (ω), one above and one below (ω_n) for which:

$$2\omega_n\delta_n = \pm\ (\omega^2-\omega_n^2) \tag{4.66}$$

substitution of this condition in Equation (4.63) shows that the amplitude of these values of (ω) is $1/\sqrt{2}$ of its peak value.

The difference between these frequencies and the natural frequency is small, and if it is assumed that there is no off-resonant contribution from neighbouring modes, then they are equidistant from the natural frequency. (FIG.4.3).

Let:

$$\omega_2 - \omega_1 = \Delta\omega$$

then:

$$\omega_1 = \omega_n - \frac{\Delta\omega}{2}$$

and .

$$\omega_2 = \omega_n + \frac{\Delta\omega}{2}$$

substitution in Equation (4.66):

$$2\omega_n\delta_n = -\left(\left(\omega_n - \frac{\Delta\omega}{2}\right)^2 - \omega_n^2\right) = \omega_n\Delta\omega + \frac{\Delta\omega^2}{4}$$

and

$$2\omega_n\delta_n = +\left(\left(\omega_n + \frac{\Delta\omega}{2}\right)^2 - \omega_n^2\right) = \omega_n\Delta\omega + \frac{\Delta\omega^2}{4}$$

add together:

$$4\omega_n\delta_n = 2\omega_n\Delta\omega$$

therefore

$$\delta_n = \frac{\Delta\omega}{2} = \frac{\omega_2-\omega_1}{2} \tag{4.67}$$

It follows that the damping constant δ_n can be found by measuring the frequency band between points on the amplitude resonance curve corresponding to the amplitude being equal to $1/\sqrt{2}$ of its peak value at these frequencies i.e. to half-power width on an energy resonance curve.

However, the main obstacle in the analysis of the peak-amplitude plot is to find which part of a given peak is due to the resonant mode, and which to the off-resonant vibrations from neighbouring modes. The latter, therefore, introduces some error into the use of this method.

It has been pointed out[41,43] that it is not easy to apply the assumption made by Kennedy and Pancu in their method (to be dealt with later in this Chapter) to the peak-amplitude technique. They assume that the off-resonant contribution is constant (in amplitude and phase) as the mode passes through resonance, and could be extracted from the overall response. But this does not mean that the contribution to the amplitude from the off-resonant vibration is constant. In fact, because of the marked change in phase of the response in the resonant mode when passing through resonance, the effect of the off-resonant vibration on the amplitude above the resonant frequency will differ from the effect below, and the resonance curve will not be symmetrical.

If, however, it is assumed that the effect of the off-resonant vibration on the amplitude is constant, then the contribution to the peak from the resonant mode may be found by the method described by Gladwell[45] as a refined estimate of the damping constant. The validity of this assumption has been examined by Bishop and Gladwell[41]. A theoretical analysis and practical appraisal of the method of Gladwell are given by Pendered and Bishop[43], and their conclusion is that although the refined method is theoretically more attractive than the simple method, it has serious practical drawbacks arising from the cumulative error in defining the exact location of the peak and the value of the natural frequency.

4.3.4.5) DETERMINATION OF MODAL SHAPES:-

The shape of the mode is identified from the ratios of the amplitudes at various points in the room when the system is being driven at its natural frequency.

If it is assumed that only a single pure mode is excited, the variation in magnitude of the response with distance is expressed by equation (4.48):

$$A^2 = \left|\frac{P}{P_o}\right|^2 = \cosh^2\left(\frac{\delta_x x}{c}\right) - \sin^2\left(\frac{\omega_x x}{c}\right) \qquad (4.48)$$

the maximum value of A is

$$A_{max} = \cosh\left(\frac{\delta_x x}{c}\right) \qquad (4.68)$$

and the minimum is

$$A_{min} = \sinh\left(\frac{\delta_x x}{c}\right) \qquad (4.69)$$

Therefore, equation (4.68) gives the maximum response and defines the antinodal planes, while equation (4.69) gives the minimum and defines the nodal planes.

Since more than one mode may be represented at any particular frequency, and for a particular souce and received locations, the measured response will not represent a true pure mode, and unless other modes are eliminated, there will be off-resonant contributions in the response amplitudes. These, in turn, will give rise to considerable inaccuracy in the modal shape. This can only be achieved by some systematic procedure of mode excitation involving the locations of the source and receiver. This will be discussed later in this Chapter.

4.3.4.6) APPRAISAL OF METHOD:

The greatest advance of the Peak-Amplitude technique is its simplicity in terms of the time and instrumentation required for running the test. It is satisfactorily applicable to the study of enclosures with lightly damped modes with widely-spaced natural frequencies.

For more complicated conditions, the drawbacks of this technique are that "too little is measured, and what is measured is badly displayed."(41). This may be aggravated more by a number of related factors.

The closeness of the natural frequencies is the most important factor contributing to the distortion of the amplitude-frequency curve. One of its effects(17) is that a mode may be missed altogether due to mutual in-phase superpositions and cancellations of modes. A second effect is to produce greater discrepancies between the natural frequencies and the frequencies at which the peaks occur.

The presence of heavy damping is the second factor that distorts the amplitude-frequency curve. A heavily-damped mode may be obscured altogether from the response curve.

Also at the resonant frequency of a heavily damped mode extraneous vibration from other modes may be comparable in magnitude to the vibration in the resonant mode, and unless accurate analysis of the response into its resonant and off-resonant components is made, there will be large errors in the relevant damping constants and modal shapes.

Finally, all these limitations are accentuated when the effects of heavy damping and close natural frequencies are combined.

4.3.5) THE PHASE-ANGLE METHOD:-

4.3.5.1) INTRODUCTION:-

This technique of measurement is similar to that used in the Peak-Amplitude method, but instead of measuring the amplitude of the response, its angle of phase lag behind the driving signal is measured.

As a marked change in phase lag occurs at resonance, the analysis of a plot of phase angle variation with frequency can yield information on the natural frequency and damping constant of a mode of vibration in the same way an analysis of an amplitude plot will.

4.3.5.2) DISPLAY OF RESULTS:-

The variation in phase angle with frequency, is plotted against the driving frequency (FIG.4.4).

If both the source and receiver are located at anti-nodal points, it can be shown that resonance is accompanied by a phase change from nearly $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ radians or from nearly $\frac{3\pi}{2}$ to $\frac{\pi}{2}$ radians, depending on the location of the source and the receiver with respect to the nodal structure in space of the mode under consideration.

If both source and receiver are at points of maxima, the phase change will range from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ radians, with resonance occurring at an angle of π i.e. the response is in antiphase with the driving signal. An identical effect will occur if both points are of minima.

If either of the two points is of a maxima or minima, the phase change will range from $\frac{3\pi}{2}$ to $\frac{\pi}{2}$ radians, with resonance occurring at an angle of 0 i.e. the response is in phase with the driving signal.

An analytical view of this condition may be obtained from considering the signs of the products $\psi_{n(r)}\ \psi_{n(r_o)}$ in equation (4.58) representing the characteristic functions of the receiver and the source respectively. If the sign of the product is positive, the response is in phase at resonance. If the sign is negative, an anti-phase resonance occurs.

4.3.5.3) DETERMINATION OF NATURAL FREQUENCIES:-

From Equation (4.64), the phase angle of the response is given by

$$\tan\alpha = \frac{\omega^2 - \omega_n^2}{2\omega_n \delta_n} \qquad (4.64)$$

at resonance:

$\omega = \omega_n$ and

$\tan\alpha = 0$ i.e. $\alpha = 0$ or π radians.

Thus for a pure mode, the natural frequency may be calculated by the intersection of the phase angle curve with the line $\alpha = 0$ or $\alpha = \pi$.

For a linear system, the phase angle is independent of the amplitudes of the driving signal and the response of the system.

Unlike the peak-amplitude method, there is no discrepancy between the natural frequency of the mode and the frequency at which resonance occurs.

However, the presence of off-resonant vibration associated with heavy damping or closely-spaced natural frequencies will, as in the peak-amplitude plot, distort the location of resonance on the phase-angle plot.

4.3.5.4) DETERMINATION OF DAMPING CONSTANTS:-

The rate of change of phase angle with respect to frequency can be utilized to give a measure of damping in a manner analogous to the sharpness of peaks in amplitude-frequency plots.

Consider the particular value of (ω), one above and one below ω_n for which:

$$2\omega_n\delta_n = \pm (\omega^2-\omega_n{}^2)$$

Substitution of this condition in equation (4.64) shows that the phase angle at these values of (ω) is:

$$\tan\alpha = \pm 1 \qquad (4.70)$$

i.e. $\alpha = \frac{3\pi}{4}$ and $\frac{5\pi}{4}$ or $\frac{\pi}{4}$ and $\frac{7\pi}{4}$ radians.

It follows that the damping constant (δ_n) can be found by measuring the width of the frequency band between the points on the phase-angle plot corresponding to the values of the phase angle α given by equation (4.70).

The damping constant is represented by Equation (4.67):

$$\delta_n = \frac{\Delta\omega}{2} = \frac{\omega_2-\omega_1}{2} \qquad (4.67)$$

as derived in discussing the Peak-Amplitude technique.

Likewise it is subject to the same approximations.

An alternative method, reported by Pendered and Bishop[43], and claimed to be more exact method, is derived from Equation (4.64):

$$\alpha = \tan^{-1} \frac{\omega^2-\omega_n{}^2}{2\omega_n\delta_n} \qquad (4.64)$$

Let $\alpha = \tan^{-1}u$ where $u = \dfrac{\omega^2-\omega_n{}^2}{2\omega_n\delta_n}$

$$\therefore \tan\alpha = u \qquad (4.71)$$

The slope of the phase angle curve is obtained by differentiating both sides of Equation (4.71) with respect to (ω). Hence:

$$\sec^2\alpha . \frac{d\alpha}{d\omega} = \frac{du}{d\omega}$$

$$\frac{d\alpha}{d\omega} = \frac{1}{1+\tan^2\alpha} . \frac{du}{d\omega} = \frac{1}{1+u^2} . \frac{du}{d\omega}$$

$$\frac{d\alpha}{d\omega} = \frac{1}{1+\left(\frac{\omega^2-\omega_n^2}{2\omega_n\delta_n}\right)^2} . \frac{2\omega}{2\omega_n\delta_n}$$

$$= \frac{4\omega\omega_n\delta_n}{4\omega_n^2\delta_n^2+(\omega^2-\omega_n^2)^2}$$

at resonance:

$\omega = \omega_n$, and:

$$\frac{d\alpha}{d\omega} = \frac{1}{\delta_n} \qquad (4.72)$$

Thus the damping constant is given by:

$$\delta_n = \frac{1}{(\text{slope at } \omega_n)} \qquad (4.73)$$

If the assumption that, between points satisfying the condition $\tan\alpha = \pm 1$, the phase angle curve is approximately linear, then the slope is given by:

$$\text{slope} \simeq \frac{\pi}{2} \div \Delta\omega = \frac{\pi}{2\Delta\omega}$$

substitution in equation (4.73):

$$\delta_n = \frac{2\Delta\omega}{\pi} \qquad (4.74)$$

In this way the second method gives a value of (δ_n) higher than that given by the first method by a factor $\frac{4}{\pi}$. Pendered and Bishop[43] report that comparison of practical results confirm a factor of this order and attribute the error to the assumption of linearity,discussed above, and to the approximations inherent in the first method.

The validity of both methods is affected by the presence of off-resonance vibrations.

4.3.5.5) DETERMINATION OF MODAL SHAPES:-

Considering the variation in the location of the receiver along the x-direction, the component phase angle is, from the expanded form of equation (4.47):-

$$\tan\alpha_x = \tanh\left(\frac{\delta_x x}{c}\right) \tan\left(\frac{\omega_x x}{c}\right) \qquad (4.75)$$

where:

$$\alpha = \alpha_x + \alpha_y + \alpha_z$$

is the total phase angle of lag behind the driving signal.

The trigonometric function in the right-hand side of equation (4.75) assumes alternating values of 0 and ∞ depending on the value of (x). Accordingly, the component phase angle (α_x) is equal to $\left(\frac{n\pi}{2}\right)$ radians; n = (0,1,2,3 ...)

Even values of (n) correspond to an in-phase or antiphase response, while an odd value indicates a quadrature response.

Therefore, from the plot of response phase with distance, the nodal and antinodal planes can be identified.

However, the method is subject to the same limitations discussed under the Peak-Amplitude method for identification of modal shapes.

4.3.5.6) APPRAISAL OF METHOD:-

In the presence of off-resonant vibration, due to heavy damping or closely-spaced natural frequencies, the method, like the Peak-Amplitude Technique, tends to be unreliable as errors attributed to those, will affect the results.

However, there are some apparent advantanges.

Analytically, the identification of the natural frequency does not depend upon the accurate location of a peak.

Experimentally, only a limited number of readings is required for determination of both natural frequencies and damping constants.

Its main drawback is exhibited in the difficulty associated with measurement of phase angles, and this seems to be the reason why this method has not found wide application in resonance testing measurements in the past[43].

4.3.6) THE KENNEDY AND PANCU METHOD:-

4.3.6.1) INTRODUCTION:-

As in other forms of resonance testing, the system under investigation is excited harmonically, and the variation of the transfer function of the response with the driving frequency measured. In this technique its components in-phase with and in-quadrature with the driving signal are measured, and from their plots information is derived on modes identification and determination of resonant frequencies and damping.

Some of the assumptions inherent in the Kennedy and Pancu Method are[42]:-

(1) Each normal mode responds to an applied force as a single-degree of freedom system, i.e. there is no coupling between normal modes of a system.

(2) The shape of each normal mode is fixed for a given system and is independent of the magnitude, frequency or location and direction in space of the applied force.

(3) The properties of normal modes hold rigorously only for zero damping, but if the damping is small, the mode shapes, phase relations and coupling are assumed to be unaffected.

(4) In some range of frequency around the natural frequency of a resonating mode, the contribution from off-resonant modes is effectively constant and can be extracted and measured from complex receptance plots.

Broadbent and Havtely[41] state that practical experience seems to show that this type of plot (a vector response curve) considerably reduces the likelihood of missing a resonance and also improves the accuracy of determining the natural frequency and damping.

4.3.6.2) DISPLAY OF RESULTS:-

The vector response diagram (Figure 4.5) is obtained by plotting the receptance components in-phase with an in-quadrature to the acoustic exciting signal.

The receptance of a single, isolated mode, when plotted in this way, gives a pure circle, centred on the in-phase axis of the complex plane.

Several modes of different natural frequencies and damping combine to give a complex system of loops, curves and knots (Figure 4.6); a number of them can be identified as circles or parts of circles of varying diameters, and by examining the rate, with respect to frequency, at which the vector moves round the circle, the modes of the system can be identified.

A detailed analysis of a particular loop can yield information on both the natural frequency and damping of the mode.

4.3.6.3) VECTOR PRESENTATION OF THE RECEPTANCE.

(A) CONVENTION:-

Complex quantities are represented on the complex plane as rotating vectors according to the sign convention adopted in Figure (4.7).

An input function is represented by the vector $V_e e^{j\omega t}$, the factor $e^{j\omega t}$ signifies a rotation of the vector V_e, through an angle (ωt) with respect to the real axis, in an anti-clockwise direction.

The response function of the system is given by another vector $V_a e^{j(\omega t-\alpha)}$, also rotating in an anticlockwise direction, and lagging behind the input function by the angle (α).

The transfer function of the system is then:

$$F = \frac{V_a e^{j(\omega t-\alpha)}}{V_e e^{j\omega t}} = V_R e^{-j\alpha}$$

which is a vector of magnitude V_R and a phase angle (α) rotating in the clockwise direction.

If:

$$F = (x+jy) = V_R e^{-j\alpha}$$

then: $|F| = V_R = (x^2+y^2)^{\frac{1}{2}}$

and $\alpha = \tan^{-1} \frac{y}{x}$

This convention will be adhered to throughout this thesis.

(B) THEORY:-

For a pure mode, the complex receptance is represented by Equation (4.62)

$$P_n = (x_n+jy_n) = \frac{-R_n \omega_n \delta_n}{(\omega^2-\omega_n^2)^2+4\omega_n^2\delta_n^2} + j \frac{R_n(\omega^2-\omega_n^2)}{(\omega^2-\omega_n^2)^2+4\omega_n^2\delta_n^2} \qquad (4.62)$$

Let:

$$v = -2\omega_n \delta_n$$

and:

$$u = (\omega^2-\omega_n^2)$$

then:

$$x_n = \frac{vR_n}{u^2+v^2} \qquad (4.76)$$

$$y_n = \frac{uR_n}{u^2+v^2} \qquad (4.77)$$

From Equation (4.76) and (4.77)

$$ux_n = vy_n$$

and:-

$$u^2 = \frac{v^2 y_n^{\,2}}{x_n^{\,2}} \tag{4.78}$$

From Equation (4.76):

$$u^2 x_n + v^2 x_n = vR_n \tag{4.79}$$

substituting Equation (4.78) into Equation (4.79)

$$v^2 x_n^{\,2} + v^2 y_n^{\,2} = vR_n x_n \tag{4.80}$$

and replacing v by $-2\omega_n\delta_n$:

$$x_n^{\,2} + y_n^{\,2} + \frac{R_n x_n}{2\omega_n\delta_n} = 0 \tag{4.81}$$

The standard form for the equation of a circle is

$$x^2+y^2-2hx - 2gy + (h^2+g^2-r^2) = 0 \tag{4.82}$$

where (h,g) is the centre of the circle and r its radius.

Comparing Equation (4.81) and Equation (4.82) and equating terms:

$$g = 0 \text{ and } h = -\frac{R_n}{4\omega_n\delta_n}$$

$h^2+g^2-r^2 = 0$

i.e. $$r = \frac{R_n}{4\omega_n\delta_n} \tag{4.83}$$

Therefore Equation (4.81) represents a circle, and it may be concluded that the receptance of a mode, when plotted in the complex plane, can be represented by a circle of radius $\left(\frac{R_n}{4\omega_n\delta_n}\right)$ and centre $\left(-\frac{R_n}{4\omega_n\delta_n}, 0\right)$, and passing through the origin of coordinate system.

If the receptance is plotted as a function of increasing positive angular frequency (ω) the locus of the tip of the vector (P_n) is represented by Equation (4.81) which gives a circle centred on the real axis with the vector moving in a clockwise direction.

If (R_n) is positive, the circle will be entirely contained in the negative real half of the complex plane. If (R_n) is negative, it will be on the positive half.

In considering systems having more than one degree-of-freedom, like a room, neighbouring modes to the particular one of interest, are likewise excited to a degree depending on their closeness in frequency and spatial distribution to the resonating mode.

As the driving source is sweeping through a certain frequency range, and in the immediate vicinity of resonance of a mode, the off-resonant contribution is both small and constant compared to the resonant vibration, and the tip of the total receptance vector will tend to describe partly a circle corresponding to that mode. As the frequency departs from the resonant range of that particular mode, the off-resonant contribution increases and the circular plot becomes distorted.

If the off-resonant vibration is not small, but assumed to be constant over the resonant frequency range of the mode, all points will be shifted due to the extraneous vibration, and a new displaced origin has to be assumed.

This outlines the theory of Kennedy and Pancu Method upon which crtieria will be derived for identification of the modes and determination of their natural frequencies, damping constants and modal shapes.

4.3.6.4) DETERMINATION OF NATURAL FREQUENCIES:-

For a pure, isolated mode, representing a single degree-of-freedom system, the receptance plot on the complex plane is a circle, centred on the in-phase axis and passing through the origin.

From Equations (4.62), (4.63) and (4.64), the natural frequency of the mode corresponds on the diagram to the point where the circle cuts the in-phase axis, the response being wholely in-phase or anti-phase - depending on the location of source and receiver - with the exciting signal.

This is also the point where the amplitude of the response is greatest, and where, for any one given damping, the frequency scale is most expanded; hence the spacing of the isochrones (equal frequency contours) is greatest.

In the general case of multi degree-of-freedom systems, e.g. where room modes are closely spaced in a particular frequency range, the response is characterized by the presence of significant off-resonant contribution from neighbouring modes, and the above definitions do not necessarily coincide.

However, the natural frequency is always taken to be the point at which there is maximum spacing of equal frequency increment points[41,42,43,44].

Kennedy and Pancu[42] utilized this property in deriving a method for identifying the natural frequencies from complex response measurements based on the following analysis of vector differentiation.

In Figure (4.8), curve C represents the locus of a point Q moving in the complex plane. The initial position of Q is given by the tip of the vector V(t), which is a function of a certain parameter (t). After an incremental time (Δt) the position of Q is given by $V(t+\Delta t)$.

The derivative of the vector is:

$$V'(t) = \frac{dV}{dt} = \lim_{\Delta t \to 0} \frac{dV}{\Delta t}$$

As shown by Figure(4.9), V'(t) is a vector itself, having the direction of the tangent to the curve C and a magnitude equal to the speed of the tip of the vector V(t), $\frac{ds}{dt}$ along the path C.

Equation (4.60) for the receptance can be written as:

$$P = Ae^{-j\alpha} = |R_n|A_n e^{-j\alpha} \qquad (4.84)$$

where

$$|R_n| = \pm \frac{|RC^2\psi_{n(r)}\psi_{n(r_0)}}{V\Lambda_n}$$

And:

$$A_n = \frac{1}{[(\omega^2-\omega_n{}^2)^2+4\omega_n{}^2\delta_n{}^2]^{\frac{1}{2}}} \qquad (4.85)$$

$$\alpha = \tan^{-1} + \frac{(\omega^2-\omega_n{}^2)}{2\omega_n\delta_n} \qquad (4.64)$$

In applying the above analysis to differentiating the vector P with respect to (ω^2), the properties of the circular plot, (Figure (4.9)) are used.

The angle of the tangent, being the angle of the derivative is θ which is:

$$\theta = \left(2\alpha \pm \frac{\pi}{2}\right) \text{ radians} \qquad (4.86)$$

The length ds of the arc on the curve is the radius times the angle subtended.

hence: $ds = r \times 2d\alpha$

where, from Equation (4.83),

$$r = \frac{|R_n|}{4\omega_n\delta_n}$$

therefore:

$$ds = \frac{|R_n|}{2\omega_n\delta_n} .d\alpha$$

and the magnitude of the derivative:

$$\frac{ds}{d(\omega^2)} = \frac{|R_n|}{2\omega_n\delta_n} \cdot \frac{d\alpha}{d(\omega^2)} = \frac{|R_n|}{2\omega_n\delta_n} \cdot \frac{d}{d(\omega^2)}\left[\tan^{-1} \frac{(\omega^2-\omega_n{}^2)}{2\omega_n\delta_n}\right]$$

$$= \frac{|R_n|}{(\omega^2-\omega_n{}^2)^2+4\omega_n{}^2\delta_n{}^2} = |R_n|A_n{}^2 \qquad (4.87)$$

From Equations (4.86) and (4.87), the derivative vector P' is:

$$P' = |R_n'|A_n^2 e^{-j(2\alpha \pm \pi/2)} \qquad (4.88)$$

Comparing Equation (4.88) with (4.84)

$$P' = \frac{jP^2}{|R_n|} \qquad (4.89)$$

i.e. the first derivative P' of the complex function P is proportional to the square of the original function.

On plots of amplitude and phase of the derivative versus (ω) resonance would also be indicated by a maximum or an inflection point, respectively, in the two curves.

The peak in the amplitude plot, however, is sharper, and the slope on the phase plot is steeper than for the original function.

It may therefore be concluded that the derivative is more sensitive to resonance than the original quantity.

In theory[42], vector differentiation can be repeated a number of times, each successive differentiation will sharpen the peak in amplitude and steepen the slope of the phase response; in other words, 'the sensitization to resonance is increased by differentiation'. However, more than one differentiation will be impractical with the uncertainty present in practical measurements.

This technique of plotting the rate of change of reponse vs. the driving frequency shall be adopted, in this work, as the most reliable method for identification of the natural frequencies of room modes.

4.3.6.5) <u>DETERMINATION OF DAMPING CONSTANTS</u>:-

The inaccuracy in estimating the damping by the peak-amplitude and phase-angle techniques results from the presence near the resonant frequencies of unknown amount of vibration in the off-resonant modes.

The advantage of the Kennedy and Pancu method is that it provides a means of extracting the resonant contribution from the total response, from which the damping constant can be determined.

The method of finding the damping constant from empirical vector plots (obtained from experimental measurements) consists of drawing a best-fit circle through the empirical curve in the vicinity of the natural frequency as determined by the method outlined in Section (4.3.6.4). On the basis of a comparison with the response of a single degree-of-freedom system, the relationship between the angle subtended by that part of the arc of the circle which matches the empirical curve and the frequency change between these extremes yields the damping constant[46].

The number of points lying on the arc is governed by the range of frequency over which the assumed constancy of the off-resonant contribution applies.

With reference to Figure (4.10), if a satisfactory match occurs in the vicinity of f_n only between points A and B, then:

$$\theta_1 = (\pi - \alpha_1) \quad \text{and} \quad \theta_2 = (\alpha_2 - \pi)$$

From Equation (4.64):

$$\tan\alpha_2 = \frac{\omega_2^2 - \omega_n^2}{2\omega_n \delta_n} = \tan\theta_2$$

$$\tan\alpha_1 = \frac{\omega_1^2 - \omega_n^2}{2\omega_n \delta_n} = -\tan\theta_1$$

therefore:

$$\tan\theta_2 + \tan\theta_1 = \tan\alpha_2 - \tan\alpha_1$$

$$= \frac{\omega_2^2 - \omega_1^2}{2\omega_n \delta_n} = \frac{(\omega_2 - \omega_1)(\omega_2 - \omega_1)}{2\omega_n \delta_n} \qquad (4.90)$$

for all practical purposes,

$$\omega_n \simeq \frac{\omega_2+\omega_1}{2}$$

and Equation (4.90) can be written as:

$$\delta_n = \frac{(\omega_2-\omega_1)}{\tan\theta_1 + \tan\theta_2} = \frac{2\pi(f_2-f_1)}{\tan\theta_1 + \tan\theta_2} \qquad (4.91)$$

The most accurate condition, as long as the degree of match of the circular arc permits is where $\theta_1=\theta_2=\frac{\pi}{4}$ radians, the points A and B being equally distributed about (f_n), thereby defining the so-called half-power width.

In this case the expression in Equation (4.91) reduces to:

$$\delta_n = \frac{\omega_2-\omega_1}{2} = \pi(f_2-f_1) \qquad (4.92)$$

which is equivalent to that represented by Equation (4.67) and derived by the Peak-Amplitude and Phase-Angle methods.

4.3.6.6) DETERMINATION OF MODAL SHAPES:-

It has already been pointed out that the Kennedy and Pancu method provides a means of extracting the resonant part from the total measured response. On the best-fit circle drawn on the empirical curve, the resonant amplitude is given by the diameter of the circle passing through the point corresponding to the natural frequency of the mode, (Figure (4.11)). At its other end it will determine the displaced origin, which, had it not been for the extraneous vibration, will coincide with the origin of the coordinate system.

The vector joining the true origin and the displaced origin will give the value of the off-resonant vibration.

The magnitude of the resonant amplitude is equal to $2r = \frac{|R_n|}{2\omega_n\delta_n}$ (from Equation (4.83) and (R_n) is given by Equation (4.61).

Its sign can be determined from the way in which the circle is atttached to the curve relative to the coordinate system. If the circle is attached at the point of least value of real component, then the sign is positive; if, on the other hand, it is attached to the point of greatest real component, it will be negative[41].

Once the true amplitudes are obtained, in magnitude and sign, the shape of the mode can be found from the ratios of peak amplitudes at various locations of the receiver along a particular line, due to a given excitation at a fixed position.

4.3.6.7) APPRAISAL OF METHOD:-

The basic advantage of this technique is its ability to show up the existence of modes and provide means of locating their natural frequencies with an accuracy unmatched by other techniques.

A second advantage is that, by giving a means of extracting the vibration in the resonating mode from the total vibration, it allows a better estimate to be made of the damping of the mode.

Its third merit lies in the fact that the modal shapes can be determined more accurately from the extracted true peak amplitudes.

The main experimental difficulty in the application of the Kennedy and Pancu technique is associated with the technical devices of eliminating undesirable modes and thus exciting a pure mode separately, with a simple system of excitation. Accounts of such devices are reported by some authors[42], but the application of these poses considerable problems in room acoustics measurements.

Another difficulty is that phase differences between the exciting signal and responses, which form a vital part of the information required for the application of the technique, change very rapidly as the driving frequency passes through resonance, and if they are to be measured, the frequency must be accurately controllable and measured. Marples[46] has pin-pointed the advantages of the use of digital filtering as compared to analogue filtering in this respect as a means of overcoming this limitation in the application of this technique.

A source of error, arising from the interference of closely-spaced modes, could throw some uncertainty upon the accuracy of derived results. Some qualitative guidance on this aspect has been given by Graggs and North[47] and later Marples[44] established criteria for a'simplified interference boundary' relating the damping ratios and the ratios of the natural frequencies of closely-spaced modes to some quantified limits.

4.3.7) EXCITATION AND RESPONSE EXTRACTION OF PURE MODES.

4.3.7.1) INTRODUCTION.

So far in this chapter, different techniques for the study of the modal sound field in rooms have been surveyed. According to Bishop and Pendered[50], these are essentially of two sorts:

(1) A simple system of excitation produces response data which is usually complicated to analyze.

(2) A complicated system of excitation produces a response which occurs accurately in a particular mode. In this case, little effort is required by way of analyzing results.

The governing factor in choosing either of the two alternative approaches for a particular situation is dependent mainly on the availability and degree of accuracy of instrumentation systems for excitation of modes or measurement and analysis of extracted results.

An optimum condition will always be sought throughout this work with regard to the choice of instrumentation systems, with the sole purpose of avoiding too elaborate situations difficult to meet in practical conditions in the study of studio acoustics.

In the light of this consideration, the problem of excitation of pure modes, and extraction of their responses will be examined.

4.3.7.2) THE NULL SURFACE STRUCTURE OF MODAL FIELDS.

The spatial distribution of the steady-state response in a three-dimensional sound field in a room, in the presence of a single mode, excited at its natural frequency, the point of excitation being fixed and the receiver's position varied, is governed by the boundary conditions, the mode orders (the values of the n_x, n_y, n_z) and the directional orientation of the mode.

In a rectangular room, with fairly hard walls, the response will oscillate with position in a fairly regular manner, with alternate nodal and antinodal planes in each direction of the coordinate system, forming a set of equidistant surfaces. The mode order (e.g. n_x) indicates the number of nodal (null) surfaces perpendicular to the respective coordinate axis.

The Nodes and antinodes consist of alternating regions of maxima and minima of the response.

Three types of null surfaces can be loosely defined as follows[12]:

(1) A one-dimensional null surface system is equivalent in a topological sense to the family of planes

$$x = n_x . \lambda/2 \ , \ n_x = 1,2,3 \ldots$$

(2) A two-dimensional null surface system is similarly equivalent to two one-dimensional systems intersecting each other so that a typical space bounded by a null surface is a cylinder of infinite length with cross-sectional area of the order $(\lambda/2)^2$.

(3) A three-dimensional null surface is similarly equivalent to three one-dimensional systems intersecting each other so that a typical space bounded by a null surface is a cell with a volume of order $(\lambda/2)^3$.

These definitions correspond to the null surface systems of axial, tangential and oblique modes respectively.

The spacing of consecutive null surfaces in any direction in a rectangular room, with fairly hard walls, can be derived in terms of the wavelength and the directional cosines of the mode in the following way:

Let S_x be the distance between consecutive null surfaces in the x-direction and

θ_x the angle of incidence of the mode with respect to the x-axis.

$$S_x = \frac{L_x}{n_x} \qquad (L_x = \text{the room length, } n_x = \text{the mode x-order})$$

the component angular frequency:

$$\omega_x = \frac{n_x \pi c}{L_x} = \frac{\pi c}{S_x}$$

the directional cosine:

$$\cos\theta_x = \frac{\omega_x}{\omega_n}$$ (ω_n: being the natural frequency of the mode.)

$$\therefore \quad \cos\theta_x = \frac{\pi c}{S_x \omega_n} = \frac{\lambda_n}{2S_x}$$

i.e. $$S_x = \frac{\lambda_n}{2\cos\theta_x} \qquad (4.93)$$

for an x-axial mode ($n_y = n_z = 0$):

$$S_x = \frac{\lambda_n}{2}$$

If the walls are no longer hard, but furnish some degree of admittance, the spatial distribution of the mode will be distorted. The type and degree of distortion is dependent on the nature of the impedance of the wall's surfaces. Under such conditions the value of the spacing of the null surfaces can still be evaluated in terms of the wavelength of the mode and its directional cosine, from equation (4.93). However, θ_x is given by[22]:

$$\cos \theta_x = \frac{(\omega_x^2 - \delta_x^2)^{\frac{1}{2}}}{\omega_n} \tag{4.94}$$

where

$$(\omega_x^2 - \delta_x^2) = \text{Real}(c^2 k_x^2)$$

k_x = the component characteristic value of the mode in the x-direction, its value can be computed from a knowledge of the impedance of the corresponding pair of walls using equations (3.31)-(3.34).

For limiting cases, if the impedance is all imaginary there will be no loss of energy, and the nodal surfaces will only be shifted together by a certain amount, but the shape of the mode remains unaltered. On the other hand, if the impedance is all real, the shape will be distorted.

The implications of the type of the impedance on the nature of distortion of the mode spatial structure have been discussed in great detail by many authors[22,24,17].

4.3.7.3) ISOLATION OF MODES IN SPACE:

It must be remarked that the nature of the reponse of a room depends on the position of the exciting source and receiver, as expressed by the values of the product of the characteristic functions $\psi_{n(r_o)}$, $\psi_{n(r)}$ in equation (3.56).

The difficulty that arises in connection with the analysis of measured responses is due to the fact that vibrations take place in several modes simultaneously for a fixed location of the source and receiver.

The first step in trying to simplify a complex total response is to eliminate as many undesired modes as possible by locating a sound source at a point where it does not excite those modes, and at the same time gives maximum excitation for the particular mode under consideration. Thus the location of the source will have to correspond to a point of antinode in all three axes for the mode to be fully excited (the product of $\psi_{n(r_o)} = \psi_{n(x_o)} \cdot \psi_{n(y_o)} \cdot \psi_{n(z_o)}$ governing the spatial distribution of the response as a function of the source position, should be a maximum.)

For a mode to be eliminated, the source needs to be located at a point on only one nodal plane, since $\psi_{n(r_o)} = 0$ for only one term to be zero (e.g. $\psi_{n(x_o)} = 0$).

The above discussion applies identically to elimination of responses of undesirable modes and excitation of the mode of interest by similarly locating the receiver.

This can be demonstrated by the simple case of placing either the source or the receiver at the centre of the room and the other at a corner. The modes with even orders (n_x, n_y, n_z:even) will only be fully excited, while all other odes modes with an odd order (either n_x, n_y, n_z: odd) will, in theory, be completely eliminated.

So far an account of a systematic system of excitation and elimination of modes has been given in terms of a simple method, based on the concept of the null surfaces of modes, and using a simple system of a single exciter and receiver.

Accounts of technical devices based on considering the orientation of modes, as well as the location of the sound sources and receivers, and extended to the use of multipoint exciters and receivers with adjustable amplitudes and phases, have been given by several authors.

Bishop and Gladwell[41] have reported and discussed the technique presented by Lewis and Wrisley[48] and pointed out the complicated instrumentation required for their method to be applied. They also discussed the Traill-Nash[49] Method and showed the great amount of computation involved as a 'grave disadvantage in the method.'

4.3.7.4) SEPARATION OF MODES IN FREQUENCY:-

The response of the room as expressed by the Green's Function depends, in addition to the location of the source and receiver, on the value of the resonance denominator for each mode (equation (4.63))

$$[(\omega^2-\omega_n^{\,2})^2 + 4\omega_n^{\,2}\delta_n^{\,2}]^{\frac{1}{2}}$$

The response, in amplitude, is inversely proportional to the closeness of the natural frequency to the exciting frequency and the value of the damping constant.

For low frequencies, the natural frequencies of modes are widely spaced and thus the maxima on the frequency-response curve can, in general, be associated with individual modes. In this region it is possible to identify and study each mode separately by driving the room at the corresponding natural frequency.

With high frequency sounds, the modes are closely spaced in frequency, and a criterion for modes identification must be considered.

For the purpose of identification, it will be sufficient to ensure that resonance peaks of adjacent modes will not overlap, and thus each mode can be identified by a peak on a frequency-response curve.

A theoretical limiting condition[18] can be based on the spacing between successive peaks, corresponding to individual modes, being larger than half of the half-width of resonance (Fig.4.12).

The spacing between adjacent modes can be obtained from equation (3.21) for the modal density

$$\frac{dN}{df} \simeq \frac{4\pi V}{c^3} . f^2 \qquad (4.95)$$

by setting dN equal to unity, the corresponding (df) is the average minimum frequency spacing:

$$df_n \simeq \frac{c^3}{4\pi V f_n^{\ 2}} \qquad (4.96)$$

The limiting condition for modes identification is thus this spacing should be larger than ½ half-width of resonance i.e.

$$df_n \geqslant \frac{\delta_n}{2\pi} \qquad (4.97)$$

The limiting frequency below which individual modes can be excited separately can be obtained from (4.96) and (4.97):

$$f_{max} < \left(\frac{c^3}{2V\bar{\delta}_n} \right)^{\frac{1}{2}} \qquad (4.98)$$

where

$\bar{\delta}_n$: the average damping constant.

For frequencies higher than the limiting frequency, f_{max}, the maxima of frequency-response curves cannot be identified with single resonance peaks[17]; the latter would be situated, if they could be detected experimentally, much closer. The average separation $(\Delta S)_{max}$ of two adjacent maxima can be calculated, then, by applying the following formula[51]:

$$\Delta S_{max} = \frac{\bar{\delta}_n}{(3)^{\frac{1}{2}}} \qquad (4.99)$$

where

$(\bar{\delta}_n)$ is the mean value of many damping constants, which are associated with neighbouring modes.

Hence, the average separation of two adjacent maxima is somewhat larger than the average half-width of resonance, and, remembering that the inequality (4.97) for frequencies above f_{max} is:

$$df \leqslant \frac{d_n}{2\pi} \qquad (4.100)$$

and combining (4.99) and (4.100) results in:

$$\Delta S_{max} = \frac{\bar{\delta}_n}{(3)^{\frac{1}{2}}} \geqslant \frac{\delta_n}{2\pi} > df$$

i.e. $\Delta_{max} >> df$

The separation of successive maxima is by far greater than the actual spacing of resonance peaks.

In this case there are enough modes contributing to the response that statistical rather than wave analysis can be applied to the study of room response, and the summation in the expression for the Green's Function equation (3.56) can be transformed into an integral[18].

Another governing condition for modes identification is the frequency range of the exciting signal. The narrower the bandwidth that the source sweeps, the less is the number of modes contributing to the measured response.

If (df) in equation (4.96) can be interpreted in terms of the driving frequency then the number of excited modes is:

$$dN = \frac{4\pi V}{c^3} \cdot f^2 \cdot df \qquad (4.101)$$

and, incorporating the fact that the response of a particular mode has a spread of:

$$\Delta f_n = \frac{\delta_n}{\pi}$$

the effective range of frequency of contributing modes is:

$$df = (\Delta f + \Delta f_n) = \left(\Delta f + \frac{\delta_n}{\pi}\right) \qquad (4.102)$$

where:

df : is the effective range of response spread

Δf : is the sweeping frequency range

Δf_n : is the half-width of resonance

Substituting (4.102) in (4.101):

$$dN = \frac{4\pi V}{c^3} \cdot f^2 \cdot \left(\Delta f + \frac{\bar{\delta}_n}{\pi}\right) \qquad (4.103)$$

from which it can be concluded that the number of excited modes is a function of the bandwidth of the exciting frequency, the centre frequency and the average damping constant.

So far a theoretical account has been given of the conditions necessary for modes identification.

Attempts to establish some interference boundary limits which will ensure that, in the measured response of an excited mode, the off-resonant contribution due to other modes is small in comparison with the resonant component, have been made by some experimentors.

Craggs and North(47) have given some qualitative guidance on modes interference and reiterated the need for extreme care when interpreting plots relating to situations with close natural frequencies.

Marples(44) has investigated the problem in view of the mutual effect of a pair of adjacent pure modes, equally excited, and proposed ways for a quantified assessment for limits of interference between them. His criteria can be summarised in:

(1) the amplitude of one mode at its natural frequency being much smaller than that of an adjacent mode at its own natural frequency;

(2) the contribution of the first mode at the natural frequency of the second mode being much smaller than that of the resonant amplitude of the second mode.

(3) The change in the off-resonant contribution from the first mode, between the half-power width of the second mode, being small in comparison to the resonant amplitude of the second mode.

These criteria are derived in terms of the modal damping ratios and the separation of the natural frequencies of the modes.

The extension of the above theory to room modes imposes two necessary modifications:

(1) the relative degree of excitation of the modes as represented by the characteristic functions $\psi_{n(r_o)} \cdot \psi_{n(r)}$ should be considered.

(2) The interference on a resonant mode should be viewed as a summation from all possible off-resonant modes as represented by the series of equation (3.56).

4.3.7.5) <u>EXTRACTION OF OFF-RESONANT CONTRIBUTION</u>:-

The measured total complex receptance of the room can be represented from equation (4.59) by:

$$P_T = A_T e^{-j\alpha_T}$$

The total receptance is:

$$P_N = A_N e^{-j\alpha_N}$$

the resonant component of the receptance can be extracted by vectorially substracting the off-resonant compoent from the total receptance

$$\text{i.e. } P_N = P_T - \sum_{n=1}^{n-1} P_n = A_N e^{-j\alpha_N} = A_T e^{-j\alpha_T} - \sum_{n=1}^{n-1} A_n e^{-j\alpha_n} \qquad (4.104)$$

Kennedy and Pancu Method[42] provides a means of extracting the resonance component from the complex measured response, graphically. This has been discussed in Section (4.3.6.6) and is illustrated by Fig.(4.11).

If only the magnitude of the receptance is measured, thenan analytical method suggested by Gladwell[45], a full treatment of which is given in a published paper, can be employed.

From Figure (4.13):

Let $\dfrac{y_0-y_1}{y_1-y_2} = \alpha$

$$\frac{\omega_2^2-\omega_n^2}{\omega_1^2-\omega_n^2} = \beta$$

$$\frac{y_2-d}{y_1-y_2} = \gamma$$

where (d) is the (constant) off-resonant contribution.

It is always possible to choose $\alpha = 1$

$$\text{i.e. } y_1 = \tfrac{1}{2}(y_0+y_2)$$

then the equation for γ is

$$(2\beta^2-4)\gamma^3 + (\beta^2-4)\gamma^2 - 12\gamma - 4 = 0$$

which has just one positive root for:

$$\alpha > 0 \ , \quad \beta^2 > \frac{\alpha+1}{\alpha}$$

Then the true resonant peak is given by:

$$y_n = y_0 - d = (2+\gamma)/y_1-y_2) \qquad (4.105)$$

The method, as suggested by Pendered and Bishop[43], is theoretically attractive, but the assumption of a constant off-resonant contribution in amplitude is a serious drawback; still it is an improvement over the assumption that the peak arises from the resonant vibration only.

4.3.7.6) ELIMINATION OF RESPONSE OF MEASUREMENT SYSTEM:

In practical measurements of room response, the measurement set-up imposes its own characteristics on the true room response, in amplitude and phase. This can be expressed as:

$$P_T = P_N . P_S$$

where:

P_T = complex measuredpressure response in a modal field

P_N = true complex modal response

P_S = measuring set-up complex response.

The modal response can be obtained by eliminating the response of the measuring set-up from the total measured response by:

$$P_N = \frac{P_T}{P_S} \qquad (4.106)$$

The factor (P_S) due to the measurement set-up can be determined from free-field (anechoic) measurements.

The free-field pressure at a point (r) from the source is expressed by:

$$P_r = - \frac{j\omega\rho Q_o}{4\pi r} e^{jkr} \qquad (3.66)$$

If the response of the system is included, then the total measured free field response is:

$$P_F = P_r . P_S \qquad (4.107)$$

Inserting equation (4.107) into equation (4.106):

$$P_N = \frac{P_T . P_r}{P_F} \qquad (4.108)$$

or, in polar coordinates:

$$A_n e^{-j\alpha_N} = \frac{A_T . A_r}{A_F} e^{-j(\alpha_T - \alpha_F + \alpha_r)} \qquad (4.109)$$

4.3.8) CONCLUSION:

In this section, the various techniques of resonance testing have been discussed, and their applications to the analysis of modal parameters in room acoustics, assessed. The conditions necessary for the implementation of these techniques have been surveyed by considering the means of pure mode excitation and response extraction.

In the experimental work in this thesis, the choice of a particular technique will mainly depend on the prevalence of the conditions necessary for its application.

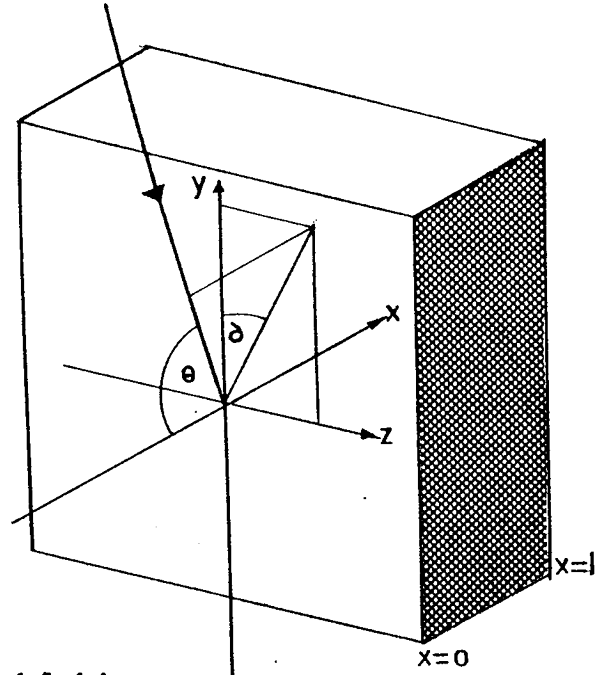


FIG.(4.1.): RELATION OF WAVE DIRECTION AND AXES ON AN ABSORBING SLAB

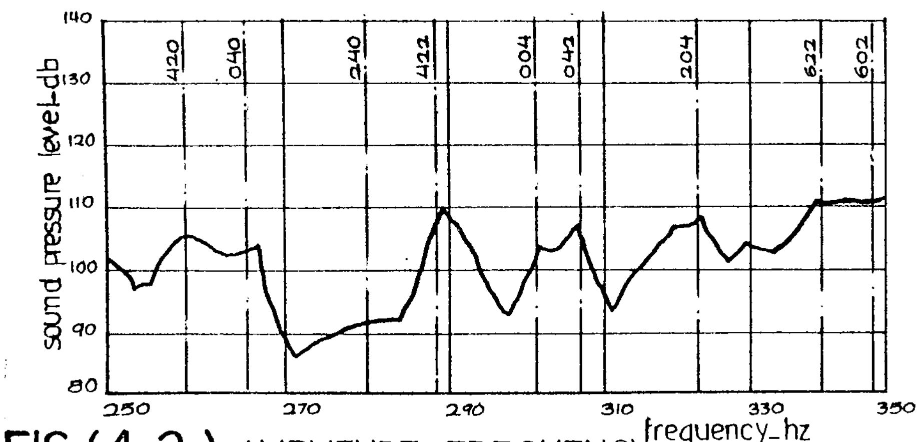


FIG.(4.2.): AMPLITUDE_FREQUENCY RESPONSE.

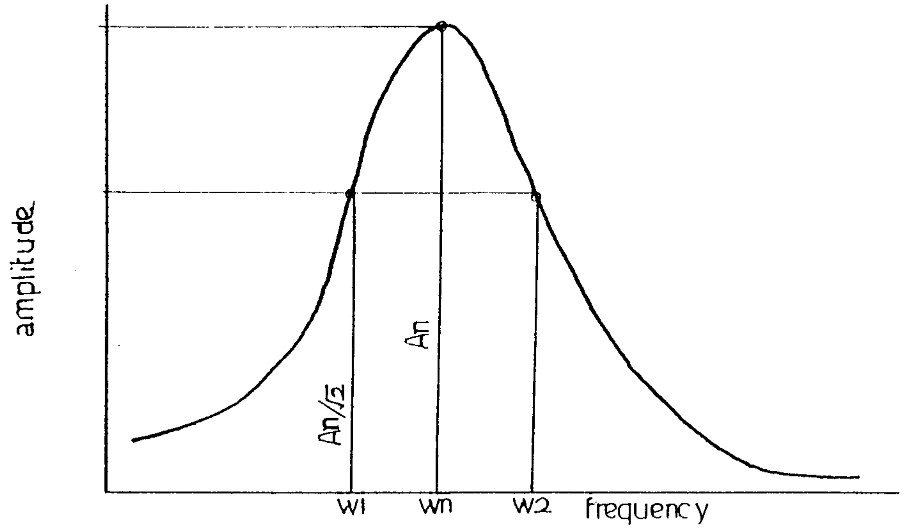


FIG.(4.3.): DETERMINATION OF NATURAL FREQUENCY & DAMPING CONSTANT BY PEAK_AMPLITUDE Mᴰ

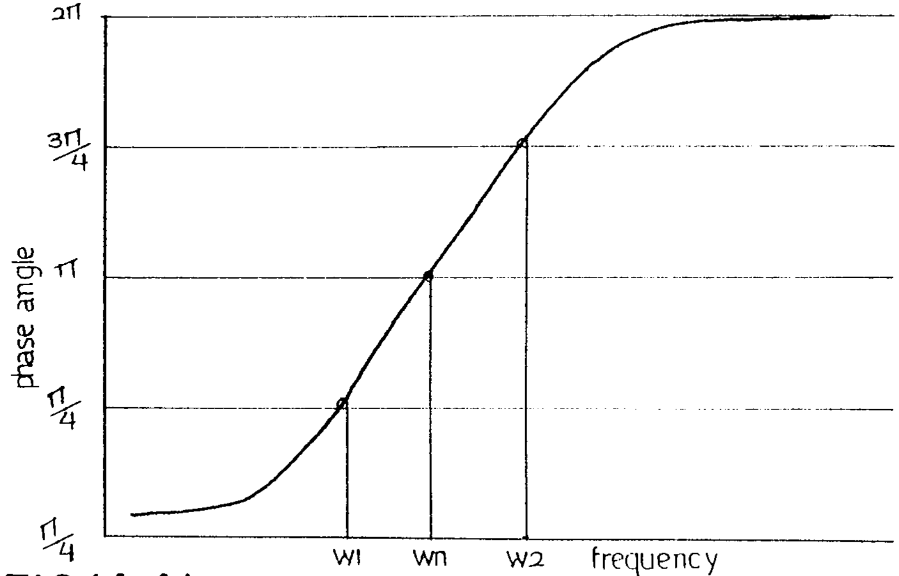


FIG.(4.4.): DETERMINATION OF NATURAL FREQUENCY & DAMPING CONSTANT BY PHASE_ANGLE Mᴰ

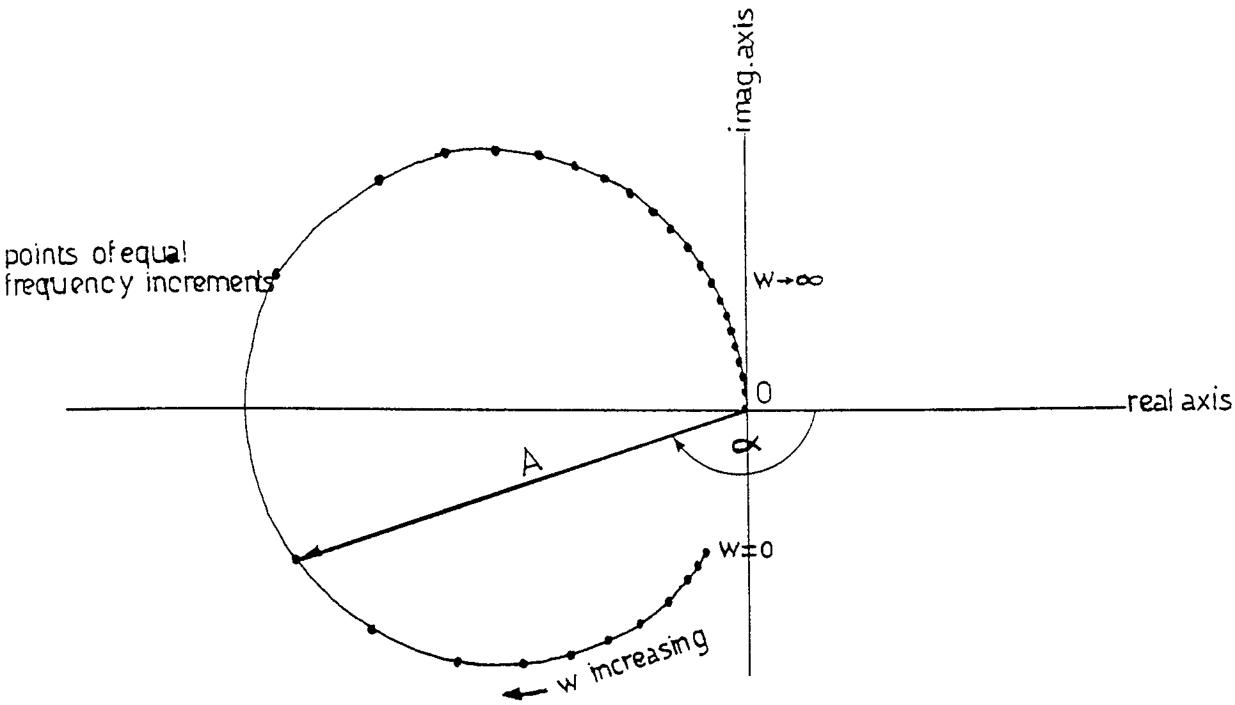


FIG.(4.5.): COMPLEX_FREQUENCY RESPONSE OF A PURE MODE

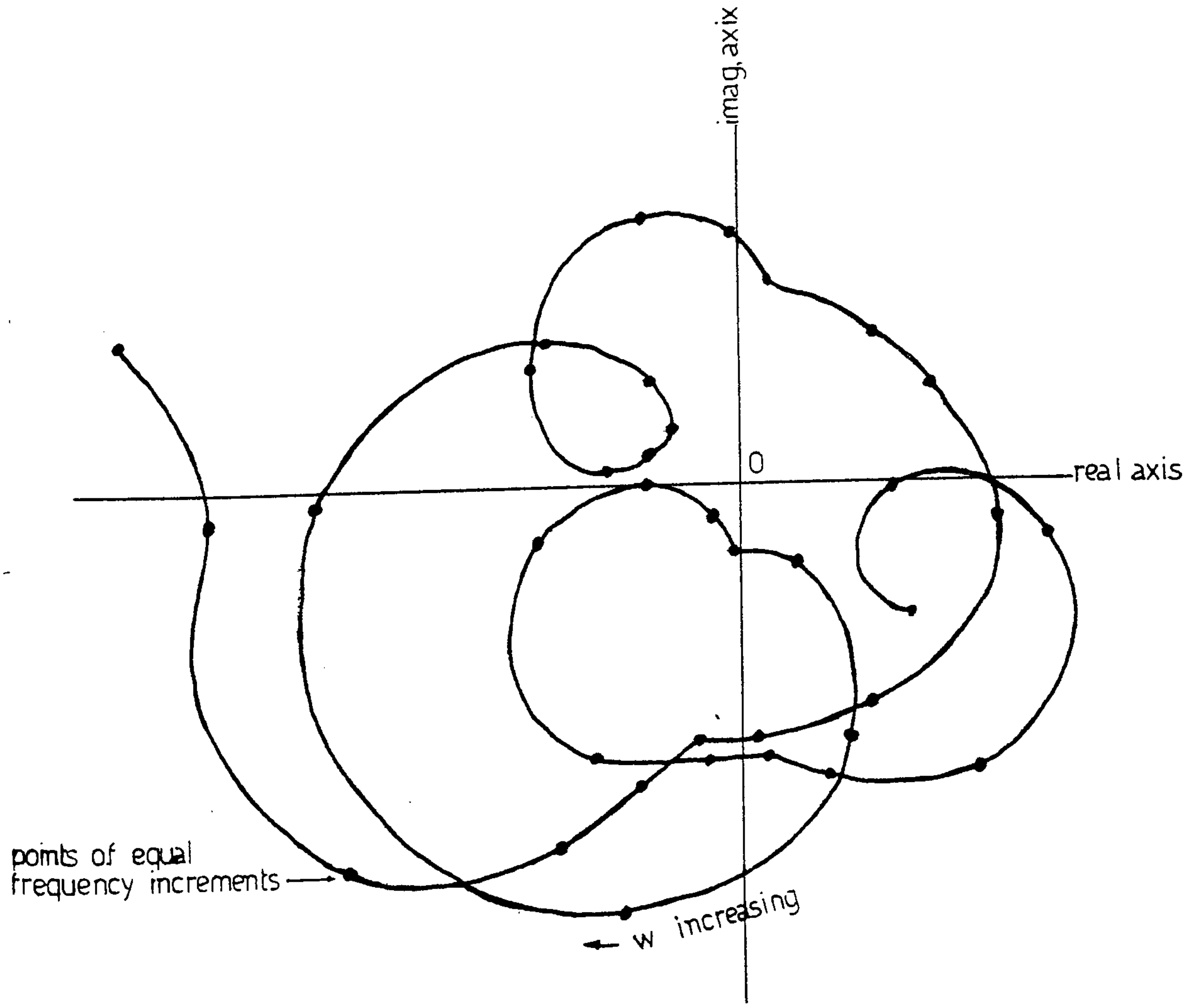


FIG.(4.6.): EMPIRICAL COMPLEX_FREQUENCY RESPONSE OF MULTI_MODAL SYSTEM

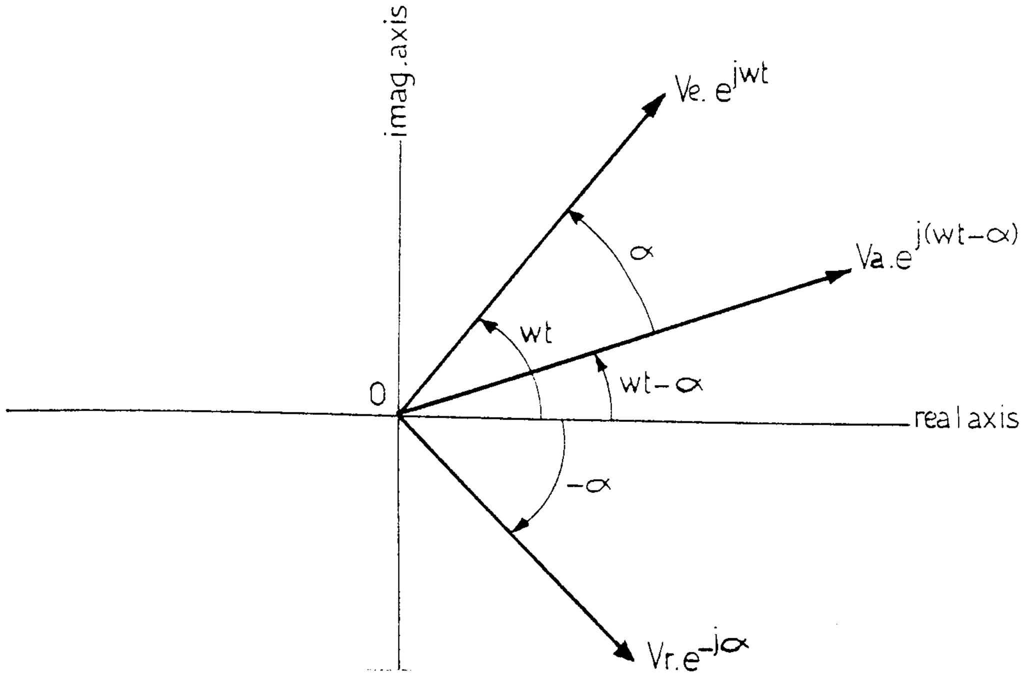


FIG.(4.7.): SIGN CONVENTION OF VECTOR REPRESENTATION

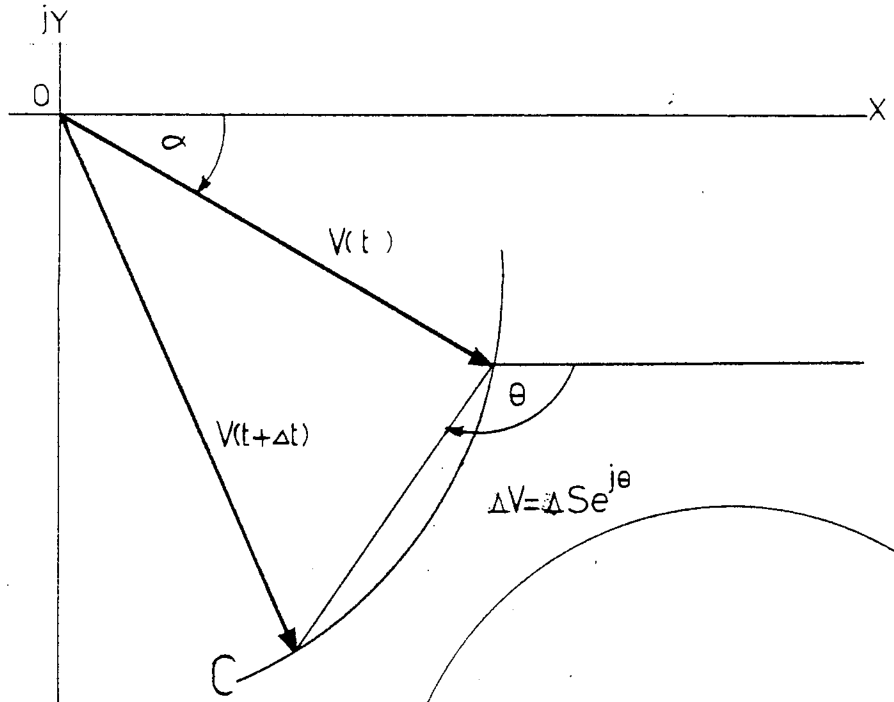


FIG.(4.8.): VECTOR DERIVATIVE

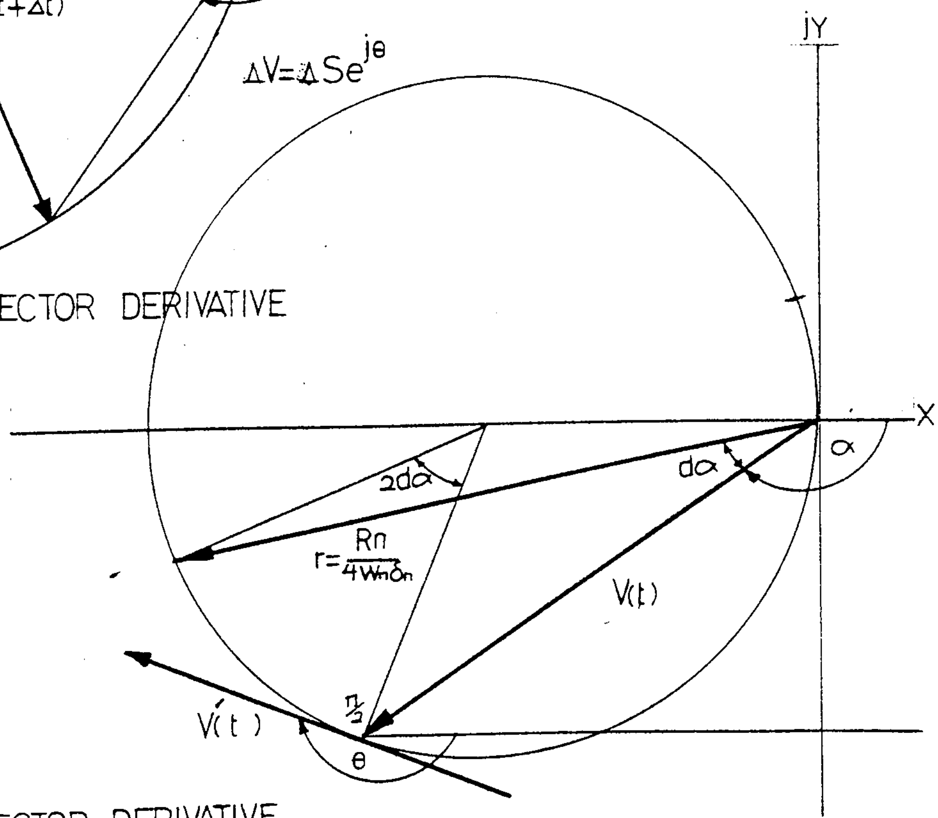


FIG.(4.9.): VECTOR DERIVATIVE

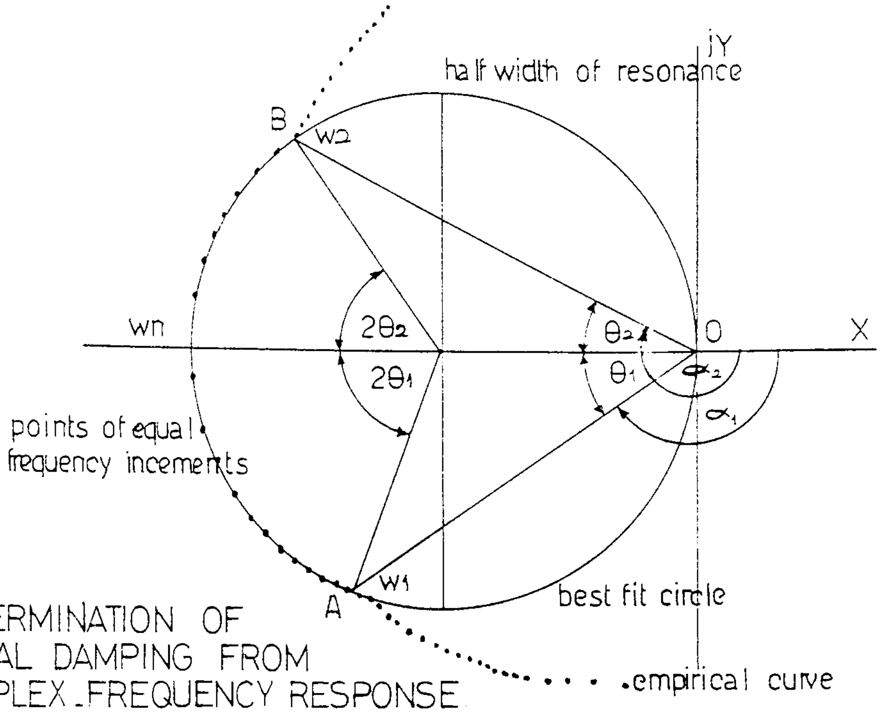


FIG.(4.10): DETERMINATION OF MODAL DAMPING FROM COMPLEX-FREQUENCY RESPONSE MEASUREMENTS

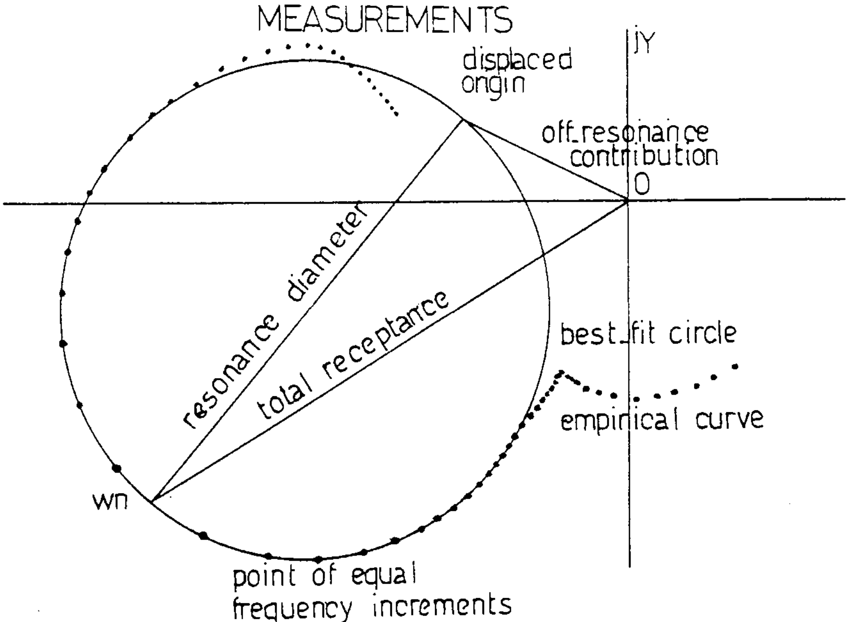


FIG.(4.11): DETERMINATION OF RESONANT AMPLITUDE BY KENNEDY AND PANCU METHOD

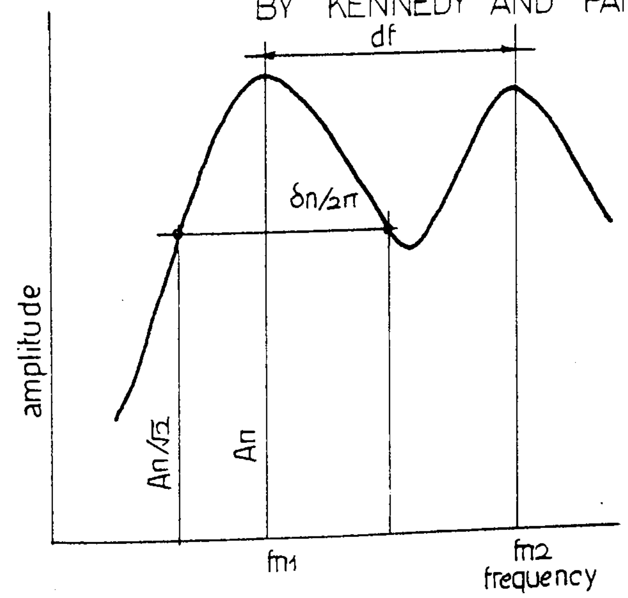


FIG.(4.12): CRITERIA FOR MODES IDENTIFICATION

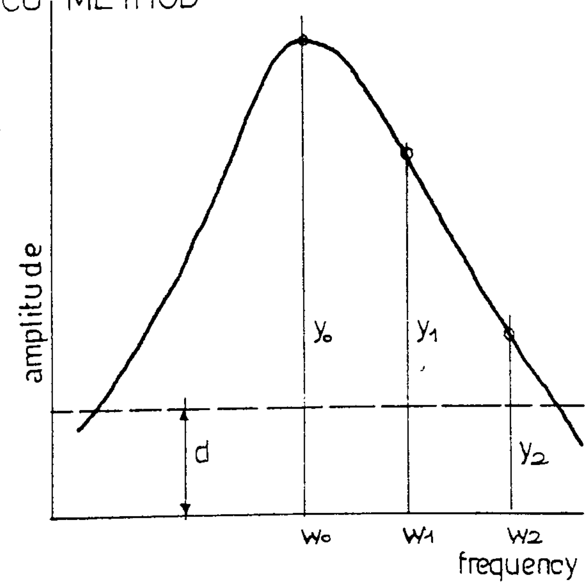


FIG.(4.13): ANALYSIS OF AMPLITUDE-FREQUENCY PLOT BY GLADWELL'S METHOD

CHAPTER FIVE

METHODS OF NUMERICAL ANALYSIS AND THEIR IMPLEMENTATION

5.1) INTRODUCTION:-

The implementation of the theoretical analysis presented in Chapter Three provides means of predicting the acoustical behaviour of small rooms, prior to construction. The method uses objective design data based on the choice of room size, dimensions and the absorbing qualities of the bounding surfaces. The pattern of the distribution of absorbers for the case where different surfaces are uniformly covered with different, locally reacting, sound absorbents, can be handled. That such predicted room behaviour can be verified by experimental measurements of unique modal parameters representative of the modal response to acoustic excitation, has been discussed in Chapter Four, where alternative techniques have been described for measurement of modal shapes, natural frequencies and damping constants as the parameters for verification.

Previously, such work has been of theoretical interest only due to the laborious complexity of the mathematical calculations required(52). However, the use of digital computers has opened possibilities for overcoming these difficulties.

Computer simulation techniques have been utilized for similar purposes of theoretical predictions, with geometrical analysis methods(11), and of experimental data analysis and presentation with resonance testing techniques(47) in vibration studies.

In this work, where a wave analysis approach is followed in the theoretical predictions, and resonance testing is employed in room acoustics studies, the digital computer is utilized for the twofold objective of:-

(1) Manipulation of theoretical predictions by simulating the room modal response and the associated parameters by mathematical models. The computational processes are

(1) contd.

carried out by employment of the relevant techniques of numerical analysis.

(2) Analysis of raw data extracted from experimental measurements, and the presentation of processed data in the form of tabulated results, graphs or stored files. The latter will enable intercommunications between different computer programs.

5.2) THE THEORETICAL MODEL:

5.2.1) ROOM SIZE AND DIMENSIONS:-

A small broadcasting or recording studio is simulated by a rectangular parallelopiped enclosure of the following dimensions:

length: 3.27 m, width: 2.57 m, height: 2.29 m and volume: 19.25 cum.

The choice has been dictated by the availability of an existing room of these dimensions rather than an optimum size for a hypothetical studio, such that the calculated theoretical predictions can be verified by experimental measurements in the room available.

5.2.2) FREQUENCY RANGE AND MODE ORDER LIMIT:

The frequency of the exciting signal is confined to the range 20 Hz to 600 Hz. The upper limit is approximately set by formula (Equation 4.59):

$$f_{max} < \left(\frac{c^3}{2V\bar{\delta}_n}\right)^{\frac{1}{2}} \qquad (4.98)$$

defining the frequency below which individual modes of vibration can be excited separately, and thus the modal wave analysis can be applied. Above this limiting frequency, the room modes are closely spaced, and it is possible to study the room response by statistical geometrical methods.

The range of mode order investigated (i.e. n_x, n_y, n_z) is restricted to a maximum of 10 (i.e. $n_{xmax} = n_{ymax} = n_{zmax} = 10$).

This quantification is based on the discussion in Section (4.3.7.2) of Chapter Four. The separation of two consecutive minima or maxima planes in the spatial distribution of sound pressure amplitude in the room is:

$$S_x = \frac{L_x}{n_x}$$ for the x-direction (with similar expressions for the y-direction and z-direction).

A maximum separation of approximately 30 cms $\left(\frac{L_x}{n_{xmax}}\right)$ is considered as the limit below which the pressure fluctuations with distance will not be detectable in consideration of the binaural nature of our hearing mechanism.

5.2.3) SURFACE TREATMENT:-

The choice of sound absorbents for covering the room bounding surfaces is restricted to locally reacting materials whose absorbing properties, as expressed by their specific acoustic admittance, is independent of the angle of incident sound - an assumption underlying the validity of application of the Wave Modal Theory.

These are materials which are characterized by some degree of surface porosity. To some approximation, a bare plastered wall is assumed to fall in this category.

Three different types of absorbents are considered:

(1) bare plastered brick wall, representing the bounding surfaces of the room as it stands. The floor and ceiling of the room are approximated to this type.

(2) fibre-glass mat, of one inch thickness and 200 Kg/m^3 density.

(3) perforated Celotex tiles of one inch thickness (10% perforation)

Two study models of the room are made up with these absorbents forming the surface coverings:-

(1) a room with all six surfaces made up of bare walls

(2) a room with uniform but different coverage for different walls made up as follows (Figure 3.1):

Y2 and X2 walls covered with fibre glass mats

Y1 and X1 walls left as bare plastered surfaces

Z1 ceiling covered with perforated celotex tiles

Z2 floor left as bare plastered surface

5.2.4) CHOICE OF REPRESENTATIVE MODES FOR STUDY:

The underlying assumption in the wave modal approach is that the response of the room to an impressed signal can be reproduced from the response from several individual modes whose number and distribution in frequency and space is a function of the room size, its dimensions and the type and location of the absorbing materials forming its bounding surfaces.

However, it is necessary, from practical considerations, to subdivide the large number of excited modes into a manageable number of sub-groups having common properties. By utilizing the concept of frequency space, described in Chapter Three, the number and types of modes having characteristic frequencies lying within any specified frequency interval, can be readily computed. The modal shapes, following the null surface structure of modal sound fields, described in Chapter Four, yield yet another method of classifying the modes with respect to their spatial distribution as axial, tangential and oblique waves. This classification is not without physical meaning, for since different classes have different energy content (equation 3.15) their decay rates will be different.

The way the modes are damped and their natural frequencies shifted as the result of application of a particular type of surface treatment in the room, gives rise to further means of classification of modes.

For the purpose of study, representative samples of those subgroups and classes will be considered for demonstration of methods of theoretical predictions and techniques of experimental verifications.

5.3.) THE SCHEMATIC MODEL:

In pursuit of a comprehensive approach to the study of sound fields in small rooms that would yield a design technique applicable in practical conditions, the following schematic model will be followed:-

(1) Predictions of the normal modes of vibration and their corresponding natural frequencies for a hypothetical rigid-walled room of the size and dimensions suggested earlier, for the frequency range 20 Hz to 600 Hz, and for modes of maximum order: $n_x, n_y, n_z \leq 10$.

The results obtained will be used as perturbation values in determination of modal parameters for rooms of practical design set-ups.

(2) Predictions of the spatial structure of modal sound fields in the three coordinate planes for representative modes of different groups with the purpose of determining the most suitable locations for excitation and elimination of modes in experimental measurements. Plots of actual modal shapes for non-rigid rooms will be obtained experimentally, and compared with the predicted shapes, the distortion that might

(2) contd.

occur, will be discussed.

(3) Computations of the specific acoustic admittance of the three different types of surface treatments (i.e. bare plastered brick wall, fibre glass mat and perforated celotex tiles.)

The specific acoustic admittance is determined for the frequency range: 100-700 Hz. For the existing bare walls of the room, exploration of the spatial distribution of modal sound fields, by the Transmission Characteristic method, yields values of the component characteristic values which can be used to determine the admittance. The admittance of fibre glass and celotex is determined from measurements, on small cut samples by the Standing Wave Tube method, of the normal absorption coefficient and the position of sound pressure minima relative to the surface of the sample.

(4) Assignment of admittance values for each type of material to each mode at its corresponding natural frequency as computed for a rigid-walled room, from the value of admittance already determined at discrete frequency intervals. These are obtained by the use of linear interpolation techniques(53).

(5) Defining the boundary conditions for each of the two room models by assigning each of the six boundary surfaces the respective value of admittance as a function of frequency, to build up the model.

(6) Computing the natural frequencies and damping constants for each mode by solving the three basic transcendental equations of Section 3.4.2, Chapter 3, expressing the component characteristic values as functions of the admittance of the respective pair of room surfaces. A group relaxation method(53) is employed in obtaining the

(6) contd.

solutions from successive iteration of the value of the natural frequency.

The measurements of these two parameters for a representative group of modes by resonance testing techniques will be performed and the predicted and measured values compared.

(7) Prediction of the complex frequency response and amplitude-frequency response of the room over a wide frequency band (250 Hz-350 Hz) for a fixed point of excitation at alternate receiver's locations with the purpose of determining the most suitable location and frequency range for the excitation and response extraction of a specific pure mode. The effect of these variables will be examined experimentally, with reference to the criteria for excitation and response extraction of pure modes, discussed in Section (4.3.7) of Chapter Four.

(8) Accurate measurements of complex-frequency response over a narrow frequency range for selected modes are carried out. The results, which are obtained in digital form are processed and analysized by the computer. Different resonance testing techniques are applied in their analysis and presentation in graphical form. The modal parameters (natural frequencies and damping constants) are then extracted from the graphical plots. The results from each technique are compared with one another and with the predicted values.

However, it is necassary to eliminate the response of the measuring instrumentation set-up from the measured response in order that the plots will represent only the room response. Such corrections are obtained from free field measurements of the response of the measurement set-up.

(9) Accurate plots of modal shapes for the two design models when the room is excited at the corresponding natural frequencies of the modes are obtained experimentally. The results will serve to help identify the modes when compared with the plots predicted for rigid wall conditions, the distortions that may be effected by the presence of non-rigid bounding conditions will become apparent.

The modal shape plots for the bare-walled room are employed in extracting the admittance values by the Transmission Characteristic method.

A mental picture of the interrelations between the theoretical and experimental work described in this schematic model can be obtained from the flow chart diagram of Figure (5.1) and Table (5.1).

5.4) DIGITAL COMPUTING AND NUMERICAL METHODS:-

5.4.1) INTRODUCTION:-

The task of manipulating the theoretical predictions and processing of experimental digitally-recorded data and its presentation in tabulated or graphical form are carried out by employing a digital computer of the University of Aston in Birmingham.

The computer[54] is a 1904S type, controlled by GEORGE 3 operating system, which provides facilities for running jobs on the background queue, the use of multiaccess systems (MOP) by accessing the computer from a console and storing information in the form of data and programs in file stores.

The language used in writing all programs is FORTRAN IV procedure-oriented language. Jobs are compiled and run using the macro instruction UAFORTRAN with its associated parameters[55].

Use is made of the computer software in obtaining graphical presentation of results by employing the GINO-F graphics package interfaced the GINOGRAPH Library of subroutines. Both have been developed at the Computer Aided Design Centre, Cambridge[55].

Experimental results, from complex response measurements, are registered, using a B and K Digital Event Recorder Type 7502. The analogue signal is sampled and converted into digital form - 8 Bit Binary Coded Words - which are clocked into the two K words memory of the event recorder. The recorded signal is then read out, in its digital form, to a digital data receiver and stored in punched paper tape. This is fed to the computer for further processing.

In the following pages, a brief account of the computer programs and subroutines, used in this thesis, is contained. A concise description of the structure and functions of each prgram or subroutine is given; detailed program listings are included in the appendices. The results of the theoretical predictions are reported and discussed with program descriptions, whereas the results of experimental measurements, analyzed and presented by the computer, are appended with the experimental work described in Chapter Six.

5.4.2) Program; ROOMODES:

5.4.2.1) DESCRIPTION:-

The function of this program is to permute room modes, as specified by their mode orders (n_x,n_y,n_z), by successively incrementing a set of integers from zero to ten (the significance of an upper limit of ten for a room of this size has been discussed earlier in this chapter).

The natural frequency, corresponding to each mode, when all the bounding surfaces of the room are assumed to be perfectly rigid, are calculated from the mode orders, room dimensions and the speed of sound in air by the application of Equation (3.12).

The modes are arranged in an ascending order of increasing natural frequencies by calls to subroutine SORT(A,B,L,NS). This is a one pass sorting routine, written by Dr.K.A.Mulholland[56].

The results are stored in a file DATA1.

A full listing of the program and file DATA1 is found in Appendix (A.1).

5.4.2.2) RESULTS AND DISCUSSION.

The mode orders (n_x,n_y,n_z) serve to label the different modes of vibration of the room. Their physical interpretation lies in the fact that they signify the number of nodal planes, along the corresponding coordinate direction, in the sound pressure spatial field when a pure mode is excited separately.

The calculated natural frequencies, as pointed out earlier, are only hypothetical, since no actual room has perfectly rigid boundaries, and their main significance is that they will be used in conjunction with relaxation techniques as iteratives in deriving solutions for the transcendental equations expressing the modal parameters as functions of the boundary conditions in real rooms.

5.4.3) Program: SPACEMODE:

5.4.3.1) DESCRIPTION:

The program calculates the spatial distribution of sound pressure in the room as a function of location of the receiver when a pure mode of vibration is excited, the sound source being located at an antinodal point.

For a room with a rigid or fairly hard boundaries, the pressure at the point (x,y,z) is described by the characteristic functions, given by Equation (3.21).

Subroutine CONTOUR of the graphic package SCONTOUR, devised by Dr.K.A.Mulholland(56), is employed in drawing two dimensional contour maps of the sound pressure on x-y, x-z and y-z planes in the room, taken as cross-sections on antinodal points in the normal direction. Contours are drawn joining points of equal pressure, as calculated by SPACEMODE.

A full listing of SPACEMODE is included in Appendix (A.2).

5.4.3.2) RESULTS AND DISCUSSION:

Examples of the results obtained for an oblique mode (Mode:324), a tangential mode (Mode:240) and an axial mode (Mode:300) are shown in Figures (5.2A),(5.2B) and (5.2C).

From the plots, the modal shapes corresponding to the nodal surface structure of a pure mode can be readily identified. Experimental plots for real rooms will be made and compared with these to assess the degree of distortion in modal shapes as the result of the introduction of some absorption in the surfaces of the room.

However, graphical plots of the spatial pressure distribution of a mode, obtained in this way, for the rigid walled room, can furnish first approximation information as to the most suitable locations for excitation and response reception of the mode for experimental purposes. For degenerate modes or modes closely space in frequency, this principle of space separation can allow each mode to be separately excited and studied.

5.4.4) Programs: ADMITTANCE1, ADMITTANCE:

5.4.4.1) DESCRIPTION:

The two programs evaluate the specific normal impedance and admittance as functions of frequency for three types of sound absorbents. These are:

(1) Plastered brick wall

(2) Fibre-glass mat

(3) Perforated celotex tiles

Both parameters are determined for plastered brick surfaces from measurements on existing walls by the Transmission-Characteristic method, discussed in Section (4.2.3.3). The measured quantities are the sound pressure minima and its distance from the surface of the wall. By applying the spatial distribution analysis, the component characteristic values and consequently the impedance and admittance are determined.

For fibre-glass and celotex, measurements are carried on small-cut samples by the Standing Wave Tube method discussed in Section (4.2.3.2). From measurements of the normal absorption coefficient and the location of the sound pressure minima both the modulus and phase angle of the complex pressure reflection factor are evaluated. From the complex reflection factor the specific acoustic impedance and admittance are computed.

Program ADMITTANCE1 determines the values of impedance and admittance of the bare wall frequencies corresponding to the natural frequencies of a selected number of room modes. The values are then linearly interpolated at approximately equal frequency intervals and stored in file ADMITBARE.

Program ADMITTANCE evaluates the values of impedance and admittance for fibre-glass and celotex, and reads in the corresponding values for the barewall form ADMITBARE. All results are stored in file ADMITTANCE for intercommunications with other programs. Graphical forms of the results are obtained by GINO-F interfaced with GINOGRAF routines.

A full listing of the two programs are included in Appendix (A.3).

5.4.4.2) RESULTS AND DISCUSSION:

Table (5.2) gives the admittance of the three samples as a function of frequency (for the range 100 Hz to 700 Hz).

Graphical plots of the impedance and admittance as functions of frequency for the three samples are shown in Fig.(5.3A), Fig.(5.3B) and Fig.(5.3C).

The results obtained for small-cut samples of fibre glass and celotex are in good agreement with previous measurements performed by analogous methods[10,32]. The impedance has a small, positive resistive part which is nearly independent of frequency, and a relatively larger, negative reactive part which increases linearly with frequency. Both the conductance and susceptance of the admittance, for both samples, are positive and increase with frequency.

For the plastered wall, the impedance has large resistance and reactance terms in the low frequency range. However succëssive pits and rises on the impedance curve could be attributed to wall resonances at its natural frequencies, for in this case the assumption, that the acoustical behaviour of the material is not dependent on the size of the sample, is hardly plausible.

The conductive part of the admittance is small and fairly constant with frequency, while the susceptance term exhibits sharp fluctuations with frequency in magnitude and sign.

5.4.5) Program: TRANSCEQNS:

5.4.5.1) DESCRIPTION:

The object of this program is to compute solutions corresponding to room modes, for the set of three transcendental equations of the form:

$$jk_N L_N + \coth^{-1}\left(\frac{k_N}{k\beta_{NM}}\right) + \coth^{-1}\left(\frac{k_N}{k.\beta_{NM}}\right) = 0 \qquad (3.29)$$

where the subscripts:

(N = x,y,z) (M = 1,2)

These basic equations,-discussed in Chapter Three - express the component characteristic values (k_x,k_y,k_z) in terms of the specific acoustic admittance ($\beta 1$ and $\beta 2$) of the corresponding pair of opposite walls and the frequency (contained in K).

The solutions are obtained in the form of power series of the admittance and frequency, as discussed in Section (3.3.2). Alternative formulae are presented, and the validity of each is constrained by certain convergence limits which depend on the mode order, the room dimension and frequency and admittance of each of the pair of opposite wqlls.

The program reads in the modes, as identified by their orders, from File DATA1 and assign each mode corresponding values of admittance of the different types of absorbents likely to be used for surface treatment. These admittance values are read in from file ADMITTANCE at discrete frequencies and then linearly-interpolated at the natural frequency (for the rigid walled room) of each mode.

The study models of the room - described in Section (5.2.3) are then constructed of various arrangements of different types of absorbents for uniform coverage of individual surfaces,each being designated by the value of its specific admittance.

Solutions are computed, using the formula that satisfies the convergence limits, for the component characteristic values separately which are then inserted in Equation (3.38) and Equation (3.39) to yield the frequency and damping constant of the mode.

A group relaxation technique is followed, in which successive iteration of the computed characteristic value ($K_n = \omega_n + j\delta_n$) is made. Each time the computed value is inserted back into the original solution in place of (k), the wave number, until a self-consistent solution, which will result in the same value for (k_n) is obtained. The real part of this value yields the natural frequency of the mode and the imaginary part its damping constant[22,23].

A computer function CACOTH(A), which follows the main segment of the program, can be employed to determine the component phase angles (ϕ_x, ϕ_y, ϕ_z) which are used as corrections in the arguments of the hyperbolic functions of Equation (3.23) whose product gives the characteristic functions of the mode.

If an exact account of the spatial distribution of sound in the room is required, then these values of the characteristic functions should be used, rather than the approximate functions of Equation (3.21). A full listing of the program is included in Appendix (A.4), together with an example of the application of the relaxation method.

5.4.5.2) RESULTS AND DISCUSSION:

The computed values of the natural frequency and damping constant for the study model of a bare walled room are stored in File DATA2, while those for a treated walled room model are in file DATA3. Table (5.3) contains samples of the computed results.

The calculated values, for both the natural frequency and damping constant, tend to fall in line with the expected pattern.

In the majority of cases, the shifts in the natural frequencies are towards lower values than those for rigid wall conditions, as may be caused by the introduction of materials with impedance having a negative imaginary term.

For the small range of frequency over which the imaginary part of the impedance is positive, the shifts are towards higher values of frequencies than of the rigid wall case.

Different ranges of damping can be associated with different classes of modes. In all cases, modes travelling at grazing incidence to the most absorbing surfaces (those with large real terms of admittance) have the least damping constants, while modes which travel at oblique incidence to those absorbing surfaces are the most highly damped.

The variation in damping rates - apart from the effect of the variation of impedance with frequency, is due to the fact that the mean energy contained in different types of modes is different. The energy in an axial mode is twice that in a tangential mode and four times the energy in an oblique one. This is because the mean square of $\left(\cos\frac{\pi n_x x}{L_x}\right)$ is one when $(n = 0)$, but is $(\frac{1}{2})$ when $(n > 0)$. Since the rate of withdrawal of energy at a particular wall is the same in all cases, the damping constant, defining this rate of withdrawal, is different for different classes of modes[25].

5.4.6) Programs: MODESEPARATN, MODESPLOT:-

5.4.6.1) DESCRIPTION:-

The effect of location of the sound source and receiver in the room on the excitation and response extraction of a pure mode in preference to other neighbouring, closely-spaced (in frequency) modes, is investigated.

The procedure, for such investigation, is outlined here by considering two alternative locations for the receiver while the source is kept at a fixed point. The frequency of the source is swept over the range 250 Hz to 350 Hz, but the effects of all room modes up to 600 Hz upper limit (see Section 5.2.1) are included.

Two alternative configurations of boundary conditions, formulated as the room study models, (discussed in Section 5.2.3), are considered.

Program MODESEPARATN reads in the modes, the natural frequencies and damping constants for a particular set-up of boundary conditions from the files series DATA2, DATA3.

The total complex pressure response of the room is calculated by the Green's function formula (Equation 3.56), as a function of the driving frequency, for the two different locations of the receiver, the source assumed at an antinodal point of a specific mode.

The complex response of that mode, assumed to be separately excited, is also calculated.

Similarly the amplitude response, expressed as sound pressure levels, are calculated for the summation of the series and for a pure mode only.

The computed values of the inphase and quadrature components of the complex response together with the sound pressure levels, as functions of the driving frequency, are stored in the files series RESULTS2, RESULTS3, for intercommunication with program MODESPLOT.

This is a graph plotting program built up of GINO-F and GINOGRAF routines library.

The total complex response is presented as a vector on the Argand plane with its real and imaginary parts as the X and Y coordinates.

The response of the pure mode is likewise, presented on the same coordinate system.

Similarly, the sound pressure levels are plotted against the driving frequency, for the total amplitude response and the pure mode response.

Program listings of MODESEPARN and MODESPLOT are included in Appendix (A.5).

5.4.6.2) RESULTS AND DISCUSSION:

In Figures (5.4A) and (5.4B) the complex and amplitude pressure responses of a pure axial mode-mode : 004 - are plotted along with the total responses of the room for the bare-walled condition. The only difference is in the location of the receiver, one being at the room corner (x = 0 ,y = 0 , z = 0) while the other is at the point ($x = \frac{1}{4}L_x$, $y = \frac{1}{4}L_y$, $z = \frac{1}{4}L_z$). In both cases, the sound source is positioned at the centre of the room ($x = \frac{1}{2}L_x$, $y = \frac{1}{2}L_y$, $z = \frac{1}{2}L_z$).

Figures (5.4B) and (5.4D) are similar plots for the treated walled condition.

For the first position of the receiver, the total response, in its complex of amplitude form, is the summation of contributions from a large number of modes and it would be erroneous to associate the response at a certain frequency with a particular mode.

For the second position of the receiver, where contributions from a few number of modes are picked, the response plots are much simpler and the total response can readily be attributed to the particular mode under consideration. Only then in the latter case can the different resonance testing techniques be applied, without grave errors, for the determination of modal parameters, since the off-resonant contribution is much smaller in comparison with the resonant vibration in the principal mode.

Experimental measured plots will be obtained for the cases considered theoretically. The experimental and predicted plots will be compared in order to establish the ranges of validity of the theories of modes separation discussed in Chapter Four.

5.4.7) Program: MODERESPONSE:

5.4.7.1) DESCRIPTION:-

The function of this program and the incorporated subroutines is the analysis and presentation, in graphical form of the experimental results derived from complex response measurements.

The measured in-phase and quadrature components of the signal are obtained as sampled digital data in the Binary Two's Complement Code, and input into the computer's system, using subroutine BUDMACRO[57] in the file series MODESN.

Subroutine BUD[57] reads in the digital data, from file MODESN, and returns the analogue values of the signals representing the measured response of the room.

Since the inphase and quadrature components of the signal are ordered sequentially in the recorded data, the program separates them and evaluates the magnitude and phase angle of the signal. All these quantities are evaluated as functions of the driving frequency which is assumed to be a linear function of time.

As the measurement set-up imposes its own response characteristics on the room response, it will be necessary to extract the room response from the total measured response. This is achieved by employing subroutine CORRECT which reads in correction values for the measurement set-up (see Program: FREEFIELD) and eliminates them from the total response.

Graphical plots of the complex receptance i.e. response per unit amplitude of the driving signal, as a function of frequency, are obtained by using the GINO-F and GINOGRAF routines.

The rate of change of the spacing of equal frequency increments, on the complex plane, with frequency (outlined in Section 4.3.6 of Chapter Four) is calculated and averaged to give a smooth varying function with frequency. From a plot of the spacing versus frequency - using Program AMPHASEPLOT (to be described later) - the position of the natural frequency corresponding to a resonating mode can be determined. On feeding back this information in Program MODERESPONSE, a best-fit circle is constructed on the vector response plot. By applying the Kennedy and Pancu Method[42] in the analysis of the plot, the damping constant and resonant amplitude of the mode can be accurately determined.

A full listing of the program is in Appendix (B.1).

5.4.7.2) <u>RESULTS</u>:-

The vector plots, representing the response of the modes under investigation, will be presented and analyzed with the experimental work on complex response measurement to be discussed in Chapter Six.

The values of the resolved components, amplitude and phase angle of the signals representing the room response are stored in file CVALUES. Those of the spacings of equal frequency increments are in the CSPACING.

5.4.8) Program: FREEFIELD:-

5.4.8.1) DESCRIPTION:-

The twofold object of this program is:

(1) to assess the linearity of the pressure frequency response of the set-up employed in complex response measurements in modal sound fields.

(2) To establish correction data for the set-up in terms of calculated and measured free field responses.

In the experimental work - to be discussed in Chapter Six - two types of sound sources, a low frequency box loudspeaker and a high frequency pressure unit, are used with the measurement set-up for room excitation. This is in order to cover a wide range of frequency while still maintaining linear frequency response characteristics of the source and a small overall size compared to the wavelength of the emitted signal such that the sound field in the room is not distorted by its presence.

A source coefficient, as a function of frequency, is determined from measured and calculated values of the set-up response - with either of the two sources - according to the procedure discussed in Section (4.3.7.6) and defined by Equation (3.66).

A correction factor, equal to the product of the source coefficient and the source strength, is determined.

The measured results of the free field experiments are read in the computer as binary data and stored in files ANECHBLS and ANECHPUS for the box loudspeaker and pressure unit, respectively. Subroutine BUD[57] is used to return their analogue values.

A full listing of program FREEFIELD is found in Appendix (B.2).

5.4.8.2) RESULTS AND DISCUSSION:

The linearity of the pressure frequency response of either of the two sources can be determined from the variation of the correction factor with frequency. While the response for the former is a smooth varying function of frequency, the latter shows an inconsistent response in the low frequency range. In view of this, a minimum frequency limit of 200 Hz is set for the use of the pressure unit as a sound source. This is also an upper frequency limit for the use of the box loudspeaker above which the effect of its size compared to the wavelength, in distorting the sound field, cannot be ignored.

The results of the correction factor for the box loudspeaker are stored in file CORRBLS. Those for the pressure unit are in file CORRPUS. These files are read in, using subroutine CORRECT, in Program MODERESPONSE, as discussed before. The results, in both files, are demonstrated with experimental work.

5.4.9) Program: AMPHASEPLOT:-

5.4.9.1) DESCRIPTION:-

This program comprises a group of interfaced GINO-F and GINOGRAF routines for graphical presentation of the results derived from modal complex response measurements as analyzed and processed by program MODERESPONSE.

The spacings of equal frequency increments are read in from file CSPACING and plotted against the driving frequency in the form of a histogram.

The resolved components of the signal representing the complex response, its amplitude and phase angle are read in from file CVALUES and plotted, separately, against the frequency.

5.4.9.2) RESULTS AND DISCUSSION:-

The determination of modal parameters (i.e. the natural frequency, damping constants and modal shapes) is achieved by application of the various resonance testing techniques - discussed in Chapter Four - on the corresponding plots.

The Kennedy and Pancu Technique is based on identification of the natural frequency from the point of maximum spacing on the equal frequency increments plot. Alternatively, the point of maximum (or minimum) in-phase component can indicate the position of the natural frequency. Once the natural frequency is identified, the vector plot - discussed in Section (5.4.7) - can be used for determining the damping constant and resonance response.

According to the peak amplitude method, the natural frequency is defined as the value of frequency at which the peak is attained. The damping is then calculated from the shaprness of the peak by the half-power width measurement on the amplitude frequency plot.

From the phase angle plot, the natural frequency can be identified as the frequency at which the response is in-phase (or antiphase) with the driving signal. The damping can be obtained from the rate of change of phase angle with frequency according to the methods discussed in Section (4.3.5.4) of Chapter Four.

The results of those modal parameters, extracted by the various methods discussed above, will be reported and analyzed with the experimental work in Chapter Six.

5.5) CONCLUSION:-

In this chapter, an implementation of the theoretical analysis, presented in Chapters Three and Four, in the acoustical design of small rooms, is sought.

Defined modal parameters are evaluated in terms of the geometry of the room and the properties of different configurations of its boundary conditions as described by a single index of their absorbing qualities - the acoustic impedance.

A schematic model for a design approach is suggested.

Mathematical simulation techniques and numerical analysis methods are employed in manipulation of theoretical predictions and analysis and presentation of experimental results.

An account of the computer programs incorporated in the work is given, with the results reported and discussed.

In the following chapter the schematic model will be followed in verifying these results experimentally.

PROGRAM	SUBROUTINE	DATA FILES READ-IN	DATA FILES WRITE-TO	GRAPHICAL OUTPUT
ROOMODES	SORT	-	DATA1	
SPACEMODE	SCONTOUR	-	-	SPATIAL VARIATIONS PLOTS
ADMITTANCE1	-	-	ADMITBARE	
ADMITTANCE	-	ADMITBARE	ADMITNCE	IMPEDANCE & ADMITTANCE PLOTS
TRANSCEQNS	CACOTH	DATA1,	DATA2,	-
		ADMITNCE	DATA3	
MODESEPERTN	-	DATA2,	RESULTS2	
		DATA3	RESULTS3	
MODESPLOT	-	RESULTS2,	-	COMPLEX & AMPLITURE-
		RESULTS3	-	FREQUENCY RESPONSE PLOTS
MODERESPONSE	BUD,	MODESN, CORRBLS,	CVALUES,	VECTOR RESPONSE PLOTS
	CORRECT	CORRPUS	CSPACING	
AMPHASEPLOT	-	CVALUES,	-	AMPLITUDE,PHASE & FREQ.
		CSPACING	-	SPACING PLOTS
FREEFIELD	BUD	ANECHBLS,	CORRBLS,	-
		ANECHPUS	CORRPUS	

TABLE (5.1) A Directory of:
Computer programs, subroutines, data files
and graphical output.

TABLE (5.2) DERIVED SPECIFIC ACOUSTIC ADMITTANCE OF VARIOUS TYPES OF SOUND ABSORBENTS, FROM IMPEDANCE MEASUREMENTS

Frequency (HZ)	Bare Walls		Fibre-Glass		Perforated Celotex	
	Real	Imaginary	Real	Imaginary	Real	Imaginary
100.0	0.025	-0.020	0.007	0.060	0.018	0.039
110.0	0.034	-0.024	0.007	0.070	0.018	0.035
120.0	0.035	-0.013	0.007	0.072	0.018	0.047
130.0	0.035	-0.006	0.008	0.073	0.019	0.044
145.0	0.036	-0.002	0.010	0.084	0.024	0.051
160.0	0.037	0.003	0.010	0.041	0.026	0.053
175.0	0.031	0.021	0.011	0.099	0.026	0.055
190.0	0.016	0.041	0.010	0.109	0.026	0.067
210.0	0.013	0.022	0.013	0.121	0.032	0.067
230.0	0.012	0.002	0.013	0.134	0.036	0.071
250.0	0.028	-0.021	0.015	0.143	0.041	0.078
275.0	0.015	0.007	0.020	0.150	0.049	0.100
300.0	0.041	0.020	0.016	0.191	0.056	0.131
330.0	0.039	-0.007	0.044	0.215	0.076	0.105
365.0	0.036	-0.038	0.039	0.204	0.079	0.117
400.0	0.051	0.020	0.040	0.228	0.081	0.120
435.0	0.060	0.011	0.050	0.255	0.090	0.107
480.0	0.023	-0.011	0.067	0.287	0.109	0.130
525.0	0.056	-0.047	0.072	0.296	0.122	0.147
576.0	0.024	0.031	0.081	0.314	0.140	0.140
630.0	0.024	0.106	0.101	0.363	0.162	0.108
690.0	0.024	0.106	0.114	0.380	0.183	0.087

TABLE (5.3) PREDICTED NATURAL FREQUENCIES AND DAMPING CONSTANTS FOR A BARE-WALLED ROOM AND A TREATED-WALLED ROOM

Mode	Rigid-Walled Room	Bare-Walled Room		Treated Walled Room	
	Natural Frequency (HZ)	Natural Frequency (HZ)	Damping Constant	Natural Frequency (HZ)	Damping Constant
100	52.4	53.9	12.8	51.0	8.4
001	75.1	76.5	13.9	73.5	9.8
101	91.5	93.4	16.4	89.8	11.7
200	104.7	106.5	15.0	103.4	9.3
210	124.3	125.3	21.9	121.5	13.9
300	157.1	157.2	18.4	155.3	12.7
220	169.9	168.3	20.8	165.4	14.4
202	183.0	179.6	14.2	177.2	12.7
400	209.5	208.1	6.1	205.3	6.7
302	217.3	215.8	8.0	211.9	10.9
222	226.7	225.8	9.6	219.4	13.1
420	248.5	250.5	16.8	242.9	15.8
322	255.1	256.8	19.4	248.4	19.8
232	271.5	270.8	13.2	262.9	18.4
240	287.2	285.4	17.1	278.7	14.6
332	295.7	293.2	28.1	284.3	26.1
004	300.2	297.0	20.7	297.1	28.2
620	341.5	343.6	23.2	332.6	32.4
324	364.3	368.3	27.8	355.4	35.2
252	381.1	381.8	33.2	370.1	38.4
524	420.2	418.0	43.8	406.8	45.8
462	476.8	477.3	19.8	462.8	42.2
226	481.3	481.8	18.4	507.2	45.9
662	531.2	535.4	39.7	518.3	56.8
654	548.3	549.1	31.9	533.4	53.3
446	564.1	561.4	24.2	547.0	51.2
480	574.4	570.5	14.6	560.5	34.5
664	591.4	584.2	18.6	571.9	51.7

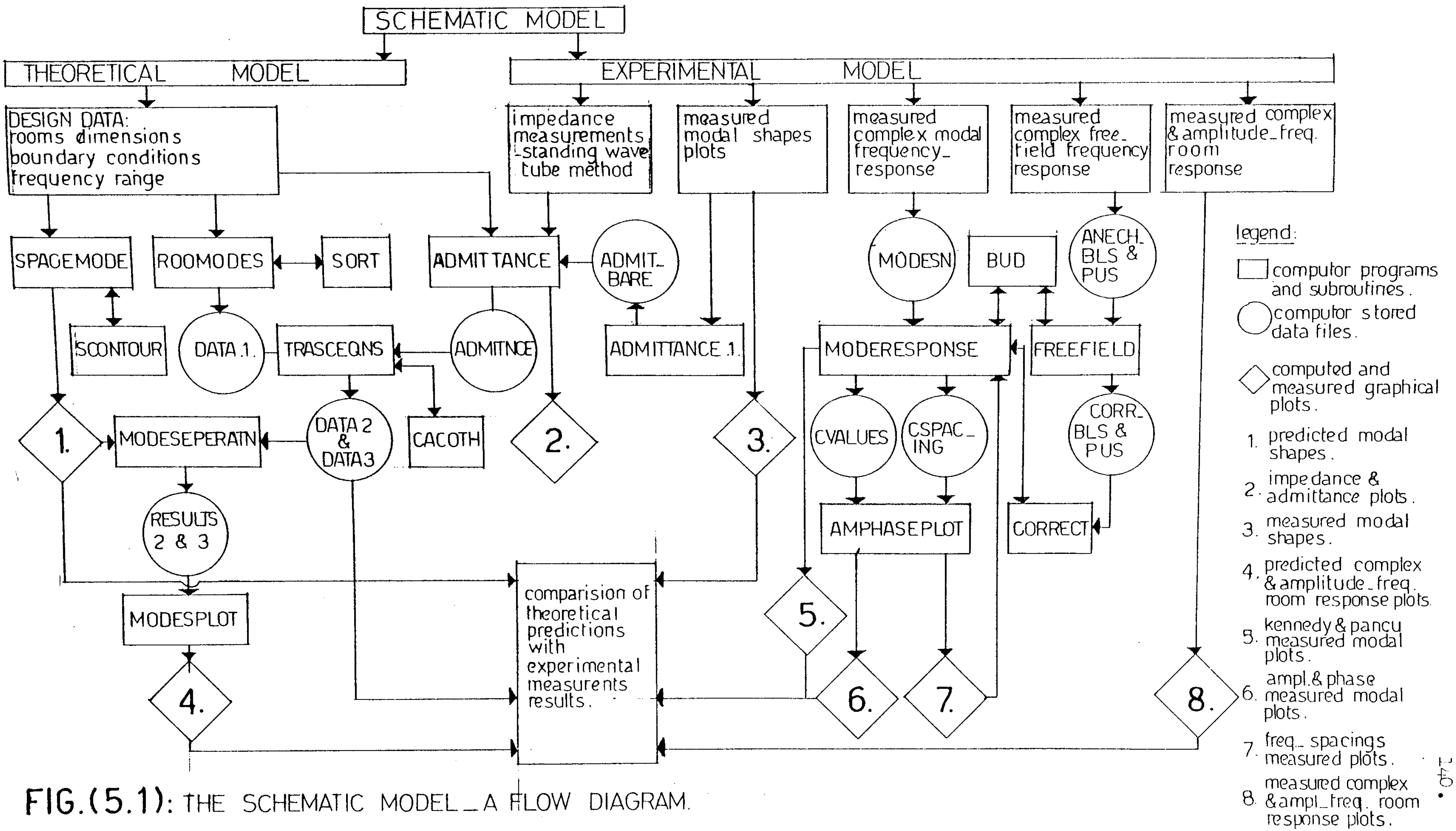


FIG.(5.1): THE SCHEMATIC MODEL_A FLOW DIAGRAM.

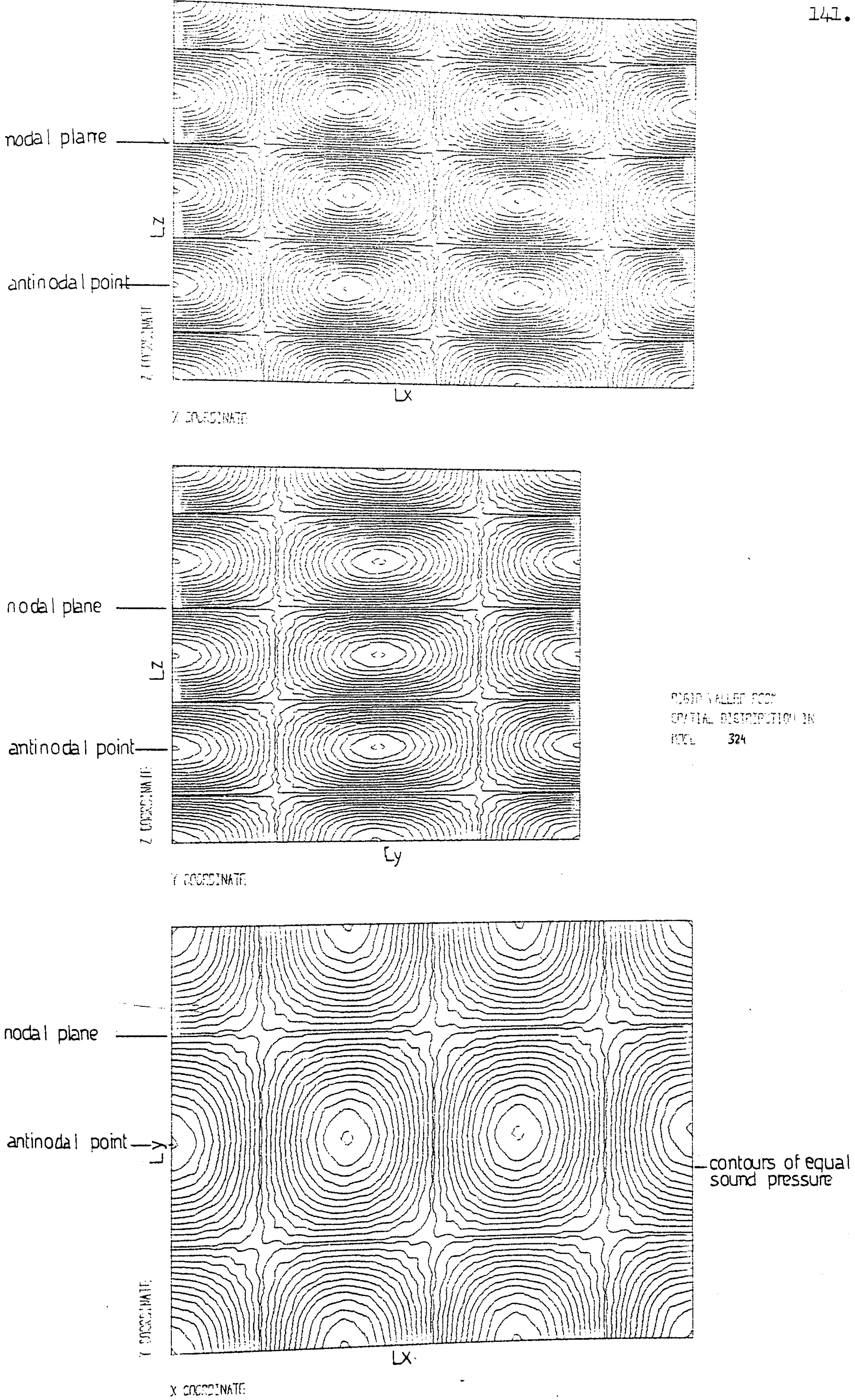


FIG.(5.2A): PREDICTED SPATIAL DISTRIBUTION IN AN OBLIQUE MODE

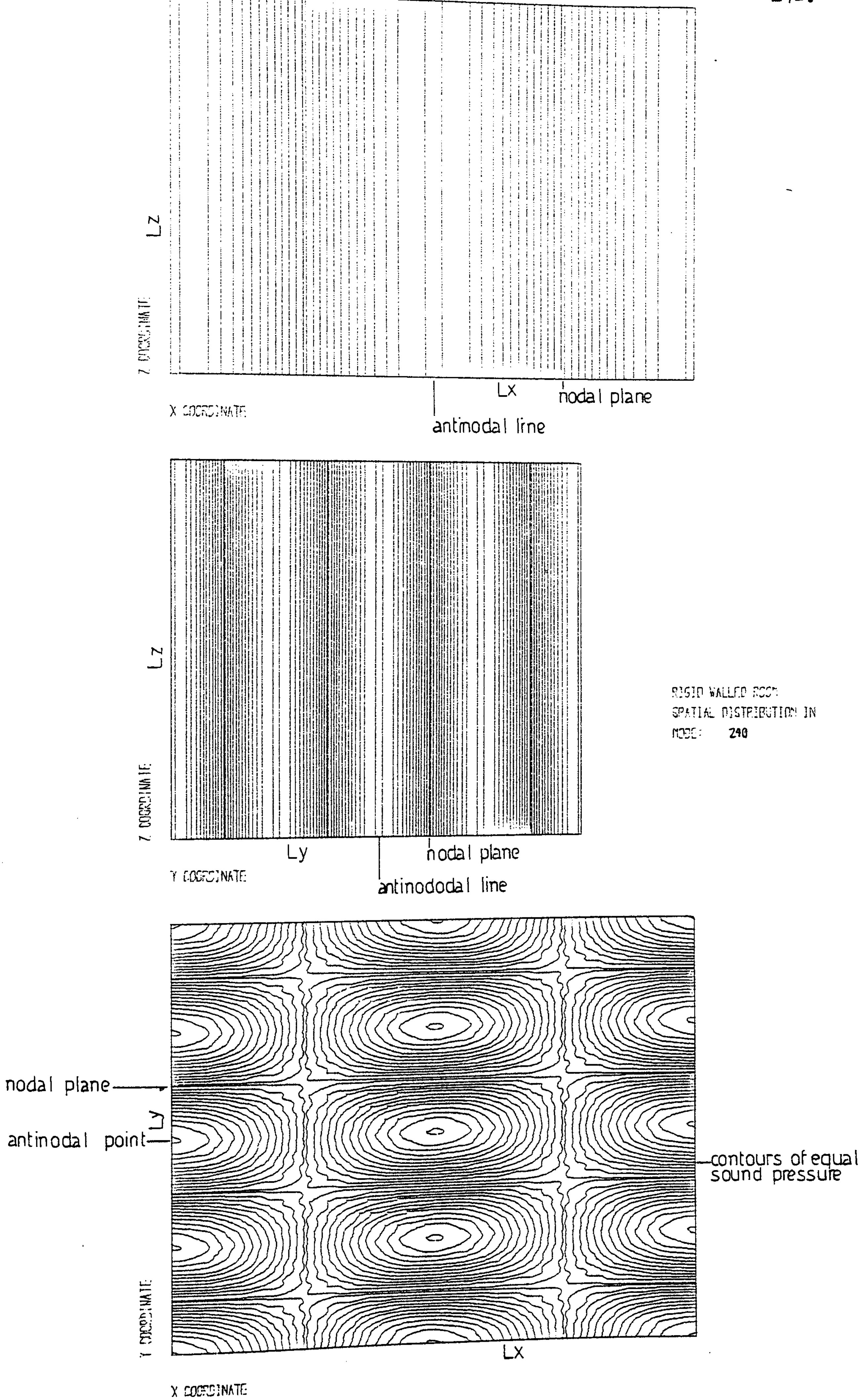


FIG.(5.2B): PREDICTED SPATIAL DISTRIBUTION IN A TANGENTIAL MODE

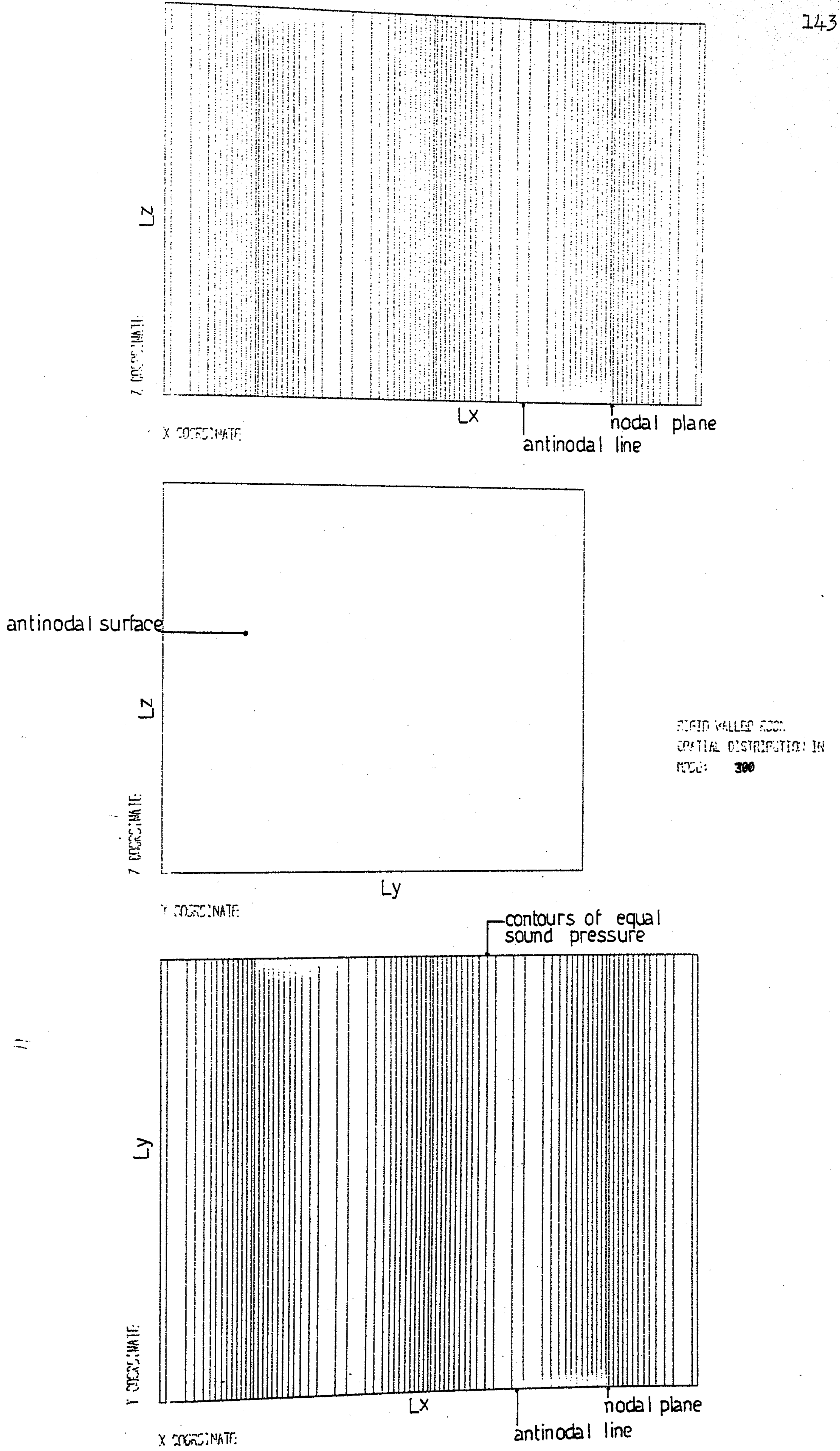


FIG.(5.2C):PREDICTED SPATIAL DISTRIBUTION IN AN AXIAL MODE

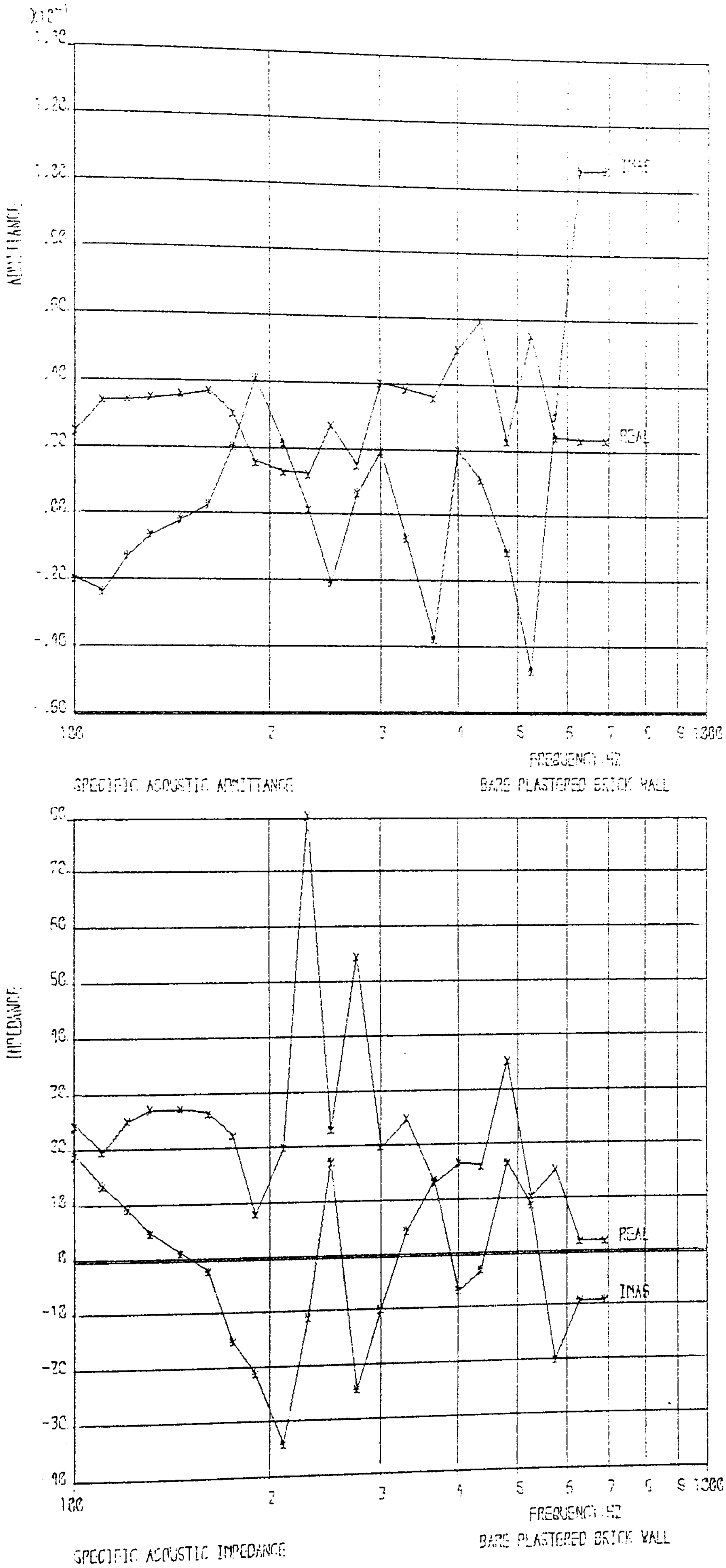


FIG.(5.3A): VARIATION OF IMPEDANCE AND ADMITTANCE WITH FREQUENCY FOR A BARE WALL

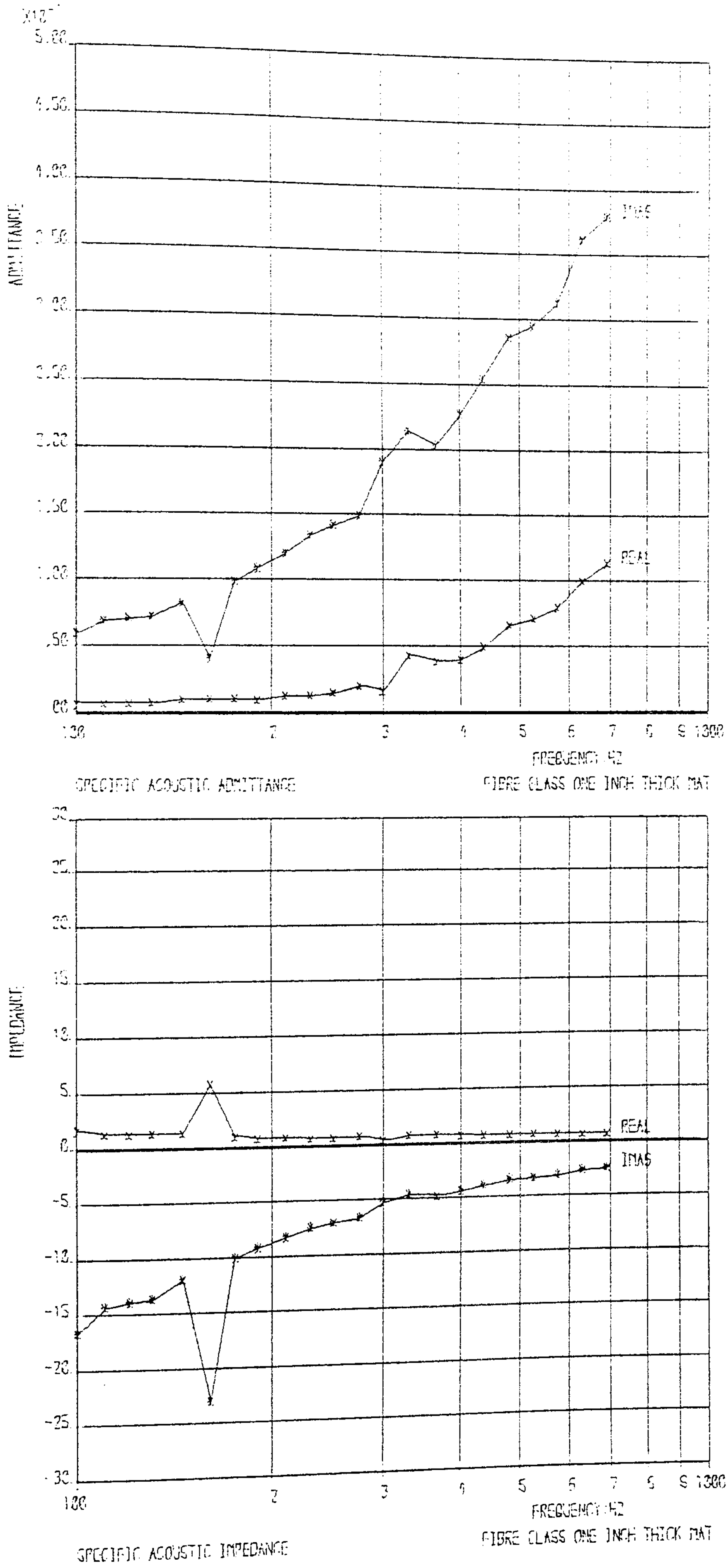


FIG.(5.3B): VARIATION OF IMPEDANCE AND ADMITTANCE WITH FREQUENCY FOR FIBRE_GLASS SAMPLE

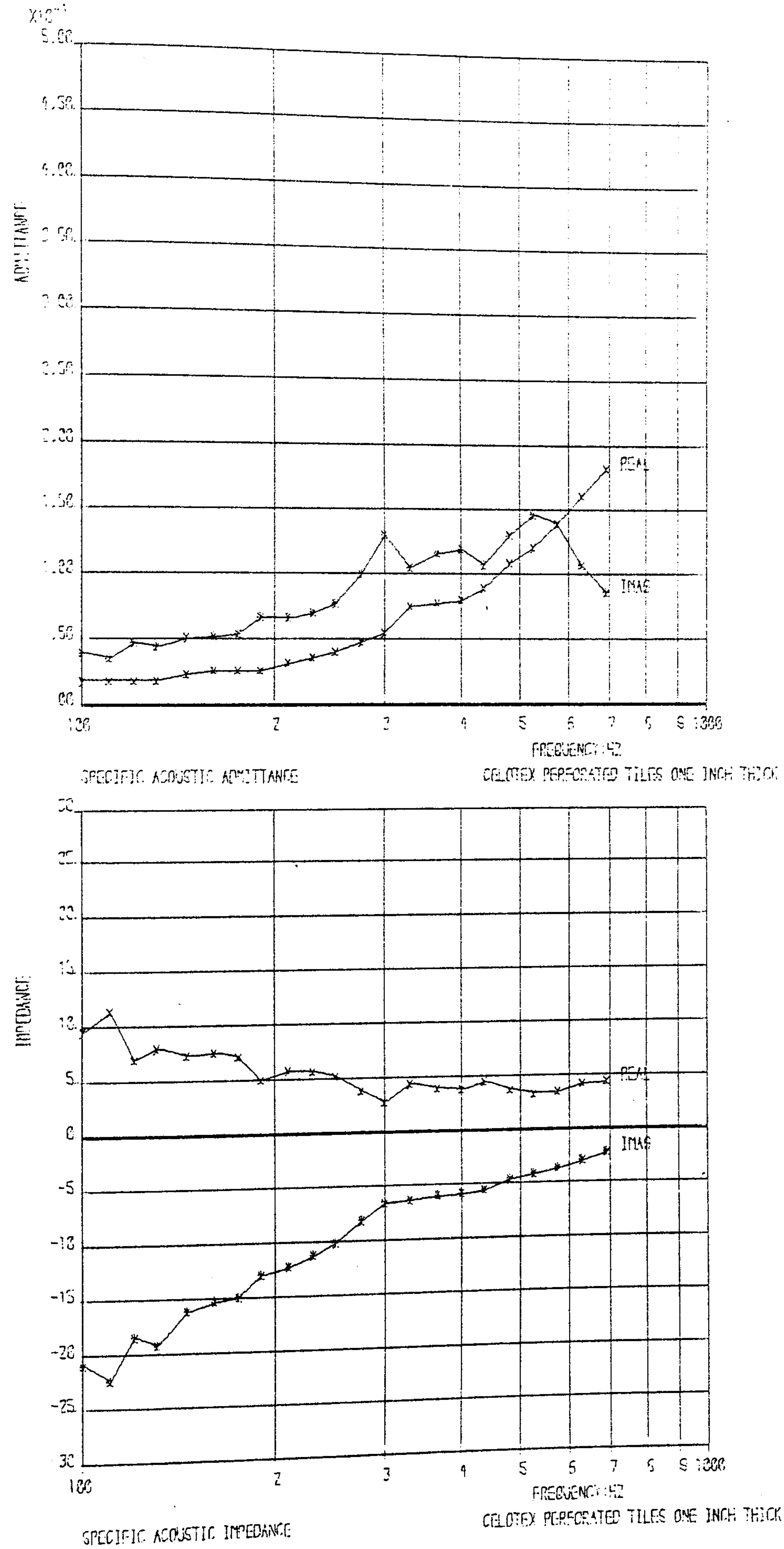


FIG.(5.3C): VARIATION OF IMPEDANCE AND ADMITTANCE WITH FREQUENCY FOR PERFORATED CELOTEX SAMPLE

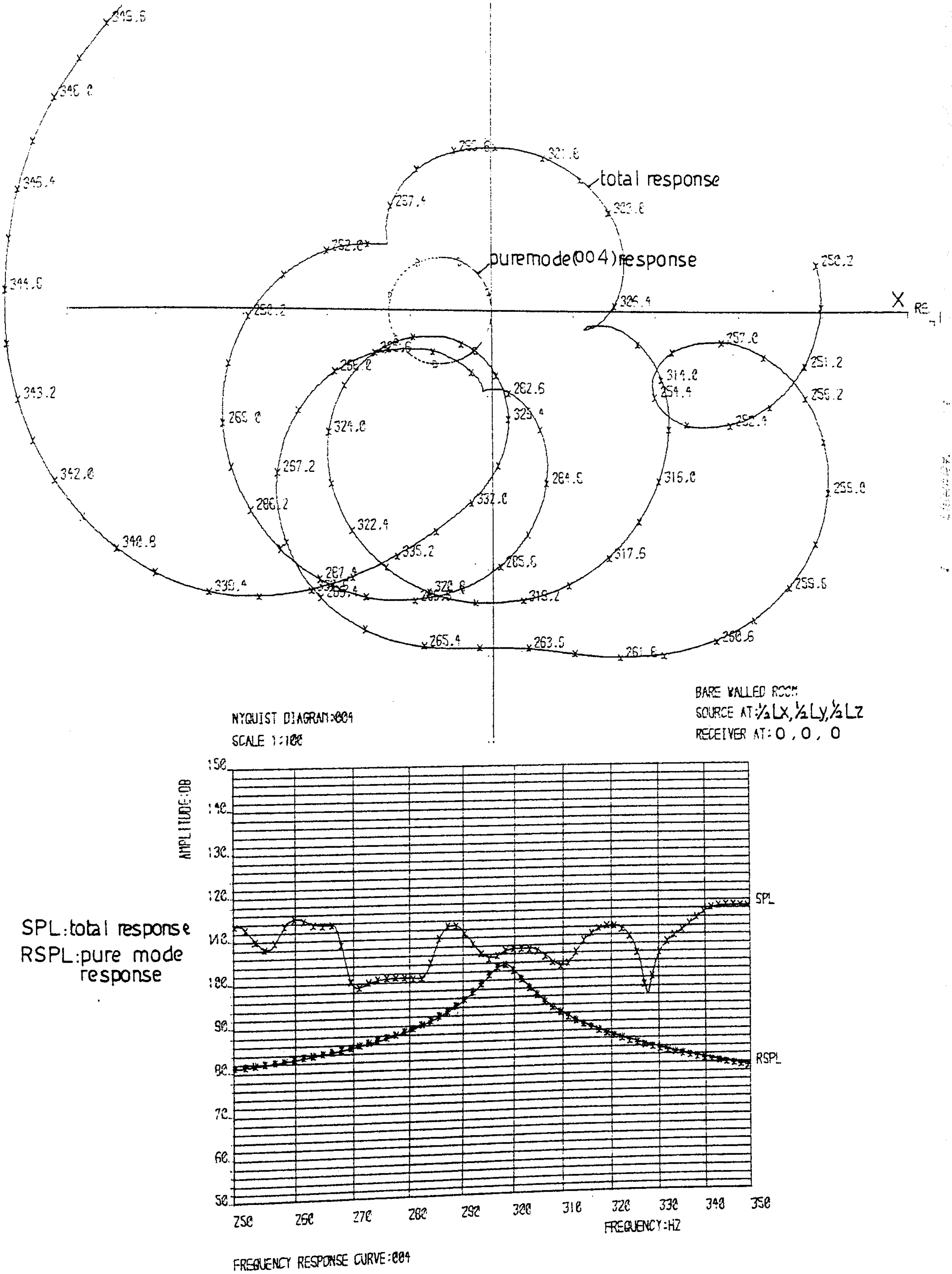


FIG.(5.4A): PREDICTED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A BARE WALLED ROOM_RECEIVER LOCATED AT A ROOM CORNER.

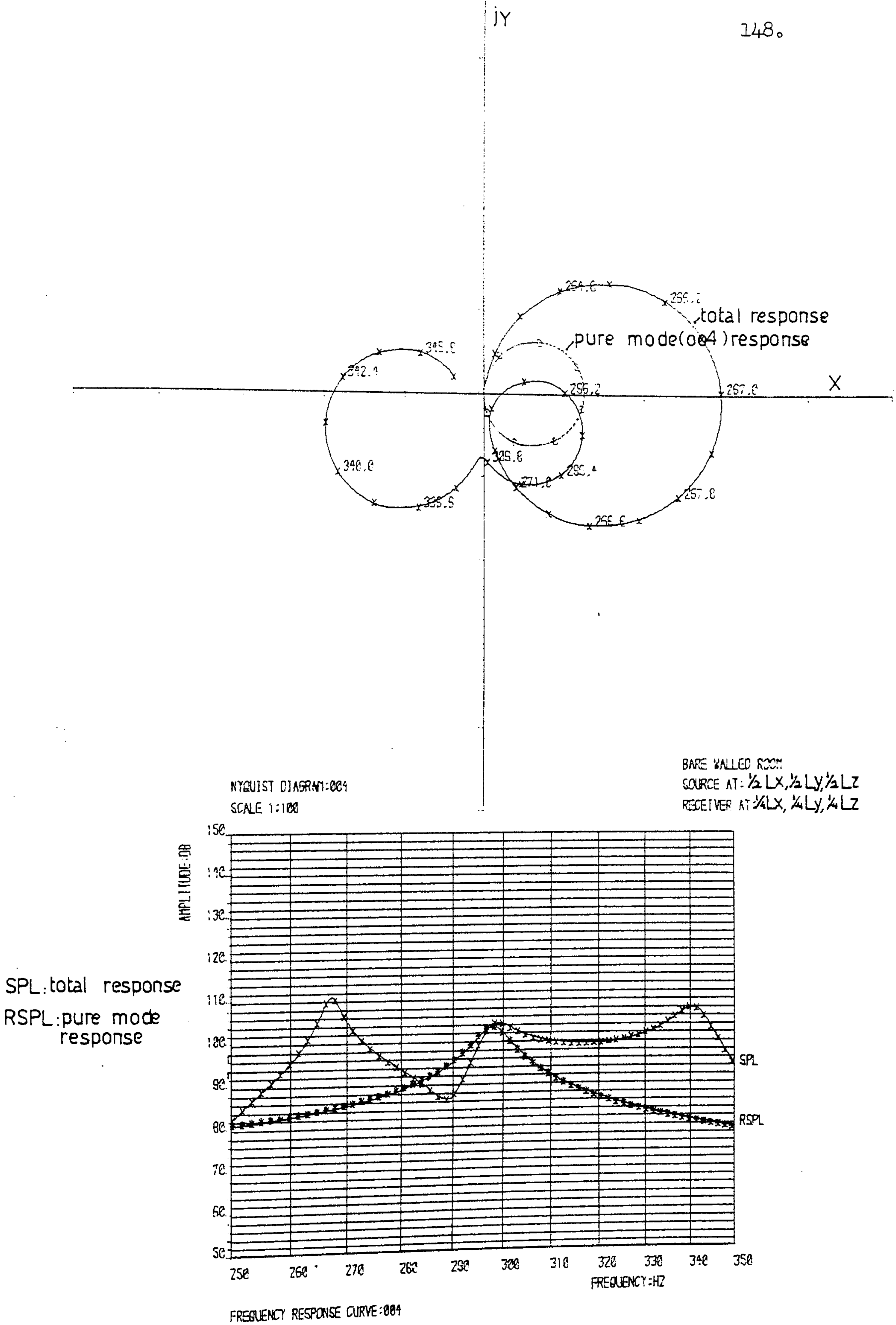


FIG. (5.4B): PREDICTED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A BARE WALLED ROOM — RECEIVER LOCATED AT ANTINODES FOR MODE(004)

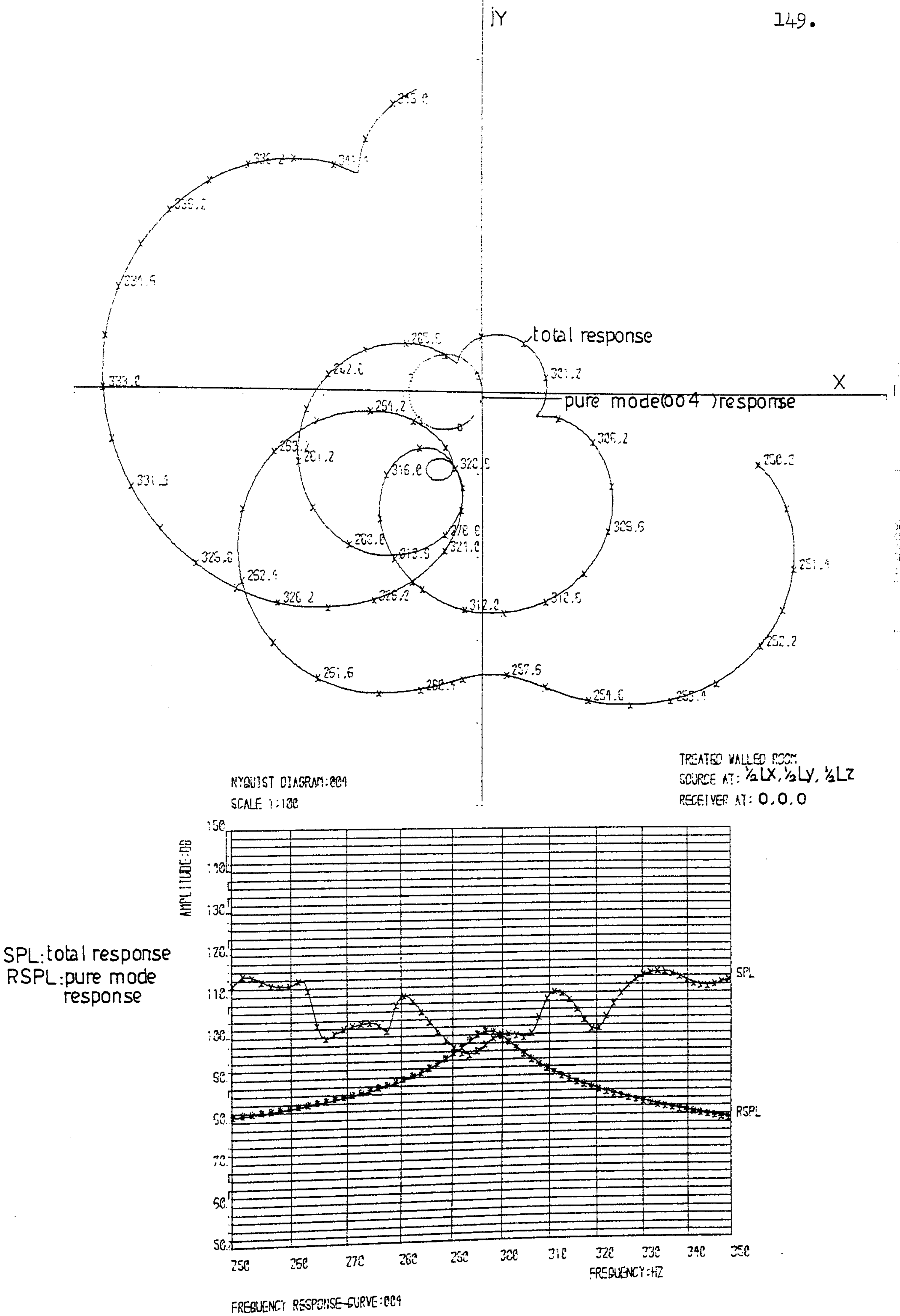
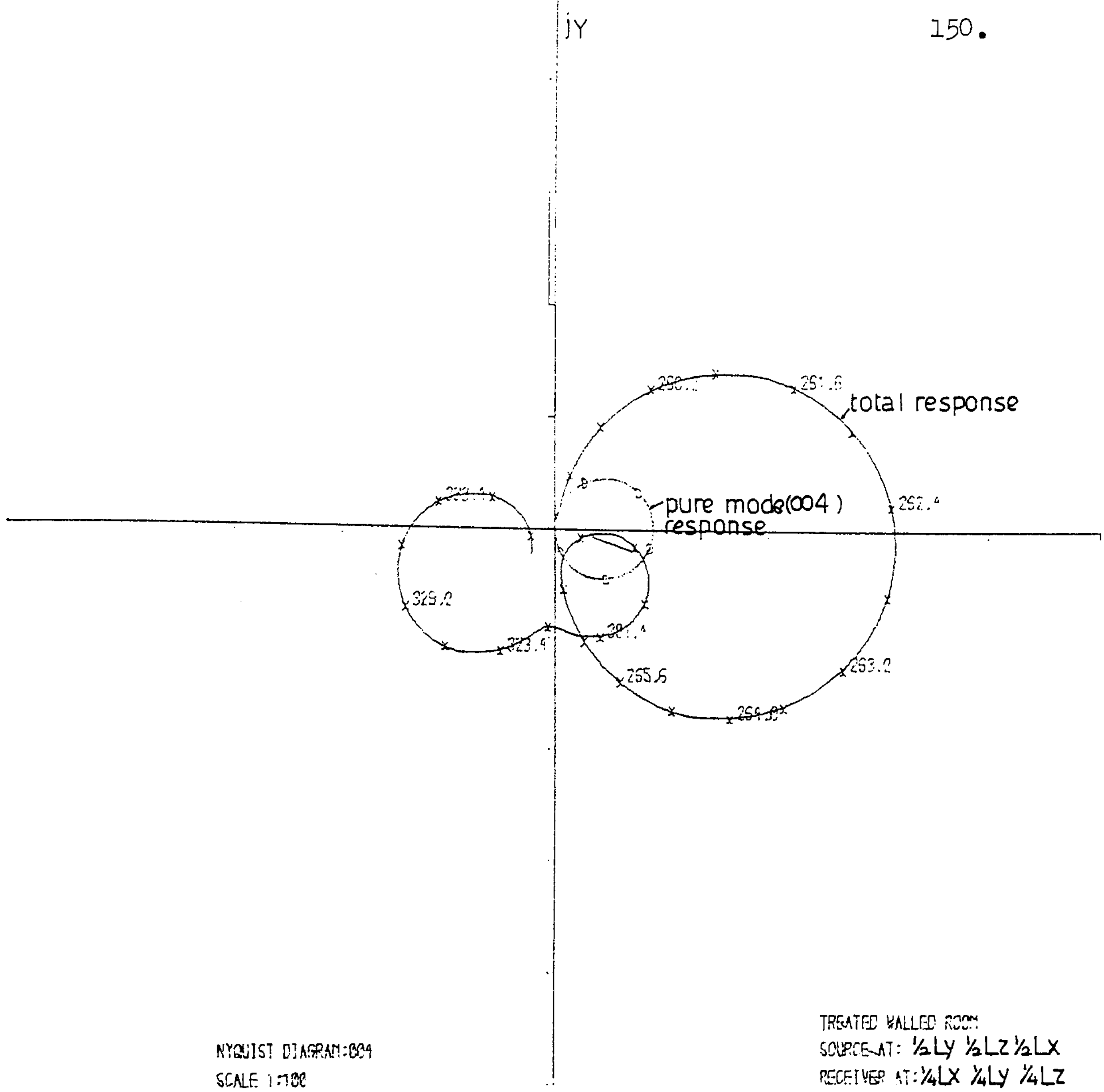


FIG. (5.4C): PREDICTED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A TREATED WALLED ROOM___RECEIVER LOCATED AT A ROOM CORNER



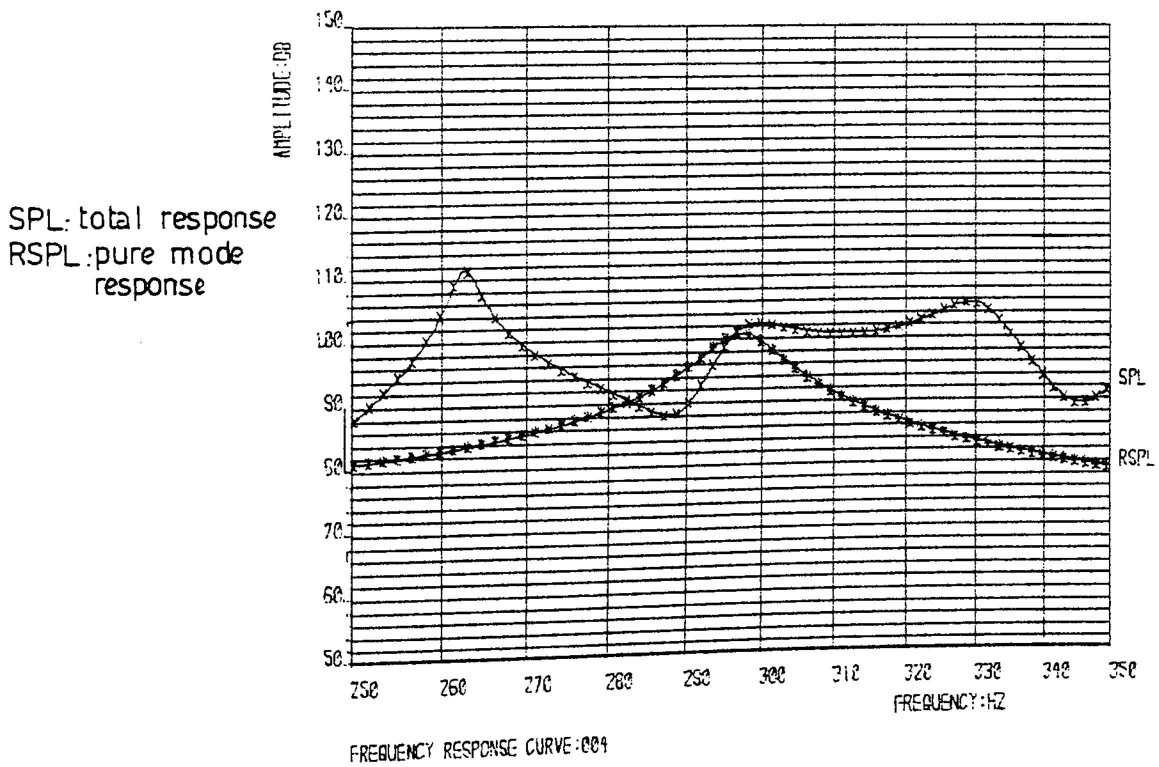


FIG.(5.4D): PREDICTED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A TREATED WALLED ROOM — RECEIVER LOCATED AT ANTINODES FOR MODE(004)

CHAPTER SIX

EXPERIMENTAL MEASUREMENTS

6.1) INTRODUCTION:

The experimental work covered in this Chapter, is aimed at:

(1) Verification of the theoretical prediction of the room performance, prior to construction, as reviewed in Chapter Five. This comprises methods of separation of modes in space and frequency, and means of extracting their characteristic properties from both complex amplitude and absolute measurements.

(2) Establishment of experimental measurement techniques whereby, at an after construction stage, the modal response of the room can be investigated and tested for what could, subjectively, constitute 'good acoustics.'

Simple techniques for determining the acoustic impedance- as an index of the physical properties of sound absorbents- are described.

Measurements are confined to steady-stage conditions of room excitation as transient response measurements are only meaningful for conditions of sound diffusion unlikely to be satisfied in the situation under investigation. Moreover, the modal parameters - discussed in the previous chapters - as determined by steady-state measurements are the acoustical constants that determine completely both the steady-state and transient characteristics of the room.

This paradox has been discussed by F.V.Hunt(40) who pointed out the fact that 'since virtually all of our acoustical experiences in enclosed spaces do involve the transmission of sound between a source and a receiver, so that making measurements upon a similar path would appear to be the most direct approach for investigation of the effect of the enclosure upon the transmission'.

Complex response data, containing information on phase as well as amplitude of the sound signal, are experimentally derived wherever their analysis is considered to yield more information than mere amplitude response data.

The experimental results are presented, analyzed and compared with their theoretically predicted counterparts.

6.2.) THE EXPERIMENTAL MODEL:-

The experiments, pertaining to the investigations of modal sound fields, are conducted in a small room adjoining the Acoustic Laboratory of the Department of Construction and Environmental Health at the University of Aston in Birmingham.

The room is of the same size and dimensions as the theoretical model discussed in Chapter Five. The walls are one and a half brick thick, plastered on both sides. The floor is a concrete slab finished with a cement screed. The roof is of timber joists with hardboard ceiling. The door is of two inches thick, double-leaves with flush and honey-combed structure, with rubber gaskets and cork lining over the jambs.

Measurements for the bare-walled room are made in the room as it exists.

For the treated-walled room, one of each opposite walls (X2 and Y2 walls in Figure (3.1)) are lined with one inch thick sheets of fibre glass. The floor, (representing the ceiling in the actual model), is covered with one inch thick perforated celotex tiles.

The loudspeaker is supported on a light mobile stand of adjustable height.

Gliding microphone tracks are fitted to the ceiling which enable movement of the microphone along the x and y directions by a small mobile motor, with the microphone suspended from a rod on which its vertical position can be adjusted.

Measurement set-ups for complex response measurements are calibrated in the anechoic chamber of the Department of Physics. This has the following clear dimensions:
length: 400 cms, width: 360 cms, height: 205 cms
with fibre glass wedges 70 cms deep.

6.3) INSTRUMENTION.

The equipment utilized in the experimental measurements, carried out in this work, can be described briefly under the following classification:

6.3.1) THE SOUND SOURCE:-

Two types of loudspeakers are used as sound sources for different ranges of frequency.

(1) A GOODMAN MAXIM direct radiator speaker mounted in a 26.0 x 18.0 x 13.5 cms cabinet, housing a four inch low frequency base unit coupled with a tweeter for high frequency sound reproduction.

(2) A VITAVOX pressure unit speaker, 20.0 cms in diameter, for mid and high frequency sound reproduction.

The former type is employed in measurements involving low frequency sounds below 200 Hz. The latter is used for frequencies above this limit.

The implications of this distinction have been discussed in Section (5.4.8) of Chapter Five. The discussion will be further persued with the analysis of the results of calibration measurements.

6.3.2) THE DRIVING GENERATOR:

Two types of sine-generators are used to supply electrical input to the loudspeakers, the choice being governed by the sort of response to be measured.

(1) For complex-response measurements, two modules of the plug-in units of AIM system (Advanced Instrumentation Modules) type 5.15B are used[58]. A programmable filter oscillator type PFO166B, driven by a low scan generator type SSG 212, supplies the electrical input to the loudspeaker through a RADFORD power amplifier. The output of the slow scan generator is a triangular waveform the amplitude of which can be vafied from 0 volt to 20 volts, and its sweep rate is manually controlled. The frequency range swept by the programmable filter oscillator is controlled by the voltage output of the generator.

Two reference signals of approximately one volt r.m.s., and having a phase difference of 90° are supplied by the oscillator.

(2) For absolute amplitude response measurements, a B and K sine random generator type 1024 is used. A sine wave signal is produced, with the frequency of the sinusoidal output tunable manually or automatically from an external motor. In the latter case a synchronized frequency sweep with the level recorder type 2305 can be obtained.

A built-in compressor circuit is included in the instrument. This can be controlled from an external voltage, whereby it is possible to keep the sound pressure level constant[59] by feeding back the output from a microphone located in the near field of the loudspeaker to the compressor input.

6.3.3) THE RECEIVER:

A B & K one inch condenser microphone type 4145 is used as the sound receiver. It has a linear free field frequency response and omnidirectional characteristics.

6.3.4) PHASE RESOLUTION:

The resolved components of the output signal are measured using a pair of phase sensitive detectors type PSD122A of AIM system.

The output signal of the system under investigation (i.e. the room) is amplified using a B & K spectrometer type 2112 and connected to the input sockets of the phase detectors. Two reference signals from the programmable filter oscillator are connected to the in-phase and quadrature phase-reference input sockets of the two phase detectors. The resolved signals are amplified and presented at output sockets and also displayed on indicating meters. The output signals provide simultaneous readings of the real and imaginary components of the transfer function of the response of the system under investigation[58], thus allowing Nyquist plots to be displayed on the screen of an oscilloscope, made by an x-y plotter or stored in the memory of a digital event recorder.

6.3.5) DATA RECORDING AND STORAGE:

The recording of the signals representing the measured response of the room and their storage in the form of a permanent record are facilitated by employment of equipment utilizing both analogue and digital recording systems.

A MOSELEY x-y Recorder (Model 7035B), fed by the resolved components of the signal from the output sockets of the two phase sensitive detectors, is used to produce a Nyquist plot of the transfer function of the room response as a function of frequency. A circuit is devised which, when operated with a square waveform WEST HYDE pulse generator, lowers the x-y plotter pen momentarily on the paper at equal intervals thus making the plot provide information on the rate of charge of the vector with frequency. In this fashion a first approximation is obtained on the locations of the natural frequencies of the system under investigation.

For amplitude response measurements a B & K level recorder type 2305 is used to obtain graphical plots of response measurements as functions of frequency or distance on a logarithmic scale.

Digital recording of resolved phase signals is carried out using a B & K digital event recorder type 7502, with a 2K memory. The sampling frequency of the input recording which is controlled by an external clock provided by the square waveform output of the WEST HYDE pulse generator, is chosen such that the time window defining the recorded signal is equal to the sweep time of the simulating oscillator PFO 166B. The phenomenon of aliasing, associated with the sampling frequency being of the order of magnitude of the highest frequency of interest in the sampled function, is avoided[60].

Simultaneous recording in the memory of the digital event recorder are made possible by use of the B & K multiplexer type 5699 which enables the former to record from two channels at the same time.

The memory content of the event recorder is then read out sequentially to a digital data receiver - a SOLARTRON paper tape punch type 324S. The signal, stored on a punch paper tape, is fed to the computer for data processing and presentation in graphical form.

6.3.6) READ-OUT AND VISUAL DISPLAY:

The presentation of measured results is achieved by visual display in addition to graphical plot methods, already described. The indicating meter on a B & K spectrometer type 2112 is employed to give direct readings of the absorption coefficient of small cut samples of sound absorbents using the Standing Wave Apparatus.

A large screen, long persistence RACAL Oscilloscope type (382-2) is utilized to give a visual display of the Nyquist diagram of the transfer function in complex response measurements when connected to the output sockets of the phase sensitive detectors. A BRADLEY dual channel oscilloscope type 200 fed from the output socket of the digital event recorder or the multiplexer displays the waveform of the inphase and quadrature components of the recorded signal.

6.3.7) IMPEDANCE MEASUREMENT:

Measurements of the standing wave ratio and the distance between the surface of the sample and the sound pressure minima - two parameters required for determination of the acoustic normal impedance - are conducted using the B & K standing wave Apparatus type 4002.

The complete measuring apparatus comprises[35] a measuring tube terminated by a loudspeaker at one end and a small cut sample at the other. The loudspeaker is operated to the desired test frequency from the sine-random Generator type 1024. The sound field is explored by means of a probe microphone movable on a track equipped with a scale on which the exact distance between the probe entrance and the test sample can be read. The microphone output is fed to the spectrometer 2112 used as a selective amplifier and a direct indicator of the value of the absorption coefficient on its meter scale.

6.4) DETERMINATION OF SPECIFIC ACOUSTIC IMPEDANCE OF POROUS ABSORBENTS BY THE STANDING WAVE TUBE METHOD:

6.4.1) EXPERIMENTAL PROCEDURE:

The measurement arrangement, shown in Figure (6.1), is used for measurements on small-cut samples of fibre-glass and perforated celotex employing the Standing Wave Apparatus described in Section (6.3.7).

Readings of the normal absorption coefficient and the distance of the first and second sound pressure minima from the face of the sample are taken at approximate logarithmic frequency intervals for the frequency range 100 Hz to 700 Hz.

6.4.2) RESULTS OF EXPERIMENT:

Using Program ADMITTANCE, described in Section (5.4.4) the specific acoustic impedance and admittance are evaluated from the measured data according to the standing wave analysis described in Section (4.2.3.2.1).

The measured data and computed results for fibre-glass and perforated celotex are shown in Tables (6.1A) and (6.1B) respectively. An analysis of these results has been given in Section (5.4.4.2).

6.4.3) CONCLUSIONS:

The advantages of the method are that only small samples are needed, and the measurements are easy to carry out and are perfectly reproducible.

The limitation is that only porous absorbents can be investigated by the method. For large samples, which allow panel vibration, alternative techniques should be sought.

The use of the tube is limited to sounds of frequencies higher than a minimum frequency limit below which no pressure minimum is formed.

6.5) DETERMINATION OF SPECIFIC ACOUSTIC IMPEDANCE OF BARE WALLS BY THE TRANSMISSION-CHARACTERISTIC METHOD:-

6.5.1) EXPERIMENTAL PROCEDURE:-

A block diagram of the measurement set-up is shown in Figure (6.2). The method involves exploring the spatial distribution of the pressure amplitude resonance response when the room is excited at frequencies corresponding to the natural frequencies of its normal modes of vibration.

The spatial sound field is explored by a microphone moved along the gliding track arrangement described in Section (6.2). The location of both microphone and loudspeaker is fitted at antinodal points of the mode under consideration with the intention of eliminating as many as possible of the neighbouring modes. The frequency of the oscillator is adjusted to give maximum response on the indicating meter of the one-third octave frequency analyzer. The position of the loudspeaker is then kept fixed while the microphone is moved along the x-direction from one of the x-walls towards the other. The level recorder output gives a plot of the amplitude response (in dB) against the distance from the wall.

A list of the modes investigated and the respective locations of microphone and loudspeaker are shown in Table (6.3).

6.5.2) RESULTS OF EXPERIMENT:-

The plots of the spatial distribution of modes for the bare-walled room are shown in Figures (6.10A) to (6.10E). From measurements, on the plots, of the sound pressure level (in dB) at the wall, the sound pressure level at the first minimum and the distance from the face of the wall, of this minimum, the value and position of the sound pressure level at any other point, values of impedance and admittance are derived using program ADMITTANCE1 according to the spatial distribution analysis discussed in Section (4.2.3.3.1).

The measured data and evaluated results are shown in Table (6.10). Comments on the results have been given in Section (5.4.4.2).

6.5.3) CONCLUSIONS:

The method has the advantage of providing measurements on large area that vibrates as a panel and under actual mounting conditions. Its limitation is that, being based on the peak-amplitude technique, the measured data are likely to contain considerable error due to the off-resonant contributions in the extracted response. Also the complexity of the mathematical analysis renders the method only useful for the conditions of large impedance.

6.6) CALIBRATION OF MEASUREMENT SET-UP BY FREE FIELD MEASUREMENTS:

6.6.1) EXPERIMENTAL PROCEDURE:

The measurement arrangement is shown in Figure (6.3). The characteristics of the measurement set-up are investigated with the two sound sources described in Section (6.3.1). The box loudspeaker is tested for the frequency range 38 Hz to 401Hz, and the pressure unit for the range 153 Hz to 660 Hz. The frequency sweep method[61] is employed in investigating the free field response of the set-up. The experiment is carried out in the anechoic chamber described in Section (6.2).

The sound source is placed at a corner of the chamber while the receiver is mounted at the centre of the chamber at 3.05m from the source[15].

6.6.2) RESULTS OF EXPERIMENT:

From the measured and calculated free field complex response, the correction factor, expressing the response in amplitude and phase of the measurement set-up in terms of the source strength (Q_o) is evaluated using program FREEFIELD discussed in Section (5.4.8).

The computed values of the correction factor as a function of frequency for the measurement set-up with each of sound sources are stored in computer data files to be used in conjunction with complex response measurements in modal sound fields.

The results are demonstrated in Figures (6.11A) and (6.11B).

6.6.3) CONCLUSION:

Inspection of the results indicates that the linearity of the set-up with the pressure unit as a sound source is maintained only for frequencies above 200 Hz; hence it's use must be confined to frequencies above this limit.

The box loudspeaker shows an approximately linear response over all frequencies, but it's dimensions in comparison to the wavelength imposes a limit on it's usefulness for frequencies above the 200 Hz limit.

6.7.) MODES SEPARATION BY FREQUENCY-RESPONSE MEASUREMENTS:

The complex-frequency response and amplitude frequency response of the room are measured for the frequency range 250 Hz to 350 Hz with two alternate locations of the sound receiver. Both locations correspond to antinodal points with respect to a particular mode (mode 004), but whereas at the first point ($x = 0$, $y = 0$, $z = 0$) a considerable number of modes with natural frequencies between 250 Hz and 350 Hz are contributing to the total response, at the point ($x=\frac{1}{4}L_x, y=\frac{1}{4}L_y, z=\frac{1}{4}L_z$) all modes neighbouring to (004) are eliminated, since this point corresponds to one or more of their nodal planes.

The bare-walled room and the treated-walled room are both investigated in this manner.

6.7.1) EXPERIMENTAL PROCEDURE:

6.7.1.1) COMPLEX-FREQUENCY RESPONSE:-

The measurement arrangement is similar to the one shown in Figure (6.3) except that the digital recording part, comprising the event recorder, the multiplexer and the tape punch, is replaced by the x-y plotter described in Section (6.3.5).

The oscillator is adjusted to make a slow sweep of the frequency range 250 Hz to 350 Hz while the response is being plotted by the x-y plotted using the pen lift device.

6.7.1.2) AMPLITUDE-FREQUENCY RESPONSE:

The measurement set-up is shown in Figure (6.4). The amplitude response is recorded as sound pressure level (in dB) as a function of frequency by the level recorder type 2305 which is synchronized with the sine-random generator type 1024, to sweep the frequency range 250 Hz to 350 Hz.

6.7.2) RESULTS OF EXPERIMENT:

Plots for the bare-walled room are shown in Figures (6.5A) and (6.5B) for the two locations of the receiver.

The upper plot on each figure shows the complex frequency response, while the lower one is the amplitude frequency response.

For the treated-walled room, the results are shown in Figures (6.5C) and (6.5D) in a similar way.

6.7.3) DISCUSSION OF RESULTS AND COMPARISON WITH THEORETICAL PREDICTIONS:

A study of the experimental results reveals that:

(1) The location of the sound receiver (and of the sound source) has a marked effect on modes separation. For the position where many modes contribute to the measured response, the complex plot consists of numerous loops and knots which are very difficult to identify with particular modes.

The amplitude plot likewise consists of successive maxima and minima which are not particularly related to specific modes.

For the second position of the receiver, the complex response is much simplified, the circles are almost pure and the few modes present are clearly associated with these circles.

On the amplitude plot, the maxima correspond to these modes.

(2) In either case of locating the receiver, the complex response plot is more informative than just the amplitude plot.

For the situation where many modes are present, the complex plot, though much distorted, can still furnish information on the presence of each mode which can be detected from an inspection of the rate of change of spacing along the plot of equal frequency intervals. This is not so apparent on the amplitude plot, where the recognition of a mode may be concealed due to in-phase superposition[17] of closely spaced excited modes and the peaks tend to merge together.

(3) The effect of the boundary conditions on amplitude reductions and phase shifts in the modal response is demonstrated by comparing both the complex and amplitude response plots, for the bare-walled room and the treated walled room. The circles are more pure on the complex plot and the peaks more sharp on the amplitude plot for the bare walled conditions. For the treated walled condition, the circles tend to merge and the peaks are blurred.

In general, the experimental results are similar in style to those obtained from theoretical predictions and demonstrated by Figures (5.4A) and (5.4B) for the bare-walled room, and Figures (5.4C) and (5.4D) for the treated-walled room.

It may be concluded that a theoretical assessment on the effect of location of the receiver (and,or the source) is a powerful tool in achieving mode separation. It is also worthy to emphasize the complex response plots are more informative than amplitude response plots in this respect.

6.8) DETERMINATION OF NATURAL FREQUENCIES AND DAMPING CONSTANTS BY COMPLEX-FREQUENCY RESPONSE MEASUREMENTS:

6.8.1) EXPERIMENTAL PROCEDURE:-

The resolved components of the room response, when the oscillator is adjusted to sweep a narrow frequency range centred approximately on the natural frequency of a normal mode, are measured using the measurement set-up shown in Figure (6.3). The sound source and the receiver are located at antinodal points for the mode in order that in the measured response, the contribution of the mode under consideration is maximum in comparison to the off-resonant vibration. The locus of the tip of vector representing the response is observed on the screen of the long persistence oscilloscope to give an almost true circle. Meanwhile, the sweep rate of the oscillator is adjusted so that the time window, given by the quotient of the event recorder memory size and the sampling rate, will contain the entire response of the mode for the frequency range below and above its resonance point. This can be checked by playing back the contents of the event recorder memory on the screen of a dual channel oscilloscope.

The resolved components are recorded in the digital event memory through the multi-channel multiplexer. The recorded signals are then read out to the digital data receiver and stored in punched paper tape according to the manner described in Section (6.3.5).

Measurements are carried out for a number of modes for the bare-walled room. The experiment is then repeated for the treated-walled room.

6.8.2) RESULTS OF EXPERIMENT:-

The digitally recorded data containing the modal response of the room, is reconstructed into its analogue form by the computer and the receptance of the response extracted by the procedure outlined in Section (4.3.3).

Plots of the complex receptance, similar to those shown in Figures (6.6A) to (6.9A) are obtained for the bare walled room, and to those shown in Figures (6.6C) to (6.9C) for the treated-walled room.

Plots for the frequency spacing, amplitude, phase and inphase and quadrature components of the receptance are shown in Figures (6.6B) to (6.9B) for the bare walled room, while those for the treated-walled room are shown in Figures (6.6D) to (6.9D).

6.8.3) ANALYSIS OF RESULTS:-

The extraction of the natural frequencies and damping constants from these plots is achieved by applying different techniques of resonance testing for their analysis. The techniques have been discussed in Section (4.3) and are basically:

(1) The Peak-Amplitude Method.

(2) The Phase Angle Plot Method.

(3) The Kennedy and Pancu Method.

6.8.3.1) THE PEAK-AMPLITUDE METHOD:

The natural frequency is taken as the frequency at which the peak of the receptance is attained.

The damping constant is calculated by Equation (4.67) from the frequency band between points on the amplitude-frequency curve corresponding to the amplitude being $1/\sqrt{2}$ of it's peak value at these frequencies.

6.8.3.2) THE PHASE-ANGLE PLOT METHOD:

The natural frequency is obtained from the point of intersection of the phase angle curve with the line where the phase angle is equal to zero or π radians, depending on the respective locations of the sound source and receiver.

The damping constant is measured according to Equation (4.67) from the frequency band between points on the phase angle curve removed from the natural frequency by $\pi/4$ radians on either side.

6.8.3.3) THE KENNEDY AND PANCU METHOD:

The natural frequency is taken to be the point at which there is maximum spacing of equal frequency intervals. This can be derived from the vector response plot or the spacing versus frequency plot.

The damping constants are derived from a best-fit circle construction in the vicinity of the natural frequency on the complex receptance plot, and evaluated by Equations (4.91) or (4.92) as discussed in Section (4.3.6.5).

The extracted values of the natural frequencies and damping constants for a number of modes, using the different techniques outlined above, are shown in Table (6.2A) for the bare-walled room. Similar results for the treated-walled room are shown in Table (6.2B).

6.8.4) DISCUSSION OF RESULTS.

Comparisons of the values obtained for the natural frequencies and damping constants, by the three different techniques, for the two alternative room models reveal that:

1) In almost all cases, where modes are identifiable, the Kennedy and Pancu method produces results for both modal parameters. The peak-amplitude method is equally successful in determination of natural frequencies, but fails, on many occasions to give results on damping when there is significant off-resonant contribution in the measured response.

   The phase-angle method is the least reliable in this respect. Often the condition that the response is in-phase or antiphase with the driving signal at resonance is not satisfied, and the natural frequency cannot be extracted. The measurement of damping is even more difficult as the change of phase in the vicinity of resonance is not

1) contd.

large enough and accordingly a half-power width cannot be measured.

2) The results obtained for the natural frequency, by the three-methods, are in close agreement with one another, within the allowable experimental error. However, values of the damping constant, so obtained, exhibit wide divergence.

The peak-amplitude method gives the largest values, followed by the Kennedy and Pancu method, while the phase-angle method gives the smallest. The error in the results of the first method can be attributed to the presence of the off-resonant contribution within the measured response, while the error in the results of the last method are due to the difficulty associated with phase measurements. In this respect, results obtained by the Kennedy and Pancu method are considered more reliable.

3) In comparing the results obtained for the two different room models (i.e. the bare-walled room and treated-walled room) shown in Tables (6.2A) and (6.2B), the following factors can be pin-pointed:

a) The shifts in the natural frequencies in the bare-walled room are small and are, in general, towards the upper frequency values. For the treated-walled room, they are small in the low frequency range, but increase towards the upper limit of the frequency range considered. The inclination is towards lower frequency values than those of the predicted rigid-walled room.

b) Larger values for the damping constant are obtained in the treated-walled room than the bare-walled one.

c) In all cases, oblique modes are the most damped, followed by tangential ones, while axial modes have the least damping.

6.8.5) COMPARISON WITH THEORETICAL PREDICTIONS.

Inspection of the predicted values, shown in Table (5.3), of modal parameters for the two room models in the light of the preceding discussion shows that:-

1) The predicted shifts in natural frequencies for the bare-walled room are within the range of the measured values but show sharp fluctuations above and below the rigid-walled room values. For the treated-walled room the predicted shifts are greater than the measured ones but both agree in being towards lower frequency values when compared to those of the rigid-walled room.

2) The predicted damping for the bare-walled room is larger than the measured one by a factor of two or sometimes three. Those for the treated-walled room are in better agreement for low frequency ranges. At higher frequencies the predicted damping increases much more rapidly with frequency than the measured one.

The discrepancies between the measured and predicted values can be attributed to the following possible factors:

1) The inherent limitations in the transmission-characteristic method for determination of bare walls impedance, both theoretical and experimental. This point has been discussed in Section (4.2.3.3.5).

2) The condition of local reaction, theoretically assumed, is less valid in practical conditions, and thus the possible variation of impedance with the angle of incidence cannot be ruled out altogether.

6.9) DETERMINATION OF MODAL SHAPES BY EXPLORING THE SPATIAL DISTRIBUTION OF RESONANCE AMPLITUDES:-

6.9.1) EXPERIMENTAL PROCEDURE:

The measurement set-up is shown in Figure (6.2). The experimental procedure is identical to that employed in measurements of impedance by the Transmission-Characteristic method and described in Section (6.5.1).

The experiments are carried out in the bare-walled and the treated-walled rooms for the modes listed in Table (6.3), using the method of selective source and receiver locations as means of mode separation.

6.9.2) RESULTS OF EXPERIMENT

The plots of modal shapes obtained from both sets of measurements are shown in Figures (6.10A), (6.10B), (6.10C), (6.10D) and (6.10E).

6.9.3) ANALYSIS OF RESULTS AND COMPARISON WITH PREDICTED RIGID-WALLED ROOM PLOTS

The effect of introducing absorptive material in the room on shifting the nodal planes and blurring and broadening the maxima and minima can be seen from comparison of the plots for the two room models with each other and with the predicted rigid-walled room plots shown in Figures (5.2A),(5.2B) and (5.2C).

In most cases, the modal shapes can be identified, though with the presence of heavy damping and (or) close natural frequencies, this is sometimes not possible.

6.10) CONCLUSIONS

In this chapter different techniques for experimental determination of the indices characterising the absorptive properties of acoustical materials, and parameters describing modal sound fields in small rooms, have been demonstrated.

Applications of impedance measurements by the standing wave tube method are reliable except for the error due to the possible invalidity of the assumption of local reaction.

Measurements of impedance by the transmission-characteristic method reveal large errors due to the theoretical limitations and experimental shortcomings inherent in the method.

Measurements of modal parameters demonstrate that resonance testing techniques can be applied successfully to room acoustic measurements. The natural frequencies and damping constants of room modes can be measured with great accuracy by three different techniques. In this respect, complex response measurements are the most useful.

Modal shapes can be determined from exploration of spatial sound fields when modes are not heavily-damped.

Complete agreement between predicted and measured values has not been achieved due to limitations associated with the methods employed in impedance measurements.

Frequency (Hz)	Absorption Coefficient	Position of First Pressure Minimum (CMS)	Position of Second Pressure Minimum (CMS)	Special Acoustic Impedance (ρc) Real	Imaginary
100.0	0.026	82.5	-*	1.838	-16.574
110.0	0.027	74.5	-	1.401	-14.205
120.0	0.028	68.2	-	1.373	-13.802
130.0	0.030	62.9	-	1.420	-13.544
145.0	0.040	56.0	-	1.451	-11.795
160.0	0.041	52.2	-	5.889	-22.958
175.0	0.042	45.9	-	1.077	- 9.914
190.0	0.038	42.0	-	0.812	- 9.067
210.0	0.050	37.7	-	0.877	- 8.163
230.0	0.050	34.1	-	0.711	- 7.345
250.0	0.058	31.2	-	0.738	- 6.919
275.0	0.077	28.1	90.2	0.890	- 6.532
300.0	0.060	25.2	82.5	0.435	- 5.188
330.0	0.155	22.7	75.2	0.912	- 4.458
365.0	0.138	20.4	67.2	0.898	- 4.735
400.0	0.142	18.3	61.0	0.752	- 4.255
435.0	0.172	16.6	56.1	0.742	- 3.771
480.0	0.220	14.7	50.5	0.772	- 3.301
525.0	0.230	13.3	45.9	0.776	- 3.189
575.0	0.255	12.0	41.8	0.769	- 2.989
630.0	0.300	10.6	37.9	0.710	- 2.559
690.0	0.330	9.5	34.3	0.725	- 2.411

* A second minimum cannot be measured.

TABLE (6.1A) Results of Impedance measurements of fibre glass mat - one inch thick.

Frequency (Hz)	Absorption Coefficient	Position of First Pressure Minimum (CMS)	Position of Second Pressure Minimum (CMS)	Special Acoustic Impedance (ρc) Real	Imaginary
100.0	0.070	83.6	-*	9.655	-20.933
110.0	0.070	76.2	-	11.495	-22.377
120.0	0.070	69.3	-	7.038	-18.373
130.0	0.072	64.1	-	8.079	-19.141
145.0	0.090	57.2	-	7.363	-16.041
160.0	0.100	51.8	-	7.626	-15.187
175.0	0.100	47.3	-	7.198	-14.857
190.0	0.100	43.2	-	5.061	-12.873
210.0	0.121	39.1	-	5.888	-12.133
230.0	0.135	35.6	-	5.736	-11.158
250.0	0.150	32.6	-	5.282	-10.069
275.0	0.175	29.3	91.9	3.899	- 8.069
300.0	0.197	26.5	84.3	2.757	- 6.471
330.0	0.260	24.3	76.4	4.505	- 6.245
265.0	0.267	21.8	68.9	3.977	- 5.900
400.0	0.273	19.9	63.0	3.855	- 5.737
435.0	0.300	18.3	57.6	4.580	- 5.471
480.0	0.350	16.4	52.2	3.759	- 4.507
525.0	0.380	14.9	47.8	3.342	- 4.041
575.0	0.425	13.6	43.5	3.570	- 3.566
630.0	0.475	12.6	39.7	4.284	- 2.856
690.0	0.520	11.6	36.2	4.466	- 2.116

*A second minimum cannot be measured.

TABLE (6.1B) Results of Impedance measurements of perforated celotex files - an inch thick.

TABLE (6.1C) RESULTS OF IMPEDANCE MEASUREMENTS OF BARE WALLS BY THE TRANSMISSION-CHARACTERISTIC METHOD

Mode	Measured Resonance Frequency (HZ)	$X_{(-20dB)}$ (Cms)	X_{min} (Cms)	Specific Acoustic Impedance (ρc)	
				Real	Imaginary
101	92.1	152.3	162.5	73.647	37.112
200	105.1	75.9	81.0	17.008	14.361
210	124.3	76.4	81.5	27.321	6.216
220	170.2	77.0	82.0	25.539	-4.201
202	184.0	78.7	84.0	6.602	-18.770
222	227.5	76.8	82.0	81.955	-35.451
322	255.5	50.6	54.0	18.377	15.517
232	272.4	76.4	81.5	59.873	13.622
240	287.1	80.6	86.0	3.812	-17.208
332	296.5	51.6	55.0	18.606	-10.468
324	364.9	50.2	53.5	13.135	13.783
252	380.7	77.2	82.3	61.805	-26.594
443	407.8	38.7	41.2	13.078	-5.154
462	476.1	38.2	40.7	18.734	4.230
226	481.7	75.5	80.5	47.063	46.173
662	530.3	25.3	27.0	9.535	8.051
654	547.7	25.3	27.0	9.848	8.316
446	563.4	39.0	41.6	11.808	-15.783
480	572.7	38.7	41.3	21.094	-20.516
456	597.7	40.3	43.0	1.984	-8.956

$X_{(-20dB)}$: the distance from the wall to a point along the X-direction where the sound pressure level relative to that at the wall is (-20dB)

X_{min} : the distance from the wall to a point along the X-direction where the first sound pressure minimum occurs.

TABLE (6.2A) DERIVED NATURAL FREQUENCIES AND DAMPING CONSTANTS FOR A BARE-WALLED ROOM, FROM RESONANCE TESTING MEASUREMENTS

Mode	Rigid Wall Room	Kennedy & Pancu Method		Peak-Amplitude Method		Phase-Angle Method	
	Natural Frequency	Natural Frequency	Damping Constant	Natural Frequency	Damping Constant	Natural Frequency	Damping Constant
100	52.4	53.1	6.2	53.6	-	53.3	3.1
001	75.1	75.7	2.9	75.6	2.8	75.7	2.5
101	91.5	92.4	6.9	92.7	4.4	-	-
200	104.7	105.6	3.8	105.5	4.4	-	-
210	124.3	125.6	7.1	126.3	4.4	124.8	2.8
300	157.1	160.2	12.1	159.2	9.4	157.7	-
220	169.9	172.0	4.1	171.7	6.6	-	-
202	183.0	184.5	7.5	184.2	7.5	-	-
400	209.5	209.8	4.8	210.5	4.7	209.3	2.5
302	217.3	219.1	5.4	219.4	5.0	-	-
222	226.7	229.2	9.0	229.0	7.2	-	-
420	248.5	250.4	3.6	250.2	5.0	250.5	2.2
322	255.1	259.4	6.6	259.2	6.3	259.2	6.9
232	271.5	274.3	5.3	273.7	8.5	274.7	3.5
240	287.2	288.9	4.1	288.7	5.0	288.8	4.1
332	295.7	299.4	4.3	299.0	5.3	299.2	5.7
004	300.2	303.0	4.3	302.8	6.0	-	-
620	341.5	344.0	4.4	343.6	5.7	341.9	-
324	364.3	366.9	4.5	367.4	4.7	366.5	2.2
252	381.1	383.1	4.6	382.6	4.4	383.7	3.5
524	420.2	424.6	4.2	424.2	5.7	-	-
462	476.8	479.5	4.2	478.8	5.0	479.3	-
226	481.3	484.6	5.3	484.2	5.3	484.2	6.9
662	531.2	533.7	5.5	534.2	5.0	533.4	-
654	548.3	547.1	4.7	546.6	-	547.0	2.2
446	564.1	563.3	4.2	563.5	3.0	562.4	3.2
480	574.4	570.8	3.0	570.2	3.5	569.6	-
664	591.4	594.7	8.5	594.4	7.9	593.2	-

- values cannot be determined from measurements

TABLE (6.2B) DERIVED NATURAL FREQUENCIES AND DAMPING CONSTANTS FOR A TREATED-WALLED ROOM, FROM RESONANCE TESTING MEASUREMENTS

Mode	Rigid-Walled Room	Kennedy & Pancu Method		Peak-Amplitude Method		Phase-Angle Method	
	Natural Frequency	Natural Frequency	Damping Constant	Natural Frequency	Damping Constant	Natural Frequency	Damping Constant
100	52.4	52.5	12.3	53.3	7.2	52.7	-
001	75.1	75.6	7.8	75.6	7.9	-	-
101	91.5	91.1	6.9	91.4	6.6	91.8	6.0
200	104.7	105.4	5.3	105.5	5.0	105.9	-
210	124.3	124.5	8.2	123.8	10.1	122.2	-
300	157.1	156.6	10.8	157.6	11.0	155.2	6.9
220	169.9	168.8	12.6	169.6	15.7	171.2	-
202	183.0	182.5	17.3	183.4	-	-	-
400	209.5	208.1	15.4	208.8	-	208.5	15.1
302	217.3	216.5	16.7	215.8	24.5	218.0	-
222	226.7	224.2	23.3	224.4	21.4	223.1	31.4
420	248.5	247.1	19.2	247.2	18.2	247.8	-
322	255.1	254.2	20.7	253.8	18.9	-	-
232	271.5	269.5	20.3	268.0	40.8	271.3	15.7
240	287.2	282.9	18.9	283.0	16.7	-	-
332	295.7	294.5	24.2	293.3	23.9	295.8	-
620	341.5	341.3	-	338.8	43.9	-	-
324	364.3	362.6	25.5	362.4	32.4	362.0	15.7
252	381.1	376.7	38.1	378.4	37.7	380.4	-
524	420.2	419.2	36.7	420.1	-	-	-
462	476.8	478.7	24.8	478.9	30.2	-	-
226	481.3	467.8	24.5	469.1	34.6	467.0	35.8
662	531.2	529.6	26.3	-	-	531.0	-
654	548.3	540.5	33.0	540.3	40.3	543.8	25.1

- values cannot be determined from measurements

TABLE (6.3): LOCATIONS OF LOUDSPEAKER AND MICROPHONE FOR EXCITATION AND RESPONSE EXTRACTION OF PURE MODES

Mode	Measured Resonance Frequency (HZ)		Position of Loudspeaker			Position of Microphone		
	Bare-Walled Room	Treated-Walled Room	X/L_x	Y/L_y	Z/L_z	X/L_x	X/L_y	Z/L_z
100	52.8	52.3	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
001	75.6	75.0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	1
101	92.1	90.8	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1
200	105.1	104.2	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
210	124.3	122.5	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$
300	157.1	155.5	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{4}$
220	170.2	167.9	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
202	184.0	182.2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$
400	209.6	208.1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$
302	218.2	215.3	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$
222	227.5	222.7	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
420	248.6	245.4	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
322	255.5	251.8	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
232	272.4	268.6	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$
240	287.1	282.2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$
332	296.5	292.0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
004	300.2	– *	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$
620	341.4	336.6	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$
324	364.9	360.8	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{4}$
252	380.7	374.9	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{1}{2}$
524	420.4	416.2	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{2}$
462	476.1	474.8	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{2}$
226	481.7	470.0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$
662	530.3	527.0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
654	547.7	536.4	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{4}$
446	563.4	– *	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$
480	572.7	– *	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$
664	589.9	– *	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{4}$

*Modal shapes cannot be identified

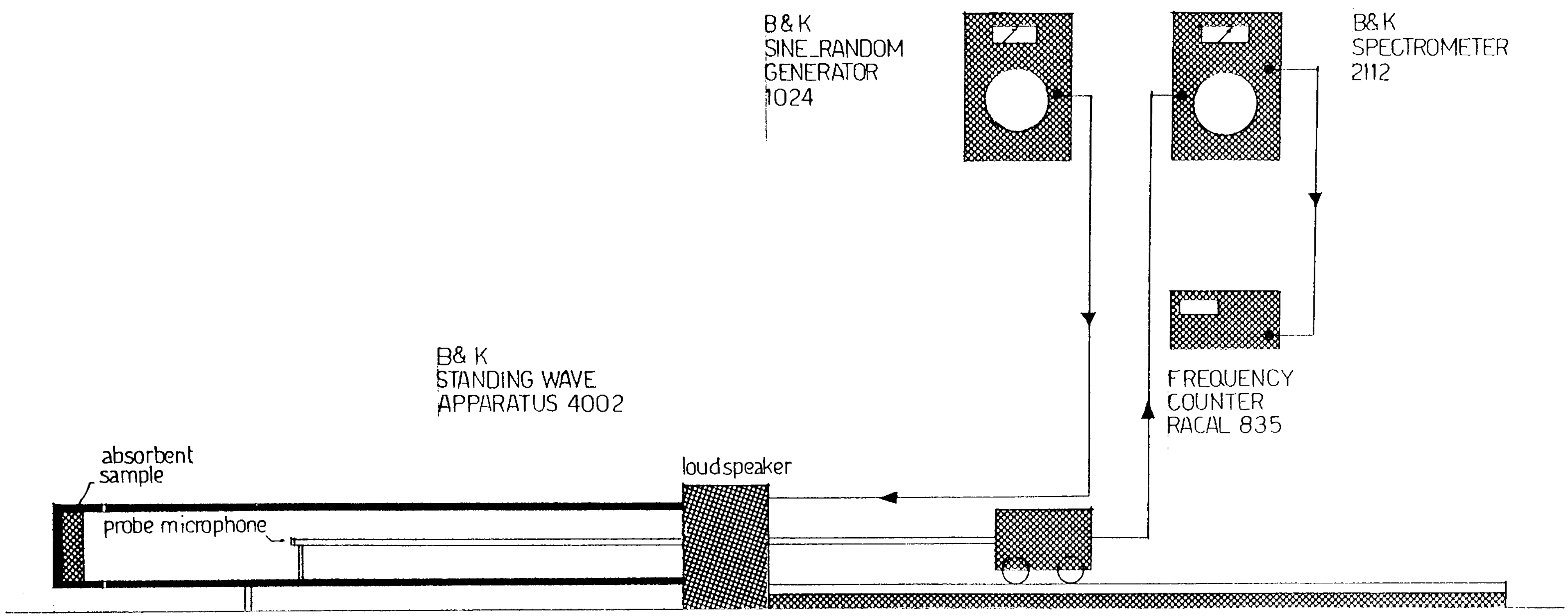


FIG.(6.1): INSTRUMENTATION SET_UP FOR DETERMINATION OF ACOUSTIC NORMAL IMPEDANCE BY THE STANDING WAVE APPARATUS

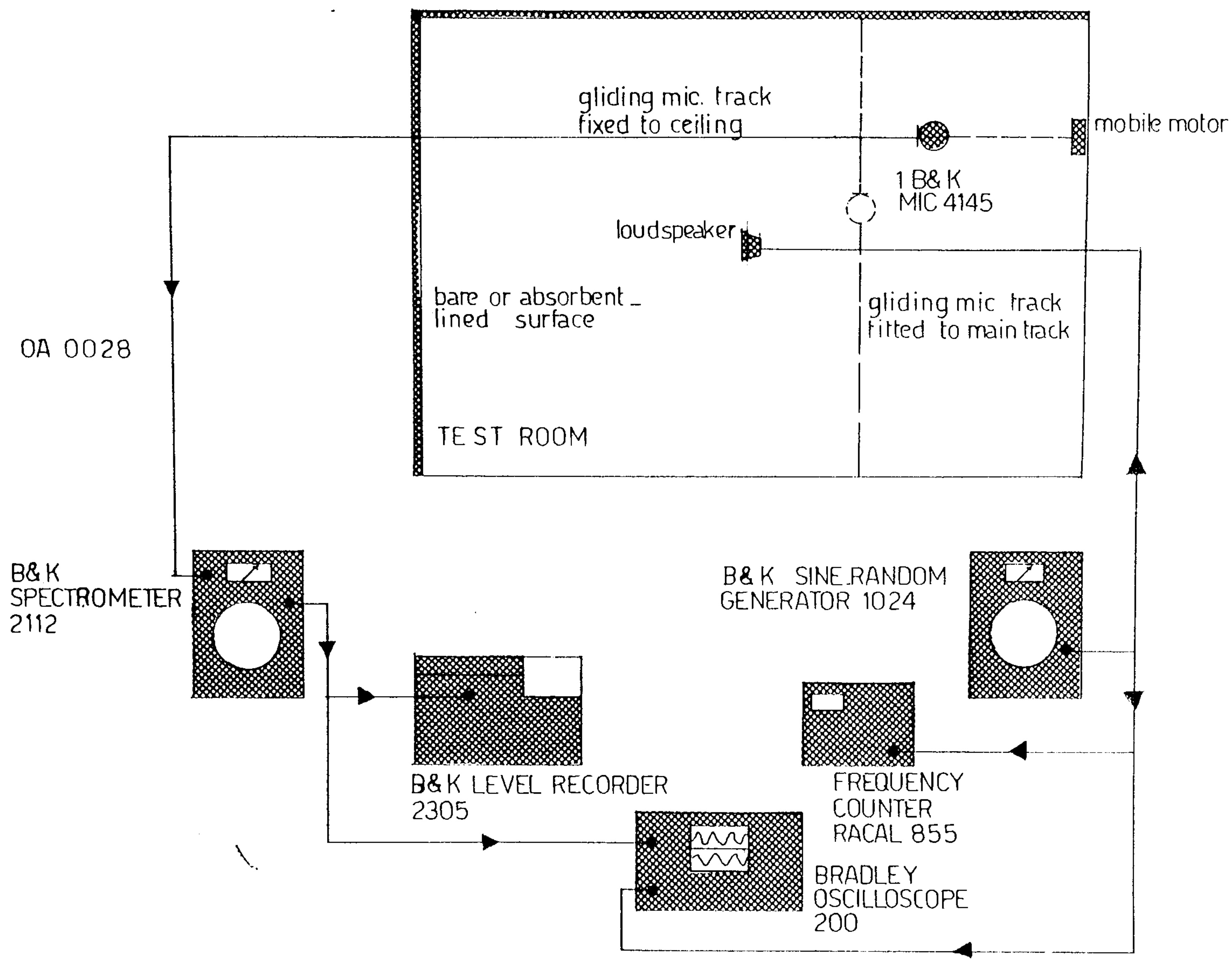


FIG.(6.2): INSTUMENTATION SET_UP FOR MEASUREMENTS OF THE SPATIAL DISTRIBUTION OF RESONANCE AMPLITUDE RESPONSE

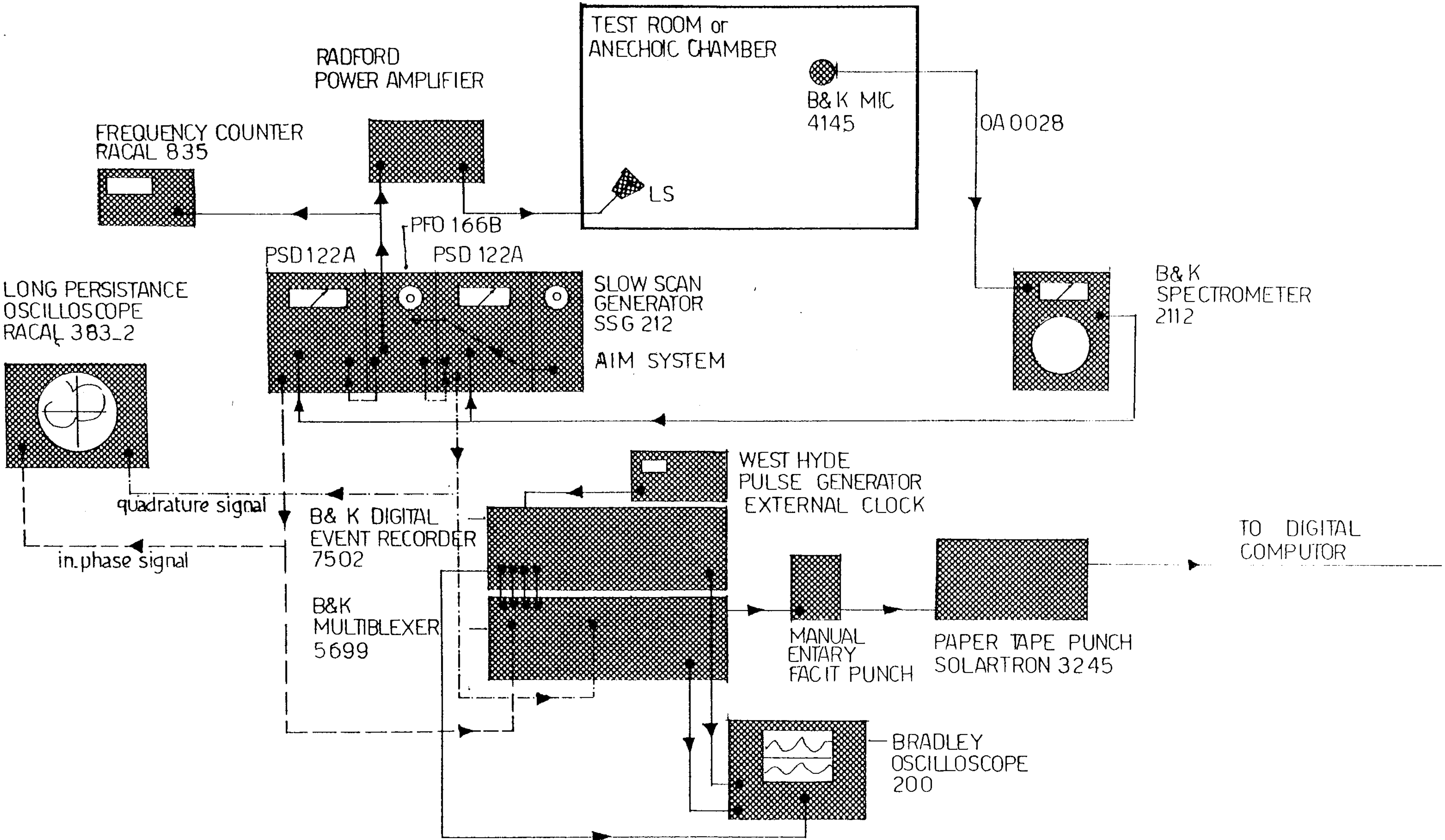


FIG.(6.3): INSTRUMENTATION SET_UP FOR COMPLEX_FREQUENCY RESPONSE MEASUREMENTS

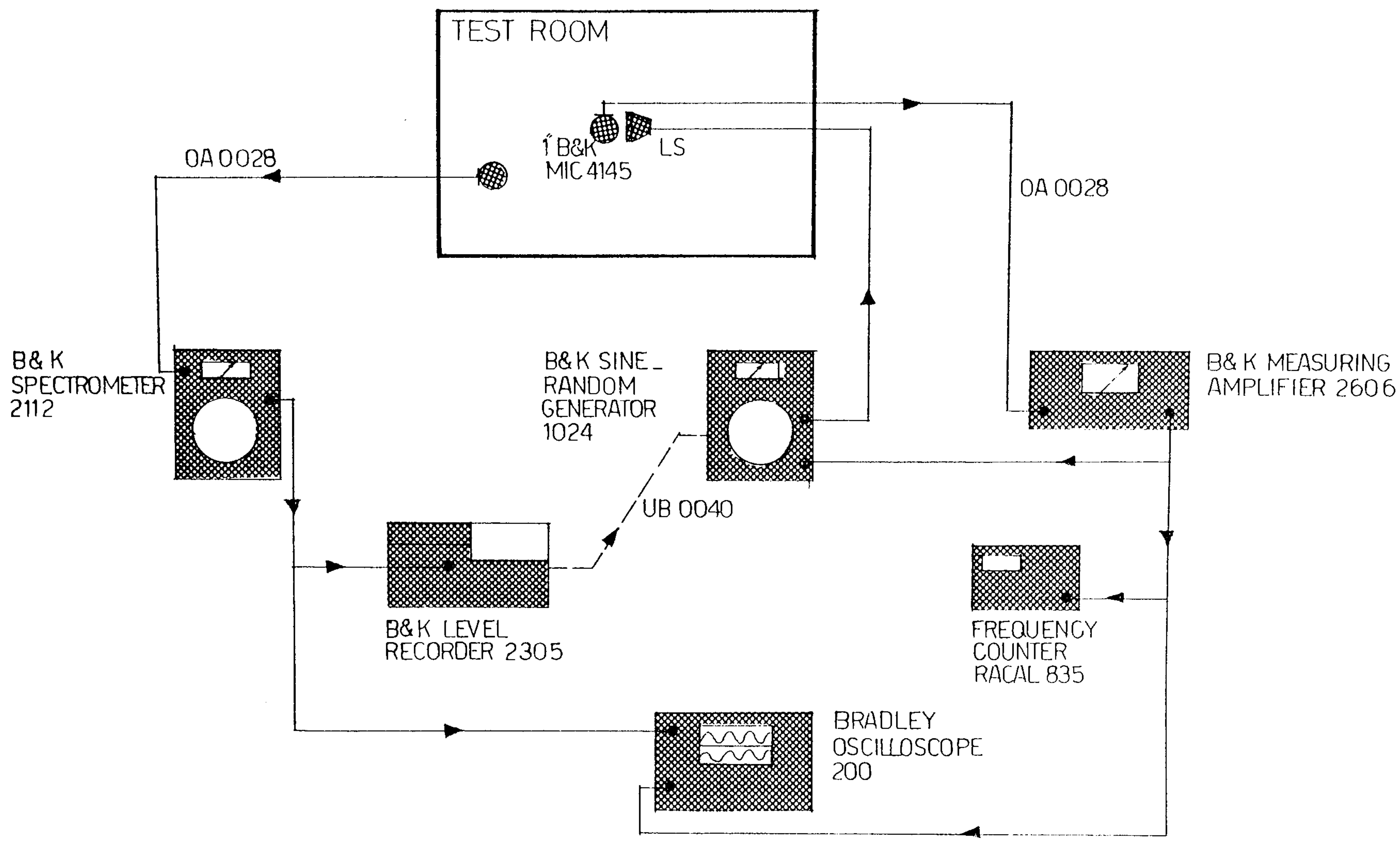


FIG.(6.4): INSTRUMENTATION SET_UP FOR AMPLITUDE_FREQUENCY RESPONSE MEASUREMENTS

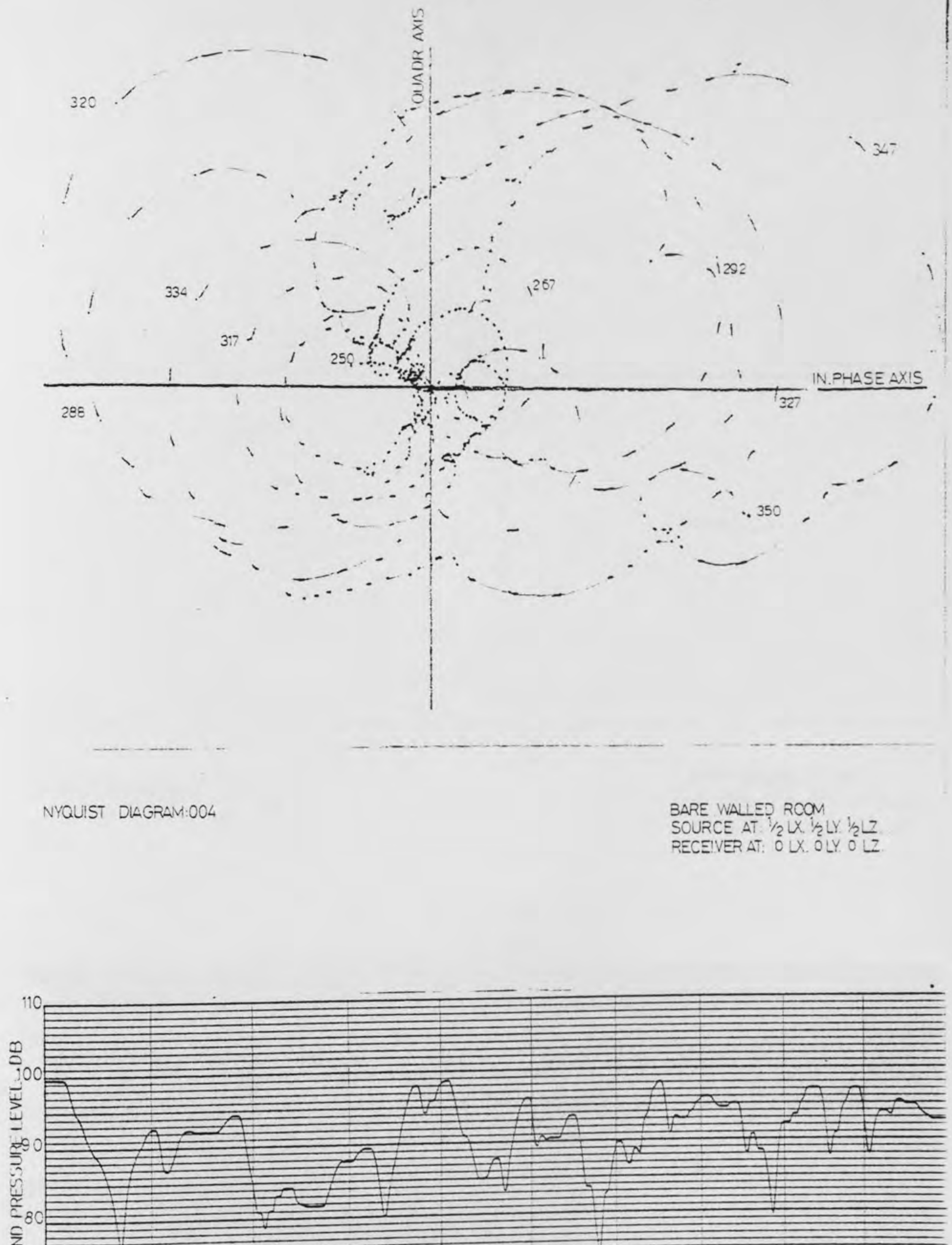
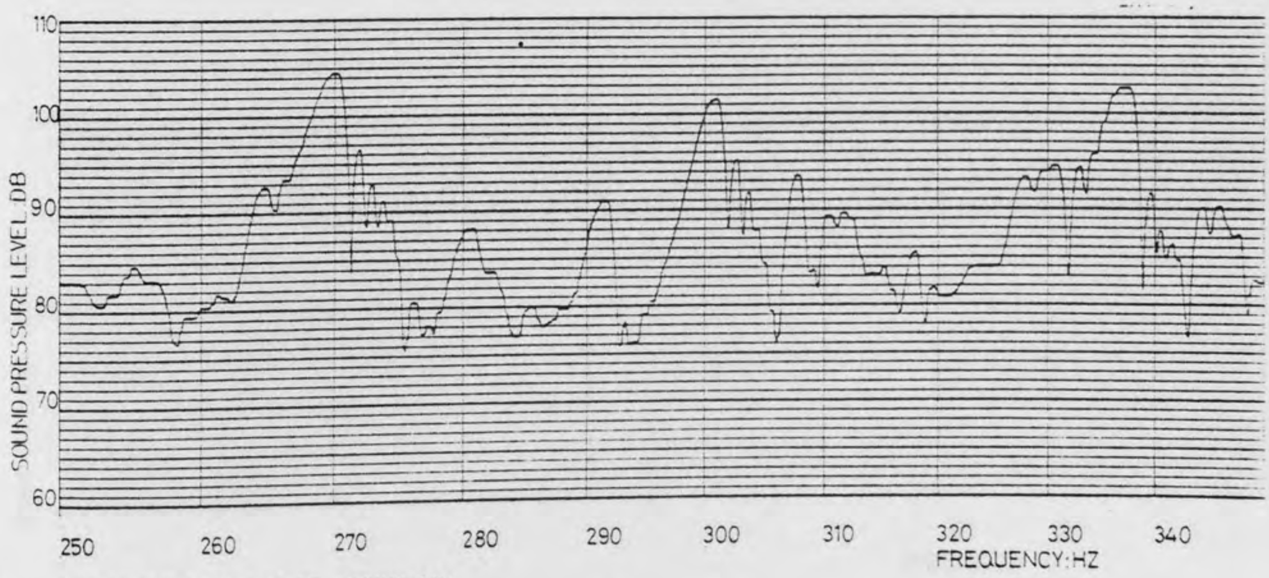


FIG.(6.5A): MEASURED COMPLEX AND AMPLITUDE–FREQUENCY RESPONSE OF A BARE WALLED ROOM – RECEIVER LOCATED AT A ROOM CORNER.

NYQUIST DIAGRAM:004

BARE WALLED ROOM.
SOURSE AT ½ LX, ½ LY, ½ LZ.
RECEIVER AT ¼ LX, ¼ LY, ¼ LZ.



FREQUENCY RESPONSE CURVE:004

FIG.(6.5B): MEASURED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A BARE WALLED ROOM_RECEIVER LOCATED AT ANTINODES FOR MODE(004).

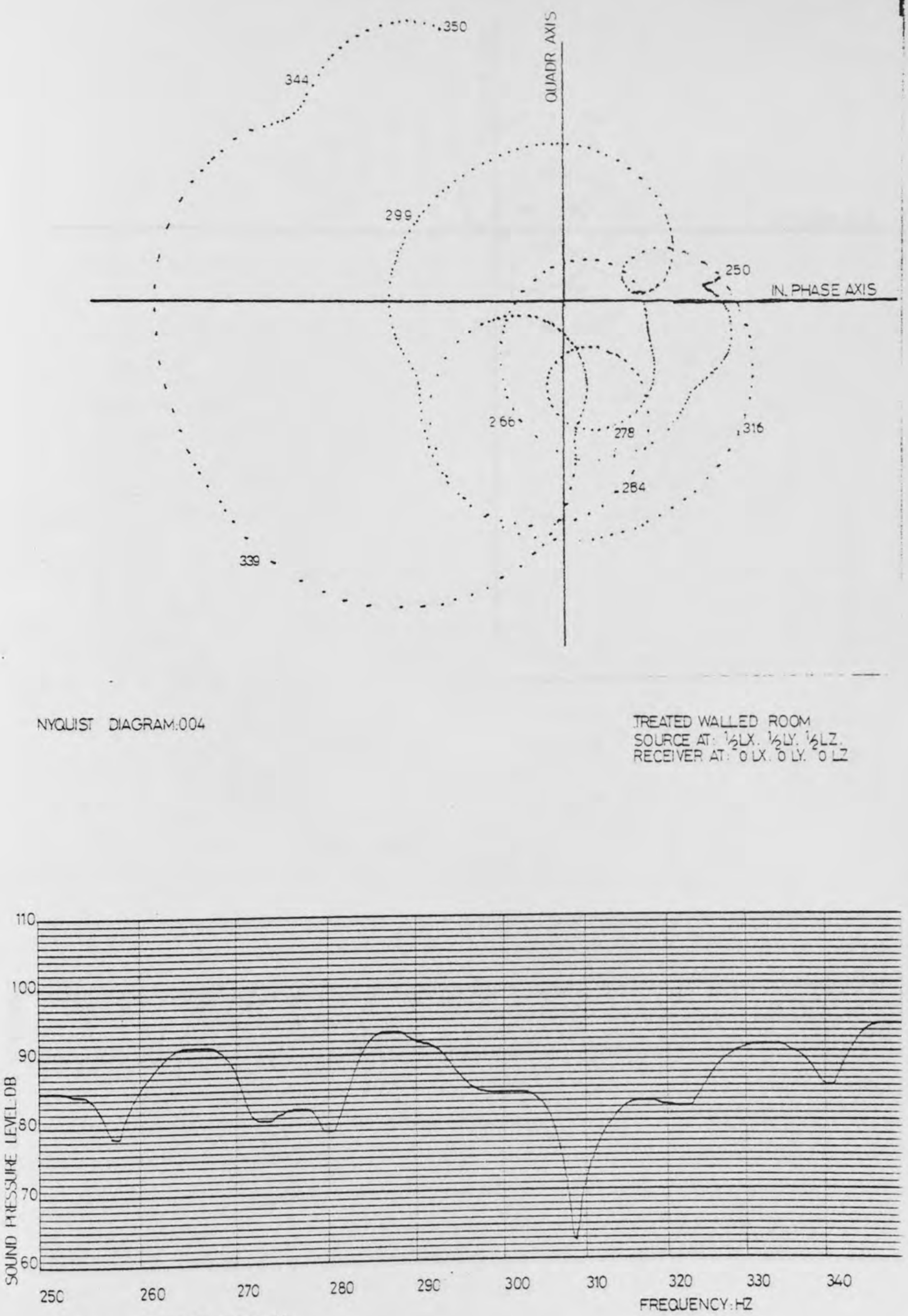
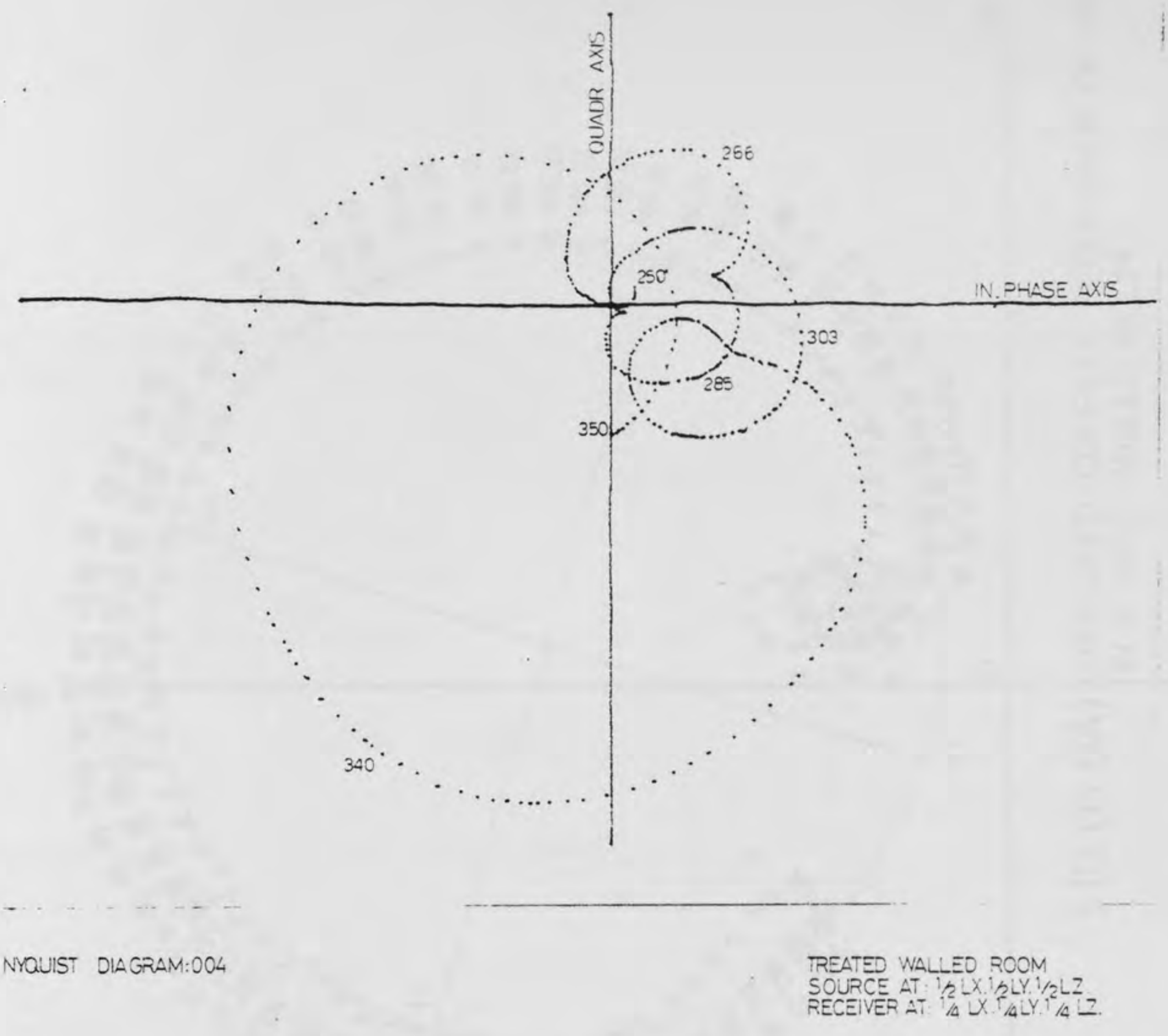


FIG.(6.5C): MEASURED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A TREATED WALLED ROOM _ RECEIVER LOCATED AT A ROOM CORNER.



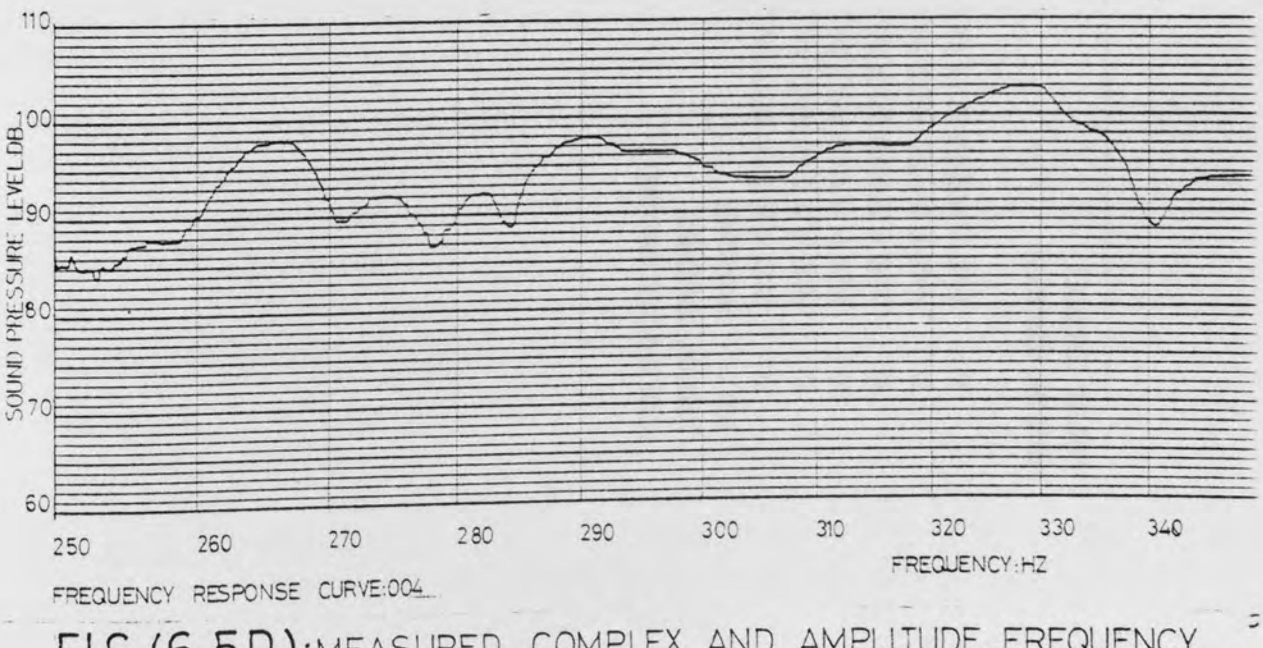


FIG.(6.5D): MEASURED COMPLEX AND AMPLITUDE_FREQUENCY RESPONSE OF A TREATED WALLED ROOM_RECEIVER LOCATED AT ANTINODES FOR MODE(004).

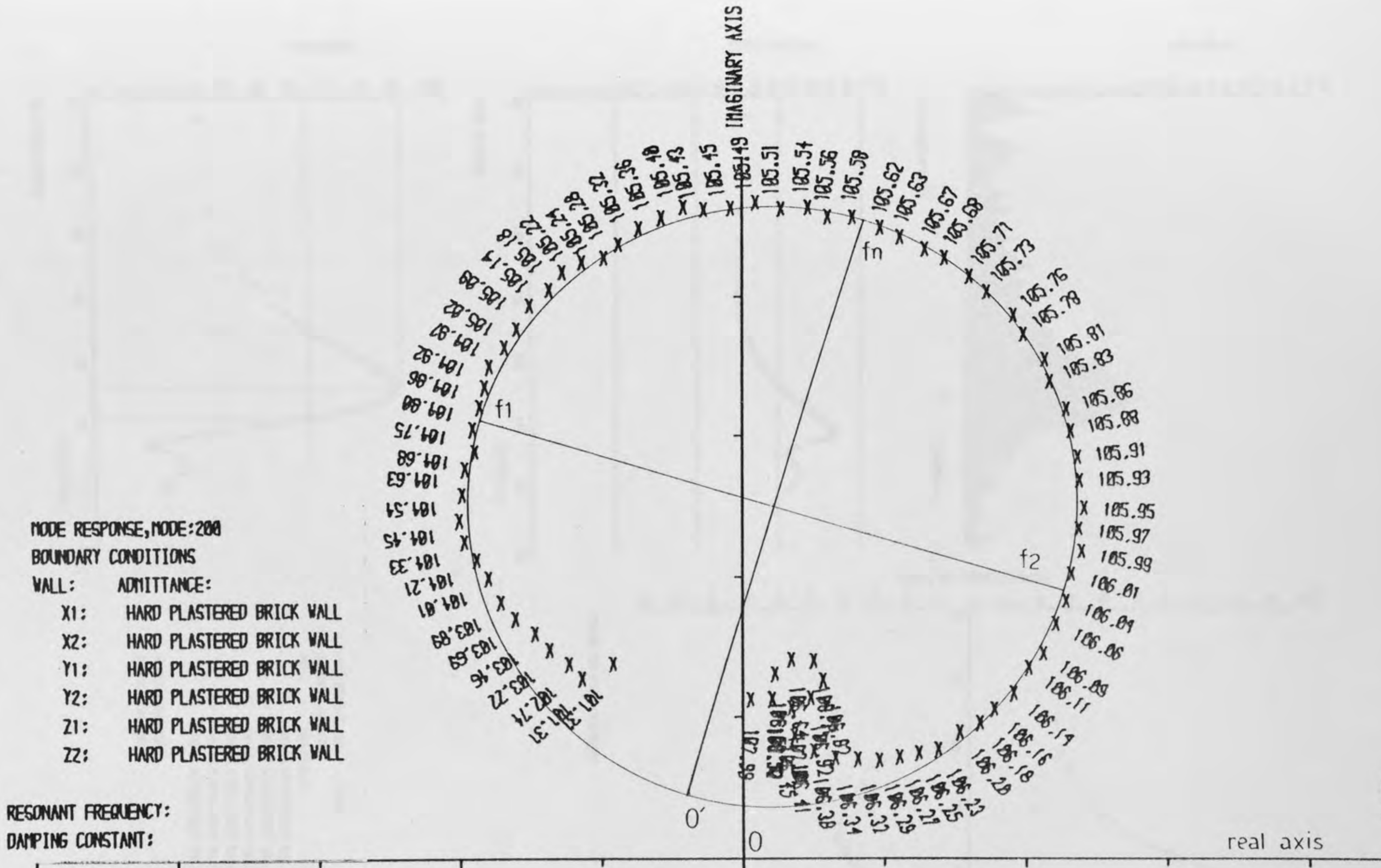


FIG.(6.6A) MEASURED COMPLEX RECEPTANCE OF MODE(200) IN A BARE WALLED ROOM.

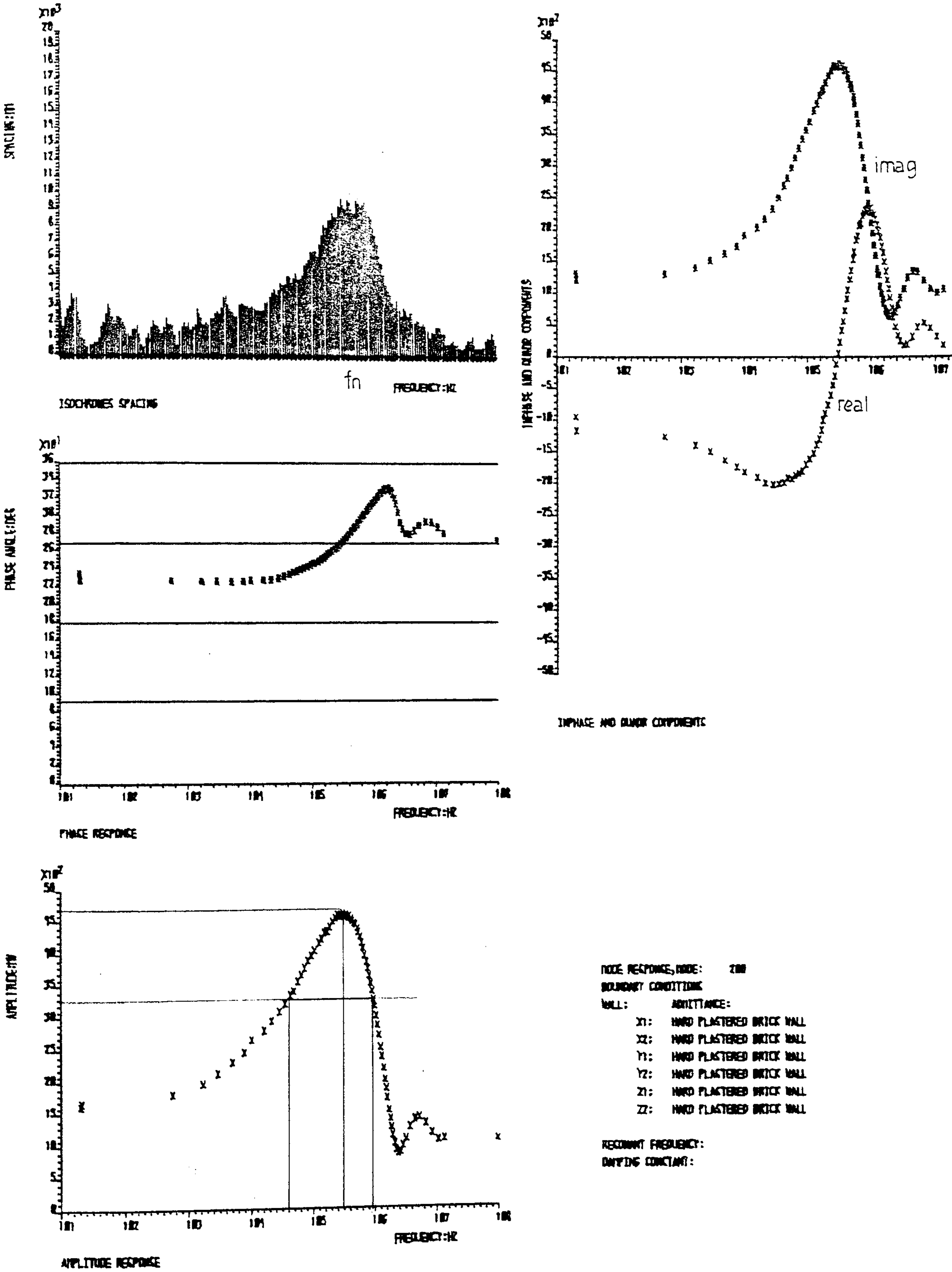
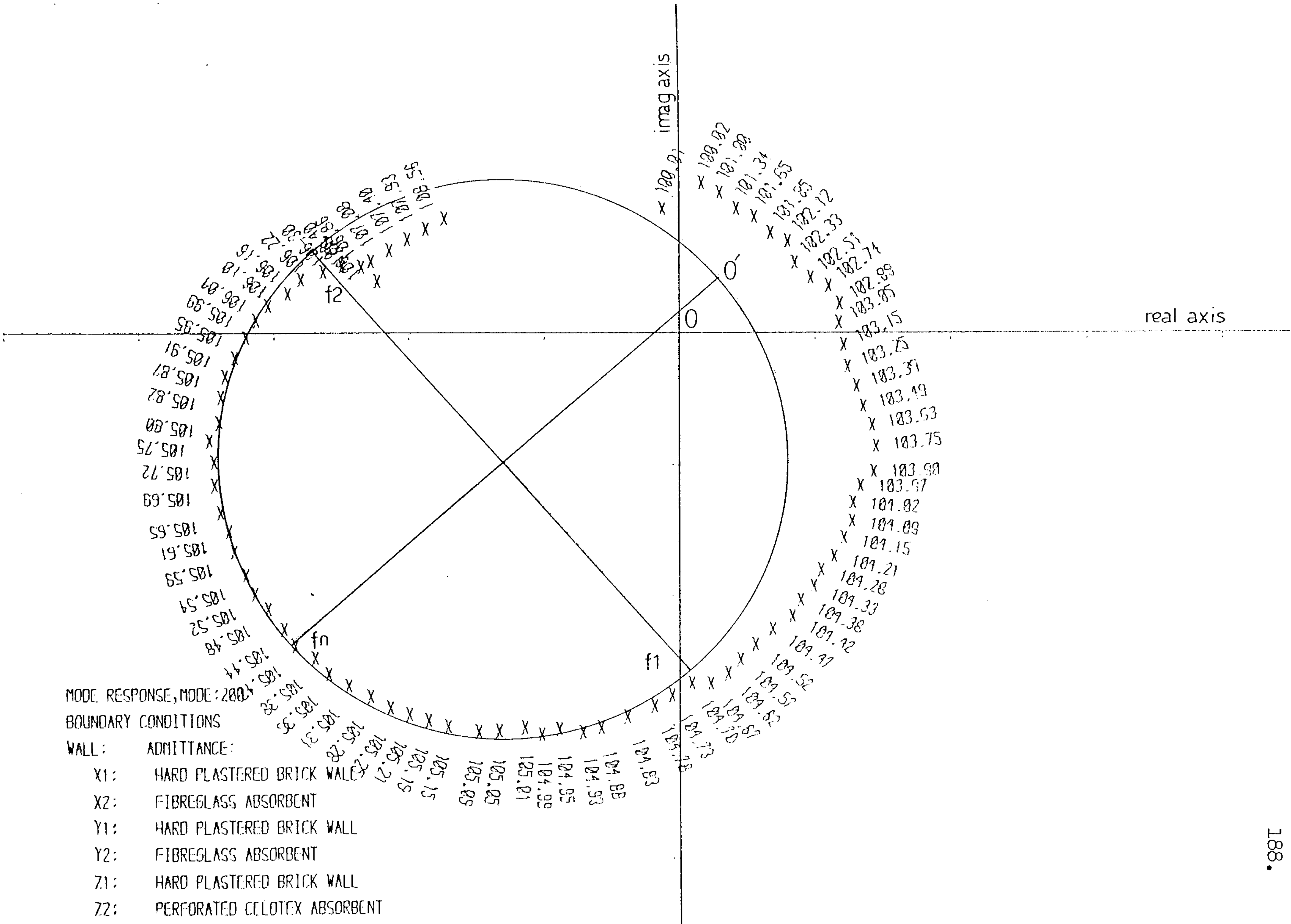


FIG.(6.6B): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(200) IN A BARE WALLED ROOM.



RESONANT FREQUENCY:
DAMPING CONSTANT:

FIG.(6.6C): MEASURED COMPLEX RECEPTANCE OF MODE(200) IN A TREATED WALLED ROOM.

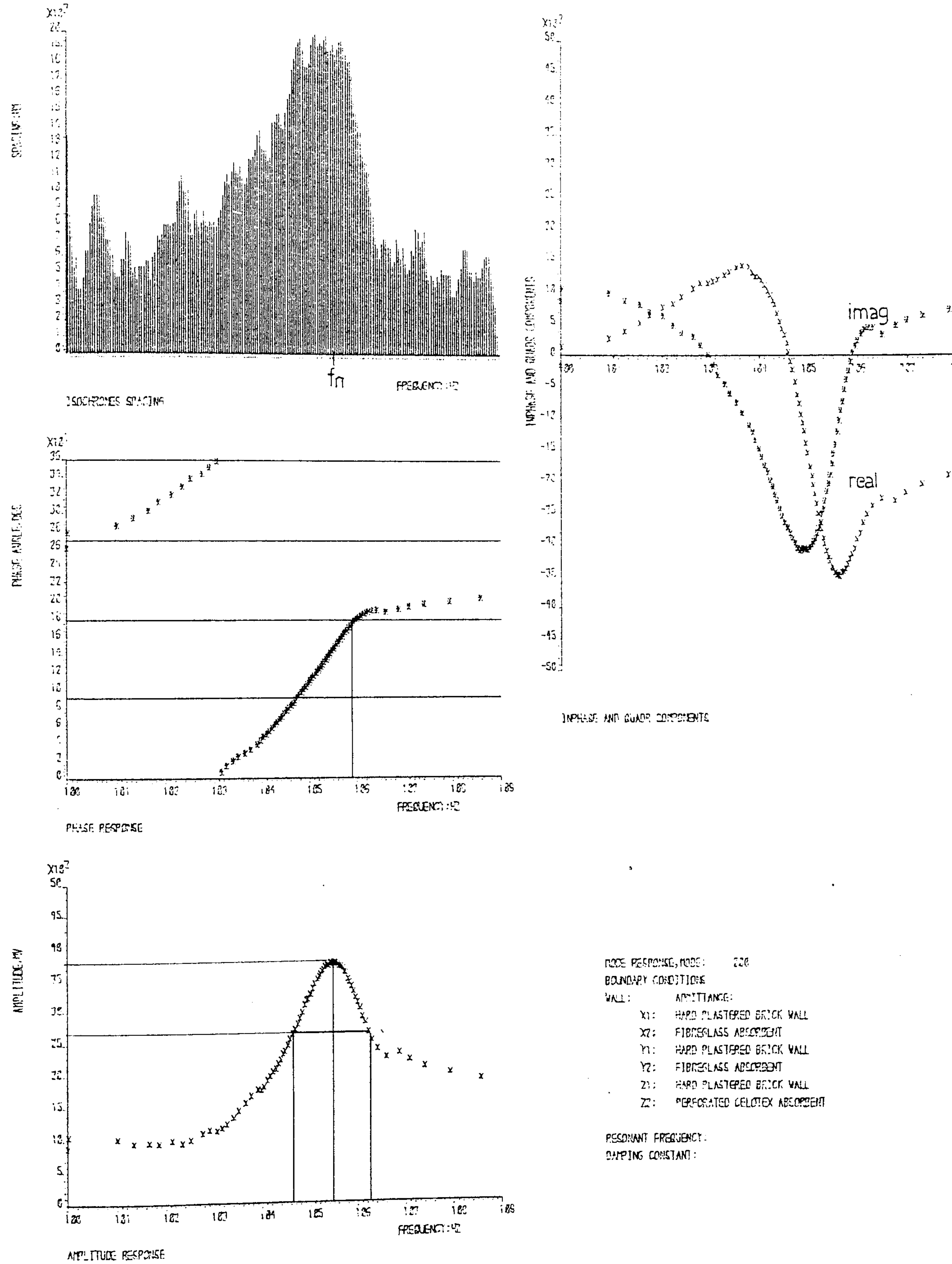


FIG. (6.6D): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(200) IN A TREATED WALLED ROOM.

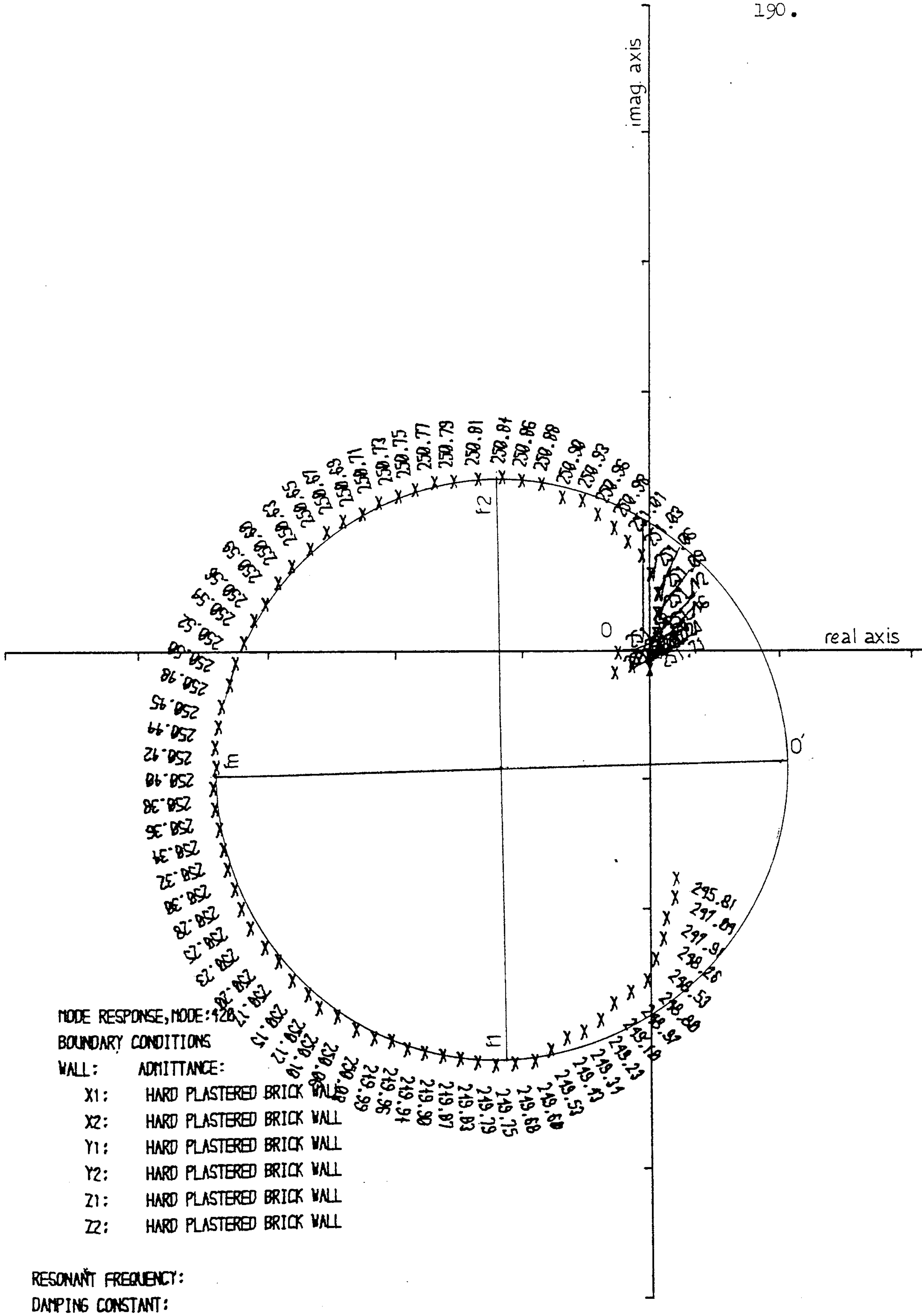


FIG. (6.7A): MEASURED COMPLEX RECEPTANCE OF MODE (420) IN A BARE WALLED ROOM.

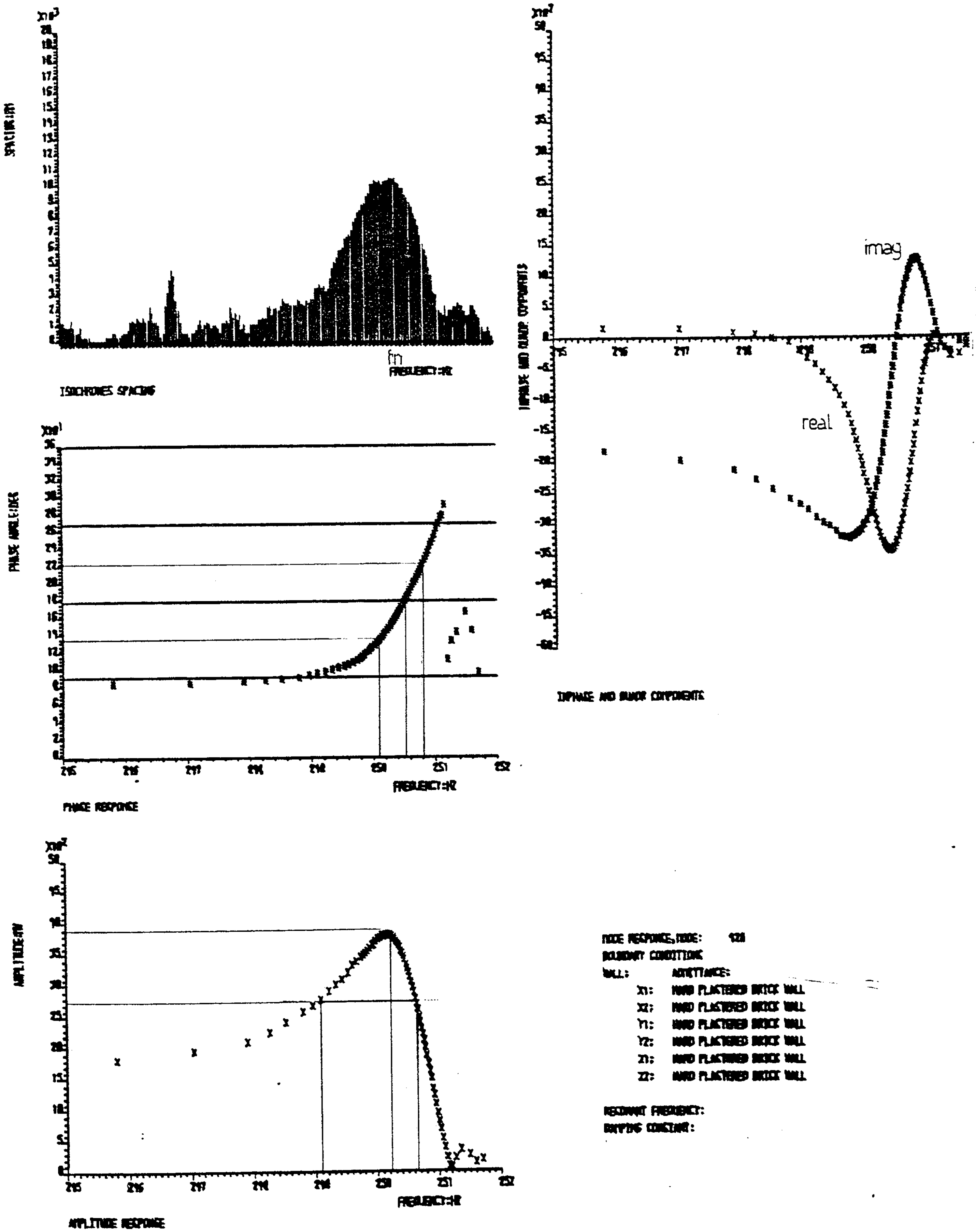
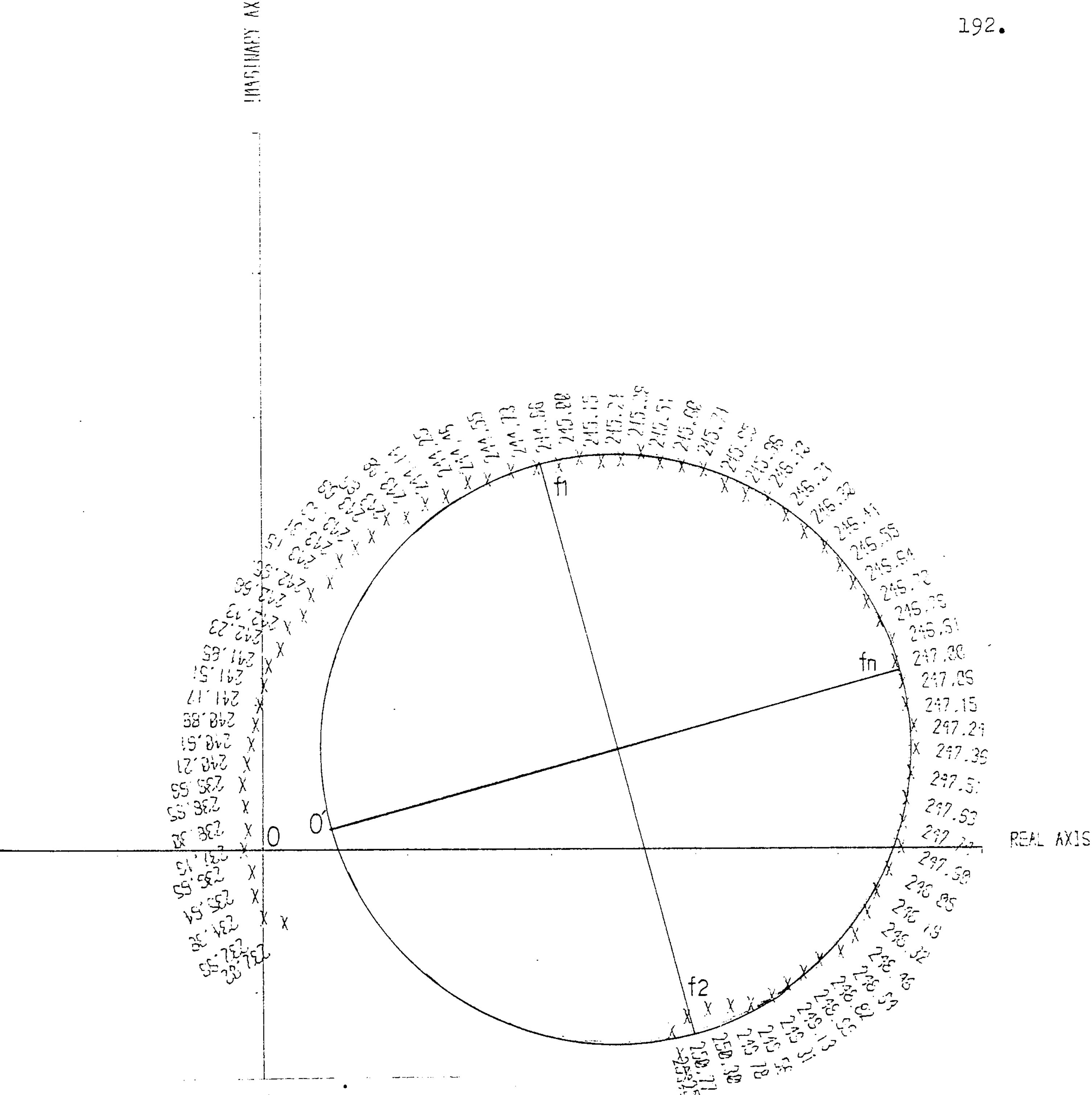


FIG.(6.7B): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MOE(420) IN A BARE WALLED ROOM.



MODE RESPONSE, MODE:420

BOUNDARY CONDITIONS

WALL: ADMITTANCE:

X1: HARD PLASTERED BRICK WALL

X2: FIBREGLASS ABSORBENT

Y1: HARD PLASTERED BRICK WALL

Y2: FIBREGLASS ABSORBENT

Z1: HARD PLASTERED BRICK WALL

Z2: PERFORATED CELOTEX ABSORBENT

RESONANT FREQUENCY:

DAMPING CONSTANT:

FIG.(6.7C): MEASURED COMPLEX RECEPTANCE OF MODE (420) IN A TREATED WALLED ROOM.

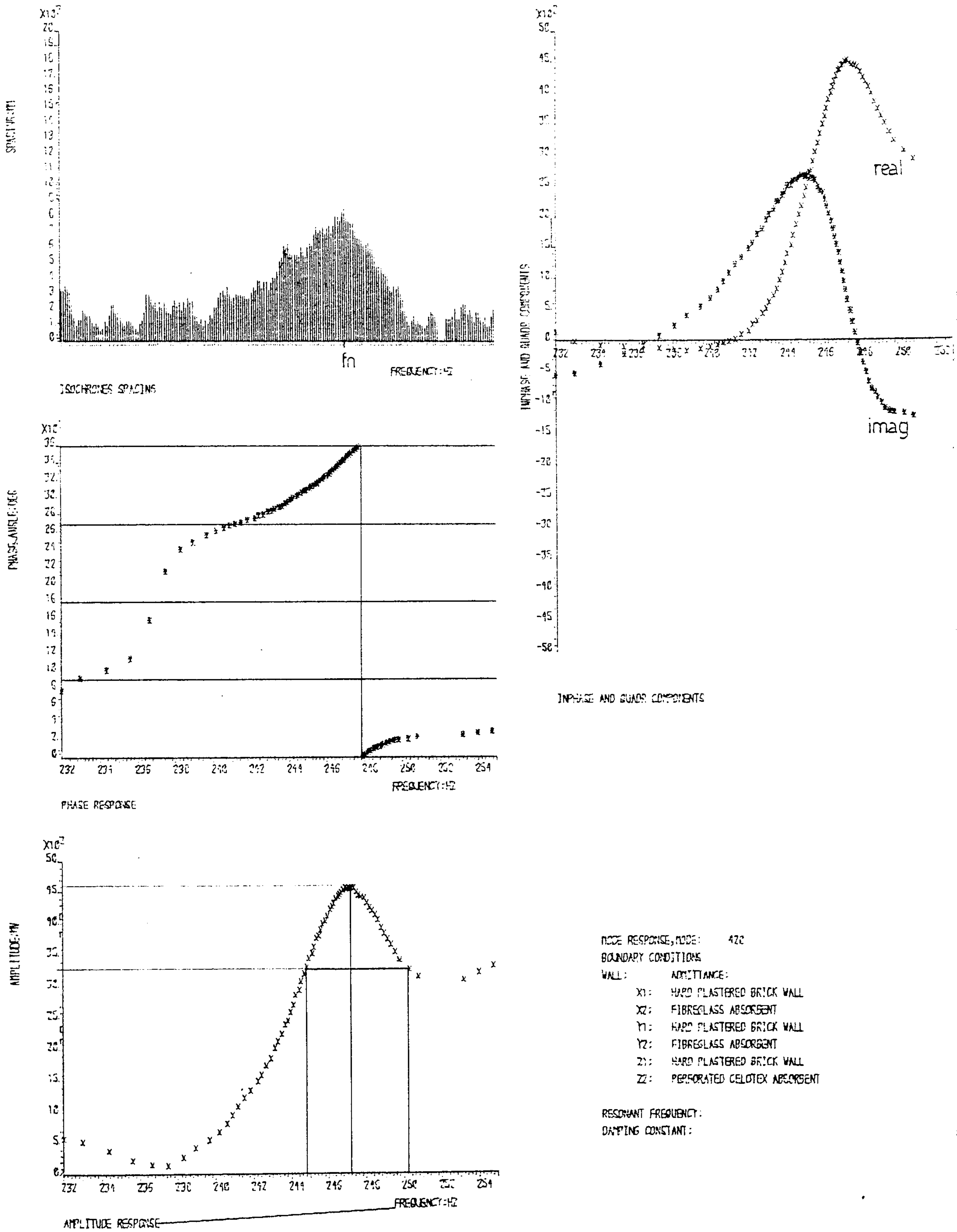
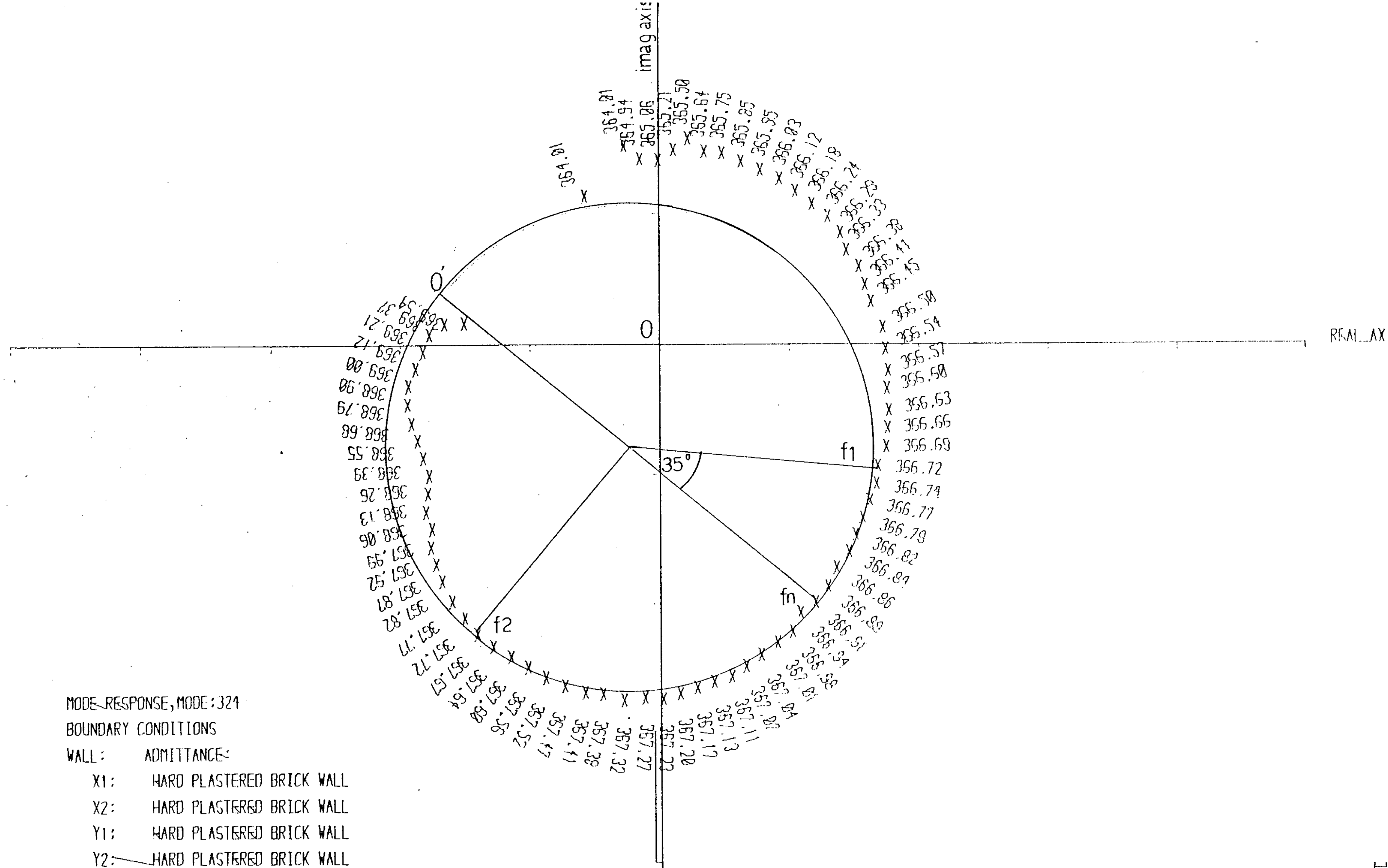


FIG.(6.7D): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(420) IN A TREATED WALLED ROOM.



MODE RESPONSE, MODE: 324

BOUNDARY CONDITIONS

WALL: ADMITTANCE:

X1: HARD PLASTERED BRICK WALL

X2: HARD PLASTERED BRICK WALL

Y1: HARD PLASTERED BRICK WALL

Y2: HARD PLASTERED BRICK WALL

Z1: HARD PLASTERED BRICK WALL

Z2: HARD PLASTERED BRICK WALL

RESONANT FREQUENCY:

DAMPING CONSTANT:

FIG.(6.8A): MEASURED COMPLEX RECEPTANCE OF MODE (324) IN A BARE WALLED ROOM.

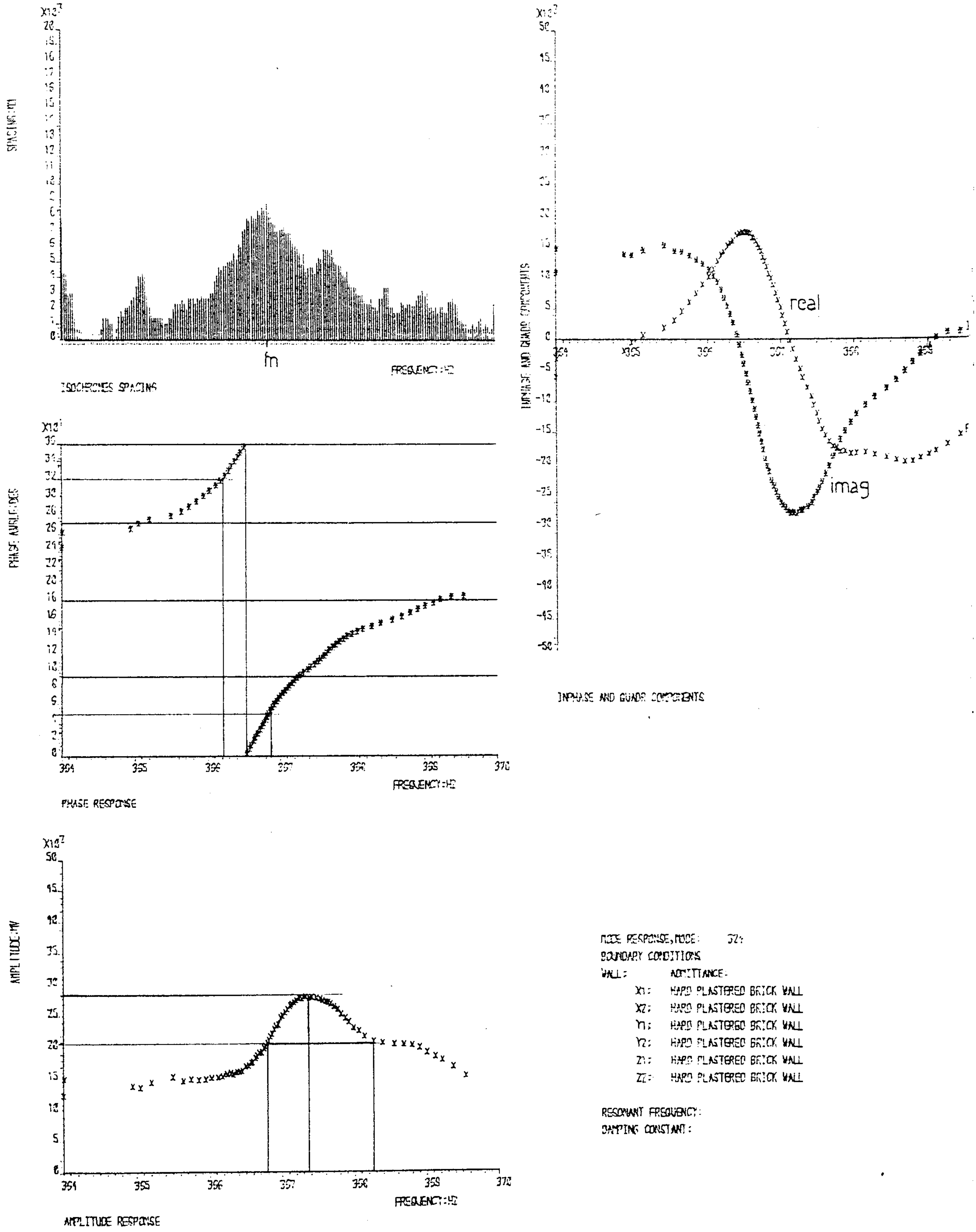


FIG.(6.8B): FREQUENCY SPACING, AMPLITUDE, PHASE AND RESOLVED COMPONENTS OF RECEPTANCE OF MODE(324) IN A BARE WALLED ROOM.

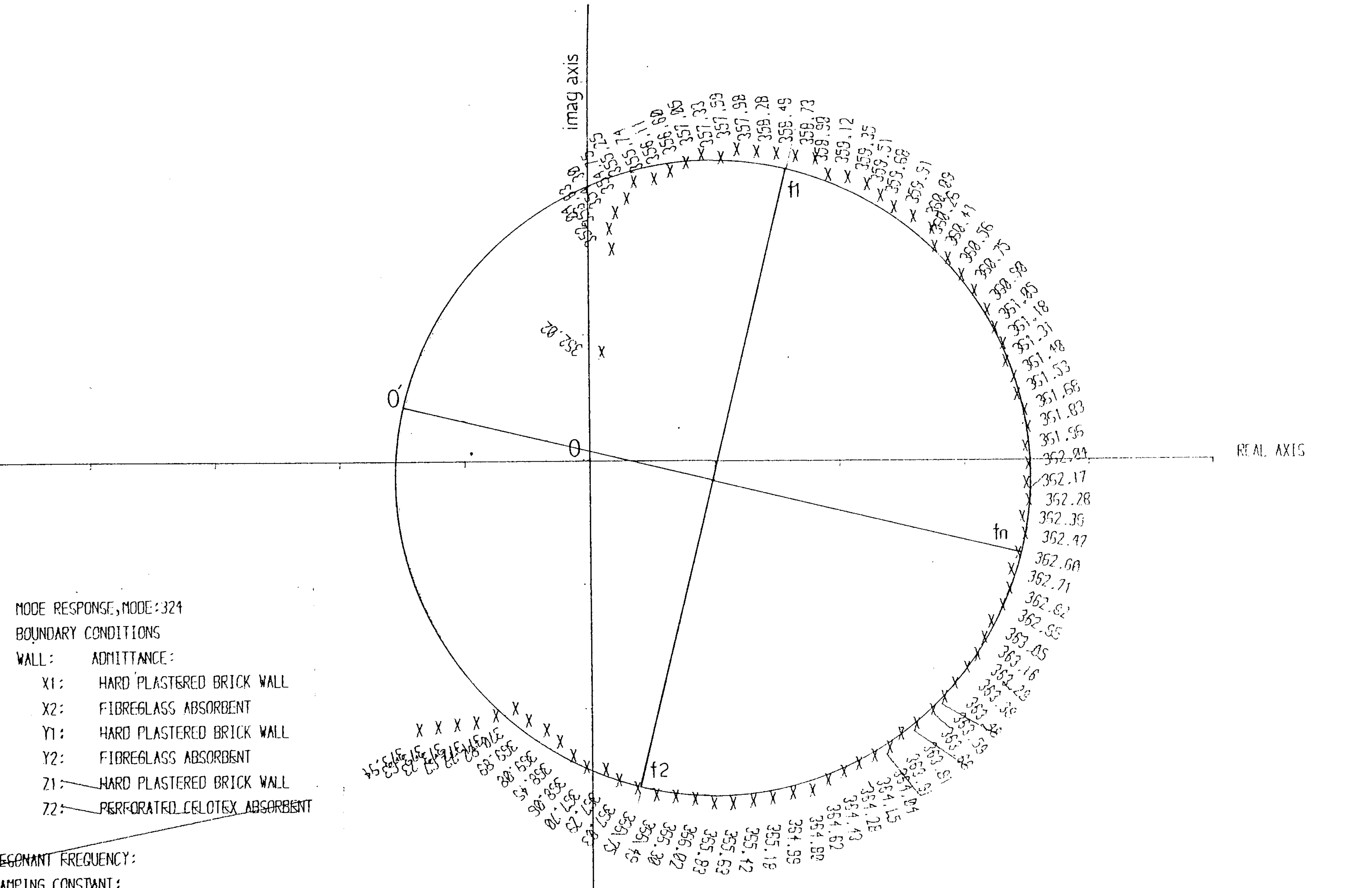


FIG.(6.8C): MEASURED COMPLEX RECEPTANCE OF MODE (324) IN A TREATED WALLED ROOM.

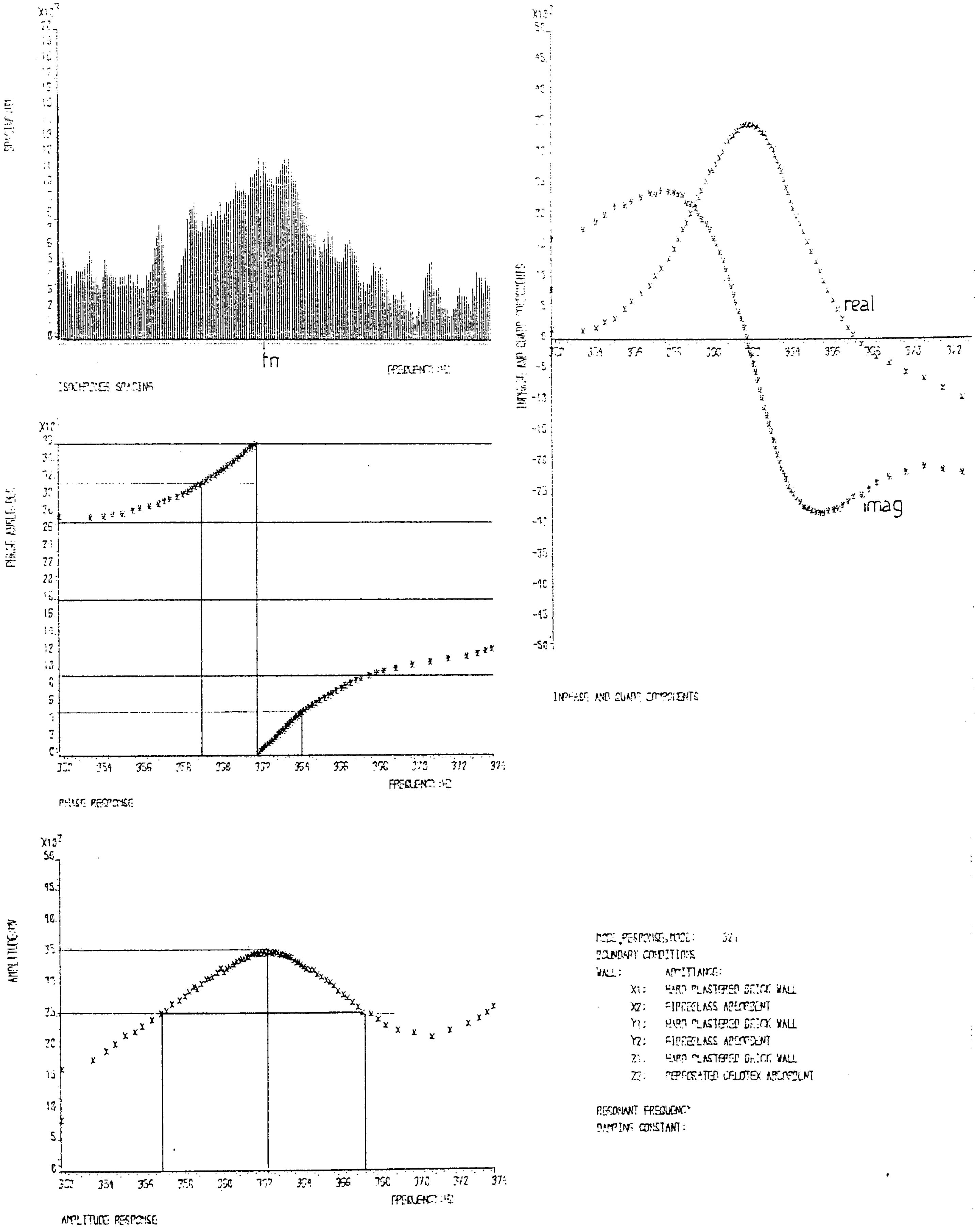
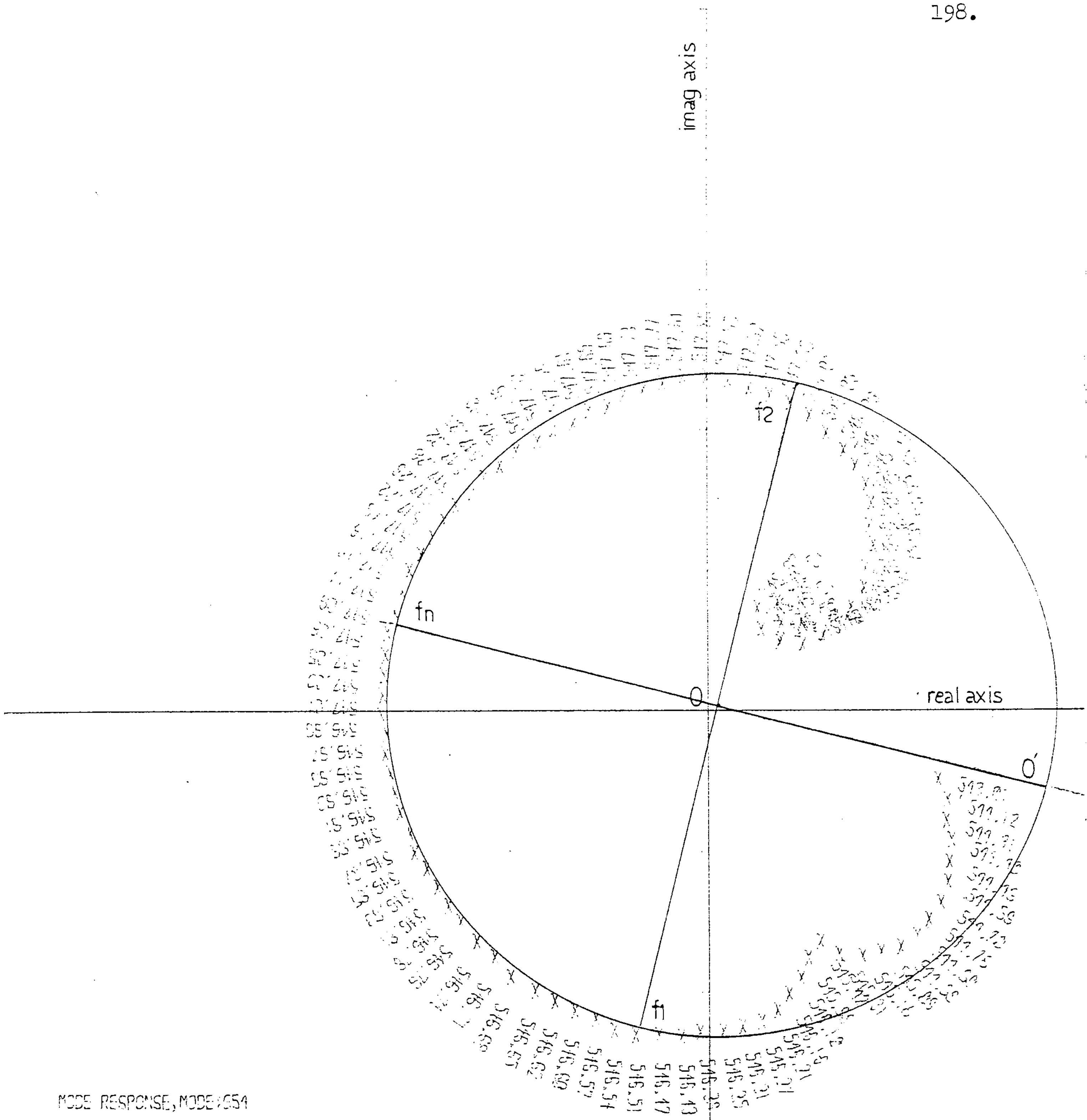


FIG.(6.8D): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(324) IN A TREATED WALLED ROOM.



MODE RESPONSE, MODE:654
BOUNDARY CONDITIONS
WALL: ADMITTANCE:
X1. HARD PLASTERED BRICK WALL
X2. HARD PLASTERED BRICK WALL
Y1: HARD PLASTERED BRICK WALL
Y2: HARD PLASTERED BRICK WALL
Z1: HARD PLASTERED BRICK WALL
Z2: HARD PLASTERED BRICK WALL

RESONANT FREQUENCY.
DAMPING CONSTANT:

FIG.(6.9A): MEASURED COMPLEX RECEPTANCE OF MODE(654) IN A BARE WALLED ROOM.

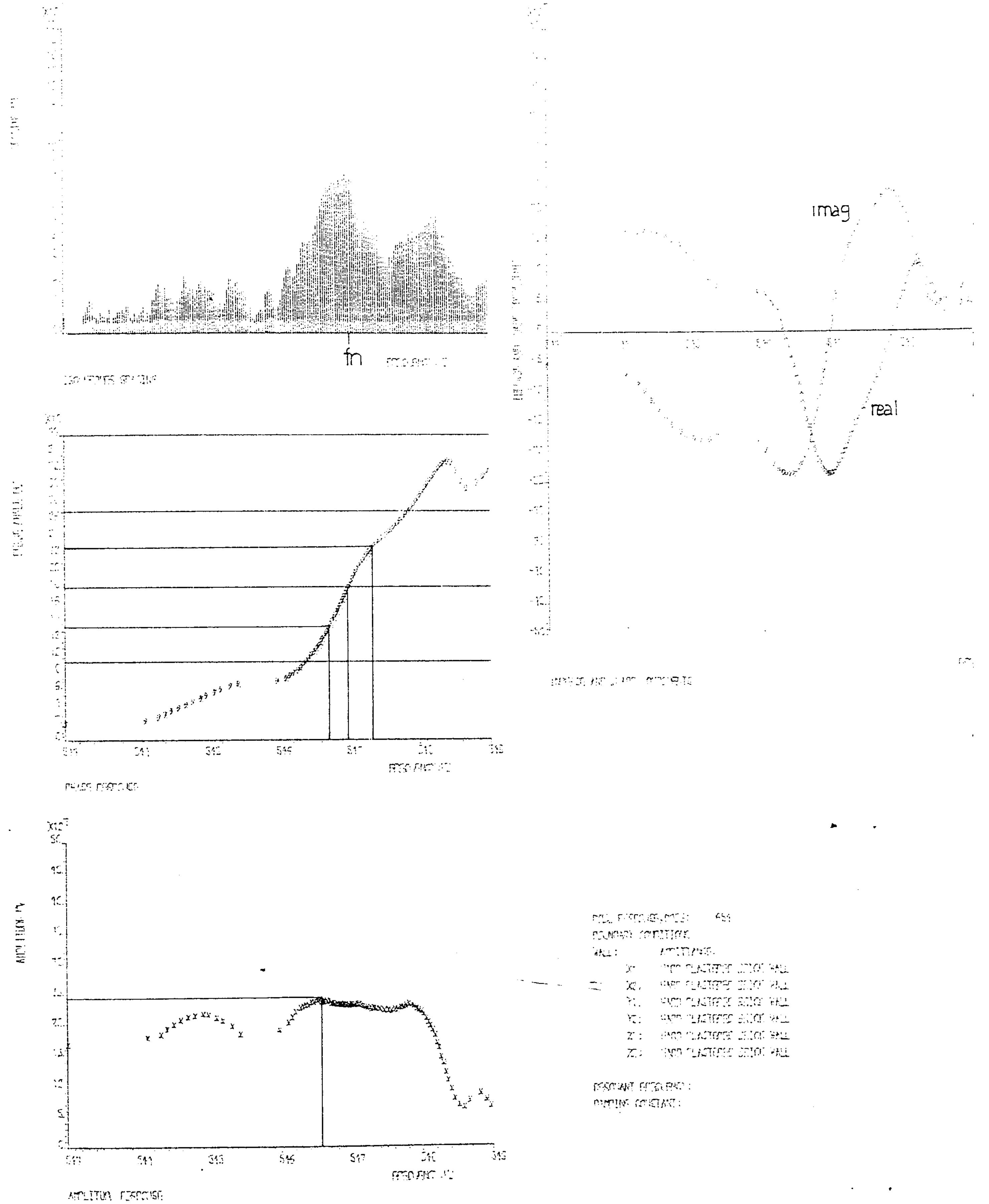
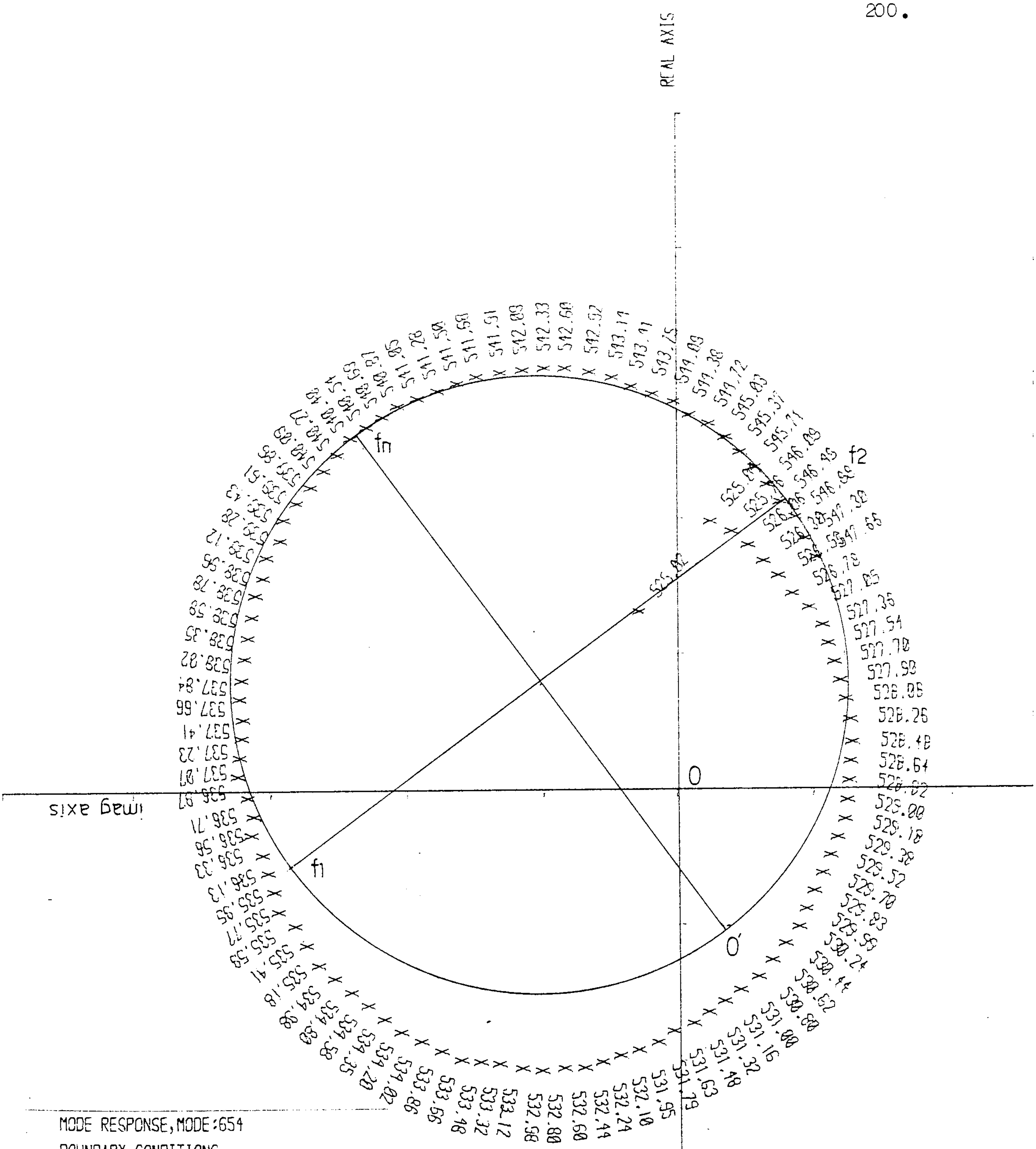


FIG.(6.9B): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(654) IN A BARE WALLED ROOM.



MODE RESPONSE, MODE:654

BOUNDARY CONDITIONS

WALL: ADMITTANCE:

X1: HARD PLASTERED BRICK WALL

X2: FIBREGLASS ABSORBENT

Y1: HARD PLASTERED BRICK WALL

Y2: FIBREGLASS ABSORBENT

Z1: HARD PLASTERED BRICK WALL

Z2: PERFORATED CELOTEX ABSORBENT

RESONANT FREQUENCY:

DAMPING CONSTANT:

FIG.(6.9C): MEASURED COMPLEX RECEPTANCE OF MODE(654) IN A TREATED WALLED ROOM.

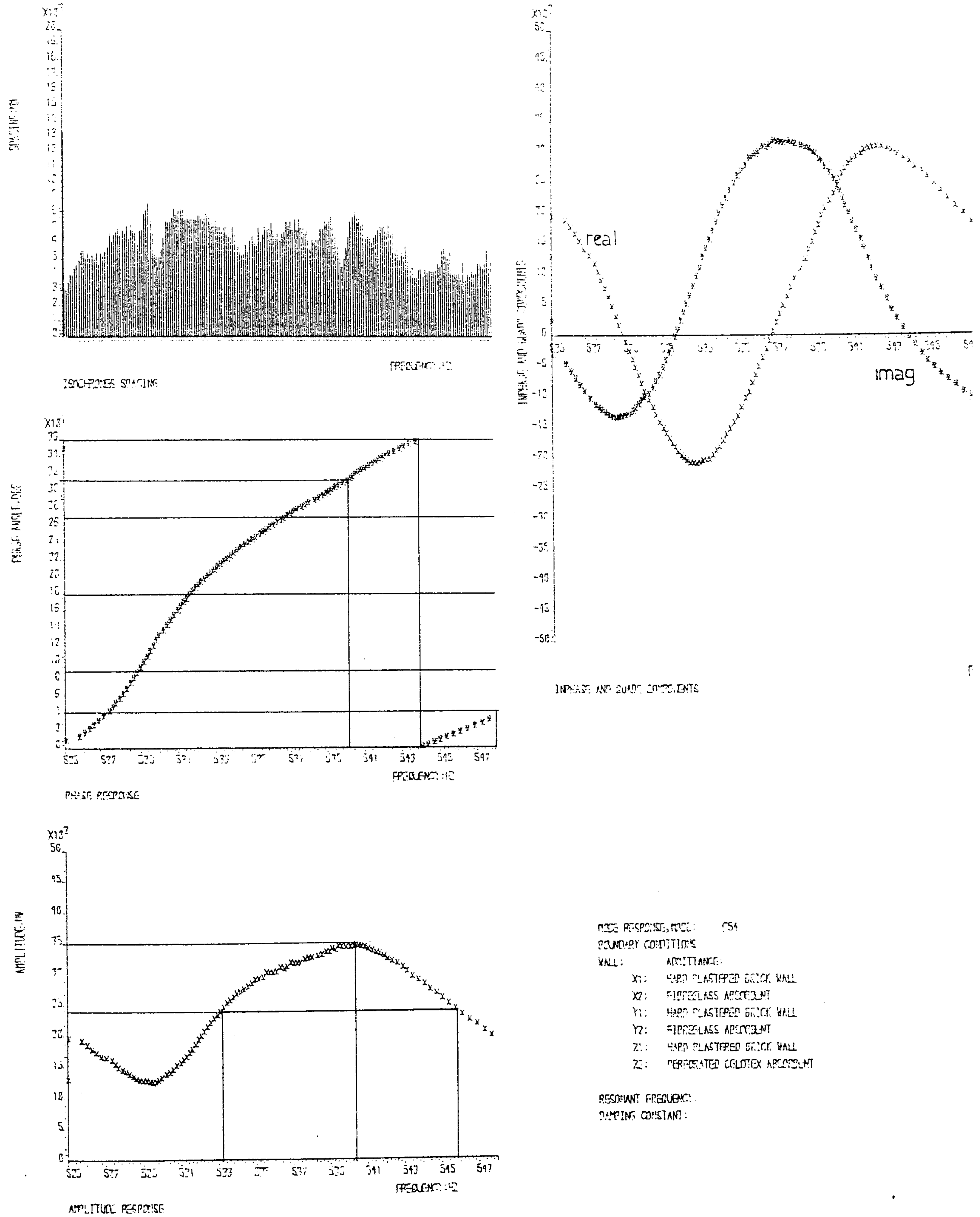


FIG.(6.9D): FREQUENCY SPACING, AMPLITUDE, PHASE & RESOLVED COMPONENTS OF RECEPTANCE OF MODE(654) IN A TREATED WALLED ROOM.

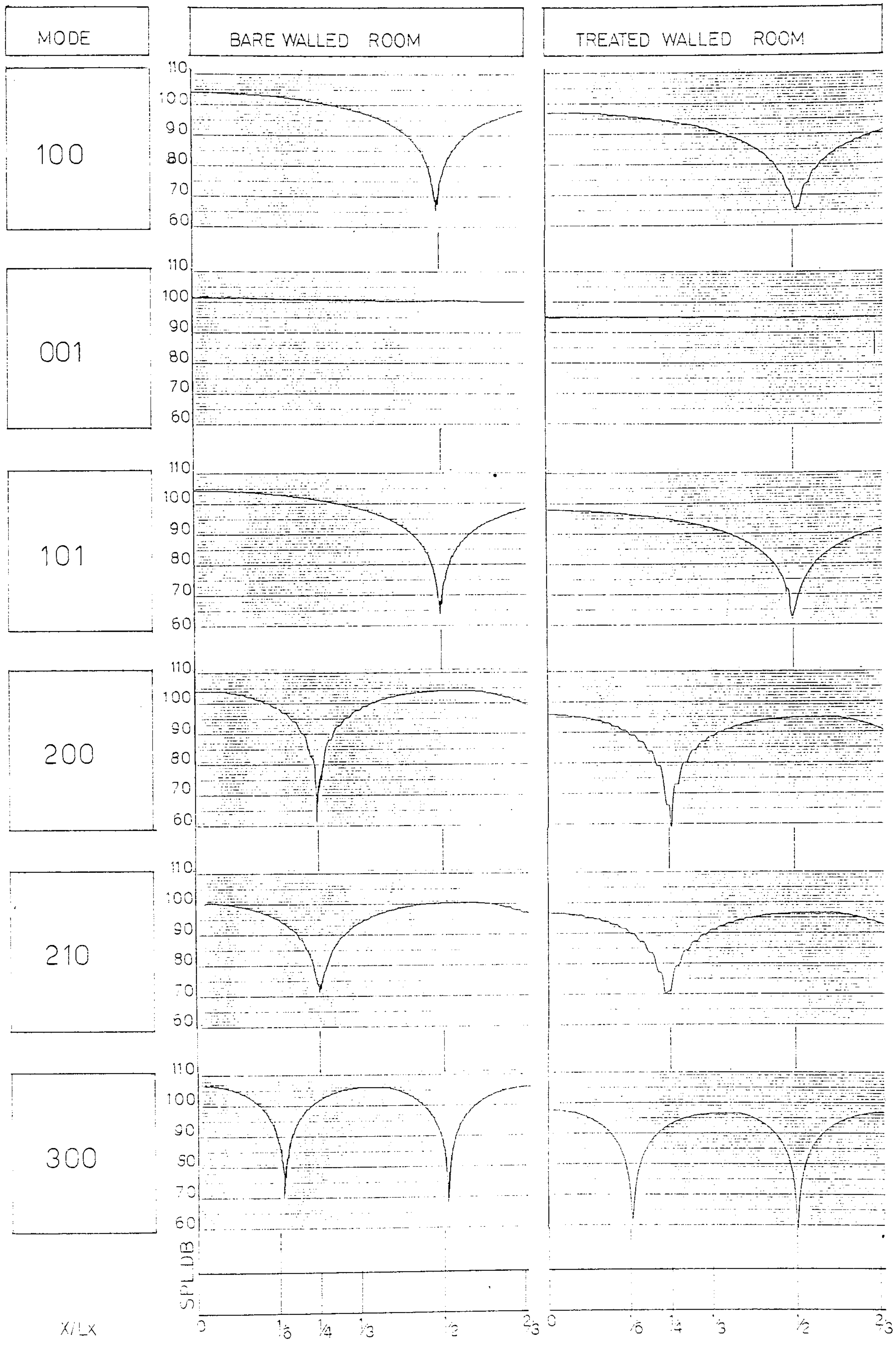


FIG. (6.10A): MODAL SHAPES IN BARE WALLED AND TREATED WALLED ROOMS.

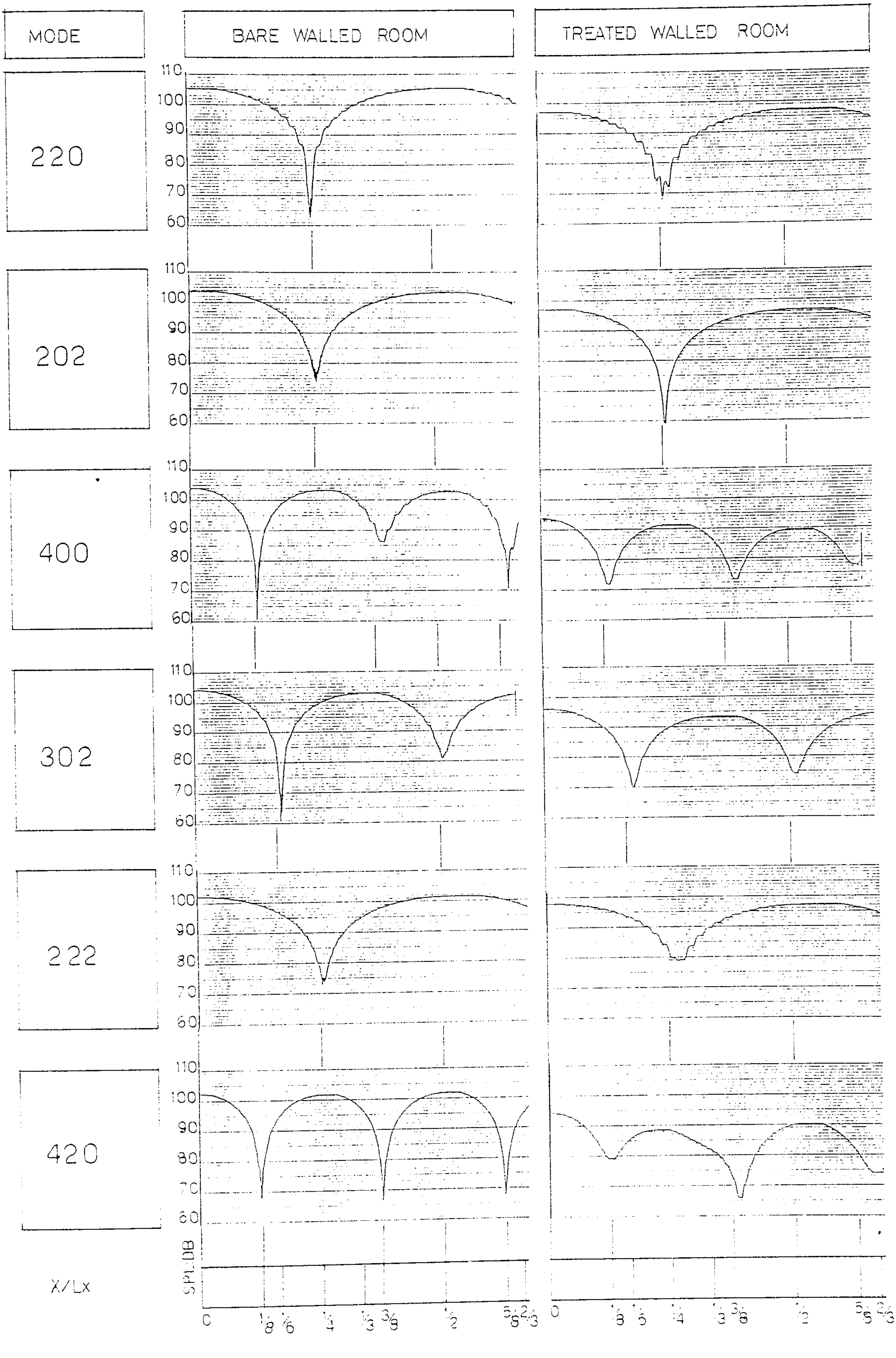


FIG.(6.10B): MODAL SHAPES IN BARE WALLED AND TREATED WALLED ROOMS

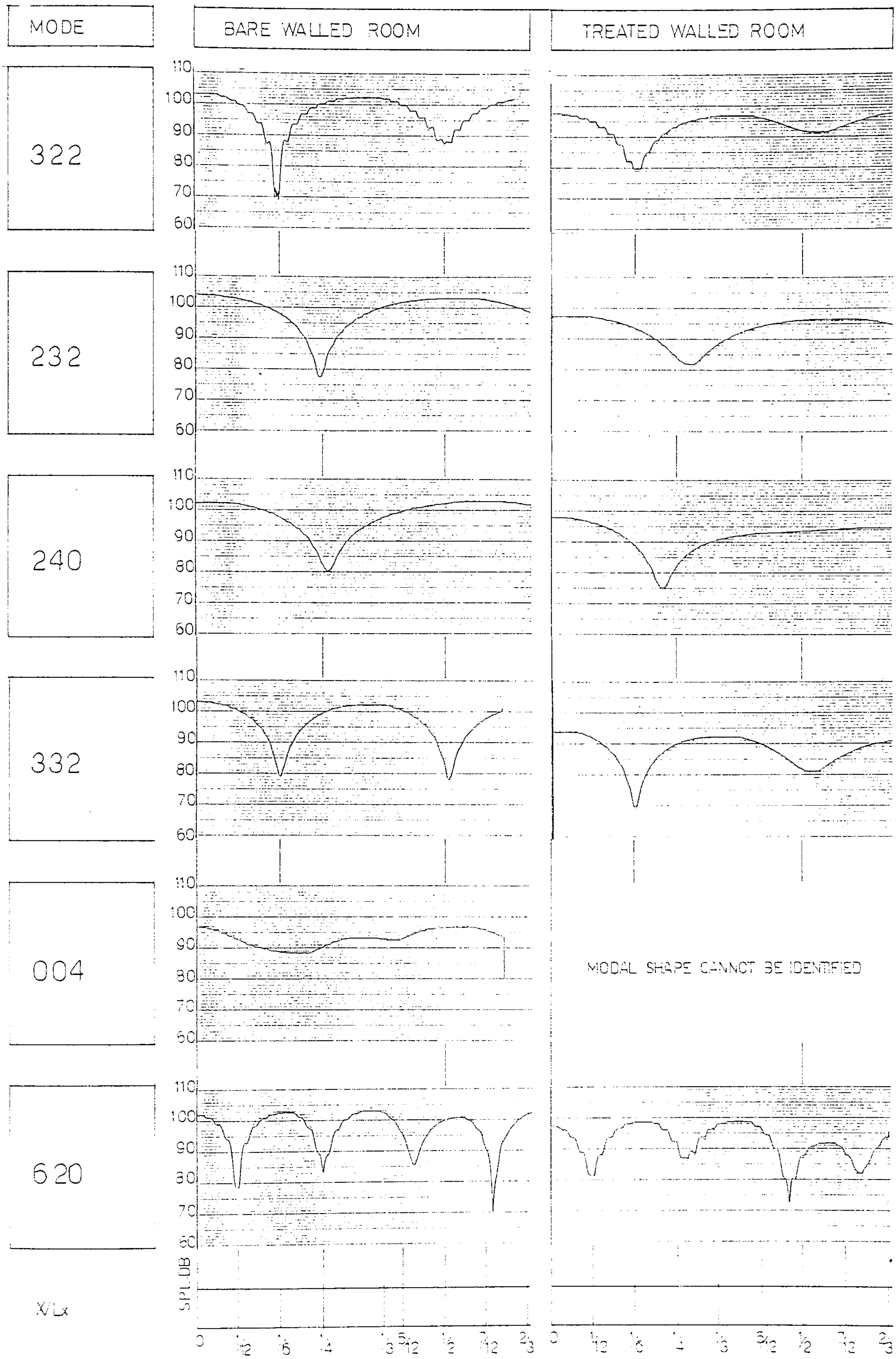


FIG.(6.10C): MODAL SHAPES IN BARE WALLED AND TREATED WALLED ROOMS

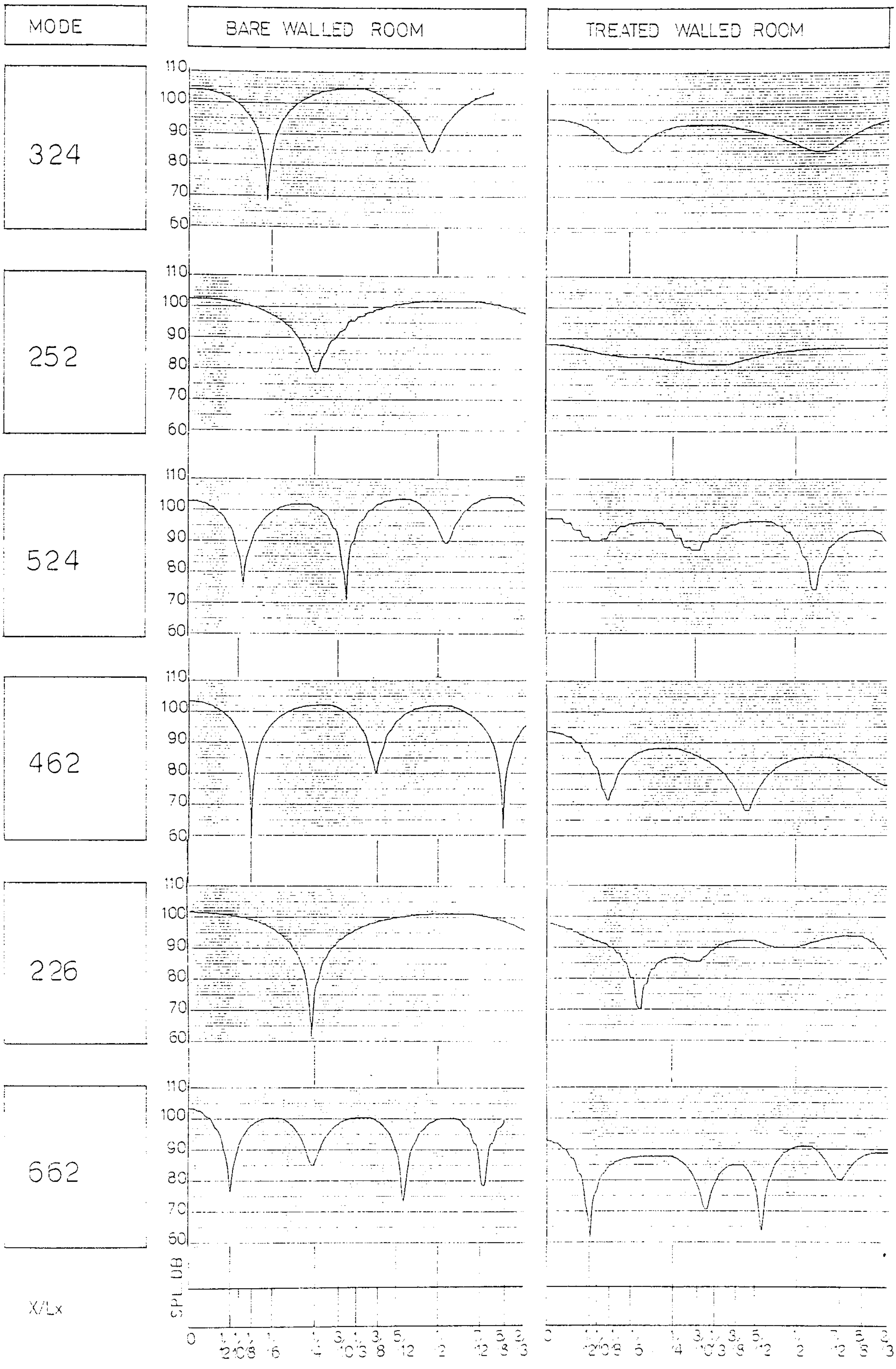


FIG.(6.10D): MODAL SHAPES IN BARE WALLED AND TREATED WALLED ROOMS

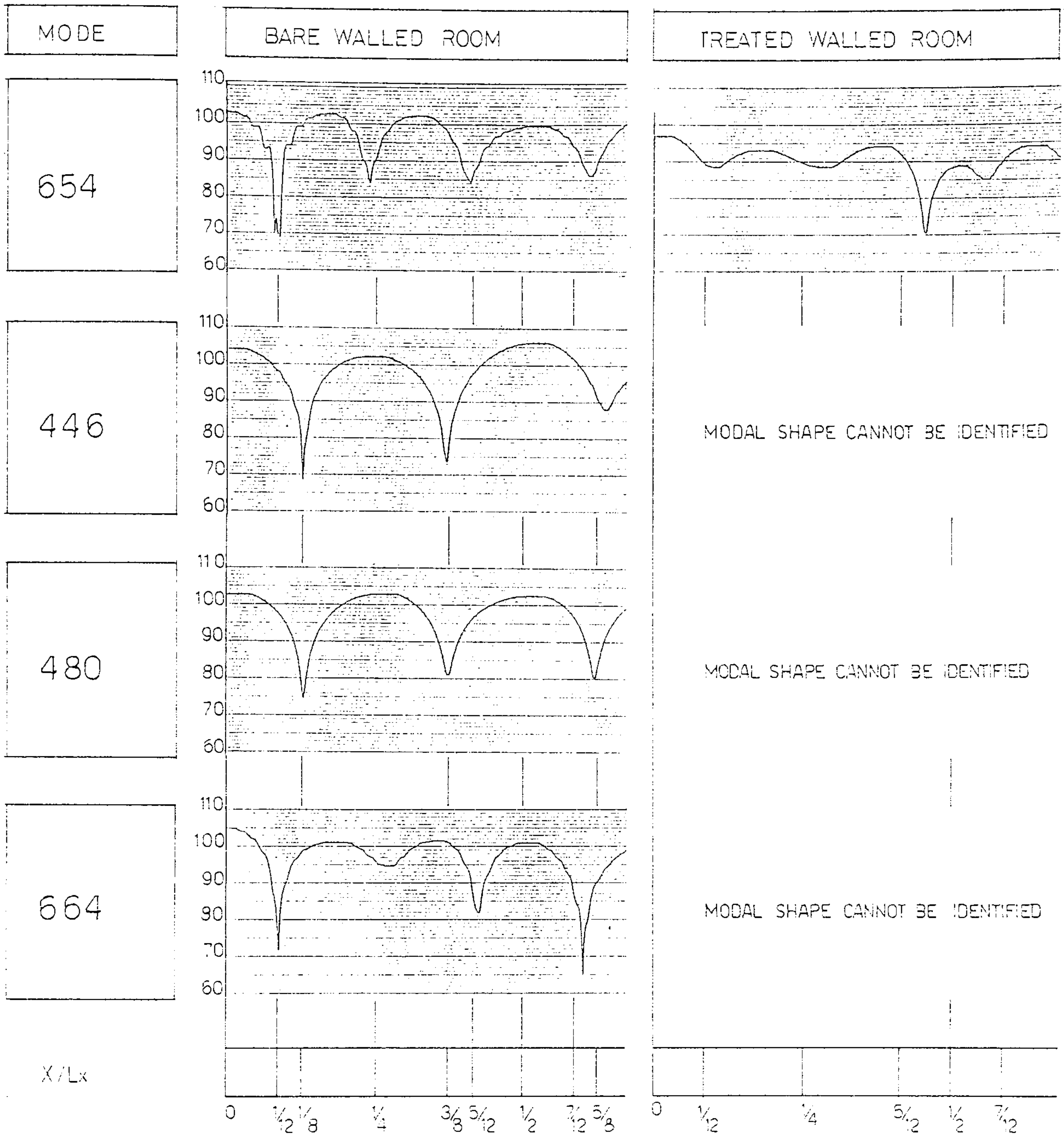


FIG.(6.10E): MODAL SHAPES IN BARE WALLED AND TREATED WALLED ROOMS.

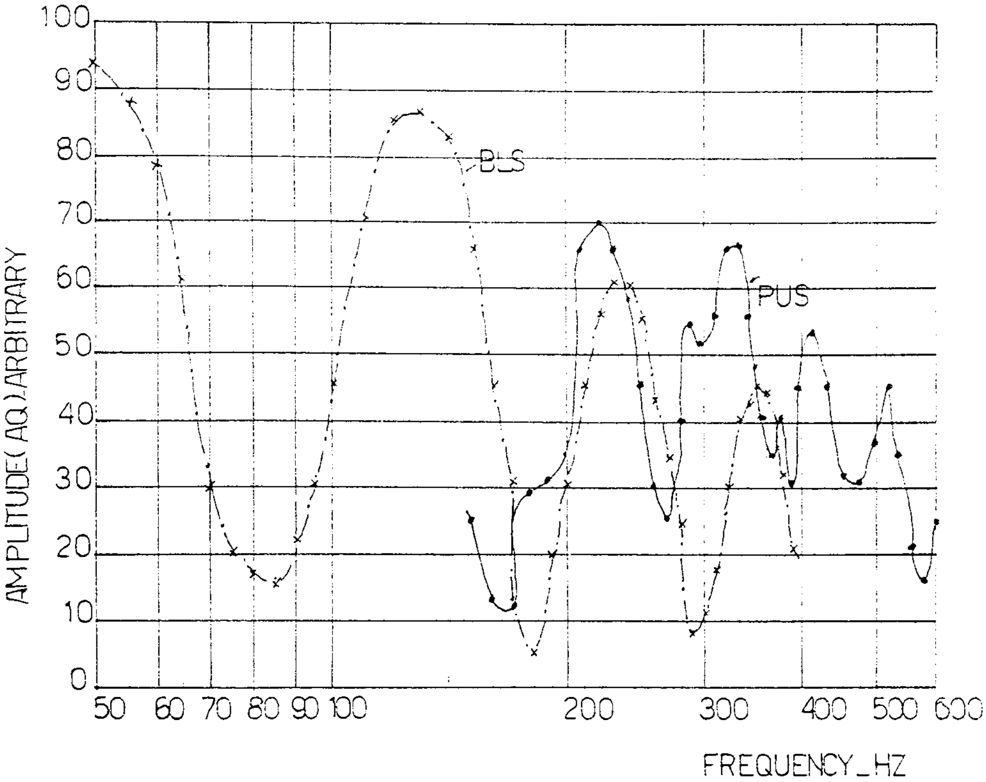


FIG.(6.11A): CALIBRATION MEASUREMENTS— AMPLITUDE_FREQUENCY RESPONSE

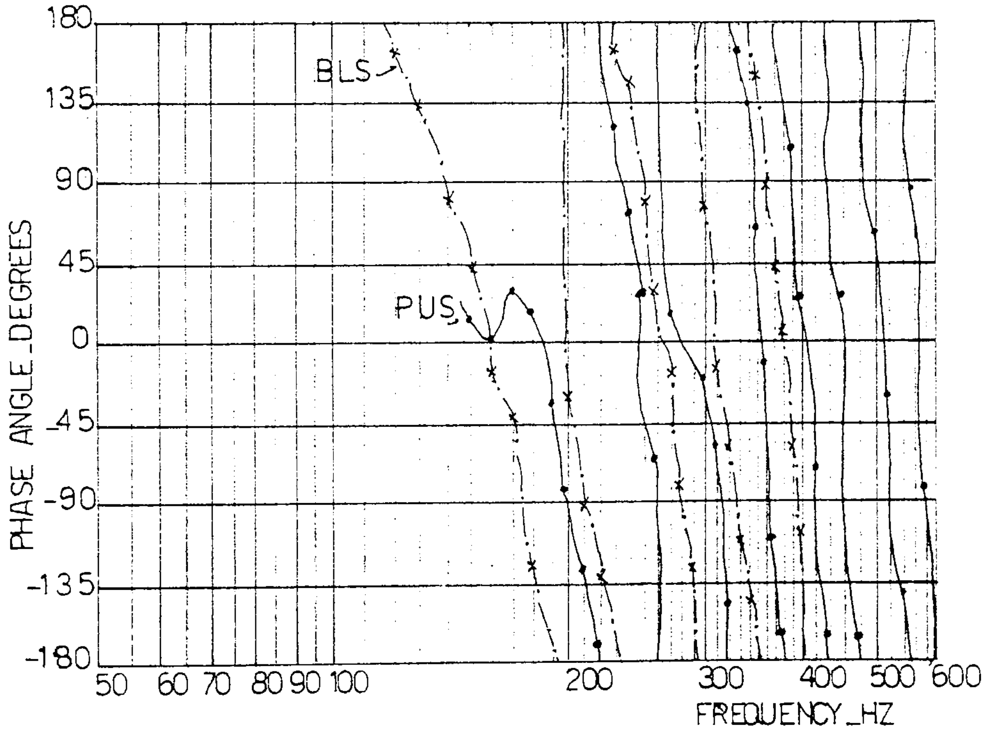


FIG.(6.11B): CALIBRATION MEASUREMENTS— PHASE_FREQUENCY RESPONSE

BLS: complex response measurement system with box loudspeaker as sound source

PUS: complex response measurement system with pressure unit as sound source

CHAPTER SEVEN

APPLICATIONS TO REAL ROOMS.

7.1) INTRODUCTION:

So far in this thesis, the prime concern has been devoted to deriving theoretical predictions and establishing experimental measurements of modal parameters characterizing the physical properties of sound fields in small rooms.

It is a major difficulty with problems of room acoustics that the final judgement of the room performance is almost entirely subjective[62], for the 'final consumers'[17] of acoustics are the listener and performer. We must therefore attempt to relate these physical properties of the sound fields to certain hearing impressions upon which the subjective judgements of the room performance are likely to be made.

Our concern will be focu ed on a particular set of small rooms, those used primarily for microphone pick-up. These include radio broadcasting, television and sound-recording studios. The special nature of the problem is primarily a consequence of the differences between monoaural and binaural hearing[16,63]. Consequently, microphone pick-up is subject to about the same pecularities as monoaural hearing, and therefore a studio should have about the same acoustical properties as would be required for the most favourable conditions for listening with one ear.

However the introduction of stereophonic systems in sound transmission and recording is an attempt to overcome this limitation. The use of multi-channel systems fed from several pick-up microphones leads to a change in the ratio of reflected and direct sound energy in reproduced sounds that helps in the location of the apparent sound source correctly and gives naturalness and fullness to the sound[5]. The reproduced sounds have speicific time shifts which change with changes in the position of the performer relative to the microphone system during recording and this helps to enhance the spatial character of the sound[16].

In this chapter the following aspects will be considered:

(1) The subjective qualities favoured by the listener and performer for the assessment of studio performance.

(2) The objective requirements for good acoustics in studios based on those subjective qualities.

(3) How the objective requirements can be related to the physical properties of sound fields in small rooms discussed in the previous chapters.

(4) How the physical properties can be controlled by some design and constructional data expressed in terms of the room geometry and the absorptive properties of its bounding surfaces.

7.2) THE SUBJECTIVE QUALITIES OF SMALL STUDIOS:-

In view of differences between speech and music, the requirements for speech studios and for music studios are not identical and accordingly, the qualities for their subjective judgements are different.

In studios, intended for speech, the listener expects the room to render speech intelligible and therefore the prime objective in the design of such studios would be the realization of the conditions that will provide intelligibility of speech.

In studios, used for music, the room is expected to support the music being performed and the objective is the creation of the most favourable conditions for the preservation and enrichment of the formal quality of the music.

The intelligibility of speech can be rated quantitatively by articulation testing techniques[16].

The rating of music is more elusive and therefore some suitable criteria must be obtained for the most preferred listening conditions in music rooms.

However there are certain broad features that apply to both types of rooms, and the emphasis on these common features is justifiable since it is not uncommon for studios to be used on a program sharing basis, especially in television studios.

The problem is aggravated by the numerous factors which influence the making of subjective value judgement on the acoustical qualities of rooms used for speech or music presentation. Among these factors are[64]:

(1) The physiological properties of our hearing organ and the manner in which hearing sensations are processed by the brain. The power of frequency discrimination and localization of the probable sound source are the most prominent of these properties of the ear.

(2) Our hearing habits are governed by the individual and group background in associating different musical styles with particular acoustical environments.

(3) The personal aesthetic sensitivity of the listener and the fact that one judges the qualities of a certain performance against preconceived norms which lie in experience.

In judging the acoustical qualities of a studio one would then expect a difference of opinion not only on which qualities play an important role in the overall assessment, but also on the manner those very individual qualities are judged[65].

As may be expected, the number of individual qualities which contribute to the overall assessment, as reported in the literature, is huge, and there are wide differences of opinion on their relative significance. A few of these qualities are[64,65,66]:-

(1) Fullness of tone and the effect of the room as compared to outdoors in listening with ease to speech or music.

(2) Blend, in hearing all players as a homogeneous source.

(3) Liveness and richness.

(4) Spatial impression, broadening of the source and the sensation of feeling 'inside the music'.

(5) Definition and clarity in hearing distinct speech syllables and separate musical notes.

(6) Tonal warmth and intimacy.

(7) Freedom from tonal colouration and echo disturbance.

The subjective qualities of rooms can only be investigated experimentally by using suitable test subjects. The tests are performed either by judging the acoustical qualities of completed rooms or by synthesizing sound fields with well defined objectives in anechoic rooms and judging them subjectively. Factor analysis methods[65] are employed to correlate the individual qualities to form an overall assessment.

7.3) THE CONDITIONS FOR 'GOOD ACOUSTICS' IN SMALL STUDIOS:-

The most important requirements that should be considered in the design of studios are:

(1) Spatial diffusion and uniformity of the distribution of sound energy throughout the room: Diffusion improves the liveness of the room and enhances the natural qualities of speech and music[16]. In practical considerations it reduces the hazards of improper microphone locations. Alternative techniques for measurement of diffusion have been worked out and reported[19,67].

(2) Optimum ratio of direct to reverberant sound energy: Reverberation is considered to contribute to the loudness of direct sound in speech rooms. It blends musical sounds

(2) contd.

and masks their imperfections.

(3) Lateral Reflections: the provision of a spatial impression of the sound field is affected by the directional distribution of the sound components. In this respect lateral reflections are more significant than horizontal ones, and the ratio of the lateral to the non-lateral energy is a measure of the spaciousness of the room.

(4) Early reflections: the ratio of the early sound, which is a combination of direct sound and first reflections to the reverberant sound defines the rooms liveness[75]. It is the sound signal to reach the listener first which, subjectively determines the direction from which the sound comes. This fact is called - according to Cremer - the law of the first wave front[17]. The conditions under which this rule is valid has been investigated by Haas[17] and his findings may be summarized as follows: for our hearing sensation, the primary sound determines the impression of direction, even when the reflection - provided it has a suitable delay time - is stronger by 10 dB. Delays up to 50 ms seconds are considered tolerable[68].

(5) Freedom from acoustical defects: the superposition of a strong isolated component on the sound can cause an undesirable effect, especially with music, namely a colouration. This is represented by a characteristic change of the signal spectrum, which takes the form of an unnatural and often monotonous emphasis of a certain frequency of sound[63]. This occurs at low frequencies.

The phenomena of flutter echo, by contrast, occurs at mid and high frequencies. The listener hears harmonic series of sound components which remain separate and distinct up

(5) contd.

to very high frequencies and are periodically repeated.

Both of these defects require proper treatment.

7.4) THE PHYSICAL PROPERTIES OF SOUND FIELDS IN SMALL ROOMS:

In order that the objective requirements for 'good acoustics' in small studios can be related to some physical properties of sound fields in small rooms, we must derive these physical properties in terms of the measurable modal parameters discussed earlier in the thesis i.e. the natural frequencies of the normal modes, their damping constants and modal shapes.

It is worth mentioning that, in considering small rooms, there are some basic differences in their acoustical behaviour between large and small rooms. Large rooms, by virtue of their size, are characterized by reverberation and echoes with delays of recognizable time scales, whereas in small studios reverberation is no longer appreciable as a time extension of the original sound. Instead, small studios are characterized by phenomena recognizable as frequency effects[63]. And whereas the wavelength of sounds in the low frequency end of the available spectrum is in the order of or even greater than the dimensions of the room, a large room has dimensions large compared to wavelengths of sound. Consequently, a small room will have well defined resonances. This can be seen from the predicted and measured modal frequencies derived in Chapters Five and Six.

The physical properties of sound fields in small rooms that will have influence on the basic requirements for good acoustics, discussed above, can be summarized as follows:

(1) The relative importance of different classes of modes is signified by special properties of each type of mode. This point has been dealt with by Mayo[69], who showed that axial

(1) contd.

modes alone are likely to have sufficient amplitudes and durations and their occurrence in low frequency regions is responsible for such acoustical defects as colouration. At high frequencies other tangential and oblique modes, which though virtually absent at low frequencies, are far more numerous than the axial modes, and their presence reduce the intensity of the axial modes.

The measurements of shifts in frequencies and damping reported in Tables (6.2A) and (6.2B) is in evidence of this fact.

(2) The effect of the spacing of the modes on the acoustical quality has been widely investigated. Mayo[70] considered 'It is the absence of modes rather than their presence, that spoils a small studio'. In his view, a prominent single mode is annoying because it lacks the necessary 'help' of other modes.

The effect of mode spacing (as discussed in Chapter Three) on diffusion has been investigated by Bolt[9] and others[10,71,72]. Equal spacing is considered favourable as it reproduces the sound faithfully and with time delays that extend the original sound[70]. The occurrence of clusters of closely-spaced modes with wide gaps in frequency should be avoided. Several ideal spacing incidences have been proposed by the above mentioned authors.

(3) The damping of individual modes affects both the steady-state amplitudes of sound as well as the decay rate of modes in the transient stage. Hence, equal damping constants will improve the uniformity of the spatial diffusion of sound and contribute to the useful early reverberation.

(4) The directional distribution of sound energy in a modal field is affected by the directional pattern of the standing waves excited. In this respect lateral axial modes help in enhancing the lateral reflections and increasing the spaciousness in the room.

(5) The spatial distribution of modal fields affects the diffusion of sound in the room. Measurements of modal shapes in a bare-walled and a treated-walled room as shown in Figures (6.10A) to (6.10E) demonstrate the effect of the presence of absorption. This effect has been discussed in Section (6.9.3). The spatial irregularity in small rooms has been investigated by Doak[12] and an index based on the ratio of pressure maxima and minima is introduced by him.

7.5) STUDIOS DESIGN AND CONSTRUCTIONAL DATA:-

Next we must consider the most important elements in the design and construction of small rooms which have direct implications of the physical properties of sounds reproduced in them.

The energy spectrum of sound for which the room is used as a studio for broadcasting or recording must first be considered. For speech, the maximum energy is in the neighbourhood of 300-400 Hz and the distribution of its fundamentals is limited to the frequency range 124-141 Hz.[63] It is the coincidence of widely-spaced isolated modes within these ranges that produces colouration in speech.

The choice of the studio size is governed by the function and the number of performers on the one hand and by economical considerations on the other. The effect of the size is to cause audible sounds in the lower frequency spectrum to occur in the form of isolated modes. This is shown by Equation (3.21) for calculating the modal density. Increasing the volume tends to decrease this effect.

The shape of the studio is another factor which comes into consideration. Rectangular shapes are most common, for constructional reasons, but non-rectangular shapes such as triangular or pentagonal prisms have been used. It has been pointed out(63) that by their unfamiliarity in ordinary homes, such shapes can impart somewhat an unusual quality to reproduced sounds.

The dimensional ratio of the sides of a studio is the most important factor that governs the frequency distribution and spacing of modes. Dimensions that bear simple ratios to one another have specially bad response. The problem has been investigated(73) and some optimum ratios for small studios investigated(74,5,16). A ratio of length to width to height of 1.6:1.3:1 is one of the most favourable in the design of small studios.

The type of sound absorptive material employed in the construction or finishes of the room boundaries influence the frequency distribution of modes, their decay rates and spatial patterns. This point has been discussed in detail in the previous chapters. It is worth mentioning that the complex acoustic impedance describes the physical properties of the material. The magnitudes and signs of the complex components of acoustic impedance and their variation with frequency affect the way the modes are shifted and their amplitudes damped.

The pattern of distribution of absorptive materials is another factor in controlling modal shaping and spatial variations. The effect of uniform coverage of room surfaces has been considered only, but the application of patchy non-symmetrical arrangements of absorbents has been studied[33] and recommended[68] for the provision of diffusion.

The phenomena of flutter echoes has been attributed to strong undamped axial modes formed between a pair of untreated surfaces.

The use of projections in wall surfaces for breaking up the standing wave patterns and providing diffusion is rendered of little value in small rooms if consideration is given to the fact that projections are only useful if their size is comparable to the wavelength of sound.

7.6) CONCLUSIONS:

The special nature of small rooms arise from their small size, simple geometrical shapes and the little absorption involved in their construction.

Their performance in the low frequency range of sound is characterized by the standing wave patterns that dominate the acoustical reponse.

Small studios have their special subjective qualities, as preferred by the listener, and these should be formulated in terms of objective design requirements.

A design and constructional data could then be derived in terms of the room size, dimensions and properties of sound absorbents and the pattern of their distribution in the room. The effects of these factors are best expressed in terms of some predictable and measurable modal parameters related to the physical properties of sound fields in small rooms.

CHAPTER EIGHT

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

8.1) CONCLUSIONS:

In this thesis, the problem of the acoustics of small rooms intended for use as studios has been tackled.

In Chapters One and Two the validity of the application of a statistical geometrical approach to the study of sound fields in small rooms has been questioned. An alternative method, based on the wave analysis theory has been proposed. For experimental investigations, steady-state methods have been favoured in preference to transient-state methods, as the former are considered to better describe the physical properties of sound fields in small rooms.

The wave theory of room acoustics has been presented in Chapter Three. A modal analysis has been formulated in the form of characteristic parameters that can be predicted in terms of the geometry of the room and the properties of its bounding surfaces. The predicted characteristic parameters can be used to evaluate both the steady-state and transient response of the room to acoustic excitation. The statistics of room modes have been reported and such indices as the modal density and mode spacing described.

In Chapter Four, the theories of experimental techniques have been discussed. Two methods for determination of impedance have been outlined; the first is considered to be suitable for measurements on small samples of porous materials, and is based on the standard standing wave analysis. The second is a new modified version of the transmission characteristic method. This is based on the spatial exploration of modal sound fields and is specially suitable for measurements on large panel-type samples investigated under actual mounting conditions.

Also in this Chapter, the application of resonance testing techniques, previously limited to vibrational measurements, to room acoustics study, has been explored. Their application for measurement of modal parameters (i.e. natural frequencies, damping constants and modal shapes) and for the excitation and response extraction of pure modes has been discussed.

In Chapter Five, the implementation of digital computing techniques and methods of numerical analysis has been conducted in an effort to overcome the complexity of the mathematics of the wave theory, and in the analysis and presentation of experimental results.

A schematic model has been proposed for the investigation of small room acoustics, based on the theoretical analysis presented in Chapter Three, and the experimental techniques described in Chapter Four.

Experimental measurements for the purpose of determining the impedance of absorbing materials, and the modal parameters of an existing small room with different boundary configurations have been reported in Chapter Six. The analysis of the results has been performed according to different experimental techniques discussed in Chapter Four.

In Chapter Seven, a qualitative assessment of the subjective qualities of small studios has been made. The feasibility of application of the theoretical analysis and experimental methods on the acoustics of small rooms, as presented in the previous chapter, to the design and construction of small broadcasting and recording studios is discussed.

The main findings in this thesis can be summarized in the following:

(1) A modal approach, based on the wave theory can be successfully applied in the field of investigating the acoustic properties of small rooms. This results in defined modal parameters which are both predictable and measurable quantities, and which describe both the steady-state and transient performance of small rooms.

(2) The absorbing properties of acoustical materials can be best described by their complex acoustic impedance. However, measurements of impedance are fraught with possible experimental errors, which should be eliminated if good agreement between the predicted and measured values of modal parameters is to be expected.

(3) Steady-state measurements, based on resonance testing techniques are invaluable tools in experimental investigations of room response to acoustic excitation. However, complex response measurements, containing information on phase as well as amplitude, are most useful in this respect.

(4) The use of digital computer techniques in room acoustics help to overcome the complexity of theoretical analysis and their employment in conjunction with digital recording facilitates the analysis of experimental results and their presentation in tabulated or graphical forms.

(5) The conditions for good acoustics in small studios and the subjective qualities on which they are based, can be explained in terms of the physical properties of sound in small rooms as described by the modal parameters. These, in turn, can be interpreted in the form of design and constructional data based on the room geometry and the properties of its bounding surfaces.

8.2) SUGGESTIONS FOR FURTHER WORK:

The main discrepancies between theoretical predictions and experimental measurements of the modal parameters describing the room performance have been attributed to errors involved in impedance determination, particularly for bare walls as derived from the Transmission-Characteristic method.

The refinement of the techniques requires accurate determination of distances from the walls to the points where the sound pressure levels are taken. A possible method is to resort to digital recording of resonance amplitude as a function of distance from the wall. From the analysis of such recording the distance can be accurately determined.

Measurements of impedance by the standing wave tube method are only meaningful for locally reacting materials. However, the discrepancies between the theory and measurements of modal parameters suggest the need for an investigation of the possible variation of impedance with angle of incidence of sound.

An extension of the transmission-characteristic method, based on a more profound theoretical analysis, to impedance measurements on large samples of soft materials, is a field that requires further study.

The work in this thesis has been confined to considering rooms with surfaces that have uniform, but different coverage of absorbing material. The investigation of the performance of rooms with non-symmetrical patchy treatment has been studied theoretically(76). An experimental investigation on parallel basis is a field to be considered for further work.

In the field of application of theoretical analysis and experimental techniques to real rooms, a qualitative approach has been adopted. A quantitative assessment of the effect of the physical properties of sound fields in small rooms, based on the wave theory, on the acoustic conditions in small studios should be sought.

APPENDIX A

LISTINGS OF COMPUTER PROGRAMS EMPLOYED IN THEORETICAL ANALYSIS

A1. :program:ROOMODES

```
ROOMODES
UAFORTRAN OWNPD,LIB :DBS6007.SSORT,*LP1 DATA1,EXIT
      LIST(LP)
      PROGRAM(RMOD)
      INPUT  5= CR0
      OUTPUT 6= LP0
      OUTPUT 7= LP1
      TRACE 2
      END
      MASTER ROOMODES
      REAL A(1000),L1,L2,L3
      INTEGER B(1000),II(1000,3),B1,B2,B3
      NS=1000
   4  READ(5,100) N1,N2,N3,L1,L2,L3,B1,B2,B3,C,FLOW,FHIGH,MORE
 100  FORMAT(3I0,3F0.0,3I0,3F0.0,I0)
      WRITE(6,101) L1,L2,L3
 101  FORMAT(25H ROOM DIMENSIONS          ,3F12.2)
 102  FORMAT(25H MODE LIMITS              ,3I12)
      WRITE(6,102) N1,N2,N3
 103  FORMAT(13H     HARD/HARD)
 104  FORMAT(13H     HARD/FREE)
 105  FORMAT(13H     FREE/FREE)
 106   FORMAT(25H BOUNDARY CONDITIONS      )
      WRITE(6,106)
      IF (B1.EQ.1) WRITE(6,103)
      IF (B1.EQ.2) WRITE(6,104)
      IF (B1.EQ.3) WRITE(6,105)
      IF (B2.EQ.1) WRITE(6,103)
      IF (B2.EQ.2) WRITE(6,104)
      IF (B2.EQ.3) WRITE(6,105)
      IF (B3.EQ.1) WRITE(6,103)
      IF (B3.EQ.2) WRITE(6,104)
      IF (B3.EQ.3) WRITE(6,105)
      WRITE(6,107) C
 107  FORMAT(25H SPEED OF SOUND IN MEDIUM,F12.2)
      WRITE(6,108)FLOW,FHIGH
 108  FORMAT(25H FREQUENCY LIMITS         ,2F12.2)
      N1=N1+1
      N2=N2+1
      N3=N3+1
      M1=1
      IF (B1.NE.1) M1=2
      M2=1
      IF (B2.NE.1) M2=2
      M3=1
      IF (B3.NE.1) M3=2
      IF (B1.EQ.2) L1=L1*2.0
      IF (B2.EQ.2) L2=L2*2.0
      IF (B3.EQ.2) L3=L3*2.0
      K1=1
      IF (B1.EQ.2) K1=2
      K2=1
      IF (B2.EQ.2) K2=2
      K3=1
```

```
      IF (B3.EQ.2) K3=2
      IF (B1.EQ.2) N1=N1*2
      IF (B2.EQ.2) N2=N2*2
      IF (B3.EQ.2) N3=N3*2
      L=0
      DO 1 I1=M1,N1,K1
      I=I1-1
      RI=I
      DO 1 J1=M2,N2,K2
      J=J1-1
      RJ=J
      DO 1 J2=M3,N3,K3
      K=J2-1
      RK=K
      RES=C/2.0*SQRT((RI/L1)**2.0+(RJ/L2)**2.0+(RK/L3)**2.0)
      IF ((RES.LT.FLOW).OR.(RES.GT.FHIGH))  GOTO 1
      L=L+1
      II(L,1)=I
      II(L,2)=J
      II(L,3)=K
      A(L)=RES
      IF (L.EQ.1000) GOTO 2
    1 CONTINUE
      WRITE(6,110)
  110 FORMAT(1H1)
    2 CALL SORT(A,B,L,NS)
      DO 3 I=1,L
      J=L-I+1
      J=B(J)
      WRITE(6,109) II(J,1),II(J,2),II(J,3),A(J)
      WRITE(7,113)(II(J,K),K=1,3),A(J)
    3 CONTINUE
  109 FORMAT(3I5,5X,F10.3)
  113 FORMAT(1X,3I5,F10.3)
      IF (MORE.NE.0) GOTO 4
      NQ1=0
      WRITE(7,119)NQ1,NQ1,NQ1
  119 FORMAT(1X,3I5)
      STOP
      END
      FINISH
```

12.32.21←

A.2. program: SPACEMODE

```
SPACEMODE
MAFORTRAN OUNPD,LIB :EE39003.SCONTOUR,*GP GINO
      LIST(LP)
      PROGRAM(SPMD)
      INPUT  5= CR0
      OUTPUT 6=LP0
      EXTENDED DATA
      TRACE 2
      END
      MASTER SPMODE
      INTEGER IC(65,65)
      INTEGER N(10)
      REAL X(65),Y(65),Z(65),FNXY(65,65)
      REAL FNXZ(65,65),FNYZ(65,65),B(50),KX,KY,KZ,R,A(65,65)
      LOGICAL ANOT,CROSSES,CELLS
      COMMON/CONT/SIZEX,SIZEY,ANOT,TX,TY,CROSSES,CELLS,INX,INY,NDEL
      ANOT=.FALSE.
      CELLS=.FALSE.
      CROSSES=.FALSE.
      NDEL=0
      PI=3.14159265
      READ(5,100)IMODE
  100 FORMAT(I0)
      READ(5,101)AL,AW,AH,R
  101 FORMAT(3F6.2,F7.2)
      READ(5,102)(N(J1),J1=1,3)
  102 FORMAT(3I5)
      NEX=N(1)
      NEY=N(2)
      NEZ=N(3)
      IN=8*NEX+1
      IF(IN.EQ.1)IN=13
      JN=8*NEY+1
      IF(JN.EQ.1)JN=13
      KN=8*NEZ+1
      IF(KN.EQ.1)KN=13
      KX=PI*FLOAT(NEX)/AL
      KY=PI*FLOAT(NEY)/AW
      KZ=PI*FLOAT(NEZ)/AH
      DO2I=1,IN
    2 X(I)=(I-1)*AL/(IN-1)
      DO3J=1,JN
    3 Y(J)=(J-1)*AW/(JN-1)
      DO4K=1,KN
    4 Z(K)=(K-1)*AH/(KN-1)
      DO51I=1,IN
      DO51J=1,JN
      FNXY(I,J)=R*COS(KX*X(I))*COS(KY*Y(J))
   51 FNXY(I,J)=ABS(FNXY(I,J))
      DO52I=1,IN
      DO52K=1,KN
      FNXZ(I,K)=R*COS(KX*X(I))*COS(KZ*Z(K))
   52 FNXZ(I,K)=ABS(FNXZ(I,K))
```

```
      DO53J=1,JN
      DO53K=1,KN
      FNYZ(J,K)=D*COS(KY*Y(J))*COS(KZ*Z(K))
   53 FNYZ(J,K)=ABS(FNYZ(J,K))
      WRITE(6,106)(N(J1),J1=1,3)
  106 FORMAT(7X,9HFNYYMODE:,3I1/)
      DO6J=1,JN,2
      WRITE(6,103)(FNXY(I,J),I=1,IN,2)
    6 CONTINUE
      WRITE(6,107)(N(J1),J1=1,3)
  107 FORMAT(7X,9HFNYZMODE:,3I1/)
      DO7K=1,KN,2
      WRITE(6,103)(FNYZ(J,K),J=1,JN,2)
    7 CONTINUE
      WRITE(6,108)(N(J1),J1=1,3)
  108 FORMAT(7X,9HFNXZMODE:,3I1/)
      DO8K=1,KN,2
      WRITE(6,103)(FNXZ(I,K),I=1,IN,2)
    8 CONTINUE
  103 FORMAT(2X,25(F5.1))
      DO9I1=1,13
      B(I1)=4.0*FLOAT(I1-1)+1.0
    9 CONTINUE
      CALL OPEN GINOGP
      CALL CHASWI(1)
      CALL SCALE(0.4)
      CALL SHIFT2(300.,350.)
      CALL MOVTO2(0.,0.)
      CALL CHAHOL(19HRIGID WALLED ROOM*.)
      CALL MOVTO2(0.,-10.)
      CALL CHAHOL(25HSPATIAL DISTRIBUTION IN*.)
      CALL MOVTO2(0.,-20.)
      CALL CHAHOL(7HMODE:*.)
      CALL MOVTO2(25.,-20.)
      CALL CHAINT(IMODE,4)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(-300.,-350.)
      CALL MOVTO2(0.,0.)
      NC=13
      NX=IN
      NY=JN
      DO20I=1,NX
      DO20J=1,NY
      A(I,J)=FNXY(I,J)
   20 CONTINUE
      DO10I=1,NDBL
      IF(NDBL.EQ.0)GOTO10
      CALL DOUBLE(A,NX,65,NY,65)
   10 CONTINUE
      SIZEX=AL*8.0
      SIZEY=AW*8.0
      TX=SIZEX*10.0/(NX-1)
      TY=SIZEY*10.0/(NY-1)
      CALL SHIFT2(50.0,40.0)
      CALL MOVTO2(0.,-20.)
```

```
      CALL CHAHOL(14HX COORDINATE*.)
      CALL MOVTO2(-10.,0.)
      CALL ROTAT2(90.)
      CALL CHAHOL(14HY COORDINATE*.)
      CALL ROTAT2(-90.)
      CALL MOVTO2(0.,0.)
      CALL SCALE2(TX,TY)
      CALL CONTOUR(A,NX,65,NY,65,B,NC,50,IC)
      CALL SCALE2(1.0/TX,1.0/TY)
      NX=JN
      NY=KN
      DO21I=1,NX
      DO21J=1,NY
      A(I,J)=FNYZ(I,J)
   21 CONTINUE
      DO11I=1,NDBL
      IF(NDBL.EQ.0)GOTO11
      CALL DOUBLE(A,NX,65,NY,65)
   11 CONTINUE
      SIZEX=AW*8.0
      SIZEY=AH*8.0
      TX=SIZEX*10.0/(NX-1)
      TY=SIZEY*10.0/(NY-1)
      CALL SHIFT2(0.0,AW*80.0+40.0)
      CALL MOVTO2(0.,-20.)
      CALL CHAHOL(14HY COORDINATE*.)
      CALL MOVTO2(-10.,0.)
      CALL ROTAT2(90.)
      CALL CHAHOL(14HZ COORDINATE*.)
      CALL ROTAT2(-90.)
      CALL MOVTO2(0.,0.)
      CALL SCALE2(TX,TY)
      CALL CONTOUR(A,NX,65,NY,65,B,NC,50,IC)
      CALL SCALE2(1.0/TX,1.0/TY)
      NX=IN
      NY=KN
      DO22I=1,NX
      DO22J=1,NY
      A(I,J)=FNXZ(I,J)
   22 CONTINUE
      DO12I=1,NDBL
      IF(NDBL.EQ.0)GOTO12
      CALL DOUBLE(A,NX,65,NY,65)
   12 CONTINUE
      SIZEX=AL*8.0
      SIZEY=AH*8.0
      TX=SIZEX*10.0/(NX-1)
      TY=SIZEY*10.0/(NY-1)
      CALL SHIFT2(0.0,AH*80.0+40.0)
      CALL MOVTO2(0.,-20.)
      CALL CHAHOL(14HX COORDINATE*.)
      CALL MOVTO2(-10.,0.)
      CALL ROTAT2(90.)
      CALL CHAHOL(14HZ COORDINATE*.)
      CALL ROTAT2(-90.)
```

```
CALL MOVTO2(0.,0.)
CALL SCALE2(TY,TY)
CALL CONTUR(A,NX,65,NY,65,5,70,50,IC)
CALL SCALE(2.5)
CALL CHASTI(0)
CALL DEVEND
STOP
END
FINISH
```

A.3. programs: ADMITTANCE1, ADMITTANCE

```
ADMITTANCE1
UAFORTRAN DVNPD,*CP1 ADMITBARE
      LF(LP)
      PROGRAM(ADMT1)
      INPUT  5= CR0
      OUTPUT 6=LP0
      OUTPUT 7=CP1
      COMPRESS INTEGER AND LOGICAL
      TRACE 2
      END
      MASTER ADMITTANCE
      COMPLEX ADMIT(30),IMP(30),AD(30),IM(30)
      REAL F(30),V(30),X(30),Y(30),Z(30),L,FD(30),Y1(30),Z1(30)
      REAL V1(30),X1(30)
      C=34300.0
      PI=4.0*ATAN(1.0)
      L=327.0
      IC=20
      KC=22
      DO1I=1,IC
      READ(5,100)NX,NY,NZ,FR,X20,XMI
  100 FORMAT(3I3,3F6.1)
      F(I)=FR
      W=2.0*PI*FR
      C0=PI*X20/XMI
      CA=C0*180.0/PI
      C1=COS(C0)
      C2=0.02-C1
      IF(C2.LT.1.0)GOTO11
      T1=ACOSH(C2)
      DN=C*T1/(2.0*X20)
      WN=C*PI/(2.0*XMI)
      GW=WN**2.0-DN**2.0
      GD= 2.0*WN*DN
      GR=(PI*NX*C/L)**2.0
      WRITE(6,200)CA,C1,C2,T1,DN,WN,GW,GR,GD
  200 FORMAT(1X,4F10.5,5F10.1/)
      BR=GD*L/(4.0*W*C)
      BI=L*((PI*NX*C/L)**2.0-GW)/(4.0*W*C)
      Y(I)=BR
      Z(I)=BI
      ADMIT(I)=CMPLX(BR,BI)
      IMP(I)=1.0/ADMIT(I)
      V(I)=REAL(IMP(I))
      X(I)=AIMAG(IMP(I))
      WRITE(6,101)NX,NY,NZ,F(I),X20,XMI,V(I),X(I),Y(I),Z(I)
  101 FORMAT(1X,3I3,3F6.1,2F12.3,2F12.6/)
      GOTO1
   11 W RITE(6,99)X20,XMI,CA,C1
   99 FORMAT(1X,3F10.1,F10.6/)
    1 CONTINUE
      FDB=F(1)
      FDE=F(IC)
      READ(5,102)(FD(K),K=1,KC)
```

```
102 FORMAT(9F7.1)
    DO3K=1,KC
    IF(FD(K).LE.FDB)GOTO52
    IF(FD(K).GT.FDE)GOTO53
    DO4I=2,IC
    IF(FD(K).LE.F(I).AND.FD(K).GE.F(I-1))GOTO31
  4 CONTINUE
 52 Y1(K)=Y(1)
    Z1(K)=Z(1)
    GOTO3
 53 Y1(K)=Y(IC)
    Z1(K)=Z(IC)
    GOTO3
 31 F1=F(I-1)
    F2=F(I)
    RA1=Y(I-1)
    RA2=Y(I)
    AI1=Z(I-1)
    AI2=Z(I)
    Y1(K)=RA1+(FD(K)-F1)*(RA2-RA1)/(F2-F1)
    Z1(K)=AI1+(FD(K)-F1)*(AI2-AI1)/(F2-F1)
  3 CONTINUE
    WRITE(6,109)
109 FORMAT(1H1 )
    DO5K=1,KC
    AD(K)=CMPLX(Y1(K),Z1(K))
    IM(K)=1.0/AD(K)
    V1(K)=REAL(IM(K))
    X1(K)=AIMAG(IM(K))
    WRITE(6,103)FD(K),V1(K),X1(K),Y1(K),Z1(K)
    WRITE(7,103)FD(K),V1(K),X1(K),Y1(K),Z1(K)
103 FORMAT(F7.1,4F12.6)
  5 CONTINUE
    STOP
    END
    FINISH
```

```
ADMITTANCE
UAFORTRAN OWNPD,*GP GINO,*CP1 ADMITNCE,*CR2 ADMITBARE
      LF(LP)
      PROGRAM(ADMT)
      INPUT  4= CR1
      INPUT  5= CR0
      INPUT  9= CR2
      OU TPUT 6=LP0
      OUTPUT 7=CP1
      COMPRESS INTEGER AND LOGICAL
      TRACE 2
      END
      MASTER ADMITTANCE
      COMPLEX R(30),IMP(30),ADMIT(30),ST(30,3)
      REAL MODR,ALPHA(30),Y1(30),LAMDA(30),C,FREQ(30),XC(9),Y2(30)
      REAL V(30),X(30),Y(30),Z(30),F(30)
      INTEGER TEXT(20)
      DIMENSION IHOL(10)
      DATA IHOL/4H100 ,4H1000/
      CALL OPEN GINOGP
      C=34300.0
      PI=4.0*ATAN(1.0)
      IN=22
      JC=3
      DO 11 J=1,JC
      IF(J.GT.1)GOTO8
      READ(9,71)(F(I),V(I),X(I),Y(I),Z(I),I=1,IN)
   71 FORMAT(F7.1,4F12.6)
      V B1=-40.0
      V E1=80.0
      V B2=-0.06
      VE2=0.14
      DO 61 I=1,IN
      ST(I,J)=CMPLX(Y(I),Z(I))
   61 CONTINUE
      GOTO9
    3 V B1=-30.0
      VE1=30.0
      VB2=0.0
      V E2=0.50
      DO1 I=1,IN
      READ(5,100)FREQ(I),ALPHA(I),Y1(I),Y2(I)
  100 FORMAT(2X,4F6.1)
      IF(Y2(I).EQ.0.0)GOTO2
      LAMDA(I)=2.0*(Y2(I)-Y1(I))
      GOTO 3
    2 LAMDA(I)=C/FREQ(I)
    3 ALPHA(I)=ALPHA(I)/100.0
      MODR=SQRT(1.0-ALPHA(I))
      DELTA=(4.0*Y1(I)/LAMDA(I)-1.0)*PI
      A=MODR*COS(DELTA)
      B=MODR*SIN(DELTA)
      R(I)=CMPLX(A,B)
      IMP(I)=(1.0+R(I))/(1.0-R(I))
      ADMIT(I)=1.0/IMP(I)
      ST(I,J)=ADMIT(I)
```

```
      V(I)=REAL(IMP(I))
      X(I)=AIMAG(IMP(I))
      Y(I)=REAL(ADMIT(I))
      Z(I)=AIMAG(ADMIT(I))
      F(I)=FREQ(I)
    1 CONTINUE
    9 READ(5,98)(TEXT(I),I=1,20)
   98 FORMAT(20A4)
      IF(J.EQ.1)GOTO72
      WRITE(6,102)(TEXT(I),I=1,20)
  102 FORMAT(2X,32HSPECIFIC ACOUSTIC ADMITTANCE OF:,20A4/)
      WRITE(6,101)
  101 FORMAT(1X,9HFREQUENCY,1X,5HALPHA,4X,2HY1,5X,2HY2,11X,
     19HIMPEDANCE,15X,10HADMITTANCE/)
      WRITE(6,103)(FREQ(I),ALPHA(I),Y1(I),Y2(I),
     +IMP(I),ADMIT(I),I=1,IN)
  103 FORMAT(1X,F6.1,3X,F6.3,2X,2F6.1,2F12.3,2F12.6)
   72 CALL CHASWI(1)
      CALL SCALE(0.4)
      CALL SHIFT2(0.,100.)
      CALL AXIPOS(1,40.,40.,250.,1)
      CALL AXIPOS(1,40.,40.,250.,2)
      CALL AXISCA(4,0,100.,1000.,1)
      CALL AXISCA(3,12,VE1,VE1,2)
      CALL AXIDRA(2,0,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRID(3,0,0)
      CALL GRASYM(F,V,IN,4,0)
      CALL GRAPOL(F,V,IN)
      CALL GRASYM(F,X,IN,8,0)
      CALL GRAPOL(F,X,IN)
      CALL GRAMOV(F(IN),V(IN))
      CALL CHAHOL(8H  REAL*.)
      CALL GRAMOV(F(IN),X(IN))
      CALL CHAHOL(8H  IMAG*.)
      CALL GRAMOV(100.,0.)
      CALL GRALIN(1000.,0.)
      CALL AXILAB(IHOL,2,4,1,30.,1)
      CALL MOVTO2(0.,0.)
      DO5I=2,9
      AI=FLOAT(I)
      XC(I)=40.0+250.0*ALOG10(AI)
      CALL MOVTO2(XC(I),40.)
      CALL LINBY2(0.,250.)
      INT=I
      CALL MOVTO2(XC(I),30.)
      CALL CHAINT(INT,-2)
    5 CONTINUE
      CALL MOVTO2(0.,0.)
      CALL MOVBY2(40.,10.)
      CALL CHAHOL(29HSPECIFIC ACOUSTIC IMPEDANCE*.)
      CALL MOVTO2(220.,20.)
      CALL CHAHOL(14HFREQUENCY:HZ*.)
      CALL MOVTO2(20.,200.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(11HIMPEDANCE*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,0.)
      CALL MOVBY2(200.,10.)
      CALL CHAARR(TEXT,20,4)
```

```
      CALL SHIFT2(0.,-70.)
      CALL AXIPOS(1,40.,400.,250.,1)
      CALL AXIPOS(1,40.,400.,250.,2)
      CALL AXISCA(4,0,100.,1000.,1)
      CALL AXISCA(3,10,VB2,VE2,2)
      CALL AXIDRA(2,0,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRID(3,0,0)
      CALL GRASYM(F,Y,IN,4,0)
      CALL GRAPOL(F,Y,IN)
      CALL GRASYM(F,Z,IN,8,0)
      CALL GRAPOL(F,Z,IN)
      CALL GRAMOV(F(IN),Y(IN))
      CALL CHAHOL(8H  REAL*.)
      CALL GRAMOV(F(IN),Z(IN))
      CALL CHAHOL(8H  IMAG*.)
      CALL AXILAB(IHOL,2,4,1,390.,1)
      CALL MOVTO2(0.,360.)
      DO6I=2,9
      AI=FLOAT(I)
      XC(I)=40.0+250.0*ALOG10(AI)
      CALL MOVTO2(XC(I),400.)
      CALL LINBY2(0.,250.)
      INT=I
      CALL MOVTO2(XC(I),390.)
      CALL CHAINT(INT,-2)
    6 CONTINUE
      CALL MOVTO2(0.,360.)
      CALL MOVBY2(40.,10.)
      CALL CHAHOL(30HSPECIFIC ACOUSTIC ADMITTANCE*.)
      CALL MOVTO2(220.,380.)
      CALL CHAHOL(14HFREQUENCY:HZ*.)
      CALL MOVTO2(20.,560.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(12HADMITTANCE*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,360.)
      CALL MOVBY2(200.,10.)
      CALL CHAARR(TEXT,20,4)
      CALL MOVTO2(0.,360.)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(0.,-30.)
      CALL MOVTO2(0.,0.)
      CALL SCALE(2.5)
      CALL CHASWI(0)
      CALL MOVTO2(0.,0.)
      IF(J.NE.JC)CALL PICCLE
   11 CONTINUE
      WRITE(6,105)
  105 FORMAT(2X,32HSPECIFIC ACOUSTIC ADMITTANCE OF:/)
      W RITE(6,106)
  106 FORMAT(1X,9HFREQUENCY,5X,10HBARE WALLS,14X,11HFIBRE GLASS,9X,
     518HPERFORATED CELOTEX/)
      DO 10 I=1,IN
      WRITE(6,133)F(I),(ST(I,J),J=1,JC)
      WRITE(7,104)F(I),(ST(I,J),J=1,JC)
  133 FORMAT(F7.1,6F11.6/)
  104 FORMAT(F7.1,6F12.6)
   10 CONTINUE
      CALL DEVEND
      STOP
      END
      FINISH
```

A.4. program: TRANSCEQNS

```
TRANSCEQNS
UAFORTRAN EWNPD,*CR1 DATA1,*CR2 ADMITNCE,*CP1 DATA2,-
LINES 10000
      LF(LP)
      PROGRAM(TRQN)
      INPUT  3=CR0
      INPUT  4=CR1
      INPU T  5=CR2
      OUTPUT 6=LP0
      OUTPUT 7=CP1
      COMPRESS INTEGER AND LOGICAL
      TRACE 2
      END
      MASTER SOLUTION
      INTEGER NI(650,3),N(3),IS(650,3)
      REAL LD(3),FD(30),RAD(650,3),AAD(650,3),FN(650),
     +RAA(3),AIA(3)
      REAL FS(650),EW1(3),EW2(3),MK,NS,NR,MR,LX,LY,LZ
      COMPLEX ADMIT1(3),ADMIT2(3),E1(3),E2(3),QC,CT0,CT1,
     +CT2,EE,KNN(3),
     1KNX,KNY,KNZ,X,Y,Z,A(3),PHI(3),CACOTH,KN(3),WN,
     3XN,YN,ZN,ADMIT(650,3),PHIX,PHIY,PHIZ
      WRITE(6,98)
      WRITE(6,99)
   98 FORMAT('  EXAMPLE OF SOLUTION PROCEDURE')
   99 FORMAT(4X,7HPRVFREQ,2X,7HRESFREQ,3X,7HDAMPING,
     +1X,11HXCHARCVALUE,
     13X,11HYCHARCVALUE,3X,11HZCHARCVALUE,4X,11HXPHASEANGLE,
     24X,11HYPHASEANGLE,4X,11HZPHASEANGLE/)
      READ(3,102)(LD(J),J=1,3)
  102 FORMAT(3F6.2)
      KC=22
      DO 9 K=1,KC
    9 READ(5,100)FD(K),(RAD(K,J),AAD(K,J),J=1,3)
  100 FORMAT(F7.1,6F12.6)
      FDB=FD(1)
      FDE=FD(KC)
      DO 24 J=1,3
      RAA(J)=RAD(1,J)
      AIA(J)=AAD(1,J)
   24 CONTINUE
      DO 2 I=1,650
      READ(4,101)(NI(I,J),J=1,3),FN(I)
  101 FORMAT(3I5,F10.3)
      IF(NI(I,1).EQ.0.AND.NI(I,2).EQ.0.AND.NI(I,3).EQ.0)
     +GOTO 10
    2 CONTINUE
   10 IC=I-1
      DO 3 I=1,IC
      IF(FN(I).LE.FDB)GOTO52
      IF(FN(I).GT.FDE)GOTO 41
      DO 4 K=2,KC
      IF(FN(I).LE.FD(K).AND.FN(I).GE.FD(K-1))GOTO 31
    4 CONTINUE
```

```
52 DO 25 J=1,3
   ADMIT(I,J)=CMPLX(RAA(J),AIA(J))
25 CONTINUE
   GOTO 3
31 DO 5 J=1,3
   F1=FD(K-1)
   F2=FD(K)
   RA1=RAD((K-1),J)
   RA2=RAD(K,J)
   AI1=AAD((K-1),J)
   AI2=AAD(K,J)
   RADM=RA1+(FN(I)-F1)*(RA2-RA1)/(F2-F1)
   AIADM=AI1+(FN(I)-F1)*(AI2-AI1)/(F2-F1)
   ADMIT(I,J)=CMPLX(RADM,AIADM)
 5 CONTINUE
 3 CONTINUE
   IFLAG=1
   C=343.00
   PI=4.0*ATAN(1.0)
   GOTO 42
41 IC=I-1
42 ICOUNT=0
   DO 7 I=1,IC
   IF(IFLAG.EQ.0)GOTO19
   ADMIT1(1)=ADMIT(I,1)
   ADMIT2(1)=ADMIT(I,2)
   ADMIT1(2)=ADMIT(I,1)
   ADMIT2(2)=ADMIT(I,2)
   ADMIT1(3)=ADMIT(I,1)
   ADMIT2(3)=ADMIT(I,3)
   GOTO 20
19 ADMIT1(1)=ADMIT(I,1)
   ADMIT2(1)=ADMIT(I,1)
   ADMIT1(2)=ADMIT(I,1)
   ADMIT2(2)=ADMIT(I,1)
   ADMIT1(3)=ADMIT(I,1)
   ADMIT2(3)=ADMIT(I,1)
20 DN=0.0
   DH=0.0
   FG=FN(I)
   FR=FN(I)
   WR=FR*PI*2.0
   DO 6 J=1,3
   E1(J)=1.0/ADMIT1(J)
   E2(J)=1.0/ADMIT2(J)
   EW1(J)=CABS(E1(J))
   EW2(J)=CABS(E2(J))
 6 CONTINUE
 1 WP=WR
   FP=FR
   WN=CMPLX(WR,DN)
   CH=0.0
   DO 8 J=1,3
   CT0=CMPLX(0.0,1.0)
   QC=WN*LD(J)/(PI*C)
   NR=FLOAT(NI(I,J))
   MR=(NR+0.5)
   MK=(NR+1.0)
```

```
      NS=NR
      IF(NI(I,J).EQ.0)NS=1.0
      QS=CABS(WN)*LD(J)/(PI*C)
      R1=QS/(NS*EW1(J))
      R2=QS/(NS*EW2(J))
      IF((R1.LT.1.0).AND.(R2.LT.1.0))GOTO71
      IF((R1.LT.1.0).AND.(R2.GE.1.0))GOTO73
      IF((R1.GE.1.0).AND.(R2.GE.1.0))GOTO 74
      CH=-1.0
      IJ=J
      IF(IJ.EQ.1)WRITE(6,401)(NI(I,L),L=1,3),FN(I),R1,R2
      IF(IJ.EQ.2)WRITE(6,402)(NI(I,L),L=1,3),FN(I),R1,R2
      IF(IJ.EQ.3)WRITE(6,403)(NI(I,L),L=1,3),FN(I),R1,R2
  401 FORMAT(1X,3I3,F9.3,2X,26HINTERMEDIATE RANGE:X WALLS,2X,2F8.3)
  402 FORMAT(1X,3I3,F9.3,2X,26HINTERMEDIATE RANGE:Y WALLS,2X,2F8.3)
  403 FORMAT(1X,3I3,F9.3,2X,26HINTERMEDIATE RANGE:Z WALLS,2X,2F8.3)
       GOTO 8
   71 IF(NI(I,J).EQ.0)GOTO72
      EE=(1.0/E1(J)+1.0/E2(J))
      KNN(J)=NR+CT0*QC*EE/(PI*NR)+(QC*EE)**2/(PI**2*NR**3)
      IS(I,J)=71
      GOTO16
   72 CT1=CMPLX(1.0,1.0)
      CT2=CMPLX(1.0,-1.0)
      KNN(J)=CSQRT(QC*(E1(J)+E2(J))/(2.0*PI*E1(J)*E2(J)))*CT1
     6+CSQRT(QC**3*PI/(72.0*E1(J)**3*E2(J)**3*(E1(J)+E2(J))))
     7*(E1(J)**2+E2(J)**2-E1(J)*E2(J))*CT2
      IS(I,J)=72
      GOTO16
   73 KNN(J)=MR+CT0*(QC/(E1(J)*MR)+E2(J)*MR/QC)/PI+
     7((QC/(E1(J)*MR))**2-(E2(J)*MR/QC)**2)/(PI**2*MR**2)
      IS(I,J)=73
      GOTO16
   74 KNN(J)=MK+CT0*MK*(E1(J)+E2(J))/(QC*PI)+(E1(J)+E2(J))**2*MK/
     3(QC**2*PI**2)
      IS(I,J)=74
   16 KN(J)=KNN(J)
      A(J)=((-1.0*KN(J))/(QC*ADMIT1(J)))
      PHI(J)=CACOTH(A(J))
    8 CONTINUE
      IF(CH.LT.0.0)GOTO 7
      KNX=KN(1)
      KNY=KN(2)
      KNZ=KN(3)
      PHIX=PHI(1)
      PHIY=PHI(2)
      PHIZ=PHI(3)
      XN=KNX**2
      YN=KNY**2
      ZN=KNZ**2
      LX=LD(1)
      LY=LD(2)
      LZ=LD(3)
      XN=REAL(XN)
      XD=AIMAG(XN)
      YN=REAL(YN)
      YD=AIMAG(YN)
      ZN=REAL(ZN)
```

```
      ZD=AIMAG(ZN)
      WR=PI*C*SQRT(XW/(LX**2)+YW/(LY**2)+ZW/(LZ**2))
      FR=WR/(2.0*PI)
      DN=(XD/(LX**2)+YD/(LY**2)+ZD/(LZ**2))*PI**2*C**2/(2.0*WR)
      IF(ICOUNT.GE.5)GOTO 11
      WRITE(6,103)FP,FR,DN,KNX,KNY,KNZ,PHIX,PHIY,PHIZ
  103 FORMAT(1X,3F9.3,3(2X,2F6.3),3(2X,2F7.3))
   11 IF(ABS(WP-WR).LT.0.1)GOTO12
      GOTO 1
   12 IF(ICOUNT.GE.5)GOTO 13
      WRITE(6,105)
  105 FORMAT('  COMPUTATIONS OF SOLUTION')
      IF(IFLAG.EQ.0)GOTO97
      WRITE(6,106)
  106 FORMAT(19X,17HRIGID WALLED ROOM,5X,19HTREATED WALLED ROOM)
      GOTO 96
   97 WRITE(6,113)
  113 FORMAT(19X,17HRIGID WALLED ROOM,5X,16HBARE WALLED ROOM)
   96 WRITE(6,107)
  107 FORMAT(6X,4HMODE,9X,7HRESFREQ,4X,8HDAMPCONST,3X,7HRESFREQ,3X,
     78HDAMPCONST/)
   13 WRITE(6,108)(NI(I,J),J=1,3),FG,DH,FR,DN,(IS(I,J),J=1,3)
  108 FORMAT(6X,3I3,2X,2F9.3,4X,2F9.3,20X,3I5)
      WRITE(7,109)(NI(I,J),J=1,3),FG,DH,FR,DN
  109 FORMAT(1X,3I3,4F9.3)
      ICOU NT=ICOUNT+1
    7 CONTINUE
      NQ1=0
      WRITE(7,109)NQ1,NQ1,NQ1
      STOP
      END
      COMPLEX FUNCTION CACOTH(A)
      COMPLEX A
      CACOTH=0.5*CLOG((A+1.0)/(A-1.0))
      RETURN
      END
      FINISH
```

```
DOCUMENT    1

EXAMPLE OF SOLUTION PROCEDURE
  PPVFREQ   RESFREQ    DAMPING XCHARCVALUE    YCHARCVALUE    ZCHARCVALUE      XPHASEANGLE     YPHASEANGLE     ZPHASEANGLE

  52.366    51.046     8.345   0.987 0.010    0.034 0.110    0.055 0.088     -0.025  0.020    0.074  0.202    0.023  0.212
  51.046    51.046     8.374   0.987 0.010    0.035 0.109    0.053 0.087     -0.025  0.019    0.071  0.200    0.020  0.209
COMPUTATIONS OF SOLUTION
                   RIGID WALLED ROOM      TREATED WALLED ROOM
   MODE            RESFREQ    DAMPCNST    RESFREQ   DAMPCNST

    1  0  0        52.366     0.000       51.046    8.374                        71   72     72
  66.862    65.220     9.232   0.042 0.143    0.987 0.010    0.061 0.100      0.096  0.251   -0.025  0.020    0.026  0.238
  65.220    65.227     8.332   0.040 0.141    0.987 0.010    0.060 0.099      0.092  0.249   -0.025  0.019    0.043  0.235
COMPUTATIONS OF SOLUTION
                   RIGID WALLED ROOM      TREATED WALLED ROOM
   MODE            RESFREQ    DAMPCNST    RESFREQ   DAMPCNST

    0  1  0        66.862     0.000       65.227    8.832                        72   71     72
  75.055    73.541    10.239   0.044 0.152    0.040 0.133    0.994 0.014      0.102  0.264    0.090  0.237   -0.025  0.020
  73.541    73.536     9.337   0.041 0.151    0.038 0.132    0.994 0.014      0.090  0.263    0.087  0.236   -0.025  0.019
COMPUTATIONS OF SOLUTION
                   RIGID WALLED ROOM      TREATED WALLED ROOM
   MODE            RESFREQ    DAMPCNST    RESFREQ   DAMPCNST

    0  0  1        75.055     0.000       73.536    9.837                        72   72     71
  84.928    83.072    11.155   0.979 0.017    0.984 0.013    0.069 0.113     -0.041  0.033   -0.032  0.026    0.031  0.266
  83.072    83.075    10.665   0.979 0.016    0.984 0.013    0.067 0.112     -0.041  0.032   -0.032  0.025    0.028  0.264
COMPUTATIONS OF SOLUTION
                   RIGID WALLED ROOM      TREATED WALLED ROOM
   MODE            RESFREQ    DAMPCNST    RESFREQ   DAMPCNST

    1  1  0        84.928     0.000       83.075    10.665                       71   71     72
  91.517    89.780    12.107   0.978 0.018    0.043 0.149    0.993 0.017     -0.044  0.036    0.100  0.259   -0.030  0.025
  89.780    89.781    11.653   0.978 0.018    0.041 0.148    0.992 0.017     -0.044  0.034    0.097  0.258   -0.030  0.024
COMPUTATIONS OF SOLUTION
                   RIGID WALLED ROOM      TREATED WALLED ROOM
   MODE            RESFREQ    DAMPCNST    RESFREQ   DAMPCNST

    1  0  1        91.517     0.000       89.781    11.653                       71   72     71
```

A.4.1. EXAMPLE OF SOLUTION PROCEDURE.

A.4.2. file: DATA (DATA2 AND DATA3)

FILE DATA

FNR:NATURAL FREQUENCY-RIGID WALLED ROOM
FNB:NATURAL FREQUENCY-BARE WALLED ROOM
FNT:NATURAL FREQUENCY-TREATED WALLED ROOM
DNB:DAMPING CONSTANT-BARE WALLED ROOM
DNT:DAMPING CONSTANT-TREATED WALLED ROOM

MODE			RIGID WALLED ROOM FNR	BARE WALLED ROOM FNB	DNB	TREATED WALLED ROOM FNT	DNT
1	0	0	52.4	53.9	12.8	51.0	8.4
0	1	0	66.9	68.3	13.5	65.2	8.8
0	0	1	75.1	76.5	13.9	73.5	9.8
1	1	0	84.9	86.8	16.0	83.1	10.7
1	0	1	91.5	93.4	16.4	89.8	11.7
0	1	1	100.5	102.4	17.4	98.6	12.3
2	0	0	104.7	106.5	15.0	103.4	9.3
1	1	1	113.3	115.6	26.8	110.5	17.3
2	1	0	124.3	125.3	21.9	121.5	13.9
2	0	1	128.8	129.6	22.6	126.3	15.5
0	2	0	133.7	133.9	13.5	131.0	12.4
1	2	0	143.6	143.6	22.7	139.8	15.4
2	1	1	145.2	145.3	28.3	140.8	20.1
0	0	2	150.1	149.8	19.7	147.4	14.9
0	2	1	153.3	152.9	24.8	150.0	18.2
3	0	0	157.1	157.2	18.4	155.3	12.7
1	0	2	159.0	158.4	24.1	156.1	17.8
1	2	1	162.0	161.1	28.5	158.3	20.8
0	1	2	164.3	163.1	24.0	160.6	17.9
2	2	0	169.9	168.3	20.8	165.4	14.4
3	1	0	170.7	169.3	20.3	166.3	14.4
1	1	2	172.5	169.9	24.9	166.3	19.0
3	0	1	174.1	172.2	19.8	169.5	15.8
2	0	2	183.0	179.6	14.2	177.2	12.7
2	2	1	185.7	181.1	15.6	177.8	14.1
3	1	1	186.5	182.0	15.1	173.7	13.9
2	1	2	194.9	190.1	11.3	186.5	12.5
0	3	0	200.6	197.7	7.0	195.1	7.1
0	2	2	201.0	197.4	9.3	194.3	10.9
3	2	0	206.3	203.8	8.2	199.9	8.7
1	3	0	207.3	204.5	8.2	200.4	8.8
1	2	2	207.7	204.4	10.3	199.7	12.8
4	0	0	209.5	208.1	6.1	205.3	6.7
0	3	1	214.2	211.9	8.5	207.9	11.4
3	0	2	217.3	215.8	8.0	211.9	10.9
3	2	1	219.5	218.0	9.8	212.2	13.2
4	1	0	219.9	219.0	7.3	213.9	4.3
1	3	1	220.5	218.7	9.3	216.8	13.3
4	0	1	222.5	221.8	8.0	217.6	11.3
0	0	3	225.2	224.1	6.6	221.3	9.3
2	3	0	226.3	225.5	7.7	228.0	8.7
2	2	2	226.7	225.3	9.6	219.4	13.1
3	1	2	227.3	226.7	9.6	229.9	13.2
1	0	3	231.2	230.7	8.4	225.8	11.6
4	1	1	232.3	232.7	10.9	225.6	14.4
0	1	3	234.9	234.8	10.7	229.3	13.2
2	3	1	238.4	239.1	14.4	231.5	16.5
1	1	3	240.6	241.5	15.6	233.5	17.3
2	0	3	243.3	249.3	17.2	240.1	16.5
4	2	0	248.5	250.5	16.3	243.0	15.3
0	3	2	250.5	252.1	14.6	243.9	17.3
3	3	0	254.8	256.1	16.0	243.6	16.0
3	2	2	255.1	256.3	19.4	243.4	19.9
1	3	2	255.9	257.2	18.9	245.3	19.9
2	1	3	257.2	258.4	18.5	249.9	19.9
4	0	2	257.7	259.0	15.4	251.3	16.8
4	2	1	259.6	260.9	17.6	252.7	19.9
5	0	0	261.8	262.9	10.5	258.0	12.7
0	2	3	261.9	262.1	14.5	256.0	16.6
3	3	1	265.6	265.8	15.4	257.8	19.3
4	1	2	266.2	266.6	15.2	253.5	19.0
1	2	3	267.1	266.8	14.9	253.7	18.8
0	4	0	267.4	267.1	9.9	262.6	9.4
5	1	0	270.2	270.5	11.1	263.9	11.9
2	3	2	271.5	270.3	13.2	262.9	13.4
5	0	1	272.4	272.3	10.7	266.8	15.5
1	4	0	272.5	271.6	10.4	265.2	11.5
3	0	3	274.6	273.6	10.0	268.3	14.7
0	4	1	277.3	276.4	12.3	270.5	16.8
5	1	1	280.5	279.6	16.4	271.3	20.7
2	2	3	282.0	280.3	17.4	272.1	20.8
3	1	3	282.6	281.0	17.9	272.7	20.9
1	4	1	282.7	280.9	19.2	272.5	21.3
2	4	0	287.2	285.4	17.1	273.7	14.6
4	3	0	290.0	288.5	18.8	281.7	15.9
4	2	2	290.3	288.4	23.9	279.3	24.1
5	2	0	294.0	292.5	21.3	285.3	16.4
3	3	2	295.7	293.6	[illegible]	[illegible]	[illegible]
2	4	1	296.9	294.2	22.5	[illegible]	27.3
4	3	1	299.6	297.1	31.7	[illegible]	[illegible]
0	0	4	299.2	297.7	[illegible]	[illegible]	[illegible]
0	3	3	301.6	299.0	27.6	295.8	32.8
5	0	2	301.8	300.2	26.5	294.9	24.2
5	2	1	303.4	301.5	31.9	291.9	29.6
1	0	4	304.8	302.6	26.3	312.5	[illegible]
3	2	3	305.4	303.1	31.3	293.6	29.3
1	3	3	306.1	303.7	31.7	310.1	34.1
0	4	2	306.7	304.6	27.3	309.4	33.9

4	0	3	307.5	306.1	26.1	299.8	24.8
0	1	4	307.6	305.6	27.7	301.1	32.6
5	1	2	309.1	307.3	31.5	297.9	30.1
3	4	0	310.2	303.8	25.3	300.8	20.1
1	4	2	311.1	309.3	31.4	314.9	36.0
1	1	4	312.0	310.3	31.7	315.5	35.5
6	0	0	314.2	314.2	19.9	309.5	14.8
4	1	3	314.7	313.7	31.1	303.4	31.4
2	0	4	318.0	317.1	25.4	310.3	26.6
3	4	1	319.1	318.5	30.7	307.9	34.5
2	3	3	319.2	318.3	30.5	307.8	33.5
6	1	0	321.2	321.7	24.8	337.9	35.2
6	0	1	323.0	323.7	25.2	321.4	37.8
2	4	2	324.1	323.7	30.3	312.9	35.2
2	1	4	324.9	324.7	30.3	338.4	43.0
4	3	2	326.6	326.8	30.1	315.9	35.7
0	2	4	328.7	328.7	25.7	323.4	35.0
5	3	0	329.8	330.6	24.0	320.4	32.1
6	1	1	329.9	331.3	29.9	343.7	50.4
5	2	2	330.1	331.0	29.9	319.8	37.0
1	2	4	332.8	333.4	29.7	335.8	40.9
0	5	0	334.3	334.7	19.4	330.2	29.7
4	2	3	335.3	336.5	29.6	325.1	36.5
5	3	1	338.3	340.1	29.2	328.4	37.5
1	5	0	338.4	339.3	23.1	342.2	36.4
3	0	4	338.8	339.9	24.0	337.0	34.7
4	4	0	339.7	341.1	23.4	330.3	32.1
3	3	3	340.0	341.5	29.4	329.8	36.1
6	2	0	341.5	343.6	23.2	332.6	32.4
0	5	1	342.6	344.0	24.4	338.2	33.5
3	4	2	344.6	346.6	29.1	334.7	36.1
2	2	4	344.9	346.8	29.0	334.8	36.3
5	0	3	345.3	347.3	24.7	344.3	33.0
3	1	4	345.4	347.5	23.5	358.3	47.2
1	5	1	346.6	348.3	27.3	350.0	41.0
4	4	1	347.9	350.5	28.3	338.4	37.1
6	0	2	348.2	350.8	24.5	347.6	32.6
0	4	3	349.6	351.4	25.7	345.5	30.4
6	2	1	349.6	353.0	28.0	340.7	37.3
2	5	0	350.3	352.2	23.9	340.9	32.3
5	1	3	351.7	355.0	27.9	365.1	46.4
1	4	3	353.5	356.3	27.7	357.1	39.0
6	1	2	354.6	358.4	27.5	363.2	46.3
2	5	1	358.3	361.7	27.1	349.0	36.9
0	3	4	361.1	363.9	25.4	357.2	27.6
5	3	2	362.4	366.7	28.0	354.0	35.2
3	2	4	364.3	368.3	27.3	355.4	35.2
1	3	4	364.8	363.8	26.4	363.6	38.2
2	4	3	365.0	362.0	27.8	356.0	35.1
4	0	4	366.1	369.4	24.6	365.2	34.4
2	5	2	366.5	369.5	25.6	362.7	27.5
7	0	0	366.6	369.8	20.3	366.5	26.0

4	3	3	367.2	371.0	23.7	358.3	25.3
3	5	0	369.4	371.9	24.7	360.2	30.7
1	5	2	370.2	373.2	28.5	373.1	38.9
5	2	3	370.3	373.8	29.6	[illegible]	26.1
4	4	2	371.4	374.4	30.0	365.0	36.2
4	1	4	372.1	375.0	29.5	333.5	45.0
7	1	0	372.6	375.7	24.6	[illegible]	42.2
6	3	0	372.8	375.4	25.5	363.9	33.2
6	2	2	373.0	376.2	30.4	362.9	36.9
7	0	1	374.2	377.0	25.7	[illegible]	33.1
5	4	0	374.3	376.3	25.9	365.0	33.3
0	0	5	375.3	376.0	23.1	375.7	27.5
2	3	4	375.9	377.7	31.5	365.5	37.1
3	5	1	376.9	373.7	31.4	366.5	33.8
1	0	5	378.9	379.8	26.2	383.1	36.5
7	1	1	380.1	382.4	32.5	390.3	47.0
6	3	1	380.2	382.1	32.3	370.0	32.4
2	5	2	381.1	381.3	33.3	370.1	38.4
0	1	5	381.2	381.3	28.3	395.8	42.1
5	4	1	381.7	383.0	33.5	371.0	39.6
3	4	3	383.3	383.6	34.1	372.1	33.3
1	1	5	384.8	384.8	34.9	404.0	46.0
6	0	3	386.5	387.0	29.1	333.6	32.1
2	0	5	389.6	388.7	30.1	[illegible]	41.2
4	2	4	389.7	388.9	36.3	377.7	39.7
7	2	0	390.2	390.4	29.6	381.0	25.8
6	1	3	392.3	391.5	37.8	392.7	45.9
3	3	4	393.8	391.9	37.6	391.1	40.2
4	5	0	394.5	393.1	30.9	334.1	25.9
2	1	5	395.3	393.1	39.2	401.2	46.4
7	0	2	396.1	395.2	32.4	391.3	44.9
7	2	1	397.3	395.8	40.0	384.8	42.3
5	0	4	398.4	396.4	33.1	393.0	45.5
0	2	5	398.4	395.8	34.7	399.1	45.6
3	5	2	398.7	395.9	39.4	395.5	41.5
5	3	3	399.4	396.7	39.5	396.4	41.2
0	6	0	401.2	399.0	23.4	395.0	31.6
4	5	1	401.6	398.7	41.2	388.0	43.1
7	1	2	401.7	399.7	41.4	407.7	47.1
1	2	5	401.8	398.5	41.6	399.3	42.8
6	3	2	401.9	399.4	49.2	389.1	42.2
0	4	4	402.1	399.2	35.4	393.6	46.5
0	5	3	403.1	400.3	35.6	394.7	46.8
5	4	2	403.3	400.6	40.4	399.2	42.3
5	1	4	403.9	401.3	41.7	409.2	46.8
1	6	0	404.6	402.1	35.0	404.3	27.9
1	4	4	405.5	402.3	42.2	403.0	43.0
1	5	3	406.5	403.4	42.4	404.0	43.2
3	0	5	406.8	404.1	34.2	409.4	47.9
4	4	2	407.6	404.3	41.2	394.2	42.8
0	6	1	408.1	405.6	37.3	399.4	49.9
6	5	3	409.0	406.8	41.6	396.1	43.5

1	6	1	411.5	408.7	44.3	408.4	45.8
2	2	5	411.9	409.0	42.4	397.9	44.5
3	1	5	412.3	409.5	43.5	416.5	48.0
6	4	0	412.6	411.1	34.8	401.8	28.5
2	6	0	414.6	412.4	35.3	402.9	29.3
2	4	4	415.5	412.7	42.9	401.7	44.7
2	5	3	416.4	413.7	43.2	402.7	45.0
4	3	4	417.4	414.9	43.2	404.0	44.9
7	3	0	417.9	416.8	35.6	407.2	29.5
7	2	2	418.1	416.4	43.6	405.3	46.0
8	0	0	418.9	418.7	29.0	416.4	34.4
6	4	1	419.4	417.6	44.9	406.1	47.4
5	2	4	420.2	418.0	43.8	406.8	45.8
2	6	1	421.4	418.9	45.5	425.6	52.7
4	5	2	422.1	419.8	44.2	408.7	46.3
3	1	0	424.2	423.8	38.1	431.0	33.1
7	3	1	424.5	423.2	46.0	411.4	48.8
5	5	0	424.6	423.2	36.6	413.3	30.4
0	3	5	425.5	423.3	39.4	417.5	52.5
3	0	1	425.6	425.1	39.2	420.8	54.0
3	2	5	428.2	425.9	45.5	414.3	47.7
0	6	2	428.3	426.3	40.9	420.5	53.6
1	3	5	428.7	426.3	47.3	425.5	46.7
4	0	5	429.8	428.1	38.4	424.3	52.3
7	0	3	430.2	429.3	38.5	425.6	52.5
8	1	1	430.8	430.2	49.0	434.9	52.7
3	6	0	430.8	429.2	37.6	419.0	31.7
5	5	1	431.2	429.5	47.4	417.6	50.2
1	6	2	431.5	429.3	48.0	428.4	47.6
3	4	4	431.7	429.4	46.0	417.9	47.9
3	5	3	432.6	430.4	46.2	418.9	48.2
6	0	4	434.6	433.5	39.1	429.7	53.2
4	1	5	434.9	433.1	48.3	438.1	50.5
7	1	3	435.4	434.3	48.1	439.3	50.4
6	3	3	435.5	434.0	46.4	422.4	48.6
5	4	3	436.8	435.1	45.9	423.4	48.2
3	6	1	437.3	435.6	46.6	423.2	51.1
2	3	5	438.2	436.2	44.8	441.7	51.6
6	4	2	439.1	437.8	44.2	426.0	48.3
6	1	4	439.7	438.6	45.0	443.1	49.1
8	2	0	439.8	439.6	35.7	423.9	31.7
2	6	2	441.0	439.2	43.1	444.4	51.4
7	3	2	444.0	443.3	41.0	431.1	47.8
8	0	2	445.0	445.1	33.7	440.7	48.0
5	3	4	446.0	444.9	39.6	432.5	46.8
8	2	1	446.1	446.1	40.5	432.9	49.9
0	5	4	449.3	448.1	32.5	441.5	45.9
8	1	2	450.0	450.2	37.9	453.5	47.1
4	2	5	450.1	449.0	37.0	436.1	46.5
0	0	6	450.3	449.3	25.8	450.7	44.2
5	5	2	450.4	449.6	36.8	436.9	46.5
7	2	3	450.5	450.2	36.3	437.3	46.5

1	5	4	452.4	451.3	36.3	448.6	43.4
4	6	0	452.6	451.9	28.6	433.8	43.8
4	4	4	453.4	452.5	34.9	439.6	45.5
1	0	6	453.4	452.5	29.7	457.7	40.7
3	3	5	453.6	452.6	34.3	439.5	45.8
7	4	0	453.8	453.8	28.0	441.7	43.3
4	5	3	454.2	453.4	34.3	440.5	45.4
6	2	4	454.7	454.3	34.1	441.2	45.6
0	1	6	455.3	454.4	22.6	463.3	42.8
3	6	2	456.2	455.4	33.1	449.3	45.3
5	0	5	457.6	457.2	26.6	452.1	41.0
1	1	6	458.3	457.6	32.5	478.1	41.2
4	6	1	458.7	459.4	31.7	444.6	47.1
6	5	0	458.8	458.8	25.3	446.3	44.1
7	4	1	459.9	460.3	30.9	446.4	46.3
0	6	3	460.0	459.3	26.4	459.1	40.9
0	4	5	460.8	460.1	26.0	452.3	40.5
2	5	4	461.4	460.7	22.3	463.6	45.7
2	0	6	462.3	461.8	24.9	472.5	40.3
5	1	5	462.4	462.4	29.2	464.6	43.1
1	6	3	463.0	462.5	23.9	459.1	41.1
1	4	5	463.8	463.3	23.4	459.7	40.8
3	3	0	464.5	465.3	22.3	459.4	44.9
8	2	2	464.7	465.5	27.6	451.6	44.7
6	5	1	464.9	465.2	27.6	451.1	45.9
2	1	6	467.2	466.9	26.1	434.3	42.2
0	7	0	468.0	467.7	16.8	465.4	34.0
0	2	6	469.8	469.4	20.9	457.7	43.8
6	4	3	470.1	470.6	24.9	456.5	42.3
8	3	1	470.5	471.7	23.3	474.4	30.1
1	7	0	471.0	470.9	18.3	472.3	34.2
9	0	0	471.3	472.7	14.5	472.2	30.4
2	6	3	471.8	471.8	23.0	473.6	43.4
2	4	5	472.6	472.6	22.5	473.9	43.1
1	2	6	472.7	472.7	22.4	494.1	44.7
7	0	4	473.8	474.7	17.3	463.5	34.7
0	7	1	474.0	474.1	13.3	475.4	69.9
4	3	5	474.3	474.6	21.5	460.9	42.8
7	3	3	474.7	475.7	21.9	461.3	42.2
8	0	3	475.6	476.9	16.9	470.6	34.3
3	5	4	476.0	476.3	20.4	461.3	41.9
9	1	0	476.0	477.7	16.0	484.2	32.5
5	2	5	476.7	477.4	19.9	483.6	47.7
4	6	2	476.3	477.3	19.3	462.3	49.2
1	7	1	476.9	477.3	19.4	439.0	69.3
3	0	6	476.9	477.1	16.1	470.6	34.9
9	0	1	477.2	479.0	15.3	488.9	65.3
7	4	2	477.9	479.2	19.1	464.6	42.0
7	1	4	478.5	479.8	13.5	480.6	39.9
6	3	4	478.6	479.6	18.7	464.9	41.4
5	6	0	479.1	479.7	14.6	479.7	34.2
2	7	0	479.6	479.9	14.3	486.4	33.4

5	4	4	479.8	480.6	17.9	465.9	41.1
9	1	3	480.3	482.0	17.7	482.6	38.3
5	5	3	480.6	481.6	18.2	466.8	41.3
2	2	6	481.3	481.8	18.4	507.2	45.9
3	1	6	481.6	482.3	18.4	482.7	39.3
9	1	1	481.9	484.2	18.2	500.4	66.6
6	5	2	482.7	484.1	19.3	469.2	42.6
5	6	1	484.9	486.3	20.2	487.4	72.6
2	7	1	485.4	486.5	20.3	502.5	69.3
3	6	3	486.1	487.3	21.2	472.1	43.5
3	4	5	486.9	488.0	21.7	472.8	43.7
3	3	2	488.1	490.6	22.3	475.2	44.8
6	0	5	489.4	491.0	19.3	484.2	34.4
9	2	0	489.9	492.4	19.4	507.3	33.7
0	7	2	491.5	492.6	21.6	433.7	36.4
7	2	4	492.3	494.8	24.6	504.1	50.4
0	3	6	493.0	494.1	22.5	485.1	36.0
3	7	0	493.7	495.0	21.4	485.6	35.4
6	1	5	494.0	496.5	24.8	495.8	43.0
3	2	3	494.0	497.0	25.6	506.1	51.1
1	7	2	494.3	496.2	24.7	490.4	42.7
9	0	2	494.6	497.5	22.2	490.5	35.7
3	2	6	495.3	497.3	26.2	506.1	51.6
9	2	1	495.6	499.2	25.4	522.8	72.0
4	5	4	495.8	497.9	26.7	432.2	46.6
1	3	6	495.8	497.8	25.5	491.6	42.7
7	5	0	496.1	498.4	22.6	439.1	35.0
4	0	6	496.7	498.3	23.5	491.3	35.1
8	4	0	497.0	499.7	23.1	490.3	35.1
9	1	2	499.1	503.1	27.0	501.9	45.5
3	7	1	499.4	502.0	27.4	501.4	72.9
5	3	5	499.6	502.3	28.9	486.2	48.1
0	6	4	501.1	502.9	26.9	493.7	35.7
4	1	6	501.1	504.0	23.2	502.6	45.5
7	5	1	501.8	505.4	28.6	504.6	72.4
5	6	2	502.0	505.0	30.0	438.9	49.5
2	7	2	502.5	505.3	30.0	504.0	49.3
0	5	5	502.6	504.6	27.7	495.3	35.5
3	4	1	502.7	506.6	29.0	505.7	72.5
1	6	4	503.8	506.7	29.4	500.0	45.1
2	3	6	504.0	506.7	31.9	505.3	49.3
1	5	5	505.3	508.3	30.1	501.5	45.5
4	6	3	505.5	508.6	32.1	492.2	50.2
4	4	5	506.2	509.4	32.6	492.9	50.3
7	4	3	506.6	510.4	32.7	494.0	50.5
6	2	5	507.4	511.1	33.0	513.5	54.8
6	6	0	509.6	512.6	29.6	503.1	33.7
6	4	4	510.3	514.3	34.9	497.5	51.7
6	5	3	511.1	515.1	35.3	498.4	52.0
2	6	4	511.9	515.5	35.4	513.4	51.5
9	3	0	512.2	516.3	30.9	506.6	33.6
9	2	2	512.4	517.5	35.2	524.3	57.3

4	7	0	512.8	515.7	31.3	506.0	33.3
2	5	5	513.4	517.1	36.2	514.8	51.9
4	2	6	514.3	518.3	36.7	524.9	57.1
7	3	4	514.5	519.2	37.2	502.1	53.4
6	6	1	515.1	519.8	34.6	517.6	73.1
8	0	4	515.4	519.4	33.5	511.9	33.0
3	7	2	516.0	520.1	37.5	509.9	51.9
8	3	3	516.2	521.3	38.0	504.1	54.2
3	3	6	517.4	521.5	38.6	504.1	54.9
9	3	1	517.7	523.5	35.5	520.9	73.6
4	7	1	518.2	522.9	35.9	520.3	73.5
7	5	2	518.3	523.4	33.9	506.1	55.3
8	4	9	519.2	524.6	39.4	507.3	55.8
0	7	3	519.4	522.8	37.1	512.7	31.4
8	1	4	519.7	525.5	36.4	522.3	51.4
5	5	4	520.0	524.8	40.4	507.4	55.1
5	0	6	520.9	524.6	36.6	516.9	31.3
1	7	3	522.0	526.9	36.7	518.7	51.4
9	0	3	522.3	527.2	37.2	519.5	30.9
10	0	0	523.7	527.3	32.9	527.6	1.5
0	4	6	523.8	527.4	39.3	517.0	23.1
7	0	5	524.6	529.0	38.7	521.1	29.6
3	6	4	525.1	530.0	42.9	532.5	60.5
5	1	6	525.9	530.7	38.4	527.0	53.1
0	0	7	525.4	527.9	36.0	527.6	0.4
1	4	6	526.4	531.3	37.7	529.9	52.8
3	5	5	526.6	531.1	42.3	533.7	60.2
9	1	3	526.6	532.9	37.9	[illegible]	[illegible]
10	1	0	527.9	533.2	39.4	537.5	[illegible]
1	0	7	528.0	531.5	32.9	533.0	56.5
7	1	5	528.8	534.1	37.6	529.7	59.2
6	3	5	528.9	533.5	41.2	516.1	56.5
10	0	1	529.0	534.2	33.1	540.7	47.1
5	6	3	529.3	533.7	41.0	516.4	56.4
0	1	7	529.6	532.9	24.4	537.9	17.4
2	7	3	529.8	533.3	40.1	530.8	55.0
5	4	5	530.0	534.2	40.3	516.9	56.1
6	6	2	531.2	535.5	39.7	518.3	56.3
1	1	7	532.2	536.3	33.6	542.1	36.4
3	2	4	532.5	536.9	39.1	542.6	53.4
10	1	1	533.2	538.9	33.0	542.4	64.5
9	3	2	533.7	538.3	33.4	551.3	57.0
9	4	6	534.1	537.2	33.3	534.4	54.0
4	7	2	534.3	537.6	38.3	520.6	56.6
0	3	0	534.9	536.1	29.5	534.5	15.3
4	3	6	535.6	538.5	37.9	551.6	55.7
2	0	7	535.7	537.6	32.3	544.7	35.4
3	5	0	536.0	533.9	39.3	529.5	35.3
5	7	0	536.3	538.5	39.1	522.1	36.0
1	3	0	537.5	539.4	29.6	539.2	30.9
5	2	6	537.3	540.5	36.7	516.3	57.2
9	2	3	539.2	542.6	36.0	543.6	57.1

2	1	7	539.9	542.1	34.3	553.3	47.7
0	3	1	540.1	541.7	30.9	546.6	53.7
10	2	0	540.5	543.3	29.3	556.3	38.7
3	5	1	541.2	544.2	34.2	541.5	67.9
7	2	5	541.4	543.8	35.0	549.8	56.2
5	7	1	541.5	543.8	34.1	541.1	68.0
9	4	0	541.9	544.4	27.2	534.8	38.5
0	2	7	542.1	543.3	23.9	556.6	43.4
3	7	3	542.6	544.1	34.5	547.4	57.1
1	8	1	542.7	544.6	32.8	551.0	61.2
4	6	4	543.1	544.6	34.4	528.4	54.3
7	6	0	543.4	545.2	26.8	535.6	38.8
7	4	4	544.1	546.0	33.9	529.9	54.1
4	5	5	544.5	545.7	33.7	529.6	54.0
1	2	7	544.7	545.8	32.9	560.9	49.8
10	0	2	544.8	547.3	26.9	540.3	43.8
7	5	3	544.8	546.6	33.5	530.6	54.1
2	8	0	545.0	545.8	26.1	550.1	36.7
8	4	3	545.6	547.5	33.1	531.6	54.1
10	2	1	545.7	548.4	32.5	567.5	64.8
3	4	6	546.8	547.4	32.6	550.8	56.0
9	4	1	547.1	549.2	32.1	546.7	66.0
6	5	4	548.3	549.1	31.9	533.4	53.3
3	0	7	548.4	548.5	25.8	560.9	45.7
7	6	1	548.6	549.8	31.6	547.5	65.3
10	1	2	548.8	551.0	31.4	549.0	49.5
6	0	6	549.1	549.7	25.6	542.8	44.0
0	6	5	549.3	549.3	26.7	540.5	45.6
2	8	1	550.2	550.4	30.9	561.6	60.7
1	6	5	551.8	551.5	30.4	545.3	47.2
2	2	7	552.2	551.6	30.0	571.4	50.5
3	1	7	552.4	552.1	30.1	569.2	48.7
8	3	4	553.1	553.4	29.6	538.1	53.0
6	1	6	553.2	553.1	29.3	551.6	47.3
0	8	2	555.6	554.5	24.6	545.8	45.5
0	7	4	556.0	554.9	24.3	546.2	47.3
3	5	2	556.6	556.3	27.9	541.3	53.3
5	7	2	556.9	555.9	27.7	540.9	53.3
6	6	3	557.1	556.2	27.6	541.3	52.3
3	8	0	557.5	556.3	22.3	565.3	41.5
6	4	5	557.7	556.7	27.3	541.3	52.0
1	8	2	558.0	556.5	27.9	550.8	47.5
5	3	5	558.2	556.8	27.1	541.9	52.2
1	7	4	558.5	556.8	27.6	551.3	46.1
9	0	4	558.8	558.5	22.4	551.8	45.4
2	6	5	559.2	557.3	26.6	556.3	43.9
9	3	3	559.5	558.9	26.5	544.1	52.4
4	7	3	560.0	558.2	26.2	543.5	52.1
10	3	0	560.8	560.6	21.2	550.6	40.2
0	5	6	560.9	558.8	22.6	550.0	47.2
10	2	2	560.9	560.4	25.9	567.9	53.2
7	3	5	561.6	560.0	25.4	545.4	51.7

9	4	2	562.3	561.2	25.9	546.6	52.7
0	3	7	562.4	560.0	22.1	551.1	47.4
3	0	5	562.4	561.3	21.1	[illegible]	45.9
3	8	1	562.5	560.2	25.7	[illegible]	59.4
9	1	4	562.8	561.6	25.5	560.9	45.5
1	5	6	563.3	560.5	25.3	[illegible]	45.2
6	7	0	563.7	561.9	20.1	[illegible]	33.9
7	6	2	563.8	561.8	24.5	547.1	52.2
4	4	6	564.1	561.4	24.2	547.8	51.2
3	2	7	564.4	561.6	24.2	536.6	51.0
1	3	7	564.8	561.7	24.6	556.6	45.3
6	2	6	565.2	562.6	23.8	562.7	50.2
2	8	2	565.3	562.3	23.9	561.6	49.0
5	6	4	565.4	562.6	23.6	548.4	50.9
4	0	7	565.6	562.9	18.8	556.1	44.6
10	3	1	565.8	564.3	24.1	562.7	52.6
2	7	4	565.8	562.7	23.5	562.0	47.7
8	1	5	566.4	564.1	23.8	563.7	44.6
5	5	5	566.7	563.7	22.9	549.6	50.6
6	7	1	568.7	565.5	22.6	564.3	53.2
4	1	7	569.5	565.7	22.2	565.5	44.2
10	0	3	570.0	568.1	16.9	561.5	43.7
2	5	6	570.5	566.4	21.2	566.1	46.7
3	6	5	571.4	567.2	20.7	571.9	50.7
2	3	7	572.0	567.6	20.5	567.2	46.8
10	1	3	573.9	570.7	20.0	570.9	43.6
4	8	0	574.4	570.6	14.6	569.5	34.5
9	2	4	574.6	571.0	19.3	573.6	48.9
3	8	2	577.3	572.1	19.1	577.0	51.5
3	7	4	577.8	572.4	13.9	577.5	50.2
9	5	0	577.8	574.6	14.2	564.5	33.5
8	2	5	578.1	573.6	12.0	581.3	43.1
4	8	1	579.3	573.7	12.4	579.9	56.4
7	5	4	579.9	574.8	13.8	561.3	50.1
8	6	0	580.0	576.2	14.1	566.1	32.9
0	8	3	580.4	575.2	15.3	566.3	41.3
10	3	2	580.5	576.2	12.1	563.0	51.2
8	4	4	580.7	575.7	13.3	562.7	50.4
7	0	6	580.7	576.3	14.5	562.7	49.3
4	2	7	581.2	575.3	12.2	532.0	49.1
8	5	3	581.3	576.3	13.9	563.4	58.7
5	7	3	581.6	575.9	13.9	563.0	59.3
3	5	6	582.4	576.2	12.9	531.4	59.3
9	5	1	582.7	577.6	12.3	576.8	56.2
1	3	3	582.7	576.9	13.9	579.8	44.9
6	7	2	583.4	577.5	12.0	564.6	51.5
3	3	7	583.9	577.4	13.9	532.4	50.2
7	1	6	584.5	578.5	12.0	579.4	43.4
6	3	6	584.6	578.5	13.8	565.5	51.0
8	6	1	584.9	579.1	12.2	578.4	56.2
10	2	3	585.5	580.3	12.1	532.2	49.6
5	4	6	585.6	579.1	12.3	566.3	51.9

A.5 programs: MODESEPERATN, MODESPLOT.

```
MODE SEPERATN
UAFORTRAN CWNPD,*CR1 DATA3,*CP1 RESULTS3,LINES 5000
      LIST(LP)
      PROGRAM(SPRT)
      INPUT  4= CR1
      INPUT  5= CR0
      OUTPUT 6= LP0
      OUTPUT 7= CP1
      TRACE 1
      END
      MASTER PEAKS
      REAL DFREQ(600),RFREQ(650),SPL,RSPL,K,LX,LY,LZ,NUM
      REAL XT,YT,XR,YR,DCN(650)
      INTEGER ENX,ENY,ENZ,N(650,3)
      COMPLEX SIGMA,TERM,RTERM
      C=343.0
      ROW=1.21
      PI=4.0*ATAN(1.0)
      LX=3.270
      LY=2.570
      LZ=2.290
      READ(5,101)N1,N2,N3
  101 FORMAT(3I3)
      READ(5,100)XS,YS,ZS
  100 FORMAT(3F8.5)
      READ(5,99)F1,F2,Q
   99 FORMAT(3F8.3)
      V=LX*LY*LZ
      K=ROW*Q*C**2/(SQRT(2.0)*V*2.0*(10.0**(-5)))
      DO 1 I=1,650
      READ(4,97)(N(I,J),J=1,3),FR0,DN0,FR1,DN1
      RFREQ(I)=FR1
      DCN(I)=DN1
      IF(N(I,1).EQ.0.AND.N(I,2).EQ.0.AND.N(I,3).EQ.0)GOTO96
   97 FORMAT(1X,3I3,4F9.3)
    1 CONTINUE
   96 IC=I-1
      DO 2 J=1,600
      DFREQ(J)=F1+FLOAT(J)*0.2
      IF(DFREQ(J).GE.F2)GOTO95
    2 CONTINUE
   95 JC=J
   94 READ(5,93)N0,XP,YP,ZP
      IF(N0.EQ.0)GOTO16
   93 FORMAT(I3,3F8.5)
      DO 3 J=1,JC
      SIGMA=CMPLX(0.0,0.0)
      DOMEGA=2.0*PI*DFREQ(J)
      DO 4 I=1,IC
      ROMEGA=2.0*PI*RFREQ(I)
      ENX=2
      IF(N(I,1).EQ.0)ENX=1
      ENY=2
      IF(N(I,2).EQ.0)ENY=1
      ENZ=2
      IF(N(I,3).EQ.0)ENZ=1
```

```
      DN=DCN(I)
      FSR=COS(PI*N(I,1)*XS/LX)*COS(PI*N(I,2)*YS/LY)*COS(PI*N
     +(I,3)*ZS/LZ)
      FPT=COS(PI*N(I,1)*XP/LX)*COS(PI*N(I,2)*YP/LY)*COS(PI*N
     +(I,3)*ZP/LZ)
      CONS=ENX*ENY*ENZ
      NUM=FSR*FPT*CONS*K
      DEM=(DOMEGA**2.0-ROMEGA**2.0)**2.0+4.0*ROMEGA**2.0*DN**2.0
      A=-2.0*ROMEGA*DN*NUM/DEM
      B=(DOMEGA**2.0-ROMEGA**2.0)*NUM/DEM
      TERM=CMPLX(A,B)
      IF(N1.EQ.N(I,1).AND.N2.EQ.N(I,2).AND.N3.EQ.N(I,3))RTERM=TERM
      SIGMA=SIGMA+TERM
    4 CONTINUE
      XT=REAL(SIGMA)/100.0
      YT=AIMAG(SIGMA)/100.0
      XR=REAL(RTERM)/100.0
      YR=AIMAG(RTERM)/100.0
      PRESS=CABS(SIGMA)
      IF(PRESS.EQ.0.0)PRESS=1.0
      PR=CABS(RTERM)
      IF(PR.EQ.0.0)PR=1.0
      SPL=20.0*ALOG10(PRESS)
      IF(SPL.LT.0.0)SPL=0.0
      RSPL=20.0*ALOG10(PR)
      IF(RSPL.LT.0.0)RSPL=0.0
      WRITE(7,92)DFREQ(J),XT,YT,SPL,XR,YR,RSPL
   92 FORMAT(1X,F8.1,2F12.1,F10.3,2F12.1,F10.3)
      GOTO3
      WRITE(6,92)DFREQ(J),XT,YT,SPL,XR,YR,RSPL
    3 CONTINUE
      FT=-1.0
      WRITE(7,92)FT
      GOTO94
   16 FS=0.0
      WRITE(7,92)FS
      STOP
      END
      FINISH
```

```
MODESPLOT
UAFORTRAN OWNPD,*CR1 RESULTS5,*GP GINO,LINES 5000
       LIST(LP)
       PROGRAM(SPRT)
       INPUT  4= CR1
       INPUT  5= CR0
       OUTPUT 6= LP0
       COMPRESS INTEGER AND LOGICAL
       TRACE 2
       END
       MASTER PEAKS
       REAL F(600),X(600),Y(600),XC(11),G(600),XW(11)
       REAL X1(600),Y1(600),X2(600),Y2(600),XX(600),YY(600)
       INTEGER IF(11),JF(600),IMODE(1)
       CALL OPEN GINOGP
       READ(5,99)IMODE(1)
    99 FORMAT(A4)
C      IFLAG IS 1 FOR TREATED WALLS AND ZERO FOR BARE WALLS
C      CS IS 40 FOR TREATED WALLS AND 400 FOR BARE WALLS
C      OMIT DATA CARDS WITH TREATED WALLS
       IFLAG=1
    71 CALL CHASWI(1)
       CALL SCALE(0.4)
       JC=0
     1 READ(4,100)DFREQ,XT,YT,SPL,XR,YR,RSPL
   100 FORMAT(1X,F8.1,2F12.1,F10.3,2F12.1,F10.3)
       IF(DFREQ.LT.0.0)GOTO2
       IF(DFREQ.EQ.0.0)GOTO16
       JC=JC+1
       G(JC)=DFREQ
       JF(JC)=DFREQ
       F(JC)=ALOG10(DFREQ)
       X1(JC)=XT
       Y1(JC)=YT
       X(JC)=SPL
       X2(JC)=XR
       Y2(JC)=YR
       Y(JC)=RSPL
       GOTO1
     2 F1=ALOG10(G(1)-0.2)
       F2=ALOG10(G(JC))
       G1=G(1)-0.2
       G2=G(JC)
       W=250.0
       C=F2-F1
       CALL SHIFT2(100.,100.)
       CALL AXIPOS(1,40.,40.,W,1)
       CALL AXIPOS(1,40.,40.,200.,2)
       CALL AXISCA(3,10,F1,F2,1)
       CALL AXISCA(3,10,50.,150.,2)
       CALL AXIDRA(0,0,1)
       CALL AXIDRA(-2,-1,2)
       CALL GRASYM(F,X,JC,4,7)
       CALL GRAPOL(F,X,JC)
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      CALL GRASYM(F,Y,JC,8,7)
      CALL GRAPOL(F,Y,JC)
      CALL GRAMOV(F(JC),X(JC))
      CALL CHAHOL(6H SPL*.)
      CALL GRAMOV(F(JC),Y(JC))
      CALL CHAHOL(7H RSPL*.)
      CALL MOVTO2(0.,0.)
      DO 7 L=1,11
      U=(G2-G1)/10.0
      FC=G1+U*(L-1)
      IF(L)=FC
      XC(L)=40.0+(ALOG10(FC)-F1)*W/C
      XW(L)=XC(L)-3.048
      CALL MOVTO2(XW(L),30.0)
      CALL CHAINT(IF(L),4)
      CALL MOVTO2(0.,0.)
    7 CONTINUE
      DO 8 K=1,11
      CALL MOVTO2(XC(K),40.0)
      CALL LINBY2(0.,200.)
    8 CONTINUE
      DO 9 N=1,51
      YC=40.0+4.0*(N-1)
      CALL MOVTO2(40.,YC)
      CALL LINBY2(W,0.)
    9 CONTINUE
      CALL MOVTO2(0.,0.)
      CALL MOVBY2(40.,10.)
      CALL CHAHOL(27HFREQUENCY RESPONSE CURVE:*.)
      CALL CHAARR(IMODE,1,4)
      CALL MOVTO2(220.,20.)
      CALL CHAHOL(14HFREQUENCY:HZ*.)
      CALL MOVTO2(20.,200.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(14HAMPLITUDE:DB*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(165.,450.)
      CS=30.0
      CU=20.0*CS
      SC=1.0/CS
      LC=1
      JLC=JC
      IF(IFLAG.GT.0)GOTO83
   81 READ(5,98)LC,JLC
   98 FORMAT(2I5)
      IF(LC.EQ.0)GOTO82
   83 X1P=0.0
      Y1P=0.0
      IPM=2
      IP=1
      I=0
      K=0
      DO 11 J=LC,JLC
      K=K+1
      XX(K)=X1(J)
      YY(K)=Y1(J)
      IF(SQRT((X1(J)-X1P)**2+(Y1(J)-Y1P)**2).LT.CU)GOTO 11
```

```
      I=I+1
      CALL SCALE(SC)
      CALL MOVTO2(X1(J),Y1(J))
      CALL SCALE(CS)
      CALL SYMBOL(4)
      IF(I.EQ.IP)GOTO20
      GOTO 25
   20 IF((ABS(X1(J)).LT.1.5*CU).AND.(ABS(Y1(J)).LT.1.5*CU))GOTO35
      CALL CHAFIX(G(J),6,1)
   35 IP=IP+IPM
   25 X1P=X1(J)
      Y1P=Y1(J)
   11 CONTINUE
      KC=K
      CALL SCALE(SC)
      CALL MOVTO2(XX(1),YY(1))
      CALL CURTO2(XX,YY,KC,0.,0.)
      CALL SCALE(CS)
      CALL MOVTO2(0.,0.)
      IF(IFLAG.GT.0)GOTO82
      GOTO81
   82 CALL MOVTO2(0.,0.)
      X2P=0.0
      Y2P=0.0
      I=0
      DO 12 J=1,JC
      IF(SQRT((X2(J)-X2P)**2+(Y2(J)-Y2P)**2).LT.CU)GOTO 12
      I=I+1
      CALL SCALE(SC)
      CALL MOVTO2(X2(J),Y2(J))
      CALL SCALE(CS)
      CALL SYMBOL(7)
      X2P=X2(J)
      Y2P=Y2(J)
   12 CONTINUE
      CALL DASHED(1,10.,5.,0)
      CALL SCALE(SC)
      CALL MOVTO2(X2(1),Y2(1))
      CALL CURTO2(X2,Y2,JC,0.,0.)
      CALL SCALE(CS)
      CALL DASHED(0,0.,0.,0)
      CALL MOVTO2(0.,0.)
      CALL AXIPOS(0,0.,0.,400.,1)
      CALL AXIPOS(0,0.,0.,400.,2)
      CALL AXISCA(3,10,-5000.,5000.,1)
      CALL AXISCA(3,10,-5000.,5000.,2)
      CALL AXIDRA(1,0,1)
      CALL AXIDRA(-1,0,2)
      CALL MOVTO2(204.,0.)
      CALL CHAHOL(11HREAL AXIS*.)
      CALL MOVTO2(0.,204.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(16HIMAGINARY AXIS*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,0.)
      CALL MOVTO2(-125.,-190.)
      CALL CHAHOL(18HNYQUIST DIAGRAM:*.)
      CALL CHAARR(IMODE,1,4)
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      CALL MOVTO2(-125.,-200.)
      CALL CHAHOL(13HSCALE 1:100*.)
      CALL MOVTO2(100.,-150.)
      CALL CHAHOL(21HTREATED WALLED ROOM:*.)
      CALL MOVTO2(100.,-190.)
      CALL CHAHOL(12HSOURCE AT:*.)
      CALL MOVTO2(100.,-200.)
      CALL CHAHOL(14HRECEIVER AT:*.)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(-265.,-550.)
      CALL MOVTO2(0.,0.)
      CALL SCALE(2.5)
      CALL CHASWI(0)
      DO 6 J=1,JC,10
      WRITE(6,101)S(J),X(J),Y(J)
  101 FORMAT(1X,F8.1,2F10.3)
    6 CONTINUE
      CALL PICCLE
      IFLAG=IFLAG+1
      GOTO 71
   16 CALL DEVEND
      STOP
      END
      FINISH
```

APPENDIX B

LISTINGS OF COMPUTER PROGRAMS EMPLOYED IN ANALYSIS AND PRESENTATION OF EXPERIMENTAL DATA

APPENDIX B

LISTINGS OF COMPUTER PROGRAMS EMPLOYED IN ANALYSIS AND PRESENTATION OF EXPERIMENTAL DATA

B.1. program: MODERESPONSE

```
MODERESPONSE
UAFORTRAN OWNPD,*LP1 CVALUES,*LP2 CSPACING,*LP3 MTEXTS,-
*GP GINO,LIB :EBS7005.GLIB,LINES 10000,*CR1 CORRELS,-
*TR0 MODESN
      LF(LP)
      PROGRAM(MSRS)
      INPUT  3=TR0
      INPUT  4=CR1
      INPUT  5=CR0
      OUTPUT 6=LP0
      OUTPUT 7=LP1
      OUTPUT 8=LP2
      OUTPUT 9=LP3
      TRACE 2
      EXTENDED DATA
      END
      TRACE 1
      MASTER VECTOR
      REAL X(1024),F(1024),MOD(1024),VOLT(2048),ALPHA(1024)
      REAL YA(1024),SP(1024),FP(1024)
      REAL SG(1024),G(1024)
      INTEGER IWX1(20),IWX2(20),IWY1(20),IWY2(20),IWZ1(20),IWZ2(20)
      LOGICAL FIRST
      COMMON/ST/FIRST
      FIRST=.TRUE.
      CALL OPEN GINOGP
      CALL ERRMAX(50)
      READ(5,200)(IWX1(I),I=1,10)
      WRITE(9,200)(IWX1(I),I=1,10)
      READ(5,200)(IWX2(I),I=1,10)
      WRITE(9,200)(IWX2(I),I=1,10)
      READ(5,200)(IWY1(I),I=1,10)
      WRITE(9,200)(IWY1(I),I=1,10)
      READ(5,200)(IWY2(I),I=1,10)
      WRITE(9,200)(IWY2(I),I=1,10)
      READ(5,200)(IWZ1(I),I=1,10)
      WRITE(9,200)(IWZ1(I),I=1,10)
      READ(5,200)(IWZ2(I),I=1,10)
      WRITE(9,200)(IWZ2(I),I=1,10)
  200 FORMAT(10A8)
   15 READ(5,100)IMODE,IF1,IF2
  100 FORMAT(3I0)
      WRITE(9,101)IMODE,IF1,IF2
      WRITE(6,101)IMODE,IF1,IF2
      IF(IMODE.EQ.0000)GOTO 16
  101 FORMAT(1X,3I5)
      CALL CHASWI(1)
      CALL SCALE(0.5)
      CALL SHIFT2(300.0,300.0)
      CALL SCALE(0.05)
      F1=IF1/10.0
      F2=IF2/10.0
      XP=1000.0
      YP=1000.0
```

```
      VA=-1.0
      CALL BUD(VOLT,2048,VA,VB,SRATE,NSAMP,.FALSE.,.TRUE.)
      PI=4.0*ATAN(1.0)
      IC=1024
      YAA=0.0
      DO 12 I=2,IC
      J=2*I
      K=J-1
      F(I)=F1+(I-1)*(F2-F1)/(IC-1)
      X(I)=VOLT(K)*1000.0*(F1+F2)/(2.0*F(I))
      YA(I)=VOLT(J)*1000.0*(F1+F2)/(2.0*F(I))
      YAB=YA(I)
      YA(I)=(YA(I)+YAA)/2.0
      YAA=YAB
   12 CONTINUE
      CALL CORRECT(X,YA,F)
      READ(5,303)DXO,DYO,DXC,DYC
      IF(DXC.EQ.1000.0)GOTO106
  303 FORMAT(4F7.1)
      DXC=DXC*20.0
      DYC=DYC*20.0
      DXO=DXO*20.0
      DYO=DYO*20.0
      DXF=DXO+2.0*(DXC-DXO)
      DYF=DYO+2.0*(DYC-DYO)
      IC=1024
      CALL MOVTO2(DXO,DYO)
      CALL SCALE(20.0)
      CALL CHAHOL(20H  DISPLACED ORIGIN*.)
      CALL SCALE(0.05)
      XD1=DXC+DYC-DYF
      YD1=DYC-DXC+DXF
      XD2=DXC-DYC+DYF
      YD2=DXC+DYC-DXF
      CALL MOVTO2(XD1,YD1)
      CALL LINTO2(XD2,YD2)
      CALL MOVTO2(DXO,DYO)
      CALL ARCTO2(DXC,DYC,DXO,DYO,0)
      CALL MOVTO2(DXO,DYO)
      CALL LINTO2(DXF,DYF)
      CALL MOVTO2(00.0,00.0)
  106 XPR=1000.0
      YPR=1000.0
      DO19I=2,IC
      J=(I-1)
      SP(J)=SQRT((X(I)-XPR)**2+(YA(I)-YPR)**2)
      FP(J)=(F(I)+F(I-1))/2.0
      XPR=X(I)
      YPR=YA(I)
   19 CONTINUE
      L=1
      DO20I=6,IC,5
      SG(L)=(SP(I)+SP(I-1)+SP(I-2)+SP(I-3)+SP(I-4))/
     +((FP(I)-FP(I-4))**2)
      G(L)=(FP(I)+FP(I-4))/2.0
      L=L+1
   20 CONTINUE
      LC=L
```

```
      LE=LC-3
      DO31 L=3,LE
      SPACE=(SG(L-2)+SG(L-1)+SG(L)+SG(L+1)+SG(L+2))/5.0
      CNFR=G(L)
      WRITE(8,103)CNFR,SPACE
  103 FORMAT(2(1X,2F12.4))
   31 CONTINUE
      FREQ=10000.0000
      WRITE(8,103)FREQ
      DO 18 I=2,IC
      YC=YA(I)
      IF(SQRT((X(I)-XP)**2+(YA(I)-YP)**2).LT.140.0)GOTO18
      MOD(I)=SQRT(X(I)**2+YA(I)**2)
      IF(X(I).EQ.0.0.AND.YA(I).EQ.0.0)GOTO11
      ALPHA(I)=ATAN2(-YA(I),X(I))*180.0/PI
      GOTO21
   11 ALPHA(I)=0.0
   21 IF(ALPHA(I).LT.0.0)ALPHA(I)=360.0+ALPHA(I)
      IF(ALPHA(I).LT.0.0)GOTO21
      WRITE(7,102)F(I),X(I),YA(I),MOD(I),ALPHA(I)
  102 FORMAT(1X,5F12.4)
      XP=X(I)
      YP=YA(I)
   18 CONTINUE
      FT=10000.0000
      WRITE(7,102)FT
      DR=100.0
      XS=0.0
      YS=0.0
      ICOUNT=0
      DO 17 I=2,IC
      YC=YA(I)
      IF(SQRT((X(I)-XP)**2+(YC-YP)**2).LT.140.0)GOTO17
      XP=X(I)
      YP=YC
      XS=XS+X(I)
      YS=YS+YC
      ICOUNT=ICOUNT+1
   17 CONTINUE
      XP=1000.0
      YP=1000.0
      XS=XS/ICOUNT
      YS=YS/ICOUNT
      DO 13 I=2,IC
      YC=YA(I)
      IF(SQRT((X(I)-XP)**2+(YC-YP)**2).LT.140.0)GOTO13
      XP=X(I)
      YP=YC
      IF((YC-YS).EQ.0.0.AND.(X(I)-XS).EQ.0.0)GOTO7
      TH=ATAN2(YC-YS,X(I)-XS)*180.0/PI
      GOTO8
    7 TH=0.0
    8 R=SQRT((YC-YS)**2+(X(I)-XS)**2)
      R=R+DR
      XN=R*COS((TH)*PI/180.0)+XS
      YN=R*SIN((TH)*PI/180.0)+YS
      CALL MOVTO2(X(I),YC)
      CALL SCALE(20.0)
```

```
      CALL CHAHOL(3HY*.)
      CALL SCALE(0.05)
      CALL MOVTO2(XN,YN)
      CALL SCALE(20.0)
      CALL ROTAT2(TN)
      CALL CHAFIX(F(I),7,2)
      CALL MOVTO2(XN,YN)
      CALL ROTAT2(-TN)
      CALL SCALE(0.05)
   13 CONTINUE
      CALL MOVTO2(00.0,00.0)
      CALL SCALE(20.0)
      CALL AXIPOS(0,0.,0.,500.,1)
      CALL AXIPOS(0,0.,0.,500.,2)
      CALL AXISCA(3,10,-5000.,5000.,1)
      CALL AXISCA(3,10,-5000.,5000.,2)
      CALL AXIDRA(1,0,1)
      CALL AXIDRA(-1,0,2)
      CALL MOVTO2(260.,0.)
      CALL CHAHOL(11HREAL AXIS*.)
      CALL MOVTO2(0.,260.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(16HIMAGINARY AXIS*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(-300.0,-300.0)
      CALL MOVTO2(00.0,00.0)
      CALL MOVTO2(70.,160.)
      CALL CHAHOL(21HMODE RESPONSE,MODE:*.)
      CALL MOVTO2(125.,160.)
      CALL CHAINT(IMODE,4)
      CALL MOVTO2(70.,150.)
      CALL CHAHOL(21HBOUNDARY CONDITIONS*.)
      CALL MOVTO2(70.,140.)
      CALL CHAHOL(7HWALL:*.)
      CALL MOVTO2(100.,140.)
      CALL CHAHOL(13HADMITTANCE:*.)
      CALL MOVTO2(80.,130.)
      CALL CHAHOL(5HX1:*.)
      CALL MOVTO2(100.,130.)
      CALL CHAARR(IWX1,10,8)
      CALL MOVTO2(80.,120.)
      CALL CHAHOL(5HX2:*.)
      CALL MOVTO2(100.,120.)
      CALL CHAARR(IWX2,10,8)
      CALL MOVTO2(80.,110.)
      CALL CHAHOL(5HY1:*.)
      CALL MOVTO2(100.,110.)
      CALL CHAARR(IWY1,10,8)
      CALL MOVTO2(80.,100.)
      CALL CHAHOL(5HY2:*.)
      CALL MOVTO2(100.,100.)
      CALL CHAARR(IWY2,10,8)
      CALL MOVTO2(80.,90.)
      CALL CHAHOL(5HZ1:*.)
      CALL MOV TO2(100.,90.)
      CALL CHAARR(IWZ1,10,8)
      CALL MOVTO2(80.,80.)
```

```
      CALL CHAHOL(5HZ2:*.)
      CALL MOVTO2(100.,80.)
      CALL CHAARR(IMZ2,10,3)
      CALL MOVTO2(60.,60.)
      CALL CHAHOL(21HRESONANT FREQUENCY:*.)
      CALL MOVTO2(60.,50.)
      CALL CHAHOL(19HDAMPING CONSTANT:*.)
      CALL MOVTO2(50.,50.)
      CALL SCALE(2.0)
      CALL CHASWI(0)
      CALL PICCLE
      GOTO 15
 16   CALL DEVEND
      FS=0.0
      WRITE(7,102)FS
      STOP
      END


      TRACE 1
      SU BROUTINE CORRECT(X,YA,F)
      REAL IA,IA500,IAQ1,IAQ2
      REAL X(1024),YA(1024),F(1024),STORE(5000,3)
      COMPLEX VEC,ZVEC,CAQ
      LOGICAL FIRST
      COMMON/ST/FIRST,STORE,JCK
      IF(.NOT.FIRST)GOTO2
      DO 24 I=1,4500
      READ(4,104)(STORE(I,J),J=1,3)
104   FORMAT(1X,F7.2,2F12.6)
      IF(STORE(I,1).EQ.0.0)GOTO 98
 24   CONTINUE
 98   JCK=I
      WRITE(6,150)JCK
15 0  FORMAT(2X,3HJCK,2X,I6)
      FIRST=.FALSE.
  2   READ(5,301)FRSN
      IF(FRSN.EQ.00.0)GOTO153
301   FORMAT(1X,F6.2)
      DO 23 J=2,JCK
      IF(FRSN.GE.STORE((J-1),1).AND.FRSN.LT.STORE(J,1))GOTO 99
 23   CONTINUE
 99   FF1=STORE((J-1),1)
      FF2=STORE(J,1)
      RAQ1=STORE((J-1),2)
      RAQ2=STORE(J,2)
      IAQ1=STORE((J-1),3)
      IAQ2=STORE(J,3)
      RA=RAQ1+(FRSN-FF1)*(RAQ2-RAQ1)/(FF2-FF1)
      IA=IAQ1+(FRSN-FF1)*(IAQ2-IAQ1)/(FF2-FF1)
      CAQ=CMPLX(RA,IA)
      SC=SQRT(RA**2+IA**2)
      IC=1024
      DO 22 I=2,IC
      VEC=CMPLX(X(I),YA(I))
      ZVEC=VEC/CAQ
      X(I)=REAL(ZVEC)*SC
      YA(I)=AIMAG(ZVEC)*SC
 22   CONTINUE
153   RETU RN
      END
      FINISH
```

B.2. program: FREEFIELD

```
FREEFIELD
UAFORTRAN DUMPO,LIB :EBS8007.GLIB,LINES 10000,-
*TR0 ANECHELS,*CP1 CORRBLS,*CP2 BLS
      LIST(LP)
      PROGRAM(ANCH)
      INPUT   3=TR0
      INPUT   5=CR0
      OUTPUT  6=LP0
      OUTPUT  7=CP1
      OUTPUT  8=CP2
      EXTENDED DATA
      TRACE 2
      END
      TRACE 1
      MASTER QSOURCE
      REAL X(1024),YA(1024),F(1024),VOLT(2048)
      COMPLEX CR(1024),RES(1024),AQ(1024)
      FT=0.0
      FR=0.0
      FMIN=0.0
      WRITE(6,104)
      WRITE(8,104)
  104 FORMAT(2X,4HFREQ,4X,4HREAL,4X,4HIMAG,6X,2HAQ,4X,
     17HMEASANG,2X,6HCALANG,3X,4HDIFF/)
   15 READ(5,100)IMODE,IF1,IF2,IR
  100 FORMAT(4I0)
      IF(IMODE.EQ.0000)GOTO 16
      F1=IF1/10.0
      F2=IF2/10.0
      R=IR/100.0
      WA=-1.0
      CALL BUD(VOLT,2048,WA,WB,SRATE,NSAMP,.FALSE.,.TRUE.)
      PI=4.0*ATAN(1.0)
      C=344.00
      ROW=1.21
      LC=1024
      YAA=0.0
      DO 12 L=2,LC
      J=2*L
      K=J-1
      X(L)=VOLT(K)*1000.0
      YA(L)=VOLT(J)*1000.0
      YAB=YA(L)
      YA(L)=(YA(L)+YAA)/2.0
      YAA=YAB
      F(L)=F1+(L-1)*(F2-F1)/(LC-1)
   12 CONTINUE
      I=1
      DO 13 L=5,LC,4
      F(I)=(F(L)+F(L-1)+F(L-2)+F(L-3))/4.0
      YA(I)=(YA(L)+YA(L-1)+YA(L-2)+YA(L-3))/4.0
      X(I)=(X(L)+X(L-1)+X(L-2)+X(L-3))/4.0
      ARG=PI/2.0-2.0*PI*R*F(I)/C
      A=F(I)*ROW*COS(ARG)/(2.0*R)
      B=F(I)*ROW*SIN(ARG)/(2.0*R)
      IF(A.EQ.0.0.AND.B.EQ.0.0)GOTO13
```

```
      CR(I)=CMPLX(A,B)
      RES(I)=CMPLX(X(I),YA(I))
      AQ(I)=RES(I)/CR(I)
      XAQ=REAL(AQ(I))
      YAQ=AIMAG(AQ(I))
      IF(XAQ.EQ.0.0.AND.YAQ.EQ.0.0)GOTO 13
      IF(F(I).LT.FT)GOTO13
      FT=F(I)
      I=I+1
   13 CONTINUE
      IC=I
      DO 18 I=2,IC,50
      FG=F(I)
      XN=REAL(RES(I))
      YN=AIMAG(RES(I))
      IF(XN.EQ.0.0.AND.YN.EQ.0.0)GOTO11
      ALPHA=ATAN2(-YN,XN)*180.0/PI
      GOTO10
   11 ALPHA=0.0
   10 IF(ALPHA.LT.0.0)ALPHA=360.0+ALPHA
      IF(ALPHA.LT.0.0)GOTO10
      XK=REAL(CR(I))
      YK=AIMAG(CR(I))
      IF(XK.EQ.0.0.AND.YK.EQ.0.0)GOTO17
      BETA=ATAN2(-YK,XK)*180.0/PI
      GOTO30
   17 BETA=0.0
   30 IF(BETA.LT.0.0)BETA=360.0+BETA
      IF(BETA.LT.0.0)GOTO30
      GAMA=ALPHA-BETA
   31 IF(GAMA.LT.0.0)GAMA=360.0+GAMA
      IF(GAMA.LT.0.0)GOTO31
      XAQ=RE AL(AQ(I))
      YAQ=AIMAG(AQ(I))
      AAQ=CABS(AQ(I))
      IF(XAQ.EQ.0.0.AND.YAQ.EQ.0.0)GOTO 18
      IF(FG.LT.FR)GOTO18
      WRITE(6,102)FG,XAQ,YAQ,AAQ,ALPHA,BETA,GAMA
      WRITE(8,102)FG,XAQ,YAQ,AAQ,ALPHA,BETA,GAMA
  102.FORMAT(1X,F7.2,2F9.2,F7.2,3F8.2)
      FR=F(I)
   18 CONTINUE
      DO 19 I=1,IC
      IF(F(I).LT.FMIN)GOTO19
      WRITE(7,103)F(I),AQ(I)
  103 FORMAT(1X,F7.2,2F12.6)
   19 CONTINUE
      GOTO 15
   16 FS=0.0
      WRITE(7,103)FS
      STOP
      END
      FINISH
```

B.3. program: AMPHASEPLOT

```
AMPHASEPLOT
UAFORTRAN OUNFD,*CR0 CVALUES,*CR1 MTEXTS,*GP GINO,-
*CR2 CSPACING,-
LINE S 10000
      LF(LP)
      PROGRAM(AMPH)
      INPUT   3=CR0
      INPUT   5=CR1
      INPUT   6=CR2
      OUTPUT 7=LP1
      COMPRESS INTEGER AND LOGICAL
      EXTENDED DATA
      TRACE2
      END
      MASTER AMPHASE
      REAL FS(1024),MODS(1024),ALPHAS(1024),FP(1024),SP(1024)
      REAL MOD,XS(1024),YS(1024)
      INTEGER IWX1(20),IWX2(20),IWY1(20),IWY2(20),IWZ1(20),IWZ2(20)
      CALL OPEN GINOGP
      READ(5,200)(IWX1(I),I=1,20)
      READ(5,200)(IWX2(I),I=1,20)
      READ(5,200)(IWY1(I),I=1,20)
      READ(5,200)(IWY2(I),I=1,20)
      READ(5,200)(IWZ1(I),I=1,20)
      READ(5,200)(IWZ2(I),I=1,20)
  200 FORMAT(20A4)
   15 READ(5,103)IMODE,IF1,IF2
      IF(IMODE.EQ.000)GOTO16
  103 FORMAT(3I0)
      JC=0
    1 READ(3,100)F,X,Y,MOD,ALPHA
  100 FORMAT(1X,5F12.4)
      IF(F.EQ.10000.0000)GOTO3
      IF(F.EQ.0.00)GOTO16
      JC=JC+1
      XS(JC)=X
      YS(JC)=Y
      MODS(JC)=MOD
      FS(JC)=F
      ALPHAS(JC)=ALPHA
      GOTO1
    3 KC=0
    2 READ(6,101)CNFREQ,SPACE
  101 FORMAT(2(1X,2F12.4))
      IF(CNFREQ.EQ.10000.0000)GOTO4
      KC=KC+1
      FP(KC)=CNFREQ
      SP(KC)=SPACE/3.0
      GOTO2
    4 NB=FS(1)
      NE=FS(JC)
      NE=NE+1
      NT=(NE-NB)
      FB=FLOAT(NB)
```

```
FE=FLOAT(NE)
CALL CHASWI(1)
CALL SHIFT2(0.,50.)
CALL SCALE(0.320)
CALL AXIPOS(1,50.,40.,250.,1)
CALL AXIPOS(1,50.,40.,180.,2)
CALL AXISCA(1,NT,FB,FE,1)
CALL AXISCA(1,10,0.,5000.,2)
CALL AXIDRA(2,1,1)
CALL AXIDRA(-2,-1,2)
CALL GRASYM(FS,MODS,JC,4,0)
CALL MOVTO2(0.,0.)
CALL MOVBY2(50.,10.)
CALL CHAHOL(20HAMPLITUDE RESPONSE*.)
CALL MOVTO2(00.0,00.0)
CALL MOVTO2(240.,20.)
CALL CHAHOL(14HFREQUENCY:HZ*.)
CALL MOVTO2(25.,150.)
CALL ROTAT2(90.0)
CALL CHAHOL(14HAMPLITUDE:MV*.)
CALL ROTAT2(-90.0)
CALL MOVTO2(0.,0.)
CALL AXIPOS(1,50.,280.,250.,1)
CALL AXIPOS(1,50.,280.,180.,2)
CALL AXISCA(1,NT,FB,FE,1)
CALL AXISCA(1,18,0.,360.,2)
CALL AXIDRA(2,1,1)
CALL AXIDRA(-2,-1,2)
CALL GRASYM(FS,ALPHAS,JC,8,0)
CALL GRAMOV(FB,90.)
CALL GRALIN(FE,90.)
CALL GRAMOV(FB,180.)
CALL GRALIN(FE,180.)
CALL GRAMOV(FB,270.)
CALL GRALIN(FE,270.)
CALL GRAMOV(FB,360.)
CALL GRALIN(FE,360.)
CALL MOVTO2(0.,240.)
CALL MOVBY2(50.,10.)
CALL CHAHOL(16HPHASE RESPONSE*.)
CALL MOVTO2(0.,240.)
CALL MOVTO2(240.,260.)
CALL CHAHOL(14HFREQUENCY:HZ*.)
CALL MOVTO2(25.,390.)
CALL ROTAT2(90.0)
CALL CHAHOL(17HPHASE ANGLE:DEG*.)
CALL ROTAT2(-90.0)
CALL MOVTO2(0.,240.)
CALL AXIPOS(1,50.,520.,250.,1)
CALL AXIPOS(1,50.,520.,180.,2)
CALL AXISCA(5,KC,FB,FE,1)
CALL AXISCA(1,20,0.,20000.,2)
CALL AXIDRA(1,0,1)
CALL AXIDRA(-2,-1,2)
CALL GRAHIS(SP,KC,0.0)
CALL MOVTO2(0.,480.)
CALL MOVBY2(50.,10.)
CALL CHAHOL(20HISOCHRONES SPACING*.)
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```
      CALL MOVTO2(0.,480.)
      CALL MOVTO2(240.,500.)
      CALL CHAHOL(14HFREQUENCY:HZ*.)
      CALL MOVTO2(25.,630.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(12HSPACING:MM*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(0.,480.)
      CALL SHIFT2(-65.,0.)
      CALL MOVTO2(0.,0.)
      CALL AXIPOS(1,400.,520.,250.,1)
      CALL AXIPOS(0,400.,520.,360.,2)
      CALL AXISCA(1,NT,FB,FE,1)
      CALL AXISCA(1,20,-5000.,5000.,2)
      CALL AXIDRA(2,1,1)
      CALL AXIDRA(-2,-1,2)
      CALL GRASYM(FS,XS,JC,4,0)
      CALL GRASYM(FS,YS,JC,8,0)
      CALL GRAMOV(FS(JC),XS(JC))
      CALL CHAHOL(7H REAL*.)
      CALL GRAMOV(FS(JC),YS(JC))
      CALL CHAHOL(7H IMAG*.)
      CALL MOVTO2(400.,310.)
      CALL CHAHOL(30HINPHASE AND QUADR COMPONENTS*.)
      CALL MOVTO2(640.,320.)
      CALL CHAHOL(14HFREQUENCY:HZ*.)
      CALL MOVTO2(385.,480.)
      CALL ROTAT2(90.0)
      CALL CHAHOL(30HINPHASE AND QUADR COMPONENTS*.)
      CALL ROTAT2(-90.0)
      CALL MOVTO2(350.,240.)
      CALL MOVTO2(0.,0.)
      CALL SHIFT2(65.,0.)
      CALL MOVTO2(0.,0.)
      CALL MOVTO2(360.,170.)
      CALL CHAHOL(21HMODE RESPONSE,MODE:*.)
      CALL MOVTO2(430.,170.)
      CALL CHAINT(IMODE,4)
      CALL MOVTO2(360.,160.)
      CALL CHAHOL(21HBOUNDARY CONDITIONS*.)
      CALL MOVTO2(360.,150.)
      CALL CHAHOL(7HWALL:*.)
      CALL MOVTO2(400.,150.)
      CALL CHAHOL(13HADMITTANCE:*.)
      CALL MOVTO2(380.,140.)
      CALL CHAHOL(5HX1:*.)
      CALL MOVTO2(400.,140.)
      CALL CHAARR(IWX1,20,4)
      CALL MOVTO2(380.,130.)
      CALL CHAHOL(5HX2:*.)
      CALL MOVTO2(400.,130.)
      CALL CHAARR(IWX2,20,4)
      CALL MOVTO2(380.,120.)
      CALL CHAHOL(5HY1:*.)
      CALL MOVTO2(400.,120.)
      CALL CHAARR(IWY1,20,4)
      CALL MOVTO2(380.,110.)
      CALL CHAHOL(5HY2:*.)
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      CALL MOVTO2(400.,110.)
      CALL CHAARR(IVY2,20,4)
      CALL MOVTO2(380.,100.)
      CALL CHAHOL(5HZ1:*.)
      CALL MOVTO2(400.,100.)
      CALL CHAARR(IVZ1,20,4)
      CALL MOVTO2(380.,90.)
      CALL CHAHOL(5HZ2:*.)
      CALL MOVTO2(400.,90.)
      CALL CHAARR(IVZ2,20,4)
      CALL MOVTO2(360.,70.)
      CALL CHAHOL(21HRESONANT FREQUENCY:*.)
      CALL MOVTO2(360.,60.)
      CALL CHAHOL(19HDAMPING CONSTANT:*.)
      CALL MOVTO2(350.,0.)
      CALL MOVTO2(00.0,00.0)
      CALL SCALE(3.125)
      CALL SHIFT2(0.,-50.)
      CALL CHASWI(0)
      CALL PICCLE
      GOTO15
   16 CALL DEVEND
      STOP
      END
      FINISH
```

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