MAINTENANCE STRATEGIES AFFECTING

EQUIPMENT PERFORMANCE

A Thesis submitted for the degree of

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by

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SUMMARY

Proper maintenance of plant items is crucial for the safe and profitable operation of process plants. The relevant maintenance policies fall into the following four categories:

(i) preventive/opportunistic/breakdown replacement policies,
(ii) inspection/inspection-repair-replacement policies,
(iii) restorative maintenance policies, and
(iv) condition based maintenance policies.

For correlating failure times of component equipment and complete systems, the Weibull failure distribution has been used. A new powerful method, SEQLIM, has been proposed for the estimation of the Weibull parameters; particularly, when maintenance records contain very few failures and many successful operation times.

When a system consists of a number of replaceable, ageing components, an opportunistic replacement policy has been found to be cost-effective. A simple opportunistic model has been developed.

Inspection models with various objective functions have been investigated. It was found that, on the assumption of a negative exponential failure distribution, all models converge to the same optimal inspection interval; provided the safety components are very reliable and the demand rate is low. When deterioration becomes a contributory factor to some failures, periodic inspections, calculated from above models, are too frequent. A case of safety trip systems has been studied.

A highly effective restorative maintenance policy can be developed if the performance of the equipment under this category can be related to some predictive modelling. A novel fouling model has been proposed to determine cleaning strategies of condensers.

Condition-based maintenance policies have been investigated. A simple gauge has been designed for condition monitoring of relief valve springs.

A typical case of an exothermic inert gas generation plant has been studied, to demonstrate how various policies can be applied to devise overall maintenance actions.

Key Words

Maintenance, reliability, process plant, safety, Weibull distribution.

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CHAPTER 1

INTRODUCTION

1.1 Maintenance in Perspective

Maintenance management is vitally important, not only for industrial profitability but also for the national economy. Hundreds of millions of pounds are spent annually in the UK on maintenance of equipments. Failure to develop effective maintenance planning for plants and industries can lead to high penalty costs from downtime. The importance of competent maintenance has increased greatly with the increase in mechanisation and automation in every industry. The long held 'Cinderella' image of maintenance is diminishing with the growing awareness of its importance to production, finance and general management. This process can be accentuated by maintenance managers adopting a more scientific basis for planning their operations.

Although maintenance of equipments is concerned primarily with keeping equipments operational, the development of an effective maintenance plan or programme should start at the design stage or at the stage when acquisition of the equipment is to be decided, having due regard to maintainability considerations and the logistics of support schemes. The effectiveness of the maintenance plan needs to be re-evaluated throughout the life of the equipment . Recently, this life-cycle view has been recognised in the concept of 'Terotechnology', which has been defined as 'a combination of management, financial and other practices applied to physical assets in pursuit of economic life-cycle cost'. It is explained as giving life-long care to all forms of technological assets, building upon maintenance, but reaching back into design, planning and most importantly, accountancy. So, it can be seen that terotechnology recognizes the multidisciplinary nature of maintenance activities.

Now, regarding the mathematical techniques in formulating maintenance policies for individual plant items or the plant as a whole, the advent of reliability engineering and operations research has contributed enormously in the sophistication of these techniques and has put maintenance engineering on a sounder scientific basis. Consequently, maintenance engineering now is similar to an exact science.

1.2 Maintenance Engineering Techniques in the Process Industries

The concept of maintenance and maintenance engineering analysis are the same for process industries as for other industries. But, in applying the mathematical techniques already used in industries like manufacturing, nuclear, aerospace and electronic, one has to recognise the differences between these industries; for example, many process plants are run continuously and similar equipments are subjected to a variety of environments. As a result, the frequency and interval of maintenance of an equipment may depend strongly on the process and the plant environment. However, in developing a maintenance plan, one is concerned more with the effect of the failure of an equipment than the failure itself. It may be convenient to classify the effects of failures by their severity. A possible scheme is:

(i) catastropic failure - results in danger to people, extensive damage to property, etc.

(ii) major failure - results in plant shutdown or other condition that prevents production objectives being met.

(iii) minor failure - results in need for unscheduled maintenance and so increases chances of not meeting production objectives, but does not prevent objectives being met.

(iv) unimportant failure - action can be taken during normal plant operation or during scheduled maintenance period.

One may work through a process flow sheet and list the possible ways in which each component can fail. The next step is to determine the effect of failures. At this stage, most attention must be paid to catastropic and major failures. However, the prime benefit from this sort of analysis is that it draws attention to points of weakness and suggests the remedy; eg. redundancy, rearranging maintenance schedules and so on.

1.3 Maintenance Policies for the Process Industries

This dissertation is concerned primarily with the investigation and development of maintenance policies associated with the process industries. But before describing these relevant policies, it is worth looking at the typical equipments used in a process industry.

In spite of their differences, all process plants have automatic instrumentation and control. The use of sophisticated instrumentation and control schemes is directed partly to improve the operation of complex units and partly to simplify the duties of the plant operators. As the tendency is to reduce the number of human operators in process plants, the importance of the reliability of instrumentation and control cannot be over emphasized. However, most of the equipments utilized for control purposes are either mechanical or electrical or both. Safety shutdown systems are an example of this; where many of the protective devices are trips, relief valves, bursting discs, lutes, vents, relays and so on.

Rotating machinery such as pumps, compressors, fans, blowers, etc. are indispensable in process plant operations. Most mechanical failures, causing plant shutdown, involve this class of equipment and more specifically compressors, their drivers and auxiliaries.

In a process plant, many common process equipments include pressure vessels, heat exchangers, fired heaters, reactors, separating columns, etc. Usually, pressure vessels and columns give long trouble-free service; but accelerated corrosion or metallurgical degradation can cause catastrophic failure and necessitates periodic inspection and NDT. Attention should be focussed on brittle fracture, stress concentration and hydrogen induced deterioration under operating conditions. Heat exchangers are prone to fouling which is an accelerating process which, if not rectified by cleaning, can limit production rates and increase energy costs. As for the reactors, catalysts may be deactivated by sintering or poisoning of active sites or by gradual fouling. The former usually requires a catalyst change, the latter can be reversed by regenerating the catalyst in situ.

It is quite conceivable that the nature and effect of the failure of each of the above mentioned process equipments are likely to be different. So, there is unlikely to be a single policy which is applicable to all the equipments in a process plant. However, the more likely maintenance policies can be classified in four ways, which are convenient also for mathematical modelling:

(1) Preventive/Opportunistic/Breakdown replacement policies - mostly applicable to mechanical or electrical equipments.

(2) Inspection/Inspection - repair - replacement policies - for the equipments which must be maintained at high reliability; eg. safety equipments.

(3) Restorative maintenance policies - can be applied in cases where equipments do not fail in the usual sense, but their performance deteriorates during operation; eg. boilers, heat exchangers, catalytic reactors, etc.

(4) Condition based maintenance policies - can be applied to continuously or periodically monitor the equipments which can cause catastropic or major failures; eg. wear of bearing or compressor seals, structural weakening of process equipment due to corrosion or excursions in process conditions.

Broadly, preventive/opportunistic/breakdown replacement policies are classed as stochastic replacement problems. The failures of equipments in this category cannot be predicted with certainty, but it is assumed that when a failure occurs, it will be known immediately and a replacement will be carried out. In the construction of the models for these policies, it is assumed furthermore, that the replacement action returns the equipment to the as new condition; thus continuing to provide the same services as before. The various objective functions for the modelling may be maximizing profit, minimizing total maintenance cost, minimizing downtime, etc.

The decision regarding the feasibility of preventive replacement actions; that is, actions taken before equipment reaches a failed state, can be made apriori. The two necessary conditions for preventive replacements are:

(i) The total cost of the replacement must be greater after failure than before, if cost is the appropriate criterion. When downtime is the criterion, downtime due to failure should be greater.

(ii) The failure rate of the equipment must be increasing. If there is constant failure rate, replacement before failure does not affect the probability that the equipment will fail in the next instant given that it is good now. Similarly, when the failure rate is decreasing, no benefit can be derived from preventive replacements.

Obviously, when these two conditions are not met, the correct policy would be to operate the equipment to failure and then carry out overhaul or replacement. But in some circumstances, neither preventive nor breakdown replacement policies would be effective. For example, when a system consists of two or more parts and the failure and distributions of the parts are stochastically independent, application of a preventive replacement model to the parts separately, may not result in an optimal replacement age or interval. But if the replacement costs of two or more parts together are less than the replacement cost for each part taken separately, there exist some critical ages when an unfailed part can be replaced with a failed part with cost-effectiveness, if cost is the appropriate criterion. This type of policy can be characterized as an opportunistic replacement policy.

In contrast to the replacement problems mentioned above, the basic purpose behind an inspection is to determine the state of the equipment. Once indicators to describe the state have been specified, and the inspection made to determine the values of these indicators, then some further maintenance action such as repair or replacement may be taken, depending upon the perceived state. The inspection programme should be influenced by the objective function considered to be appropriate for the problem. For example, in the case of safety components, the objective functions may be the minimization of downtime or in other words maximization of availability, maximizing probability of safety over a campaign time, minimization of total cost, etc. In constructing the inspection model, it is assumed that inspection will reveal the state of the equipment with certainty.

It has been mentioned earlier that restorative maintenance policies are applicable to equipments which do not cease to function in the usual sense, but their performance deteriorates over a period of time. The failed state of this type of equipments may be defined as the performance level which does not conform with specified tolerance limits. From cost or availability considerations, it may be advantageous to take some forms of maintenance actions before the equipment passes to its failed state. Usually, for the restorative maintenance problems, the maintenance action is called an overhaul. The treatment of these problems is similar to preventive replacement. Nevertheless, a separate category for restorative maintenance policies can be justified, on the basis that a time dependent performance criterion has to be formulated before the construction of any model.

Condition based maintenance is defined as maintenance carried out in response to a significant deterioration in an equipment as indicated by a change in a monitored parameter of the equipment condition. It differs from both failure maintenance and fixed-interval replacement in that it requires monitoring of some condition-indicating parameter of the equipment being maintained. In general, condition-based maintenance will be more efficient and adaptable than either of the other maintenance actions. On indication of deterioration, the equipment can be scheduled for shutdown at a time chosen in advance of failure or can be left to run to failure, if the production policy dictates. Condition-based maintenance has some notable advantages. It reduces the amount of unnecessary preventive replacements and thus increases the production time of the equipment. In cases of catastrophic failures, condition monitoring can be employed to indicate possible impending failure well before it becomes a significant probability. In spite of its advantages, condition-based maintenance cannot be employed in every circumstance. The two major reasons are that not all causes of plant failure can be detected in advance and that by its very nature, condition monitoring is costly in manpower, monitoring equipment or both. So, before applying condition-based maintenance, one has to consider its cost-effectiveness as well as the safety criterion of the plant.

Condition monitoring falls into two distinct classes: (a) monitoring which can be carried out without interrupting the operation of the unit, and (b) monitoring which requires the shutdown of the unit, or at least, the release of the unit from its prime duty. Clearly, the former category has significant advantages over the latter, since no interruption to plant output is involved. However, there are many situations where the monitored unit is shutdown regularly or frequently, as part of the normal operating policy.

1.4 Scope of this Research

The main objective of this research is the investigation and development of maintenance policies related to the process industries and to demonstrate how mathematical methods can help formulate an optimal policy for maintaining plant items. In the previous section, the relevant policies have been outlined. Within this general outline, various models will be studied. So, the scope of this research can be demonstrated in the following ways:

(i) In the construction of models for replacement, inspection and restorative maintenance and in the subsequent evaluation of optimal maintenance schedules, the determination of correct failure distributions is very important. Because of its capability for accommodating both random and wearout failures, the Weibull distribution has been used. The methods available for estimating Weibull distribution parameters, fail to produce good estimates with very few failures; particularly when maintenance records contain also many successful operation times. So, a method has to be developed to estimate Weibull distribution parameters for this type of situation.

(ii) Review of various preventive replacement models was carried out and a simple opportunistic replacement model has been developed. A case study is presented which demonstrates the effectiveness of the models.

(iii) New inspection models have been developed for maximizing the probability of safety over a campaign time. Comparison of these new models with the available models has been done. A new model has been constructed on the basis of the Weibull failure distribution. The effectiveness of this model is demonstrated with a case study of a safety shutdown systems.

(iv) For the application of the restorative maintenance policy, a case study of condenser fouling by saltwater was undertaken. Relationships for the deterioration of condenser performance with time have been formulated.

(v) Various techniques for condition-based maintenance were reviewed. On-line condition monitoring of relief valve springs is important; so, a gauge has been designed for checking the correct set pressure of relief valves.

(vi) Finally, application of all the maintenance policies has been demonstrated for a typical process plant.

In judging the success of the above mentioned tasks, it is important to be clear what is a complete solution of a maintenance problem. In general, a complete solution of a problem can be regarded as a description of the physical situation, a definition and characterization of an optimal maintenance policy for this physical situation and a derivation of the operating characteristics of the optimal maintenance policy. An alternative approach may be to describe the physical situation in detail and provide a computing scheme for systematically examining all possible policies that might apply.

CHAPTER 2

LITERATURE REVIEW

2.1 General

Over the last two decades, literature involving the theory of maintenance has grown extensively. The firm establishment of operations research and reliability engineering disciplines, during this period, has helped the development of many maintenance policies. Moreover, the practical need for subtle and delicate maintenance policies has stimulated theoretical interest and led to the development of policies that possess theoretical novelty as well as practical importance. Owing to the wide range of mathematical techniques utilised in this development process and the diversity of applications, the underlying structure common to all these policies has sometimes been ignored.

Jorgenson, Radner and McCall (1,2) have attempted to give a comprehensive review of maintenance policies. But their work involves basically equipment replacement policies, which have been categorized as preventive maintenance policies, preparedness maintenance policies and maintenance policies for uncertain failure distributions. The last category includes policies like minimax policies, policies by bounds on costs and adaptive policies. Perhaps the most interesting feature of their work is the inclusion of opportunistic maintenance in the first two categories. Jardine (3) also has discussed various maintenance models in a very straightforward and comprehensive way. In his book, he has given overhaul and restorative maintenance policies as separate categories; although he mentions that these may form a part of the overall preventive maintenance policy. Kelly, Lewis and Harris (4,5), in their classification of major maintenance policies, have included (i) fixed time maintenance, (ii) condition-based-maintenance (continuous or periodic, etc) (iii) operate-to-failure (corrective maintenance by repair/ replacement in situ), (iv) opportunity maintenance (for complex items and/or continuously operating plant) and (v) design-out-maintenance. They have given a procedure for selecting a suitable maintenance policy for complex plant items. Recently, Sims (6) has given a similar decision tree for single stream chemical plant. In his paper, he has discussed some basic features associated with each of these policies in the context of chemical plants.

In contrast to the mechanical, electrical, aerospace or nuclear industry, the chemical industry has not explored nor exploited, sophisticated maintenance policies. Very little has been published about the maintenance of chemical plants. One of the early papers on this subject was by Hooper (7); who dealt with the organisational aspects of maintenance planning. Williams and Russell (8) have discussed the application of NASA reliability techniques in the chemical industry. Buffham, Freshwater, Lees, Pan and Aird

(9,10,11) have drawn attention to the potential cost-effectiveness of applying reliability engineering to process plant maintenance.

Although the chemical industry has been reluctant to develop delicate maintenance policies, many such policies developed in other industries are applicable to this industry; because it uses many mechanical and electrical equipments as well as process vessels.

2.2 Preventive/Opportunistic/Breakdown Replacement Policies

Replacement problems can be classed as either deterministic or stochastic. In the earliest literature on the theory of maintenance, attention was confined to deterministic problems; but during the last two decades, a substantial body of literature has developed on maintenance in stochastic situations.

Perhaps the first deterministic replacement theory can be credited to Taylor and Hotelling (12,13) in the 1920's. Their work was concerned mainly with the statistical theory of depreciation. Later, Terborgh and Alchian (14,15) dealt with optimal equipment replacement policies in the presence of obsolescence and technological change. The early work on Operations Research (16) also included equipment replacement policies. Bellman and Dreyfus (17) used applied dynamic programming to cater for technological improvement.

Preventive, opportunistic and breakdown replacement policies fall into the category of stochastic replacement problems. Replacement involves a problem of decision-making under one main source of uncertainty; that is, the instant of failure is unpredictable, or more generally the transition time from one state to another (3). But, it is assumed that changes of state are immediately detected. However, these problems are closely connected with the reliability area. So, the early literature which considered the mathematical theory of reliability is relevant also to replacement problems.

One of the earliest treatments of stochastic replacement problems was by A.J.Lotka (18). Another contemporary worker was Campbell (19), who discussed the comparative advantages of replacing a number of street lamps either all at once or as they failed. This is the classic example of a group replacement problem.

After a dormant period, literature started to appear again in the 1950's. Weiss (20), in a series of reports, considered the effects of both age replacements and random age replacement policies on system reliability and on maintenance costs. In one of the reports, he was interested particularly in the time to the first system failure. He proposed that an age replacement policy would be beneficial if the expected time to an inservice failure was a decreasing function of the age of replacement. Herd (21) also considered the benefit of a replacement policy, basing his considerations solely on the failure rate. In his opinion, replacement should be considered if the failure rate is increasing. This recommendation was proved later by Barlow, Proschan and Hunter (22). With any replacement model, the operating characteristics, like the expected number of failures and the expected number of planned replacements, are of great interest also. A detailed study to this respect was given by Flehinger (23). Similar analysis was presented also by Jorgenson, Radner and McCall (1).

Age replacement policies, for an infinite time span, have received extensive attention in the literature (24, 25, 26, 27, 28). Morse (29) shows how to determine the replacement interval minimising expected cost per unit time, when such an optimal interval exists. The derivation of an optimum age replacement interval, corresponding to a finite time span, is basically a much more difficult problem. The maintenance policy for this finite time span problem, is known as a sequential preventive maintenance policy (2). This policy differs from the age replacement policy in that the replacement interval is re-calculated at each replacement. This optimal interval minimises the expected cost of operating the equipment over the remaining finite time span; that is, until the equipment becomes obsolete. Applications of sequential policies appear to be most important in those industries where equipment is subject to rapid technological change and therefore possesses a relatively short time span. Barlow and Proschan (25) prove the existence of an optimal sequential policy, in which the optimal sequence is non-random.

In addition to age-based and sequential preventive maintenance policies, there is the type of preventive maintenance policy called the block replacement policy. This policy was designed for the maintenance of many similar equipments. Each equipment is replaced at failure and all equipments are replaced together at periodic intervals. No claim regarding optimality is made for this policy. Nevertheless, the policy is easily executed and requires no information regarding the age of each equipment. Block replacement policies for an infinite time span, within a more general setting, have been studied by Savage (30). His formulation does not seem readily applicable since he leaves the cost expression as a general function of the replacement interval. These policies have been investigated also by Flehinger (23), Drenick (27), Welker (31) and Barlow and Proschan (32).

A comparison of the individual and block replacement policies, when each is governed by the same replacement interval, has been made by Barlow and Proschan (32). Between age-based and fixed interval policies, the former is more cost-optimal than the later policy. Regarding the selection of the correct preventive maintenance policy, an interesting study has been made by Parsons (33).

When an equipment consists of several parts, it may not be cost-effective to apply an age-based or a sequential preventive maintenance policy to individual parts. The principal reasons are that failure distributions of the several parts may not be stochastically independent and secondly,that the cost of replacing several parts jointly may be less than the cost of several separate replacements. Under either condition, it may be advantageous to apply an opportunistic replacement policy, in which the replacement decision of one part depends upon the states of the remaining parts of the system. Radner, Jorgenson and McCall (1,2,34,35) have developed an opportunistic preventive maintenance model and demonstrated its operating characteristics. The model allows for economic inter-dependence in the replacement of parts but the parts are assumed to be stochastically independent. Duncan and Scholnick (36) also, have studied interrupt and opportunistic replacement strategies for systems of deteriorating components. It seems that their study is concerned mainly with systems of similar components. Despite the attractiveness of opportunistic maintenance policies, very few papers describing their industrial application have appeared.

Regarding the industrial applications of preventive replacement policies, graphical methods of determining the optimal preventive replacement interval is always attractive. Glasser (37) has produced two graphs; one for an age-based policy and the other for a block replacement policy.

Renewal theory is a very important subject in the study of preventive maintenance. Most of the general theory may be found in the classical paper by Smith (38). A comprehensive study has been done also by Cox (39). A more recent theory based on an underlying increasing failure rate distribution, may be found in a paper by Barlow, Marshall and Proschan (40).

2.3. Inspection/Inspection-Repair - Replacement Policies

Generally, inspection policies are applicable to stochastically failing equipments which are placed in storage or on stand by and will be called upon to perform some given tasks only if a specific but unpredictable emergency occurs. The distinctive feature of inspection models is that the state of the equipment is ascertained only at the time of inspection or replacement/repair. Like the replacement policies, inspection policies are classified as periodic, sequential or opportunistic. Jorgenson, Radner and McCall (1,2,41,42) have given a good comprehensive review of these policies.

The development of an inspection model requires an assumption regarding the distribution of the time-to-failure of the equipment. Owing to its mathematical simplicity, the exponential distribution has been used widely for this purpose. The periodic inspection policy based on this failure distribution was given by Barlow and Proschan (43). Because the failure rate of an exponential distribution is constant, the resulting optimal inspection interval is constant. But, when the failure rate is not constant, ie., the failure distribution is other than exponential, the optimal inspection will be age dependent. Barlow, Proschan and Hunter (22) prove that for an increasing failure rate, there will be a sequence of inspection intervals decreasing with age.

Conversely, for a decreasing failure rate, each subsequent inspection interval will be longer.

In all the studies mentioned above, it has been assumed that the inspection is perfect; that is, inspection will determine the state of the equipment with certainty. But in reality, this may not be the case. Kamins (44) and Coleman and Abrams (45) have included this possibility in their studies. They have recognised two imperfections; firstly, the inspection procedure may fail to detect an existing failure and secondly, inspection may register a failure when the equipment is good.

On the assumption of perfect inspections, Munford and Shahani (46) have proposed a novel inspection policy. Their policy is based upon a single parameter, 'inter-inspectional hazard'. It has been shown that this policy compares quite well with the optimal policy. Based on the same concept, they have presented an inspection policy for the Weibull failure distribution (47).

An opportunistic inspection policy has been devised by Radner, Jorgenson and McCall (1,2,35) for a single part in the presence of another continuously inspected part. The failure distribution for both parts was assumed to be exponential. Radner and Jorgenson (42) have proved the optimality of such a policy.

In all the studies mentioned so far, the principal objective has been the development of cost-optimal inspection policy. But for safety systems, cost-optimality may not be an over-riding factor. In many applications, the objective for an inspection policy is the minimisation of a downtime function or the maximisation of the probability of safety over a campaign period. In the process industries, the downtime function is often taken as fractional dead time. Gibson and Knowles (48) have produced some simple models for determining the optimal inspection interval, based upon the minimisation of fractional dead time. They have considered also the case of safety systems with redundant elements. In modern process plants, it is fairly common practice to design safety systems with two-out-of-three majority voting logic to decrease the chance of spurious shutdowns while attaining the required reliability of the safety system. Stewart (49) gives an analysis of 'high integrity protective systems'. Sometimes, instead of minimising fractional dead time, a target fdt is set and the inspection interval is obtained to achieve that target. Chay and Mazumdar (50) have given some interesting formulations for the determination of test intervals by this way. Evaluation of risk to plant personnel and the application of hazard analysis are important for the proper instrumentation of the protective systems. Kletz (51,52) has produced two enlightening papers on these subjects from the practical point of view, in which he uses the concept of 'fatal accident frequency rates' (FAFR) to quantify the residual risk to employees. FAFR is the number of industrial fatalities in 10^8 man hours of exposure to the risk.

Human error throughout the inspection process, involving disarmament and resetting of safety system elements, is an important matter. In a recent paper, Gibson (53) claims that a major cause of unreliability in the conventional safety system is human error. Recently, Embrey (54) has produced an interesting paper on using predisposing factors to quantify human reliability.

2.4 Restorative Maintenance Policies

2.4.1 General

Restorative maintenance policies are applicable to equipments which do not fail in the usual sense but their performance deteriorates over a period of time. Owing to the similarity between restorative and preventive maintenance policies, the literature on restorative maintenance is sparse. Davidson (55) has studied an overhaul policy for determining the optimal overhaul time for air heaters in a steam generating unit. Dynamic programming is a very useful tool for the determination of a cost-optimal overhaul policy. Jardine (3) has presented a few models, with the objective of determining a combined overhaul/repair/replace policy to minimise the total cost associated with these actions and any consequential production losses over the remaining periods (finite time horizon) or over a long period of time (infinite time horizon). He has discussed also the problem of optimal overhaul cost limits for equipment.

In the models by Jardine, the maintenance actions considered are overhaul, repair and replacements of the equipment. But there are many equipments for which only overhaul is possible before their ultimate replacement. These equipments are likely to be operational over a long period of time, eg., heat exchangers, calalytic reactors, filters, etc. For this type of equipment, an overhaul policy can be determined simply by formulating a time dependent performance relationship or a relationship describing the deterioration mechanism.

2.4.2 Fouling Models

A condenser fouling model is presented in chapter 6, in connection with a case study of restorative maintenance. Since the late 1950's, the fculing mechanism of heat transfer equipment and the predictive methods of fouling behaviour have been studied extensively. A systematic investigation of fouling, applicable to the process industry, has been given by Taborek and his colleagues (56,57). They have listed a number of fouling mechanisms, eg., crystallisation, sedimentation, chemical reaction and polymerisation, coking, organic material growth, etc. The fouling process has been shown to be dependent upon the following system characteristics: (i) heat transfer process involved, (ii) operational variables like flow velocity, surface temperature and fluid bulk temperature, (iii) surface material and structure, and (iv) equipment design geometry and flow patterns.

The need for predictive methods of fouling behaviour of industrial streams, which shows the effects of controllable variables upon the fouling process, is obvious. Despite the efforts of many workers, a general fouling model has not emerged. However, it has been accepted that the fouling process is asymptotic in time and that the net fouling rate can be expressed as the difference between a deposition rate and an erosion rate.

With this basic concept, Kern and Seaton (58) have developed a fouling model which is applicable mainly to sedimentation type fouling. Watkinson and Epstein (59) have postulated a 'transfer adhesion' model to correlate their experimental results for sand deposition from water. In many industrial situations, biological fouling is significant. Bott and Pinheiro (60) give a good review of slime formation mechanisms. They have studied also the effects of velocity and temperature on algae growth. Their experiments indicated that wall temperatures at about 310K led to maximum growth. Another author Beal (61) has postulated an eddy diffusion model for dust deposition from gas streams. But his model is questionable for high velocity and loose material because of neglecting the removal rate function.

All the models mentioned above, have been developed for specific fouling mechanisms. A model of general applicability has been postulated by Taborek, et al (57), including methods of ascertaining various constants. For design purposes, this model is useful, but from the maintenance point of view, the model seems to be complicated.

2.5 Condition-based Maintenance Policies

Condition-based maintenance is a relatively new philosophy. Its adoption into the overall maintenance strategy has been stimulated by the possibility of significant savings in direct maintenance costs and costs of lost production caused by plant failure. But condition monitoring should be used only in the right circumstances. A justification for condition-based maintenance has been given by Collacott (62).

Over the last decade, condition monitoring techniques based on non-destructive testing, have attracted a lot of attention from many authors. Kelly and Harris (5) have given an extensive review on the subject. They have included the advantages of condition monitoring, in addition to the various monitoring techniques. In a recent paper, Sims (6), has discussed also various monitoring techniques with indications of the operator skill required and the costs of implementing monitoring policies. In his book, (63), Collacott has discussed methods of fault diagnosis and condition monitoring. He has described also a few examples of computer applications for monitoring the conditions of certain equipments.

Vibration monitoring of any moving equipment, particularly compressor bearings, has attracted wide interest, because of the potential hazards from failure of these equipments. Collacott (64) has discussed a few vibration monitoring techniques with the objective of cost-effective plant maintenance.

Because condition monitoring can be employed to indicate possible impending failure well before it becomes a significant probability, any development of instrumentation or method is a step in the right direction.

2.6 Estimation of Weibull Distribution Parameters

The determination of the actual failure distribution of equipment is crucial to devising appropriate maintenance plans. Owing to the versatility of the Weibull distribution in accommodating a wide variety of distributions, it was used throughout this work.

Weibull (65) postulated his statistical distribution in 1951. Since then, it has generated considerable interest among many workers; particularly in the reliability field. Many practical applications of this distribution have been published (66,67,68,69), but most of the literature is concerned with the estimation of the parameters of this distribution.

For the two-parameter Weibull distribution, the cumulative distribution function is defined as:

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta_w}\right] \qquad (2.6.1)$$

The probability density function is then given by:

$$f(t) = \frac{\beta_w}{\eta}\left(\frac{t}{\eta}\right)^{\beta_w - 1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta_w}\right] \qquad (2.6.2)$$

In equations (2.6.1) and (2.6.1), β_w is the shape parameter, η is the scale parameter and t is the time. η is also called 'the characteristic life' and measures the period during which 63.2% of the components are expected to fail.

The published methods for the estimation of the parameters β_w and η, include the maximum likelihood estimators (MLE), the best linear invariant estimators (BLIE), the good linear unbiased estimators (GLUE) and the least squares. In addition to these methods, some graphical techniques such as hazard plotting and Weibull paper plotting are available also. All of these methods will be reviewed briefly, in the subsequent discussion.

Of all the methods mentioned above, MLE has attracted most attention. For life testing situations, where r failures have been observed from a sample of n items, Bain (70) gives the following equation for estimating $\hat{\beta}_w$:

$$\frac{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} \ln t_{i:n} + (n-r)\, t_{r:n} \ln t_{r:n}^{\hat{\beta}_w}}{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} + (n-r)\, t_{r:n}^{\hat{\beta}_w}} - \frac{1}{\hat{\beta}_w} = \frac{1}{r} \sum_{i=1}^{r} \ln t_{i:n} \qquad (2.6.3)$$

After determining $\hat{\beta}_w$, the scale parameter $\hat{\eta}$ can be estimated by:

$$\hat{\eta} = \left[\frac{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} + (n-r) t_{r:n}^{\hat{\beta}_w}}{r} \right]^{1/\hat{\beta}_w} \qquad (2.6.4)$$

For a complete sample case, $r=n$, the equations reduce to the forms given by Cohen (71). Equation (6.2.3) for $\hat{\beta}_W$ can be solved by a numerical method, such as the Newton-Raphson. It has been customary to obtain the initial value of $\hat{\beta}_W$ required for the N-R technique, from another method; for example, the least squares, GLUE, etc.

From equation (2.6.3), it is clear that $\hat{\beta}_W$ cannot be obtained with a single failure, in a life-testing situation. Therefore, estimation of $\hat{\beta}_W$ and $\hat{\eta}$ can only start after the second failure. But when the unfailed components have different ages; that is, if they are put into service at different times, the equation for $\hat{\beta}_W$ remains valid. The major problem with MLE is that the estimate of $\hat{\beta}_W$ is highly biased when there are only a few failures. Bain (70) has produced unbiasing factors for five or more failures and Harter and Moore (72) for 3 and 4 failures. Thus, in effect, MLE does not become effective before the third failure.

The method of best linear invariant estimators (BLIE) has been developed by Mann, et al (73). They have given the following relationships for evaluating estimates $\hat{\beta}_W$ and $\hat{\eta}$.

$$\hat{\eta} = \sum_{i=1}^{r} A(n,r,i)t_i \qquad (2.6.5)$$

and

$$1/\hat{\beta}_W = \sum_{i=1}^{r} C(n,r,i)t_i \qquad (2.6.6)$$

Mann, et al. have given the weighting factors $A(n,r,i)$ and

$C(n,r,i)$ for $n \geq 2$ and $r \geq 2$; r is the number of observed failures and n is the number of items on test. Obviously, these formulations are valid for life-testing situations only. The main disadvantages of this method are that it cannot be used before the second failure and the estimate $\hat{\beta}_W$ is highly biased for the first few failures.

The method of good linear unbiased estimators (GLUE) has been discussed by Bain (70). The equations to be solved are:

(i) for $r < n$ (incomplete data),

$$1/\tilde{\beta}_W = \frac{r \ln t_{r:n} - \sum_{i=1}^{r} \ln t_{i:n}}{n k_{r,n}} \qquad (2.6.7)$$

$$\ln \tilde{\eta} = \ln t_{r:n} - (C_{r,n}/\tilde{\beta}_W) \qquad (2.6.8)$$

where the constants $k_{r,n}$ and $C_{r,n}$ are given in tabular forms for $n \geq 5$ or may be calculated by quadratic approximations provided by Bain (70).

(ii) for $r=n$ (Complete data),

$$1/\tilde{\beta}_W = \frac{-\sum_{i=1}^{s} \ln t_{i:n} + \{s/(n-s)\} \sum_{i=s+1}^{n} \ln t_{i:n}}{n k_n} \qquad (2.6.9)$$

$$\ln \tilde{\eta} = (\frac{1}{n} \sum_{i=1}^{n} \ln t_{i:n}) + r^*/\tilde{\beta}_W \qquad (2.6.10)$$

where k_n is an unbiasing constant given in tablular form,

$s = [0.84\ n]$ is the largest integer $\leq 0.84n$, and $r^* = 0.5772$ is the Euler constant.

Unlike MLE and BLIE, this method provides a good estimate of β_w for few failures. But, it also cannot be used before the second failure. Another shortcoming with GLUE is that it is inapplicable when replacements are carried out on failure; because there is no single value of $t_{r:n}$.

The method of least squares is applicable only after all the equipments in the sample have failed; ie., no censoring is allowed. Nevertheless, the method is effective in dealing with few failures within engineering approximations. There is little bias in the estimated shape parameter β_w; nevertheless Hinds et al (74) have produced unbiasing factors in tabular form. The determination of β_w and η by this method, is based on the inversion and double logarithmic transformation of the equation for the cumulative distribution function F(t) given by equation (2.6.1), which on transformation becomes,

$$\ln \ln \left[\frac{1}{1-F(t)}\right] = \beta_w \ln t - \beta_w \ln\eta \qquad (2.6.11)$$

This is of linear form $Y=mX-Z$,

where $Y = \ln \ln \left[\frac{1}{1-F(t)}\right]$

$m = \beta_w$

$X = \ln t$

$Z = \beta_w \ln\eta$

There are several ways of calculating $F(t_i)$ for the ith failure (73,74); three of which are as follows:

Choice	$F(t_i)$	
Mean rank	$i/(n+1)$	
Benard's approximation	$(i-0.3)/(n+0.9)$	
Symmetrical sample cdf	$(i-0.5)/n$	(2.6.12)

Estimates of β_w and η can be obtained either graphically or by using a computer programme. In the programme, equations (2.6.11) and (2.6.12) can be solved by the least squares technique to estimate β_w, while the scale parameter η is estimated by the following equation.

$$\hat{\hat{\eta}} = \exp \{Y(\text{at } 63.2\%) + Z\} / \hat{\hat{\beta}}_w \qquad (2.6.13)$$

Equation (2.6.11) can also be solved graphically. Some special graph papers called 'Weibull Probability' papers are commonly used for plotting Y vs. X. In using such papers, the usual technique is to produce a 'best' fit line by eye. As an aid to this type of fitting (74), it should be noted that the line must pass through $(\bar{X},\bar{Y})$, where

$$\bar{X} = \frac{\sum_{i=1}^{n} \ln t_i}{n}$$

$$\bar{Y} = \frac{\sum_{i=1}^{n} \ln\ln\left[\frac{1}{1-F(t_i)}\right]}{n}$$

Like all other methods mentioned before, the least squares method also cannot be used for $n < 2$.

Another graphical technique for estimating the Weibull parameters is 'hazard plotting' which has been developed by Nelson (75). The main difference of this method from the earlier one is that it uses cumulative hazard function $H(t)$ instead of cumulative distribution function $F(t)$ for a linear fit.

The hazard function $h(t)$ for a distribution of times t to failure is defined in terms of the cumulative distribution function $F(t)$ and its derivative, which is the probability density $f(t)$, as follows:

$$h(t) = f(t)/(1-F(t)) \tag{2.6.14}$$

The cumulative hazard function $H(t)$ of a distribution is the integral of the hazard function upto time t ; that is:

$$H(t) = \int_{-\infty}^{t} h(t)dt = -\ln(1-F(t)) \tag{2.6.15}$$

For the Weibull distribution, the cumulative hazard function $H(t)$ can be written in the following form from equations (2.6.1) and (2.6.15):

$$H(t) = (t/\eta)^{\beta_w}, \quad t > 0 \tag{2.6.16}$$

This can be rewritten to express time as a function of the cumulative hazard, namely:

$$\ln t = (1/\beta_w) \ln H + \ln \eta \qquad (2.6.17)$$

By this relationship, ln t is a linear function of lnH. Thus, Weibull hazard paper is simply log-log graph paper. From equation (2.6.17), it is obvious that the slope of the straight line plot equals $1/\beta_w$; this fact is used to estimate β_w with the aid of the shape parameter scale from a Weibull hazard plot. For H = 1 (100%) the corresponding time t equals η; this fact is used to estimate η graphically.

For hazard plotting, the basic steps involved are as follows:

(i) Order the n sample times (both failed and unfailed times) from smallest to largest.

(ii) Label the times with reverse ranks, that is, the first time is labelled n, the second n-1, and the nth is labelled 1.

(iii) Mark the failure times to distinguish from the unfailed times, because the hazard value is calculated involving the failed items only.

(iv) Calculate the hazard value for each failure, omitting unfailed items, as $1/k\left(\frac{100\%}{k}\right)$, where k is its reverse rank.

(v) Calculate the cumulative hazard value for each failure as the sum of its hazard value and the hazard values of all preceeding failure times. The sample cumulative hazard values provide a non-parametric estimate of the cumulative hazard function of the true distribution.

(vi) Plot cumulative hazard values vs. sample failure times on a log-log graph paper. Nelson (75) mentions that Weibull hazard paper is available commercially.

(vii) By eye, fit a straight line through the data points. If the plot is reasonably straight, one may conclude that the distribution adequately fits the data.

(viii) Estimate β_w and η as mentioned before.

As an alternative to plotting, equation (2.6.17) can be solved by computer programme also. Like the least squares method mentioned earlier, this method also cannot be used before the second failure. In addition, estimates of the parameters are likely to be biased.

2.7 Estimation of Weibull Distribution Parameters by Sequential Probability Ratio Test (SPRT)

The sequential probability ratio test (SPRT) as devised by Wald (76) is basically a method of reliability test of an equipment within limited resources and time. Since its introduction, the technique has been used primarily for procurement of military equipments. Nevertheless, it generated enormous interest among many reliability workers and a considerable literature has been published widening its applications and the method itself.

Fundamentally, SPRT is a method of accepting or rejecting a specified failure rate or mean time between failures (MTBF), on the basis of hypothesis testing, with pre-assigned values of the risks involved. Statistically, these risks are α and β; the producer's risk and customer's risk, respectively. The operating characteristic (OC) curve and the average sample number (ASN) curve, or the average time to terminate testing (ATT) curve, are essential features of SPRT plans, which enable users to determine the true uncertainty involved in any decision and to obtain the expected length of the test, respectively.

Epstein and Sobel (77), Aroian and his co-workers (78, 79) and many other authors have made extensive contribution to the further development of SPRT. Nicolae and Obreja (80) and Harter and Moore (72) have devised test plans for the Weibull distribution. In the Military Standard MIL-STD 781 (81), a number of test plans have been given for the exponential distribution. But, only Harter and Moore (72) have discussed a method for the determination of both β_W and η simultaneously. Their method requires the use of the

MLE with unbiasing factors to determine β_w at each step. It is applicable only when 3 or more items are on test and can only be as good as the MLE. However, in the absence of any prior knowledge of the β_w and η values, the present methods of sequential testing are ineffective for systems with few recorded failures. But, one significant advantage of SPRT is that inference can be drawn from maintenance records showing the equipment being in the good state as well as the failed state.

In addition to the literature cited above, a comprehensive description of the basic principles of SPRT, has been given by Bazovsky (82) and Kapur and Lamberson (83). However, the construction of SPRT plans is based on testing two hypotheses, $H_o:\lambda=\lambda_o$ against $H_1:\lambda=\lambda_1$, where $\lambda_1 > \lambda_o$. λ_1 is the specified higher value of the failure rate for the exponential distribution and λ_o is some chosen lower value. The two error probabilities - α (producer's risk) and β (customer's risk) are designated by

$\alpha = P(H_1/H_o)$, ie., probability of accepting H_1 when H_o is true,

and

$\beta = P(H_o/H_1)$, ie., probability of accepting H_o when H_1 is true.

The rules for executing SPRT are stated in terms of the ratio of the likelihood functions.

(i) Accept H_o, if

$$\ln \frac{\prod_{i=1}^{r} f(t_i, \lambda_1)}{\prod_{i=1}^{r} f(t_i, \lambda_o)} \leq \ln \frac{\beta}{1-\alpha}$$

(ii) Reject H_o, if

$$\ln \frac{\prod_{i=1}^{r} f(t_i, \lambda_1)}{\prod_{i=1}^{r} f(t_i, \lambda_o)} \leq \ln \frac{1-\beta}{\alpha}$$

(iii) Continue testing, if

$$\ln \frac{\beta}{1-\alpha} < \ln \frac{\prod_{i=1}^{r} f(t_i, \lambda_1)}{\prod_{i=1}^{r} f(t_i, \lambda_o)} < \ln \frac{1-\beta}{\alpha}$$

For the exponential failure distribition, the inequalities become,

$$\frac{\ln B}{\ln(\lambda_1/\lambda_o)} + \frac{(\lambda_1-\lambda_o)}{\ln(\lambda_1/\lambda_o)} \sum_{i=1}^{r} t_i < r < \frac{\ln A}{\ln(\lambda_1/\lambda_o)} + \frac{(\lambda_1-\lambda_o)}{\ln(\lambda_1/\lambda_o)} \sum_{i=1}^{r} t_i \tag{2.7.1}$$

where $B = \beta/(1-\alpha)$, $A = (1-\beta)/\alpha$ and r is the number of observed failures. The inequalities of (2.7.1) can be written also in the form,

$$a + bT < r < c + bT \qquad (2.7.2)$$

where,

$$a = \ln B/\ln(\lambda_1/\lambda_o)$$

$$b = (\lambda_1 - \lambda_o)/\ln(\lambda_1/\lambda_o)$$

$$c = \ln A/\ln(\lambda_1/\lambda_o)$$

$$T = \sum_{i=1}^{r} t_i$$

This expression for T is valid, when only one item is being tested at a time. If n items are on test,

$T = \sum_{i=1}^{n} t_i$, where t_i denotes failure as well as successful operation time.

It is clear from inequalities (2.7.1) and (2.7.2), that the left and right sides represent equations of two parallel straight lines, called the 'acceptance line' and the 'rejection line', respectively. Commonly, graphical procedures have been used to plot observed failures and compare these with the two decision lines for various test plans.

SPRT plans for the Weibull distribution follow the same pattern as the exponential distribution. However, the time scale is changed from t to t^{β_w}, whereupon the two-parameter Weibull distribution is transformed to a one-parameter exponential distribution with a mean ψ related to the η of the Weibull distribution as follows: $\psi = \eta^{\beta_w}$. Following Nicolae and Obreja (80) and Harter and Moore (72), the test plans for the Weibull distribution can be stated as follows:

$$\frac{\ln B}{\ln d^{\beta_w}} + \frac{d^{\beta_w}-1}{\ln d^{\beta_w}} \sum_{i=1}^{r} \left(\frac{t_i}{\eta_o}\right)^{\beta_w} < r < \frac{\ln A}{\ln d^{\beta_w}} + \frac{d^{\beta_w}-1}{\ln d^{\beta_w}} \sum_{i=1}^{r} \left(\frac{t_i}{\eta_o}\right)^{\beta_w} \tag{2.7.3}$$

where η_o and η_1 are the higher and lower values of the scale parameter, respectively, and d is the discrimination ratio (η_o/η_1). The inequalities (2.7.3) can be arranged also in the form:

$$r\,\frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln A}{d^{\beta_w}-1} < \frac{1}{\eta_o^{\beta_w}} \sum_{i=1}^{r} t_i^{\beta_w} < r\,\frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln B}{d^{\beta_w}-1} \tag{2.7.4}$$

When the shape parameter β_w is known, graphical test plans of (2.7.3) or (2.7.4) can be obtained readily for any pre-estimated values of η_o and η_1. Harter and Moore (72) have given an adaptive procedure for cases where β_w is unknown. The difficulty with this method is that the hypotheses can be tested only against a specified value of η, which may be difficult to decide before testing begins.

The arrival at an appropriate decision by a SPRT plan is that when a plot of the number of observed failures against the cumulative operating time (for Weibull distribution, the transformed time scale) first crosses either of the two decision lines. In order to limit the maximum time to arrive at a decision, it may be agreed in advance to truncate the tests after a certain time T_o or a certain number of failures N. At the arrival of T_o or N, the decision is taken to accept or reject, depending upon whether the point giving the cumulative operating time vs. the number of failures is nearer the 'accept line' or the 'reject line' respectively. The decision taken from SPRT cannot be reversed by subsequent upcrossing of the accept line or downcrossing of the reject line (84).

After any decision, the true risks α' and β' of the decision being incorrect, owing to the limited number of failures and/or the limited test time, are not necessarily equal to the specified risks α and β. The maximum allowable values of α' and β' are related to α and β as follows:

$$\alpha'_{max} = \frac{\alpha}{1-\beta} \qquad \text{and } \beta'_{max} = \frac{\beta}{1-\alpha}$$

Kapur and Lamberson (83) state that the true risks must satisfy the following two conditions,

(i) at least one of the inequalities $\alpha' \leq \alpha$ and $\beta' \leq \beta$ must hold, and

(ii) $\alpha' + \beta' \leq \alpha + \beta$.

In the case of truncated sequential tests, some authors (72) seem to have disregarded these conditions and have accepted higher values of α' and β'.

Now, the power and length of the SPRT are very much dependent on the choice of α, β and the discrimination ratio d. In MIL-STD 781 (81), various combinations of $\alpha = \beta = 0.1$, 0.2, 0.3 and d = 1.5, 2,3 have been used. In general, any value of α and β can be used, provided these are less than 0.5, in order to avoid the triviality of A=B. But, there is virtually no restriction on the value of d.

From the above description of SPRT, it is clear that it cannot be used independently to determine the Weibull parameters, unless the possible values of β_W and η are known. But SPRT can be effective if it is used in conjunction with a method of validating the estimated distribution parameters.

2.8 Validation of Weibull Distribution Parameters

It is common practice in reliability studies to calculate the confidence limits of the distribution parameters. But in some circumstances, particularly in maintenance planning, the wide range of the limiting values of the distribution parameters can prove a major drawback. Dubey (85) has proposed the theory of Probability Limits for validating assumed failure distribution parameters (β_w and η). By setting any desired single-sided or double-sided probability limits for the rth order failure time t_r, one can check whether ordered observed failure times are within these limits and thus accept or reject these assumed parameters.

By the theory of probability limits, the 100ε per cent probability limit for the rth failure time t_r, in a sample of size n ($1 \leq r \leq n$) from the Weibull distribution is given by

$$t_\varepsilon (r) = \eta \{-\ln(1-b_\varepsilon)\}^{1/\beta_w} \qquad (2.8.1)$$

In equation (2.8.1) b_ε is the 100ε per cent percentage point of the beta distribution, with parameters r and n-r+1, respectively. Harter (86) has tabulated b_ε values for n=1 to 40.

Dubey has compared his method with the Kolmogorov-Smirnov test and found it very effective.

CHAPTER 3

CORRELATIONS OF FAILURE AND REPAIR TIMES

3.1 Introduction

It has been mentioned in the introduction to this thesis that the correlations of failure and repair times have considerable bearing on decisions regarding the right kind of maintenance policy. In spite of this, in many industrial organisations, mean time between failures and mean time to repair are used in the mathematical modelling of maintenance planning. As a consequence, the cause of failure, the types of failures and repair time distributions have been ignored. in formulating maintenance plans. The principal causes of this failure to seek the right distribution function, are probably that most of the parametric distributions, and the widely published methods to determine the parameters of these distributions, require quite a number of failures and also complicated mathematical procedures.

Concerning the best choice of distribution, it should have the property of incorporating all types of failures whether these are due to random causes and/or due to wearout situations. The most versatile distribution in this respect, is the Weibull distribution (65), which has the probability density function defined as:

$$f(t) = \frac{\beta_w}{\eta} \left(\frac{t}{\eta}\right)^{\beta_w - 1} \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta_w}\right\} \qquad (3.1.1)$$

where β_w is the shape parameter, η is the scale parameter or characteristic life and t is the time between successive failures. When β_w is equal to 0.5, 1.0, 2.0 and 3.4, the Weibull distribution simulates the hyper-exponential, the negative exponential, the log-normal and the normal distributions, respectively. The literature contains a number of methods for estimating β_w and η, providing that there are sufficient failure times or repair times recorded. These methods include maximum likelihood estimators (MLE), the best linear invariant estimators (BLIE), the good linear unbiased estimators (GLUE) and the least squares with unbiasing coefficients. There are some graphical methods also; for example, hazard plotting and Weibull plotting. Weibull plotting requires special graph paper. All these methods can be quite effective if the failure data comes from life testing situations. But for data containing records of failures as well as successful operation times, most of the methods become ineffective. To overcome this difficulty and to incorporate simple numerical procedure, the author has proposed a new method called SEQLIM. In the following section, SEQLIM is briefly discussed. A detailed description is given in Appendix A1.

3.2 Estimation of β_w and η by SEQLIM

The method of SEQLIM utilizes the basic acceptance principles of sequential probability ratio test (SPRT) to estimate sets of Weibull distribution parameters (β_w, η); followed by the theory of probability limits to select the most suitable values of these parameters. The SPRT plan for the Weibull distribution can be stated as:

$$r \frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln A}{d^{\beta_w}-1} < \frac{1}{\eta_o^{\beta_w}} \sum_{i=1}^{r} t_i^{\beta_w} < r \frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln B}{d^{\beta_w}-1} \quad (3.2.1)$$

where d is the discrimination ratio (η_o/η_1), η_o and η_1 are the higher and lower values being considered for the scale parameter, r is the number of observed failures and t_i denotes the ith failure time. Constants A and B are given by $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$, where α is the "producer's risk" and β is the "consumer's risk". In the inequalities (3.2.1), the right hand expression represents an acceptance line and the left hand expression represents a rejection line. For fixed β_w and d, the acceptance points for a given value of η_o, with r number of failures, are given by:

$$v_r = r \frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln B}{d^{\beta_w}-1} \quad (3.2.2)$$

The value of this expression is very dependent upon the choice of the values of the two error probabilities α, β and the discrimination ration d. Considering that even with few failures, the method should minimise the error in the estimation of β_w and η , it is recommended that $\alpha = 0.10$, $\beta = 0.05$ and d = 2.0 should be used. Appendix A2 gives the acceptance points for r = 0 to 10 and for various values of shape parameter β_w .

For untruncated sequential testing, η_o is accepted when $\frac{1}{\eta_o^{\beta_w}} \sum_{i=1}^{n} t_i^{\beta_w}$ reaches the acceptance line. But, for SEQLIM, separate values of η_o and η_1 are not required; v_r is determined from a discrimination ratio d, as shown by equation (3.2.2). The value of η which satisfies equations (3.2.2) and (3.2.3) is accepted.

$$\frac{1}{\eta^{\beta_w}} \sum_{i=1}^{n} t_i^{\beta_w} = v_r \qquad (3.2.3)$$

In equation (3.2.3) t_i denotes the operating time for the ith item on test (including failure time if any), and n is the total number of items on test to date. From equation (3.2.3), the accepted estimate of η for any value of β_w, is obtained as:

$$\eta = \left\{ \left(\sum_{i=1}^{n} t_i^{\beta_w} \right) / v_r \right\}^{1/\beta_w} \qquad (3.2.4)$$

So, for a set of β_w values, one can get a set of (β_w, η) pairs. Now, to pick the most suitable pair which will fit the recorded data best, the theory of probability limits is applied.

By the theory of probability limits (85), the 100 ε per cent probability limit for the rth failure time t_r, in a sample of n ($1 \leqslant r \leqslant n$) from the Weibull distribution, is given by:

$$t_\varepsilon (r) = \eta \{ - \ln (1 - b_\varepsilon) \}^{1/\beta_w} \qquad (3.2.5)$$

where b_ε is the 100 ε per cent percentage point of the beta distribution, with parameters r and n - r + 1, respectively. Appendix A3 gives a number of tables which list $t_\varepsilon (r)/\eta$ values for a range of β_w values and for n = 1 to 10 ($r \leqslant n$); each table is for a different value of 100 ε per cent, in the range 5% to 95%.

Having obtained a set of (β_w,η) pairs, one needs to compute t_r/η for the r observed failure times and to check whether these values are within the desired probability limits. The pair, which has no 'outlier', is accepted as the best estimate of β_w and η .

For both replacement and non-replacement situations, the fundamental approach for applying probability limits is the same, as explained above, but it is proposed that one should first use the 40% lower - 60% upper limits (double sided). If at this stage, no pair of (β_w , η) can be selected, then one should progressively increase the limits to 30% - 70%, 20% - 80%, 10% - 90% and so on, until a pair can be selected. When a number of pairs lie within the selection zone, the pair with the smallest value of β_w should be selected. On selecting β_w and η values from data with r failures, one can predict the lower and upper time limits for the arrival of the (r + 1) th failure, at the same accepted levels of probability limits. If the (r + 1) th failure arrives within this time limit, the values of β_w and η remain unchanged. If this (r + 1) th failure arrives outside the limited period, one may increase the probability limits to check if the previous estimates are acceptable or restart the whole exercise with recalculation of (β_w , η) pairs. However, it is recommended that for the first few failures, it is better to restart with 40% - 60% limits. For replacement situations, one has to remember that the observed failures are not in natural order, as in life testing situations. So, one needs to order the failures before checking for outliers.

3.3 Case Study: Evaluation of Weibull Distribution Parameters by SEQLIM

In Appendix A1, four hypothetical examples are given to demonstrate the effectiveness of the SEQLIM method in comparison with other methods. The working of one of the examples is shown also. In this section, five more examples are presented, all of which come from real situations. Tables 3.1 and 3.2 give the maintenance records of two types of safety trips LA115 and PA163 respectively. These trips are used to protect certain pressure vessels in three ammonia plants. Table 3.3 gives the records of the operating times of the three ammonia plants in a year. It is desired to find the distributions of the failure times of the trips and also the distributions of the operating times between shutdowns for the three plants. The Weibull distribution parameters have been evaluated for all these cases by the methods of SEQLIM, maximum likelihood estimators (MLE) and good linear unbiased estimators (GLUE). Since the method of GLUE is applicable only to life-testing situations, it cannot be applied to the trip data. For plant 1 data the GLUE method has been used after leaving out planned shutdown time.

Table 3.4 lists the results of the five examples considered in this study.

3.4 Discussion

From the results of the five examples given in Table 3.4, it can be seen that the SEQLIM method gives similar estimates to MLE, which is widely considered as the best method for estimating Weibull distribution parameters. For both replacement and non-replacement situations and with very few failures recorded, SEQLIM provides a more powerful method. Furthermore, with a large number of items on test, it rapidly converges to good estimates of β_w and η.

In many industrial organisations, the negative exponential distribution is used in devising maintenance plans, principally because of the complexity of established methods in finding the parameters of more realistic distribution. In this respect SEQLIM can prove very handy, requiring simple straight forward calculations.

In Appendix A1, further conclusions about the SEQLIM method are listed.

Table 3.1 Maintenance Records of LA115 trips (1500 Psi; Blow down Vessel; 106 ^{O}F; Ex H_I Level) in three ammonia plants

Plant 1			Plant 2			Plant 3		
Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results
						14.1.75	-	-
27.1.75	-	-						
			12.2.75	-	-			
						27.2.75	44	OK (on-line)
19.3.75	51	OK (on-line)						
			14.4.75	61	OK (off)			
22.4.75	34	OK (off)						
						7.5.75	69	OK (off)
						16.5.75	9	OK (off)
			23.7.75	100	Fault (electrical)			
24.7.75	93	Fault (Float Switch)						
12.9.75	50	OK (on)				4.11.75	172	OK (on)
			8.11.75	108	Fault (solenoid)			
21.11.75	70	OK (on)						
						2.12.75	28	OK (on)
			10.12.75	32	OK (on)			
29.12.75	38	OK (on)						
						7.1.76	36	OK (on)
19.1.76	21	OK (on)						
			26.1.76	47	OK (on)			
22.3.76	63	OK (on)				22.3.76	75	OK (on)

Table 3.1 (continued)

Plant 1			Plant 2			Plant 3		
Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results
			23.3.76	57	OK (on)			
17.5.76	56	OK (on)				17.5.76	56	OK (on)
						12.7.76	56	OK (on)
13.7.76	57	OK (on)						
			21.7.76	120	OK (on)			
7.9.76	56	OK (on)				7.9.76	57	OK (on)
			29.9.76	70	OK (off)			
1.11.76	55	OK (on)				11.11.76	65	OK (on)
25.11.76	24	OK (off-line)						
			29.11.76	61	OK (on)			
						14.1.77	64	Fault (Float Switch)
16.1.77	52	OK (off-line)						
			25.1.77	57	OK (on)			
						27.1.77	13	OK (on)
22.2.77	37	OK (on)						
			21.3.77	55	OK (on)			
						31.3.77	63	OK (on)
19.4.77	56	OK (on)						
			16.5.77	56	OK (on)			
						17.5.77	47	OK (on)
15.6.77	57	OK (on)						
						20.6.77	34	OK
			20.7.77	65	Fault (Solenoid)			

Table 3.1 (continued)

Plant 1			Plant 2			Plant 3		
Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results
8.8.77	54	OK (on)						
			17.8.77	28	OK (on)			
						23.9.77	95	Fault (Elecrical)
5.10.77	58	OK (on)						
28.11.77	54	OK (on)						
			6.1.78	142	OK (on)	6.1.78	105	OK (on)
24.1.78	57	OK (on)						

Table 3.2 Maintenance Records of PA163 trips (Stripper N_2 Blanket) in two ammonia plants

Plant 1			Plant 3		
Date	Interval between tests (Days)	Test Results	Date	Interval between tests (Days)	Test Results
2.11.76	-	-			
26.11.76	24	OK (off-line)			
			30.11.76	-	-
25.1.77	60	Fault (Electrical)			
			31.3.77	121	OK (on)
19.4.77	84	OK (on)			
			17.5.77	47	OK (on)
15.6.77	57	OK (on)			
8.8.77	54	OK (on)			
			6.9.77	112	Fault (Vent Valve)
3.10.77	56	OK (on)			
28.11.77	56	OK (on)			
			13.1.78	129	OK (on)
24.1.78	57	OK (on)			

Table 3.3 Operating times between shutdowns for three ammonia plants

PLANT 1		PLANT 2		PLANT 3	
Date	Length of run (days)	Date	Length of run (days)	Date	Length of run (days)
8.2.78	98	15.3.78	98	22.2.78	61
25.4.78	66	5.4.78	19	16.3.78	9
6.5.78	0.33	10.6.78	66	6.7.78	111
15.6.78	2.67	10.8.78	52		
21.7.78	35	17.8.78	6		
28.9.78	65	27.11.78	84		
10.10.78	5				
20.1.78	8				
11.11.78	12 (planned)				

Table 3.4 Evaluation of Weibull distribution parameters by SEQLIM, MLE and GLUE

	SEQLIM			MLE			GLUE		
Example	β_w	η	Acceptance level	β_w	η	Acceptance level	β_w	η	Acceptance level
LA 115	1.3	18.4 months	10% - 90%	1.3	17.626 months	10% - 90%	-	-	-
PA 163	1.6	10.186 months	20% - 80%	1.65	11.668 months	20% - 80%	-	-	-
Plant 1	0.90	32.45. days	10% - 90%	0.75	32.406 days	10% - 90%	0.668	29.158	10% - 90%
Plant 2	1.3	60.03 days	20% - 80%	1.5	59.427 days	20% - 80%	1.45	57.27	20% - 80%
Plant 3	1.3	53.95 days	20% - 80%	1.275	64.68 days	30% - 70%	0.946	72.44 days	30% - 70%

CHAPTER 4

PREVENTIVE/BREAKDOWN/OPPORTUNISTIC REPLACEMENT POLICIES

4.1 Introduction

The conditions for carrying out preventive replacement, failure replacement or opportunistic replacement of any equipment are discussed in chapter 1, in which various objective functions are presented. In determining an optimal policy for replacement, the models used assume that there are only two possible states for the equipment, GOOD and FAILED and that the state of the equipment is known with certainty that is, failure to provide the desired service will be immediately revealed. The optimal policy derived from these models will remain valid as long as the various costs, failure distributions, etc., are valid. Whenever technologically improved equipment is incorporated into the system, the optimal replacement policy will have to be re-evaluated.

4.2 Models for Preventive Replacement Policies

Basically, there are two types of preventive replacement plans; one is a 'periodic' or 'fixed interval' replacement plan and the other is an 'age-based replacement' plan. In the first type, preventive replacements occur at fixed intervals and failure replacements are carried out whenever necessary. For the second type, the policy is to replace either on failure or at a predetermined age depending upon which one comes first. With this policy, the possibility of performing a preventive replacement shortly after a failure replacement is avoided.

Both types of plan, with various objective functions, have been covered widely in the literature. (1, 2 , 3). A brief mention of some of these will be made here.

For a fixed interval policy, the optimal interval between preventive replacements subject to the minimization of the total expected cost per unit time is determined from the following model (3):

$$C(t_p) = \frac{C_p + C_f \, H(t_p)}{t_p} , \qquad (4.2.1)$$

where, $C(t_p)$ = Total expected cost/unit time

C_p = Cost of a preventive replacement

C_f = Cost of a failure replacement

t_p = Preventive replacement interval

$H(t_p)$ = Expected number of failures in interval $(0, t_p)$.

$H(t_p)$ can be calculated either by the 'renewal theory approach' or by 'discrete approach' (3).

A similar model for the minimization of total downtime per unit time is given in the following way (3):

$$D(t_p) = \frac{T_f\ H(t_p) + T_p}{t_p + T_p} \tag{4.2.2}$$

where, $D(t_p)$ = Total downtime/unit time

T_p = Downtime for a preventive replacement

T_f = Downtime for a failure replacement

The cost model (3) for the age-based policy to determine the optimal replacement age of the equipment is:

$$C(t_p) = \frac{C_p \times R(t_p) + C_f \times [1 - R(t_p)]}{(t_p + T_p)\ R(t_p) + [M(t_p) + T_f]\ [1 - R(t_p)]} \tag{4.2.3}$$

where, $R(t_p)$ = Probability of a preventive cycle

$1 - R(t_p)$ = Probability of a failure cycle

$M(t_p)$ = Expected length of a failure cycle, which is given by $\int_{-\infty}^{t_p} t\ f(t)dt/[1-R(t_p)]$

$f(t)$ = Probability density function of the equipment's failure times

Now, this model can also be expressed in multiples of planned replacement cost (87), which is given by:

$$\frac{C(t_p)}{C_p} = \frac{R(t_p) + \rho[1 - R(t_p)]}{T_p \times R(t_p) + T_f\ [1 - R(t_p)] + \int_o^{t_p} R(t)dt} \tag{4.2.4}$$

where, ρ = Relative failure cost (= C_f/C_p)

The saving of an optimal age replacement policy from a 'replace at failure only' policy is calculated in the following way:

$$\text{Percentage Saving} = \left[1 - \frac{\frac{C(t_p)}{C_p} \times \text{Mean failure time}}{\rho} \right] \times 100\% \qquad (4.2.5)$$

The downtime model (3) is similarly given by

$$D(t_p) = \frac{T_p\, R(t_p) + T_f\, [1 - R(t_p)]}{(t_p + T_p)\, R(t_p) + [M(t_p) + T_f]\, [1 - R(t_p)]} \qquad (4.2.6)$$

All the models are solved numerically by an iterative procedure.

4.3 Models for Opportunistic Replacement Policies

4.3.1 Review of Opportunistic Model by Radner and Jorgenson

This model (1) is applicable in devising a replacement policy for an equipment which is composed of M + 1 parts; part 0 has an increasing failure rate and parts i, i = 1, ..., M, have constant failure rates. The policy has an (n_i, N) structure in the sense that there are M + 1 critcal ages, n_i, i = 1, ..., M and N, where N is the preventive replacement age of part 0 and n_i is the lower bound of time after which opportunistic replacement of part 0 is profitable, on failure of any of the parts i. Denoting by t, the time since the last replacement of part 0, the policy can be stated simply in the following way:

(i) If part i fails at a time t such that $n_i \leqslant t < N$, then part 0 and part i are replaced and the equipment is as good as new.

(ii) If part O fails in the interval $O \leq t < N$ or has not been replaced at t = N, it is replaced and the equipment is returned to the new condition.

Now, the M + 1 critical ages are evaluated by minimizing the total expected cost per unit good time, which is given by the following model.

$$\frac{C(n_1, n_2, \ldots, n_M, N)}{G\ (n_1, n_2 \ldots n_M, N)} = \frac{AL + \sum_{i=1}^{M} \lambda_i E(W_i) C_i + \sum_{i=0}^{M} q_{oi} C_{oi} + d_{M+1} C_o}{E(T)} \tag{4.3.1.1}$$

where,

$C(n_1, n_2, \ldots, n_M, N)$ = Total expected cost over a cycle

$G\ (n_1, n_2, \ldots, n_M, N)$ = E(T) = Expected on-stream time over the cycle

A = Time rate of amortization

L = Length of the cycle

λ_i = Failure rate of part i/unit time

$E(W_i)$ = Expected length of the interval over which part i alone will be replaced

C_i = Cost of replacing part i alone

q_{oi} = Probability that the cycle ends with a replacement of part O and part i together

C_{oi} = Cost of replacing part O and part i together

d_{M+1} = Likelihood that part O is not replaced until t=N

C_o = Cost of replacing part O alone

The model is to be solved by some standard numerical methods.

4.3.2 Opportunistic Replacement Model for a System Where One or More Components Have Increasing Failure Rates

It is clear that the opportunistic model proposed by Radner and Jorgenson has very limited applicability; because it cannot deal with the typical system which has more than one component with time-dependent failure rate. With this type of system, the replacement opportunity will not only be created by the failure of a component with constant failure rate but also by a component with increasing failure rate, either from a failure or from the arrival of the preventive replacement age, if there is any. To take this factor into account, an alternative opportunistic model has been proposed (Appendix B1). Here, only the main features of the model are given.

(i) By using the models presented in section 4.2, the preventive replacement age for a component with increasing failure rate is determined.

(ii) Any component is replaced either on failure or at the arrival of the preventive replacement age, whichever arrives first.

(iii) When a replacement opportunity is created by condition (ii), it provides an opportunity for replacing other unfailed component(s). Now, this opportunity is used only if there is an advantage in doing so. From the cost point of view, the advantage may be interpreted as the minimization of overall replacement cost. In other words, it is a cost balance between the cost of replacing now and the cost of replacing this component alone at a later date, prior to a subsequent opportunity.

For the construction of the model, let the component which must be replaced now be denoted by i and an unfailed component by k. The cost advantage for replacing k with i, is given by the following:

Cost advantage = [Replacement costs which include cost of production loss, engineer's cost, etc.]
x Probability that k will fail before another opportunity
÷ [Cost of component k]
x Proportion of useful life that will be lost through early retirement of k with i

(4.3.2.1)

For component k, denoting the present age by T_k and the conditional mean failure time, (having survived upto T_k) by μ_k, the proportion of useful life lost through early retirement can be calculated from the ratio $(\mu_k - T_k)/\mu_k$, where μ_k is given by the following equations:

If k has no sensible preventive replacement age

$$\mu_k = T_k + \frac{1}{R(T_k)} \int_{T_k}^{\infty} R(t)dt, \tag{4.3.2.2}$$

If k has a preventive replacement age of N_k

$$\mu_k = \left[\{T_k R(T_k) - N_k R(N_k)\} + \int_{T_k}^{N_k} R(t)dt\right] / \{R(T_k) - R(N_k)\}$$

(4.3.2.3)

The function R(·) denotes the reliability function for component k.

The probability that component k will fail, before another replacement opportunity, can be calculated in the following way. Let R_{ki} denote the probability that k will survive until the next failure of i. This survival probability of component k is given by

$$R_{ki} = \int_0^\infty \frac{f_k(x + T_k)}{R(T_k)} F_i(x)\, dx \qquad (4.3.2.4)$$

The functions f(·) and F(·) represent the probability density function of failure times and the cumulative distribution function, respectively. The term $f_k(x + T_k)/R(T_k)$ represents the conditional probability density function for component k which has survived upto age T_k.

Since there may be other unfailed component(s) in the system, the survival probability of k with respect to those unfailed component(s) will have to be taken into account also. Denoting these later components by j, j=1,2,3,...

and their present ages by T_j, the survival probabilities can be calculated as

$$R_{kj} = \int_o^{\infty} \frac{f_k(x+T_k)}{R(T_k)} \left\{ \frac{F_j(x+T_j) - F_j(T_j)}{R(T_j)} \right\} dx, \quad \begin{array}{l} j=1,2,3\cdots \\ j \neq k \\ j \neq i \end{array} \tag{4.3.2.5}$$

Since the probabilities of success and failure add up to unity, the respective failure probabilities of component k with respect to components i and j's are

$$F_{ki} = 1 - R_{ki} \text{ and } F_{kj} = 1 - R_{kj} \tag{4.3.2.6}$$

Clearly, each of the events of component k failure before components i and j's are independent. So, the probability that k will fail before i and j's is simply,

$$P\ [k \text{ failure before i and j's}] = F_{ki} \times \prod_j F_{kj}, \quad \begin{array}{l} j=1,2,3\cdots \\ j \neq i \\ j \neq k \end{array} \tag{4.3.2.7}$$

In reality, there may be more than one component failing at the same time, eg., common mode failure. So, the equation (4.3.2.7) can be re-written as,

$$P\ [k \text{ failure before i's and j's}] = \prod_i F_{ki} \times \prod_j K_{kj}, \quad \begin{array}{l} j \neq i \\ j \neq k \end{array} \tag{4.3.2.8}$$

Putting the relevant costs and the values of $(\mu_k - T_k)/\mu_k$ and P[k failure] into equation (4.3.2.1), the cost advantage can be calculated. If there is any positive advantage, k should be replaced.

When an unfailed component is being opportunistically replaced, this component will start in the 'as new' condition in the next cycle of operation. Consequently, the probability of failure before another opportunity for each of the other unfailed components as in equation (4.3.2.7) or (4.3.2.8) will have to be re-calculated.

4.4 Case Study: Application of Opportunistic Replacement Policy

4.4.1 General

To illustrate the application of a preventive/breakdown/opportunistic replacement policy, a case study was undertaken involving a fleet of office photocopying machines. Most of the results have been published in the paper given in Appendix B1. Here, the results from one such machine with some further illustration and modified computational procedures, are given.

From the engineer's estimates, the following times were obtained:

Replacement of LSR = 45 minutes

Changing MIX = 20 minutes

Advancing OPC = 5 minutes

Changing OPC after 12 advances = 25 minutes

Travelling to site = 30 minutes

Average response time to attend a machine after a failure call = 5.365 hours

Average rate of copying = 500 pages/day (8 hours working time ≡ 1 day)

The scaled cost data supplied by the company was:

Price of OPC = 192.00 for 12 advances

Price of LSR = 42.48

Price of MIX = 11.00

Engineer's cost = 20 per hour

There was no penalty for failure of the equipment nor for discontinuity of the service during waiting time.

The Weibull distribution was chosen for the correlation of failure times of the components and of the machines. The distribution parameters for the components of the selected machine, which is machine No. 1 from the non-commercial zone, are given in Table 4.1.

The objective for this problem was the minimization of total replacement costs. It can be seen that in the event of no profit loss nor penalty due to the failure of any component, there is no point in carrying out preventive replacement of any component. But, some excess maintenance cost is incurred when the engineer has to attend the machine again shortly after replacing a failed component.

Concerning the application of an opportunistic replacement policy, it is worth remembering that this

Table 4.1

Weibull Distribution Parameters for the

Components of Machine No.1 (Non-Commerical Zone)

Components	Shape Parameter (β_W)	Scale Parameter (η) in day unit
OPC	1.994	19.084
LSR	2.794	28.164
MIX	1.281	63.278

policy is applicable for unfailed components only; because the component which has failed or arrived at its preventive replacement age will have to be replaced any way.

For the equipments of this study, opportunistic replacements can occur in the following ways:

(i) OPC has failed; opportunistic replacements of LSR and/or MIX.

(ii) LSR has failed; opportunistic replacements of OPC and/or MIX.

(iii) MIX has failed; opportunistic replacements of OPC and/or LSR.

To find the opportunistic replacement ages for the unfailed components in each of the cases mentioned above, the opportunistic replacement model of section 4.3.2 was applied separately. The cost advantage function for the opportunistic replacement of an unfailed component in this problem can be evaluated in the following way.

[Cost advantage]$_{LSR}$ = [Engineer's cost]
x Probability that LSR will fail before another opportunity
- [Price of LSR] x Proportion of useful life lost if LSR is replaced with the advancement/replacement of OPC or the replacement of MIX, which must be carried out now.

[Cost advantage]$_{MIX}$ = [Engineer's cost]

x Probability that MIX will fail before another opportunity

- [Price of MIX] x Proportion of useful life lost if MIX is replaced with the advancement/replacement of OPC or the replacement of LSR, which must be carried out now.

[Cost advantage]$_{OPC}$ = [Engineer's cost]

x Probability that OPC will have to be advanced/replaced before another opportunity

- [Price of OPC for one advance] x Proportion of useful life lost if OPC is advanced/replaced with LSR/MIX now.

4.4.2 Computational Procedures for the Opportunistic Model

The basic steps to carry out computation for the solution of the opportunistic model given in section 4.3.2, are as follows:

(i) Define the cost advantage function as in equation (4.3.2.1)

(ii)(a) Read the various cost and time variables of the cost function.

(b) Read the number of components and the component numbers

(c) Read the number of failed components and the failed component numbers.

(d) Read the unfailed component numbers or, screen out the unfailed component numbers

(e) Read the Weibull distribution parameters for each component

(f) Read preventive replacement age for each component, wherever appropriate

(g) Read the range of time for each component, over which the model is to be solved. For a component, which can be replaced on a preventive basis, put the upper limit at the preventive replacement age.

(iii) Calculate for each component, the probability density function, cumulative distribution function and the reliability function. Since the calculation cannot be done upto infinity, the suitable truncation point may be at, say 98% or 99% or 99.5% of the failure probability limit.

(iv) Calculate for each unfailed component, the proportion of life that will be lost by early retirement, using equations (4.3.2.2) or (4.3.2.3), over the time range considered in (ii)(g). Set the upper limit of the integration in (4.3.2.2) at the truncation point chosen in (iii).

(v) Calculate for each unfailed component, the probability that this component will not survive any component (failed) which is being replaced at the present opportunity, by equations (4.3.2.4) and (4.3.2.6). Set the upper limit of integration in (4.3.2.4) at the lower time range to reach a truncation point chosen as in (iii) for any of the two components under consideration.

(vi) Calculate for each unfailed component, the probability that this component will not survive another unfailed component, using equations (4.3.2.5) and (4.3.2.6). The limit of integration is as in (v).

(vii) Calculate for each unfailed component, the probability that this component will fail before another opportunity for its replacement, using equation (4.3.2.8).

(viii) Calculate for each unfailed component, the value of the cost advantage function as defined in step (i). If this value is postive for any component, that particular component should be replaced at the present opportunity. If no unfailed compoenent can be replaced, stop computation.

(ix) Take the component to be opportunistically replaced into the failed component category.

(x) Recalculate for other unfailed components, the failure probabilities as in steps (v) and (vii).

(xi) Re-evaluate the value of the cost advantage function for each of the remaining unfailed components, by following step (viii). If no other unfailed components can be replaced, stop computation. If replacement of one or more of the remaining unfailed components is possible, re-evaluation is to be carried out by steps (ix) through (xi).

A general programme called 'OPPORTUNIST' has been written using FORTRAN language. The programme list for the ICL 1904S computer is given in Appendix B2.

4.5 Results and Discussion

Figures 4.1, 4.2 and 4.3 give the opportunistic replacement times of the other two components when OPC or LSR or MIX has failed, respectively. In Figures 4.1 and 4.2, replacement plans are straightforward. But, in Figure 4.3 when MIX has failed, the opportunistic replacement demarcation lines for the OPC and LSR are not uniform. The principal reason for this non-uniformity is the evaluation of failure probabilities given in equation (4.3.2.8), which in turn, is dependent upon the evaluations of integrals given in equations (4.3.2.4) and (4.3.2.5). Evaluation of conditional mean failure times in equations (4.3.2.2) and (4.3.2.3) also play a part. In the 'OPPORTUNIST' programme for this study, the truncation point for the upper limit of the integrals for each component was chosen to be the time when there was at least 98% probability of failure of the component concerned. Possibly, a better algorithm than the one given in section 4.4.2 would have produced more uniform decision lines on Figure 4.3.

Figures 4.1 through 4.3 show that opportunistic replacement of a component will, in many circumstances, give a cost-optimal policy. When there is a large number of components in the system, it will be cumbersome to present decision rules by a priorisolution of the model for a time range and for each component. In this situation, one can use the present age of each unfailed component and determine the replacement policy for the present situation. Other remarks and conclusions related to the case study are given in Appendix B1.

The principal advantage of this simple opportunistic model is its applicability to systems containing components with any type of failure characteristics.

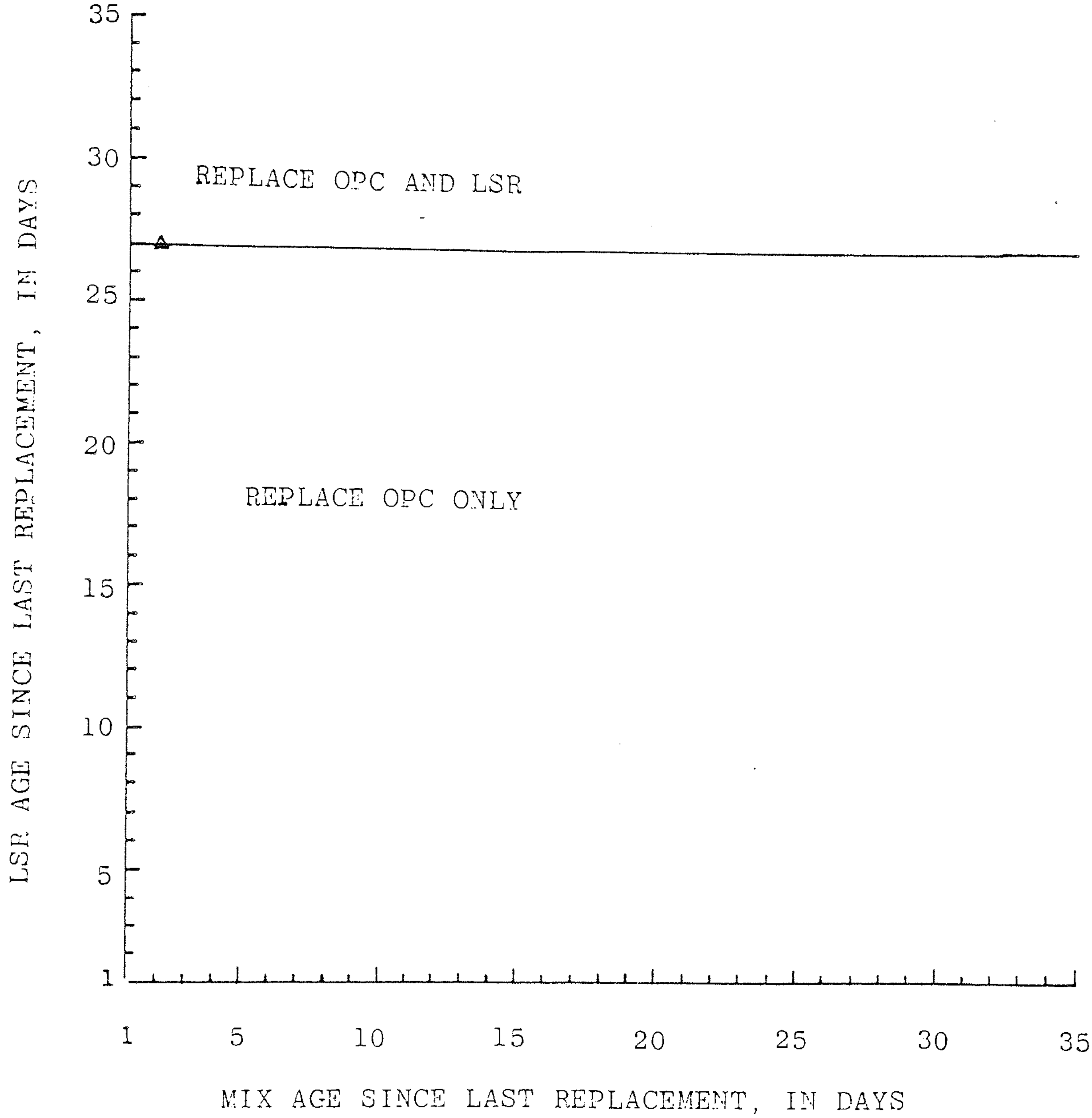


Fig 4.1 Opportunistic replacement policy for components (OPC, LSR and MIX) of a photocopier machine. This graph gives the opportunistic replacement times of LSR and MIX when OPC has failed. The Weibull distribution parameters for the components are as follows:

Component	β_w	η(days)
OPC	1.994	19.084
LSR	2.794	28.164
MIX	1.281	63.278

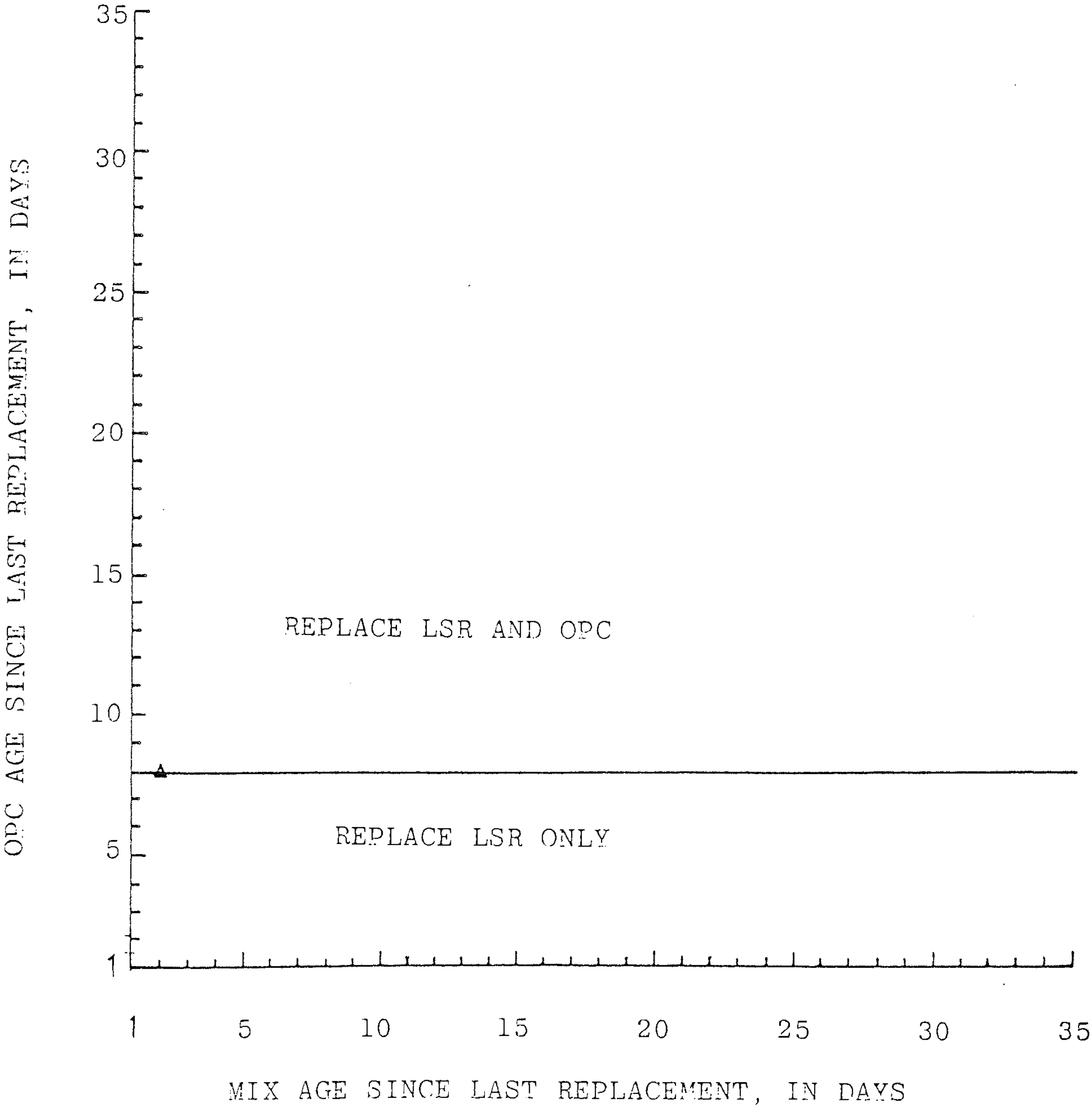


Fig 4.2 Opportunistic replacement policy for components (OPC, LSR and MIX) of a photocopier machine. This graph gives the opportunistic replacement times of OPC and MIX when LSR has failed. The Weibull distribution parameters for the components are as follows:

Component	β_w	η(days)
OPC	1.994	19.084
LSR	2.794	28.164
MIX	1.281	63.278

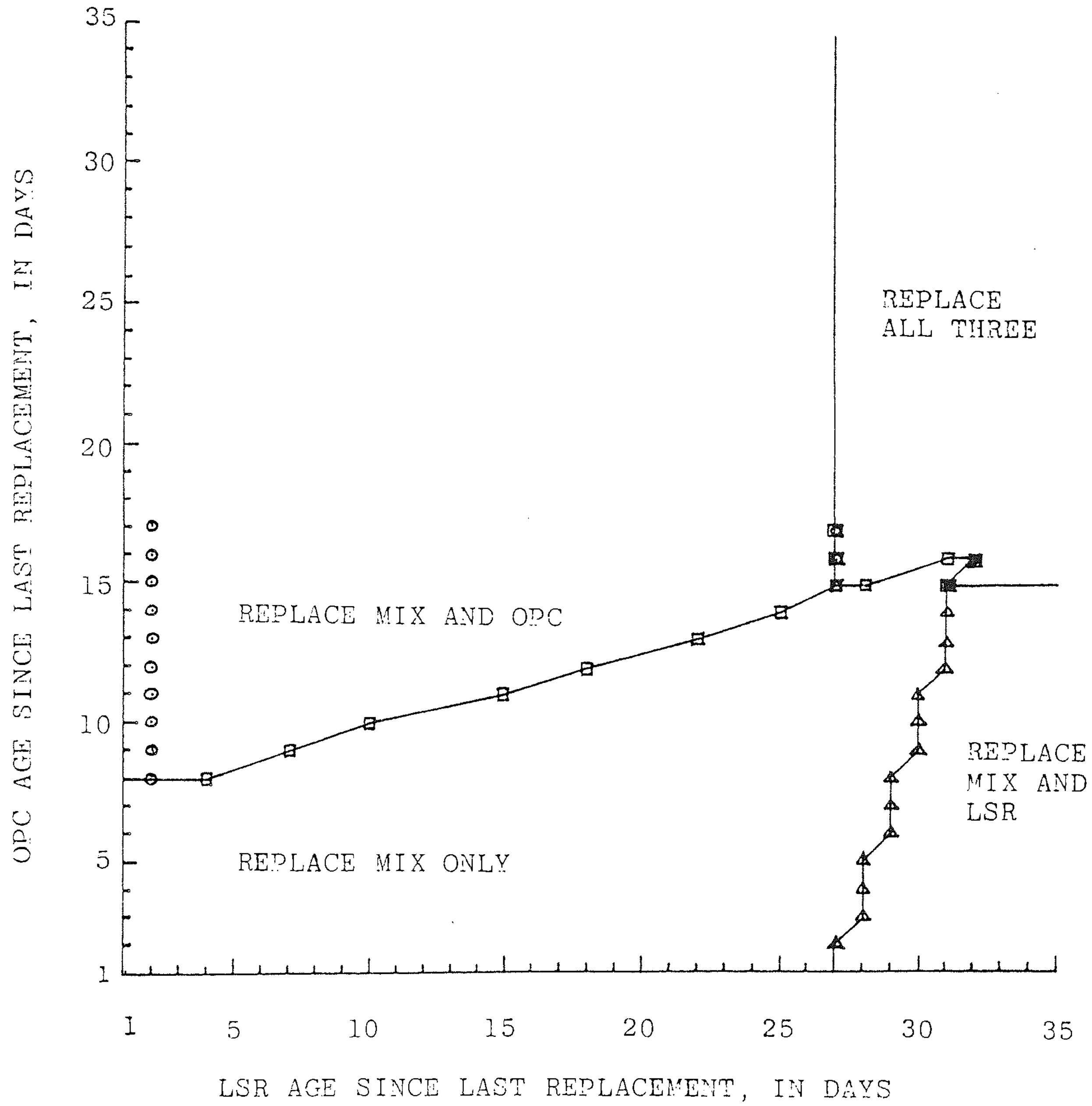


Fig 4.3 Opportunistic replacement policy for components (OPC, LSR, and MIX) of a photocopier machine. This graph gives the opportunistic replacement times of OPC and LSR when MIX has failed. The Weibull distribution parameters for the components are as follows:

Component	β_w	η(days)
OPC	1.994	19.084
LSR	2.794	28.164
MIX	1.231	63.278

CHAPTER 5

INSPECTION/INSPECTION-REPAIR-REPLACEMENT POLICIES

5.1 Introduction

As explained in chapter 1, inspection of safety systems is the only case considered in this research work. The possibilities of various objectives have also been mentioned in chapter 1.

As with replacement problems, the development of an inspection model and the consequent optimal policy are very much dependent upon the failure distribution of the equipment concerned. If the failure rate of the equipment is constant, the logical inspection policy will be of periodic type, ie., inspection will be carried out at fixed intervals. On the other hand, if the failure rate of the equipment is decreasing or increasing with time, the successive inspection intervals will be longer or shorter, respectively. Inspection models are generally constructed in two ways; either solely from reliability considerations or from cost and reliability considerations, to provide cost-optimal reliability. Detailed discussion of some of the models is given in Appendix C1.

5.2 Inspection Models from Reliability Approaches

Models based on the reliability approach have objective functions defined as the minimization of a downtime function, or the minimization of the probability of danger, or the maximization of probability of safety over a period of time. Apart from these differences in objective functions, the reliability models may or may not take into account the demand rate on the equipment. Usually, simple models published in the literature, do not consider demand rate. Besides, these models have been developed on the assumption that the negative exponential distribution fits the failure distribution of the equipment. To overcome this defficiency and to see the effect of the demand rate on the inspection interval of the equipment, the author has developed two additional models. All these models will be discussed briefly in subsequent sections.

5.2.1 Model Based on Fractional Dead Time (fdt)

Fractional dead time, which is defined as the proportion of time for which the system is in the failed state, is in fact a downtime function. In many industrial situations, it is regarded as a good practical measure of the unreliability of the safety system. For a single component in a safety system, for which the inspection interval is to be determined, the total fractional dead time may be considered to be composed of three contributions (48):

(i) fdt due to the failure of the component at sometime between successive inspections;

(ii) fdt due to disarming the component for inspection;

(iii) fdt due to errors in inspection operations and/or resetting of the components after inspection.

For an exponential failure distribution, which is often assumed to be valid for safety components, the total fdt is given (48) by

$$fdt = \tfrac{1}{2}\lambda t_r + t'/t_r + ne \qquad (5.2.1.1)$$

where $\lambda t_r << 1$, λ is the 'fail to danger' failure rate of the component, t' is the duration for which the component is disarmed for inspection, n is the number of operations in the inspection procedure, e is the probability of human error per operation and t_r is the length of inspection interval.

The optimal inspection interval to minimize the fractional dead time is then given (48) by,

$$t_r^* = (2t'/\lambda)^{\frac{1}{2}} \qquad (5.2.1.2)$$

5.2.2 <u>Model Based on Inter-inspection Hazard</u>

This model also has been developed (47) to minimize a downtime function, but the important feature is that the inspection interval has been taken as a function of the average failure/hazard rate between successive inspections. In terms of the cumulative density function F of failure times, the hazard rate r is expressed as

$$r = \frac{F(x_i) - F(x_{i-1})}{1 - F(x_{i-1})} , \quad i = 1,2,3 \ldots. \qquad (5.2.2.1)$$

where x_i is the time of the ith inspection, $x_o=0$ at the beginning of the operation and F(0)=0. In physical terms, r is the probability of transition from a 'good state' to a 'failed state' during the interval (x_{i-1}, x_i) given it was in the 'good state' at time x_{i-1}. Transforming equation (5.2.2.1) in terms of r only, the ith inspection time x_i can be expressed as:

$$x_i = F^{-1}(1-q^i), \text{ where } q = 1 - r \qquad (5.2.2.2)$$

Now, if I is taken as a random variable, which denotes the probable number of inspections which will be carried out before a failure will be detected, the expected value of I is given by:

$$E[I] = \frac{1}{r} \qquad (5.2.2.3)$$

The downtime for this model is considered to be composed of downtime due to inspections and the time for which the equipment is in the failed state. So, the total downtime D until a failure is detected, is given by:

$$D = it' + (x_i - t) \qquad (5.2.2.4)$$

where i is the number of inspections to detect a failure,

t' is the inspection time as before and t is the time to failure. Now, the mean downtime τ due to equipment failure is:

$$\tau = \sum_i \int_{x_{i-1}}^{x_i} (x_i - t) f(t)\,dt = \sum_i x_i q^{i-1} r - E(T) \qquad (5.2.2.5)$$

where E(T) is the expected instant of failure.

From equations (5.2.2.3) through (5.2.2.5), the expected downtime can be given as:

$$E(D) = t'/r + \sum_i x_i q^{i-1} r - E(T) \qquad (5.2.2.6)$$

Now, for the exponential failure distribution, all inspection intervals are equal and so the problem reduces to finding the first inspection time x_1. Subsequent inspection times are multiples of x_1; namely $x_i = ix_1$, $i=2,3..$ For this distribution, r is expressed as:

$$r = 1 - e^{-\lambda x_1} \qquad (5.2.2.7)$$

which gives

$$x_1 = -\{\ln(1-r)\}/\lambda = -\{\ln(q)\}/\lambda \qquad (5.2.2.8)$$

The expected downtime is then given by

$$E(D) = t'/r - \{\ln(q)\}/\lambda r - 1/\lambda \qquad (5.2.2.9)$$

and since $\lambda x_1 \ll 1$ for all practical purposes, the optimal inspection interval to minimize this downtime can be found as:

$$x_1 = \{2t'/\lambda\}^{\frac{1}{2}} \qquad (5.2.2.10)$$

The expected downtime per unit time can be obtained as:

$$E(D)/\text{unit time} = t'/x_1 + \lambda x_1/2 \qquad (5.2.2.11)$$

5.2.3 Model Based on Maximization of Probability of Safety and Consideration of Demand Rate

The development of this model is also based on the assumptions that the failure of the equipment and the demand on the equipment are both randomly distributed. Consequently, each inspection interval can be treated independently. Other assumptions for the construction of this model are:

(i) The plant is operational over a campaign time T, after which a statutory shutdown will take place and all equipments will be checked and returned "as new".

(ii) During the campaign time, there will be a number of inspections of each equipment at periodic intervals.

(iii) If there is any hazard, the whole plant will be shut down and all equipments will be checked before start-up. The campaign time will then be counted from this start-up.

(iv) If there is a safe shutdown of the plant by the activation of one or more equipments, following a genuine demand, then this will constitute verification of the safety system and the next inspection cycle will start after resetting the equipments involved. Since a genuine demand breaks the operating cycle before its full duration, this cycle will obviously be shorter. Now, one may add this time difference to the next cycle and thus maintain an average cycle length t_i (inclusive of time for inspection). This may be a reasonable assumption when the demand rate is not very high. On the other hand, one may maintain the pre-determined operating length t_r, which is valid for all cases. The first situation is treated in case (a) and the second in case (b).

The probability of a system malfunction or the danger situation consists of two terms:

(i) The equipment has failed at some time t between two successive inspections and following this failure there is a demand.

(ii) Demand occurs during inspection, when the equipment has been disarmed. Inspection only takes place if no hazard has occured during the operating interval $(0,t_r)$. If there is any hazard, failure of the equipment will be immediately revealed and attended to.

So, the probability of danger in a cycle can be expressed as:

$$P\,[\text{Danger}] = \{P\,[\text{Equipment failed at time } t] \text{ AND } P\,[\text{Demand subsequent to failure } (t, t_r)]\}_{o}^{t_r} \text{ OR } \{P\,[\text{Demand during testing, } t'] \text{ AND } P\,[\text{No hazard during } (0,t_r)]\} \qquad (5.2.3.1)$$

The probability of safety can be written as:

$$P\,[\text{Safety}] = 1 - P\,[\text{Danger}] \qquad (5.2.3.2)$$

Expressing the demand rate on the equipment by d and keeping other notations as before, equation (5.2.3.1) can be written as:

$$P\,[\text{Danger}] = \int_{o}^{t_r} \lambda e^{-\lambda t}(1 - e^{-d(t_r-t)})dt$$

$$+ (1-e^{-dt'})\ \{1 - \int_{o}^{t_r} \lambda e^{-\lambda t}(1-e^{-d(t_r-t)})dt\}$$

$$(5.2.3.3)$$

which on integration becomes

$$P\,[\text{Danger}] = 1 - e^{-dt'}\ \{\frac{\lambda e^{-dt_r} - de^{-\lambda t_r}}{\lambda - d}\} \qquad \lambda \neq d \quad (5.2.3.4)$$

$$P\,[\text{Danger}] = 1 - e^{-dt'}\ \{e^{-\lambda t_r}\{1 + \lambda t_r\}\ ; \qquad \lambda = d \quad (5.2.3.5)$$

So, the probability of safety for a single cycle is:

$$P\,[\text{Safety}] = e^{-dt'}\ \{\frac{\lambda e^{-dt_r} - de^{-\lambda t_r}}{\lambda - d}\}; \qquad \lambda \neq d \ (5.2.3.6)$$

$$P\,[\text{Safety}] = e^{-dt'} e^{-\lambda t_r}\{1 + \lambda t_r\} \quad ; \qquad \lambda = d \ (5.2.3.7)$$

Now, let N be the total number of equal inspection intervals over the whole campaign time T, so that there will be N-1 inspections. The probability of safety over the campaign time is then given by

$$P\ [\text{Safety during campaign time}] = P\ [\text{Safety during one interval}]^N \quad (5.2.3.8)$$

Using equations (5.2.3.6) and (5.2.3.7), the probability of safety at or before shutdown time T, is

$$P\ [\text{Safety}]_T = \{e^{-dt'}\}^{N-1} \left[\frac{\lambda e^{-dt_r} - de^{-\lambda t_r}}{\lambda - d}\right]^N \quad ; \lambda \neq d \quad (5.2.3.9)$$

$$P\ [\text{Safety}]_T = \{e^{-dt'}\}^{N-1} [e^{-\lambda t_r}\ \{1+\lambda t_r\}]^N \quad ; \lambda = d \quad (5.2.3.10)$$

From this point, the maximization of the probability of safety for the two cases are treated separately.

Case (a):

This is the case where the occurence of a demand replaces the next scheduled inspection of the equipment and the average length of inspection interval t_i is not altered. For finding the optimal t_i^*, equations (5.2.3.9) and (5.2.3.10) are differentiated with respect to t_i. Replacing t_r by $(t_i - t')$ and N by T/t_i, t_i^* can be derived as:

$$t_i^* = \left[\ln\left\{ \frac{\lambda e^{-d(t_i^*-t')} - d e^{-\lambda(t_i^* - t')}}{\lambda - d} \right\} - dt'\right] \times$$

$$\left[\frac{\lambda e^{-d(t_i^* - t')} - d e^{-\lambda(t_i^*-t')}}{\lambda d \{e^{-\lambda(t_i^*-t')} - e^{-d(t_i^*-t')}\}}\right] \quad ; \quad \lambda \neq d \quad (5.2.3.11)$$

$$t_i^* = \left[\lambda(t_i^*-t') + dt' - \ln\{1+\lambda(t_i^*-t')\}\right] \times \left[\{1+\lambda(t_i^*-t')\} / \{\lambda^2(t_i^*-t')\}\right]; \quad \lambda = d \quad (5.2.3.12)$$

Equations (5.2.3.11) and (5.2.3.12) will have to be solved by an iterative procedure like the Newton-Raphson technique. But a simplified solution can be found by approximating the exponentials upto second order terms. This is reasonable for the usual situations where λt_i, dt_i and dt' are much less than unity and t' is much less than t_i. The approximate optimal test interval t_i^* for both $\lambda \neq d$ and $\lambda = d$ is then given by

$$t_i^* = \{2t'/\lambda\}^{\frac{1}{2}}$$

Case (b):

In this case, the predetermined operating period t_r is maintained following a genuine demand situation. Since some of the cycles are shortened, the expected cycle length is given by:

Expected cycle length $=$ [(Length of the cycle when no demand occurs) x (Probability of no demand)]

+ [(Mean Length of a safe-operating cycle in a demand situation) x (Probability of a demand)]

(5.2.3.13)

Using the notations as before, equation (5.2.3.13) can be expressed as:

$$t_i = (t_r + t') e^{-dt_r} + \left[\frac{d\{1-e^{-t_r(\lambda+d)}[1+t_r(\lambda+d)]\}}{(\lambda+d)^2 (1-e^{-dt_r})} + t' \right] x \; (1-e^{-dt_r})$$

which on simplification becomes

$$t_i = t_r e^{-dt_r} + t' + \left[\frac{d\{1-e^{-t_r(\lambda+d)}[1+t_r(\lambda+d)]\}}{(\lambda+d)^2} \right] \tag{5.2.3.14}$$

for both $\lambda \neq d$ and $\lambda = d$.

Now, from equations (5.2.3.9), (5.2.3.10) and (5.2.3.14), the optimal inspection interval can be obtained as:

$$t_i^* = \frac{[e^{-dt_r^*}(1-dt_r^*) + dt_r^* e^{-t_r^*(\lambda+d)}] [\ln\{\frac{\lambda e^{-dt_r^*} - de^{-\lambda t_r^*}}{\lambda - d}\} - dt] [\lambda e^{-dt_r^*} - de^{-\lambda t_r^*} \ldots}{\lambda d \{e^{-\lambda t_r^*} - e^{-dt_r^*}\}}$$

for $\lambda \neq d$ (5.2.3.15)

$$t_i^* = \frac{\left[e^{-dt_r^*}(1-dt_r^*) + dt_r^* \, e^{-t_r^*(\lambda+d)}\right]\left[\lambda t_r^* + dt' - \ln\{1+\lambda t_r^*\}\right](1+\lambda t_r^*)}{\lambda^2 t_r^*} ;$$

for $\lambda = d$ (5.2.3.16)

These equations also will have to be solved by a numerical method. The explicit expression for t_r^* by approximating the exponentials in equations (5.2.3.14) through (5.2.3.16), can be obtained as

$$t_r^* = \{2t'/\lambda\}^{\frac{1}{2}} \text{ for both } \lambda \neq d \text{ and } \lambda = d.$$

The approximate value of the expected cycle length is

$$t_i^* = t_r^* + t' - dt_r^{*2}/2 \qquad (5.2.3.17)$$

5.2.4 Model Based on the Weibull Distribution and Specified Fractional Dead Time

For the models described earlier, the failure of the safety equipment were assumed to be randomly distributed in time. But, if there is a slow deterioration, which may be the case when the equipment is placed in an aggressive environment, some failures may be due to deterioration of the components of the equipment. The combined effects of deterioration and random failures can be correlated by the Weibull distribution, for which the probability density function f(t), the cumulative distribution function F(t) and the reliability function R(t) are expressed as:

$$f(t) = \frac{\beta_w}{\eta} \left(\frac{t}{\eta}\right)^{\beta_w - 1} \exp\left[- \left(\frac{t}{\eta}\right)^{\beta_w}\right] \qquad (5.2.4.1)$$

$$F(t) = 1 - \exp\left\{- \left({}^{t}/_{\eta}\right)^{\beta_w}\right\} \qquad (5.2.4.2)$$

$$R(t) = \exp\left\{- \left(\frac{t}{\eta}\right)^{\beta_w}\right\} \qquad (5.2.4.3)$$

where β_w is the shape parameter and η is the characteristic life. When β_w is greater than 1, a contribution of wearout failure is accommodated.

Since fractional dead time (fdt) is regarded as a good practical measure of reliability, the construction of the model is based on meeting target values of fdt. The expression for fdt, on the other hand, is dependent upon whether or not inspection returns the equipment to the 'as new' condition. When an in-depth inspection is performed such that all worn parts are detected and replaced during inspection, then the equipment can be assumed to be restored to the "as new" condition after each inspection. Consequently, each inspection interval can be considered as independent of the previous one and fdt for for this case can be expressed as:

$$fdt = \frac{1}{t_i} \int_o^{t_i} F(t)\, dt \qquad (5.2.4.4)$$

Using equation (5.2.4.2) and retaining only the first term of the exponential, fdt can be given as:

$$fdt \simeq ({}^{t}i/_{\eta})^{\beta_w}/(\beta_w+1) \qquad (5.2.4.5)$$

Now, if inspection is not followed by complete overhaul, the ageing phenomonon will continue throughout the subsequent test intervals. Then, the expression for fdt will have to be based on conditional failure probability during an inspection interval. For this situation, fdt can be expressed as:

$$fdt = \frac{1}{t_i} \int_o^{t_i} \{1 - \frac{R(T+t)}{R(T)}\} \; dt \qquad (5.2.4.6)$$

where T is the age at the last inspection. Using equation (5.2.4.3) in equation (5.2.4.6) and retaining only the first term of the exponential, fdt becomes

$$fdt = \frac{1}{\eta^{\beta}{}_{w}(\beta_w+1)t_i} \quad \{(T+t_i)^{\beta_w+1} - (T)^{\beta_w+1}\} - (\frac{T}{\eta})^{\beta_w}$$

(5.2.4.7)

which after re-arrangement can be written as:

$$(T+t_i)^{\beta_w+1} = \eta^{\beta_w}(\beta_w+1)\ t_i\left[fdt + \left(\frac{T}{\eta}\right)^{\beta_w}\right] + (T)^{\beta_w+1}$$

(5.2.4.8)

From equations (5.2.4.5) and (5.2.4.8), the first inspection interval, based on fdt due to equipment failure only, can be given as:

$$t_1 = \eta\{(\beta_w+1)\ fdt\}^{1/\beta_w} \qquad (5.2.4.9)$$

For subsequent inspection times, where inspection does not return the equipment to the "as new" condition, equation (5.2.4.8) can be generalised as:

$$(T_i)^{\beta_w+1} = \eta^{\beta_w}(\beta_w+1)(T_i-T_{i-1})\left[fdt + \left(\frac{T_{i-1}}{\eta}\right)^{\beta_w}\right] + (T_{i-1})^{\beta_w+1}$$

for $i = 2,3,\ldots$ and $T_1=t_1$

(5.2.4.10)

This equation can easily be solved by an iterative procedure. Figures 5.1 through 5.5 give five sequential inspection times for various values of β_w and η to achieve a target fdt of 0.02.

This model can easily be extended to take into account fdt due to inspection and fdt due to errors during inspection. The total fdt can then be written as:

$$fdt = \frac{1}{\eta^{\beta_w}(\beta_w+1)t_i}\{(T+t_i)^{\beta_w+1} - (T)^{\beta_w+1}\} - (\frac{T}{\eta})^{\beta_w} + \frac{t'}{t_i} + ne$$

(5.2.4.11)

where, t' is the duration of inspection, n is the number of operations in the inspection procedures, and e is the probability of human error per operation. The equation can be solved for minimum fdt to determine an optimal inspection interval.

Differentiating equation (5.2.4.11) with respect to t_i and then equating to zero, the following relationship for t_i can be obtained:

$$t_i = \frac{\{(T+t_i)^{\beta_w+1} - (T)^{\beta_w+1}\} + \eta^{\beta_w}(\beta_w+1)t'}{(\beta_w+1)(T+t_i)^{\beta_w}} \qquad (5.2.4.12)$$

Since T is zero at the beginning, the first inspection time or interval is given by:

$$t_1 = \{\frac{\beta_w+1}{\beta_w} \eta^{\beta_w} t'\}^{1/(\beta_w+1)} \qquad (5.2.4.13)$$

Subsequent inspection times can be obtained by the following general relationship:

$$T_i^{\beta_w} = \frac{\eta^{\beta_w}(\beta_w+1)t' - (T_{i-1})^{\beta_w+1}}{\beta_w T_i - (\beta_w+1)T_{i-1}} \qquad (5.2.4.14)$$

for $i = 2,3....$ and $T_1 = t_1$

This equation can also be easily solved by an iterative procedure.

5.3 Inspection Models For Cost Optimization

The determination of inspection intervals by reliability criteria often results in frequent inspections. Consequently, high labour costs are incurred. In order to provide cost-optimal reliability; by striking a balance between various costs and high reliability that must be ensured, cost models have been developed. In this section, two such models will be discussed briefly.

5.3.1 Model Based on Hazard Rate Calculated from Fractional Dead Time

In this model (48), hazard rate is simply calculated as:

$$\text{Hazard rate} = d \times fdt$$
$$= d \times \{\tfrac{1}{2}\lambda t_c + t'/t_c + ne\} \qquad (5.3.1.1)$$

where t_c is the inspection interval incorporating costs, and all other notations are the same as before. The breakdown of various costs associated with the failure analysis of the safety equipment can be given as:

(i) cost of a hazard:

$$\text{Cost} = C_H \times d \times\{\tfrac{1}{2}\lambda t_c + t'/t_c + ne\} \qquad (5.3.1.2)$$

where C_H is the average cost incurred from a hazardous event.

(ii) cost of spurious operation caused either by fail safe fault of the equipment or by a human error during inspection.

$$\text{Cost} = (S + ne/t_c)C_s \qquad (5.3.1.3)$$

where S is the spurious fault rate and C_s is the cost of a spurious operation. Generally, this cost is insignificant for many plants and may not be considered in the construction of the model.

(iii) capital cost of the safety system as an annual cost.

$$\text{Cost} = C_c \qquad (5.3.1.4)$$

where C_c is the annual cost of a single trip channel.

(v) cost of inspection.

$$\text{Cost} = r_t\, t'(8760/t_c) \qquad (5.3.1.5)$$

where r_t is the hourly cost of inspection.

Using equations (5.3.1.2) through (5.3.15), the total cost is given by:

$$\text{Total cost} = C_T = C_H d\,\{\tfrac{1}{2}\lambda t_c + t'/t_c + ne\} + (S + {}^{ne}/t_c)\,C_s + C_c$$

$$+ \frac{8760 r_t t'}{t_c} \qquad (5.3.1.6)$$

The optimal inspection interval t_c^* for cost minimization can be derived as:

$$t_c^* = \{\frac{2}{dC_H\lambda}\,(C_H dt' + C_s ne + 8760 r_t t')\}^{\frac{1}{2}} \qquad (5.3.1.7)$$

Neglecting the cost of spurious operation, equation (5.3.1.7) becomes:

$$t_c^* = \{\frac{2t'}{\lambda}\,(1 + \frac{8760 r_t}{C_H d})\}^{\frac{1}{2}} \qquad (5.3.1.8)$$

In contrast to the optimal inspection interval derived from reliability considerations only, this optimal is a function of failure rate, demand rate, inspection time, cost of a hazardous event and the hourly cost of inspection. Furthermore, t_c^* will always be longer than t_r^*.

5.3.2 Model Based on Hazard Rate Calculated from the Exact Probability of Hazard

The hazard rate for this model is calculated from the exact probability of hazard in an inspection interval derived in equations (5.2.3.4) and (5.2.3.5), while the cost function is the same as in the previous model. When the probability of hazard is much smaller than unity, hazard rate can be reasonably approximated as the probability of hazard per unit time, which is:

$$h = \left[1 - e^{-dt'}\left\{\frac{\lambda e^{-dt_{rc}} - de^{-\lambda t_{rc}}}{\lambda - d}\right\}\right]/t_c \quad ; \lambda \neq d \qquad (5.3.2.1)$$

$$h = \left[1 - e^{-dt'} e^{-\lambda t_{rc}}(1 + \lambda t_{rc})\right] /t_c \quad ; \lambda = d \qquad (5.3.2.2)$$

where t_{rc} is the operating interval for the cost model. Putting these two equations in the cost function, the total cost can be obtained as:

$$C_T = \frac{C_H}{t_c}\left\{1 - e^{-dt'}\left(\frac{\lambda e^{-dt_{rc}} - de^{-\lambda t_{rc}}}{\lambda - d}\right)\right\} + C_c + \frac{8760 r_t t'}{t_c} \quad ; \lambda \neq d$$

(5.3.2.3)

$$C_T = \frac{C_H}{t_c}\left\{1 - e^{-dt'} e^{-\lambda t_{rc}}(1+\lambda t_{rc})\right\} + C_c + \frac{8760 r_t t'}{t_c} \quad ; \lambda = d$$

(5.3.2.4)

neglecting the spurious cost term as insignificant. Since t_{rc} is $(t_c - t')$, the optimal inspection intervals for minimum costs can be derived as:

$$t_c^* = \frac{\left\{1-e^{-dt'}\left(\frac{\lambda e^{-dt_{rc}^*} - d e^{-\lambda t_{rc}^*}}{\lambda - d}\right)\right\} + \frac{8760 r_t t'}{C_H}}{\frac{\lambda d}{\lambda - d}\, e^{-dt'}\left(e^{-\lambda t_{rc}^*} - e^{-dt_{rc}^*}\right)} \; ; \quad \lambda \neq d$$

(5.3.2.5)

$$t_c^* = \frac{\left\{1-e^{-dt'}\, e^{-t_{rc}^*}\left(1+\lambda t_{rc}^*\right)\right\} + \frac{8760 r_t t'}{C_H}}{\lambda^2 t_{rc}^*\; e^{-dt'} \times e^{-\lambda t_{rc}^*}} \; ; \quad \lambda = d$$

(5.3.2.6)

Neglecting the higher order terms of the exponentials, an explicit expression for t_c^* can be obtained as:

$$t_c^* = \left\{\frac{2t'}{\lambda}\left(1+\frac{8760 r_t}{C_H d}\right)\right\}^{\frac{1}{2}} \qquad (5.3.2.7)$$

for both $\lambda \neq d$ and $\lambda = d$.

5.4 Case Study: Optimal Test Intervals for Safety Trips

The safety equipments investigated in this case study were trip systems used in three Ammonia Plants. These trips are provided for the protection of process equipment, furnaces, vessels, etc. They are required to be working whenever a demand occurs. The trips on this plant function by opening a solenoid valve on the pneumatic line to a trip and/or control valve in a process line. Since trips are fairly common and widely used protective devices, their reliable performance is very important in the context of continuous hazard free running of the plant.

Basically, the trip systems (48) consist of three functional units:

(i) the initiator - which monitors a process variable and gives a signal when the set point, ie., the error condition, is passed.

(ii) the activator - which consists of final elements, receives a signal and carries out the required action; eg., shutting off the process stream.

(iii) the link - which transfers the signal from the initiator to the activator in the required form. It is usually composed of relay(s) or solid-state electronics.

Although the above-mentioned functions are the same for all trips, the process operations and the equipments they consist of, are by no means standard. Moreover, they are much more complex in reality. For example, on the ammonia plant the initiators may be level switch, pressure switch, float switch, DP cell, etc., the activators are solenoid valve/trip valve, solenoid valve/control valve, used either as a single channel or in parallel, stop valve, nozzle valve etc. ; the links are electrical relays, ratio relay etc.

Here the analyses and results of two types of trips will be given. These trips are LA115 and PA163, for which maintenance records have been extracted in Tables 3.1 and 3.2. The fail to danger fault rates of the components and the test intervals quoted by the company are listed below:

LA115 (Ex high level in 1500 Psi blowdown drum; 106°F)

Components	Failure rate/year
Mobrey Level Switch	0.1
Electrical Relay	0.002
Solenoid Valve (Max seal; En-to-trip; Hand relatch)	0.1
Trip Valve	0.04

Fractional dead time (fdt) for the system tested completely every 2 months = 0.02 .

PA163 (Stripper N_2 Blanket)

Components	Failure rate/year
Delta Pressure Switch (Operating on falling gas pressure)	0.1
Electrical Relays	0.002
Solenoid Valve (Max seal, En-to-trip, Hand relatch)	0.2
Control Valve	0.1

fdt for the system tested monthly = 0.016

Table 5.1 gives the results of the failure analysis for both the trips on the basis of the maintenance records to date. It can be seen that for both trip systems, the shape parameter, β_w is greater than unity implying that some failures must have been due to deterioration of components.

The first testing time, t_1 was determined from equation (5.2.4.9); for the LA115 system, it is t_1 = 52 days and for the PA163 system it is t_1 = 42 days. Now, if an in-depth test is performed such that worn components are detected and replaced or repaired at each test, the test intervals can be regarded as fixed and equal to first test interval. Otherwise, subsequent testing times will have to be determined from equation (5.2.4.10). The reason for the calculated test intervals not being greatly different from the quoted ones, is that the average

Table 5.1 Estimation of Weibull Distribution Parameters (β_w, η) by SEQLIM for LA115 and PA163 Trips from Maintenance Records Listed in Tables 3.1 and 3.2

System/Components/ Others	No of failures	Total Numbers	Shape Parameter β_W	Scale Parameter η (months)
LA 115 system	6	9	1.3	18.4
Mobrey Level Switch	2	5	1.4	32.794
Solenoid Valve	2	5	1.5	34.92
Trip Valve (*)	0	3	2.6	67.223
Electrical Relay (*)	0	3	2.6	67.223
Electrical Fault	2	5	1.5	35.632
PA163 System	2	4	1.6	10.186
Pressure Switch (*)	0	2	2.5	22.531
Solenoid Valve (*)	0	2	2.5	22.531
Control Valve (*)	0	2	2.5	22.531
Electrical Relay (*)	0	2	2.5	22.531
Electrical Fault	1	3	1.1	9.005
Vent Valve	1	3	2.7	17.196
Common Components (LA115 and PA163)				
Solenoid	2	7	1.7	40.97
Electrical Relay (*)	0	5	1.5	60.322
Electrical Fault	3	8	1.5	34.755

* Parameters have been estimated on the assumption that the component with lowest operating time fails first.

failure rates for both trips are much higher than the quoted failure rates.

If the design of the trip system is such that individual components can be tested separately, trip test intervals can be greatly increased. For example, the test interval for the trip valve ($\beta_w = 2.6$, $\eta = 67.223$ months) to achieve a target fdt of 1/3 x 0.02, is 16 months; similarly, for a solenoid valve ($\beta_w = 1.7$, $\eta = 40.97$ months), test interval for fdt = 1/3 x 0.02, is 3.856 months ($\simeq$ 117 days).

5.5 Discussion

The general conclusions regarding the various inspection models described in this chapter have been listed in Appendix C.1. The most important point which has been observed, is that the failure modes of each trip should be carefully monitored for wear out or environmental influence. If such influence exists, initially test/inspection intervals can be greatly increased. The failure modes and subsequent failure analysis of individual components are also important. When it is not necessary to test the system completely at one time, fractional dead time (fdt) for each component can be separately specified and consequently separate test intervals can be determined.

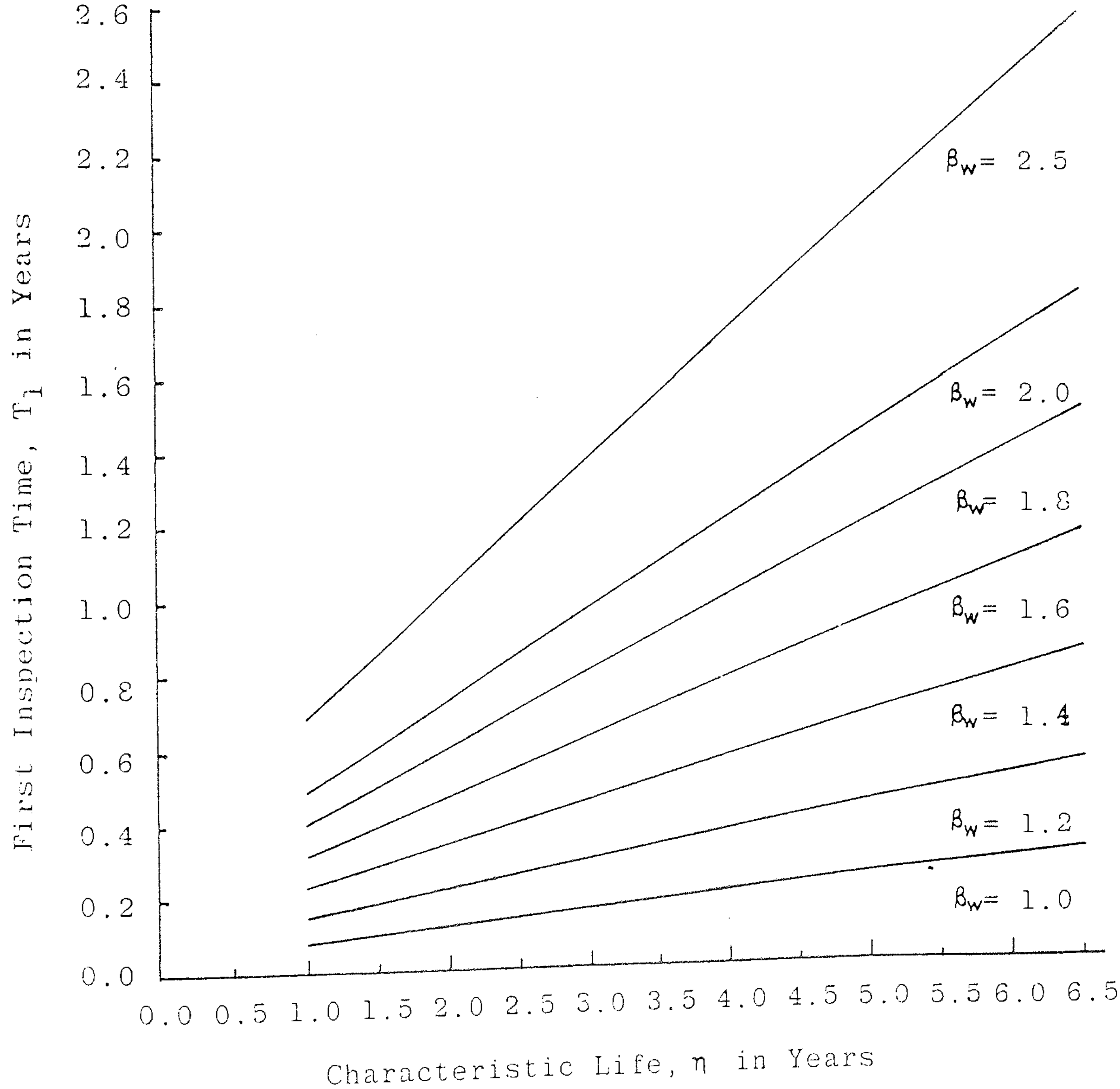


Fig 5.1 Plot of first inspection times T_1 vs. Characteristic life η of Weibull distribution for different values of shape parameter β_w and for fractional dead time (fdt) of 0.02.

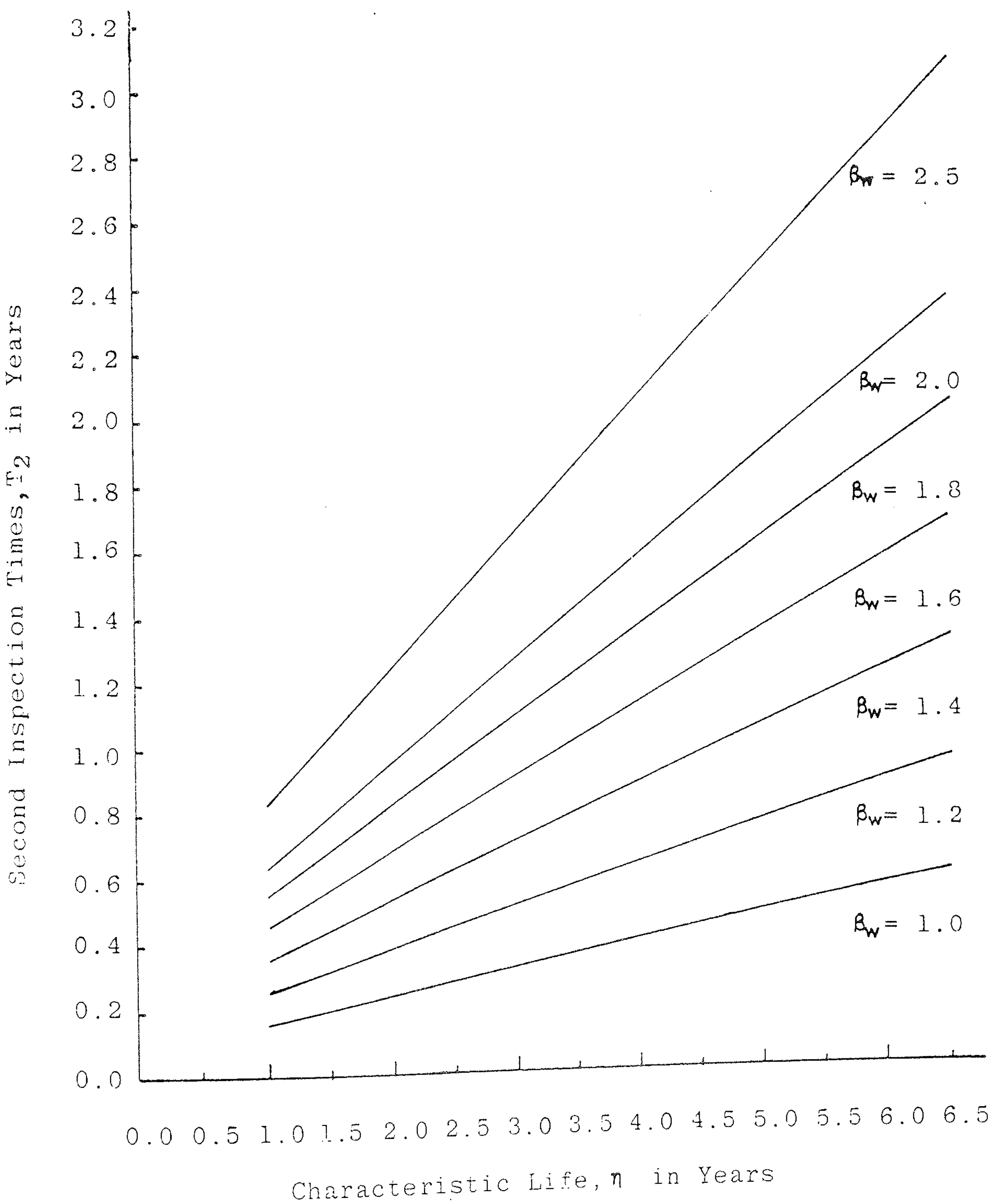


Fig 5.2 Plot of second inspection times T_2 vs characteristic life η of Weibull distribution for different values of shape parameter β_w and for fractional dead time (fdt) of 0.02.

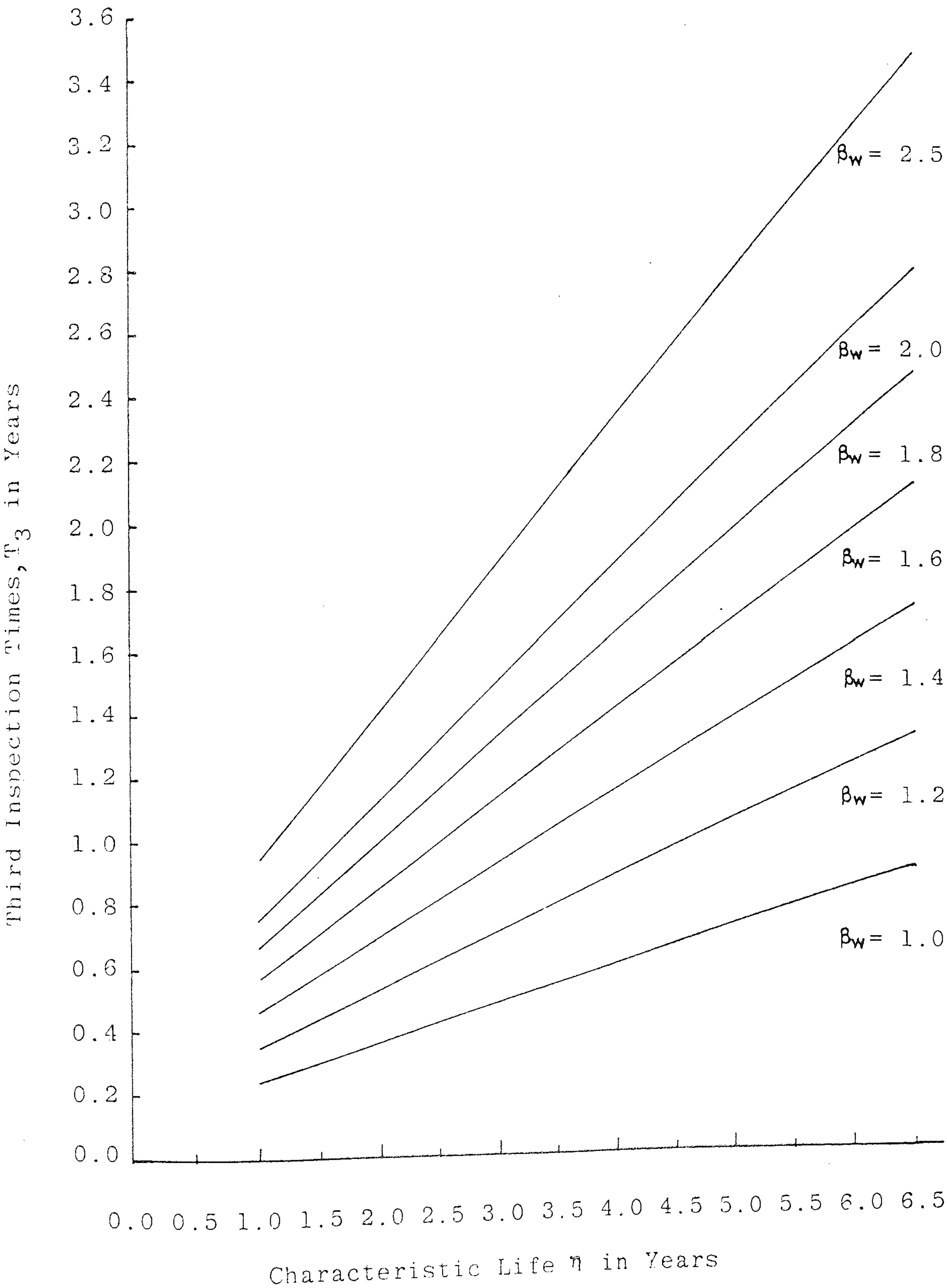


Fig 5.3 Plot of third inspection times T_3 vs characteristics life η of Weibul distribution for different values of shape parameter β_w and for fractional dead time (fdt) of 0.02.

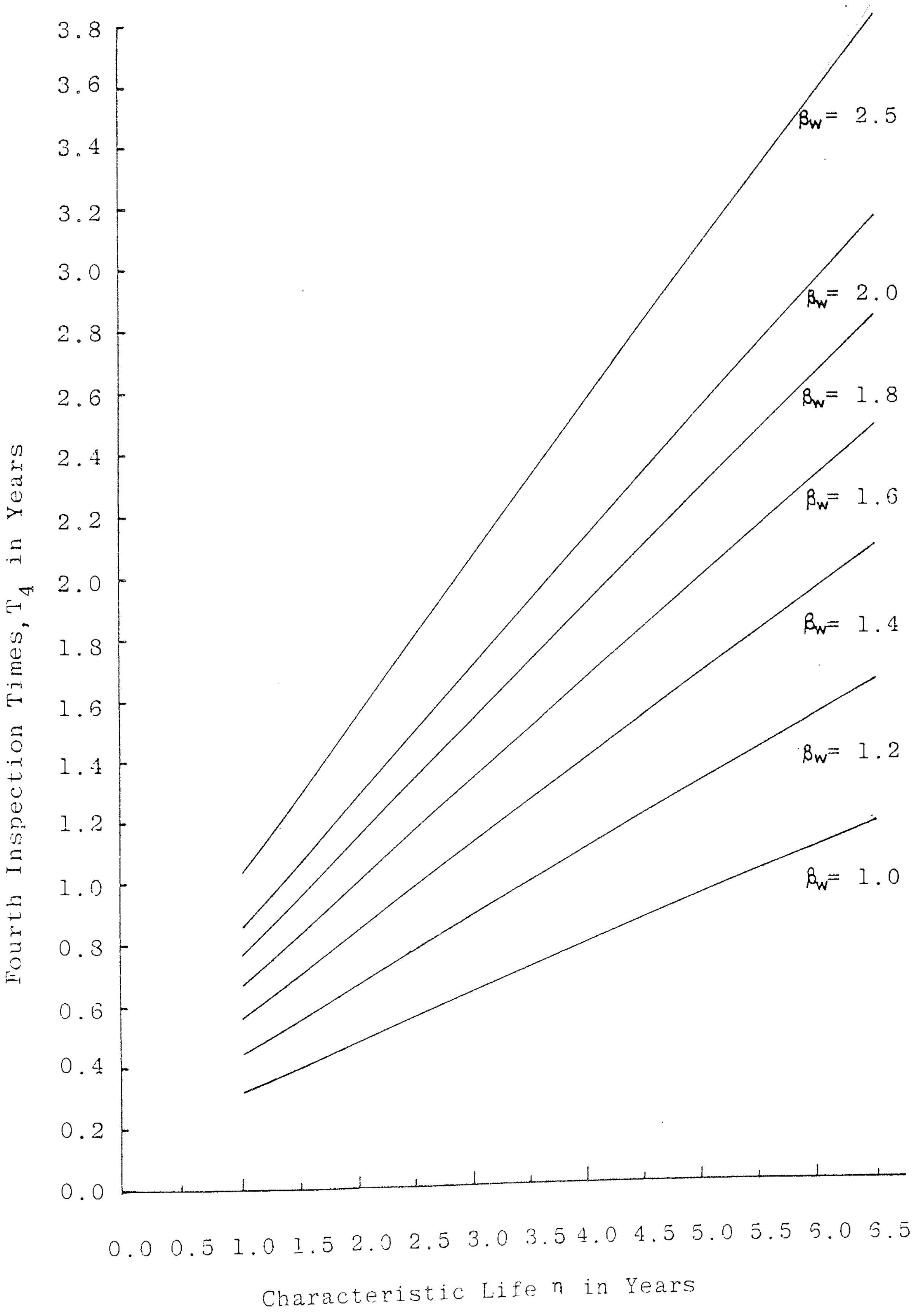


Fig 5.4 Plot of fourth inspection times T_4 vs characteristic life η of Weibull distribution for different values of shape parameter β_w and for fractional dead time (fdt) of 0.02.

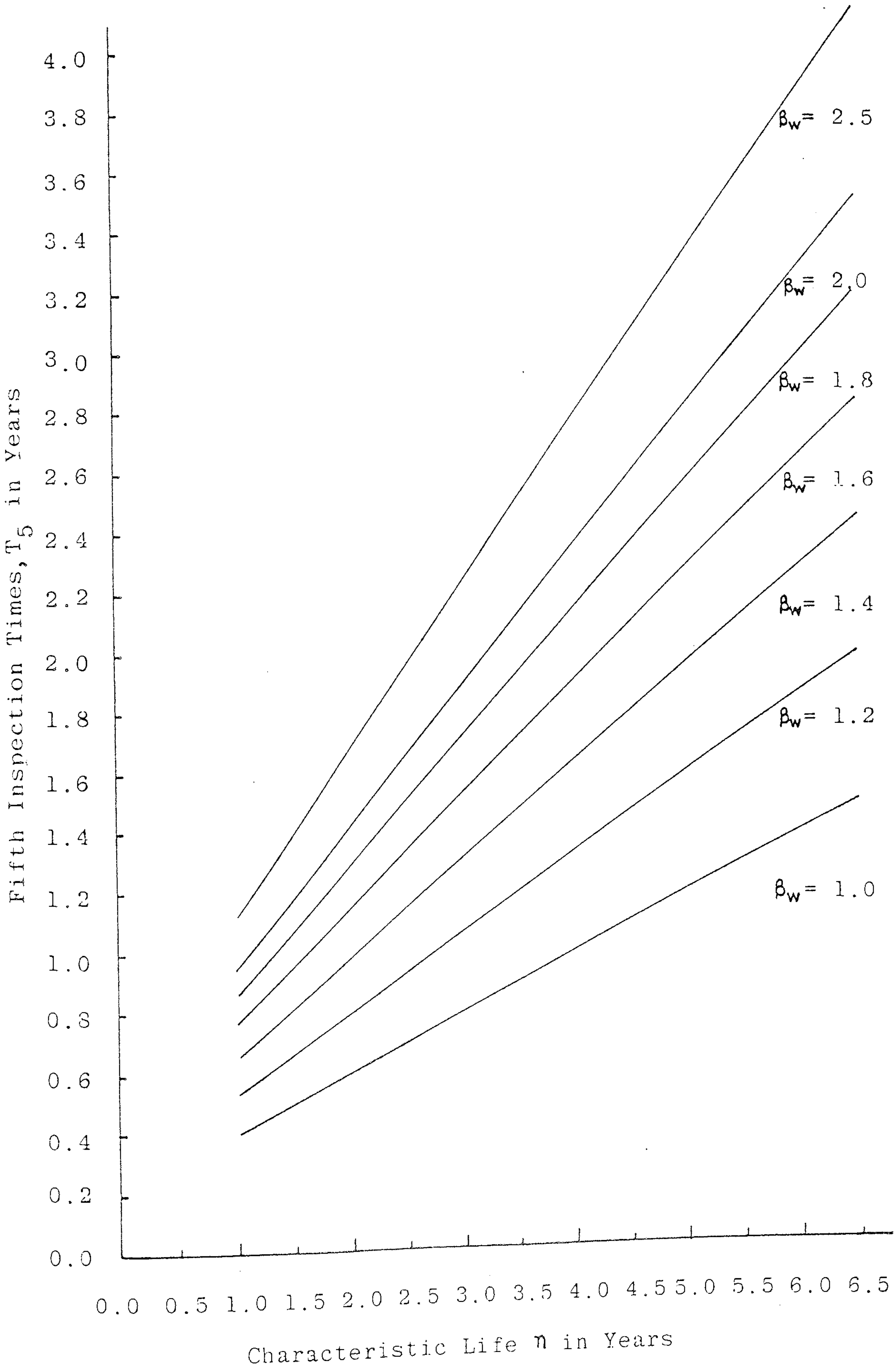


Fig 5.5 Plot of fifth inspection times T_5 vs characteristic life η of Weibull distribution for different values of shape parameter β_w and for fractional dead time (fdt) of 0.02.

CHAPTER 6

RESTORATIVE MAINTENANCE POLICIES

6.1 Introduction

It has been explained in chapter 1 that restorative maintenance policies are applicable to equipments which do not fail in the usual sense, but their performance deteriorates over a period of time. These equipments may be operational over a long time before their ultimate replacement can be contemplated. It has been mentioned also that in order to maintain a reasonable performance for these equipments, some forms of maintenance actions are necessary, eg., cleaning, repairing or replacements of some components. The principal decision associated with this kind of policy is the interval between overhauls which return the equipment to the reasonable performance level. It is possible that when the equipment is aged, it cannot be restored to its original condition, because gradual deterioration sets in and at some point of time, it will have to be replaced completely.

If the performance of the equipment can be related to operating time, its behaviour over a period of time can be predicted very conveniently. Then, subject to operational constraints, if any, a cost-optimal overhaul time can be determined. It is thought that because of its wide applicability, the Weibull distribution will be more convenient than any other distribution, to demonstrate the performance of an equipment subject to restorative maintenance. As mentioned in chapter 3, the cumulative distribution function F(t) for this distribtuion is given by:

$$F(t) = 1 - \exp\left(-\left({}^{t}/\eta\right)^{\beta_w}\right) \qquad (6.1.1)$$

where β_w is the shape parameter and η is the scale parameter.

In this chapter, a novel fouling model in terms of the Weibull distribution is given. Later, a case study concerning an optimal strategy for cleaning a set of four condensers subject to rapid fouling by salt water, is briefly described.

6.2. Correlation of Time Dependent Fouling Resistance by the Weibull Distribution

The fouling resistance R over a period of time t, correlated by the two-parameter Weibull model, can be postulated as:

$$R = R_\infty \left(1 - \exp - \left({}^{t}/\eta\right)^{\beta_w}\right. \qquad (6.2.1)$$

where R_∞ is the ultimate resistance which would be obtained after a very long time. When the shape parameter β_w is 1, equation (6.2.1) becomes the negative exponential relationship which has been suggested previously (58).

By differentiating equation (6.2.1), the rate of fouling can be given by:

$$\frac{dR}{dt} = (R_\infty - R) \left\{\frac{\beta_w}{\eta}\right\} \left\{\frac{t}{\eta}\right\}^{\beta_w - 1} \qquad (6.2.2.)$$

Now the onset of fouling can be conceived as the silt particles filling the cavities between the surface perturbations in the tube wall. These surface irregularities would be supplemented by colonies of marine organisms (algae) adhering to the wall. The rate of growth of the colonies should depend on their concentration in the water, with a negative exponential term containing an inhibition factor and the water temperature. It is thought that all these factors are instrumental in determining the shape of the fouling resistance curve. So, the equation for β_w in dimensionless form can be postulated as:

$$\beta_w = \frac{d_s}{\varepsilon_w} \left\{1 - \frac{\varepsilon_w}{\varepsilon_a} p_a \exp\left(-\frac{I}{T}\right)\right\} \qquad (6.2.3)$$

where ε_w and ε_a are the roughness of the wall and algae, respectively; d_s is the diameter of the silt particles; p_a is the concentration of algae expressed as parts per million (ppm) by weight; I is an inhibition factor and T is the absolute temperature of the wall. The inhibition factor I would be increased by chemical treatment of the cooling water with a biocide and by a deficiency of nutrients and oxygen.

Most of the models in the literature (57,58) have been postulated on the basis that the rate of fouling is the result of the difference between the rate of deposition of silt and its rate of erosion from the deposit. It is conceivable that the deposit should start with a large value of coherence, which may diminish as the layer gets deeper; but may attain a higher, constant value independent of depth, if colonies of algae are intermingled with the silt deposit. The value of coherence at dynamic equilibrium can be considered as an effective kinematic viscosity μ_d.

$$\mu_d = \tau \delta_\infty / V = \tau K R_\infty / V \qquad (6.2.4)$$

In this equation, τ is the shear stress at the surface of the deposit; V is the superficial velocity of water and the ultimate depth of deposit is replaced by the ultimate resistance R_∞ and the thermal conductivity K of the deposit. The shear stress τ is related to the rate of flow by means of the Fanning friction factor f.

$$\tau = 2f\rho_m V^2 \qquad (6.2.5)$$

ρ_m is the bulk density of water and silt. Combining equations (6.2.4) and (6.2.5) gives a relationship between R_∞ and operating parameters.

$$R_\infty = \mu_d / 2fV\rho_m K \qquad (6.2.6)$$

Considering that the rate of increase of R due to deposition is proportional to R_∞ while the rate of erosion is proportional to R, the deposition rate may be deduced from equation (6.2.2)

$$\left(\frac{d\delta}{dt}\right)_D = \frac{KR_\infty \beta_w}{\eta} \left\{\frac{t}{\eta}\right\}^{\beta_w - 1} \qquad (6.2.7)$$

When β_w is 1, the deposition rate is independent of time and equal to the initial rate of increase of R. When β_w is not equal to 1, equation (6.2.7) shows that the rate of growth of the fouling deposit depends upon β_w.

If silt is transported to the surface of the deposit by eddy diffusion only, the mass transfer coefficient is related to the Fanning friction factor as follows:

$$\frac{k}{V} = 0.023\ Re^{-0.2} = \frac{f}{2} \qquad (6.2.8)$$

where k is the mass transfer coefficient of silt to the deposit layer and Re is the Reynolds number. In order to allow for the time dependence of the deposition rate, the mass transfer rate is scaled by $(0.582\delta/(\delta_\infty-\delta))^{\beta_w-1}$. Furthermore, a numerical factor of 8.64×10^4 s/day is to be included when V is measured in m/s and t in days. To make the scale parameter η similar to experimental values, deposition rate is scaled by a factor $(\delta_\infty/D)^{1.5}$, where D is the inner diameter of the tubes. So, the mass flux rate of silt may be predicted by:

$$\rho_s \left\{\frac{\partial \delta}{\partial t}\right\}_D = 8.64 \times 10^4 k C_s \left\{\frac{\delta_\infty}{D}\right\}^{1.5} \left\{\frac{0.582\,\delta}{\delta_\infty - \delta}\right\}^{\beta_w - 1} \qquad (6.2.9)$$

where C_s is the mass of silt per unit volume of water and ρ_s is the bulk density of the silt deposit. It is to be noted that the function of δ in equation (6.2.9) meets all the criteria of the deposition rate variations with different values of β_w. The inclusion of factor 0.582 also ensures that for all values of β_w, $\delta = 0.632\delta_\infty$ when $t = \eta$, required by the Weibull distribution.

It is assumed that the rates predicted by equations (6.2.7) and (6.2.9) become equal at $t = \eta$. Thus, substituting $\delta = KR$ in equation (6.2.9) and equating the rate of increase of R due to the deposition at $t = \eta$, with k coming from equation (6.2.8), η can be given by:

$$\eta = \frac{\beta_w D}{4.32 \times 10^4 f V} \left\{\frac{\rho_s}{C_s}\right\} \left\{\frac{D}{KR_\infty}\right\}^{0.5} \qquad (6.2.10)$$

Equations (6.2.1), (6.2.3) and (6.2.10) completely describe the fouling model in terms of the Weibull distribution.

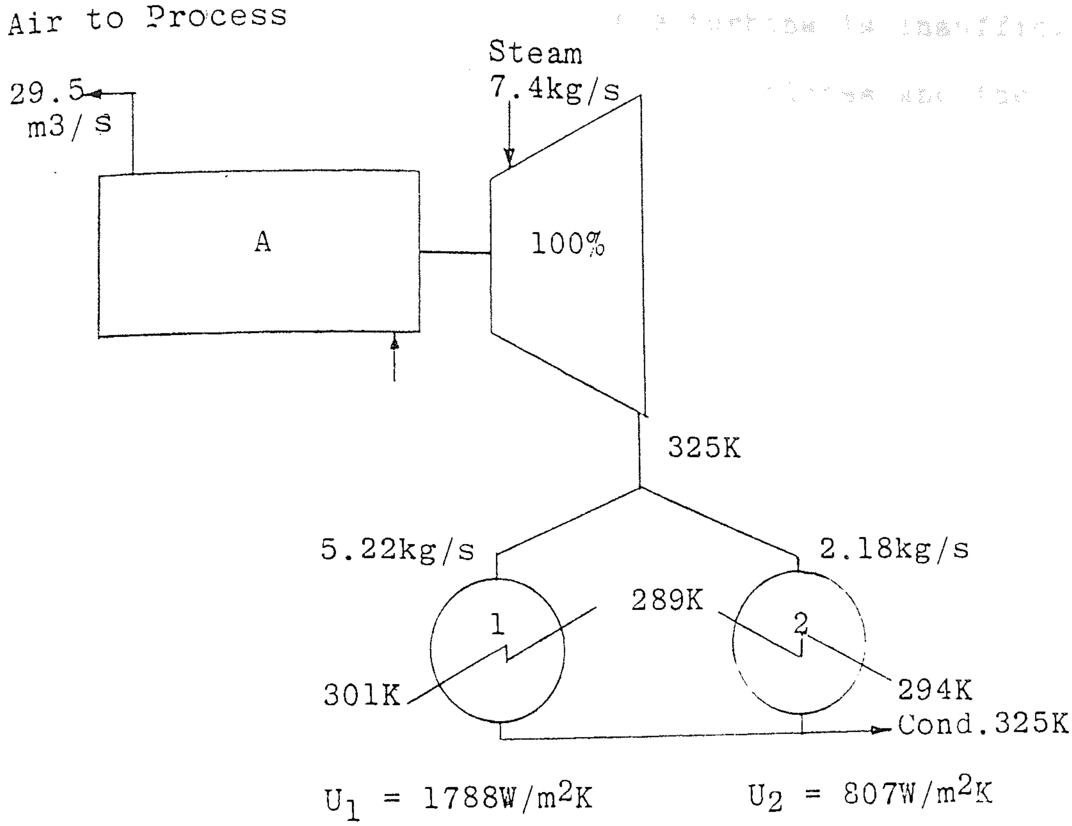
6.3 Case Study: Strategy for Cleaning a Set of Four Condensers Subject to Rapid Fouling by Saltwater

6.3.1 Cleaning Strategy

The case study which is briefly described here, is not a direct application of the Weibull fouling model postulated in the previous section. The data was collected by others (88); so many operating parameters, which would have been of interest in determining the fouling model parameters, were not measured. Nevertheless, the study demonstrates the effects of various model parameters on the cleaning strategy. In this section, a brief discussion of the cleaning strategy is given. The detailed discussion can be found in (89).

The company operate a process which requires about 52 m^3/s of air, this is provided by two turbo-blowers driven by steam turbines at 3480 revolutions per minute. The steam from each turbine is condensed by two identical condensers in parallel. The condensers are cooled by estuary water, and require tube-side cleaning every one or two months. Figure 6.1 demonstrates the arrangement.

With the condensers becoming more fouled, the inlet pressure of steam to the turbines is increased, in order to maintain the turbine speed and to deliver the requisite air. But the steam pressure cannot be increased beyond 650kN/m^2. If the pressure exceeds this limit, the turbine blades and thrust bearings will be damaged. However, the more common operating constraint



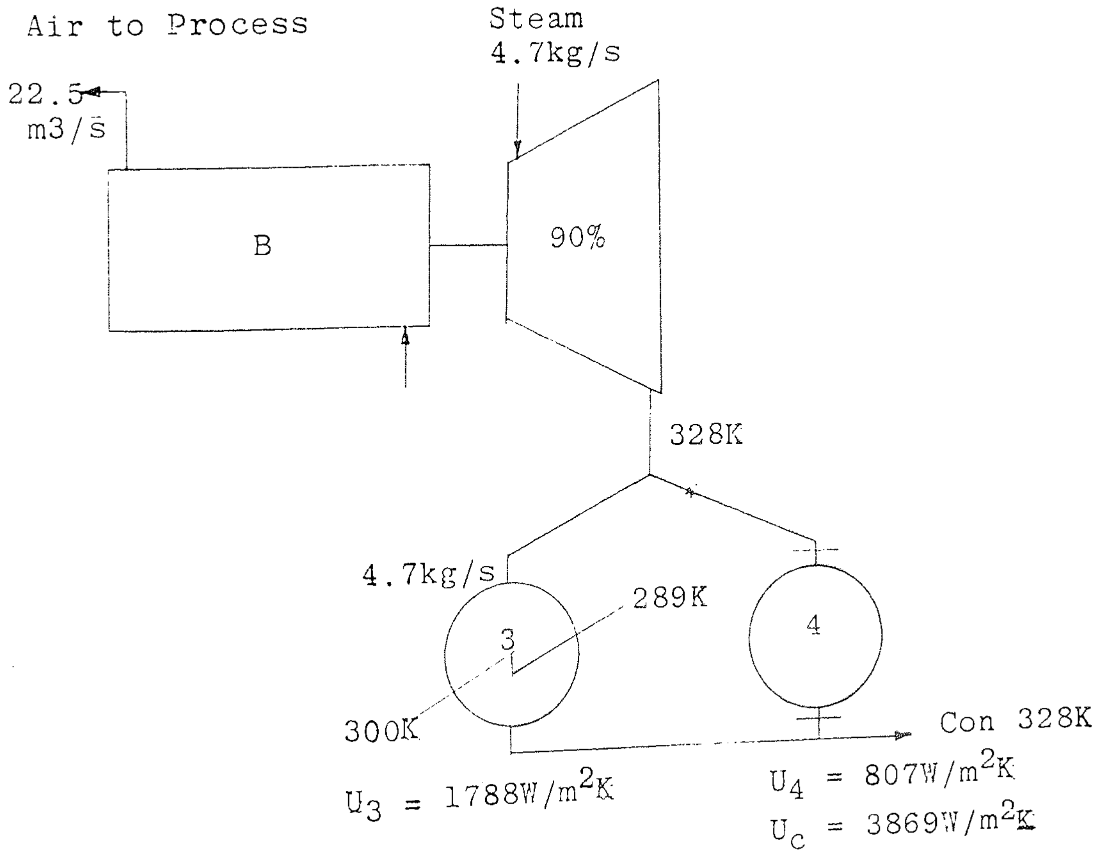


Fig 6.1 Schematic diagram of the condenser arrangements. Numbers associated with different process variables are valid only when condenser 4 is being serviced.

is the condensate temperature. At about 325K, the pressure drop across the last stage of the turbine is insufficient to maintain the boundary layer on the blades and the stage effectively stalls. This causes the turbine to slow down; its speed is typically 90% when the condenser hotwell temperature is 328K.

Under the above mentioned operating constraints, the company's cleaning strategy was to wait until the hotwell temperature reached 325K and then to isolate and clean each of the four condensers on consecutive days. Because all four condensers were fouled and the hotwell temperature was at the maximum value for 100% turbine speed, the steam supply had to be reduced during the four day cleaning operation. The resultant loss of profit from the plant was estimated to be £12,000. The cleaning costs were about £100 per condenser. So, there was considerable financial incentive to develop a cleaning strategy which did not involve reducing the air flow rate to the plant.

The cleaning strategy proposed, enabled three condensers to handle the steam, equivalent to a total air flow rate of $52m^3/s$. At maximum speed, either blower can deliver $29.5m^3/s$; so that the blower with only one condenser in service, must deliver $22.5m^3/s$. This blower can be operated at 90% speed with the condenser hotwell temperature at 328K. This condenser must condense 4.7kg/s of steam. The two condensers serving the blower which is delivering $29.5m^3/s$ of air, will be condensing jointly 7.4kg/s of steam at 325K.

Numbering the condensers 1,2,3,4 and assuming that 1 and 2 are connected in parallel to blower A, with condensers 3 and 4 serving blower B. For the situation when condenser 4 is isolated for cleaning, 3 will be condensing 4.7kg/s and B will be delivering $22.5m^3/s$. Condensers 1 and 2 will be condensing 7.4kg/s, which is less than twice 4.7kg/s; so condenser 3 has a greater heat load than the average of 1 and 2. Figure 6.1 shows the relevant numbers for each equipment.

The strategy which maximises the operating cycle but minimises the number of calls on the cleaning crew, is to clean one condenser from each pair consecutively. Thus if 2 and 4 are due for cleaning, 1 and 3 are at the half-cycle stage. Condensers 1 and 3 are said to have attained the 'limiting fouling resistance'. The overall cycle time will be twice as long as it takes to attain limiting fouling.

The 'maximum fouling' which is permitted can be calculated by considering a pair of condensers, one at limiting fouling and the other at maximum fouling, able to condense 7.4kg/s of steam with a hotwell temperature of 325K.

However, an optimal fouling model will be the one, which allows all four condensers to attain the appropriate critical values of fouling, simultaneously.

6.3.2 Effects of Various Model Parameters on the Cleaning Strategy

Because of the unavailability of many operating parameters, the actual parameters of the Weibull fouling model could not be directly determined. However, a number of curves have been fitted through the data to see the effects of various model parameters. In Figure 6.2, a Weibull model, curve A, has been fitted through the data except for days 28 to 30. Conversely, curve B has been fitted through the data for days 28 to 35, but ignores the earlier data. The values of R which are plotted on Figure 6.2, were calculated from the reported average overall coefficients and a clean overall coefficient of 3869 W/m^2K.

The observations that can be made about the curves are as follows:

(1) If curve A represents the growth of fouling, the limiting resistance (which has been calculated as $3 \times 10^{-4} m^2K/W$) would be attained in 21 days. With the staggered cleaning strategy, one condenser from each pair would be cleaned every 21 days and the overall cycle time for each condenser would be 42 days.

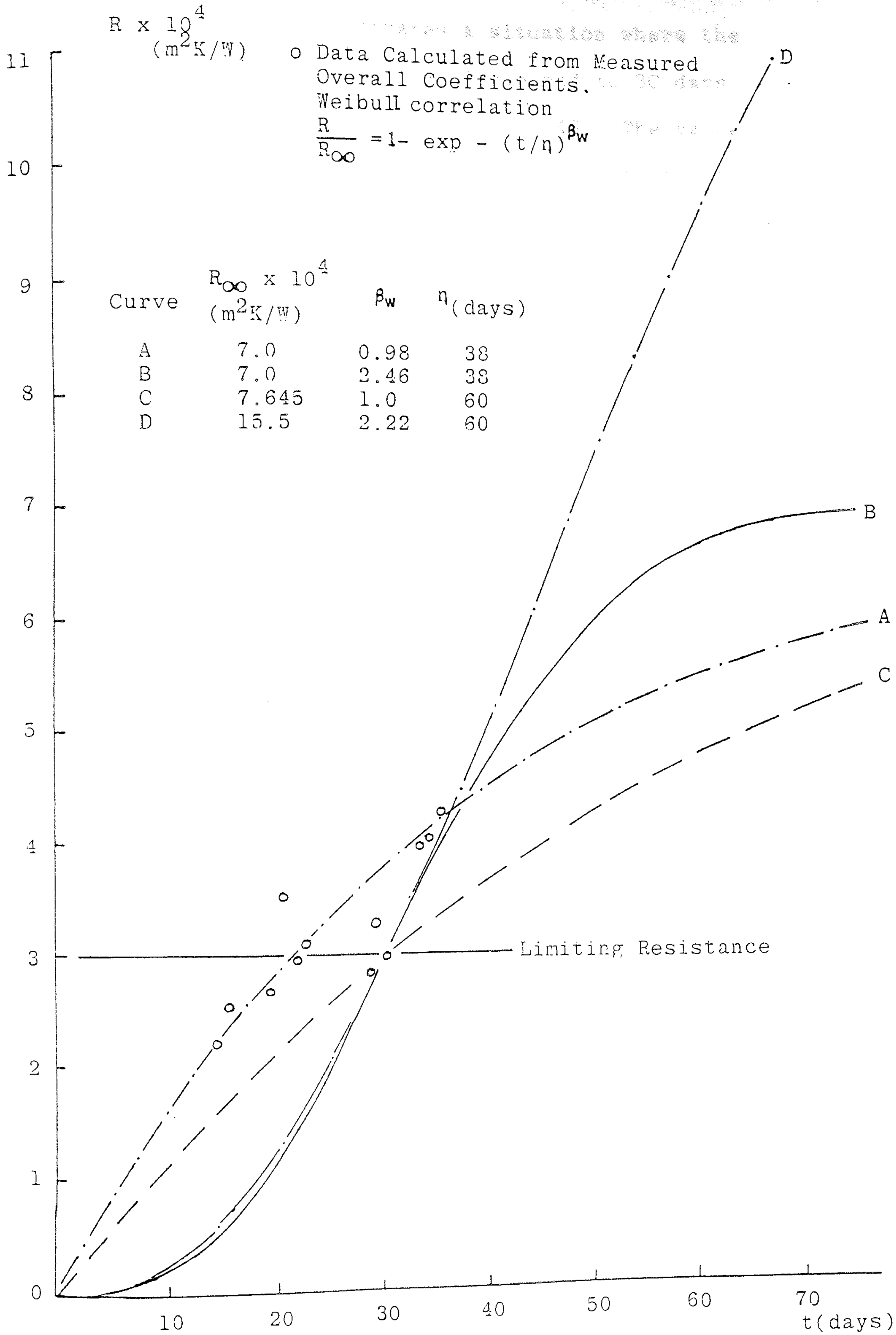


Fig 6.2 Illustration of Various Weibull Fouling Models (Condenser fouling by Saltwater). Limiting resistance is defined as the resistance at the half-time of the cleaning clyle.

(2) Curve B illustrates a situation where the half-cycle time has been increased to 30 days, by increasing β_w from 0.98 to 2.46. The value of the scale parameter η remains unaltered at 38 days and the ultimate fouling resistance is still $7 \times 10^{-4} m^2 K/W$. It will be seen that for curve B, the initial rate of fouling is much less than for curve A. This may be achieved by adding a biocide to the cooling water before it enters those condensers which have been cleaned recently, in order to delay the growth of marine organisms on the tubes. These organisms could provide the necessary roughness for silt to adhere.

(3) Curve D represents an optimal fouling model, because the condensers would attain the maximum permitted fouling resistance ($9.8 \times 10^{-4} m^2 K/W$) just before they were cleaned at 60 days. Also, the ultimate fouling resistance has been allowed to rise to $15.5 \times 10^{-4} m^2 K/W$. The scale factor η has been put equal to the overall cycle time for curve D. This means that the maximum fouling resistance is 63.2% of the ultimate fouling resistance.

(4) Curve C illustrates the exponential model of fouling, where β_w is 1. The scale parameter is equal to the overall cycle time, as with curve D.

But the fouling resistance just before cleaning is only $4.8 \times 10^{-4} m^2 K/W$.

(5) Curves B, C and D all have the same length of half-cycle (30 days). But, just before cleaning, the fouling resistance is higher for curve D than for curve B, which in turn, is higher than for curve C.

On Figure 6.3, curve E is the best fit by the least squares method, of a Weibull model, through all the data. For this curve, the scale parameter η was found to be equal to the half-cycle time, at which limiting fouling is achieved. The ultimate fouling resistance was found to be $4.65 \times 10^{-4} m^2 K/W$ and β_W is 1.19.

Curves F and G illustrate very clearly that altering β_W will have no effect on the half-cycle time, when this is equal to η. However, the larger the value of β_W the higher will be R at cleaning; thus the values of R/R_∞ at 42 days for the various curves are: curve F 86.5%, curve E 89.8%, curve G 98.2%.

6.4 Discussion

The detailed numerical figures associated with this case study of fouling models have been given in (89). It is true that because of the unavailability of many operating parameters, the Weibull fouling model

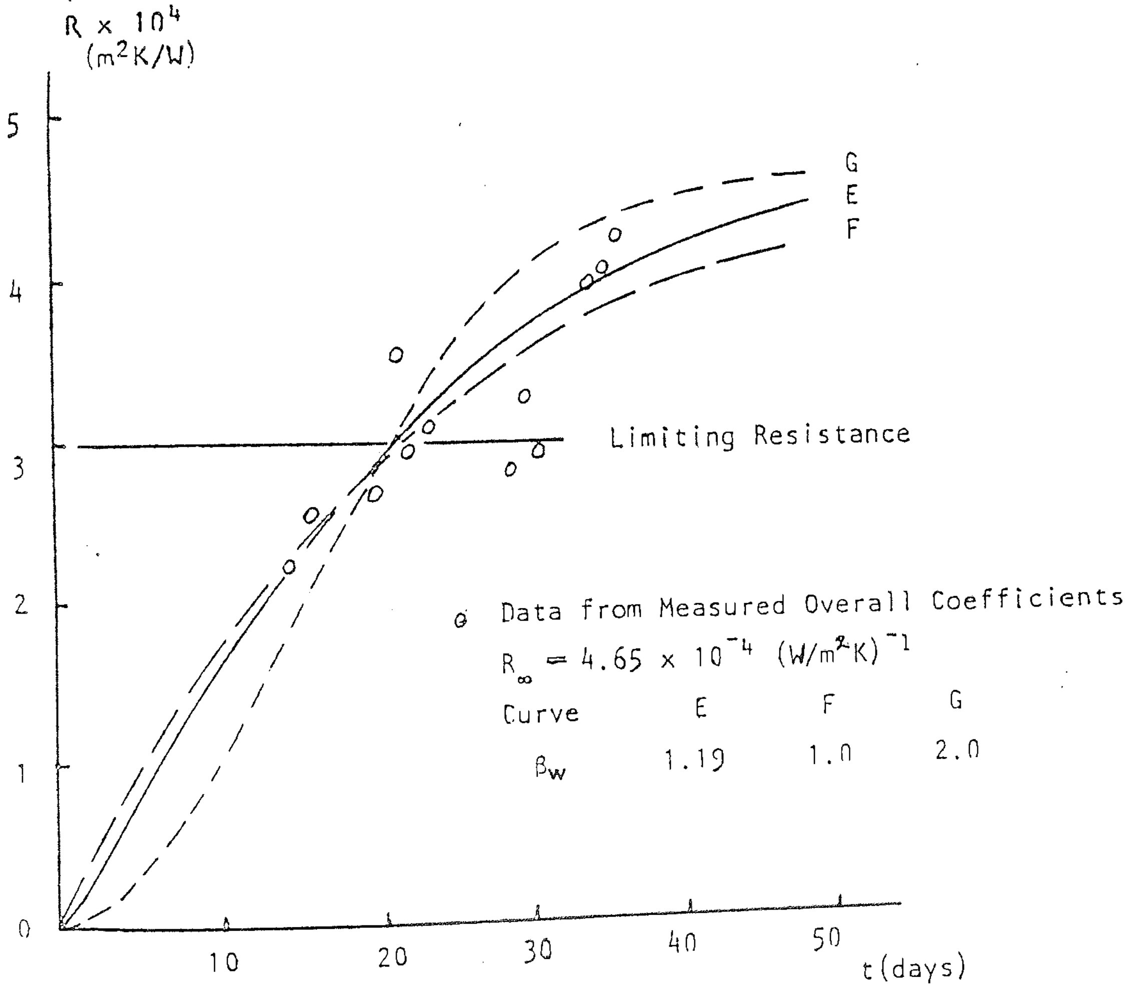


Fig. 6.3 Fouling Models with The Scale Parameter Equal to the Half-Cycle Time

put forward in this chapter, could not be compared with other fouling models published in the literature. Nevertheless, this case study demonstrates the fact that by developing a predictive model of the equipment behaviour, better restorative maintenance planning can be achieved. In the event of the whole plant being shutdown due to some other equipment(s) in the plant, the predictive model allows the maintenance planner to consider the possibility of an opportunistic restorative maintenance of an equipment classified in this category.

CHAPTER 7

CONDITION BASED MAINTENANCE

7.1 Introduction

In the introductory chapter, the general conditions of applying condition-based maintenance and its categories have been discussed. It has been mentioned also that condition monitoring under appropriate circumstances, can be part of a total maintenance strategy.

Owing to the diversity, in nature and function, of the equipments used in a plant or industry, the range of monitoring methods in common use is very wide indeed. But the basic principle of all the techniques involves a systematic application of conventional methods of fault diagnosis. The choice of a monitoring technique and the frequency of its application should depend upon the plant operating experience, historical data, analysis of the equipment and process conditions to judge how an item might fail and the time relationship of failure. In section 7.2, some currently available monitoring techniques are reviewed.

In section 7.3, a technique for monitoring the condition of a pressure relief valve spring is presented. Pressure relief valves are often used to protect process vessels from overpressure, which can cause yield or catastrophic failure.

Relief valves are safety equipment which can suffer unrevealed failure due to a variety of causes. The failure needs to be detected before there is a high pressure excursion, requiring the valve to open and limit the pressure rise in the vessel. The maintenance strategy for pressure relief valves is one of periodic tests and overhauls. But it seems, there is no accepted code of practice to determine a test and overhaul interval, which is found to vary from six months to two years. Furthermore, the testing procedure is imprecise in many companies.

In most circumstances, the workshop testing of pressure relief valves is carried out with air from compressed air bottles. The set pressure of a valve is determined by slowly increasing the pressure until the valve just 'pops' open; ie. it just lifts but no more. The onset of lift is detected either by listening or by immersing the outlet nozzle in soap water and recording the pressure which produces about two bubbles per minute. This subjective criterion of set pressure is necessary because most valves leak near the set pressure. If the valve is found to be in the 'seized open' or the 'seized closed' condition, it is overhauled completely and the necessary maintenance actions are taken before re-setting and re-testing.

However, from the failure characteristics of pressure relief valves, which are discussed later, the major cause of the valve failure is variations in the force provided by the valve spring. In section 7.3, the design of a gauge to detect the spring compression and thereby the set pressure, is discussed. It should be noted that there are few on-line surveillance methods for relief valves at present. The development and introduction of this gauge in industries could facilitate periodic on-line condition monitoring of valve springs.

7.2 A Review of Some Currently Available Condition Monitoring Techniques

There is a wide range of monitoring techniques available, which vary in nature and price and in the skills required for their application. Kelly and Harris (5) and Sims (6) have listed many monitoring techniques with skill and cost indicators.

Some low-cost (less than £1000) monitoring systems have wide applications on chemical plants; these systems can be applied easily by technical personnel without special training. A few examples are:

(i) Portable vibration monitoring equipments - these can help determine the general condition of most common machinery, eg. balance, alignment, bearing failure, gear defects, rolling element bearing defects, etc.

(ii) Ultrasonic thickness testing equipments - these can be used for process vessels subject to thinning by corrosion. The equipments can be used for pipeworks also.

(iii) Interpretation of operating records - in a number of circumstances, proper interpretation of operating records may help predict an impending failure. For example, efficiency indication from plant records of temperature, pressure, flow, etc. can be used to monitor wear of valve gear, impellers, etc. Another example is, when fouling of heat transfer surfaces needs to be monitored, calculation of heat transfer coefficients from flow rates and temperatures can be helpful.

(iv) Oil sample analysis - in case of monitoring wear of rubbing or bearing surfaces, a common technique is the examination of lubricating oil for wear particle contamination.

If an industrial organization can sustain the cost of a specialist service, they can apply higher technology and improved monitoring systems. Some examples may be:

(i) Full frequency vibration analysis.

(ii) Radiography for cracks or flaws in metal, stress corrosion cracking, local pitting, etc.

(iii) Ultrasonic or eddy current for crack detection.

(iv) Thermographic imaging for insulation and refractory defects.

At the high-cost end of the range come acoustic emission testing, radioisotope measurement techniques, etc. These are some of the most sophisticated equipments available for non-destructive testing and their use for condition monitoring is still novel. Few process companies can afford to develop these facilities in-house. But there are specialist services provided by a few contractors (6).

7.3 Case Study: Condition Monitoring of Pressure Relief Valve Spring

7.3.1 Pressure Relief Valve Problems

Regarding maintenance, pressure relief valve problems (90) fall into two categories - (i) plant problems and (ii) workshop problems. Generally, plant problems occur in three ways:

(a) fail to lift on demand, thus causing over-pressuring of equipment.

(b) lift spuriously, chatter, fail to reseat, thus causing production loss.

(c) leak so that process material is lost.

The first type of problem creates very significant potential hazard. The second and third types of problems, although potentially less hazardous, cause economic loss and potential environmental pollution.

The principal causes of a pressure relief valve lifting light are either incorrect set pressure or the relaxation or failure of the valve spring. The latter may arise from using the wrong type of spring or spring material, or from thermal damage to the spring by hot fluids being released through the valve.

Failure to reseat after relieving upstream pressure may be caused also by spring overheating. The other causes include deposition of solid residue on the seat or plug, groves in the seat, etc. If the valve is used to perform dirty duty, deposition of residue is likely. Failure of the valve to lift at the expected pressure may be caused either by sticking of the spindle, valve blockage or by too strong a spring.

In the workshop, some relief valve problems arise in the following ways:

(1) inadequate spring data - this causes incorrect selection of the proper spring or spring material for a particular duty.

(2) imprecise determination of lift pressure.

(3) problems in determining the correct set pressure.

Figure 7.1 (90) summarizes the major operating and workshop problems associated with pressure relief valves.

7.3.2 Checking the Effectiveness of Bubble Tests to Determine Set Pressure

A simple laboratory experiment was performed to

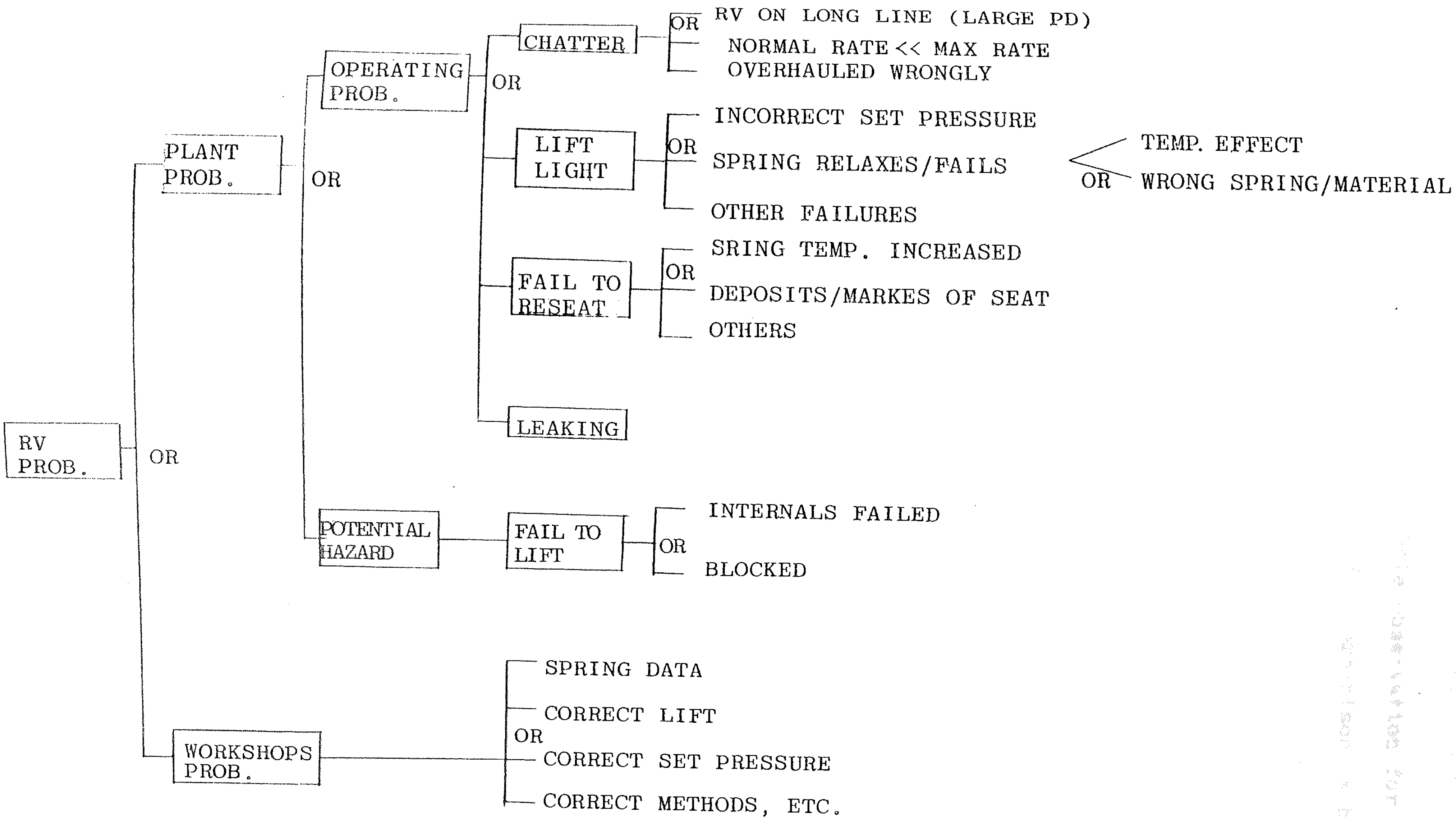
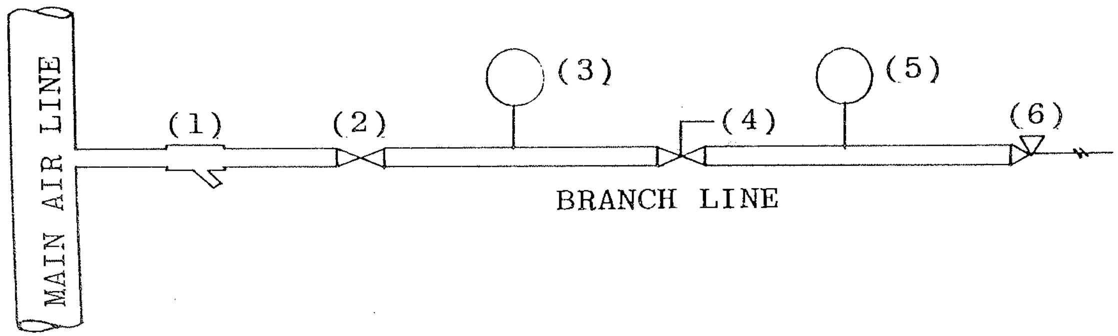


Fig 7.1 Potential Causes of Relief Valve Problems

determine the effectiveness of bubble observation for determining relief valve set pressure. For comparison, a hot wire anemometer was used to detect the flow of air from the valve.

Figure 7.2 shows the experimental arrangement. A branch line of about 6 feet length was taken from the compressed air main. At the end of the branch, a relief valve was fitted. The other instruments were: a strainer for intercepting large dirt particles, a control valve for controlling air flow and thereby the line pressure, a pressure gauge for measuring the main pressure, a pressure reducing valve for controlling the pressure upstream of the relief valve and a pressure gauge 3 feet upstream of the relief valve. Pressure drop between this gauge and the relief valve was negligible. When the anemometer was used to detect air flow, the relief valve discharged to the atmosphere; otherwise, the relief valve discharged via a short plastic hose immersed to a depth of 1 inch under water.

Figures 7.3 and 7.4 show the variations in the number of bubbles appearing and the hot wire anemometer reading in feet per minute (fpm), respectively, with the variation in line pressure. For both cases, readings were taken, first increasing the pressure and then decreasing the pressure. It was found that flowrate was somewhat higher while decreasing the pressure than when the pressure was being increased. This 'hysteresis' behaviour is to be expected for compression and relaxation of the spring. However, the average flowrate



(1) Strainer

(2) Control valve

(3) Pressure gauge

(4) Pressure reducing valve

(5) Pressure gauge

(6) Relief valve

Fig 7.2 Diagram Showing the Experimental Arrangement for Determining Set Pressure of a Pressure Relief Valve.

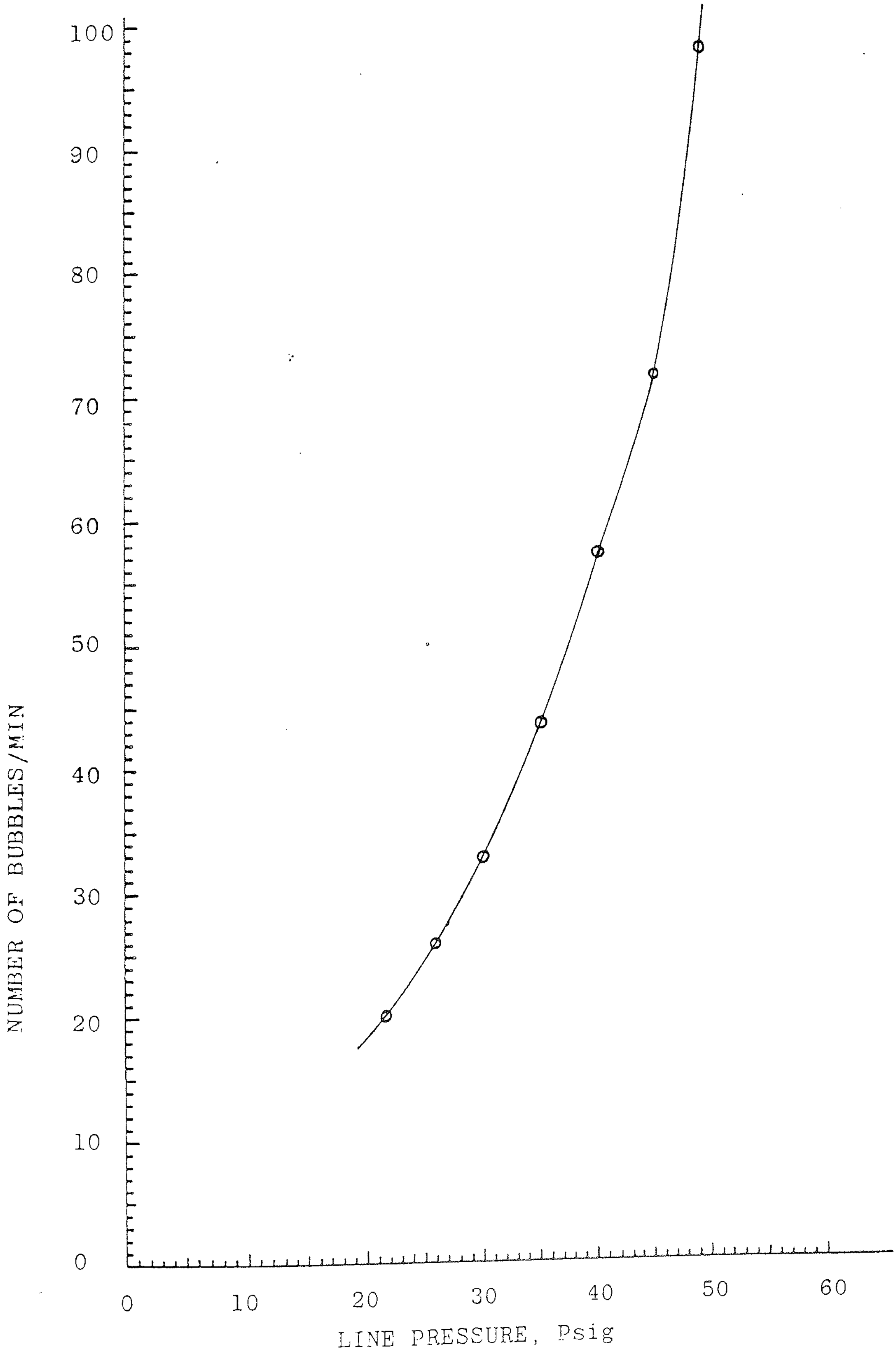


Fig 7.3 Plot of Number of Bubbles/min vs. Operating Line Pressure. The graph shows the average number of bubbles that appeared at a particular line pressure. (Determination of set pressure of a pressure relief valve).

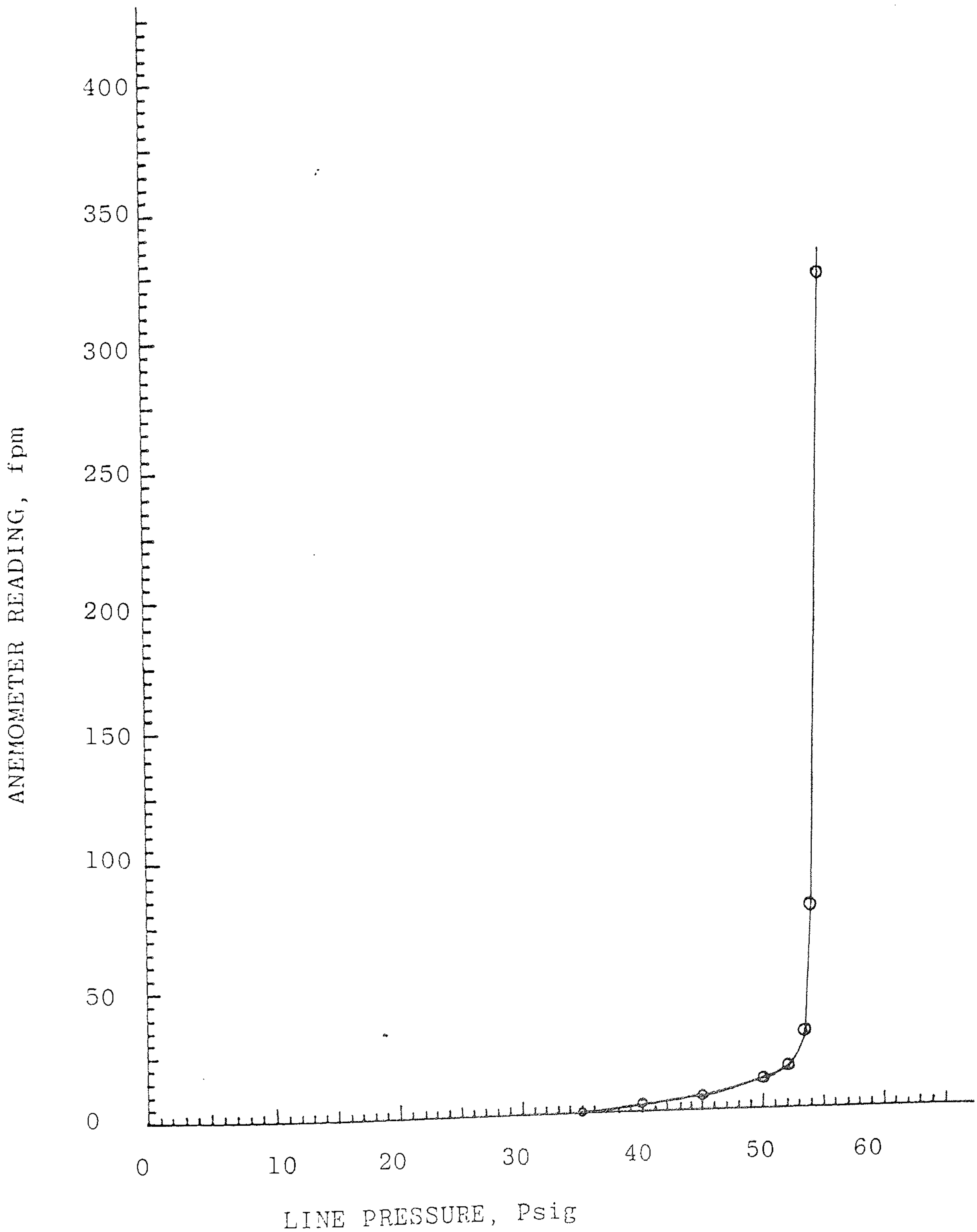


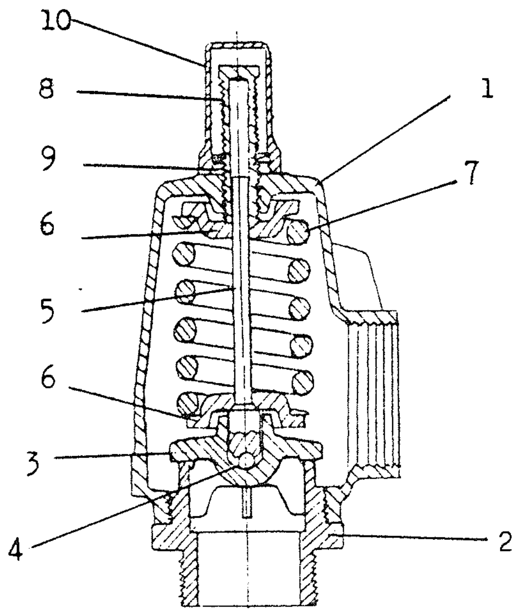
Fig 7.4 Plot of Hot wire Anemometer Reading vs. Line Pressure. The graph shows the air flow reading of anemometer at a particular line pressure. (Determination of set pressure of a pressure relief valve).

at each line pressure is shown on Figures 7.3 and 7.4, because the main objective of this experiment was to show the effectiveness of the bubble test. From the figures, it can be seen that the detection of air flow by the anemometer is more sensitive than counting the number of bubbles; because, in the former case, the curve is much steeper.

7.3.3 Design of a Gauge for Monitoring the Condition of a Relief Valve Spring and for Determining Set Pressure

In the previous section, determination of set pressure of a relief valve by hot wire anemometer has been described. The method can be applied in workshop tests and also for on-line condition monitoring if the gas or vapour released is not hazardous. Otherwise, the relief valve will be piped into a vent header system and flow from the valve cannot be measured easily. To overcome this problem, a gauge has been designed to determine the set pressure and the spring condition, both on-line and off-line. The main consideration in the design was that the gauge would not disturb the setting of the relief valve, nor would it open the valve under test.

The gauge has been designed for a particular type of pressure relief valve, the schematic diagram of which is given in Figure 7.5. It was thought that fitting the gauge between the lower spring plate and the valve would serve the purpose well. Figure 7.6 shows the schematic diagrams of the gauge with relevant dimensions on the drawings.

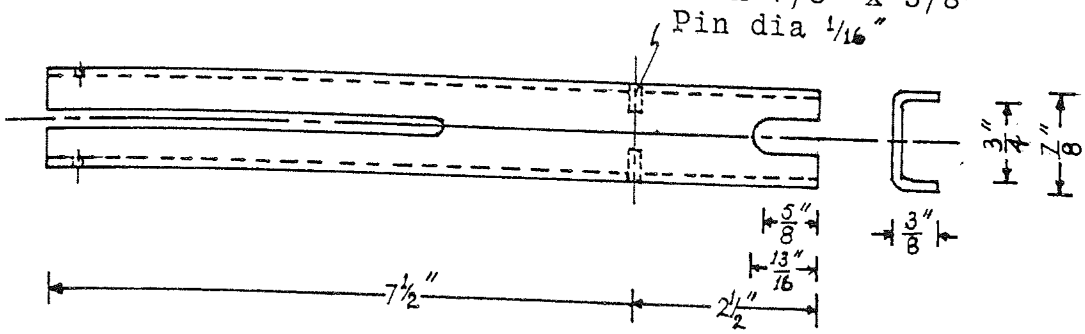


1. Body
2. Valve Holder
3. Valve
4. Ball
5. Spindle
6. Spring plates
7. Spring
8. Adjusting screw
9. Locknut
10. Cap

Fig 7.5 Schematic diagram of a pressure relief valve for which set pressure measuring gauge has been designed.

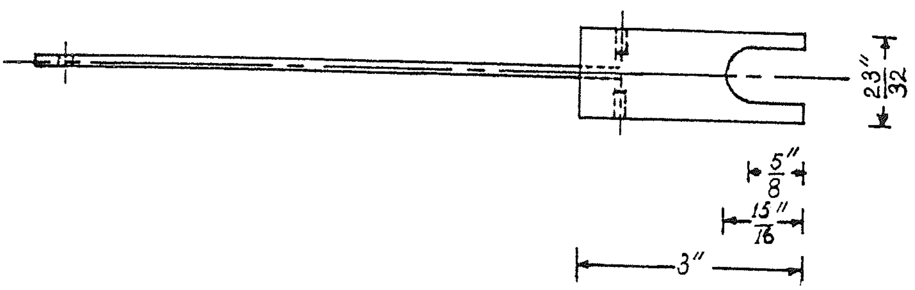
(a) Upper Channel;

- Steel Plate; Dimensions - 10" x 7/8" x 3/8"



(b) Lower Plate;

- Steel Plate; Dimensions - 3" x 23/32" x 3/16"



(c) Joined view of the gauge

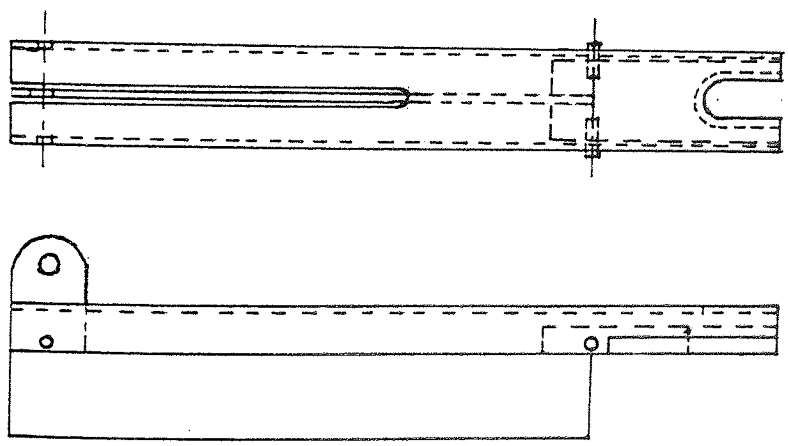


Fig 7.6 Design Drawings of the Gauge for Measuring the Set Pressure of a Relief Valve (Drawings not to the scale).

The working principle of the gauge is similar to that of a lever as explained by Figure 7.7 below, F_1 is the

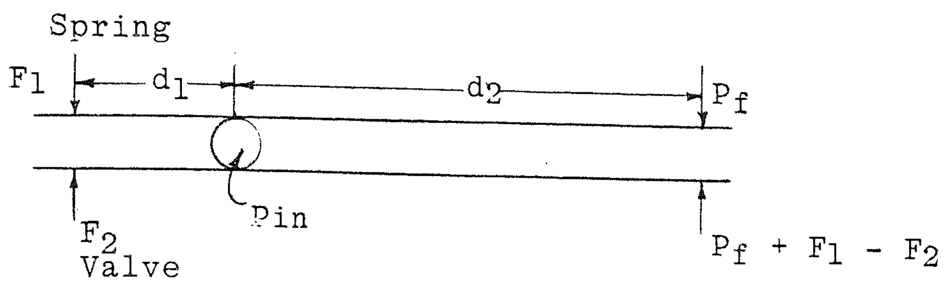


Fig 7.7 Illustration of the working principle of relief valve gauge.

force exerted by the spring on the spring plate, F_2 is the upward thrust by the fluid on the valve plate and P_f is the weight to be suspended. To determine the set pressure, only forces F_1 and P_f are relevant; because the counter force $P_f + F_1 - F_2$ is required only to keep the lower plate of the gauge in a horizontal position (Fig 7.6). From Figure 7.7, it can be seen that F_1 is given by the following relationship:

$$F_1 = \frac{d_2}{d_1} P_f \qquad (7.3.3.1)$$

Denoting by P_s the set pressure and by A_s the cross-sectional area of the valve, F_1 can be given as:

$$F_1 = P_s \times A_s \qquad (7.3.3.2)$$

From equations (7.3.3.1) and (7.3.3.2), P_s can be determined as:

$$P_s = \left(\frac{d_2}{d_1} \times \frac{1}{A_s} \right) P_f = RP_f \qquad (7.3.3.3)$$

R is the factor by which the suspended weight should be multiplied to calculate the set pressure. For the valve of this study, R is equal to 8.324.

The procedure to determine set pressure of the relief valve is to suspend that amount of weight which just lowers the upper plate of the gauge (Figure 7.6) from the horizontal line and then multiply this weight by the factor R. The method can be applicable to relief valves, in general, if the valves discharge to the atmosphere or if the outlet nozzles can be isolated in order to insert the gauge. Since the valves of different makes were not available, the method could not be tested generally. However, it is assumed that the basic principle will be valid for all makes of relief valve.

CHAPTER 8

APPLICATION OF MAINTENANCE POLICIES TO AN INERT GAS GENERATOR PLANT

8.1 Introduction

In this chapter, applications of all the maintenance policies developed in chapters 4, 5, 6 are given. The example chosen, is a typical process plant given by Lihou (91), with a slight modification. Figure 8.1 shows a schematic diagram of the plant.

In this plant, which produces inert gas as a service for the main process plant, both air and reagent gas are passed continuously into a catalytic reactor R1 where an oxidation reaction proceeds at 1300K and 1 bar absolute pressure. The products from the reactor are cooled to 350K in order to remove water vapour. The gas is then compressed to 8 bar for storage, from which the main plant is supplied as required. The inert gas must contain no oxygen and the residual reagent should be less than 0.1%.

The required flow rate of air is obtained by setting the flow controller FC. It receives an indication of the air flow in line L1 via the flow transmitter FTr1. Signals from the flow transmitters FTr1 and FTr2 go to the flow ratio controller FrC which adjusts the reagent flow by means of CV2, in order to achieve the required slightly-rich ratio of reagent to air.

Now, explosions can occur in the compressor due to heat of compression and in other processes using the inert gas, if the mixture contains residual oxygen. The two causes for an explosive mixture are lean feed mixture and

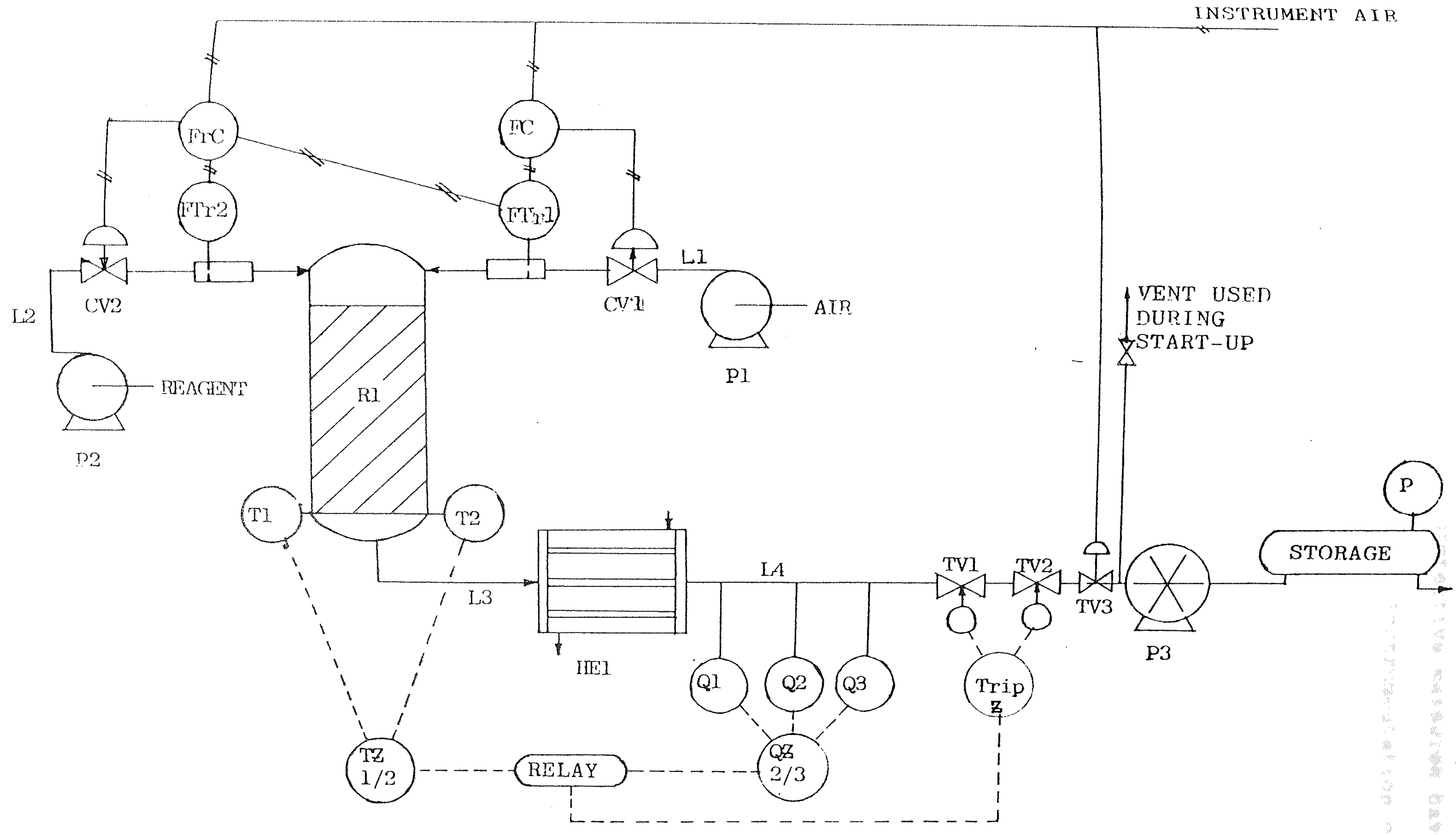


Fig 8.1 Process Flowsheet for an Inert Gas Generator Plant

incomplete reaction. So, two protective measures have been included as part of the safety instrumentation of the plant.

Firstly, identical thermocouples T1 and T2 are located near the reactor outlet to detect mal-function. The outlet temperature will be low if the feed mixture is lean or if there is incomplete reaction due to catalyst deactivation. Either T1 or T2 can operate the temperature switch TZ which, via the relay, will cause trip switch Z to close trip valves TV1 and TV2.

Secondly, the gas analysers Q1, Q2 and Q3 should detect oxygen in the product gas and send a signal to the analysis switch QZ. In order to avoid spurious trips, the logic for QZ is a 2 out of 3 majority vote system. QZ sends a signal through the relay to trip switch Z. The fault-tree analysis for oxygen escaping past TV1 and TV2 has been given by Lihou (91).

8.2 Classification of Equipment for Maintenance Planning

The maintenance policies for the equipments of the plant being considered, were developed on the basis of overall cost minimization. Since the equipments have varying characteristics, a single maintenance policy is inappropriate.

Therefore, the equipments have been divided into six classes, which take account of their function and the effect of their failure or maintenance on the operation and/or safety of the whole plant.

Class 1 Includes equipments whose failures will certainly cause shutdown of the generator plant. Apart from the breakdown maintenance (BM), there may or may not be preventive maintenance (PM), depending upon the cost-advantage. The equipments in this category are the blower (P1), the pump (P2), the compressor (P3), the reactor (R1) and the heat exchanger (HE1). It should be noted that by earlier definition, the maintenance of the reactor and the heat exchanger falls into the restorative maintenance category.

It has been considered that preventive maintenance of seals and bearings of P1, P2, P3 and replacement of the catalyst in R1 will be cost-effective. In addition to BM and PM of these equipments, opportunistic maintenance of unfailed equipments, during shutdown of the plant due to the failure or PM of another equipment, may be cost-effective also. The storage capacity is also included in this class; because when the storage is full, the generator plant will have to be shutdown. This gives an opportunity for the maintenance of all equipments if cost-advantageous.

Class 2 Includes those equipments whose failures will also cause shutdowns of the plant; but which are regularly tested and serviced to minimise breakdowns. The test intervals are determined to achieve target fractional dead times; opportunistic tests are also applicable. These components are the flow controller (FC), the flow ratio controller (FrC) and the control valves (CV1, CV2).

Class 3 - The relay and trip switch (Z) are in this category. These components are essential for the safety system to remain operational. Failures of any of these may or may not require shutdowns of the plant, depending upon a trade-off between the expected hazard cost and down-time cost. Like items of class 2, these are regularly tested to maintain high reliability and high operational availability.

Class 4 Includes safety components which have back-up systems, the failures of which may or may not require shutdown depending upon the cost balance. These items are the analyser switch (QZ) and temperature switch (TZ), which are tested regularly.

The concurrent failures of two gas analysers (2Q's) or two thermocouples (T1, T2) will leave the system with one safety system operational. So, this situation is included in class 4.

Class 5 - Gas analysers (Q1, Q2, Q3) and flow transmitters (FTr1, FTr2) are included in this class; because the failure of any of these items will not cause shutdown. Their maintenance policy is periodic on-line tests, to maintain high operational availability.

Class 6 - Trip valves (TV1, TV2, TV3) have been put into a separate category, because these can be tested only off-line, ie. during shutdown.

Figure 8.2 summarizes the maintenance actions with their relevant causes, resultant system state and system state logic.

8.3 Cost Modelling for Maintenance Actions

The maintenance cost minimization model, presented in this section, is applicable to any state of the plant when an opportunity for maintenance has been created. This opportunity may have been created by the failure of an

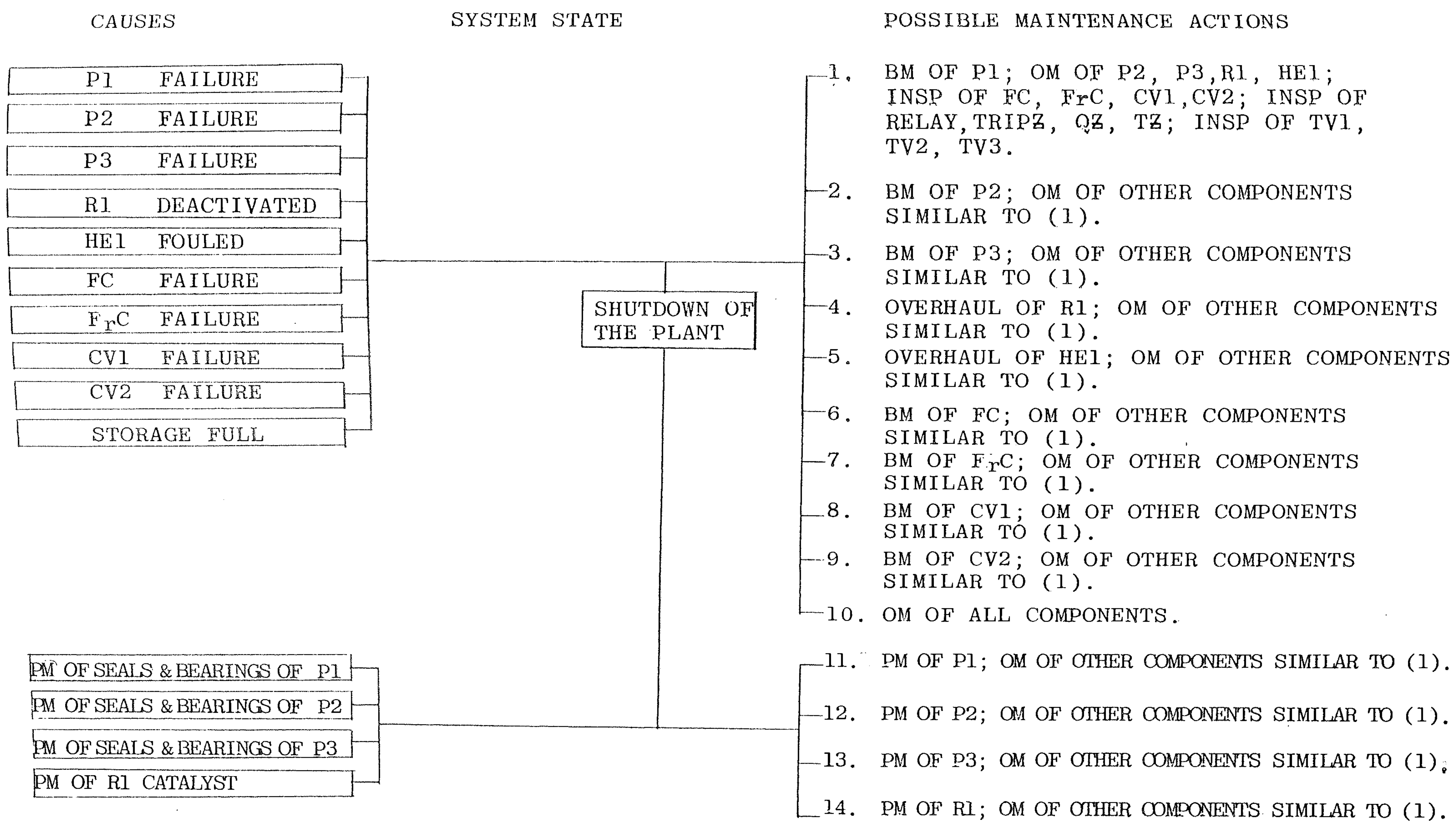


Fig 8.2 Possible Maintenance Actions for the Equipments of the Generator Plant (Fig 8.1), With Their Relevant Causes, Resultant System State and System State Logic.

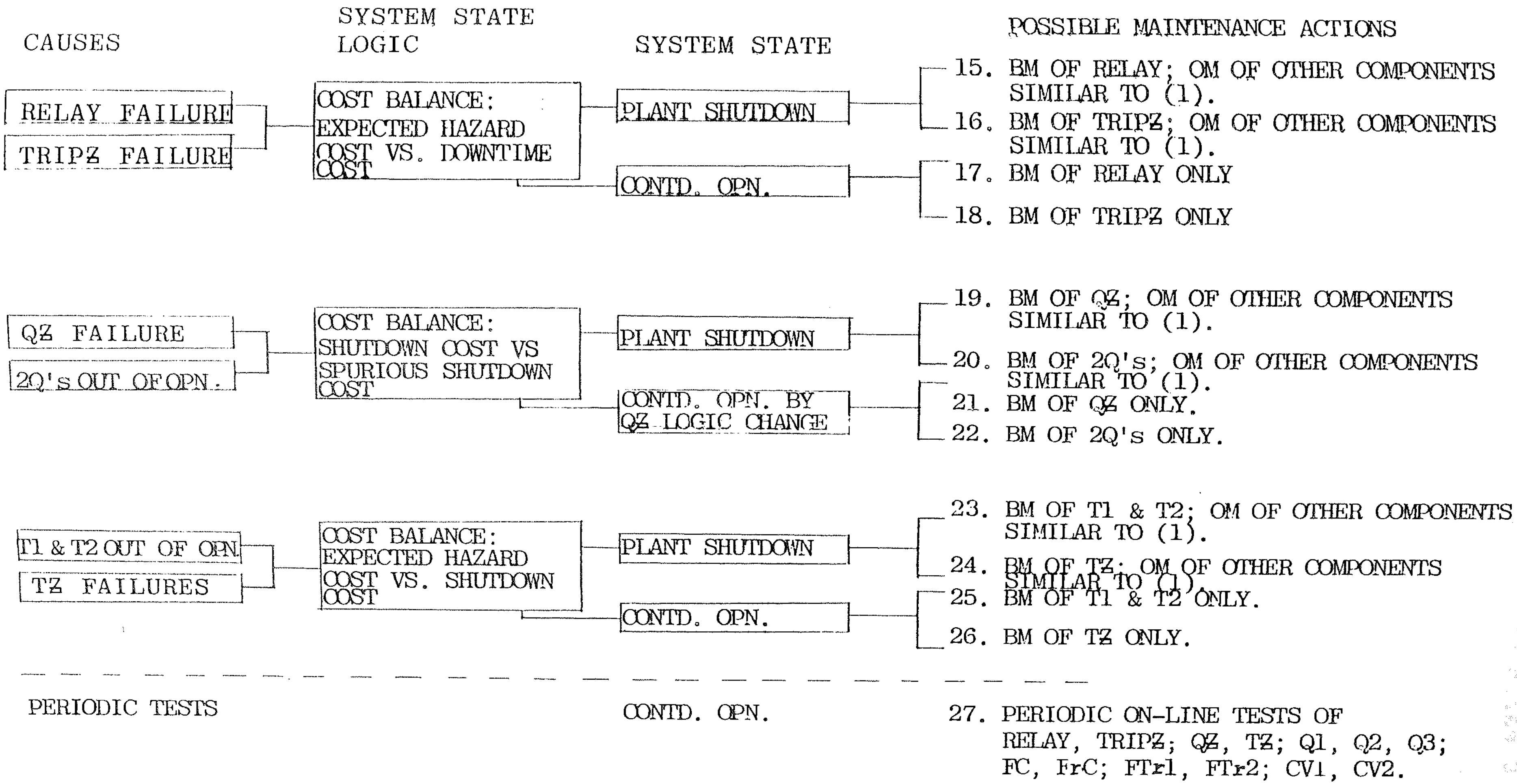


Fig 8.2 (Continued)

equipment or by the arrival of preventive maintenance time. In general, equipments from class 1 and class 2 will require shutdown of the plant upon their failures or PM; but for those from classes 3 and 4, only if cost-effective will shutdown be required.

The current cost of replacement/repair of an equipment alone can be given in the following way:

Total cost of replacement/repair (involving failed component only)

= Cost of replacing/repairing the failed component (including engineer's cost)

+ (Lost production cost for the generator plant during replacement/repair

+ Lost production cost for the main plant which requires inert gas from the generator

+ Start-up cost)

x Probability of shutdown due to this component (either zero or unity depending upon the cost balance, if any).

Mathematically, this expression can be written as follows:

$$C(i,i) = C_r(i) + \{C_p \times t_r(i) + Cmp\ (t_r(i) - t_s) + C_s\} \times P_s(i); \quad (8.3.1)$$

$(t_r(i) - t_s)$ is either zero or positive.

where,

i = the failed component.

$C(i,i)$ = total cost of replacement/repair.

$C_r(i)$ = cost of replacing/repairing failed component (including engineer's cost).

C_p = lost production cost for generator system/hour.

Cmp = lost production cost for the main plant/hour.

C_s = start-up cost (including engineer's cost, lost production during start-up and wastage of material, if any).

$t_r(i)$ = time in hours, of replacing/repairing ith component.

t_s = time in hours, for which main plant can be operated at full capacity, using inert gas from the pressurised storage.

$P_s(i)$ = probability of generator plant shutdown due to ith component failure (either zero or unity).

The shutdown creates an opportunity for maintenance (replacement, repair, tests or whatever is appropriate) of unfailed components on the plant. Now, whether this opportunity should be taken, depends upon the cost advantage. This advantage can be defined as a gain function for each class of equipments.

The gain function, Gl (i, j) for the jth unfailed item of class 1 equipments in which the ith equipment has caused shutdown, can be written as:

$$Gl\,(i,j) = \{C_p\ t_r(j) + Cmp\ t_z(j) + C_s\} \times P(j)$$

$$- C_r(j) \times S(j) - C_p\ t_x(j) - Cmp\ t_y(j) \qquad (8.3.2)$$

where,

$P(j)$ = probability of not getting another opportunity for replacement/repair of the jth component; the opportunity may be created by other components at some later date.

$S(j)$ = proportion of useful life that will be lost for the jth component through early retirement, if the opportunity created by the ith component is taken.

$t_x(j)$ = extra time in hours, for which the generator plant will be shutdown if the replacement/repair time for the jth component is greater than that of the component i which caused the shutdown.

$t_y(j)$ = extra time in hours, for which the main plant will be shutdown; this is in excess of $(t_r(i) - t_s)$ in equation (8.3.1).

$t_z(j)$ = time in hours, for which the main plant will be shutdown, if the jth component alone needs to be replaced/repaired at some later date.

The meaning of the other notations are same as before.

Class 2 equipments are periodically tested and serviced to minimise unexpected failures which will cause shutdown. So, the gain function G2(i,j) for jth unfailed component is given as:

$$G2(i,j) = \{C_r(j) + C_p\, t_r(j) + Cmp\, t_z(j) + C_s\} \times P_t(j) - \{C_r(j) + C_p\, t_x(j) + Cmp\, t_y(j)\} \times P_{1i}(j) \qquad (8.3.3)$$

where,

$P_t(j)$ = probability of failure during the interval between tests.

$P_{1i}(j)$ = probability of failure since last test.

Class 3 and class 4 equipments are basically part of the safety systems. If the plant is not shutdown when these items fail there is a probability of hazard, which must be considered in the gain function. The gain function G3(i,j) for the jth component of class 3, is:

$$G3(i,j) = C_r(j)\, P_t(j) + \{C_p\, t_r(j) + Cmp\, t_z(j) + C_s\} \times P_t(j) \times P_s(j) + [C_H \times \{1 - \exp[-D(j)\, t_r(j)]\}] \times \{1 - P_s(j)\}$$

contd.

$$- \{ C_r(j) + C_p\, t_x(j) + Cmp\, t_y(j) \} \times P_{1i}(j) \quad (8.3.4)$$

where,

C_H = estimated cost of a hazard

$D(j)$ = demand rate/hour on the jth component of the safety system.

For class 4 equipments, there is an alternative safety system; so, this factor will have to be included in the gain function. The gain function G4(i,j) for this class can be constructed as:

$$G4(i,j) = C_r(j)\, Pt(j) + \{ C_p\, t_r(j) + Cmp\, tz(j) + C_s \} \times Pt(j) \times P_s(j)$$
$$+ [C_H \times \{ 1 - \exp[-D(j)\, t_r(j)] \}] \times \{ 1 - P_s(j) \} \times Paf(j)$$
$$- \{ C_r(j) + C_p\, t_x(j) + Cmp\, t_y(j) \} \times P_{1i}(j) \quad (8.3.5)$$

where,

$Paf(j)$ = probability of alternative safety system failure during the replacement/repair of the jth component.

In equations (8.3.3) through (8.3.5), the cost of testing a component has not been included. It has been assumed that if an opportunistic test is performed, it will replace the next scheduled test, thus maintaining an average length of test interval.

In equations (8.3.1), (8.3.4) and (8.3.5), there is the probability $P_s(j)$ of shutdown due to the failure of the jth component. These components are the relay, trip switch (Z), analyser switch (QZ) and temperature switch (TZ). When two analysers (2Q's) are out of service, one has to decide whether or not to shutdown. However, the determination of P_s varies from item to item, depending upon the relevant cost balance. These are :

For the relay and trip switch (Z); $P_S = 1$ if

$$C_H \{1 - \exp(-D\ t_r)\} > (C_p\ t_r + Cmp\ t_z + C_S) \quad (8.3.6)$$

For the analyser switch (QZ); $P_S = 1$ if

$$C_H \{1 - \exp(-D\ t_r)\} > (C_p\ t_r + Cmp\ t_z + C_S) \times$$
(Probability of temperature system failure during the repair of QZ) (8.3.7)

For the temperature switch (TZ); $P_S = 1$ if

$$C_H \{1 - \exp(-D\ t_r)\} > \{C_p\ t_r + Cmp\ t_x + C_S\} \times$$
(Probability of analyser system failure during the repair of TZ) (8.3.8)

With two analysers out of service, it may be possible to operate the analyser system by changing the logic of switch QZ to act on receiving a signal from the remaining analyser. In that situation, the probability of shutdown P_S will be unity, if

$$[C_H \{1 - \exp(-D\ t_r)\} + \{C_S \times \text{(Probability of spurious signal during repair of 2Q's)}\}] > \{C_p\ t_r + Cmp\ t_z + C_S\} \times$$
(Probability of temperature system failure during repair of 2Q's) (8.3.9)

In the inequalities (8.3.6) through (8.3.9), the values of D and t_r are not constant; they vary from item to item. The terms $p(j)$ and $S(j)$ in equation (8.3.2) can be determined by the relationships given in chapter 4.

The values of $P_t(j)$, $P_{1i}(j)$ and $Paf(j)$ for any jth component can also be determined very easily, if an exponential failure distribution is accepted as valid.

For determining currently valid overall maintenance actions one has to solve equations (8.3.2) through (8.3.5) and the total expected saving is given by:

$$\text{Total expected saving} = \Sigma G1 + \Sigma G2 + \Sigma G3 + \Sigma G4 \qquad (8.3.10)$$

8.4 Computational Procedures for Determining Overall Maintenance Plan

Clearly, there is not a single procedure for carrying out the computations of the cost-model described in the previous section, for all circumstances. However, the equations (8.3.6) through (8.3.9) can be solved separately from the main computations, for evaluating the probability of shutdown due to the failure of the relay, the trip switch and the other relevant components.

The preventive replacement ages for class 1 equipments can also be determined apriori. A computer programme called 'PREMAIN' given in Appendix D1, has been written using FORTRAN language. It is usable in ICL 1904S computers. Test intervals of the relevant components can be determined apriori also by the methods given in chapter 5.

The basic steps to carry out the main computations involving the feasibily of appropriate maintenance actions during a shutdown of the plant, are as follows:

1. Read -
   - (i) the number of equipment classes
   - (ii) total number of equipments in each class
   - (iii) equipments' identification numbers in each class
   - (iv) total number of failed equipments in each class
   - (v) the failed equipments' identification numbers in each relevant class.
2. Screen out the unfailed equipments in each class.

3. Read -

(i) the time in hours, for which the generator plant will be shutdown, while the failed equipments are replaced/repaired

(ii) time in hours, for which the main plant can be run by using inert gas from storage

(iii) lost production cost for reactor plant, C_p, per hour

(iv) lost production cost for the main plant, Cmp, per hour

(v) cost of start-up (including labour and materials), C_S

(vi) cost of a hazard, C_H

4. For class 1 equipments:

(i) find out by equation (8.3.2) if any unfailed component should undergo an appropriate maintenance action

(ii) if any component is thus maintained, check whether the previous shutdown times for the generator and the main plant are still valid. If not, put the new values in store for subsequent operations.

5. For class 2 equipments:

(i) find out using equation (8.3.3), if any unfailed component should be tested and replaced or repaired if found to be failed

(ii) check whether the shutdown times from step 4, are still valid. If not, put the new values in store.

6. For class 3 equipments, perform the same operations as in step 5, using equation (8.3.4).

7. For class 4 equipments, carry out the same operations as in step 5 or step 6, using equation (8.3.5).

8. Print the resultant maintenance plan and shutdown times in hours.

9. If the total maintenance cost is required, maintenance cost of failed equipments (equation 8.3.1) and the respective values of the maintenance cost for each of the opportunistically maintained items in steps 4 through 7, will have to be calculated. Then the total cost can be calculated as the sum of all these costs.

A general programme called STRATEGY has been written using FORTRAN language. It contains a separate subroutine for the determination of an opportunistic maintenance plan for each class of equipment (SUBROUTINES CLASS 1, CLASS 2, CLASS 3 and CLASS 4). The programme list for the ICL 1904S computer is given in Appendix D2.

8.5 Data Input for Computer Programme STRATEGY

A case has been chosen that the catalyst in reactor R1 has been deactivated and needs to be replaced. As mentioned in earlier sections, the plant must be shutdown to effect this replacement. To determine an overall maintenance plan, the following data is put in FORMAT-free statements to the computer programme.

A. Input to the master programme:-

(1)	number of classes of equipments:	4				
(2)	total number of equipments in each class					
	(in respective order):	5	4	2	2	
(3)	identification numbers of equipments for each class-					
	class 1 (P1, P2, P3, R1, HE1):	1	2	3	4	5
	class 2 (FC, F_rC, CV1, CV2):	6	7	8	9	
	class 3 (Relay, trip switch Z):	11	12			
	class 4 (QZ, TZ):	13	14			
(4)	number of failed equipments in each class					
	(in order):	1	0	0	0	
(5)	identification number of failed equipment:	4				
(6)	reactor shutdown time in hours	72.0				
(7)	time in hours, for which main plant can run					
	without stoppage:	48.0				
(8)	lost production costs (reactor and main plant),					
	start-up cost and hazard cost in pounds (in order)	500.0	1000.0	500.0	100000.0	

B. Input to SUBROUTINE CLASS 1:-

(Components: P1, P2, P3, R1, HE1)

(1)	replacement/repair/cleaning costs in pounds for each item (in respective order):	100.0	100.0	100.0	500.0	200.0
(2)	time in hours, to complete maintenance of each item (in respective order):	26.0	26.0	26.0	72.0	48.0
(3)	present age of each item, in days, (in respective order); for the failed item, the age is put as 1, for numerical simplicity.	20	20	20	1	25
(4)	preventive replacement time in days; for the item with no such time, it is indexed as zero.	30	30	30	30	0
(5)	shape parameter and scale parameter (Weibull) in days, for each item:					
	P1:	2.0	30.0			
	P2:	2.0	30.0			
	P3:	2.0	30.0			
	R1:	2.0	40.0			
	HE1:	2.0	40.0			

C. Input to SUBROUTINE CLASS 2:-

(Components: FC, F_rC, CV1, CV2)

(1)	cost of replacement/repair for each item:	50.0	50.0	50.0	50.0
(2)	time for testing and replacing/repairing on failure, in hours:	27.0	27.0	27.0	27.0
(3)	test interval in days, for each item:	60.0	60.0	45.0	45.0
(4)	time elapsed since last test to date (in days)	30.0	30.0	25.0	25.0
(5)	failure rate per year, for each item:	0.29	0.29	0.65	0.65

D. Input to SUBROUTINE CLASS 3:-

(Components: Relay and trip Z)

(1)	cost of replacement/repair for each item:	50.0	50.0
(2)	time for testing and replacing/repairing on failure (in hours): "	15.5	15.5
(3)	test interval in days, for each item:	30.0	30.0
(4)	time elapsed since last test to date (in days):	15.0	15.0
(5)	failure rate of each item (per year):	0.10	0.30
(6)	demand rate of each item (per year):	3.34	3.34

(7) probability of shutdown due to the failure of any item, P_S:	0.0	0.0
E. Input to SUBROUTINE CLASS 4:- (Components: QZ, TZ)		
(1) cost of replacement/repair for each item:	50.0	50.0
(2) time for testing and replacing/repairing on failure (in hours):	27.0	27.0
(3) test interval in days:	30.0	30.0
(4) time elapsed since last test to date:	15.0	15.0
(5) failure rate/year:	0.75	0.75
(6) demand rate/year:	3.34	2.61
(7) probability of shutdown due to failure:	0.0	0.0
(8) probability of alternative safety system failure during maintenance of the failed item:	0.0008	0.00137

8.6 Results and Discussion

For the data listed in the previous section, the following maintenance plan was produced by the programme STRATEGY.

Maintenance Plan:

(1) replace reactor catalysts (R1); replace seals and bearings of blower (P1), pump (P2) and compressor (P3); clean heat exchanger (HE1); (Components:4, 1, 2, 3, 5 respectively).

(2) test and on failure replace/repair FC, F_rC, CV1 and CV2 (Components: 6, 7, 8 and 9 respectively)

(3) test and on failure replace/repair relay and trip switch Z (Components: 11 and 12)

(4) test and on failure replace/repair analyser switch QZ and temperature switch TZ (Components: 13 and 14).

To effect this maintenance plan, the generator will be shutdown for 72 hours and the main plant for 24 hours.

The maintenance plan for this example is quite straight forward. Although there was only one failed component, it was found that opportunistic maintenance of other unfailed components were cost-effective. Maintenance of these unfailed components at a later date would not have caused shutdown of the main plant; but, clearly, the shutdown of the generator in the immediate future (for the preventive replacements of seals and bearings of P1, P2, P3) would have incurred more downtime and lost production.

In reality, the values of the scale parameter and those of the preventive replacement times for P1, P2, P3, R1 and HE1 may be larger than those used in the example. The smaller values have been taken to keep the computer core size low. Nevertheless, the whole exercise demonstrates how cost-effective maintenance plans can be devised by proper analysis and mathematical modelling.

CHAPTER 9

CONCLUSIONS AND FUTURE WORK

9.1 Conclusions

(i) Quantitative methods for developing maintenance plans in the process industries can enable maintenance managers to assess the risks and benefits of alternative maintenance actions. These techniques also provide an insight into the overall maintenance strategy for the plant. If maintenance plans are based on a sound scientific and economic analysis, production, finance and general managements could be convinced about the need for certain maintenance actions. Thereby, the long held view that maintenance is a 'necessary evil' should diminish.

(ii) Classification of maintenance policies into separate categories, and the subsequent classification of plant items by the maintenance categories or by the effects of their failures, helps to formulate mathematical models easily. This will help the development of maintenance plans based upon proper analysis. The case study of chapter 8 demonstrates the effectiveness of these classifications.

(iii) Statistical analysis of failure data of plant items and determination of appropriate failure distributions have been shown to be crucial for developing maintenance plans, either for cost minimisation or for reliability maximisation. The models and case studies of chapters 4 and 5 provide examples to support this claim.

(iv) A new method SEQLIM has been developed for the estimation of the Weibull distribution parameters. The method is applicable to the analysis of maintenance data from both replacement and non-replacement cases. Because of its rapid convergence to good estimates of the parameters using straight forward calculations, it can be used easily in industrial applications. Further comments on the method are given in chapter 3 and Appendix A1.

(v) Chapters 4 and 8 demonstrate the effectiveness of opportunistic maintenance policies. When a system consists of a number of items, the failures of which are statistically independent, but where there may be an economic advantage in replacing/maintaining two or more items together, opportunistic maintenance policies have been proven to be more cost-effective than the maintenance policies based on individual items.

(vi) All inspection models based on the assumption of a negative exponential failure distribution give the same optimal inspection interval if the product of the failure rate and inspection interval and also the product of the demand rate and inspection interval are significantly less than unity. But, inspection intervals have been found to be much greater when some failures of the components are caused by wearout. This indicates that the failure modes of each component should be monitored carefully for wearout or environmental influence. Some specific conclusions regarding various models and their applications to safety trip systems have been given in chapter 5 and Appendix C1.

(vii) Effective restorative maintenance policies can be developed by formulating predictive models of equipments' performance with time. This fact has been highlighted by the case studies in chapters 6 and 8. In chapter 6, the development of fouling in a condenser has been simulated in terms of the Weibull distribution. This not only provides a generalised model of the equipment performance between cleaning, but also facilitates the mathematical analysis of optimal overhaul interval and opportunistic maintenance policies.

(viii) When it is cost-effective, condition-based maintenance can be integrated into the overall maintenance planning with resultant improvement in plant availability and better production planning. Condition monitoring of a pressure relief valve to determine the set pressure and the spring condition of the valve in situ, is feasible, if the gauge designed in chapter 7 can be further developed.

9.2 Scope for Future Work

The following list may provide a helpful guide to additional work, the need for which has become apparent during the course of this research.

(i) Identification of various types of equipments that are used in process industries, in general. Classification of these equipments according to the maintenance policy categories or their failure effects on plant availability. Consideration should be givenalso to equipments with likely 'design-out-maintenance'.

(ii) Determination of the failure characteristics of the equipments in accordance with their use in different environments. Application of a general failure distribution, eg. the Weibull distribution, and evaluation of the parameters of the chosen failure distribution for each of these equipments. Many industrial and service organizations collect failure data of equipments simply in terms of failure rate or mean time between failures. But these statistics are of limited use in assessing maintenance strategies, because they ignore the effects of wearout or degradation. Proper consideration should be given to actual failure distributions.

(iii) The effectiveness of SEQLIM as opposed to MLE, BLIE, GLUE and Least Squares should be tested with more industrial applications. In the development of SEQLIM, a discrimination ratio of 2 (cf. chapter 3 and Appendix A1) has been used. Simulation can be used to test the optimality of this choice of discrimination ratio.

(iv) Development of a general opportunistic maintenance model to take into account the statistical interdependence of components in the systems. In the model of chapter 4, conditional probability in terms of cumulative distribution function has been used. A general model using the hazard rate equation itself, may be developed and compared with the model of chapter 4.

(v) Development of inspection models for systems with redundant components, with due consideration of the failure characteristics of the components. Applications of these models for industrial situations should be undertaken.

(vi) Validation by experiments of the general fouling model postulated in chapter 6 and its comparison with other fouling models. Effort should be given to develop general models for other process equipments also which come under the restorative maintenance category.

(vii) Industrial applications of the gauge (cf. Chapter 7) to measure the set-pressure and determine spring condition of a relief valve can be undertaken. On the proof of its effectiveness, the gauge can be developed as a cheap, handy instrument. In general, cheap condition monitoring techniques should be developed wherever possible.

APPENDIX A1 (To be published in Terotechnica Vol. 2, No. 1, November 1980)

SEQLIM - A NEW METHOD FOR ESTIMATION OF WEIBULL DISTRIBUTION PARAMETERS FOR SAFETY SYSTEMS

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ABSTRACT

Unlike the negative exponential distribution, the Weibull distribution can be used to correlate failure times in systems in which gradual deterioration as well as random causes produce failure. With safety systems, failure is a rare event; it may take a long time to record sufficient failures, to estimate the two parameters of the Weibull distribution. The well-established methods for estimating these parameters, MLE, BLIE and GLUE require at least two failures. Of these, only MLE can be used for cases where replacements have occured; but MLE requires more than three failures in order to yield an unbiased estimate of the shape parameter. To offest these difficulties in the estimation of the Weibull distribution parameters (β_W and η), a new method, SEQLIM, is proposed. This method uses the acceptance points of sequential probability ratio tests to obtain a set of (β_W, η) pairs and then, the theory of probability limits to select the most appropriate pair. For life testing, SEQLIM has also an in-built characteristic of predicting the time limit for the arrival of the next failure, whereupon the previous estimates of β_W and η are

assessed. Five examples are given to demonstrate its effectiveness in comparison with other methods, and it is found to perform well. Only simple calculations are required to use SEQLIM.

1. INTRODUCTION

A previous paper [1] concerning the 'optimal test intervals for safety trip systems', established two important points. Firstly, if the time to failure of a trip is exponentially distributed, ie. deterioration is assumed to be insignificant, the required inspection interval is found to be the same, whether it is predicted by a simple model based on minimising fractional dead time (FDT), or by other more complex models, such as the Munford-Shahani model or the maximum safety model, provided the trip is very reliable and/or the demand rate is low. The second point is that if environmental factors do cause deterioration, the trip test intervals can be significantly increased, by as much as ten times longer than predicted using the negative exponential distribution, while maintaining the same specified FDT. If testing returns the trip to the "as new" condition, successive test intervals remain fixed; whereas, if testing does not return the trip to the "as new" condition after each test, then successive test intervals must be decreased or increased with age depending upon whether the failure rate is increasing or decreasing, respectively. To accommodate both random and wearout failures, the Weibull distribution was used [1]. The probability density function f(t) for this distribution is,

$$f(t) = \frac{\beta_w}{\eta} \left(\frac{t}{\eta}\right)^{\beta_w - 1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta_w}\right] \qquad (1)$$

where β_w is the shape parameter, η is the scale parameter and t is the time. The successive inspection times were shown to be given by equations (2) in which T_1 is the time to the first inspection and T_i is the time from the 'new' to the ith inspection.

$$T_1 = \eta\{(\beta_w+1)FDT\}^{1/\beta_w}$$

$$(T_i)^{\beta_w+1} = \eta^{\beta_w}(\beta_w+1)(T_i - T_{i-1})\left[FDT + \left(\frac{T_{i-1}}{\eta}\right)^{\beta_w}\right] + (T_{i-1})^{\beta_w+1}$$

$$\text{for } i = 2,3,.... \qquad (2)$$

It can be seen, that the problem of determining the optimal test intervals for a trip system is the determination of the true values of the distribution parameters (β_w and η). The literature contains a number of methods for estimating β_w and η, providing that there are sufficient failures recorded. These methods include the maximum liklihood estimators (MLE), the best linear invariant estimators (BLIE), the good linear unbiased estimators (GLUE) and the least squares with the unbiasing coefficients. But, all these methods are ineffective for trip systems or safety systems in general; because failure is a rare event for these reliable systems. Typically, there are no more than two failures over a period of four or five years. Inspection policies during this time become subjective, in so far as the exponential failure distribution is assumed for all components of the system and test intervals are determined on the basis of combined failure rates per year; without regard to whether β_w is indeed equal to unity for each component.

The sequential probability ratio test (SPRT) as devised by Wald [2], is a method of accepting or rejecting a specified failure rate or mean time between failures (MTBF), on the basis of hypothesis testing, with pre-assigned values of the risks involved; these are α and β, the producer's risk and customer's risk, respectively. The operating characteristic (OC) curve and the average sample number (ASN) curve or the average time to terminate testing (ATT) curve are essential features of sequential test plans in order to determine the true risks involved with any decision and to obtain the expected length of the test, respectively. Epstein and Sobel [3], Aroian and his co-workers [4,5] and many other authors have further developed the sequential testing method. Nocolae and Obreja [6] and Harter and Moore [7] have devised test plans for the Weibull distribution. In the Military Standard MIL-STD 781 [8], a number of test plans have been given for the exponential distribution. But, only Harter and Moore [7] have discussed a method for the determination of both β_W and η simultaneously. Their method requires the use of the MLE with unbiasing factors to determine β_W at each step. It is applicable only when 3 or more items are on test and can only be as good as the MLE. However, in the absence of any prior knowledge of the β_W and η values, the present methods of sequential testing are ineffective for systems with few recorded failures.

The theory of Probability Limits, has been proposed by Dubey [9] for validating assumed failure distribution parameters (β_w and η). By setting any desired single-sided or double-sided probability limits for the ith order failure time t_i, one can check whether ordered observed failure times are within these limits and thus accept or reject these assumed parameters.

An effective inspection policy for systems in which failure is a rare event needs a method whereby the distribution parameters can be readily re-estimated. The advantage of sequential testing under these circumstances is that inference can be drawn from inspection records showing the equipment being in the good state as well as the failed state. In this paper, a new method is proposed, named SEQLIM for convenience, which utilizes the basic acceptance principles of sequential testing, to estimate sets of Weibull distribution parameters (β_w, η); probability limits are then used to select the most suitable values of these parameters. Unlike other methods mentioned above, it requires very little computation and also, it predicts the time range within which another failure must occur in order for the estimated distribution parameters to be tenable.

In Section 2, the methods of MLE, BLIE, GLUE and least squares are described briefly. The general principles of sequential testing are outlined in Section 3 and the proposed method, SEQLIM, is presented in Section 4. The various methods are compared by means of five examples.

2. METHODS OF MLE, BLIE, GLUE AND LEAST SQUARES

All these methods are widely covered in the reliability engineering literature; the relevant equations associated with the methods are given here. Further detail may be found in the references listed.

In the maximum likelihood procedures [10], for the first r failures arranged in ascending order of time, from a sample of n equipments on test, the shape parameter $\hat{\beta}_w$ is obtained by the solution of the equation

$$\frac{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} \ln t_{i:n} + (n-r)\, t_{r:n}^{\hat{\beta}_w} \ln t_{r:n}}{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} + (n-r)\, t_{r:n}^{\hat{\beta}_w}} - \frac{1}{\hat{\beta}_w} = \frac{1}{r} \sum_{i=1}^{r} \ln t_{i:n} \qquad (3)$$

and the scale parameter $\hat{\eta}$ is given by

$$\hat{\eta} = \left[\frac{\sum_{i=1}^{r} t_{i:n}^{\hat{\beta}_w} + (n-r)\, t_{r:n}^{\hat{\beta}_w}}{r} \right]^{1/\hat{\beta}_w} \qquad (4)$$

Equation (3) for $\hat{\beta}_w$ cannot be solved readily; some numerical method, such as Newton-Raphson, must be used. It has been customary to obtain the initial value of $\hat{\beta}_w$ required for the N-R technique, from another method; for example, the least squares, GLUE, etc.

In a situation when n identical items are put on test, it can be seen that equation (3) will give $\hat{\beta}_w$ equal to infinity, just after first failure. Therefore, estimation of $\hat{\beta}_w$ and $\hat{\eta}$ can only start after the second failure. Another problem with MLE is that with few failures, $\hat{\beta}_w$ is highly biased. Bain [10] has produced unbiasing factors for five or more failues $r \geq 5$ and Harter and Moore [7] for $r = 3$ and 4. So, in effect, the maximum likelihood estimates cannot be used to any advantage before the third failure.

The method of best linear invariant estimators (BLIE) [11], gives $\hat{\beta}_W$ and $\hat{\eta}$ by the following relationships.

$$\hat{\eta} = \sum_{i=1}^{r} A(n,r,i)\, t_i \tag{5}$$

and

$$1/\hat{\beta}_W = \sum_{i=1}^{r} c(n,r,i)\, t_i \tag{6}$$

The weighting factors $A(n,r,i)$ and $c(n,r,i)$ are given by Mann et al [11] for $n \geq 2$ and $r \geq 2$. This method also cannot be used before the second failure. Another disadvantage is that for few failures, the $\hat{\beta}_W$ estimate is highly biased.

The method of good linear unbiased estimators (GLUE) has been discussed by Bain [10]. The equations to be solved are:

(i) for $r < n$,

$$1/\tilde{\beta}_W = \frac{r \cdot \ln t_{r:n} - \sum_{i=1}^{r} \ln t_{i:n}}{nk_{r,n}} \tag{7}$$

$$\ln \tilde{\eta} = \ln t_{r:n} - (C_{r,n}/\tilde{\beta}_W) \tag{8}$$

where the constants $k_{r,n}$ and $C_{r,n}$ are given in tabular forms for $n \geq 5$ or may be calculated by quadratic approximations provided [10].

(ii) for $r = n$,

$$1/\tilde{\beta}_w = \frac{-\sum_{i=1}^{s} \ln t_{i:n} + \{s/(n-s)\} \sum_{i=s+1}^{n} \ln t_{i:n}}{nk_n} \tag{9}$$

$$\ln \tilde{\eta} = \left(\frac{1}{n} \sum_{i=1}^{n} \ln t_{i:n}\right) + \gamma^*/\tilde{\beta}_w \tag{10}$$

where k_n is an unbiasing constant given in tabular form, $s = [0.84n]$ is the largest integer $\leq 0.84n$, and $\gamma^* = 0.5772$ is the Euler constant.

Unlike MLE and BLIE, this method provides a good estimate of $\tilde{\beta}_w$ for very few failures. But, it also cannot be used before the second failure. Another shortcoming with GLUE is, that it is inapplicable when replacements are carried out on failure; because there is no single value of $t_{r:n}$.

The method of least squares is applicable only after all the n equipments in the sample have failed; ie. no censoring is allowed. Nevertheless, it is mentioned here because of its effectiveness in dealing with few failures within engineering approximations. There is littlie bias for the shape parameter $\hat{\hat{\beta}}_w$; but; Hinds et al [12] have produced unbiasing factors in tabular form. The determination of β_w and η by this method, is based on the inversion and double logarithmic transformation of the equation for the cumulative distribution function F(t) for the Weibull distribution, ie.

$$F(t) = 1 - \exp[-(\frac{t}{\eta})^{\beta_w}] \tag{11}$$

which on transformation becomes,

$$\ln \ln \left[\frac{1}{1-F(t)}\right] = \beta_W \ln t - \beta_W \ln \eta \qquad (12)$$

This is of linear form $Y = mX-Z$,

where $Y = \ln \ln \left[\frac{1}{1-F(t)}\right]$

$m = \beta_W$

$X = \ln t$

$Z = \beta_W \ln \eta$

There are several ways of calculating $F(t_i)$ for the ith failure [11,12], three of which are as follows:

Choice	$F(t_i)$
Mean rank	$i/(n+1)$
Benard's approximation	$(i-0.3)/(n+0.9)$
Symmetrical sample cdf	$(i-0.5)/n$

(13)

Estimates of $\hat{\hat{\beta}}_W$ and $\hat{\hat{\eta}}$ can be obtained graphically on Weibull probability paper. Alternatively, a computer programme can be used to obtain the shape parameter $\hat{\hat{\beta}}_W$ using equations (12) and (13) while the scale parameter $\hat{\hat{\eta}}$ is estimated by the following equation.

$$\hat{\hat{\eta}} = \exp \{Y(\text{at } 63.2\%)+Z\}/\hat{\hat{\beta}}_W \qquad (14)$$

Like all other methods, the least squares method also cannot be used for $n<2$.

3. SEQUENTIAL TESTING TECHNIQUES (SPRT)

The fundamental principles of sequential probability ratio tests or in short, sequential testing, have been widely covered in the literature [2-8]. The basic principles are described by Bazovsky [13] and Kapur and Lamberson [14].

The construction of SPRT is based on testing two hypotheses, Ho : $\lambda=\lambda_0$ against H_1 : $\lambda=\lambda_1$, where $\lambda_1>\lambda_0$. λ_1 is the specified higher value of the failure rate for the exponential distribution and λ_o is some chosen lower value. The two error probabilities - α (producer's risk) and β (consumer's risk) are designated by

$\alpha = P(H_1/H_o)$, ie probability of accepting H_1 when H_o is true.

and

$\beta = P(H_o/H_1)$, ie probability of accepting H_o when H_1 is true.

The rules for executing SPRT are stated in terms of the ratio of the likelihood functions.

(i) Accept H_o, if

$$\ln \frac{\prod_{i=1}^{r} f(t_i,\lambda_1)}{\prod_{i=1}^{r} f(t_i,\lambda_o)} \leqslant \ln \frac{\beta}{1-\alpha}$$

(ii) Reject H_o, if

$$\ln \frac{\prod_{i=1}^{r} f(t_i,\lambda_1)}{\prod_{i=1}^{r} f(t_i,\lambda_o)} \geqslant \ln \frac{1-\beta}{\alpha}$$

(iii) Continue testing, if

$$\ln \frac{\beta}{1-\alpha} < \ln \frac{\prod_{i=1}^{r} f(t_i,\lambda_1)}{\prod_{i=1}^{r} f(t_i,\lambda_o)} < \ln \frac{1-\beta}{\alpha}$$

For the exponential failure distribution, the inequalities become;

$$\frac{\ln B}{\ln (\lambda_1/\lambda_o)} + \frac{(\lambda_1-\lambda_o)}{\ln (\lambda_1/\lambda_o)} \sum_{i=1}^{r} t_i < r < \frac{\ln A}{\ln (\lambda_1/\lambda_o)} + \frac{(\lambda_1-\lambda_o)}{\ln (\lambda_1/\lambda_o)} \sum_{i=1}^{r} t_i \quad (15)$$

where $B = \{\beta/(1-\alpha)\}$, $A = \{(1-\beta)/\alpha\}$ and r is the number of observed failures. The inequalities of (15) can also be written in the form,

$$a + bT < r < c+bT \quad (16)$$

where,

$$a = \ln B/\ln (\lambda_1/\lambda_o)$$

$$b = (\lambda_1-\lambda_o)/\ln (\lambda_1/\lambda_o)$$

$$c = \ln A/\ln (\lambda_1/\lambda_o)$$

$T = \sum_{i=1}^{r} t_i$, when only one item is being tested at a time. If n items are on test, $T = \sum_{i=1}^{n} t_i$, where t_i denotes failure as well as successful operation time. It is quite obvious from inequalities (15) and (16), that the left and right sides represent equations of two parallel straight lines, called the 'acceptance line' and the 'rejection line', respectively. Commonly, graphical procedures have been used to plot observed failures and compare these with the the two decision lines for various test plans.

Now, the power and length of the test are very much dependent on the choice of α, β and the discrimination ratio $d = \lambda_1/\lambda_o$. In MIL-STD 781 [8], various combinations of $\alpha = \beta = 0.1, 0.2, 0.3$ and $d = 1.5, 2, 3$ have been used. In general, any value of α and β can be used, provided these are less than 0.5, in order to avoid the triviality of $A = B$. But, there is virtually no restriction on the value of d.

The basic rule is that the appropriate decision must be accepted, when a plot of the number of observed failures against the cumulative operating time first crosses either of the two decision lines. In order to limit the maximum time to arrive at a decision, it may be agreed in advance to truncate the tests after a certain time T_o or after a certain number of failures N. At the arrival of T_o or N, the decision is taken to accept or reject, depending upon whether the cumulative operating time vs number of failures line is nearer the 'accept line' or the 'reject line', respectively. The decision taken from sequential testing cannot be reversed by subsequent upcrossing of the accept line or downcrossing of the reject line [15].

After any decision, the true risks α' and β' of the decision being incorrect, owing to the limited number of failures and/or the limited test time, are not necessarily equal to the specified risks α and β. The maximum allowable values of α' and β' are related to α and β as follows.

$$\alpha'_{max} = \frac{\alpha}{1-\beta} \text{ and } \beta'_{max} = \frac{\beta}{1-\alpha}$$

Kapur and Lamberson [14] state, the true risks must satisfy the following two conditions,

(i) at least one of the inequalities $\alpha' \leq \alpha$ and $\beta' \leq \beta$ must hold, and

(ii) $\alpha' + \beta' \leq \alpha + \beta$

In the case of truncated sequential tests, some authors [7] seem to have disregarded these conditions and have accepted higher values of α' and β'.

Sequential test plans for the Weibull distribution follow the same pattern as the exponential distribution. However, the time scale is changed from t to t^{β_w}, whereupon the two parameter Weibull distribution is transformed to a one-parameter exponential distribution, with a mean ψ related to the η of the Weibull distribution as follows: $\psi = \eta^{\beta_w}$. Following Nicolae and Obreja [6] and Harter and Moore [7], the test plans for the Weibull distribution can be stated as follows,

$$\frac{\ln B}{\ln d^{\beta_w}} + \frac{d^{\beta_w}-1}{\ln d^{\beta_w}} \sum_{i=1}^{r} \left(\frac{t_i}{\eta_o}\right)^{\beta_w} < r < \frac{\ln A}{\ln d^{\beta_w}} + \frac{d^{\beta_w}-1}{\ln d^{\beta_w}} \sum_{i=1}^{r} \left(\frac{t_i}{\eta_o}\right)^{\beta_w} \qquad (17)$$

where η_o and η_1 are the higher and lower values of scale parameter, respectively: and d is the discrimination ratio (η_o/η_1). The inequalities (17) can also be arranged in the form,

$$r\frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln A}{d^{\beta_w}-1} < \frac{1}{\eta_o^{\beta_w}} \sum_{i=1}^{r} t_i^{\beta_w} < r\frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln B}{d^{\beta_w}-1} \qquad (18)$$

When the shape parameter β_w is known, graphical test plans of (17) or (18) can be readily obtained for any pre-estimated values of η_o and η_1. Harter and Moore [7] have given an adaptive procedure for cases where β_w is unknown. The difficulty with this method is that the hypotheses can only be tested against a specified value η, which may be difficult to decide before testing begins.

4. PROPOSED METHOD (SEQLIM) FOR THE DETERMINATION OF β_w AND η

It has been explained in Section 1, that the proposed method, SEQLIM for estimating β_w and η of the Weibull distribution, is based on the acceptance points of SPRT and the theory of probability limits. The acceptance points v_r for SEQLIM are calculated by the upper limit of the inequalities (18), which is

$$v_r = r\frac{\ln d^{\beta_w}}{d^{\beta_w}-1} - \frac{\ln B}{d^{\beta_w}-1} \qquad (19)$$

where all the notations are as before. But, one needs to decide the values of the risks α and β and the discrimination ratio d, which should be used. Keeping in mind the application of SEQLIM for safety systems, it is proposed that $\alpha = 0.10$, $\beta = 0.05$ and $d = 2.0$ should be appropriate. Table 1 gives the acceptance points for $r = 0$ to 5 and for various values of β_w with the following ranges and increments (β_w = 1.0, 2.0, 0.2; β_w = 2.25, 3.0, 0.25, β_w = 3.4).

For sequential testing, η_o is accepted when $\frac{1}{\eta_o^{\beta_w}} \times \sum_{i=1}^{r} t_i^{\beta_w}$ reaches the acceptance line. But, for SEQLIM, separate values of η_o and η_1 are not required; v_r is determined from a discrimination ratio d, as shown by equation (19). The value of η which satisfies equations (19) and (20) is accepted.

$$\frac{1}{\eta^{\beta_w}} \sum_{i=1}^{n} t_i^{\beta_w} = v_r \qquad (20)$$

In equation (20) t_i denotes the operating time for ith item on test (including failure time if any), and n is the total number of items on test. From equation (20), the accepted estimate of η is obtained as

$$\eta = \{(\sum_{i=1}^{n} t_i^{\beta_w})/r\}^{1/\beta_w} \tag{21}$$

for any value of β_w. Now, for a set of β_w values, one can get a set of (β_w, η) pairs. But the question is, which pair should be most suitable for the recorded data? It is thought that the answer lies in the theory of probability limits [9]. By this theory, the 100ε per cent probability limit for the rth failure time t_r, in a sample of size n $(1 \leq r \leq n)$ from the Weibull distribution, is given by

$$t_\varepsilon(r) = \eta\{-\ln(1-b_\varepsilon)\}^{1/\beta_w} \tag{22}$$

In equation (22) b_ε is the 100ε per cent percentage point of the beta distribution, with parameters r and n-r+1, respectively. Harter [16] has tabulated b_ε values for n=1 to 40. In this paper, Tables 2 to 11 give $t_\varepsilon(r)/\eta$ values for a range of β_w values and for n=1 to 5 and $r \leq n$; each table is for a different value of 100ε in the range of 5% to 95%.

Having obtained a set of (β_w, η) pairs, one now needs to compute t_r/η for the r observed failure times and to check whether these values are within the desired probability limits. The pair, which has no outlier, is accepted as the best estimate of β_w and η.

With SEQLIM, the fundamental approach for applying probability limits is the same, as explained above, but it is proposed that one should first use the 40% lower - 60% upper limits (double sided). If at this stage, no pair of (β_W, η) can be selected, then one should step by step increase the limits to 30%-70%, 20%-80%, 10%-90%, 5%-95% and so on, until a pair can be selected. It may happen, that there are a number of pairs within the selection zone. In that case, the pair with the smaller value of β_W should be selected. On selecting β_W and η values from data with r failures, one can predict the lower and upper time limits for the arrival of (r+1)th failure, at the same accepted levels of probability limits. If the (r+1)th failure arrives within this time limit, the values of β_W and η remain unchanged. If this (r+1)th failure arrives outside the limited period, one should restart the whole exercise with recalculation of (β_W, η) pairs. In this review of the recorded failure times, one can start at the previously accepted levels of probability limits. However, it is recommended that for the first few failures, it is better to apply 40%-60% limits.

SEQLIM with Replacements

The SEQLIM procedure, so far described, is for the life testing or non-replacement case, where the observed failure times are in natural order. But, where replacements have occured, the observed failure times are not in natural order and ordering is required at the time of selecting a (β_W, η) pair. Prediction for the (r+1)th failure should also be based on the total number of items in service, including previous failures. For example, the may have been r failures from n items. If there is a single latest failure (which is the rth observation), then (n+1) is the total number of items that are now in service. New predictions for first to the (r+1)th order failure times, based upon previously selected β_W and η values, should be made on the basis of the (n+1) number of items instead of n. The level of limits for this prediction should be the one at which the selected (β_W, η) pair is acceptable. While checking the observed failure times at the accepted limiting level, one should note that (r+1)th observation is not necessarily (r+1)th ordered failure time.

Evaluation of SEQLIM Method

To ascertain the effectiveness of the SEQLIM method in comparison with other methods, let us consider the following four examples (life test data for n items) with observed failure times as listed below. Examples 1 to 3 were generated with η=48 and β_W=2, 3.4 and 1.4, respectively, using least squares with symmetrical sample cdf for $F(t_i)$. Example 4 is taken from page 210 of ref. [10]; $\beta_W = 2$, $\eta = 100$.

Example 1: 15.58, 28.667, 39.963, 52.668, 72.837 (n=5)

Example 2: 24.762, 35.445, 43.095, 50.693, 61.334 (n=5)

Example 3: 9.62, 22.985, 36.944, 54.806, 87.09 (n=5)

Example 4: 5, 10, 17, 32, 32, 33, 34, 36, 54, 55, 55, 58, 58, 61, 64, 65, 65, 66, 67, 68, 82, 85, 90, 92, 92, 102, 103, 106, 107, 114, 114, 116, 117, 124, 139, 142, 143, 151, 158, 195. (n=40)

The results for all methods discussed in this paper, are compared in Table 12. The working of Example 1 by the SEQLIM method, is shown in Appendix I.

In Appendix 2, the SEQLIM method is tested using the failure times generated for Example 1; but these times are rescheduled as follows to represent a replacement case. Here, instead of putting all 5 items in operation from the beginning, it is considered that only 2 items are in operation at any time, but that 5 items have been in service by the end of the test. Thus, no replacement is made after the fourth failure. The data of Example 1 were rescheduled as follows,

	Operating times of the items in service				
Failures	1	2	3	4	5
1	15.58	15.58	0		
2		28.667	13.087	0	
3			39.963	11.296	0
4				63.964	52.668
5				72.837	

5. CONCLUSIONS

(i) Table 12 shows that, for life testing data, the SEQLIM method is more powerful than other well-established methods such as MLE, GLUE, etc. Also for situations in which only a few equipments are in service and are replaced on failure, SEQLIM converges rapidly to give acceptable estimates of β_W and η; see Appendix II.

(ii) When all items have been in service for the same length of time, the first failure will always produce an estimated β_w equal to 2.75; using SEQLIM. For the second and subsequent failure of the surviving equipments, SEQLIM rapidly converges onto good estimates of β_w and η. None of the other methods available can give an estimate of η after only one failure. After the first failure, the value of β_w estimated by SEQLIM, for situation in which the equipments have differing ages, may not necessarily be 2.75.

(iii) SEQLIM offers a method for handling both replacement and non-replacement cases of maintenance data, containing records of failures as well as successes at each test; for example trip testing records.

(iv) Another important advantage of SEQLIM is that previous estimates of β_w and η are revised if the next failure does not arrive within a prescribed time interval. Re-estimation of β_w and η is not required if the subsequent failure arrives within this time limit.

(v) When using the proposed method, SEQLIM, the optimal value of the discrimination ratio d for rapid, stable convergence is 2.

(vi) The worked examples in the Appendices show that SEQLIM requires very straightforward calculations. These are based upon Tables 1 to 11, for up to 5 failures. Extended tables, for up to 10 failures, are available elsewhere [17].

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Table - 1

Values of acceptance points, v_r for equation (21) (r denotes number of failures)

β_w	v_0	v_1	v_2	v_3	v_4	v_5
1.0	2.890372	3.583519	4.276666	4.969813	5.66296	6.356108
1.2	2.227824	2.868936	3.510048	4.15116	4.792272	5.433384
1.4	1.76348	2.355546	2.947613	3.539679	4.131745	4.723812
1.6	1.422824	1.968761	2.514699	3.060636	3.606574	4.152511
1.8	1.164438	1.667083	2.169727	2.672371	3.175016	3.67766
2.0	.963457	1.425555	1.887653	2.349752	2.81185	3.273948
2.25	.769365	1.184497	1.59963	2.014762	2.429894	2.845027
2.5	.62067	.992782	1.364893	1.737004	2.109116	2.481227
2.75	.504677	.837504	1.17033	1.503157	1.835983	2.168810
3.0	.41291	.709973	1.007036	1.304099	1.601163	1.898226
3.4	.302465	.549083	.795701	1.04232	1.288938	1.535556

Table - 2

Limiting values of t_i/n for ith order failure time for different values of n and β_W

5% LIMITS

n	i	-βw- 1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.05127	0.08415	0.11984	0.15624	0.19203	0.22648	0.26711	0.30481	0.33757	0.37155	0.41765
2	1	0.02563	0.04723	0.07305	0.10131	0.13066	0.16015	0.17629	0.23100	0.26392	0.29490	0.34046
2	2	0.25310	0.31823	0.37478	0.42370	0.46611	0.50309	0.54297	0.57713	0.60675	0.63255	0.66757
3	1	0.01710	0.03369	0.05468	0.07863	0.10430	0.13076	0.16392	0.19642	0.22774	0.25762	0.30219
3	2	0.14543	0.20055	0.25229	0.29963	0.34262	0.38135	0.42447	0.46245	0.49603	0.52533	0.56718
3	3	0.45950	0.52309	0.57332	0.61503	0.64721	0.67727	0.70779	0.73268	0.75370	0.77157	0.79556
4	1	0.01232	0.02651	0.04452	0.06569	0.08890	0.11324	0.14425	0.17507	0.20512	0.23406	0.27767
4	2	0.10271	0.15009	0.19677	0.24113	0.28242	0.32043	0.36368	0.40239	0.43711	0.46831	0.51203
4	3	0.28522	0.35217	0.40579	0.45715	0.49869	0.53462	0.57315	0.60595	0.63419	0.65372	0.69183
4	4	0.54031	0.63970	0.72729	0.75682	0.78062	0.80019	0.82026	0.83667	0.85035	0.86191	0.87711
5	1	0.01026	0.02201	0.03796	0.05714	0.07353	0.10129	0.13063	0.16012	0.18913	0.21729	0.26003
5	2	0.07952	0.12126	0.16392	0.20549	0.24497	0.28197	0.32453	0.36324	0.39827	0.43092	0.47491
5	3	0.20980	0.27217	0.32776	0.37682	0.41993	0.45804	0.49955	0.53546	0.56674	0.59421	0.63173
5	4	0.41945	0.48480	0.53763	0.58100	0.61713	0.64765	0.67962	0.70643	0.72911	0.74856	0.77450
5	5	0.79691	0.82764	0.85031	0.86772	0.88151	0.89270	0.90403	0.91320	0.92076	0.92712	0.93541

Table - 3

Limiting values of t_i/n for ith order failure time for different values of n and β_W

10% LIMITS

n	i	-βw- 1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.10536	0.15331	0.20341	0.24500	0.28645	0.32459	0.36752	0.40651	0.44117	0.47231	0.51583
2	1	0.05268	0.08604	0.12215	0.15887	0.19490	0.22952	0.27030	0.30808	0.34283	0.37487	0.42074
2	2	0.38013	0.44663	0.50113	0.54633	0.58429	0.61655	0.65058	0.67916	0.70347	0.72440	0.75240
3	1	0.03512	0.06137	0.09143	0.12330	0.15559	0.18740	0.22573	0.26195	0.29582	0.32743	0.37344
3	2	0.21791	0.28091	0.33677	0.38585	0.42392	0.46681	0.50804	0.54364	0.57461	0.60176	0.63881
3	3	0.62392	0.67495	0.71394	0.74465	0.76745	0.78938	0.81026	0.82804	0.84237	0.85449	0.87045
4	1	0.02634	0.04829	0.07445	0.10301	0.13261	0.16230	0.19863	0.23349	0.26649	0.29754	0.34314
4	2	0.15380	0.21012	0.26258	0.31035	0.35344	0.39210	0.43516	0.47292	0.50623	0.53578	0.57660
4	3	0.38634	0.45270	0.50697	0.55189	0.58957	0.62156	0.65527	0.68358	0.70763	0.72832	0.75600
4	4	0.82532	0.85301	0.87261	0.88760	0.89944	0.90902	0.91870	0.92653	0.93298	0.93839	0.94543
5	1	0.02107	0.04009	0.06348	0.08960	0.11715	0.14516	0.17928	0.21354	0.24572	0.27621	0.32135
5	2	0.11905	0.15973	0.21868	0.26444	0.30656	0.34503	0.38834	0.42656	0.46121	0.49193	0.53475
5	3	0.28321	0.34940	0.40511	0.45453	0.49615	0.53217	0.57031	0.60373	0.63207	0.65670	0.69001
5	4	0.53804	0.59660	0.64228	0.67933	0.70369	0.73351	0.75921	0.78042	0.79821	0.81334	0.83335
5	5	0.99634	0.99737	0.99774	0.99803	0.99824	0.99842	0.99860	0.99874	0.99885	0.99895	0.99907

Table - 4

Limiting values of t_i/n for ith order failure time for different values of n and β_w

20% LIMITS

n	i	-βw- 1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.22314	0.28652	0.34253	0.39162	0.43461	0.47238	0.51343	0.54382	0.57959	0.60454	0.64329
2	1	0.11157	0.16030	0.20375	0.25393	0.29571	0.33402	0.37730	0.41593	0.45046	0.48141	0.52465
2	2	0.59278	0.64677	0.68831	0.72121	0.74783	0.76992	0.79262	0.81126	0.82683	0.84004	0.85744
3	1	0.07438	0.11470	0.15623	0.19709	0.23607	0.27273	0.31509	0.35366	0.38871	0.42055	0.46567
3	2	0.33847	0.40545	0.46126	0.50810	0.54780	0.53178	0.61787	0.64335	0.67440	0.69691	0.72715
3	3	0.87900	0.89810	0.71200	0.92256	0.95086	0.93755	0.94427	0.94972	0.95419	0.75792	0.96273
4	1	0.05579	0.09025	0.12725	0.16466	0.20120	0.23619	0.27727	0.31522	0.35010	0.38210	0.42789
4	2	0.23866	0.30303	0.35938	0.40342	0.45115	0.48853	0.52900	0.56378	0.59393	0.62029	0.65614
4	3	0.54051	0.59837	0.64438	0.68077	0.71049	0.73519	0.76075	0.78184	0.79953	0.81453	0.83447
4	4	1.10485	1.08664	1.07382	1.06430	1.05696	1.05112	1.04531	1.04069	1.03692	1.03380	1.02976
5	1	0.04463	0.07493	0.10850	0.14322	0.17774	0.21126	0.25109	0.28830	0.32231	0.35471	0.40071
5	2	0.18465	0.24470	0.29921	0.34792	0.39122	0.42971	0.47200	0.50880	0.54103	0.56945	0.60345
5	3	0.39541	0.46154	0.51544	0.55976	0.59723	0.62832	0.66205	0.68995	0.71363	0.73393	0.76118
5	4	0.71296	0.75432	0.78532	0.80940	0.82365	0.84437	0.86039	0.87342	0.88424	0.89335	0.90528
5	5	1.29018	1.23654	1.19960	1.17262	1.15205	1.13586	1.11990	1.10729	1.09708	1.08864	1.07782

Table - 5

Limiting values of t_i/n for ith order failure time for different values of n and β_w

30% LIMITS

n	i	-βw- 1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.35667	0.42354	0.47884	0.52501	0.56398	0.57722	0.63243	0.66202	0.68737	0.70918	0.7324
2	1	0.17834	0.23770	0.29186	0.34043	0.38373	0.42230	0.46475	0.50176	0.53423	0.56233	0.6022
2	2	0.79346	0.82465	0.84768	0.86537	0.87939	0.89076	0.70229	0.91161	0.91931	0.92578	0.9342
3	1	0.11339	0.16955	0.21847	0.26422	0.30633	0.34481	0.38311	0.42664	0.46099	0.49172	0.5345
3	2	0.45139	0.51538	0.56657	0.60327	0.64261	0.67136	0.70221	0.72748	0.74833	0.76710	0.7714
3	3	1.10595	1.08336	1.07527	1.06556	1.05807	1.05211	1.04619	1.04148	1.03764	1.03445	1.0303
4	1	0.08917	0.13341	0.17789	0.22074	0.26107	0.29661	0.34153	0.38026	0.41520	0.44676	0.4911
4	2	0.31778	0.38489	0.44114	0.48865	0.52912	0.56390	0.60096	0.63235	0.65726	0.68255	0.7139
4	3	0.67645	0.72201	0.75640	0.78326	0.80482	0.82246	0.84054	0.85527	0.86751	0.87784	0.8914
4	4	1.34737	1.28207	1.23736	1.20485	1.18016	1.16077	1.14170	1.12667	1.11452	1.10450	1.0916
5	1	0.07133	0.11077	0.15168	0.19201	0.23065	0.26709	0.30922	0.34779	0.38224	0.41473	0.4599
5	2	0.24593	0.31071	0.36717	0.41616	0.45874	0.49592	0.53611	0.57060	0.60045	0.62653	0.6619
5	3	0.49400	0.55561	0.60427	0.64354	0.67585	0.70285	0.73093	0.75421	0.77330	0.79051	0.8126
5	4	0.86273	0.83422	0.89990	0.91135	0.92124	0.92833	0.93648	0.94265	0.94772	0.95197	0.9575
5	5	1.54179	1.43446	1.36240	1.31074	1.27192	1.24169	1.21219	1.18908	1.17050	1.15525	1.1358

Table - 6

Limiting values of t_i/n for ith order failure time for different values of **n** and β_W

40% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.51033	0.57134	0.61890	0.65716	0.68854	0.71472	0.74190	0.76432	0.78328	0.79939	0.82073
2	1	0.25541	0.32065	0.37723	0.42611	0.46343	0.50538	0.54520	0.57929	0.60277	0.63447	0.66935
2	2	1.00091	1.00076	1.00065	1.00057	1.00051	1.00046	1.00040	1.00036	1.00033	1.00030	1.00027
3	1	0.17028	0.22871	0.28237	0.33073	0.37397	0.41264	0.45529	0.49256	0.52531	0.55426	0.59411
3	2	0.56727	0.62349	0.66702	0.70165	0.72932	0.75318	0.77728	0.79711	0.81371	0.82781	0.84642
3	3	1.33487	1.27213	1.22913	1.19784	1.17405	1.15536	1.13697	1.12247	1.11074	1.10105	1.08866
4	1	0.12771	0.17996	0.22992	0.27630	0.31375	0.35736	0.40065	0.43902	0.47314	0.50358	0.54591
4	2	0.39923	0.46525	0.51899	0.56334	0.60043	0.63185	0.66492	0.69261	0.71613	0.73634	0.76333
4	3	0.81031	0.83765	0.86085	0.87715	0.89002	0.90045	0.91100	0.91953	0.92657	0.93243	0.94018
4	4	1.58607	1.46870	1.39023	1.33414	1.29208	1.25939	1.22753	1.20262	1.18262	1.16620	1.14530
5	1	0.10217	0.14942	0.19605	0.24033	0.28159	0.31963	0.36282	0.40153	0.43626	0.46748	0.51123
5	2	0.30866	0.37546	0.43186	0.47965	0.52045	0.55557	0.59306	0.62487	0.65216	0.67531	0.70770
5	3	0.59105	0.64519	0.68627	0.71989	0.74666	0.76820	0.79157	0.81031	0.82595	0.83922	0.85670
5	4	1.00790	1.00658	1.00564	1.00493	1.00438	1.00394	1.00350	1.00315	1.00286	1.00263	1.00232
5	5	1.78709	1.62227	1.51393	1.43745	1.38064	1.33682	1.29439	1.26142	1.23506	1.21353	1.18621

Table - 7

Limiting values of t_i/n for ith order failure time for different values of **n** and β_W

60% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
1	1	0.91629	0.92974	0.93747	0.94623	0.95259	0.95723	0.96189	0.96564	0.96871	0.97123	0.97462
2	1	0.45815	0.52130	0.57281	0.61394	0.64814	0.67686	0.70686	0.73182	0.75288	0.77091	0.79487
2	2	1.43935	1.39408	1.32946	1.28297	1.24794	1.22060	1.19386	1.17239	1.15601	1.14213	1.12441
3	1	0.30543	0.37219	0.42863	0.47651	0.51742	0.55266	0.59030	0.62225	0.64967	0.67345	0.70551
3	2	0.83713	0.86234	0.88078	0.89427	0.90593	0.91497	0.92405	0.93138	0.93742	0.94248	0.94907
3	3	1.85427	1.67293	1.55436	1.47096	1.40724	1.36172	1.31577	1.28017	1.25175	1.22854	1.19915
4	1	0.22707	0.29285	0.34901	0.39809	0.44099	0.47862	0.51945	0.55461	0.58514	0.61137	0.64327
4	2	0.53789	0.64231	0.68424	0.71742	0.74444	0.76674	0.78970	0.80857	0.82434	0.83772	0.85535
4	3	1.11119	1.09184	1.07622	1.06812	1.06032	1.05413	1.04797	1.04303	1.03908	1.03577	1.03150
4	4	2.12119	1.87133	1.71109	1.59997	1.51857	1.45643	1.39684	1.35093	1.31449	1.28487	1.24754
5	1	0.18326	0.24316	0.29757	0.34627	0.38957	0.42809	0.47041	0.50725	0.53954	0.56801	0.60709
5	2	0.45411	0.51796	0.56900	0.61055	0.64496	0.67387	0.70409	0.72923	0.75046	0.76863	0.79280
5	3	0.80637	0.83625	0.85709	0.87448	0.88761	0.89826	0.90903	0.91774	0.92493	0.93097	0.93883
5	4	1.32583	1.26499	1.22322	1.19279	1.16966	1.15147	1.13357	1.11944	1.10802	1.09859	1.08650
5	5	2.33181	2.02494	1.83077	1.69750	1.60057	1.52703	1.45637	1.40306	1.36053	1.32606	1.28276

Table - 8

Limiting values of t_i/n for ith order failure time for different values of n and β_W

70% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
		-βw-										
1	1	1.20397	1.16729	1.14178	1.12301	1.10863	1.09726	1.08600	1.07708	1.06933	1.06335	1.056
2	1	0.60199	0.65512	0.69592	0.72818	0.75431	0.77588	0.79307	0.81627	0.83148	0.84436	0.861
2	2	1.81192	1.64103	1.52892	1.44590	1.37127	1.34603	1.30235	1.26840	1.24123	1.21912	1.191
3	1	0.40132	0.46723	0.52093	0.56518	0.60217	0.63350	0.66646	0.69406	0.71749	0.73762	0.764
3	2	1.01264	1.01053	1.00901	1.00788	1.00700	1.00630	1.00560	1.00504	1.00453	1.00420	1.003
3	3	2.18840	1.92061	1.74964	1.63147	1.54511	1.47932	1.41634	1.36739	1.32948	1.29830	1.259
4	1	0.30079	0.36757	0.42417	0.47217	0.51323	0.54863	0.58647	0.61862	0.64623	0.67017	0.702
4	2	0.71010	0.75179	0.78307	0.80737	0.82630	0.84267	0.85885	0.87202	0.88295	0.89215	0.904
4	3	1.30054	1.24481	1.20647	1.17850	1.15718	1.14041	1.12389	1.11084	1.10027	1.09154	1.030
4	4	2.46148	2.11834	1.90294	1.75589	1.64942	1.56891	1.49233	1.43377	1.38756	1.35020	1.303
5	1	0.24079	0.30527	0.36168	0.41070	0.45339	0.49071	0.53110	0.56579	0.59586	0.62213	0.657
5	2	0.54820	0.60596	0.65092	0.68681	0.71608	0.74040	0.76555	0.78627	0.80365	0.81842	0.837
5	3	0.94207	0.95149	0.95827	0.96339	0.96739	0.97061	0.97383	0.97641	0.97854	0.98031	0.982
5	4	1.52314	1.41978	1.35060	1.30030	1.26334	1.23415	1.20564	1.18330	1.16533	1.15057	1.131
5	5	2.67532	2.27099	2.01989	1.84995	1.72773	1.63579	1.54375	1.48246	1.43034	1.38831	1.335

Table - 9

Limiting values of t_i/n for ith order failure time for different values of n and β_W

80% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
		-βw-										
1	1	1.50944	1.48672	1.40433	1.34639	1.30262	1.26864	1.23554	1.20968	1.18392	1.17190	1.1502
2	1	0.80472	0.83439	0.85625	0.87303	0.88630	0.89706	0.90795	0.91676	0.92404	0.93014	0.9381
2	2	2.24835	1.96436	1.78374	1.65926	1.56849	1.49945	1.43346	1.38276	1.34261	1.31005	1.2690
3	1	0.53648	0.59515	0.64095	0.67760	0.70754	0.73245	0.75823	0.77951	0.79736	0.81255	0.83264
3	2	1.24778	1.20259	1.17131	1.14838	1.13036	1.11704	1.10339	1.09259	1.08383	1.07658	1.06727
3	3	2.63551	2.24244	1.99810	1.83249	1.71322	1.62343	1.53234	1.47349	1.42247	1.38130	1.32979
4	1	0.40236	0.46829	0.52189	0.56609	0.60303	0.63432	0.66723	0.69478	0.71816	0.73825	0.76509
4	2	0.87336	0.89329	0.90731	0.91825	0.92753	0.93454	0.94159	0.94728	0.95195	0.95587	0.96096
4	3	1.54967	1.44057	1.36737	1.31492	1.27553	1.24486	1.21493	1.19151	1.17268	1.15721	1.13750
4	4	2.91400	2.43823	2.14673	1.95122	1.81155	1.70704	1.60357	1.53390	1.47538	1.42833	1.36967
5	1	0.32189	0.38832	0.44500	0.49240	0.53272	0.56735	0.60423	0.63545	0.66219	0.68533	0.71649
5	2	0.67372	0.71956	0.75420	0.78127	0.80299	0.82081	0.83901	0.85337	0.86622	0.87665	0.89033
5	3	1.11903	1.09825	1.08364	1.07252	1.06447	1.05784	1.05125	1.04601	1.04174	1.03820	1.03363
5	4	1.78017	1.61703	1.50974	1.43397	1.37767	1.33423	1.29216	1.25946	1.23332	1.21196	1.18486
5	5	3.13151	2.58905	2.26005	2.04106	1.88550	1.76964	1.66089	1.57373	1.51453	1.46304	1.39899

Table - 10

Limiting values of t_i/n for ith order failure time for different vlaues of **n** and β_W

90% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
							-βw-					
1	1	2.30259	2.00376	1.81437	1.68417	1.58939	1.51743	1.44872	1.39600	1.35430	1.32050	1.278
2	1	1.15129	1.12457	1.10537	1.09205	1.08141	1.07298	1.06462	1.05797	1.05257	1.04803	1.042
2	2	2.96974	2.47703	2.17599	1.97446	1.83072	1.72329	1.62217	1.54557	1.48559	1.43738	1.377
3	1	0.76753	0.80213	0.82780	0.84759	0.86330	0.87609	0.88906	0.89958	0.90827	0.91558	0.925
3	2	1.63066	1.50304	1.41804	1.35746	1.31214	1.27697	1.24275	1.21603	1.19460	1.17703	1.154
3	3	3.36649	2.74982	2.37923	2.13543	1.96280	1.83480	1.71515	1.62507	1.55489	1.49874	1.429
4	1	0.57565	0.63115	0.67404	0.70810	0.73579	0.75871	0.78235	0.80179	0.81806	0.83186	0.850
4	2	1.13800	1.11374	1.09673	1.08415	1.07446	1.06677	1.05914	1.05307	1.04813	1.04403	1.038
4	3	1.94800	1.74311	1.61009	1.51702	1.44338	1.39571	1.34495	1.30568	1.27440	1.24891	1.216
4	4	3.64980	2.94143	2.52128	2.24604	2.05292	1.91045	1.77786	1.67845	1.60126	1.53965	1.463
5	1	0.46052	0.52405	0.57473	0.61593	0.65000	0.67861	0.70849	0.73333	0.75430	0.77223	0.796
5	2	0.87681	0.89623	0.91037	0.92112	0.92957	0.93638	0.94324	0.94877	0.95332	0.95712	0.962
5	3	1.39954	1.32352	1.27157	1.23395	1.20547	1.18315	1.16124	1.14401	1.13011	1.11865	1.103
5	4	2.18716	1.91970	1.74693	1.63089	1.54462	1.47891	1.41598	1.36758	1.32921	1.29806	1.258
5	5	3.87032	3.08380	2.62717	2.32993	2.12093	1.96731	1.82482	1.71830	1.63579	1.57006	1.488

Table - 11

Limiting values of t_i/n for ith order failure time for different values of **n** and β_W

95% LIMITS

n	i	1.00	1.20	1.40	1.60	1.80	2.00	2.25	2.50	2.75	3.00	3.40
							-βw-					
1	1	2.99573	2.49509	2.18957	1.98525	1.83960	1.73082	1.62247	1.55096	1.49030	1.44157	1.330
2	1	1.49737	1.40032	1.33456	1.23727	1.25166	1.22337	1.19671	1.17541	1.15327	1.14417	1.126
2	2	3.67614	2.95910	2.53426	2.25616	2.06114	1.91733	1.78355	1.68322	1.60545	1.54335	1.466
3	1	0.99858	0.99851	0.99898	0.99911	0.99921	0.99929	0.99937	0.99943	0.99948	0.99953	0.999
3	2	1.97939	1.78171	1.64061	1.54216	1.46969	1.41417	1.36076	1.31948	1.22664	1.25790	1.226
3	3	4.07734	3.22589	2.72383	2.40705	2.18323	2.01924	1.86758	1.75449	1.66703	1.59757	1.511
4	1	0.74393	0.78590	0.81342	0.83469	0.85162	0.86541	0.87942	0.89079	0.90021	0.90813	0.918
4	2	1.39139	1.31726	1.26641	1.22957	1.20166	1.17978	1.15331	1.14141	1.12777	1.11653	1.102
4	3	2.32676	2.02123	1.32795	1.69520	1.59864	1.52537	1.45546	1.40185	1.35945	1.32511	1.281
4	4	4.36229	3.41307	2.86406	2.51107	2.26689	2.06875	1.92462	1.30264	1.70862	1.63402	1.542
5	1	0.59915	0.65255	0.69358	0.72604	0.75233	0.77405	0.79639	0.81473	0.83005	0.84303	0.860
5	2	1.07122	1.05900	1.05037	1.04393	1.03896	1.03500	1.03105	1.02790	1.02533	1.02320	1.020
5	3	1.66406	1.52911	1.43909	1.37508	1.32727	1.29022	1.25420	1.22611	1.20360	1.18515	1.161
5	4	2.57124	2.19678	1.96318	1.80443	1.68989	1.60351	1.52155	1.45901	1.40975	1.35998	1.320
5	5	4.58476	3.55711	2.96735	2.59014	2.33023	2.14120	1.96752	1.83877	1.73972	1.66126	1.564

Table 12
Comparison of the Weibull parameter estimates (β_W, η) by different methods

Example	Failures	SEQLIM		MLE			BLIE		GLUE		Least Squares	
	r	β_W	η	$\hat{\beta}_W$	Bn $\hat{\beta}_W$	$\hat{\eta}$	$\widehat{\beta}_W$	$\widehat{\eta}$	$\tilde{\beta}_W$	$\tilde{\eta}$	$\hat{\hat{\beta}}_W$	$\hat{\hat{\eta}}$
1	1	2.75	29.836	–	–	–	–	–	–	–	–	–
	2	2.0	43.244	3.48	–	35.278	3.467	38.448	1.829	42.271	–	–
	3	1.8	48.121	2.73	1.332	42.189	2.673	45.896	1.889	42.623	–	–
	4	1.8	48.121	2.47	1.465	45.438	2.385	49.057	1.907	41.903	–	–
	5	1.8	48.121	2.30	1.608	47.511	2.183	51.002	1.864	50.3	1.875	48
2	1	2.75	47.42	–	–	–	–	–	–	–	–	–
	2	2.75	47.42	5.92	–	40.047	5.895	42.126	3.11	44.542	–	–
	3	2.75	47.42	4.65	2.269	44.491	4.543	46.751	3.211	44.76	–	–
	4	2.75	47.42	4.20	2.491	46.476	4.054	48.619	3.242	44.313	–	–
	5	2.75	47.42	3.91	2.733	47.712	3.712	49.743	3.169	49.339	3.188	48
3	1	2.75	18.423	–	–	–	–	–	–	–	–	–
	2	1.8	33.212	2.44	–	30.916	2.427	34.959	1.281	40.028	–	–
	3	1.6	40.776	1.91	.932	39.917	1.871	45.021	1.322	40.506	–	–
	4	1.4	43.183	1.73	1.026	44.383	1.669	49.518	1.335	39.534	–	–
	5	1.4	47.549	1.61	1.125	47.302	1.528	52.346	1.305	51.319	1.3125	48
4	1	2.75	20.396									
	2	2.0	45.599									
	3	2.0	68.752									
	4, 5	1.8	116.217									
	6	1.8	111.394									
	7	1.8	107.473									
	8	2.0	100.932									
	9	2.0	100.932									
	10	2.0	100.932	1.37		136.6			1.24	158.5		
	20	2.0	100.932	2.09	1.90	83.8			2.08	82.9		
	30	2.0	100.932	1.78	1.68	96.3			1.73	96.9		
	40	2.0	100.932	1.95	1.88	92.8			1.91	91.7		

APPENDIX I

Calculation steps for estimation of the Weibull distribution parameters (β_w, η) for Example 1 by the SEQLIM method.

η is calculated by equation (21); t_r/η is the ratio of the observed failure time to η, for rth failure; $t_{.40}$ and $t_{.60}$ indicate 40% lower and 60% upper probability limits; '-' indicates that the observed failure time is not an outlier, while '*' indicates an outlier; checks of outliers and non-outliers are made from Tables 2-11; '→' on the left side of β_w, indicates an incoming pair .

(i) At 15.58, 1 failure out of (fo o) 5 items)

	β_w	η	t_1/η	$t_{.40}$	$t_{.60}$
				1	1
	1.0	21.738	.7167038	-	*
	1.2	24.752	.62944682	-	*
	1.4	26.672	.58413795	-	*
	1.6	27.897	.55848867	-	*
	1.8	28.68	.54324145	-	*
	2.0	29.178	.53395786	-	*
	2.25	29.549	.52726437	-	*
	2.5	29.745	.5237856	-	*
→	2.75	29.836	.52218492	-	-
	3.0	29.864	.52170373	-	-
	3.4	29.835	.52220801	-	-

∴ Estimates after 1st failure : $\beta_w = 2.75$, $\eta = 29.836$

Prediction for 2nd failure time at 40%-60% level = 19.459 - 22.392 (Using Tables 6 & 7).

Note: Actual 2nd failure time is outside this time bound. Therefore, re-evaluation of β_w and η is necessary.

(ii) At 28.667, 2 foo 5.

β_w	η	t_1/η	t_2/η	$t_{.40}$ 1	$t_{.40}$ 2	$t_{.60}$ 1	$t_{.60}$ 2
1.8	42.102	.37005599	.68089526	-	-	-	*
→ 2.0	43.244	.36028473	.66291928	-	-	-	-
2.25	44.275	.35189468	.64748169	*	-	-	-
2.5	45.017	.34609252	.6368058	*	-	-	-
2.75	45.571	.3418845	.62906309	*	*	-	-

∴ Estimates, after 2nd failure : $\beta_w = 2.0$, $\eta = 43.244$

Prediction for 3rd failure time = 33.246 - 38.846 (at 40%-60% level)

Note: Actual 3rd failutre time violates the limit.

(iii) At 39.963, 3 foo 5

β_w	η	t_1/η	t_2/η	t_3/η	$t_{.40}$ 1	$t_{.40}$ 2	$t_{.40}$ 3	$t_{.60}$ 1	$t_{.60}$ 2	$t_{.60}$ 3
1.6	45.819	.34003007	.62565096	.87218367	-	-	-	-	*	-
→ 1.8	48.121	.3237701	.59573284	.83047655	-	-	-	-	-	-
2.0	49.92	.31209775	.57425585	.80053674	*	-	-	-	-	-

∴ Estimates after 3rd failure : $\beta_w = 1.8$, $\eta = 48.121$

Prediction for 4rth failure time = 48.333 - 56.287 (40%-60% level)

Note: Fourth failure does not violate this limit; no re-evaluation of β_w and η is necessary.

Prediction for 5th failure time = 66.436-77.022 (40%-60% level)

Note: 5th failure also does not violate the time limit; the values of β_w and η remain unchanged.

Final estimates : $\beta_w = 1.8$, $\eta = 48.121$

APPENDIX II

Calculation steps for estimation of the Weibull distribution parameters (β_w, η) for Example 1 (Replacement Case) by the SEQLIM method.

(Notations and symbols are as in Appendix I).

(i) At 15.58 , 1 foo 2

β_W	η	t_1/η	$t_{.40}$	$t_{.60}$
			1	1
2.25	19.664223	.79230182	-	*
2.5	20.61759	.75566544	-	*
→ 2.75	21.381476	.72866814	-	-
3.0	22.003757	.7080609	-	-

∴ Estimates after 1st failure : β_W = 2.75, η = 21.381476

Prediction for ith order failure time is based on (n+1) = 3 items at 40%-60% level.

1st order failure = 11.232 - 13.891

2nd order failure = 17.398 - 20.043

Note: Actual failure times violate these time bounds; re-evaluation is necessary.

(ii) At 28.667, 2 foo 3

β_W	η	t_1/η	t_2/η	$t_{.40}$		$t_{.60}$		$t_{.30}$		$t_{.70}$	
				1	2	1	2	1	2	1	2
2.25	27.230656	.57214926	1.0527473	-	-	-	*	-	-	-	*
→ 2.5	28.613772	.54449306	1.0018602	-	-	-	*	-	-	-	-
2.75	29.806147	.52271097	.96178148	*	-	-	*	-	-	-	-
3.0	30.855031	.50494196	.92908672	*	-	-	-	-	-	-	-

∴ Estimates after 2nd failure : β_W = 2.5, η = 28.613772

Prediction for ith order failure time at 30%-70% level:

1st order failure = 10.881 - 17.701

2nd order failure = 18.094 - 24.952

3rd order failure = 24.473 - 31.785

Note: 2nd and 3rd failute times are outside their bounds.

(iii) At 39.963, 3 foo 4

β_W	η	t_1/η	t_2/η	t_3/η	$t._{40}$	$t._{60}$	$t._{30}$	$t._{70}$
					1 2 3	1 2 3	1 2 3	1 2
1.8	32.443499	.48021947	.88359766	1.2317722	- - -	* * *	- - -	- *
2.0	34.452982	.4522105	.83206151	1.1599286	- - -	- * *	- - -	- -
→ 2.25	36.587427	.42582935	.78352053	1.0922605	- - -	- - *	- - -	- -
2.5	38.410651	.40561666	.74632944	1.0404145	* - -	- - -	- - -	- -

∴ Estimates after 3rd failure : β_W = 2.25, η = 36.587427

Prediction for 1st order failure at 30%-70% level = 11.316-19.432

" " 2nd " " " " " = 19.615-28.01

" " 3rd " " " " " = 26.743-35.63

" " 4th " " " " " = 34.263-44.111

Note: 2nd to 4th failures are outside their respective time limits.

(iv) At 52.668, 4 foo 5

β_W	η	t_1/η	t_2/η	t_3/η	t_4/η	$t._{40}$	$t._{60}$
						1234	1234
→ 1.6	51.947052	.29992077	.55185037	.76930256	1.0138785	----	----
1.8	55.36261	.28141737	.51780434	.72184097	.95132798	****	----
2.0	58.207563	.26766281	.49249614	.68656027	.90483088	****	----
2.25	61.260918	.25432201	.46794924	.65234086	.86329101	****	----

∴ Estimation after 4th failure : β_W = 1.6, η = 51.947052

Since, no replacement is being made after 4th failure, Prediction is made for 5th order failure straight away, at 40%-60% level. Expected time period for 5th order failure = 67.24-75.671

Note: 5th failure does not violate this limit; so previous estimates for β_W and η remain unchanged.

Final estimates : β_W = 1.6, η = 51.947052.

APPENDIX A2 Values of Acceptance Points v_r (r denotes number of failures) to be Used in SEQLIM Method

β_w	v_0	v_1	v_2	v_3	v_4	v_5
0.450	7.896322	8.748458	9.600595	10.452731	11.304868	12.157004
0.475	7.412764	8.257159	9.101554	9.945948	10.790343	11.634738
0.500	6.977975	7.814677	8.651380	9.488083	10.324785	11.161488
0.525	6.584987	7.414047	8.243107	9.072167	9.901227	10.730287
0.550	6.228099	7.049566	7.871033	8.692499	9.513966	10.335433
0.575	5.902603	6.716526	7.530449	8.344371	9.158294	9.972217
0.600	5.604574	6.411002	7.217430	8.023858	8.830286	9.636714
0.625	5.330716	6.129698	6.928681	7.727663	8.526646	9.325628
0.650	5.078238	5.869825	6.661411	7.452997	8.244583	9.036169
0.675	4.844766	5.629005	6.413244	7.197482	7.981721	8.765960
0.700	4.628262	5.405202	6.182143	6.959083	7.736024	8.512964
0.725	4.426970	5.196661	5.966352	6.736043	7.505734	8.275425
0.750	4.239370	5.001860	5.764350	6.526840	7.289331	8.051821
0.775	4.064134	4.819472	5.574811	6.330149	7.085487	7.840825
0.800	3.900104	4.648339	5.396574	6.144809	6.893044	7.641279
0.825	3.746261	4.487441	5.228621	5.969802	6.710982	7.452162
0.850	3.601705	4.335879	5.070053	5.804226	6.538400	7.272574
0.875	3.465640	4.192856	4.920071	5.647287	6.374503	7.101719
0.900	3.337357	4.057663	4.777969	5.498275	6.218581	6.938887
0.925	3.216226	3.929670	4.643115	5.356559	6.070003	6.783448
0.950	3.101681	3.808312	4.514942	5.221573	5.928204	6.634835
0.975	2.993215	3.693080	4.392945	5.092810	5.792675	6.492540
1.000	2.890372	3.583519	4.276666	4.969813	5.662960	6.356108
1.100	2.527550	3.194301	3.861053	4.527805	5.194557	5.861308
1.200	2.227824	2.868936	3.510048	4.151160	4.792272	5.433384
1.300	1.976608	2.592828	3.209048	3.825267	4.441487	5.057707
1.400	1.763480	2.355546	2.947613	3.539679	4.131745	4.723812
1.500	1.580797	2.149439	2.718081	3.286723	3.855365	4.424008
1.600	1.422824	1.968761	2.514699	3.060636	3.606574	4.152511
1.700	1.285175	1.809117	2.333059	2.857001	3.380943	3.904885
1.800	1.164438	1.667083	2.169727	2.672371	3.175016	3.677660
1.900	1.057918	1.539952	2.021985	2.504019	2.986053	3.468086
2.000	0.963457	1.425555	1.887653	2.349752	2.811850	3.273948
2.100	0.879309	1.322135	1.764960	2.207786	2.650611	3.093437
2.200	0.804044	1.228247	1.652451	2.076654	2.500858	2.925061
2.300	0.736480	1.142699	1.548918	1.955137	2.361356	2.767575
2.400	0.675631	1.064491	1.453350	1.842210	2.231069	2.619929
2.500	0.620670	0.992782	1.364893	1.737004	2.109116	2.481227
2.600	0.570896	0.926857	1.282818	1.638779	1.994740	2.350701
2.700	0.525711	0.866106	1.206501	1.546896	1.887291	2.227686
2.800	0.484604	0.810003	1.135402	1.460801	1.786200	2.111599
2.900	0.447131	0.758091	1.069051	1.380011	1.690970	2.001930
3.000	0.412910	0.709973	1.007036	1.304099	1.601163	1.893226
3.400	0.302465	0.549083	0.795701	1.042320	1.288938	1.535556

β_w	v_6	v_7	v_8	v_9	v_{10}
0.450	13.009141	13.861277	14.713413	15.565550	16.417686
0.475	12.479133	13.323528	14.167922	15.012317	15.856712
0.500	11.998151	12.834893	13.671596	14.508299	15.345001
0.525	11.559347	12.388407	13.217467	14.046527	14.875587
0.550	11.156899	11.978366	12.799833	13.621300	14.442766
0.575	10.736140	11.600062	12.413985	13.227908	14.041831
0.600	10.443143	11.249571	12.055999	12.862427	13.668855
0.625	10.124611	10.923594	11.722576	12.521559	13.320541
0.650	9.827755	10.619341	11.410928	12.202514	12.994100
0.675	9.550179	10.334438	11.118677	11.902915	12.687154
0.700	9.289905	10.066845	10.843785	11.620726	12.397666
0.725	9.045116	9.814807	10.584498	11.354189	12.123880
0.750	8.814311	9.576801	10.339292	11.101782	11.864272
0.775	8.596164	9.351502	10.106840	10.862179	11.617517
0.800	8.389514	9.137749	9.885984	10.634219	11.382454
0.825	8.193342	8.934522	9.675702	10.416882	11.158062
0.850	8.006748	8.740921	9.475095	10.209269	10.943443
0.875	7.828934	8.556150	9.283366	10.010531	10.737797
0.900	7.659193	8.379499	9.099805	9.820111	10.540417
0.925	7.496892	8.210336	8.923780	9.637225	10.350669
0.950	7.341465	8.048096	8.754727	9.461357	10.167988
0.975	7.192405	7.892270	8.592135	9.292000	9.991865
1.000	7.049255	7.742402	8.435549	9.128696	9.821844
1.100	6.528060	7.194812	7.861564	8.528315	9.195067
1.200	6.074496	6.715608	7.356720	7.997832	8.638944
1.300	5.673927	6.290147	6.906366	7.522586	8.138806
1.400	5.315878	5.907944	6.500011	7.092077	7.684143
1.500	4.992650	5.561292	6.129934	6.698576	7.267219
1.600	4.698449	5.244386	5.790324	6.336261	6.882199
1.700	4.428826	4.952768	5.476710	6.000652	6.524594
1.800	4.180305	4.682949	5.185593	5.688238	6.190882
1.900	3.950120	4.432154	4.914188	5.396221	5.873255
2.000	3.736046	4.198144	4.660242	5.122340	5.584438
2.100	3.536262	3.979088	4.421913	4.864739	5.307564
2.200	3.349265	3.773468	4.197672	4.621875	5.046078
2.300	3.173794	3.580014	3.986233	4.392452	4.798671
2.400	3.008788	3.397648	3.786507	4.175367	4.564226
2.500	2.853338	3.225449	3.597561	3.969672	4.341783
2.600	2.706662	3.062623	3.418584	3.774545	4.130506
2.700	2.568011	2.908475	3.243870	3.589265	3.929660
2.800	2.436998	2.762398	3.087797	3.413196	3.738595
2.900	2.312890	2.623850	2.934810	3.245770	3.556730
3.000	2.195239	2.492352	2.789415	3.086478	3.383541
3.400	1.782175	2.028793	2.275411	2.522030	2.768648

APPENDIX A3

100ε Per Cent Limiting Values of $t_\varepsilon(r)/n$ for rth Order Failure Time for Different Values of n and β_w

5% LIMITS

n	r	-βw- .450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
1	1	0.00136	0.00192	0.00263	0.00349	0.00451	0.00571	0.00708	0.00863	0.01036	0.01227	0.01436
2	1	0.00029	0.00045	0.00066	0.00093	0.00128	0.00171	0.00223	0.00285	0.00357	0.00440	0.00534
2	2	0.04720	0.05543	0.06406	0.07301	0.08224	0.09167	0.10127	0.11098	0.12078	0.13061	0.14046
3	1	0.00012	0.00019	0.00029	0.00043	0.00061	0.00084	0.00113	0.00149	0.00191	0.00241	0.00299
3	2	0.01378	0.01727	0.02115	0.02541	0.03003	0.03497	0.04022	0.04574	0.05150	0.05748	0.06365
3	3	0.17764	0.19455	0.21114	0.22737	0.24321	0.25863	0.27362	0.28818	0.30230	0.31600	0.32927
4	1	0.00006	0.00010	0.00016	0.00025	0.00036	0.00051	0.00070	0.00094	0.00123	0.00157	0.00198
4	2	0.00636	0.00830	0.01055	0.01310	0.01596	0.01910	0.02253	0.02622	0.03016	0.03433	0.03873
4	3	0.06165	0.07160	0.08169	0.09204	0.10259	0.11326	0.12402	0.13482	0.14562	0.15639	0.16711
4	4	0.37133	0.39120	0.41000	0.42778	0.44461	0.46056	0.47568	0.49003	0.50366	0.51662	0.52895
5	1	0.00004	0.00006	0.00011	0.00016	0.00024	0.00035	0.00048	0.00066	0.00087	0.00113	0.00144
5	2	0.00360	0.00484	0.00632	0.00805	0.01002	0.01224	0.01470	0.01741	0.02034	0.02350	0.02687
5	3	0.03111	0.03734	0.04402	0.05108	0.05847	0.06615	0.07408	0.08220	0.09050	0.09892	0.10744
5	4	0.14565	0.16056	0.17594	0.19112	0.20605	0.22069	0.23504	0.24905	0.26273	0.27606	0.28905
5	5	0.60362	0.62007	0.63506	0.64894	0.66183	0.67381	0.68499	0.69543	0.70522	0.71440	0.72303
6	1	0.00003	0.00004	0.00007	0.00012	0.00017	0.00025	0.00036	0.00049	0.00066	0.00086	0.00111
6	2	0.00229	0.00316	0.00421	0.00547	0.00693	0.00860	0.01048	0.01258	0.01489	0.01740	0.02011
6	3	0.01855	0.02288	0.02764	0.03279	0.03830	0.04413	0.05026	0.05665	0.06326	0.07007	0.07705
6	4	0.07760	0.08877	0.10020	0.11180	0.12351	0.13527	0.14703	0.15874	0.17039	0.18193	0.19335
6	5	0.25599	0.27502	0.29336	0.31100	0.32795	0.34424	0.35988	0.37490	0.38932	0.40316	0.41645
6	6	0.85891	0.86581	0.87207	0.87776	0.88299	0.88778	0.89220	0.89628	0.90006	0.90358	0.90686
7	1	0.00002	0.00003	0.00005	0.00009	0.00013	0.00019	0.00028	0.00038	0.00052	0.00069	0.00089
7	2	0.00158	0.00222	0.00301	0.00397	0.00510	0.00642	0.00792	0.00961	0.01149	0.01356	0.01581
7	3	0.01223	0.01542	0.01900	0.02294	0.02724	0.03186	0.03678	0.04197	0.04742	0.05308	0.05895
7	4	0.04812	0.05646	0.06518	0.07423	0.08355	0.09307	0.10275	0.11254	0.12240	0.13230	0.14222
7	5	0.14350	0.15894	0.17425	0.18936	0.20424	0.21885	0.23315	0.24713	0.26078	0.27409	0.28706
7	6	0.38732	0.40714	0.42585	0.44352	0.46022	0.47601	0.49097	0.50514	0.51858	0.53135	0.54348
7	7	1.12654	1.11950	1.11320	1.10753	1.10240	1.09774	1.09348	1.08958	1.08599	1.08268	1.07961
8	1	0.00001	0.00002	0.00004	0.00007	0.00010	0.00015	0.00022	0.00031	0.00042	0.00056	0.00074
8	2	0.00115	0.00164	0.00226	0.00302	0.00393	0.00500	0.00623	0.00763	0.00921	0.01095	0.01287
8	3	0.00862	0.01106	0.01387	0.01701	0.02047	0.02424	0.02830	0.03264	0.03723	0.04206	0.04710
8	4	0.03252	0.03905	0.04593	0.05319	0.06077	0.06864	0.07675	0.08505	0.09351	0.10208	0.11075
8	5	0.09181	0.10410	0.11657	0.12913	0.14172	0.15429	0.16678	0.17917	0.19142	0.20351	0.21541
8	6	0.22527	0.24365	0.26147	0.27872	0.29538	0.31147	0.32698	0.34195	0.35634	0.37022	0.38360
8	7	0.53328	0.55122	0.56788	0.58339	0.59786	0.61138	0.62404	0.63592	0.64709	0.65761	0.66753
8	8	1.40046	1.37586	1.35408	1.33468	1.31728	1.30159	1.28737	1.27443	1.26260	1.25174	1.24174
9	1	0.00001	0.00002	0.00003	0.00005	0.00008	0.00013	0.00018	0.00026	0.00035	0.00047	0.00062
9	2	0.00087	0.00126	0.00175	0.00237	0.00312	0.00401	0.00505	0.00624	0.00759	0.00909	0.01075
9	3	0.00638	0.00833	0.01058	0.01314	0.01600	0.01915	0.02258	0.02628	0.03023	0.03441	0.03881
9	4	0.02347	0.02860	0.03416	0.04012	0.04643	0.05306	0.05997	0.06711	0.07446	0.08198	0.08964
9	5	0.06363	0.07356	0.08381	0.09432	0.10500	0.11581	0.12670	0.13761	0.14852	0.15939	0.17019
9	6	0.14781	0.16346	0.17895	0.19423	0.20926	0.22398	0.23839	0.25246	0.26619	0.27956	0.29258
9	7	0.31957	0.33935	0.35819	0.37614	0.39323	0.40952	0.42504	0.43983	0.45395	0.46743	0.48030
9	8	0.68957	0.70320	0.71569	0.72718	0.73773	0.74760	0.75672	0.76521	0.77312	0.78053	0.78746
9	9	1.67673	1.63174	1.59227	1.55739	1.52634	1.49854	1.47349	1.45082	1.43020	1.41138	1.39411
10	1	0.00001	0.00002	0.00003	0.00004	0.00007	0.00010	0.00015	0.00022	0.00030	0.00040	0.00053
10	2	0.00063	0.00099	0.00140	0.00192	0.00255	0.00331	0.00419	0.00522	0.00639	0.00771	0.00917
10	3	0.00490	0.00648	0.00834	0.01047	0.01288	0.01557	0.01851	0.02172	0.02516	0.02884	0.03274
10	4	0.01765	0.02182	0.02642	0.03141	0.03677	0.04244	0.04842	0.05465	0.06111	0.06778	0.07462
10	5	0.04658	0.05474	0.06330	0.07219	0.08135	0.09073	0.10027	0.10993	0.11968	0.12947	0.13927
10	6	0.11439	0.11757	0.13065	0.14416	0.15745	0.17061	0.18365	0.19653	0.20922	0.22170	0.23396
10	7	0.21416	0.23226	0.24763	0.26490	0.28342	0.29939	0.31482	0.32971	0.34409	0.35796	0.37133
10	8	0.42364	0.44323	0.46154	0.47895	0.49524	0.51061	0.52511	0.53882	0.55179	0.56407	0.57572
10	9	0.85306	0.86024	0.86674	0.87267	0.87806	0.88306	0.88765	0.89189	0.89582	0.89948	0.90289
10	10	1.95233	1.88524	1.82541	1.77476	1.72903	1.68840	1.65195	1.61912	1.58938	1.56234	1.53764

10% LIMITS

-βν-

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
1	1	0.00673	0.00876	0.01110	0.01375	0.01671	0.01997	0.02350	0.02731	0.03136	0.03565	0.04016
2	1	0.00144	0.00204	0.00278	0.00367	0.00474	0.00598	0.00740	0.00901	0.01080	0.01277	0.01492
2	2	0.11655	0.13051	0.14450	0.15844	0.17223	0.18597	0.19947	0.21276	0.22581	0.23860	0.25113
3	1	0.00059	0.00087	0.00123	0.00170	0.00227	0.00295	0.00377	0.00471	0.00579	0.00700	0.00836
3	2	0.03384	0.04045	0.04748	0.05490	0.06264	0.07066	0.07891	0.08734	0.09593	0.10463	0.11342
3	3	0.35053	0.37042	0.38927	0.40716	0.42413	0.44025	0.45556	0.47011	0.48396	0.49715	0.50971
4	1	0.00031	0.00047	0.00069	0.00098	0.00134	0.00179	0.00233	0.00297	0.00372	0.00457	0.00554
4	2	0.01560	0.01942	0.02366	0.02827	0.03325	0.03855	0.04415	0.05002	0.05613	0.06245	0.06895
4	3	0.12082	0.13504	0.14926	0.16341	0.17743	0.19129	0.20494	0.21835	0.23151	0.24440	0.25701
4	4	0.65446	0.66922	0.68280	0.69532	0.70690	0.71764	0.72763	0.73694	0.74565	0.75380	0.76144
5	1	0.00019	0.00030	0.00044	0.00064	0.00090	0.00122	0.00161	0.00208	0.00264	0.00329	0.00403
5	2	0.00883	0.01133	0.01417	0.01736	0.02087	0.02469	0.02881	0.03320	0.03785	0.04273	0.04782
5	3	0.06060	0.07023	0.08021	0.09045	0.10089	0.11146	0.12214	0.13285	0.14358	0.15428	0.16493
5	4	0.25224	0.27120	0.28949	0.30709	0.32402	0.34029	0.35593	0.37094	0.38536	0.39922	0.41253
5	5	0.99300	0.99337	0.99370	0.99400	0.99427	0.99452	0.99474	0.99495	0.99515	0.99533	0.99549
6	1	0.00013	0.00020	0.00031	0.00045	0.00064	0.00089	0.00119	0.00155	0.00199	0.00251	0.00311
6	2	0.00562	0.00739	0.00944	0.01179	0.01443	0.01734	0.02054	0.02399	0.02769	0.03162	0.03578
6	3	0.03608	0.04298	0.05030	0.05800	0.06601	0.07429	0.08279	0.09147	0.10028	0.10920	0.11818
6	4	0.13437	0.14934	0.16423	0.17899	0.19355	0.20787	0.22193	0.23570	0.24918	0.26234	0.27518
6	5	0.41440	0.43406	0.45256	0.46997	0.48638	0.50186	0.51649	0.53032	0.54342	0.55583	0.56761
6	6	1.34709	1.32613	1.30755	1.29096	1.27606	1.26260	1.25040	1.23927	1.22909	1.21973	1.21111
7	1	0.00009	0.00015	0.00023	0.00034	0.00049	0.00068	0.00092	0.00121	0.00157	0.00200	0.00249
7	2	0.00387	0.00518	0.00674	0.00855	0.01062	0.01294	0.01551	0.01832	0.02137	0.02464	0.02812
7	3	0.02378	0.02895	0.03456	0.04056	0.04692	0.05360	0.06055	0.06774	0.07513	0.08269	0.09038
7	4	0.08316	0.09479	0.10664	0.11864	0.13071	0.14280	0.15486	0.16686	0.17876	0.19053	0.20215
7	5	0.23105	0.24957	0.26751	0.28485	0.30158	0.31771	0.33326	0.34823	0.36265	0.37654	0.38990
7	6	0.59696	0.61340	0.62857	0.64262	0.65567	0.66781	0.67914	0.68974	0.69966	0.70898	0.71774
7	7	1.70638	1.65905	1.61759	1.58096	1.54839	1.51923	1.49299	1.46924	1.44766	1.42796	1.40991
8	1	0.00007	0.00011	0.00017	0.00026	0.00038	0.00054	0.00073	0.00098	0.00128	0.00164	0.00206
8	2	0.00281	0.00383	0.00505	0.00650	0.00817	0.01007	0.01220	0.01455	0.01712	0.01991	0.02290
8	3	0.01676	0.02078	0.02523	0.03006	0.03525	0.04077	0.04658	0.05266	0.05897	0.06549	0.07219
8	4	0.05630	0.06551	0.07507	0.08492	0.09500	0.10523	0.11558	0.12600	0.13645	0.14690	0.15731
8	5	0.14744	0.16307	0.17854	0.19381	0.20852	0.22355	0.23793	0.25200	0.26572	0.27909	0.29210
8	6	0.34504	0.36491	0.38378	0.40168	0.41869	0.43484	0.45019	0.46480	0.47870	0.49194	0.50456
8	7	0.79281	0.80256	0.81144	0.81955	0.82700	0.83386	0.84019	0.84606	0.85152	0.85661	0.86135
8	8	2.06509	1.98776	1.92064	1.86187	1.81000	1.76390	1.72268	1.68561	1.65210	1.62166	1.59390
9	1	0.00005	0.00009	0.00014	0.00021	0.00031	0.00044	0.00060	0.00081	0.00107	0.00138	0.00174
9	2	0.00212	0.00294	0.00393	0.00512	0.00650	0.00810	0.00989	0.01190	0.01411	0.01652	0.01913
9	3	0.01240	0.01562	0.01923	0.02321	0.02755	0.03220	0.03716	0.04239	0.04787	0.05357	0.05947
9	4	0.04049	0.04794	0.05580	0.06402	0.07254	0.08131	0.09027	0.09938	0.10861	0.11792	0.12728
9	5	0.10204	0.11507	0.12821	0.14138	0.15453	0.16760	0.18055	0.19334	0.20596	0.21837	0.23057
9	6	0.22570	0.24410	0.26193	0.27919	0.29586	0.31194	0.32746	0.34241	0.35682	0.37070	0.38408
9	7	0.47183	0.49085	0.50863	0.52527	0.54087	0.55552	0.56929	0.58227	0.59451	0.60607	0.61700
9	8	0.99709	0.99724	0.99738	0.99750	0.99762	0.99772	0.99781	0.99790	0.99798	0.99806	0.99813
9	9	2.41999	2.31010	2.21530	2.13296	2.06077	1.99699	1.94026	1.88950	1.84381	1.80250	1.76497
10	1	0.00004	0.00007	0.00011	0.00017	0.00025	0.00036	0.00051	0.00069	0.00091	0.00118	0.00150
10	2	0.00156	0.00232	0.00314	0.00414	0.00531	0.00667	0.00821	0.00995	0.01188	0.01401	0.01631
10	3	0.00951	0.01216	0.01515	0.01850	0.02216	0.02617	0.03046	0.03503	0.03985	0.04490	0.05016
10	4	0.03043	0.03657	0.04315	0.05011	0.05742	0.06501	0.07285	0.08090	0.08912	0.09746	0.10591
10	5	0.07463	0.08556	0.09675	0.10813	0.11963	0.13120	0.14279	0.15435	0.16585	0.17727	0.18856
10	6	0.15911	0.17528	0.19122	0.20690	0.22226	0.23727	0.25193	0.26621	0.28012	0.29363	0.30677
10	7	0.31509	0.33463	0.35366	0.37161	0.38871	0.40501	0.42056	0.43538	0.44953	0.46304	0.47595
10	8	0.60800	0.62414	0.63912	0.65279	0.66537	0.67746	0.68854	0.69890	0.70859	0.71769	0.72624
10	9	1.20645	1.19459	1.18431	1.17453	1.16597	1.15821	1.15115	1.14468	1.13875	1.13328	1.12823
10	10	2.76924	2.62469	2.50104	2.39423	2.30108	2.21919	2.14669	2.08209	2.02418	1.97200	1.92475

20% LIMITS

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
						-βw-						
1	1	0.03568	0.04252	0.04979	0.05744	0.06540	0.07364	0.08209	0.09073	0.09950	0.10838	0.11733
2	1	0.00765	0.00968	0.01245	0.01554	0.01855	0.02206	0.02586	0.02993	0.03425	0.03881	0.04359
2	2	0.31284	0.33257	0.35139	0.36934	0.38644	0.40275	0.41831	0.43315	0.44731	0.46084	0.47377
3	1	0.00311	0.00421	0.00553	0.00709	0.00887	0.01090	0.01316	0.01564	0.01836	0.02129	0.02442
3	2	0.09005	0.10222	0.11456	0.12701	0.13950	0.15198	0.16439	0.17670	0.18888	0.20091	0.21276
3	3	0.75082	0.76223	0.77265	0.78220	0.79098	0.79908	0.80659	0.81355	0.82003	0.82608	0.83174
4	1	0.00164	0.00230	0.00311	0.00410	0.00526	0.00661	0.00814	0.00987	0.01179	0.01390	0.01619
4	2	0.04143	0.04898	0.05696	0.06529	0.07391	0.08277	0.09183	0.10103	0.11034	0.11973	0.12916
4	3	0.25481	0.27383	0.29215	0.30978	0.32673	0.34301	0.35865	0.37366	0.38808	0.40193	0.41523
4	4	1.24805	1.23358	1.22070	1.20916	1.19877	1.18936	1.18079	1.17297	1.16579	1.15919	1.15309
5	1	0.00100	0.00144	0.00199	0.00268	0.00351	0.00448	0.00562	0.00691	0.00837	0.00999	0.01177
5	2	0.02343	0.02854	0.03410	0.04005	0.04636	0.05298	0.05988	0.06702	0.07436	0.08187	0.08952
5	3	0.12722	0.14180	0.15635	0.17080	0.18506	0.19917	0.21302	0.22661	0.23993	0.25295	0.26568
5	4	0.47149	0.49032	0.50831	0.52495	0.54056	0.55521	0.56899	0.58197	0.59422	0.60578	0.61672
5	5	1.76154	1.70982	1.66457	1.62467	1.58922	1.55753	1.52904	1.50329	1.47990	1.45857	1.43904
6	1	0.00067	0.00098	0.00138	0.00189	0.00252	0.00326	0.00414	0.00516	0.00632	0.00762	0.00907
6	2	0.01491	0.01860	0.02271	0.02719	0.03203	0.03720	0.04267	0.04841	0.05439	0.06058	0.06696
6	3	0.07561	0.08661	0.09788	0.10934	0.12091	0.13254	0.14419	0.15580	0.16735	0.17880	0.19014
6	4	0.24957	0.26848	0.28673	0.30430	0.32121	0.33747	0.35310	0.36811	0.38253	0.39639	0.40971
6	5	0.71835	0.73097	0.74251	0.75311	0.76288	0.77191	0.78028	0.78806	0.79532	0.80209	0.80843
6	6	2.27298	2.17664	2.09380	2.02140	1.95776	1.90141	1.85117	1.80613	1.76552	1.72874	1.69527
7	1	0.00047	0.00071	0.00102	0.00141	0.00190	0.00250	0.00320	0.00403	0.00499	0.00607	0.00728
7	2	0.01025	0.01365	0.01621	0.01972	0.02358	0.02775	0.03222	0.03696	0.04196	0.04719	0.05263
7	3	0.04977	0.05828	0.06718	0.07640	0.08587	0.09555	0.10537	0.11529	0.12528	0.13530	0.14532
7	4	0.15402	0.16995	0.18570	0.20120	0.21641	0.23130	0.24585	0.26004	0.27387	0.28733	0.30041
7	5	0.39742	0.41720	0.43564	0.45342	0.47002	0.48570	0.50054	0.51459	0.52791	0.54055	0.55256
7	6	0.98215	0.98308	0.98392	0.98468	0.98537	0.98600	0.98658	0.98711	0.98761	0.98806	0.98849
7	7	2.77433	2.62927	2.50320	2.39801	2.30454	2.22239	2.14966	2.08485	2.02676	1.97442	1.92703
8	1	0.00035	0.00053	0.00078	0.00109	0.00149	0.00198	0.00257	0.00326	0.00406	0.00498	0.00602
8	2	0.00744	0.00963	0.01215	0.01499	0.01814	0.02160	0.02534	0.02936	0.03362	0.03812	0.04284
8	3	0.03506	0.04182	0.04901	0.05658	0.06447	0.07264	0.08102	0.08959	0.09830	0.10712	0.11601
8	4	0.10410	0.11727	0.13053	0.14382	0.15708	0.17024	0.18327	0.19614	0.20883	0.22130	0.23355
8	5	0.25266	0.27163	0.28992	0.30753	0.32446	0.34074	0.35637	0.37139	0.38581	0.39966	0.41297
8	6	0.56264	0.57993	0.59595	0.61082	0.62466	0.63757	0.64964	0.66095	0.67156	0.68154	0.69093
8	7	1.25485	1.23995	1.22668	1.21481	1.20411	1.19442	1.18561	1.17757	1.17019	1.16340	1.15712
8	8	3.26215	3.06533	2.89837	2.75516	2.63112	2.52274	2.42733	2.34274	2.26727	2.19956	2.13850
9	1	0.00027	0.00042	0.00061	0.00087	0.00120	0.00161	0.00211	0.00270	0.00339	0.00418	0.00508
9	2	0.00563	0.00739	0.00945	0.01180	0.01443	0.01736	0.02055	0.02400	0.02771	0.03164	0.03579
9	3	0.02593	0.03142	0.03736	0.04369	0.05037	0.05736	0.06461	0.07209	0.07977	0.08760	0.09556
9	4	0.07480	0.08574	0.09694	0.10834	0.11985	0.13143	0.14303	0.15460	0.16611	0.17753	0.18883
9	5	0.17451	0.19130	0.20779	0.22394	0.23970	0.25506	0.27000	0.28451	0.29861	0.31228	0.32553
9	6	0.36645	0.38633	0.40574	0.42296	0.43983	0.45582	0.47099	0.48539	0.49907	0.51208	0.52447
9	7	0.73954	0.75136	0.76219	0.77211	0.78124	0.78967	0.79748	0.80473	0.81148	0.81779	0.82368
9	8	1.53146	1.49751	1.46757	1.44101	1.41726	1.39595	1.37668	1.35919	1.34324	1.32864	1.31522
9	9	3.73517	3.48488	3.27430	3.09422	2.93937	2.80475	2.68678	2.58264	2.49009	2.40735	2.33299
10	1	0.00021	0.00033	0.00050	0.00072	0.00099	0.00134	0.00177	0.00228	0.00288	0.00358	0.00437
10	2	0.00439	0.00594	0.00756	0.00954	0.01178	0.01429	0.01706	0.02008	0.02333	0.02682	0.03052
10	3	0.01989	0.02444	0.02943	0.03481	0.04055	0.04661	0.05296	0.05956	0.06639	0.07341	0.08058
10	4	0.05617	0.06536	0.07491	0.08475	0.09481	0.10504	0.11538	0.12579	0.13623	0.14667	0.15708
10	5	0.12747	0.14206	0.15662	0.17108	0.18537	0.19947	0.21333	0.22692	0.24025	0.25328	0.26601
10	6	0.25758	0.27675	0.29511	0.31276	0.32973	0.34603	0.36167	0.37669	0.39110	0.40494	0.41823
10	7	0.49144	0.51017	0.52733	0.54394	0.55920	0.57351	0.58696	0.59960	0.61151	0.62275	0.63338
10	8	0.92414	0.92798	0.93146	0.93461	0.93749	0.94012	0.94255	0.94478	0.94685	0.94876	0.95055
10	9	1.80393	1.75334	1.70[illegible]	1.66204	1.62416	1.59021	1.55977	1.53226	1.50734	1.48460	1.46380
10	10	4.19317	3.88645	3.63319	3.41671	3.23112	3.07249	2.93026	2.80692	2.69768	2.60033	2.51308

30% LIMITS

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
						-β_V-						
1	1	0.10117	0.11413	0.12722	0.14034	0.15344	0.16647	0.17939	0.19215	0.20473	0.21712	0.22929
2	1	0.02166	0.02653	0.03180	0.03748	0.04351	0.04987	0.05650	0.06339	0.07048	0.07776	0.08518
2	2	0.59803	0.61443	0.62958	0.64360	0.65663	0.66874	0.68005	0.69062	0.70052	0.70982	0.71856
3	1	0.00881	0.01130	0.01414	0.01731	0.02082	0.02464	0.02875	0.03313	0.03777	0.04264	0.04773
3	2	0.17074	0.18739	0.20375	0.21979	0.23546	0.25074	0.26562	0.28008	0.29413	0.30777	0.32100
3	3	1.25331	1.23830	1.22533	1.21353	1.20290	1.19328	1.18452	1.17653	1.16919	1.16244	1.15621
4	1	0.00465	0.00616	0.00795	0.01001	0.01234	0.01494	0.01780	0.02091	0.02426	0.02785	0.03164
4	2	0.07838	0.08962	0.10111	0.11277	0.12453	0.13634	0.14814	0.15990	0.17158	0.18315	0.19460
4	3	0.41955	0.43918	0.45762	0.47498	0.49133	0.50674	0.52130	0.53506	0.54809	0.56043	0.57214
4	4	1.93983	1.87335	1.81547	1.76464	1.71966	1.67960	1.64370	1.61135	1.58205	1.55540	1.53106
5	1	0.00283	0.00385	0.00509	0.00654	0.00822	0.01013	0.01227	0.01463	0.01721	0.02001	0.02301
5	2	0.04429	0.05216	0.06048	0.06913	0.07805	0.08721	0.09654	0.10600	0.11556	0.12517	0.13482
5	3	0.20864	0.22657	0.24403	0.26099	0.27742	0.29332	0.30870	0.32356	0.33792	0.35177	0.36514
5	4	0.72028	0.73283	0.74430	0.75484	0.76456	0.77353	0.78185	0.78959	0.79679	0.80353	0.80983
5	5	2.61720	2.48797	2.37713	2.28111	2.19718	2.12326	2.05768	1.99913	1.94658	1.89914	1.85613
6	1	0.00189	0.00263	0.00353	0.00462	0.00590	0.00738	0.00905	0.01093	0.01300	0.01527	0.01773
6	2	0.02818	0.03400	0.04026	0.04692	0.05392	0.06122	0.06877	0.07654	0.08450	0.09259	0.10081
6	3	0.12378	0.13817	0.15254	0.16683	0.18098	0.19494	0.20868	0.22218	0.23541	0.24837	0.26104
6	4	0.37919	0.39934	0.41780	0.43553	0.45230	0.46817	0.48322	0.49748	0.51102	0.52389	0.53612
6	5	1.04649	1.04399	1.04174	1.03972	1.03788	1.03620	1.03467	1.03326	1.03196	1.03076	1.02964
6	6	3.27207	3.07416	2.90630	2.76233	2.63766	2.52874	2.43286	2.34786	2.27204	2.20402	2.14268
7	1	0.00134	0.00193	0.00260	0.00345	0.00446	0.00564	0.00700	0.00854	0.01026	0.01215	0.01423
7	2	0.01937	0.02384	0.02873	0.03402	0.03968	0.04565	0.05192	0.05844	0.06518	0.07212	0.07922
7	3	0.08140	0.09289	0.10461	0.11648	0.12844	0.14043	0.15240	0.16431	0.17613	0.18783	0.19938
7	4	0.23343	0.25200	0.26998	0.28735	0.30411	0.32026	0.33582	0.35081	0.36523	0.37911	0.39247
7	5	0.57519	0.59218	0.60789	0.62247	0.63603	0.64867	0.66048	0.67153	0.68189	0.69163	0.70080
7	6	1.38397	1.36050	1.33972	1.32119	1.30457	1.28958	1.27598	1.26360	1.25228	1.24189	1.23232
7	7	3.90057	3.63092	3.40421	3.21131	3.04544	2.90150	2.77553	2.66448	2.56592	2.47791	2.39890
8	1	0.00100	0.00143	0.00199	0.00267	0.00350	0.00447	0.00561	0.00690	0.00835	0.00997	0.01176
8	2	0.01406	0.01760	0.02154	0.02586	0.03053	0.03553	0.04083	0.04641	0.05222	0.05826	0.06448
8	3	0.05731	0.06652	0.07628	0.08623	0.09639	0.10671	0.11714	0.12763	0.13814	0.14865	0.15912
8	4	0.15757	0.17336	0.18955	0.20517	0.22049	0.23547	0.25009	0.26435	0.27823	0.29173	0.30485
8	5	0.36454	0.38443	0.40325	0.42107	0.43795	0.45396	0.46915	0.48357	0.49727	0.51031	0.52272
8	6	0.78699	0.79698	0.80607	0.81439	0.82203	0.82906	0.83556	0.84159	0.84719	0.85241	0.85728
8	7	1.72485	1.67806	1.63533	1.59562	1.56209	1.53208	1.50509	1.48068	1.45849	1.43825	1.41970
8	8	4.50244	4.15964	3.87349	3.63160	3.42483	3.24634	3.09090	2.95449	2.83391	2.72666	2.63071
9	1	0.00077	0.00112	0.00157	0.00214	0.00282	0.00365	0.00461	0.00571	0.00697	0.00838	0.00994
9	2	0.01063	0.01350	0.01675	0.02035	0.02429	0.02855	0.03311	0.03794	0.04303	0.04835	0.05387
9	3	0.04237	0.05004	0.05812	0.06656	0.07528	0.08424	0.09339	0.10268	0.11207	0.12154	0.13104
9	4	0.11312	0.12687	0.14067	0.15444	0.16812	0.18167	0.19506	0.20823	0.22119	0.23390	0.24636
9	5	0.25134	0.27029	0.28856	0.30615	0.32308	0.33935	0.35498	0.36999	0.38441	0.39827	0.41158
9	6	0.51072	0.52910	0.54621	0.56217	0.57706	0.59104	0.60414	0.61644	0.62802	0.63893	0.64924
9	7	1.00836	1.00792	1.00752	1.00716	1.00684	1.00654	1.00626	1.00601	1.00578	1.00557	1.00537
9	8	2.06463	1.98734	1.92025	1.86151	1.80966	1.76359	1.72239	1.68533	1.65184	1.62142	1.59367
9	9	5.07877	4.66244	4.31700	4.02657	3.77954	3.56724	3.38313	3.22215	3.08036	2.95465	2.84251
10	1	0.00061	0.00090	0.00127	0.00175	0.00233	0.00304	0.00386	0.00483	0.00593	0.00717	0.00855
10	2	0.00829	0.01067	0.01339	0.01645	0.01982	0.02351	0.02743	0.03173	0.03624	0.04097	0.04593
10	3	0.03249	0.03892	0.04577	0.05302	0.06059	0.06844	0.07653	0.08482	0.09326	0.10183	0.11048
10	4	0.08490	0.09657	0.10665	0.12076	0.13294	0.14515	0.15728	0.16936	0.18133	0.19317	0.20485
10	5	0.18339	0.20051	0.21729	0.23367	0.24963	0.26516	0.28024	0.29487	0.30905	0.32279	0.33609
10	6	0.35639	0.37826	0.39712	0.41497	0.43190	0.44796	0.46320	0.47758	0.49145	0.50455	0.51703
10	7	0.66739	0.68175	0.69493	0.70778	0.71631	0.72872	0.73839	0.74740	0.75562	0.76369	0.77108
10	8	1.23516	1.22133	1.20937	1.19647	1.18865	1.17975	1.17165	1.16425	1.15746	1.15121	1.14543
10	9	2.40072	2.29253	2.19942	2.11840	2.04733	1.98454	1.92865	1.87865	1.83364	1.79292	1.75592
10	10	5.63116	5.14154	4.73736	4.39916	4.11269	3.86745	3.65552	3.47081	3.30860	3.16516	3.03758

40% LIMITS

-β_w-

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
1	1	0.22476	0.24313	0.26094	0.27818	0.29484	0.31092	0.32643	0.34138	0.35579	0.36967	0.38304
2	1	0.04817	0.05651	0.06524	0.07429	0.08361	0.09314	0.10282	0.11261	0.12248	0.13239	0.14230
2	2	1.00203	1.00192	1.00182	1.00174	1.00166	1.00158	1.00152	1.00146	1.00140	1.00135	1.00130
3	1	0.01956	0.02406	0.02899	0.03432	0.04003	0.04601	0.05231	0.05886	0.06564	0.07260	0.07973
3	2	0.28371	0.30316	0.32160	0.33965	0.35674	0.37309	0.38874	0.40371	0.41804	0.43177	0.44491
3	3	1.89998	1.83667	1.78186	1.73352	1.69071	1.65254	1.61831	1.58745	1.55948	1.53403	1.51076
4	1	0.01032	0.01313	0.01631	0.01984	0.02371	0.02790	0.03239	0.03715	0.04216	0.04741	0.05286
4	2	0.12997	0.14470	0.15939	0.17395	0.18835	0.20253	0.21646	0.23013	0.24350	0.25658	0.26936
4	3	0.62747	0.64305	0.65741	0.67067	0.68296	0.69437	0.70501	0.71493	0.72422	0.73293	0.74111
4	4	2.78714	2.64076	2.51561	2.40749	2.31324	2.23041	2.15709	2.09177	2.03323	1.98049	1.93274
5	1	0.00629	0.00821	0.01044	0.01297	0.01580	0.01893	0.02233	0.02599	0.02991	0.03406	0.03843
5	2	0.07337	0.08418	0.09527	0.10656	0.11797	0.12946	0.14097	0.15246	0.16390	0.17526	0.18650
5	3	0.31081	0.33073	0.34934	0.36728	0.38439	0.40070	0.41627	0.43112	0.44530	0.45884	0.47179
5	4	1.01764	1.01670	1.01586	1.01510	1.01441	1.01378	1.01320	1.01267	1.01218	1.01172	1.01130
5	5	3.63350	3.39496	3.19369	3.02189	2.87374	2.74483	2.63175	2.53183	2.44297	2.36347	2.29197
6	1	0.00419	0.00559	0.00725	0.00917	0.01134	0.01378	0.01648	0.01942	0.02260	0.02600	0.02962
6	2	0.04666	0.05483	0.06339	0.07229	0.08146	0.09084	0.10039	0.11006	0.11981	0.12960	0.13942
6	3	0.18408	0.20123	0.21803	0.23443	0.25041	0.26595	0.28104	0.29567	0.30986	0.32360	0.33691
6	4	0.53293	0.55087	0.56754	0.58306	0.59754	0.61106	0.62373	0.63562	0.64680	0.65732	0.66725
6	5	1.42533	1.39918	1.37587	1.35513	1.33654	1.31979	1.30462	1.29081	1.27820	1.26663	1.25599
6	6	4.43383	4.09957	3.82033	3.58412	3.38207	3.20756	3.05551	2.92200	2.80395	2.69889	2.60487
7	1	0.00298	0.00404	0.00533	0.00683	0.00857	0.01054	0.01274	0.01517	0.01783	0.02069	0.02377
7	2	0.03207	0.03843	0.04523	0.05242	0.05993	0.06774	0.07578	0.08401	0.09241	0.10093	0.10955
7	3	0.12095	0.13513	0.14940	0.16356	0.17759	0.19145	0.20510	0.21852	0.23168	0.24457	0.25719
7	4	0.32728	0.34710	0.36596	0.38390	0.40098	0.41723	0.43271	0.44745	0.46151	0.47492	0.48771
7	5	0.77864	0.78896	0.79837	0.80697	0.81486	0.82217	0.82890	0.83515	0.84095	0.84637	0.85142
7	6	1.83723	1.77939	1.72885	1.68436	1.64491	1.60970	1.57809	1.54955	1.52367	1.50009	1.47852
7	7	5.18984	4.75899	4.40129	4.10194	3.84704	3.62816	3.43847	3.27274	3.12685	2.99757	2.88232
8	1	0.00221	0.00305	0.00408	0.00530	0.00672	0.00836	0.01020	0.01225	0.01452	0.01698	0.01964
8	2	0.02325	0.02837	0.03390	0.03983	0.04611	0.05271	0.05959	0.06671	0.07403	0.08152	0.08915
8	3	0.08512	0.09690	0.10889	0.12102	0.13321	0.14542	0.15758	0.16967	0.18165	0.19350	0.20519
8	4	0.22063	0.23890	0.25663	0.27380	0.29041	0.30645	0.32193	0.33686	0.35125	0.36513	0.37851
8	5	0.49203	0.51074	0.52819	0.54449	0.55975	0.57405	0.58748	0.60011	0.61202	0.62325	0.63386
8	6	1.03746	1.03545	1.03365	1.03202	1.03055	1.02920	1.02797	1.02683	1.02579	1.02482	1.02392
8	7	2.24587	2.15224	2.07131	2.00072	1.93863	1.88363	1.83458	1.79059	1.75092	1.71497	1.68225
8	8	5.90514	5.37823	4.94433	4.58199	4.27569	4.01395	3.78811	3.59158	3.41923	3.26703	3.13178
9	1	0.00170	0.00238	0.00322	0.00423	0.00543	0.00681	0.00838	0.01015	0.01211	0.01426	0.01660
9	2	0.01760	0.02176	0.02635	0.03134	0.03668	0.04235	0.04831	0.05454	0.06099	0.06765	0.07448
9	3	0.06290	0.07276	0.08295	0.09339	0.10401	0.11477	0.12560	0.13647	0.14734	0.15816	0.16893
9	4	0.15827	0.17440	0.19031	0.20596	0.22129	0.23629	0.25093	0.26520	0.27909	0.29260	0.30573
9	5	0.33268	0.35854	0.37741	0.39533	0.41237	0.42856	0.44396	0.45862	0.47257	0.48588	0.49857
9	6	0.67097	0.68521	0.69828	0.71035	0.72145	0.73177	0.74135	0.75028	0.75862	0.76642	0.77374
9	7	1.30299	1.28497	1.26896	1.25465	1.24176	1.23014	1.21957	1.20992	1.20109	1.19296	1.18547
9	8	2.64770	2.51544	2.40305	2.30388	2.21812	2.14260	2.07564	2.01589	1.96226	1.91387	1.87001
9	9	6.58362	5.96194	5.45273	5.02969	4.67362	4.37057	4.11006	3.88414	3.68663	3.51272	3.35858
10	1	0.00135	0.00191	0.00261	0.00346	0.00448	0.00567	0.00703	0.00858	0.01030	0.01220	0.01428
10	2	0.01373	0.01720	0.02108	0.02533	0.02994	0.03487	0.04010	0.04561	0.05136	0.05733	0.06349
10	3	0.04823	0.05657	0.06531	0.07437	0.08370	0.09323	0.10292	0.11271	0.12259	0.13250	0.14241
10	4	0.11873	0.13282	0.14692	0.16097	0.17491	0.18868	0.20226	0.21561	0.22872	0.24156	0.25414
10	5	0.24686	0.26572	0.28393	0.30147	0.31836	0.33460	0.35022	0.36523	0.37966	0.39352	0.40685
10	6	0.46992	0.48896	0.50678	0.52346	0.53909	0.55376	0.56757	0.58058	0.59284	0.60444	0.61540
10	7	0.85913	0.86613	0.87322	0.87797	0.88312	0.88796	0.89237	0.89643	0.90022	0.90373	0.90701
10	8	1.57124	1.53431	1.50162	1.47302	1.44731	1.42423	1.40340	1.38451	1.36729	1.35154	1.33708
10	9	3.04099	2.86809	2.72090	2.59425	2.48424	2.38787	2.30282	2.22726	2.15971	2.09899	2.04413
10	10	7.22892	6.51417	5.93152	5.44940	5.04520	4.70239	4.40864	4.15463	3.93317	3.73866	3.56666

60% LIMITS

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
		-βw-										
1	1	0.82344	0.83190	0.83959	0.84661	0.85304	0.85896	0.86441	0.86947	0.87416	0.87852	0.88260
2	1	0.17647	0.19334	0.20990	0.22610	0.24190	0.25730	0.27227	0.28682	0.30093	0.31462	0.32788
2	2	2.42533	2.31483	2.21969	2.13699	2.06449	2.00044	1.94347	1.89249	1.84663	1.80515	1.76747
3	1	0.07167	0.08234	0.09329	0.10444	0.11574	0.12711	0.13852	0.14992	0.16127	0.17255	0.18372
3	2	0.67372	0.68763	0.70086	0.71283	0.72388	0.73412	0.74364	0.75250	0.76078	0.76852	0.77578
3	3	3.94402	3.66923	3.43831	3.24194	3.07317	2.92676	2.79869	2.68582	2.58567	2.49628	2.41604
4	1	0.03782	0.04493	0.05247	0.06038	0.06860	0.07707	0.08576	0.09461	0.10360	0.11267	0.12181
4	2	0.30713	0.32662	0.34561	0.36355	0.38066	0.39698	0.41256	0.42743	0.44164	0.45521	0.46819
4	3	1.26402	1.24633	1.23475	1.22241	1.21130	1.20125	1.19211	1.18376	1.17610	1.16906	1.16255
4	4	5.31783	4.87010	4.49947	4.18850	3.92449	3.69799	3.50188	3.33065	3.18003	3.04665	2.92782
5	1	0.02303	0.02809	0.03358	0.03947	0.04572	0.05228	0.05913	0.06621	0.07349	0.08095	0.08856
5	2	0.17303	0.18977	0.20621	0.22231	0.23804	0.25337	0.26829	0.28278	0.29686	0.31052	0.32376
5	3	0.62272	0.63649	0.65103	0.66448	0.67694	0.68852	0.69931	0.70939	0.71882	0.72766	0.73597
5	4	1.87169	1.81094	1.75796	1.71136	1.67007	1.63325	1.60020	1.57039	1.54337	1.51876	1.49626
5	5	6.56292	5.94418	5.43735	5.01613	4.66159	4.35981	4.10037	3.87534	3.67860	3.50535	3.35179
6	1	0.01536	0.01914	0.02332	0.02789	0.03282	0.03808	0.04363	0.04946	0.05552	0.06179	0.06825
6	2	0.10995	0.12348	0.13709	0.15070	0.16423	0.17765	0.19092	0.20399	0.21685	0.22948	0.24187
6	3	0.36625	0.38614	0.40495	0.42277	0.43964	0.45563	0.47080	0.48520	0.49889	0.51190	0.52429
6	4	0.96907	0.97068	0.97212	0.97343	0.97463	0.97571	0.97671	0.97764	0.97849	0.97928	0.98001
6	5	2.47110	2.35620	2.25736	2.17152	2.09631	2.02992	1.97092	1.91814	1.87069	1.82779	1.78884
6	6	7.70069	6.91624	6.27530	5.75284	5.31303	4.94090	4.62272	4.34812	4.10914	3.89960	3.71460
7	1	0.01091	0.01363	0.01713	0.02080	0.02480	0.02912	0.03375	0.03865	0.04380	0.04918	0.05476
7	2	0.07551	0.08650	0.09777	0.10921	0.12078	0.13240	0.14404	0.15565	0.16719	0.17865	0.18998
7	3	0.24017	0.25890	0.27700	0.29446	0.31128	0.32749	0.34308	0.35808	0.37251	0.38638	0.39973
7	4	0.59206	0.60862	0.62392	0.63810	0.65126	0.66352	0.67496	0.68565	0.69568	0.70509	0.71395
7	5	1.33176	1.31183	1.29415	1.27835	1.26416	1.25134	1.23971	1.22910	1.21939	1.21046	1.20223
7	6	3.05297	2.87860	2.73055	2.60302	2.49225	2.39524	2.30963	2.23357	2.16560	2.10450	2.04931
7	7	8.74917	7.80530	7.04320	6.41800	5.89794	5.45998	5.08716	4.76668	4.48881	4.24598	4.03227
8	1	0.00811	0.01044	0.01312	0.01613	0.01945	0.02309	0.02701	0.03121	0.03566	0.04035	0.04525
8	2	0.05479	0.06384	0.07325	0.08296	0.09290	0.10301	0.11324	0.12355	0.13390	0.14425	0.15458
8	3	0.16881	0.18538	0.20168	0.21766	0.23328	0.24852	0.26336	0.27780	0.29183	0.30545	0.31867
8	4	0.39801	0.41778	0.43642	0.45400	0.47059	0.48626	0.50109	0.51514	0.52845	0.54108	0.55308
8	5	0.83626	0.84416	0.85135	0.85790	0.86389	0.86941	0.87449	0.87919	0.88356	0.88762	0.89141
8	6	1.69836	1.65166	1.61074	1.57459	1.54243	1.51364	1.48772	1.46427	1.44295	1.42348	1.40564
8	7	3.61421	3.37788	3.17643	3.00813	2.86125	2.73342	2.62126	2.52214	2.43398	2.35510	2.28414
8	8	9.72257	8.62567	7.74467	7.02538	6.42961	5.92988	5.50600	5.14283	4.82890	4.55534	4.31521
9	1	0.00624	0.00815	0.01037	0.01288	0.01570	0.01881	0.02220	0.02585	0.02975	0.03389	0.03824
9	2	0.04141	0.04896	0.05694	0.06526	0.07388	0.08274	0.09179	0.10100	0.11031	0.11969	0.12912
9	3	0.12466	0.13910	0.15352	0.16785	0.18203	0.19603	0.20980	0.22332	0.23658	0.24955	0.26223
9	4	0.28502	0.30449	0.32314	0.34100	0.35809	0.37444	0.39009	0.40506	0.41938	0.43310	0.44624
9	5	0.57360	0.59053	0.60638	0.62100	0.63460	0.64727	0.65911	0.67019	0.68058	0.69035	0.69955
9	6	1.09049	1.08553	1.08109	1.07708	1.07345	1.07015	1.06713	1.06436	1.06181	1.05945	1.05727
9	7	2.06330	1.98612	1.91913	1.86048	1.80871	1.76270	1.72156	1.68455	1.65110	1.62072	1.59301
9	8	4.15431	3.85436	3.60287	3.38955	3.20660	3.04619	2.90987	2.78817	2.68035	2.58424	2.49808
9	9	10.63207	9.38826	8.39375	7.58504	6.91767	6.35975	5.88794	5.48485	5.13730	4.83517	4.57055
10	1	0.00494	0.00653	0.00840	0.01054	0.01297	0.01566	0.01862	0.02184	0.02530	0.02899	0.03290
10	2	0.03236	0.03869	0.04553	0.05274	0.06029	0.06812	0.07619	0.08445	0.09287	0.10142	0.11006
10	3	0.09553	0.10810	0.12082	0.13361	0.14642	0.15917	0.17184	0.18438	0.19677	0.20898	0.22100
10	4	0.21357	0.23164	0.24922	0.26627	0.28277	0.29874	0.31416	0.32905	0.34342	0.35729	0.37067
10	5	0.41718	0.43682	0.45529	0.47267	0.48905	0.50455	0.51909	0.53288	0.54594	0.55832	0.57006
10	6	0.76060	0.77164	0.78170	0.79093	0.79940	0.80722	0.81446	0.82117	0.82742	0.83324	0.83869
10	7	1.34965	1.32852	1.30979	1.29306	1.27804	1.26448	1.25218	1.24097	1.23071	1.22128	1.21259
10	8	2.42354	2.31321	2.21522	2.13564	2.05324	1.99928	1.94239	1.89149	1.84568	1.80426	1.76663
10	9	4.57336	4.36933	4.00597	3.74980	3.53115	3.34267	3.17375	3.03505	2.90818	2.79544	2.69467
10	10	11.48649	10.10155	8.99847	8.10461	7.36930	6.75635	6.23937	5.79876	5.41971	5.09086	4.80340

70% LIMITS

-βw-

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
1	1	1.51060	1.47815	1.44955	1.42415	1.40144	1.38103	1.36258	1.34582	1.33053	1.31653	1.30367
2	1	0.32374	0.34354	0.36239	0.38033	0.39742	0.41369	0.42918	0.44395	0.45804	0.47148	0.48431
2	2	3.74665	3.49503	3.28306	3.10237	2.94676	2.81150	2.69297	2.58835	2.49538	2.41229	2.33760
3	1	0.13149	0.14630	0.16106	0.17569	0.19014	0.20437	0.21835	0.23206	0.24547	0.25857	0.27137
3	2	1.02831	1.02680	1.02545	1.02422	1.02311	1.02209	1.02116	1.02031	1.01952	1.01879	1.01811
3	3	5.69950	5.20063	4.78909	4.44488	4.15348	3.90413	3.68874	3.50108	3.33635	3.19074	3.06122
4	1	0.06938	0.07984	0.09060	0.10157	0.11270	0.12392	0.13518	0.14645	0.15768	0.16885	0.17992
4	2	0.46730	0.48639	0.50424	0.52095	0.53663	0.55135	0.56520	0.57824	0.59056	0.60219	0.61320
4	3	1.79312	1.73885	1.69141	1.64961	1.61250	1.57935	1.54956	1.52265	1.49822	1.47596	1.45558
4	4	7.40158	6.66147	6.05887	5.56077	5.14358	4.79006	4.48738	4.22584	3.99797	3.79795	3.62119
5	1	0.04226	0.04991	0.05798	0.06640	0.07511	0.08406	0.09320	0.10248	0.11186	0.12132	0.13081
5	2	0.26294	0.28209	0.30052	0.31823	0.33523	0.35154	0.36719	0.38221	0.39661	0.41043	0.42369
5	3	0.87561	0.88195	0.88750	0.89256	0.89719	0.90143	0.90533	0.90894	0.91229	0.91539	0.91829
5	4	2.54734	2.42501	2.31995	2.22882	2.14906	2.07877	2.01635	1.96057	1.91045	1.86520	1.82413
5	5	8.91057	7.94165	7.16003	6.51936	5.98681	5.53866	5.15738	4.82983	4.54597	4.29864	4.07994
6	1	0.02818	0.03400	0.04027	0.04692	0.05392	0.06122	0.06878	0.07655	0.08450	0.09260	0.10081
6	2	0.16694	0.18344	0.19967	0.21559	0.23117	0.24637	0.26117	0.27558	0.28959	0.30319	0.31639
6	3	0.51557	0.53385	0.55088	0.56675	0.58157	0.59544	0.60844	0.62065	0.63214	0.64297	0.65319
6	4	1.30974	1.29127	1.27487	1.26021	1.24703	1.23512	1.22430	1.21443	1.20539	1.19708	1.18941
6	5	3.27225	3.07432	2.90644	2.76246	2.63778	2.52885	2.43296	2.34795	2.27212	2.20410	2.14276
6	6	10.26912	9.08437	8.13542	7.36255	6.72385	6.18920	5.73654	5.34939	5.01525	4.72450	4.46963
7	1	0.02001	0.02458	0.02958	0.03498	0.04074	0.04682	0.05319	0.05982	0.06666	0.07369	0.08089
7	2	0.11462	0.12847	0.14235	0.15619	0.16995	0.18356	0.19700	0.21022	0.22322	0.23597	0.24846
7	3	0.33767	0.35753	0.37640	0.39433	0.41136	0.42756	0.44297	0.45763	0.47160	0.48491	0.49761
7	4	0.79767	0.80721	0.81590	0.82385	0.83114	0.83785	0.84405	0.84979	0.85513	0.86010	0.86474
7	5	1.74925	1.69852	1.65412	1.61495	1.58014	1.54902	1.52103	1.49573	1.47275	1.45178	1.43258
7	6	3.96361	3.68649	3.45568	3.25574	3.08565	2.93813	2.80911	2.69541	2.59456	2.50454	2.42375
7	7	11.50747	10.11903	9.01326	8.11730	7.38031	6.76600	6.24791	5.80638	5.42656	5.09706	4.80904
8	1	0.01487	0.01856	0.02265	0.02713	0.03196	0.03712	0.04258	0.04831	0.05428	0.06047	0.06684
8	2	0.08315	0.09478	0.10663	0.11863	0.13070	0.14279	0.15485	0.16684	0.17874	0.19051	0.20213
8	3	0.23717	0.25583	0.27388	0.29130	0.30810	0.32428	0.33986	0.35485	0.36928	0.38316	0.39651
8	4	0.53530	0.55320	0.56982	0.58529	0.59972	0.61320	0.62582	0.63766	0.64879	0.65927	0.66916
8	5	1.09423	1.08906	1.08442	1.08025	1.07646	1.07302	1.06987	1.06699	1.06433	1.06187	1.05960
8	6	2.18518	2.09710	2.02087	1.95429	1.89567	1.84368	1.79728	1.75562	1.71802	1.68393	1.65286
8	7	4.62188	4.26411	3.96585	3.71402	3.49899	3.31354	3.15220	3.01071	2.88575	2.77468	2.67537
8	8	12.64752	11.06636	9.81307	8.80189	7.97335	7.28516	6.70662	6.21504	5.79331	5.42838	5.11013
9	1	0.01144	0.01448	0.01790	0.02167	0.02580	0.03024	0.03499	0.04001	0.04529	0.05079	0.05649
9	2	0.06284	0.07269	0.08287	0.09330	0.10393	0.11468	0.12550	0.13637	0.14723	0.15806	0.16882
9	3	0.17506	0.19158	0.20839	0.22455	0.24032	0.25569	0.27064	0.28517	0.29927	0.31294	0.32620
9	4	0.38294	0.40279	0.42152	0.43922	0.45596	0.47180	0.48680	0.50102	0.51452	0.52734	0.53953
9	5	0.74895	0.76043	0.77092	0.78053	0.78937	0.79753	0.80508	0.81209	0.81862	0.82471	0.83041
9	6	1.39701	1.37264	1.35107	1.33185	1.31462	1.29907	1.28499	1.27216	1.26044	1.24968	1.23977
9	7	2.61313	2.48430	2.37330	2.27807	2.19439	2.12067	2.05528	1.99690	1.94448	1.89717	1.85428
9	8	5.24906	4.81041	4.44706	4.14202	3.88291	3.66051	3.46786	3.29958	3.15150	3.02033	2.90342
9	9	13.70554	11.94152	10.54669	9.42937	8.51506	7.75791	7.12315	6.58514	6.12466	5.72705	5.38099
10	1	0.00965	0.01160	0.01450	0.01773	0.02130	0.02518	0.02936	0.03381	0.03851	0.04345	0.04860
10	2	0.04901	0.05744	0.06626	0.07540	0.08483	0.09440	0.10416	0.11402	0.12395	0.13392	0.14389
10	3	0.13411	0.14907	0.16396	0.17370	0.19325	0.20757	0.22152	0.23539	0.24885	0.26201	0.27485
10	4	0.28674	0.30623	0.32469	0.34276	0.35986	0.37621	0.39185	0.40681	0.42113	0.43484	0.44796
10	5	0.54399	0.56171	0.57814	0.59342	0.60767	0.62097	0.63342	0.64510	0.65507	0.66639	0.67612
10	6	0.97199	0.97345	0.97476	0.97595	0.97703	0.97802	0.97392	0.97976	0.98053	0.98124	0.98191
10	7	1.70121	1.65429	1.61318	1.57586	1.54455	1.51563	1.48960	1.46604	1.44463	1.42508	1.40716
10	8	3.03104	2.85920	2.71289	2.58698	2.47759	2.38176	2.29717	2.22201	2.15481	2.09441	2.03983
10	9	5.84759	5.32856	4.90094	4.54369	4.24157	3.98330	3.76039	3.56635	3.39613	3.24577	3.11212
10	10	14.69399	12.75590	11.23119	10.00934	9.01431	8.19245	7.50597	6.92374	6.42717	5.99920	5.62735

90% LIMITS

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
						$-\beta_W-$						
1	1	2.87923	2.72335	2.59029	2.47551	2.37559	2.28738	2.21033	2.14131	2.07951	2.02388	1.97355
2	1	0.61705	0.63293	0.64757	0.66111	0.67367	0.68534	0.69621	0.70637	0.71588	0.72479	0.73317
2	2	6.05232	5.50514	5.05310	4.67971	4.36269	4.09203	3.85870	3.65581	3.47801	3.32139	3.18173
3	1	0.25061	0.26955	0.28781	0.30540	0.32231	0.33858	0.35421	0.36922	0.38364	0.39750	0.41082
3	2	1.63547	1.59367	1.55696	1.52448	1.49554	1.46960	1.44621	1.42502	1.40574	1.38812	1.37196
3	3	8.61501	7.69187	6.94593	6.33356	5.82384	5.39435	5.02854	4.71394	4.44104	4.20246	3.99242
4	1	0.13224	0.14710	0.16189	0.17656	0.19104	0.20529	0.21929	0.23301	0.24644	0.25956	0.27237
4	2	0.74015	0.75196	0.76276	0.77266	0.78177	0.79018	0.79797	0.80521	0.81195	0.81824	0.82412
4	3	2.64702	2.51482	2.40149	2.30336	2.21764	2.14217	2.07524	2.01551	1.96190	1.91354	1.86970
4	4	10.76957	9.50325	8.49138	7.66905	6.99078	6.42403	5.94496	5.53583	5.18321	4.87677	4.60846
5	1	0.08054	0.09196	0.10361	0.11542	0.12733	0.13926	0.15118	0.16305	0.17483	0.18650	0.19803
5	2	0.41576	0.43542	0.45390	0.47130	0.48769	0.50316	0.51777	0.53158	0.54466	0.55705	0.56882
5	3	1.28391	1.26713	1.25222	1.23888	1.22687	1.21602	1.20615	1.19714	1.18888	1.18129	1.17428
5	4	3.60233	3.36736	3.16902	2.99966	2.85356	2.72638	2.61479	2.51617	2.42844	2.34993	2.27931
5	5	12.63880	11.05914	9.80698	8.79669	7.96885	7.28123	6.70316	6.21195	5.79054	5.42588	5.10786
6	1	0.05371	0.06265	0.07195	0.08156	0.09140	0.10142	0.11157	0.12180	0.13207	0.14235	0.15262
6	2	0.26375	0.28292	0.30135	0.31907	0.33607	0.35239	0.36804	0.38305	0.39746	0.41127	0.42453
6	3	0.75352	0.76483	0.77515	0.78461	0.79331	0.80134	0.80876	0.81566	0.82208	0.82806	0.83366
6	4	1.83570	1.77794	1.72751	1.68312	1.64376	1.60862	1.57707	1.54859	1.52276	1.49923	1.47770
6	5	4.49657	4.15451	3.86895	3.62754	3.42118	3.24303	3.08789	2.95172	2.83136	2.72430	2.62851
6	6	14.29692	12.42911	10.95767	9.77705	8.81451	8.01868	7.35245	6.78851	6.30642	5.89063	5.52912
7	1	0.03813	0.04528	0.05286	0.06081	0.06906	0.07757	0.08629	0.09517	0.10419	0.11329	0.12245
7	2	0.18100	0.19804	0.21474	0.23106	0.24698	0.26246	0.27750	0.29210	0.30626	0.31998	0.33327
7	3	0.49274	0.51144	0.52887	0.54516	0.56041	0.57469	0.58811	0.60073	0.61262	0.62384	0.63445
7	4	1.11347	1.10719	1.10157	1.09650	1.09192	1.08775	1.08395	1.08046	1.07725	1.07428	1.07154
7	5	2.37896	2.27288	2.18147	2.10193	2.03214	1.97044	1.91554	1.86637	1.82211	1.78207	1.74567
7	6	5.33409	4.88421	4.51185	4.19947	3.93430	3.70684	3.50990	3.33798	3.18675	3.05286	2.93357
7	7	15.79139	13.65710	11.98369	10.64735	9.56174	8.66772	7.92183	7.29243	6.75593	6.29442	5.89416
8	1	0.02834	0.03419	0.04047	0.04715	0.05417	0.06150	0.06907	0.07687	0.08484	0.09296	0.10119
8	2	0.13127	0.14607	0.16082	0.17544	0.18988	0.20411	0.21808	0.23178	0.24518	0.25829	0.27108
8	3	0.34576	0.36553	0.38450	0.40240	0.41940	0.43555	0.45090	0.46550	0.47939	0.49262	0.50524
8	4	0.74559	0.75720	0.76780	0.77752	0.78647	0.79472	0.80237	0.80947	0.81607	0.82224	0.82801
8	5	1.48094	1.45064	1.42391	1.40015	1.37889	1.35976	1.34246	1.32674	1.31239	1.29925	1.28715
8	6	2.90730	2.74850	2.61301	2.49619	2.39452	2.30532	2.22647	2.15632	2.09352	2.03701	1.98590
8	7	6.12091	5.56423	5.10663	4.72513	4.40310	4.12828	3.89146	3.68560	3.50525	3.34614	3.20487
8	8	17.15650	14.77256	12.91177	11.43094	10.23265	9.24857	8.42988	7.74085	7.15492	6.65200	6.21672
9	1	0.02181	0.02668	0.03198	0.03768	0.04373	0.05011	0.05676	0.06366	0.07078	0.07807	0.08552
9	2	0.09918	0.11200	0.12496	0.13797	0.15097	0.16390	0.17673	0.18941	0.20193	0.21426	0.22638
9	3	0.25506	0.27407	0.29240	0.31003	0.32698	0.34326	0.35890	0.37392	0.38834	0.40218	0.41548
9	4	0.53266	0.55061	0.56729	0.58281	0.59729	0.61083	0.62350	0.63539	0.64658	0.65710	0.66703
9	5	1.01039	1.01031	1.00980	1.00933	1.00890	1.00851	1.00816	1.00783	1.00753	1.00725	1.00699
9	6	1.84849	1.78967	1.73634	1.69317	1.65312	1.61738	1.58530	1.55635	1.53010	1.50618	1.48431
9	7	3.41845	3.20430	3.02306	2.86793	2.73382	2.61685	2.51404	2.42302	2.34193	2.26927	2.20382
9	8	6.86284	6.20122	5.66048	5.21199	4.83516	4.51497	4.24012	4.00205	3.79418	3.61135	3.44947
9	9	18.41427	15.79674	13.76066	12.14558	10.84243	9.77509	8.88927	8.14538	7.51409	6.97327	6.50599
10	1	0.01726	0.02137	0.02590	0.03083	0.03611	0.04172	0.04762	0.05379	0.06019	0.06679	0.07357
10	2	0.07734	0.08849	0.09992	0.11146	0.12317	0.13491	0.14665	0.15836	0.16999	0.18152	0.19293
10	3	0.19532	0.21265	0.22997	0.24664	0.26284	0.27857	0.29380	0.30856	0.32283	0.33664	0.34998
10	4	0.39850	0.41827	0.43690	0.45447	0.47106	0.48673	0.50156	0.51559	0.52890	0.54152	0.55352
10	5	0.73302	0.74510	0.75614	0.76627	0.77560	0.78422	0.79220	0.79962	0.80652	0.81297	0.81901
10	6	1.28211	1.26545	1.25084	1.23739	1.22547	1.21466	1.20488	1.19593	1.18773	1.18019	1.17322
10	7	2.21217	2.12163	2.04032	1.97496	1.91480	1.86148	1.81390	1.77121	1.73269	1.69777	1.66598
10	8	3.91196	3.64097	3.41316	3.21935	3.05273	2.90814	2.78162	2.67009	2.57111	2.48274	2.40340
10	9	7.56500	6.80074	6.17914	5.66585	5.23631	4.87263	4.56149	4.29282	4.05888	3.85365	3.67239
10	10	19.58265	16.74473	14.54401	12.80320	11.40212	10.25722	9.30901	8.51427	7.84102	7.26520	6.76844

90% LIMITS

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
						$-\beta_w-$						
1	1	6.38151	5.78841	5.30190	4.89705	4.55390	4.26521	4.01507	3.79791	3.60790	3.44045	3.29193
2	1	1.36763	1.34528	1.32547	1.30781	1.29195	1.27764	1.26467	1.25284	1.24203	1.23210	1.22295
2	2	11.23272	9.39000	8.31935	7.95089	7.23582	6.63925	6.13570	5.70623	5.33653	5.01561	4.73491
3	1	0.55546	0.57292	0.58910	0.60413	0.61813	0.63120	0.64341	0.65486	0.66561	0.67572	0.68525
3	2	2.96428	2.79951	2.65906	2.53806	2.43285	2.34061	2.25912	2.18666	2.12184	2.06354	2.01084
3	3	14.84242	12.87795	11.33324	10.09594	9.08874	8.25714	7.56186	6.97402	6.47205	6.03953	5.66383
4	1	0.29310	0.31263	0.33137	0.34926	0.36637	0.38272	0.39835	0.41328	0.42757	0.44124	0.45432
4	2	1.33278	1.31278	1.29503	1.27919	1.26495	1.25209	1.24042	1.22977	1.22003	1.21108	1.20282
4	3	4.40078	4.07061	3.79469	3.56120	3.36143	3.18883	3.03842	2.90631	2.78946	2.68547	2.59237
4	4	17.76163	15.26599	13.32106	11.77577	10.52708	9.50298	8.65198	7.93654	7.32875	6.80756	6.35685
5	1	0.17851	0.19545	0.21208	0.22833	0.24419	0.25962	0.27463	0.28920	0.30333	0.31703	0.33031
5	2	0.74665	0.75822	0.76879	0.77848	0.78739	0.79561	0.80323	0.81030	0.81688	0.82303	0.82877
5	3	2.11163	2.03017	1.95955	1.89777	1.84330	1.79494	1.75172	1.71287	1.67778	1.64593	1.61690
5	4	5.69233	5.19443	4.78567	4.44609	4.14920	3.90629	3.68526	3.49791	3.33344	3.18806	3.05875
5	5	20.23508	17.27279	14.97940	13.16796	11.71200	10.52371	9.54066	8.71757	8.02096	7.42569	6.91256
6	1	0.11904	0.13315	0.14727	0.16134	0.17529	0.18907	0.20265	0.21602	0.22914	0.24199	0.25457
6	2	0.47302	0.49203	0.50979	0.52641	0.54199	0.55662	0.57037	0.58333	0.59555	0.60709	0.61801
6	3	1.23348	1.21993	1.20787	1.19705	1.18730	1.17848	1.17044	1.16309	1.15635	1.15015	1.14442
6	4	2.86123	2.70722	2.57572	2.46225	2.36343	2.27668	2.19996	2.13166	2.07050	2.01544	1.96562
6	5	6.86502	6.20309	5.66210	5.21341	4.83643	4.51610	4.24113	4.00297	3.79502	3.61211	3.45018
6	6	22.39304	19.01317	16.40974	14.36282	12.72442	11.39227	10.29401	9.37738	8.60387	7.94466	7.37785
7	1	0.08451	0.09625	0.10820	0.12029	0.13244	0.14461	0.15675	0.16881	0.18076	0.19258	0.20425
7	2	0.32436	0.34417	0.36302	0.38097	0.39805	0.41432	0.42981	0.44457	0.45865	0.47209	0.48491
7	3	0.80457	0.81363	0.82226	0.82996	0.83702	0.84352	0.84952	0.85508	0.86025	0.86505	0.86955
7	4	1.72470	1.67593	1.63321	1.59550	1.56198	1.53198	1.50499	1.48059	1.45841	1.43817	1.41962
7	5	3.57501	3.34316	3.14738	2.98015	2.83584	2.71019	2.59991	2.50242	2.41567	2.33804	2.26818
7	6	7.94031	7.11996	6.45437	5.90594	5.44792	5.06082	4.73018	4.44511	4.19725	3.98008	3.78850
7	7	24.31475	20.55536	17.67188	15.41305	13.61111	12.15049	10.94970	9.95007	9.10853	8.39292	7.77887
8	1	0.06281	0.07266	0.08284	0.09327	0.10389	0.11464	0.12547	0.13633	0.14719	0.15802	0.16878
8	2	0.23512	0.25373	0.27174	0.28914	0.30591	0.32208	0.33765	0.35264	0.36706	0.38094	0.39430
8	3	0.56374	0.58101	0.59700	0.61184	0.62566	0.63855	0.65060	0.66188	0.67247	0.68242	0.69180
8	4	1.15094	1.14245	1.13487	1.12806	1.12189	1.11630	1.11119	1.10652	1.10222	1.09825	1.09458
8	5	2.20900	2.11875	2.04069	1.97254	1.91256	1.85939	1.81195	1.76938	1.73097	1.69615	1.66444
8	6	4.25313	3.94111	3.67991	3.45854	3.26887	3.10479	2.96163	2.83576	2.72433	2.62506	2.53612
8	7	8.93471	7.96273	7.17749	6.53449	6.00008	5.55039	5.16785	4.83925	4.55450	4.30580	4.08704
8	8	26.05193	21.94432	18.80425	16.35224	14.40173	12.82474	11.53135	10.45694	9.55426	8.73807	8.13173
9	1	0.04835	0.05671	0.06546	0.07453	0.08387	0.09341	0.10311	0.11292	0.12280	0.13272	0.14264
9	2	0.17758	0.19449	0.21108	0.22731	0.24315	0.25857	0.27356	0.28811	0.30224	0.31593	0.32921
9	3	0.41546	0.43512	0.45361	0.47101	0.48741	0.50288	0.51749	0.53131	0.54439	0.55679	0.56855
9	4	0.82055	0.82913	0.83694	0.84406	0.85059	0.85660	0.86214	0.86727	0.87203	0.87647	0.88060
9	5	1.50162	1.46983	1.44180	1.41689	1.39463	1.37460	1.35650	1.34006	1.32506	1.31131	1.29868
9	6	2.68164	2.54597	2.42974	2.32916	2.24135	2.16406	2.09556	2.03445	1.97963	1.93019	1.88539
9	7	4.89782	4.50491	4.17833	3.90329	3.66900	3.46738	3.29232	3.13908	3.00396	2.88404	2.77698
9	8	9.86091	8.74190	7.84378	7.11097	6.50436	5.99581	5.56465	5.19542	4.87637	4.59845	4.35458
9	9	27.54046	23.20996	19.85512	17.20325	15.11633	13.43279	12.05478	10.91221	9.95390	9.14177	8.44711
10	1	0.03626	0.04543	0.05302	0.06098	0.06925	0.07777	0.08650	0.09540	0.10442	0.11354	0.12271
10	2	0.13843	0.15363	0.16872	0.18364	0.19834	0.21280	0.22697	0.24084	0.25439	0.26762	0.28052
10	3	0.31795	0.33771	0.35655	0.37450	0.39159	0.40789	0.42341	0.43822	0.45235	0.46584	0.47872
10	4	0.61304	0.62903	0.64378	0.65743	0.67006	0.68165	0.69261	0.70306	0.71265	0.72165	0.73011
10	5	1.08606	1.08135	1.07713	1.07333	1.06986	1.06674	1.06387	1.06124	1.05882	1.05658	1.05450
10	6	1.85115	1.79212	1.74059	1.69526	1.65507	1.61921	1.58702	1.55797	1.53162	1.50763	1.48569
10	7	3.14072	2.95713	2.80106	2.66701	2.55071	2.44895	2.35924	2.27961	2.20850	2.14463	2.08698
10	8	5.51162	5.03835	4.64692	4.31912	4.04123	3.80315	3.59735	3.41769	3.25990	3.12030	2.99603
10	9	10.72361	9.46917	8.46245	7.64413	6.96915	6.40499	5.92807	5.52074	5.16962	4.86446	4.59724
10	10	29.10641	24.37455	20.77733	17.96239	15.75917	13.98718	12.53117	11.32588	10.31646	9.46221	8.73244

95% LIMITS

-βw-

n	r	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700
1	1	11.45237	10.07312	8.97441	8.08397	7.35136	6.74064	6.22546	5.78635	5.40836	5.08078	4.79423
2	1	2.45437	2.34108	2.24360	2.15891	2.08469	2.01916	1.96090	1.90878	1.86191	1.81953	1.78105
2	2	18.04791	15.49884	13.51401	11.93816	10.66561	9.62256	8.75629	8.02837	7.41027	6.88047	6.42248
3	1	0.99684	0.99701	0.99716	0.99729	0.99742	0.99753	0.99763	0.99773	0.99781	0.99789	0.99797
3	2	4.66554	4.30226	3.99955	3.74407	3.52601	3.33801	3.17451	3.03116	2.90460	2.79212	2.69159
3	3	22.71926	19.27547	16.62473	14.54198	12.87586	11.52195	10.40628	9.47554	8.69045	8.02163	7.44677
4	1	0.52600	0.54409	0.56090	0.57656	0.59117	0.60484	0.61764	0.62966	0.64097	0.65161	0.66166
4	2	2.08508	2.00598	1.93736	1.87730	1.82432	1.77725	1.73517	1.69734	1.66315	1.63211	1.60380
4	3	6.53136	5.91710	5.41381	4.99545	4.64324	4.34339	4.08557	3.86191	3.66634	3.49410	3.34142
4	4	26.40716	22.22768	19.03485	16.54316	14.56219	12.96139	11.64907	10.55941	9.64426	8.86777	8.20283
5	1	0.32035	0.34013	0.35898	0.37692	0.39402	0.41030	0.42582	0.44061	0.45472	0.46819	0.48105
5	2	1.16518	1.15584	1.14750	1.14001	1.13324	1.12709	1.12149	1.11636	1.11164	1.10729	1.10327
5	3	3.10337	2.92360	2.77109	2.63980	2.52586	2.42612	2.33816	2.26006	2.19028	2.12759	2.07099
5	4	8.15511	7.30250	6.61130	6.04262	5.56820	5.16765	4.82583	4.53137	4.27553	4.05154	3.85406
5	5	29.48438	24.67431	21.02001	18.18236	15.93652	14.12913	12.65301	11.43158	10.40902	9.54395	8.80517
6	1	0.21363	0.23171	0.24929	0.26634	0.28285	0.29881	0.31423	0.32913	0.34350	0.35737	0.37074
6	2	0.73722	0.74914	0.76004	0.77004	0.77924	0.78774	0.79561	0.80292	0.80972	0.81608	0.82202
6	3	1.80493	1.74909	1.70143	1.65891	1.62118	1.58748	1.55720	1.52986	1.50504	1.48243	1.46173
6	4	4.04874	3.76146	3.52037	3.31559	3.13977	2.98741	2.85424	2.73697	2.63301	2.54027	2.45709
6	5	9.59941	8.52212	7.65632	6.94903	6.36289	5.87102	5.45360	5.09584	4.78647	4.51679	4.27999
6	6	32.14066	26.77541	22.71691	19.57768	17.10192	15.11591	13.49867	12.16408	11.04958	10.10888	9.30724
7	1	0.15167	0.16750	0.18315	0.19857	0.21371	0.22854	0.24304	0.25719	0.27098	0.28440	0.29746
7	2	0.50516	0.52365	0.54086	0.55693	0.57194	0.58601	0.59920	0.61160	0.62328	0.63429	0.64469
7	3	1.17461	1.16470	1.15585	1.14791	1.14073	1.13422	1.12829	1.12285	1.11786	1.11326	1.10900
7	4	2.42664	2.31602	2.22078	2.13798	2.06540	2.00128	1.94426	1.89323	1.84732	1.80580	1.76809
7	5	4.92842	4.53158	4.20182	3.92419	3.68775	3.48433	3.30774	3.15319	3.01694	2.89604	2.78812
7	6	10.90477	9.61624	8.58726	7.75150	7.06251	6.48706	6.00084	5.58578	5.22817	4.91750	4.64557
7	7	34.48693	28.62367	24.20411	20.79647	18.11678	15.97283	14.23118	12.79709	11.60192	10.59505	9.73850
8	1	0.11273	0.12645	0.14023	0.15398	0.16764	0.18118	0.19455	0.20771	0.22065	0.23336	0.24580
8	2	0.36600	0.38588	0.40470	0.42251	0.43939	0.45538	0.47056	0.48496	0.49865	0.51167	0.52406
8	3	0.82169	0.83042	0.83817	0.84524	0.85173	0.85769	0.86319	0.86829	0.87302	0.87742	0.88153
8	4	1.61432	1.57414	1.53683	1.50757	1.47970	1.45470	1.43216	1.41173	1.39313	1.37613	1.36053
8	5	3.02484	2.85366	2.70790	2.58244	2.47344	2.37794	2.29365	2.21874	2.15176	2.09155	2.03715
8	6	5.75062	5.24481	4.82773	4.47903	4.18393	3.93151	3.71352	3.52366	3.35704	3.20979	3.07885
8	7	12.09890	10.61107	9.42912	8.47360	7.68923	7.03665	6.46723	6.01973	5.61815	5.27024	4.96650
8	8	36.59422	30.27503	25.53120	21.68104	19.01760	16.73171	14.87851	13.35540	12.08822	11.02237	10.11698
9	1	0.08677	0.09868	0.11080	0.12303	0.13533	0.14762	0.15987	0.17203	0.18408	0.19599	0.20774
9	2	0.27635	0.29570	0.31428	0.33209	0.34915	0.36549	0.38115	0.39614	0.41050	0.42427	0.43746
9	3	0.60517	0.62133	0.63634	0.65019	0.66304	0.67499	0.68614	0.69655	0.70631	0.71546	0.72407
9	4	1.14872	1.14037	1.13291	1.12619	1.12013	1.11462	1.10959	1.10498	1.10075	1.09684	1.09323
9	5	2.04647	1.97259	1.90672	1.84901	1.79807	1.75278	1.71227	1.67583	1.64288	1.61294	1.58564
9	6	3.59855	3.36401	3.16603	2.99696	2.85111	2.72415	2.61274	2.51427	2.42667	2.34829	2.27777
9	7	6.52272	5.90968	5.40737	4.98978	4.63321	4.33689	4.08152	3.85824	3.66299	3.49102	3.33858
9	8	13.20167	11.52520	10.19920	9.13144	8.25806	7.53320	6.92583	6.40992	5.96788	5.58581	5.25296
9	9	38.51100	31.77647	25.75168	22.25981	19.32881	17.41375	15.45926	13.85546	12.52312	11.40398	10.45453
10	1	0.06666	0.07905	0.08974	0.10066	0.11173	0.12291	0.13412	0.14535	0.15654	0.16767	0.17871
10	2	0.21540	0.23353	0.25115	0.26823	0.28476	0.30074	0.31618	0.33109	0.34547	0.35934	0.37271
10	3	0.46265	0.48200	0.49991	0.51669	0.53244	0.54723	0.56115	0.57427	0.58665	0.59836	0.60943
10	4	0.85716	0.86414	0.87047	0.87624	0.88152	0.88636	0.89083	0.89496	0.89879	0.90235	0.90566
10	5	1.47811	1.44632	1.42146	1.39786	1.37674	1.35773	1.34054	1.32492	1.31066	1.29759	1.28557
10	6	2.47344	2.35301	2.25929	2.17326	2.09794	2.03143	1.97232	1.91945	1.87191	1.82895	1.78993
10	7	4.14652	3.84921	3.59635	3.38550	3.20294	3.04487	2.90683	2.78537	2.67776	2.58184	2.49584
10	8	7.25098	6.53330	5.94761	5.46365	5.05779	4.71361	4.41873	4.16376	3.94148	3.74626	3.57365
10	9	14.22768	12.37224	10.91003	9.72556	8.77967	7.98836	7.32580	6.76489	6.28532	5.87165	5.51194
10	10	40.27166	33.15339	27.22925	23.75265	20.56756	18.03384	15.98642	14.30874	12.91680	11.74901	10.75937

5% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						-βw-						
1	1	0.01663	0.01906	0.02166	0.02441	0.02732	0.03037	0.03356	0.03688	0.04032	0.04387	0.04753
2	1	0.00639	0.00756	0.00885	0.01026	0.01179	0.01344	0.01520	0.01707	0.01906	0.02115	0.02335
2	2	0.15030	0.16010	0.16984	0.17952	0.18911	0.19860	0.20799	0.21726	0.22641	0.23544	0.24433
3	1	0.00365	0.00440	0.00525	0.00618	0.00721	0.00834	0.00956	0.01088	0.01229	0.01380	0.01540
3	2	0.06999	0.07648	0.08309	0.08981	0.09661	0.10349	0.11042	0.11739	0.12438	0.13140	0.13842
3	3	0.34213	0.35458	0.36664	0.37832	0.38963	0.40058	0.41119	0.42147	0.43143	0.44108	0.45043
4	1	0.00246	0.00300	0.00362	0.00432	0.00509	0.00594	0.00688	0.00790	0.00901	0.01020	0.01147
4	2	0.04332	0.04810	0.05305	0.05815	0.06338	0.06874	0.07420	0.07976	0.08540	0.09112	0.09689
4	3	0.17774	0.18828	0.19870	0.20899	0.21914	0.22915	0.23900	0.24869	0.25822	0.26759	0.27679
4	4	0.54069	0.55189	0.56258	0.57278	0.58253	0.59187	0.60080	0.60937	0.61758	0.62546	0.63303
5	1	0.00181	0.00223	0.00271	0.00326	0.00388	0.00457	0.00533	0.00617	0.00708	0.00806	0.00912
5	2	0.03044	0.03420	0.03813	0.04223	0.04648	0.05087	0.05539	0.06002	0.06476	0.06960	0.07452
5	3	0.11603	0.12467	0.13333	0.14199	0.15064	0.15927	0.16785	0.17638	0.18485	0.19325	0.20157
5	4	0.30169	0.31398	0.32594	0.33756	0.34885	0.35983	0.37049	0.38085	0.39092	0.40070	0.41021
5	5	0.73116	0.73883	0.74608	0.75294	0.75944	0.76562	0.77148	0.77706	0.78238	0.78744	0.79228
6	1	0.00140	0.00175	0.00215	0.00260	0.00311	0.00369	0.00433	0.00504	0.00581	0.00665	0.00757
6	2	0.02301	0.02609	0.02934	0.03276	0.03634	0.04006	0.04392	0.04790	0.05200	0.05621	0.06052
6	3	0.08417	0.09141	0.09874	0.10615	0.11362	0.12112	0.12865	0.13620	0.14374	0.15126	0.15877
6	4	0.20462	0.21573	0.22667	0.23744	0.24801	0.25839	0.26858	0.27857	0.28836	0.29795	0.30735
6	5	0.42923	0.44150	0.45330	0.46464	0.47556	0.48607	0.49620	0.50595	0.51535	0.52442	0.53317
6	6	0.90992	0.91279	0.91548	0.91801	0.92039	0.92264	0.92476	0.92678	0.92868	0.93049	0.93221
7	1	0.00114	0.00142	0.00176	0.00214	0.00258	0.00308	0.00363	0.00424	0.00492	0.00566	0.00646
7	2	0.01824	0.02084	0.02361	0.02655	0.02963	0.03286	0.03623	0.03973	0.04335	0.04708	0.05092
7	3	0.06500	0.07120	0.07753	0.08398	0.09053	0.09716	0.10385	0.11059	0.11737	0.12418	0.13100
7	4	0.15211	0.16196	0.17176	0.18148	0.19111	0.20064	0.21007	0.21937	0.22855	0.23761	0.24652
7	5	0.29968	0.31197	0.32391	0.33553	0.34682	0.35779	0.36845	0.37881	0.38888	0.39867	0.40818
7	6	0.55503	0.56603	0.57652	0.58653	0.59609	0.60523	0.61397	0.62235	0.63038	0.63808	0.64547
7	7	1.07676	1.07411	1.07164	1.06932	1.06715	1.06511	1.06320	1.06139	1.05968	1.05806	1.05653
8	1	0.00094	0.00119	0.00148	0.00181	0.00220	0.00263	0.00312	0.00366	0.00426	0.00492	0.00563
8	2	0.01495	0.01720	0.01961	0.02218	0.02489	0.02774	0.03074	0.03386	0.03710	0.04046	0.04393
8	3	0.05233	0.05774	0.06330	0.06900	0.07483	0.08075	0.08677	0.09287	0.09903	0.10524	0.11150
8	4	0.11948	0.12825	0.13704	0.14582	0.15458	0.16330	0.17198	0.18060	0.18915	0.19762	0.20601
8	5	0.22712	0.23863	0.24992	0.26098	0.27183	0.28244	0.29283	0.30299	0.31293	0.32265	0.33214
8	6	0.39648	0.40890	0.42087	0.43241	0.44353	0.45426	0.46462	0.47462	0.48428	0.49361	0.50262
8	7	0.67690	0.68576	0.69415	0.70212	0.70968	0.71688	0.72373	0.73026	0.73649	0.74244	0.74813
8	8	1.23251	1.22395	1.21600	1.20859	1.20167	1.19519	1.18912	1.18341	1.17804	1.17297	1.16818
9	1	0.00080	0.00102	0.00127	0.00157	0.00190	0.00229	0.00272	0.00321	0.00375	0.00434	0.00499
9	2	0.01257	0.01455	0.01667	0.01895	0.02137	0.02393	0.02662	0.02944	0.03239	0.03545	0.03862
9	3	0.04341	0.04819	0.05315	0.05825	0.06349	0.06886	0.07433	0.07989	0.08554	0.09126	0.09703
9	4	0.09742	0.10528	0.11321	0.12119	0.12919	0.13720	0.14522	0.15321	0.16118	0.16911	0.17699
9	5	0.18091	0.19152	0.20201	0.21236	0.22257	0.23263	0.24252	0.25226	0.26182	0.27122	0.28045
9	6	0.30525	0.31756	0.32953	0.34117	0.35247	0.36344	0.37411	0.38447	0.39453	0.40430	0.41380
9	7	0.49260	0.50436	0.51562	0.52640	0.53674	0.54665	0.55617	0.56531	0.57409	0.58254	0.59066
9	8	0.79398	0.80011	0.80588	0.81134	0.81649	0.82138	0.82601	0.83041	0.83459	0.83857	0.84236
9	9	1.37823	1.36357	1.35000	1.33740	1.32567	1.31472	1.30448	1.29489	1.28588	1.27739	1.26940
10	1	0.00069	0.00088	0.00111	0.00137	0.00167	0.00202	0.00241	0.00285	0.00334	0.00388	0.00448
10	2	0.01078	0.01254	0.01444	0.01648	0.01867	0.02098	0.02343	0.02601	0.02871	0.03152	0.03444
10	3	0.03683	0.04112	0.04558	0.05019	0.05496	0.05985	0.06487	0.06999	0.07520	0.08050	0.08587
10	4	0.08160	0.08871	0.09592	0.10321	0.11057	0.11797	0.12539	0.13284	0.14029	0.14773	0.15515
10	5	0.14907	0.15883	0.16854	0.17819	0.18775	0.19722	0.20658	0.21583	0.22496	0.23397	0.24285
10	6	0.24598	0.25775	0.26927	0.28054	0.29156	0.30232	0.31283	0.32310	0.33311	0.34289	0.35243
10	7	0.38424	0.39669	0.40870	0.42029	0.43147	0.44227	0.45270	0.46278	0.47252	0.48193	0.49104
10	8	0.58679	0.59731	0.60732	0.61686	0.62596	0.63464	0.64294	0.65088	0.65848	0.66576	0.67274
10	9	0.90608	0.90906	0.91136	0.91449	0.91698	0.91932	0.92153	0.92362	0.92561	0.92749	0.92928
10	10	1.51500	1.49416	1.47493	1.45713	1.44060	1.42522	1.41086	1.39744	1.38485	1.37304	1.36192

10% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						−β_w−						
1	1	0.04487	0.04976	0.05482	0.06003	0.06537	0.07083	0.07639	0.08205	0.08779	0.09359	0.09945
2	1	0.01725	0.01975	0.02241	0.02524	0.02822	0.03134	0.03460	0.03798	0.04150	0.04512	0.04885
2	2	0.26339	0.27537	0.28706	0.29848	0.30962	0.32048	0.33107	0.34140	0.35146	0.36126	0.37082
3	1	0.00986	0.01150	0.01328	0.01520	0.01726	0.01945	0.02177	0.02421	0.02677	0.02944	0.03223
3	2	0.12226	0.13113	0.14001	0.14888	0.15773	0.16653	0.17528	0.18397	0.19258	0.20111	0.20956
3	3	0.52170	0.53313	0.54406	0.55451	0.56451	0.57408	0.58326	0.59206	0.60050	0.60862	0.61642
4	1	0.00663	0.00784	0.00916	0.01061	0.01218	0.01386	0.01567	0.01758	0.01961	0.02175	0.02400
4	2	0.07561	0.08241	0.08931	0.09632	0.10340	0.11053	0.11771	0.12492	0.13214	0.13937	0.14659
4	3	0.26934	0.28138	0.29313	0.30459	0.31576	0.32665	0.33726	0.34760	0.35767	0.36748	0.37703
4	4	0.76863	0.77540	0.78179	0.78783	0.79354	0.79896	0.80410	0.80898	0.81363	0.81806	0.82228
5	1	0.00487	0.00582	0.00687	0.00803	0.00929	0.01066	0.01214	0.01372	0.01541	0.01720	0.01909
5	2	0.05310	0.05856	0.06418	0.06993	0.07580	0.08177	0.08784	0.09398	0.10018	0.10643	0.11273
5	3	0.17550	0.18598	0.19635	0.20660	0.21671	0.22668	0.23650	0.24617	0.25567	0.26501	0.27419
5	4	0.42532	0.43761	0.44943	0.46081	0.47176	0.48230	0.49245	0.50223	0.51167	0.52077	0.52956
5	5	0.99565	0.99579	0.99593	0.99606	0.99617	0.99629	0.99639	0.99649	0.99659	0.99668	0.99676
6	1	0.00379	0.00456	0.00543	0.00639	0.00745	0.00860	0.00986	0.01121	0.01265	0.01419	0.01583
6	2	0.04013	0.04467	0.04938	0.05425	0.05926	0.06439	0.06964	0.07499	0.08043	0.08595	0.09153
6	3	0.12722	0.13627	0.14532	0.15434	0.16334	0.17228	0.18115	0.18996	0.19868	0.20731	0.21585
6	4	0.28770	0.29990	0.31178	0.32334	0.33460	0.34555	0.35620	0.36656	0.37664	0.38644	0.39598
6	5	0.57881	0.58945	0.59959	0.60925	0.61847	0.62727	0.63569	0.64374	0.65145	0.65883	0.66592
6	6	1.20314	1.19574	1.18887	1.18246	1.17647	1.17086	1.16559	1.16064	1.15598	1.15158	1.14742
7	1	0.00306	0.00372	0.00445	0.00527	0.00618	0.00718	0.00826	0.00944	0.01071	0.01207	0.01352
7	2	0.03181	0.03568	0.03974	0.04395	0.04831	0.05282	0.05745	0.06219	0.06704	0.07198	0.07701
7	3	0.09820	0.10609	0.11406	0.12206	0.13010	0.13814	0.14618	0.15420	0.16219	0.17014	0.17804
7	4	0.21361	0.22488	0.23597	0.24687	0.25756	0.26804	0.27831	0.28838	0.29824	0.30788	0.31733
7	5	0.40277	0.41517	0.42711	0.43862	0.44971	0.46041	0.47072	0.48068	0.49029	0.49957	0.50854
7	6	0.72599	0.73379	0.74115	0.74812	0.75473	0.76100	0.76696	0.77263	0.77804	0.78320	0.78812
7	7	1.39331	1.37799	1.36381	1.35065	1.33840	1.32698	1.31629	1.30628	1.29688	1.28804	1.27971
8	1	0.00255	0.00311	0.00375	0.00446	0.00526	0.00613	0.00710	0.00814	0.00927	0.01049	0.01179
8	2	0.02608	0.02945	0.03300	0.03671	0.04058	0.04459	0.04873	0.05300	0.05738	0.06186	0.06643
8	3	0.07904	0.08601	0.09310	0.10027	0.10750	0.11479	0.12211	0.12946	0.13681	0.14417	0.15151
8	4	0.16767	0.17796	0.18815	0.19823	0.20819	0.21803	0.22773	0.23728	0.24669	0.25594	0.26505
8	5	0.30476	0.31708	0.32905	0.34068	0.35198	0.36295	0.37362	0.38398	0.39404	0.40381	0.41331
8	6	0.51660	0.52810	0.53909	0.54960	0.55966	0.56930	0.57854	0.58740	0.59591	0.60408	0.61194
8	7	0.36580	0.86997	0.87389	0.87757	0.88105	0.88434	0.88745	0.89040	0.89320	0.89586	0.89839
8	8	1.56848	1.54513	1.52359	1.50367	1.48520	1.46802	1.45201	1.43704	1.42303	1.40988	1.39752
9	1	0.00217	0.00266	0.00322	0.00385	0.00456	0.00534	0.00620	0.00714	0.00816	0.00926	0.01044
9	2	0.02193	0.02491	0.02806	0.03137	0.03484	0.03846	0.04221	0.04609	0.05008	0.05419	0.05840
9	3	0.06555	0.07179	0.07815	0.08463	0.09121	0.09786	0.10458	0.11135	0.11816	0.12499	0.13183
9	4	0.13665	0.14603	0.15538	0.16469	0.17394	0.18312	0.19222	0.20123	0.21014	0.21895	0.22764
9	5	0.24253	0.25426	0.26574	0.27698	0.28797	0.29870	0.30920	0.31944	0.32945	0.33922	0.34875
9	6	0.39696	0.40938	0.42134	0.43288	0.44400	0.45473	0.46509	0.47508	0.48474	0.49406	0.50308
9	7	0.62736	0.63719	0.64652	0.65539	0.66384	0.67189	0.67956	0.68690	0.69390	0.70061	0.70703
9	8	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99850	0.99854	0.99858	0.99862	0.99865
9	9	1.73073	1.69937	1.67055	1.64397	1.61940	1.59660	1.57540	1.55563	1.53716	1.51987	1.50364
10	1	0.00187	0.00231	0.00281	0.00338	0.00401	0.00472	0.00550	0.00635	0.00728	0.00829	0.00938
10	2	0.01880	0.02146	0.02429	0.02729	0.03043	0.03372	0.03715	0.04071	0.04439	0.04818	0.05203
10	3	0.05561	0.06124	0.06701	0.07292	0.07894	0.08506	0.09126	0.09754	0.10387	0.11025	0.11666
10	4	0.11444	0.12302	0.13162	0.14023	0.14883	0.15741	0.16595	0.17443	0.18286	0.19123	0.19951
10	5	0.19973	0.21075	0.22160	0.23229	0.24279	0.25312	0.26325	0.27319	0.28294	0.29250	0.30187
10	6	0.31953	0.33191	0.34394	0.35560	0.36692	0.37790	0.38855	0.39869	0.40892	0.41866	0.42811
10	7	0.48829	0.50010	0.51141	0.52224	0.53262	0.54258	0.55214	0.56133	0.57016	0.57865	0.58682
10	8	0.73430	0.74190	0.74908	0.75587	0.76231	0.76842	0.77422	0.77975	0.78501	0.79002	0.79481
10	9	1.12355	1.11919	1.11513	1.11134	1.10779	1.10446	1.10133	1.09838	1.09560	1.09297	1.09048
10	10	1.38178	1.84254	1.80657	1.77348	1.74296	1.71471	1.68849	1.66410	1.64135	1.62009	1.60017

20% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						$-\beta_W-$						
1	1	0.12633	0.13535	0.14437	0.15337	0.16233	0.17125	0.18010	0.18889	0.19759	0.20621	0.21472
2	1	0.04856	0.05371	0.05903	0.06448	0.07007	0.07577	0.08156	0.08744	0.09340	0.09941	0.10547
2	2	0.48613	0.49796	0.50929	0.52014	0.53055	0.54053	0.55011	0.55932	0.56818	0.57669	0.58489
3	1	0.02776	0.03128	0.03498	0.03884	0.04286	0.04702	0.05131	0.05573	0.06025	0.06487	0.06959
3	2	0.22442	0.23588	0.24713	0.25817	0.26898	0.27957	0.28994	0.30009	0.31001	0.31971	0.32920
3	3	0.83704	0.84202	0.84670	0.85111	0.85528	0.85922	0.86296	0.86650	0.86986	0.87306	0.87610
4	1	0.01867	0.02132	0.02413	0.02711	0.03024	0.03352	0.03694	0.04048	0.04415	0.04792	0.05181
4	2	0.13860	0.14804	0.15745	0.16681	0.17611	0.18534	0.19449	0.20354	0.21249	0.22133	0.23005
4	3	0.42800	0.44028	0.45209	0.46345	0.47437	0.48489	0.49503	0.50479	0.51420	0.52328	0.53205
4	4	1.14744	1.14219	1.13730	1.13274	1.12847	1.12447	1.12070	1.11716	1.11382	1.11067	1.10768
5	1	0.01372	0.01583	0.01810	0.02051	0.02308	0.02578	0.02862	0.03159	0.03468	0.03789	0.04121
5	2	0.09729	0.10515	0.11308	0.12105	0.12904	0.13706	0.14506	0.15305	0.16102	0.16895	0.17683
5	3	0.27810	0.29022	0.30204	0.31355	0.32477	0.33569	0.34633	0.35668	0.36676	0.37657	0.38612
5	4	0.62709	0.63692	0.64626	0.65513	0.66358	0.67164	0.67932	0.68665	0.69367	0.70037	0.70680
5	5	1.42109	1.40454	1.38924	1.37504	1.36183	1.34952	1.33801	1.32723	1.31711	1.30760	1.29864
6	1	0.01067	0.01241	0.01430	0.01633	0.01850	0.02080	0.02324	0.02580	0.02848	0.03127	0.03418
6	2	0.07350	0.08018	0.08698	0.09388	0.10086	0.10790	0.11499	0.12211	0.12925	0.13640	0.14355
6	3	0.20134	0.21239	0.22328	0.23399	0.24452	0.25486	0.26501	0.27497	0.28473	0.29430	0.30368
6	4	0.42251	0.43482	0.44666	0.45806	0.46902	0.47958	0.48976	0.49957	0.50903	0.51815	0.52696
6	5	0.81438	0.81998	0.82524	0.83021	0.83491	0.83935	0.84356	0.84756	0.85135	0.85497	0.85841
6	6	1.66470	1.63666	1.61085	1.58703	1.56497	1.54449	1.52543	1.50764	1.49100	1.47541	1.46077
7	1	0.00863	0.01011	0.01172	0.01347	0.01535	0.01735	0.01948	0.02174	0.02411	0.02659	0.02918
7	2	0.05825	0.06404	0.06998	0.07605	0.08222	0.08849	0.09484	0.10125	0.10772	0.11422	0.12076
7	3	0.15531	0.16526	0.17514	0.18494	0.19464	0.20424	0.21373	0.22309	0.23232	0.24141	0.25037
7	4	0.31313	0.32549	0.33749	0.34914	0.36046	0.37144	0.38210	0.39245	0.40250	0.41225	0.42173
7	5	0.56397	0.57484	0.58520	0.59508	0.60452	0.61353	0.62216	0.63041	0.63832	0.64591	0.65319
7	6	0.98888	0.98925	0.98959	0.98992	0.99022	0.99051	0.99078	0.99103	0.99127	0.99150	0.99172
7	7	1.88393	1.84457	1.80850	1.77532	1.74471	1.71638	1.69009	1.66563	1.64282	1.62150	1.60153
8	1	0.00718	0.00846	0.00987	0.01140	0.01305	0.01483	0.01673	0.01874	0.02087	0.02310	0.02545
8	2	0.04776	0.05285	0.05811	0.06352	0.06905	0.07470	0.08044	0.08628	0.09218	0.09815	0.10417
8	3	0.12496	0.13393	0.14290	0.15186	0.16079	0.16967	0.17849	0.18724	0.19591	0.20450	0.21299
8	4	0.24556	0.25732	0.26884	0.28011	0.29112	0.30188	0.31239	0.32265	0.33267	0.34244	0.35198
8	5	0.42576	0.43805	0.44987	0.46124	0.47218	0.48272	0.49287	0.50265	0.51208	0.52118	0.52996
8	6	0.69980	0.70817	0.71610	0.72361	0.73074	0.73751	0.74396	0.75009	0.75595	0.76153	0.76687
8	7	1.15132	1.14592	1.14090	1.13621	1.13182	1.12770	1.12384	1.12020	1.11677	1.11353	1.11046
8	8	2.08318	2.03284	1.98684	1.94467	1.90587	1.87006	1.83691	1.80614	1.77751	1.75081	1.72585
9	1	0.00610	0.00723	0.00848	0.00984	0.01132	0.01291	0.01462	0.01644	0.01837	0.02041	0.02255
9	2	0.04015	0.04469	0.04941	0.05427	0.05928	0.06442	0.06967	0.07502	0.08046	0.08598	0.09156
9	3	0.10362	0.11175	0.11994	0.12816	0.13639	0.14462	0.15283	0.16102	0.16917	0.17726	0.18530
9	4	0.20001	0.21103	0.22189	0.23258	0.24309	0.25341	0.26355	0.27350	0.28325	0.29281	0.30218
9	5	0.33838	0.35082	0.36288	0.37456	0.38587	0.39683	0.40745	0.41774	0.42771	0.43738	0.44675
9	6	0.53627	0.54753	0.55827	0.56853	0.57835	0.58774	0.59673	0.60535	0.61362	0.62155	0.62918
9	7	0.82921	0.83440	0.83929	0.84390	0.84825	0.85237	0.85626	0.85996	0.86348	0.86682	0.87000
9	8	1.30286	1.29142	1.28081	1.27094	1.26174	1.25314	1.24509	1.23753	1.23042	1.22373	1.21741
9	9	2.26583	2.20489	2.14936	2.09857	2.05196	2.00904	1.96939	1.93266	1.89855	1.86678	1.83714
10	1	0.00527	0.00628	0.00740	0.00862	0.00996	0.01141	0.01296	0.01462	0.01639	0.01827	0.02024
10	2	0.03442	0.03851	0.04278	0.04720	0.05178	0.05649	0.06132	0.06627	0.07131	0.07644	0.08165
10	3	0.08789	0.09531	0.10282	0.11040	0.11802	0.12568	0.13335	0.14103	0.14869	0.15634	0.16396
10	4	0.16743	0.17771	0.18789	0.19797	0.20793	0.21776	0.22745	0.23700	0.24641	0.25566	0.26475
10	5	0.27844	0.29056	0.30238	0.31389	0.32511	0.33603	0.34667	0.35702	0.36710	0.37691	0.38646
10	6	0.43099	0.44325	0.45504	0.46638	0.47728	0.48778	0.49789	0.50763	0.51701	0.52607	0.53480
10	7	0.64343	0.65296	0.66200	0.67059	0.67875	0.68653	0.69395	0.70103	0.70779	0.71426	0.72045
10	8	0.95221	0.95377	0.95522	0.95659	0.95788	0.95909	0.96024	0.96132	0.96235	0.96332	0.96424
10	9	1.44469	1.42708	1.41080	1.39571	1.38168	1.36861	1.35639	1.34495	1.33422	1.32414	1.31464
10	10	2.43448	2.36334	2.29867	2.23965	2.18559	2.13590	2.09009	2.04772	2.00844	1.97192	1.93788

30% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						-β_N-						
1	1	0.24124	0.25295	0.26442	0.27564	0.28662	0.29735	0.30783	0.31807	0.32807	0.33784	0.34737
2	1	0.09273	0.10038	0.10811	0.11589	0.12371	0.13156	0.13940	0.14725	0.15507	0.16287	0.17063
2	2	0.72680	0.73457	0.74192	0.74887	0.75546	0.76172	0.76766	0.77332	0.77871	0.78386	0.78877
3	1	0.05301	0.05846	0.06407	0.06981	0.07568	0.08165	0.08771	0.09384	0.10004	0.10629	0.11257
3	2	0.33383	0.34626	0.35831	0.36999	0.38131	0.39227	0.40290	0.41321	0.42320	0.43288	0.44228
3	3	1.15044	1.14508	1.14008	1.13542	1.13106	1.12697	1.12313	1.11951	1.11610	1.11288	1.10983
4	1	0.03565	0.03984	0.04420	0.04873	0.05340	0.05820	0.06313	0.06817	0.07330	0.07852	0.08381
4	2	0.20590	0.21704	0.22800	0.23878	0.24937	0.25977	0.26997	0.27997	0.28977	0.29937	0.30878
4	3	0.58327	0.59384	0.60391	0.61350	0.62265	0.63139	0.63974	0.64773	0.65538	0.66270	0.66973
4	4	1.50873	1.48819	1.46923	1.45167	1.43537	1.42019	1.40603	1.39278	1.38036	1.36870	1.35773
5	1	0.02620	0.02958	0.03314	0.03687	0.04074	0.04477	0.04892	0.05320	0.05759	0.06208	0.06667
5	2	0.14446	0.15408	0.16367	0.17319	0.18264	0.19201	0.20128	0.21044	0.21950	0.22843	0.23725
5	3	0.37805	0.39051	0.40254	0.41415	0.42536	0.43619	0.44665	0.45677	0.46654	0.47600	0.48515
5	4	0.81574	0.82130	0.82653	0.83146	0.83613	0.84054	0.84472	0.84869	0.85246	0.85605	0.85947
5	5	1.81697	1.78115	1.74829	1.71804	1.69009	1.66421	1.64016	1.61777	1.59688	1.57733	1.55900
6	1	0.02038	0.02320	0.02620	0.02935	0.03267	0.03612	0.03972	0.04344	0.04729	0.05124	0.05530
6	2	0.10911	0.11747	0.12587	0.13429	0.14272	0.15113	0.15951	0.16786	0.17615	0.18439	0.19256
6	3	0.27341	0.28549	0.29727	0.30875	0.31995	0.33085	0.34148	0.35182	0.36190	0.37171	0.38126
6	4	0.54777	0.55887	0.56946	0.57957	0.58923	0.59847	0.60731	0.61578	0.62390	0.63170	0.63918
6	5	1.02861	1.02764	1.02674	1.02589	1.02509	1.02435	1.02364	1.02298	1.02235	1.02176	1.02119
6	6	2.08711	2.03654	1.99035	1.94799	1.90903	1.87307	1.83978	1.80889	1.78014	1.75333	1.72826
7	1	0.01647	0.01889	0.02147	0.02421	0.02710	0.03013	0.03330	0.03660	0.04003	0.04356	0.04721
7	2	0.08646	0.09381	0.10125	0.10877	0.11633	0.12393	0.13155	0.13917	0.14679	0.15439	0.16197
7	3	0.21079	0.22201	0.23306	0.24391	0.25457	0.26502	0.27527	0.28531	0.29515	0.30478	0.31421
7	4	0.40534	0.41772	0.42965	0.44115	0.45223	0.46290	0.47320	0.48314	0.49274	0.50200	0.51095
7	5	0.70944	0.71760	0.72533	0.73264	0.73958	0.74617	0.75244	0.75841	0.76410	0.76953	0.77472
7	6	1.22348	1.21528	1.20766	1.20056	1.19393	1.18772	1.18190	1.17642	1.17127	1.16640	1.16181
7	7	2.32759	2.26296	2.20412	2.15035	2.10104	2.05566	2.01377	1.97499	1.93899	1.90550	1.87425
8	1	0.01370	0.01581	0.01807	0.02049	0.02305	0.02575	0.02859	0.03156	0.03465	0.03785	0.04117
8	2	0.07087	0.07741	0.08407	0.09084	0.09766	0.10460	0.11157	0.11858	0.12561	0.13266	0.13971
8	3	0.16954	0.17987	0.19010	0.20022	0.21022	0.22009	0.22982	0.23940	0.24883	0.25811	0.26723
8	4	0.31760	0.32997	0.34199	0.35365	0.36497	0.37595	0.38661	0.39695	0.40698	0.41673	0.42619
8	5	0.53454	0.54582	0.55658	0.56687	0.57670	0.58612	0.59513	0.60377	0.61206	0.62002	0.62767
8	6	0.86185	0.86613	0.87015	0.87394	0.87752	0.88090	0.88410	0.88713	0.89000	0.89274	0.89534
8	7	1.40265	1.38692	1.37236	1.35885	1.34628	1.33456	1.32360	1.31333	1.30369	1.29463	1.28608
8	8	2.54442	2.46643	2.39564	2.33112	2.27209	2.21790	2.16800	2.12189	2.07919	2.03952	2.00259
9	1	0.01165	0.01351	0.01552	0.01768	0.01998	0.02242	0.02499	0.02769	0.03050	0.03344	0.03648
9	2	0.05958	0.06545	0.07147	0.07761	0.08387	0.09021	0.09663	0.10311	0.10964	0.11620	0.12280
9	3	0.14055	0.15005	0.15952	0.16894	0.17829	0.18756	0.19675	0.20584	0.21482	0.22370	0.23245
9	4	0.25855	0.27047	0.28213	0.29351	0.30461	0.31545	0.32602	0.33633	0.34639	0.35619	0.36574
9	5	0.42438	0.43668	0.44850	0.45988	0.47084	0.48139	0.49155	0.50134	0.51078	0.51989	0.52869
9	6	0.65898	0.66820	0.67695	0.68525	0.69315	0.70066	0.70782	0.71465	0.72116	0.72739	0.73336
9	7	1.00518	1.00501	1.00485	1.00469	1.00455	1.00442	1.00429	1.00417	1.00406	1.00395	1.00385
9	8	1.56826	1.54492	1.52339	1.50348	1.48502	1.46785	1.45184	1.43688	1.42287	1.40973	1.39737
9	9	2.74193	2.65128	2.56918	2.49453	2.42638	2.36394	2.30654	2.25361	2.20466	2.15927	2.11707
10	1	0.01007	0.01174	0.01355	0.01550	0.01759	0.01981	0.02215	0.02463	0.02722	0.02993	0.03275
10	2	0.05107	0.05640	0.06188	0.06750	0.07325	0.07910	0.08505	0.09107	0.09716	0.10331	0.10950
10	3	0.11920	0.12796	0.13674	0.14551	0.15426	0.16298	0.17165	0.18026	0.18880	0.19727	0.20565
10	4	0.21636	0.22769	0.23882	0.24975	0.26048	0.27099	0.28129	0.29138	0.30125	0.31092	0.32037
10	5	0.34897	0.36143	0.37349	0.38517	0.39646	0.40740	0.41799	0.42824	0.43817	0.44779	0.45711
10	6	0.52892	0.54027	0.55111	0.56147	0.57137	0.58086	0.58994	0.59866	0.60702	0.61504	0.62276
10	7	0.77803	0.78456	0.79073	0.79655	0.80206	0.80726	0.81223	0.81694	0.82142	0.82568	0.82974
10	8	1.14008	1.13511	1.13048	1.12616	1.12211	1.11831	1.11475	1.11139	1.10822	1.10523	1.10240
10	9	1.72216	1.69124	1.66281	1.63660	1.61235	1.58985	1.56893	1.54943	1.53120	1.51413	1.49811
10	10	2.92340	2.82071	2.72792	2.64370	2.56695	2.49675	2.43233	2.37301	2.31823	2.26750	2.22039

40% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						β_W						
1	1	0.39593	0.40835	0.42032	0.43186	0.44299	0.45372	0.46408	0.47409	0.48375	0.49308	0.50210
2	1	0.15220	0.16205	0.17185	0.18157	0.19121	0.20074	0.21017	0.21947	0.22866	0.23771	0.24663
2	2	1.00126	1.00121	1.00118	1.00114	1.00110	1.00107	1.00104	1.00101	1.00098	1.00096	1.00093
3	1	0.08700	0.09438	0.10184	0.10938	0.11697	0.12459	0.13223	0.13987	0.14751	0.15513	0.16272
3	2	0.45751	0.46960	0.48119	0.49231	0.50300	0.51327	0.52314	0.53264	0.54179	0.55060	0.55909
3	3	1.48942	1.46977	1.45163	1.43482	1.41921	1.40467	1.39110	1.37840	1.36649	1.35531	1.34479
4	1	0.05850	0.06431	0.07026	0.07634	0.08253	0.08881	0.09518	0.10160	0.10808	0.11460	0.12114
4	2	0.28182	0.29397	0.30581	0.31735	0.32858	0.33951	0.35016	0.36051	0.37059	0.38040	0.38994
4	3	0.74880	0.75606	0.76291	0.76939	0.77553	0.78135	0.78687	0.79213	0.79713	0.80190	0.80646
4	4	1.88932	1.84967	1.81334	1.77993	1.74910	1.72057	1.69410	1.66947	1.64651	1.62504	1.60494
5	1	0.04301	0.04776	0.05268	0.05776	0.06297	0.06831	0.07375	0.07929	0.08491	0.09061	0.09636
5	2	0.19762	0.20860	0.21941	0.23006	0.24054	0.25083	0.26094	0.27087	0.28060	0.29014	0.29949
5	3	0.48417	0.49602	0.50737	0.51824	0.52866	0.53867	0.54828	0.55750	0.56638	0.57492	0.58313
5	4	1.01091	1.01055	1.01020	1.00988	1.00958	1.00930	1.00903	1.00878	1.00854	1.00832	1.00810
5	5	2.22735	2.16868	2.11519	2.06625	2.02131	1.97990	1.94163	1.90617	1.87323	1.84254	1.81389
6	1	0.03344	0.03745	0.04164	0.04599	0.05049	0.05512	0.05988	0.06475	0.06972	0.07478	0.07993
6	2	0.14922	0.15899	0.16870	0.17835	0.18792	0.19739	0.20675	0.21601	0.22514	0.23415	0.24303
6	3	0.34979	0.36225	0.37431	0.38599	0.39728	0.40822	0.41880	0.42905	0.43898	0.44859	0.45791
6	4	0.67662	0.68549	0.69389	0.70186	0.70943	0.71663	0.72348	0.73002	0.73625	0.74221	0.74790
6	5	1.24615	1.23705	1.22859	1.22071	1.21335	1.20647	1.20002	1.19396	1.18825	1.18287	1.17779
6	6	2.52028	2.44381	2.37437	2.31107	2.25314	2.19995	2.15094	2.10567	2.06371	2.02474	1.98844
7	1	0.02704	0.03050	0.03413	0.03793	0.04188	0.04598	0.05021	0.05456	0.05902	0.06358	0.06824
7	2	0.11823	0.12695	0.13569	0.14443	0.15315	0.16184	0.17048	0.17907	0.18759	0.19604	0.20440
7	3	0.26952	0.28156	0.29331	0.30477	0.31594	0.32683	0.33744	0.34778	0.35785	0.36766	0.37722
7	4	0.49994	0.51163	0.52281	0.53351	0.54377	0.55360	0.56303	0.57209	0.58079	0.58915	0.59720
7	5	0.85616	0.86060	0.86478	0.86871	0.87243	0.87594	0.87926	0.88241	0.88539	0.88824	0.89094
7	6	1.45871	1.44047	1.42361	1.40798	1.39346	1.37993	1.36729	1.35546	1.34437	1.33394	1.32412
7	7	2.77900	2.68592	2.60166	2.52508	2.45518	2.39117	2.33235	2.27812	2.22799	2.18151	2.13831
8	1	0.02249	0.02552	0.02873	0.03210	0.03562	0.03929	0.04310	0.04704	0.05109	0.05525	0.05950
8	2	0.09691	0.10475	0.11265	0.12061	0.12859	0.13659	0.14458	0.15256	0.16051	0.16843	0.17630
8	3	0.21670	0.22804	0.23917	0.25011	0.26084	0.27135	0.28166	0.29175	0.30162	0.31129	0.32074
8	4	0.39140	0.40384	0.41582	0.42738	0.43853	0.44930	0.45969	0.46972	0.47941	0.48877	0.49783
8	5	0.64390	0.65342	0.66245	0.67103	0.67920	0.68697	0.69438	0.70145	0.70820	0.71466	0.72084
8	6	1.02309	1.02231	1.02158	1.02090	1.02026	1.01966	1.01909	1.01856	1.01805	1.01757	1.01712
8	7	1.65234	1.62491	1.59967	1.57635	1.55476	1.53471	1.51604	1.49862	1.48232	1.46705	1.45270
8	8	3.01089	2.90227	2.80421	2.71530	2.63434	2.56034	2.49249	2.43005	2.37243	2.31910	2.26962
9	1	0.01912	0.02181	0.02468	0.02770	0.03088	0.03421	0.03767	0.04127	0.04498	0.04880	0.05273
9	2	0.08146	0.08856	0.09576	0.10305	0.11039	0.11779	0.12521	0.13265	0.14009	0.14753	0.15495
9	3	0.17962	0.19019	0.20066	0.21098	0.22117	0.23121	0.24109	0.25080	0.26036	0.26974	0.27896
9	4	0.31848	0.33086	0.34288	0.35454	0.36586	0.37684	0.38749	0.39784	0.40787	0.41761	0.42707
9	5	0.51068	0.52224	0.53330	0.54388	0.55402	0.56372	0.57303	0.58196	0.59054	0.59878	0.60671
9	6	0.78061	0.78708	0.79318	0.79895	0.80440	0.80957	0.81447	0.81912	0.82355	0.82777	0.83179
9	7	1.17854	1.17210	1.16611	1.16053	1.15530	1.15041	1.14581	1.14149	1.13741	1.13356	1.12993
9	8	1.83008	1.79358	1.76010	1.72928	1.70081	1.67445	1.64997	1.62718	1.60591	1.58601	1.56737
9	9	3.22116	3.09798	2.98702	2.88660	2.79534	2.71209	2.63587	2.56586	2.50134	2.44171	2.38645
10	1	0.01653	0.01895	0.02154	0.02429	0.02718	0.03022	0.03340	0.03671	0.04014	0.04368	0.04733
10	2	0.06982	0.07630	0.08291	0.08962	0.09641	0.10328	0.11020	0.11716	0.12415	0.13116	0.13817
10	3	0.15231	0.16217	0.17198	0.18170	0.19134	0.20088	0.21030	0.21961	0.22879	0.23785	0.24677
10	4	0.26643	0.27844	0.29016	0.30160	0.31276	0.32363	0.33424	0.34457	0.35463	0.36444	0.37400
10	5	0.41966	0.43199	0.44384	0.45525	0.46624	0.47682	0.48702	0.49685	0.50633	0.51548	0.52432
10	6	0.62579	0.63565	0.64501	0.65391	0.66238	0.67045	0.67815	0.68551	0.69254	0.69927	0.70571
10	7	0.91006	0.91293	0.91561	0.91814	0.92052	0.92276	0.92489	0.92689	0.92880	0.93061	0.93232
10	8	1.32375	1.31143	1.30001	1.28940	1.27950	1.27026	1.26161	1.25349	1.24586	1.23867	1.23189
10	9	1.99435	1.94899	1.90748	1.86937	1.83427	1.80183	1.77177	1.74384	1.71783	1.69354	1.67082
10	10	3.41364	3.27675	3.15367	3.04249	2.94161	2.84973	2.75573	2.68866	2.61775	2.55229	2.49170

60% LIMITS

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
						-βw-						
1	1	0.88640	0.88997	0.89333	0.89648	0.89946	0.90226	0.90492	0.90743	0.90982	0.91208	0.91424
2	1	0.34074	0.35319	0.36525	0.37692	0.38824	0.39919	0.40980	0.42008	0.43005	0.43971	0.44907
2	2	1.73310	1.70162	1.67269	1.64601	1.62134	1.59846	1.57718	1.55735	1.53881	1.52146	1.50517
3	1	0.19478	0.20569	0.21646	0.22706	0.23749	0.24775	0.25783	0.26772	0.27743	0.28695	0.29628
3	2	0.78260	0.78902	0.79508	0.80080	0.80620	0.81133	0.81619	0.82081	0.82520	0.82938	0.83337
3	3	2.34365	2.27805	2.21835	2.16379	2.11377	2.06775	2.02527	1.98596	1.94947	1.91552	1.88386
4	1	0.13098	0.14016	0.14933	0.15848	0.16758	0.17661	0.18558	0.19447	0.20327	0.21198	0.22058
4	2	0.48060	0.49248	0.50387	0.51477	0.52524	0.53528	0.54492	0.55419	0.56310	0.57168	0.57993
4	3	1.15653	1.15094	1.14573	1.14087	1.13632	1.13206	1.12805	1.12429	1.12073	1.11737	1.11420
4	4	2.82134	2.72547	2.63873	2.55992	2.48803	2.42221	2.36176	2.30604	2.25455	2.20683	2.16249
5	1	0.09628	0.10409	0.11197	0.11990	0.12786	0.13584	0.14381	0.15177	0.15970	0.16760	0.17546
5	2	0.33660	0.34904	0.36110	0.37278	0.38409	0.39505	0.40568	0.41597	0.42595	0.43563	0.44501
5	3	0.74379	0.75117	0.75813	0.76472	0.77096	0.77688	0.78251	0.78786	0.79295	0.79780	0.80244
5	4	1.47561	1.45660	1.43903	1.42276	1.40764	1.39355	1.38040	1.36810	1.35656	1.34571	1.33551
5	5	3.21487	3.09213	2.98156	2.88149	2.79055	2.70758	2.63161	2.56182	2.49751	2.43807	2.38299
6	1	0.07487	0.08163	0.08850	0.09547	0.10251	0.10961	0.11676	0.12394	0.13113	0.13833	0.14553
6	2	0.25400	0.26587	0.27748	0.28882	0.29990	0.31071	0.32126	0.33156	0.34160	0.35139	0.36095
6	3	0.53610	0.54736	0.55810	0.56837	0.57818	0.58757	0.59657	0.60519	0.61346	0.62140	0.62903
6	4	0.98069	0.98133	0.98193	0.98248	0.98301	0.98351	0.98397	0.98442	0.98483	0.98523	0.98561
6	5	1.75333	1.72082	1.69095	1.66342	1.63796	1.61436	1.59242	1.57197	1.55287	1.53499	1.51822
6	6	3.55026	3.40344	3.27159	3.15263	3.04482	2.94672	2.85713	2.77501	2.69951	2.62987	2.56547
7	1	0.06053	0.06646	0.07254	0.07874	0.08504	0.09143	0.09790	0.10443	0.11100	0.11761	0.12425
7	2	0.20118	0.21222	0.22310	0.23381	0.24434	0.25468	0.26483	0.27479	0.28455	0.29412	0.30349
7	3	0.41257	0.42493	0.43682	0.44828	0.45931	0.46994	0.48019	0.49007	0.49961	0.50882	0.51771
7	4	0.72229	0.73016	0.73761	0.74466	0.75134	0.75769	0.76372	0.76946	0.77493	0.78014	0.78513
7	5	1.19462	1.18756	1.18099	1.17487	1.16915	1.16378	1.15875	1.15402	1.14956	1.14535	1.14137
7	6	1.99923	1.95359	1.91184	1.87351	1.83821	1.80559	1.77536	1.74728	1.72112	1.69670	1.67386
7	7	3.84299	3.67435	3.52329	3.38732	3.26437	3.15274	3.05098	2.95790	2.87246	2.79380	2.72116
8	1	0.05035	0.05562	0.06106	0.06663	0.07233	0.07814	0.08404	0.09003	0.09608	0.10219	0.10835
8	2	0.16486	0.17507	0.18519	0.19521	0.20512	0.21490	0.22455	0.23407	0.24344	0.25266	0.26173
8	3	0.33148	0.34391	0.35596	0.36764	0.37895	0.38993	0.40056	0.41087	0.42087	0.43056	0.43997
8	4	0.56449	0.57535	0.58571	0.59558	0.60501	0.61401	0.62263	0.63088	0.63878	0.64636	0.65363
8	5	0.89495	0.89826	0.90138	0.90431	0.90707	0.90967	0.91214	0.91447	0.91668	0.91878	0.92078
8	6	1.38924	1.37410	1.36008	1.34707	1.33497	1.32367	1.31311	1.30321	1.29392	1.28517	1.27693
8	7	2.22000	2.16176	2.10367	2.06007	2.01544	1.97433	1.93633	1.90111	1.86838	1.83790	1.80944
8	8	4.10304	3.91443	3.74585	3.59440	3.45772	3.33383	3.22108	3.11810	3.02372	2.93695	2.85693
9	1	0.04280	0.04754	0.05245	0.05751	0.06271	0.06803	0.07346	0.07899	0.08459	0.09028	0.09602
9	2	0.13856	0.14800	0.15741	0.16677	0.17607	0.18530	0.19444	0.20349	0.21244	0.22128	0.23000
9	3	0.27462	0.28671	0.29850	0.30999	0.32119	0.33210	0.34273	0.35308	0.36315	0.37296	0.38252
9	4	0.45883	0.47090	0.48248	0.49359	0.50427	0.51453	0.52439	0.53388	0.54301	0.55181	0.56028
9	5	0.70822	0.71641	0.72416	0.73150	0.73847	0.74508	0.75137	0.75736	0.76307	0.76852	0.77373
9	6	1.05524	1.05335	1.05159	1.04994	1.04839	1.04693	1.04556	1.04427	1.04304	1.04189	1.04079
9	7	1.56764	1.54432	1.52282	1.50294	1.48450	1.46735	1.45136	1.43642	1.42243	1.40930	1.39695
9	8	2.42045	2.35017	2.28627	2.22795	2.17452	2.12540	2.08010	2.03821	1.99936	1.96324	1.92957
9	9	4.33722	4.13020	3.94549	3.77983	3.63056	3.49545	3.37268	3.26069	3.15817	3.06403	2.97731
10	1	0.03701	0.04131	0.04578	0.05041	0.05519	0.06010	0.06513	0.07026	0.07549	0.08080	0.08618
10	2	0.11876	0.12750	0.13626	0.14502	0.15375	0.16246	0.17112	0.17972	0.18825	0.19671	0.20508
10	3	0.23281	0.24440	0.25576	0.26689	0.27779	0.28846	0.29889	0.30909	0.31905	0.32879	0.33830
10	4	0.38357	0.39602	0.40804	0.41963	0.43082	0.44162	0.45205	0.46213	0.47188	0.48129	0.49040
10	5	0.58121	0.59182	0.60192	0.61155	0.62073	0.62950	0.63787	0.64589	0.65357	0.66092	0.66798
10	6	0.84379	0.84858	0.85309	0.85734	0.86135	0.86513	0.86872	0.87213	0.87536	0.87843	0.88135
10	7	1.20456	1.19711	1.19018	1.18372	1.17769	1.17204	1.16673	1.16175	1.15705	1.15262	1.14843
10	8	1.73230	1.70086	1.67197	1.64533	1.62069	1.59784	1.57658	1.55677	1.53826	1.52092	1.50466
10	9	2.60412	2.52235	2.44618	2.38063	2.31888	2.26222	2.21006	2.16191	2.11733	2.07594	2.03742
10	10	4.55038	4.32626	4.12660	3.94780	3.78690	3.64146	3.50945	3.38917	3.27919	3.17829	3.08544

70% LIMITS

n	r	-βw- .725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
1	1	1.29180	1.28082	1.27064	1.26116	1.25233	1.24407	1.23633	1.22906	1.22223	1.21579	1.20972
2	1	0.49657	0.50829	0.51951	0.53025	0.54055	0.55041	0.55989	0.56898	0.57772	0.58612	0.59420
2	2	2.27015	2.20895	2.15319	2.10220	2.05540	2.01230	1.97250	1.93563	1.90138	1.86950	1.83975
3	1	0.28386	0.29602	0.30788	0.31943	0.33067	0.34160	0.35225	0.36261	0.37269	0.38250	0.39204
3	2	1.01748	1.01689	1.01634	1.01583	1.01535	1.01489	1.01446	1.01406	1.01368	1.01331	1.01297
3	3	2.94537	2.84120	2.74709	2.66170	2.58389	2.51275	2.44746	2.38736	2.33187	2.28049	2.23279
4	1	0.19088	0.20172	0.21241	0.22294	0.23332	0.24352	0.25355	0.26340	0.27307	0.28256	0.29187
4	2	0.62362	0.63352	0.64292	0.65185	0.66036	0.66847	0.67621	0.68360	0.69066	0.69742	0.70389
4	3	1.43686	1.41960	1.40365	1.38885	1.37510	1.36227	1.35029	1.33908	1.32855	1.31866	1.30934
4	4	3.46402	3.32349	3.19719	3.08315	2.97973	2.88556	2.79950	2.72058	2.64798	2.58098	2.51899
5	1	0.14031	0.14981	0.15927	0.16868	0.17802	0.18729	0.19648	0.20556	0.21454	0.22341	0.23216
5	2	0.43643	0.44866	0.46041	0.47170	0.48257	0.49302	0.50308	0.51278	0.52212	0.53112	0.53981
5	3	0.92099	0.92352	0.92589	0.92813	0.93023	0.93221	0.93408	0.93585	0.93753	0.93912	0.94063
5	4	1.78671	1.75248	1.72104	1.69209	1.66534	1.64054	1.61750	1.59604	1.57600	1.55724	1.53966
5	5	3.88684	3.71487	3.56089	3.42233	3.29709	3.18340	3.07980	2.98506	2.89812	2.81809	2.74421
6	1	0.10911	0.11748	0.12588	0.13430	0.14273	0.15114	0.15952	0.16787	0.17616	0.18440	0.19257
6	2	0.32920	0.34162	0.35366	0.36534	0.37666	0.38763	0.39827	0.40859	0.41859	0.42830	0.43771
6	3	0.66286	0.67201	0.68068	0.68891	0.69673	0.70418	0.71127	0.71803	0.72449	0.73066	0.73656
6	4	1.18232	1.17574	1.16961	1.16390	1.15856	1.15356	1.14886	1.14444	1.14027	1.13634	1.13262
6	5	2.08718	2.03661	1.99041	1.94805	1.90908	1.87312	1.83983	1.80894	1.78019	1.75337	1.72831
6	6	4.24471	4.04501	3.86671	3.70670	3.56242	3.43177	3.31297	3.20455	3.10525	3.01403	2.92996
7	1	0.08822	0.09565	0.10317	0.11076	0.11840	0.12607	0.13375	0.14144	0.14912	0.15678	0.16441
7	2	0.26068	0.27263	0.28430	0.29569	0.30682	0.31767	0.32825	0.33856	0.34862	0.35842	0.36798
7	3	0.50973	0.52131	0.53238	0.54298	0.55312	0.56284	0.57215	0.58110	0.58969	0.59794	0.60588
7	4	0.86908	0.87316	0.87699	0.88059	0.88399	0.88720	0.89024	0.89312	0.89585	0.89845	0.90092
7	5	1.41493	1.39866	1.38360	1.36963	1.35664	1.34452	1.33320	1.32259	1.31263	1.30327	1.29445
7	6	2.35087	2.28483	2.22474	2.16983	2.11949	2.07318	2.03044	1.99088	1.95417	1.92002	1.88817
7	7	4.55553	4.33100	4.13098	3.95186	3.79067	3.64498	3.51275	3.39227	3.28210	3.18104	3.08804
8	1	0.07338	0.08005	0.08684	0.09374	0.10071	0.10774	0.11482	0.12194	0.12907	0.13622	0.14336
8	2	0.21359	0.22487	0.23596	0.24685	0.25754	0.26802	0.27830	0.28836	0.29822	0.30787	0.31731
8	3	0.40936	0.42174	0.43365	0.44512	0.45617	0.46682	0.47710	0.48700	0.49657	0.50580	0.51472
8	4	0.67849	0.68732	0.69568	0.70362	0.71115	0.71832	0.72514	0.73164	0.73785	0.74378	0.74944
8	5	1.05749	1.05552	1.05368	1.05196	1.05035	1.04883	1.04740	1.04606	1.04478	1.04358	1.04244
8	6	1.62449	1.59842	1.57442	1.55225	1.53170	1.51261	1.49483	1.47823	1.46270	1.44814	1.43445
8	7	2.58610	2.50548	2.43234	2.36570	2.30477	2.24886	2.19739	2.14986	2.10584	2.06497	2.02693
8	8	4.83062	4.58356	4.36389	4.16752	3.99111	3.83190	3.68761	3.55633	3.43646	3.32661	3.22565
9	1	0.06237	0.06842	0.07460	0.08090	0.08731	0.09380	0.10036	0.10698	0.11364	0.12034	0.12705
9	2	0.17950	0.19008	0.20053	0.21086	0.22104	0.23108	0.24096	0.25067	0.26022	0.26961	0.27882
9	3	0.33905	0.35149	0.36355	0.37523	0.38654	0.39750	0.40812	0.41841	0.42838	0.43804	0.44741
9	4	0.55113	0.56219	0.57273	0.58279	0.59240	0.60160	0.61040	0.61882	0.62690	0.63465	0.64210
9	5	0.83575	0.84076	0.84548	0.84992	0.85412	0.85809	0.86185	0.86542	0.86881	0.87203	0.87510
9	6	1.23062	1.22214	1.21425	1.20691	1.20005	1.19363	1.18761	1.18195	1.17662	1.17160	1.16685
9	7	1.81521	1.77949	1.74672	1.71654	1.68866	1.66284	1.63885	1.61652	1.59567	1.57617	1.55789
9	8	2.79864	2.70426	2.61886	2.54124	2.47043	2.40558	2.34600	2.29108	2.24032	2.19327	2.14954
9	9	5.07761	4.80992	4.57228	4.36018	4.16990	3.99840	3.84317	3.70210	3.57342	3.45565	3.34751
10	1	0.05394	0.05945	0.06512	0.07092	0.07684	0.08287	0.08898	0.09516	0.10141	0.10770	0.11404
10	2	0.15384	0.16374	0.17358	0.18335	0.19302	0.20259	0.21204	0.22138	0.23059	0.23966	0.24860
10	3	0.28736	0.29956	0.31144	0.32300	0.33426	0.34520	0.35586	0.36622	0.37630	0.38610	0.39564
10	4	0.46054	0.47260	0.48417	0.49527	0.50592	0.51617	0.52601	0.53548	0.54460	0.55338	0.56184
10	5	0.68531	0.69399	0.70222	0.71002	0.71743	0.72447	0.73117	0.73756	0.74365	0.74947	0.75503
10	6	0.98252	0.98310	0.98364	0.98415	0.98463	0.98507	0.98550	0.98590	0.98628	0.98664	0.98698
10	7	1.39069	1.37548	1.36141	1.34835	1.33619	1.32485	1.31424	1.30430	1.29497	1.28619	1.27792
10	8	1.99030	1.94516	1.90385	1.86593	1.83099	1.79871	1.76879	1.74099	1.71509	1.69092	1.66830
10	9	2.99264	2.88527	2.78831	2.70038	2.62030	2.54710	2.47996	2.41818	2.36115	2.30837	2.25938
10	10	5.30190	5.01515	4.76095	4.53436	4.33133	4.14856	3.98330	3.83327	3.69656	3.57154	3.45684

80% LIMITS

-βw-

n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
1	1	1.92783	1.88610	1.84789	1.81277	1.78039	1.75044	1.72266	1.69683	1.67275	1.65026	1.62920
2	1	0.74106	0.74850	0.75553	0.76218	0.76847	0.77445	0.78013	0.78553	0.79067	0.79557	0.80025
2	2	3.05725	2.94546	2.84459	2.75316	2.66995	2.59393	2.52424	2.46015	2.40101	2.34630	2.29555
3	1	0.42361	0.43592	0.44775	0.45914	0.47009	0.48065	0.49081	0.50061	0.51006	0.51918	0.52798
3	2	1.35708	1.34334	1.33061	1.31878	1.30777	1.29749	1.28787	1.27885	1.27038	1.26241	1.25489
3	3	3.80630	3.64044	3.49182	3.35800	3.23698	3.12705	3.02683	2.93513	2.85095	2.77342	2.70182
4	1	0.28487	0.29704	0.30890	0.32046	0.33170	0.34264	0.35329	0.36365	0.37373	0.38353	0.39308
4	2	0.82963	0.83482	0.83969	0.84429	0.84863	0.85274	0.85663	0.86032	0.86382	0.86716	0.87033
4	3	1.82979	1.79330	1.75983	1.72902	1.70057	1.67422	1.64975	1.62697	1.60570	1.58582	1.56718
4	4	4.37195	4.16216	3.97503	3.80725	3.65609	3.51931	3.39504	3.28170	3.17798	3.08273	2.99502
5	1	0.20940	0.22060	0.23162	0.24245	0.25309	0.26353	0.27376	0.28379	0.29362	0.30325	0.31267
5	2	0.57999	0.59062	0.60074	0.61038	0.61958	0.62837	0.63676	0.64480	0.65249	0.65986	0.66693
5	3	1.16779	1.16177	1.15616	1.15093	1.14604	1.14146	1.13715	1.13310	1.12928	1.12567	1.12226
5	4	2.21547	2.15750	2.10464	2.05626	2.01183	1.97089	1.93305	1.89798	1.86539	1.83504	1.80669
5	5	4.82856	4.58167	4.36214	4.16591	3.98961	3.83050	3.68631	3.55511	3.43530	3.32553	3.22463
6	1	0.16284	0.17299	0.18307	0.19304	0.20291	0.21265	0.22227	0.23175	0.24109	0.25029	0.25934
6	2	0.43726	0.44949	0.46123	0.47252	0.48338	0.49383	0.50388	0.51357	0.52290	0.53190	0.54058
6	3	0.83891	0.84384	0.84847	0.85284	0.85696	0.86086	0.86455	0.86806	0.87138	0.87455	0.87756
6	4	1.45794	1.43973	1.42290	1.40731	1.39281	1.37930	1.36669	1.35488	1.34380	1.33339	1.32359
6	5	2.54236	2.46450	2.39383	2.32941	2.27048	2.21637	2.16654	2.12051	2.07787	2.03826	2.00138
6	6	5.21251	4.93339	4.68582	4.46502	4.26709	4.08883	3.92758	3.78113	3.64762	3.52549	3.41341
7	1	0.13165	0.14085	0.15005	0.15921	0.16833	0.17738	0.18637	0.19527	0.20409	0.21280	0.22141
7	2	0.34614	0.35860	0.37066	0.38234	0.39364	0.40459	0.41518	0.42545	0.43539	0.44502	0.45435
7	3	0.64448	0.65399	0.66301	0.67158	0.67973	0.68749	0.69489	0.70195	0.70870	0.71515	0.72132
7	4	1.06899	1.06661	1.06440	1.06232	1.06038	1.05855	1.05683	1.05521	1.05368	1.05223	1.05086
7	5	1.71246	1.68202	1.65405	1.62824	1.60436	1.58221	1.56160	1.54239	1.52443	1.50761	1.49182
7	6	2.82669	2.73046	2.64341	2.56432	2.49217	2.42613	2.36547	2.30957	2.25790	2.21002	2.16554
7	7	5.54441	5.23673	4.96438	4.72193	4.50496	4.30988	4.13369	3.97390	3.82844	3.69555	3.57374
8	1	0.10950	0.11788	0.12630	0.13473	0.14317	0.15160	0.15999	0.16835	0.17665	0.18490	0.19308
8	2	0.28356	0.29573	0.30758	0.31913	0.33036	0.34130	0.35195	0.36231	0.37239	0.38219	0.39174
8	3	0.51727	0.52877	0.53975	0.55025	0.56030	0.56993	0.57916	0.58801	0.59651	0.60468	0.61253
8	4	0.83341	0.83849	0.84327	0.84777	0.85203	0.85605	0.85986	0.86347	0.86691	0.87017	0.87328
8	5	1.27600	1.26567	1.25609	1.24717	1.23885	1.23107	1.22378	1.21694	1.21050	1.20443	1.19870
8	6	1.93947	1.89712	1.85833	1.82269	1.78983	1.75945	1.73128	1.70508	1.68067	1.65786	1.63651
8	7	3.07871	2.96544	2.86326	2.77067	2.68641	2.60946	2.53892	2.47405	2.41421	2.35886	2.30752
8	8	5.83710	5.50373	5.20913	4.94728	4.71330	4.50320	4.31369	4.14204	3.98596	3.84352	3.71310
9	1	0.09308	0.10075	0.10849	0.11629	0.12412	0.13198	0.13984	0.14770	0.15553	0.16334	0.17111
9	2	0.23828	0.24994	0.26138	0.27257	0.28352	0.29423	0.30469	0.31492	0.32491	0.33467	0.34419
9	3	0.42826	0.44054	0.45234	0.46370	0.47462	0.48514	0.49527	0.50503	0.51444	0.52352	0.53228
9	4	0.67641	0.68528	0.69369	0.70166	0.70924	0.71644	0.72330	0.72984	0.73607	0.74203	0.74773
9	5	1.00675	1.00652	1.00631	1.00611	1.00593	1.00575	1.00559	1.00543	1.00528	1.00514	1.00501
9	6	1.46423	1.44574	1.42865	1.41281	1.39809	1.38438	1.37158	1.35959	1.34835	1.33779	1.32784
9	7	2.14458	2.09073	2.04158	1.99654	1.95515	1.91697	1.88166	1.84891	1.81845	1.79006	1.76353
9	8	3.30529	3.17616	3.05993	2.95484	2.85940	2.77239	2.69279	2.61970	2.55239	2.49023	2.43264
9	9	6.09914	5.74239	5.42758	5.14813	4.89874	4.67507	4.47354	4.29119	4.12554	3.97451	3.83634
10	1	0.08049	0.08755	0.09470	0.10194	0.10924	0.11659	0.12398	0.13138	0.13879	0.14619	0.15358
10	2	0.20419	0.21530	0.22623	0.23699	0.24755	0.25793	0.26811	0.27810	0.28788	0.29747	0.30687
10	3	0.36289	0.37536	0.38741	0.39906	0.41033	0.42122	0.43176	0.44195	0.45181	0.46135	0.47060
10	4	0.56492	0.57578	0.58612	0.59599	0.60541	0.61441	0.62302	0.63127	0.63916	0.64674	0.65400
10	5	0.82466	0.82998	0.83499	0.83970	0.84416	0.84838	0.85237	0.85616	0.85976	0.86319	0.86645
10	6	1.16678	1.16079	1.15522	1.15003	1.14517	1.14061	1.13633	1.13230	1.12851	1.12492	1.12153
10	7	1.63691	1.61024	1.58569	1.56300	1.54199	1.52248	1.50430	1.48734	1.47146	1.45658	1.44260
10	8	2.33182	2.26693	2.20786	2.15389	2.10439	2.05884	2.01679	1.97787	1.94175	1.90813	1.87678
10	9	3.51130	3.36733	3.23600	3.12126	3.01544	2.91912	2.83112	2.75045	2.67626	2.60782	2.54451
10	10	6.33653	5.95831	5.62495	5.32939	5.46591	4.82983	4.61733	4.42523	4.25087	4.09203	3.94683

90% LIMITS

n	r	-βW- .725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
1	1	3.15942	3.04056	2.93343	2.83641	2.74821	2.66769	2.59395	2.52617	2.46368	2.40591	2.35236
2	1	1.21449	1.20665	1.19936	1.19257	1.18622	1.18027	1.17470	1.16946	1.16452	1.15986	1.15546
2	2	4.48771	4.26865	4.07342	3.89850	3.74104	3.59865	3.46936	3.35153	3.24375	3.14484	3.05379
3	1	0.69424	0.70274	0.71078	0.71840	0.72564	0.73252	0.73906	0.74529	0.75124	0.75691	0.76234
3	2	1.96298	1.91934	1.87939	1.84270	1.80888	1.77762	1.74864	1.72171	1.69661	1.67317	1.65124
3	3	5.33508	5.04548	4.78882	4.56007	4.35514	4.17069	4.00395	3.85259	3.71468	3.58859	3.47292
4	1	0.46685	0.47886	0.49037	0.50141	0.51201	0.52219	0.53198	0.54139	0.55044	0.55916	0.56755
4	2	1.19519	1.18810	1.18152	1.17537	1.16963	1.16425	1.15921	1.15446	1.14999	1.14576	1.14177
4	3	2.50860	2.43286	2.36408	2.30136	2.24397	2.19125	2.14268	2.09780	2.05621	2.01758	1.98159
4	4	5.96409	5.61943	5.31507	5.04472	4.80329	4.58663	4.39131	4.21448	4.05377	3.90717	3.77300
5	1	0.34317	0.35563	0.36769	0.37936	0.39067	0.40162	0.41223	0.42250	0.43246	0.44210	0.45145
5	2	0.83415	0.83921	0.84397	0.84846	0.85269	0.85670	0.86049	0.86409	0.86751	0.87076	0.87386
5	3	1.59033	1.56593	1.54343	1.52264	1.50337	1.48545	1.46875	1.45315	1.43854	1.42484	1.41196
5	4	2.94307	2.83906	2.74508	2.65981	2.58212	2.51107	2.44588	2.38586	2.33044	2.27913	2.23149
5	5	6.46675	6.07663	5.73302	5.42855	5.15728	4.91436	4.69582	4.49834	4.31919	4.15605	4.00698
6	1	0.26687	0.27888	0.29061	0.30205	0.31321	0.32409	0.33469	0.34502	0.35509	0.36490	0.37443
6	2	0.62835	0.63816	0.64747	0.65633	0.66475	0.67279	0.68045	0.68776	0.69476	0.70145	0.70785
6	3	1.13911	1.13417	1.12958	1.12528	1.12126	1.11750	1.11396	1.11062	1.10748	1.10450	1.10169
6	4	1.92034	1.87902	1.84118	1.80639	1.77431	1.74464	1.71711	1.69152	1.66766	1.64536	1.62449
6	5	3.30594	3.17677	3.06050	2.95536	2.85990	2.77286	2.69323	2.62012	2.55279	2.49060	2.43300
6	6	6.88654	6.45755	6.08046	5.74697	5.45036	5.18520	4.94702	4.73213	4.53745	4.36041	4.19884
7	1	0.21575	0.22707	0.23819	0.24911	0.25983	0.27034	0.28063	0.29071	0.30058	0.31024	0.31970
7	2	0.49717	0.50889	0.52010	0.53083	0.54112	0.55098	0.56044	0.56953	0.57826	0.58666	0.59473
7	3	0.87375	0.87769	0.88139	0.88487	0.88816	0.89126	0.89420	0.89698	0.89962	0.90213	0.90451
7	4	1.40257	1.38685	1.37229	1.35879	1.34622	1.33450	1.32354	1.31328	1.30364	1.29458	1.28603
7	5	2.20502	2.14766	2.09536	2.04747	2.00349	1.96296	1.92550	1.89077	1.85850	1.82843	1.80036
7	6	3.61843	3.46659	3.33032	3.20744	3.09614	2.99492	2.90251	2.81786	2.74005	2.66833	2.60201
7	7	7.24761	6.78456	6.37821	6.01938	5.70071	5.41621	5.16099	4.93100	4.72288	4.53382	4.36146
8	1	0.17946	0.19004	0.20049	0.21082	0.22100	0.23103	0.24091	0.25063	0.26018	0.26956	0.27878
8	2	0.40716	0.41954	0.43146	0.44294	0.45401	0.46468	0.47496	0.48489	0.49447	0.50372	0.51265
8	3	0.70065	0.70900	0.71691	0.72441	0.73152	0.73828	0.74471	0.75083	0.75667	0.76224	0.76756
8	4	1.09117	1.08801	1.08505	1.08229	1.07970	1.07726	1.07497	1.07282	1.07078	1.06886	1.06703
8	5	1.63546	1.60886	1.58437	1.56175	1.54079	1.52132	1.50319	1.48627	1.47044	1.45559	1.44165
8	6	2.45603	2.38355	2.31770	2.25761	2.20258	2.15202	2.10541	2.06231	2.02236	1.98522	1.95062
8	7	3.89337	3.72090	3.56648	3.42754	3.30195	3.18796	3.08409	2.98910	2.90194	2.82171	2.74764
8	8	7.56480	7.07138	6.63897	6.25763	5.91938	5.61775	5.34744	5.10411	4.88413	4.68448	4.50261
9	1	0.15255	0.16242	0.17222	0.18196	0.19160	0.20114	0.21057	0.21988	0.22907	0.23813	0.24705
9	2	0.34206	0.35452	0.36658	0.37826	0.38956	0.40052	0.41113	0.42140	0.43136	0.44101	0.45037
9	3	0.57973	0.59036	0.60049	0.61013	0.61934	0.62813	0.63653	0.64456	0.65226	0.65964	0.66671
9	4	0.88447	0.88810	0.89151	0.89471	0.89773	0.90059	0.90328	0.90584	0.90826	0.91057	0.91276
9	5	1.28703	1.27625	1.26625	1.25694	1.24826	1.24015	1.23254	1.22541	1.21869	1.21237	1.20639
9	6	1.84461	1.80734	1.77316	1.74171	1.71267	1.68578	1.66081	1.63757	1.61589	1.59561	1.57661
9	7	2.68088	2.59419	2.51563	2.44414	2.37884	2.31897	2.26391	2.21310	2.16609	2.12248	2.08191
9	8	4.13918	3.94776	3.77671	3.62308	3.48447	3.35886	3.24457	3.14021	3.04458	2.95667	2.87562
9	9	7.84788	7.32702	6.87110	6.46948	6.11361	5.79657	5.51272	5.25742	5.02681	4.81767	4.62731
10	1	0.13192	0.14113	0.15033	0.15950	0.16863	0.17769	0.18668	0.19559	0.20441	0.21313	0.22175
10	2	0.29309	0.30533	0.31724	0.32883	0.34010	0.35106	0.36172	0.37209	0.38216	0.39196	0.40149
10	3	0.49104	0.50282	0.51410	0.52490	0.53525	0.54516	0.55471	0.56387	0.57267	0.58113	0.58927
10	4	0.73807	0.74558	0.75267	0.75939	0.76575	0.77178	0.77751	0.78297	0.78816	0.79312	0.79784
10	5	1.05258	1.05078	1.04910	1.04753	1.04605	1.04468	1.04337	1.04214	1.04098	1.03988	1.03884
10	6	1.46554	1.44699	1.42984	1.41396	1.39919	1.38544	1.37259	1.36057	1.34929	1.33870	1.32872
10	7	2.03470	1.98709	1.94356	1.90361	1.86684	1.83288	1.80142	1.77221	1.74501	1.71963	1.69589
10	8	2.88479	2.78469	2.69420	2.61203	2.53713	2.46865	2.40568	2.34773	2.29419	2.24460	2.19835
10	9	4.36167	4.15270	3.96629	3.79914	3.64854	3.51226	3.38842	3.27549	3.17212	3.07720	2.98978
10	10	8.10369	7.55776	7.08040	6.66030	6.28839	5.95735	5.66120	5.39504	5.15479	4.93706	4.73900

95% LIMITS

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n	r	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.975
1	1	4.54198	4.31854	4.11948	3.94120	3.78076	3.63573	3.50409	3.38414	3.27445	3.17382	3.08121
2	1	1.74595	1.71382	1.68429	1.65707	1.63190	1.60857	1.58687	1.56664	1.54775	1.53006	1.51346
2	2	6.02353	5.67357	5.36461	5.09026	4.84534	4.62559	4.42754	4.24828	4.08540	3.93686	3.80093
3	1	0.99804	0.99810	0.99816	0.99822	0.99828	0.99833	0.99837	0.99842	0.99846	0.99850	0.99854
3	2	2.60124	2.51965	2.44565	2.37825	2.31662	2.26008	2.20804	2.15999	2.11549	2.07419	2.03575
3	3	6.94864	6.51383	6.13174	5.79391	5.49353	5.22506	4.98396	4.76647	4.56949	4.39038	4.22696
4	1	0.67115	0.68013	0.68864	0.69671	0.70438	0.71168	0.71863	0.72526	0.73158	0.73762	0.74340
4	2	1.57789	1.55408	1.53214	1.51184	1.49303	1.47553	1.45922	1.44398	1.42971	1.41633	1.40374
4	3	3.20527	3.08320	2.97323	2.87369	2.78322	2.70068	2.62509	2.55565	2.49166	2.43251	2.37769
4	4	7.62865	7.12907	6.69138	6.30548	5.96327	5.65817	5.38482	5.13879	4.91641	4.71462	4.53085
5	1	0.49334	0.50510	0.51635	0.52713	0.53746	0.54736	0.55687	0.56600	0.57477	0.58321	0.59133
5	2	1.09954	1.09606	1.09283	1.08980	1.08696	1.08430	1.08180	1.07944	1.07721	1.07510	1.07311
5	3	2.01965	1.97288	1.93010	1.89085	1.85470	1.82130	1.79037	1.76164	1.73488	1.70991	1.68655
5	4	3.67887	3.52255	3.38234	3.25596	3.14155	3.03754	2.94263	2.85571	2.77586	2.70228	2.63427
5	5	8.16884	7.61650	7.13365	6.70881	6.33280	5.99818	5.69889	5.42995	5.18724	4.96732	4.76731
6	1	0.38365	0.39610	0.40811	0.41970	0.43089	0.44169	0.45213	0.46221	0.47195	0.48137	0.49048
6	2	0.82760	0.83283	0.83776	0.84241	0.84680	0.85095	0.85488	0.85862	0.86216	0.86553	0.86874
6	3	1.44272	1.42520	1.40900	1.39399	1.38003	1.36701	1.35486	1.34348	1.33280	1.32276	1.31331
6	4	2.38209	2.31415	2.25236	2.19593	2.14420	2.09663	2.05275	2.01215	1.97448	1.93945	1.90678
6	5	4.07070	3.88460	3.71822	3.56872	3.43376	3.31140	3.20003	3.09829	3.00503	2.91927	2.84017
6	6	8.61814	8.02108	7.50005	7.04236	6.63789	6.27845	5.95740	5.66927	5.40956	5.17450	4.96094
7	1	0.31017	0.32251	0.33450	0.34614	0.35745	0.36843	0.37910	0.38945	0.39950	0.40927	0.41875
7	2	0.65452	0.66383	0.67267	0.68105	0.68903	0.69662	0.70385	0.71075	0.71734	0.72364	0.72966
7	3	1.10505	1.10138	1.09795	1.09475	1.09175	1.08893	1.08629	1.08379	1.08144	1.07921	1.07710
7	4	1.73368	1.70217	1.67321	1.64651	1.62182	1.59892	1.57762	1.55777	1.53922	1.52185	1.50555
7	5	2.69127	2.60390	2.52474	2.45272	2.38693	2.32663	2.27117	2.22000	2.17267	2.12875	2.08791
7	6	4.40593	4.19343	4.00393	3.83406	3.68106	3.54263	3.41689	3.30224	3.19732	3.10101	3.01231
7	7	9.00339	8.36744	7.81325	7.32708	6.89796	6.51707	6.17723	5.87256	5.59820	5.35011	5.12492
8	1	0.25799	0.26991	0.28156	0.29293	0.30404	0.31487	0.32544	0.33575	0.34580	0.35560	0.36515
8	2	0.53587	0.54713	0.55788	0.56814	0.57796	0.58736	0.59636	0.60498	0.61325	0.62120	0.62883
8	3	0.88537	0.88897	0.89235	0.89553	0.89853	0.90136	0.90404	0.90658	0.90899	0.91127	0.91344
8	4	1.34616	1.33289	1.32059	1.30916	1.29852	1.28858	1.27928	1.27056	1.26236	1.25465	1.24737
8	5	1.98777	1.94277	1.90159	1.86378	1.82895	1.79676	1.76693	1.73921	1.71339	1.68928	1.66672
8	6	2.96174	2.85646	2.76137	2.67510	2.59651	2.52465	2.45873	2.39805	2.34202	2.29015	2.24201
8	7	4.69947	4.46321	4.25296	4.06485	3.89573	3.74299	3.60446	3.47835	3.36311	3.25746	3.16030
8	8	9.34101	8.67057	8.08701	7.57564	7.12476	6.72495	6.36855	6.04932	5.76208	5.50255	5.26714
9	1	0.21931	0.23068	0.24186	0.25283	0.26359	0.27413	0.28445	0.29456	0.30446	0.31413	0.32360
9	2	0.45011	0.46225	0.47390	0.48509	0.49584	0.50618	0.51612	0.52569	0.53490	0.54379	0.55235
9	3	0.73218	0.73982	0.74705	0.75389	0.76037	0.76652	0.77237	0.77793	0.78323	0.78828	0.79310
9	4	1.08987	1.08675	1.08384	1.08111	1.07856	1.07616	1.07391	1.07179	1.06978	1.06788	1.06608
9	5	1.56063	1.53765	1.51646	1.49685	1.47567	1.46175	1.44598	1.43125	1.41744	1.40449	1.39231
9	6	2.21403	2.15614	2.10336	2.05505	2.01068	1.96979	1.93201	1.89698	1.86444	1.83412	1.80582
9	7	3.20264	3.08076	2.97094	2.87155	2.78121	2.69878	2.62331	2.55396	2.49005	2.43098	2.37624
9	8	4.96093	4.70303	4.47392	4.26927	4.08556	3.91989	3.76984	3.63341	3.50890	3.39488	3.29013
9	9	9.64175	8.94027	8.33033	7.79635	7.32596	6.90919	6.53798	6.20572	5.90698	5.63724	5.39273
10	1	0.18964	0.20045	0.21112	0.22163	0.23198	0.24217	0.25218	0.26202	0.27168	0.28116	0.29046
10	2	0.38562	0.39806	0.41007	0.42165	0.43283	0.44362	0.45405	0.46412	0.47384	0.48325	0.49235
10	3	0.61993	0.62989	0.63935	0.64835	0.65692	0.66509	0.67288	0.68033	0.68745	0.69426	0.70079
10	4	0.90876	0.91167	0.91439	0.91695	0.91936	0.92164	0.92379	0.92583	0.92776	0.92959	0.93133
10	5	1.27449	1.26422	1.25470	1.24583	1.23756	1.22983	1.22258	1.21577	1.20937	1.20334	1.19764
10	6	1.75436	1.72179	1.69138	1.66430	1.63881	1.61517	1.59320	1.57272	1.55359	1.53568	1.51888
10	7	2.41835	2.34820	2.28442	2.22620	2.17286	2.12383	2.07861	2.03679	1.99800	1.96194	1.92833
10	8	3.42010	3.28275	3.15926	3.04771	2.94650	2.85433	2.77006	2.69276	2.62163	2.55597	2.49520
10	9	5.19687	4.91908	4.67267	4.45288	4.25584	4.07836	3.91781	3.77199	3.63904	3.51742	3.40579
10	10	9.91307	9.18335	8.54942	7.99491	7.50681	7.07468	6.69005	6.34601	6.03687	5.75790	5.50516

5% LIMITS

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n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	0.05129	0.06719	0.08415	0.10180	0.11984	0.13805	0.15624	0.17427	0.19203	0.20945	0.22648
2	1	0.02565	0.03578	0.04723	0.05973	0.07305	0.08697	0.10131	0.11591	0.13066	0.14543	0.16015
2	2	0.25310	0.28677	0.31623	0.34753	0.37478	0.40012	0.42370	0.44565	0.46611	0.48522	0.50309
3	1	0.01710	0.02475	0.03369	0.04372	0.05468	0.06637	0.07863	0.09132	0.10430	0.11748	0.13076
3	2	0.14543	0.17329	0.20055	0.22693	0.25229	0.27655	0.29968	0.32170	0.34262	0.36249	0.38135
3	3	0.45950	0.49316	0.52309	0.54982	0.57382	0.59547	0.61508	0.63292	0.64921	0.66414	0.67787
4	1	0.01232	0.01905	0.02651	0.03504	0.04452	0.05479	0.06569	0.07710	0.08890	0.10097	0.11324
4	2	0.10271	0.12632	0.15009	0.17366	0.19679	0.21932	0.24113	0.26218	0.28242	0.30185	0.32048
4	3	0.28562	0.32029	0.35217	0.38160	0.40879	0.43391	0.45715	0.47869	0.49869	0.51729	0.53462
4	4	0.64031	0.66679	0.68970	0.70969	0.72729	0.74289	0.75682	0.76935	0.78062	0.79086	0.80019
5	1	0.01026	0.01556	0.02201	0.02952	0.03796	0.04721	0.05714	0.06762	0.07853	0.08979	0.10129
5	2	0.07952	0.10010	0.12126	0.14263	0.16392	0.18492	0.20549	0.22554	0.24499	0.26382	0.28199
5	3	0.20930	0.24180	0.27217	0.30083	0.32778	0.35308	0.37682	0.39908	0.41998	0.43960	0.45804
5	4	0.41945	0.45372	0.48480	0.51257	0.53763	0.56034	0.58100	0.59986	0.61713	0.63301	0.64765
5	5	0.79691	0.81353	0.82764	0.83977	0.85031	0.85955	0.86772	0.87499	0.88151	0.88736	0.89270
6	1	0.00855	0.01318	0.01891	0.02565	0.03333	0.04181	0.05099	0.06074	0.07097	0.08157	0.09246
6	2	0.06491	0.08323	0.10239	0.12201	0.14179	0.16152	0.18101	0.20016	0.21887	0.23709	0.25478
6	3	0.16624	0.19572	0.22419	0.25152	0.27758	0.30234	0.32581	0.34803	0.36905	0.38893	0.40773
6	4	0.31655	0.35144	0.38344	0.41278	0.43971	0.46447	0.48727	0.50832	0.52779	0.54585	0.56262
6	5	0.54162	0.57267	0.59990	0.62395	0.64533	0.66445	0.68165	0.69719	0.71130	0.72417	0.73595
6	6	0.93385	0.93968	0.94456	0.94872	0.95229	0.95540	0.95813	0.96054	0.96269	0.96462	0.96636
7	1	0.00733	0.01146	0.01663	0.02279	0.02985	0.03773	0.04630	0.05547	0.06514	0.07521	0.08560
7	2	0.05485	0.07142	0.08899	0.10719	0.12573	0.14437	0.16293	0.18128	0.19932	0.21698	0.23421
7	3	0.13783	0.16504	0.19178	0.21775	0.24280	0.26683	0.28980	0.31170	0.33256	0.35239	0.37126
7	4	0.25531	0.28905	0.32054	0.34986	0.37711	0.40245	0.42603	0.44793	0.46837	0.48745	0.50528
7	5	0.41743	0.45193	0.48286	0.51067	0.53578	0.55854	0.57925	0.59815	0.61548	0.63140	0.64609
7	6	0.65253	0.67839	0.70069	0.72013	0.73721	0.75235	0.76585	0.77796	0.78889	0.79880	0.80782
7	7	1.05503	1.04995	1.04570	1.04211	1.03904	1.03639	1.03408	1.03204	1.03024	1.02862	1.02717
8	1	0.00641	0.01015	0.01468	0.02056	0.02714	0.03451	0.04259	0.05128	0.06049	0.07011	0.08007
8	2	0.04750	0.06266	0.07893	0.09596	0.11344	0.13116	0.14891	0.16656	0.18400	0.20115	0.21794
8	3	0.11775	0.14307	0.16823	0.19295	0.21702	0.24028	0.26268	0.28417	0.30475	0.32441	0.34320
8	4	0.21431	0.24652	0.27704	0.30579	0.33279	0.35812	0.38186	0.40411	0.42497	0.44455	0.46294
8	5	0.34142	0.37646	0.40839	0.43752	0.46413	0.48850	0.51087	0.53145	0.55045	0.56802	0.58431
8	6	0.51134	0.54349	0.57132	0.59694	0.61935	0.63945	0.65758	0.67399	0.68893	0.70257	0.71508
8	7	0.75356	0.77321	0.78996	0.80442	0.81702	0.82811	0.83793	0.84669	0.85455	0.86165	0.86809
8	8	1.16365	1.14773	1.13462	1.12365	1.11434	1.10632	1.09936	1.09325	1.08785	1.08304	1.07873
9	1	0.00570	0.00912	0.01348	0.01878	0.02495	0.03191	0.03957	0.04785	0.05665	0.06589	0.07549
9	2	0.04139	0.05589	0.07108	0.08711	0.10370	0.12061	0.13766	0.15469	0.17159	0.18827	0.20467
9	3	0.10266	0.12649	0.15027	0.17386	0.19700	0.21953	0.24135	0.26240	0.28265	0.30209	0.32072
9	4	0.18433	0.21549	0.24489	0.27288	0.29940	0.32447	0.34812	0.37041	0.39142	0.41123	0.42991
9	5	0.28951	0.32404	0.35594	0.38538	0.41254	0.43763	0.46083	0.48231	0.50225	0.52079	0.53806
9	6	0.42303	0.45744	0.48825	0.51593	0.54091	0.56353	0.58409	0.60286	0.62005	0.63585	0.65041
9	7	0.59849	0.62708	0.65195	0.67376	0.69303	0.71018	0.72554	0.73936	0.75187	0.76324	0.77362
9	8	0.84598	0.85894	0.86990	0.87927	0.88739	0.89449	0.90074	0.90630	0.91127	0.91573	0.91977
9	9	1.26185	1.23545	1.21338	1.19591	1.18073	1.16772	1.15646	1.14661	1.13793	1.13022	1.12332
10	1	0.00512	0.00828	0.01234	0.01731	0.02312	0.02972	0.03703	0.04495	0.05341	0.06231	0.07159
10	2	0.03746	0.05050	0.06477	0.07994	0.09575	0.11196	0.12838	0.14486	0.16127	0.17753	0.19356
10	3	0.09131	0.11350	0.13607	0.15863	0.18093	0.20277	0.22404	0.24465	0.26455	0.28373	0.30217
10	4	0.16255	0.19174	0.22004	0.24721	0.27316	0.29795	0.32127	0.34346	0.36447	0.38436	0.40318
10	5	0.25160	0.28522	0.31665	0.34594	0.37319	0.39854	0.42212	0.44409	0.46455	0.48371	0.50159
10	6	0.36174	0.39677	0.42855	0.45741	0.48369	0.50769	0.52966	0.54984	0.56841	0.58557	0.60145
10	7	0.49965	0.53237	0.56109	0.58659	0.60937	0.62953	0.64829	0.66504	0.68023	0.69421	0.70700
10	8	0.67944	0.70374	0.72465	0.74282	0.75877	0.77236	0.78541	0.79665	0.80677	0.81594	0.82428
10	9	0.93099	0.93706	0.94215	0.94646	0.95021	0.95345	0.95629	0.95881	0.96105	0.96306	0.96488
10	10	1.35145	1.31445	1.28523	1.26071	1.24002	1.22236	1.20712	1.19382	1.18213	1.17177	1.16252

10% LIMITS

n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
		-βw-										
1	1	0.10536	0.12928	0.15331	0.17710	0.20041	0.22308	0.24500	0.26614	0.28645	0.30593	0.32459
2	1	0.05268	0.06834	0.08604	0.10391	0.12215	0.14053	0.15887	0.17702	0.19490	0.21241	0.22952
2	2	0.38013	0.41507	0.44663	0.47519	0.50113	0.52475	0.54633	0.56611	0.58429	0.60105	0.61655
3	1	0.03512	0.04762	0.06137	0.07607	0.09143	0.10724	0.12330	0.13946	0.15559	0.17159	0.18740
3	2	0.21791	0.25028	0.28091	0.30973	0.33677	0.36212	0.38585	0.40808	0.42892	0.44846	0.46681
3	3	0.62392	0.65126	0.67495	0.69567	0.71394	0.73016	0.74465	0.75768	0.76945	0.78014	0.78988
4	1	0.02634	0.03666	0.04829	0.06097	0.07445	0.08853	0.10301	0.11775	0.13261	0.14748	0.16230
4	2	0.15380	0.18234	0.21012	0.23691	0.26258	0.28706	0.31035	0.33246	0.35344	0.37333	0.39218
4	3	0.38634	0.42123	0.45270	0.48115	0.50697	0.53045	0.55189	0.57153	0.58957	0.60620	0.62156
4	4	0.82632	0.84077	0.85301	0.86351	0.87261	0.88057	0.88760	0.89385	0.89944	0.90447	0.90902
5	1	0.02107	0.02993	0.04009	0.05135	0.06348	0.07629	0.08960	0.10326	0.11715	0.13114	0.14516
5	2	0.11905	0.14446	0.16973	0.19454	0.21868	0.24200	0.26444	0.28596	0.30656	0.32624	0.34503
5	3	0.28321	0.31762	0.34943	0.37892	0.40611	0.43126	0.45453	0.47611	0.49615	0.51479	0.53217
5	4	0.53804	0.56923	0.59660	0.62078	0.64228	0.66152	0.67883	0.69447	0.70869	0.72165	0.73351
5	5	0.99684	0.99713	0.99737	0.99757	0.99774	0.99789	0.99803	0.99814	0.99824	0.99834	0.99842
6	1	0.01756	0.02536	0.03444	0.04463	0.05573	0.06756	0.07995	0.09276	0.10586	0.11914	0.13251
6	2	0.09717	0.12011	0.14331	0.16641	0.18915	0.21135	0.23292	0.25376	0.27385	0.29317	0.31172
6	3	0.22428	0.25693	0.28773	0.31667	0.34378	0.36914	0.39287	0.41506	0.43584	0.45531	0.47358
6	4	0.40526	0.43994	0.47110	0.49918	0.52457	0.54763	0.56863	0.58783	0.60544	0.62164	0.63660
6	5	0.67272	0.69741	0.71867	0.73717	0.75340	0.76776	0.78055	0.79200	0.80233	0.81169	0.82020
6	6	1.14348	1.12963	1.11821	1.10864	1.10050	1.09350	1.08741	1.08206	1.07733	1.07312	1.06934
7	1	0.01505	0.02204	0.03029	0.03964	0.04992	0.06096	0.07261	0.08472	0.09717	0.10986	0.12268
7	2	0.08210	0.10305	0.12454	0.14618	0.16770	0.18890	0.20964	0.22982	0.24938	0.26829	0.28654
7	3	0.18589	0.21662	0.24607	0.27409	0.30064	0.32572	0.34937	0.37167	0.39263	0.41248	0.43115
7	4	0.32656	0.36134	0.39353	0.42279	0.44961	0.47422	0.49686	0.51773	0.53701	0.55487	0.57146
7	5	0.51721	0.54916	0.57729	0.60221	0.62442	0.64434	0.66228	0.67853	0.69331	0.70681	0.71918
7	6	0.79282	0.80973	0.82410	0.83646	0.84720	0.85661	0.86494	0.87235	0.87900	0.88498	0.89041
7	7	1.27184	1.24434	1.22188	1.20319	1.18740	1.17388	1.16217	1.15194	1.14293	1.13492	1.12776
8	1	0.01317	0.01952	0.02710	0.03577	0.04538	0.05577	0.06679	0.07832	0.09022	0.10240	0.11476
8	2	0.07109	0.09041	0.11046	0.13086	0.15132	0.17162	0.19160	0.21116	0.23021	0.24872	0.26664
8	3	0.15863	0.18774	0.21562	0.24284	0.26868	0.29328	0.31665	0.33881	0.35980	0.37969	0.39853
8	4	0.27399	0.30821	0.33998	0.36939	0.39663	0.42185	0.44523	0.46694	0.48712	0.50591	0.52344
8	5	0.42254	0.45697	0.48778	0.51548	0.54046	0.56310	0.58367	0.60246	0.61966	0.63546	0.65003
8	6	0.61950	0.64706	0.67096	0.69188	0.71032	0.72671	0.74135	0.75452	0.76642	0.77722	0.78708
8	7	0.90060	0.90939	0.91662	0.92278	0.92809	0.93272	0.93679	0.94039	0.94361	0.94650	0.94910
8	8	1.38587	1.34536	1.31251	1.28534	1.26250	1.24303	1.22624	1.21162	1.19877	1.18738	1.17723
9	1	0.01171	0.01754	0.02457	0.03267	0.04172	0.05156	0.06205	0.07303	0.08451	0.09625	0.10820
9	2	0.06269	0.08064	0.09947	0.11879	0.13832	0.15782	0.17712	0.19610	0.21463	0.23279	0.25039
9	3	0.13365	0.16597	0.19276	0.21879	0.24387	0.26793	0.29092	0.31283	0.33369	0.35354	0.37240
9	4	0.23622	0.26934	0.30045	0.32957	0.35676	0.38213	0.40581	0.42792	0.44859	0.46792	0.48603
9	5	0.35806	0.39310	0.42491	0.45383	0.48017	0.50424	0.52629	0.54654	0.56519	0.58242	0.59838
9	6	0.51179	0.54393	0.57224	0.59735	0.61974	0.63983	0.65794	0.67434	0.68926	0.70290	0.71540
9	7	0.71318	0.73544	0.75452	0.77104	0.78550	0.79824	0.80956	0.81969	0.82879	0.83702	0.84450
9	8	0.99869	0.99881	0.99891	0.99899	0.99906	0.99913	0.99918	0.99923	0.99927	0.99931	0.99934
9	9	1.48339	1.43554	1.39293	1.35787	1.32852	1.30360	1.28215	1.26357	1.24725	1.23283	1.21999
10	1	0.01054	0.01594	0.02250	0.03013	0.03869	0.04806	0.05810	0.06869	0.07971	0.09106	0.10265
10	2	0.05607	0.07286	0.09063	0.10902	0.12772	0.14650	0.16518	0.18364	0.20177	0.21950	0.23679
10	3	0.12310	0.14892	0.17454	0.19962	0.22397	0.24746	0.27003	0.29165	0.31231	0.33204	0.35086
10	4	0.20772	0.23962	0.26991	0.29852	0.32545	0.35074	0.37447	0.39675	0.41765	0.43729	0.45576
10	5	0.31104	0.34588	0.37738	0.40725	0.43424	0.45907	0.48196	0.50311	0.52268	0.54083	0.55771
10	6	0.43729	0.47144	0.50193	0.52926	0.55387	0.57612	0.59632	0.61474	0.63159	0.64704	0.66128
10	7	0.59470	0.62347	0.64650	0.67047	0.68989	0.70712	0.72266	0.73660	0.74922	0.76069	0.77116
10	8	0.79939	0.81543	0.82973	0.84178	0.85220	0.86133	0.86941	0.87659	0.88303	0.88883	0.89409
10	9	1.08812	1.07980	1.07392	1.06712	1.06218	1.05792	1.05420	1.05093	1.04804	1.04545	1.04313
10	10	1.55147	1.51693	1.46516	1.42274	1.38735	1.35740	1.33172	1.30947	1.29000	1.27283	1.25757

20% LIMITS

n	r	βw 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	0.22314	0.25574	0.28652	0.31544	0.34253	0.36789	0.39162	0.41382	0.43461	0.45410	0.47238
2	1	0.11157	0.13619	0.16080	0.18508	0.20876	0.23176	0.25393	0.27526	0.29571	0.31529	0.33402
2	2	0.59278	0.62164	0.64677	0.66881	0.68831	0.70566	0.72121	0.73521	0.74788	0.75940	0.76992
3	1	0.07438	0.09420	0.11470	0.13549	0.15626	0.17687	0.19709	0.21685	0.23607	0.25470	0.27273
3	2	0.33847	0.37350	0.40545	0.43460	0.46126	0.48568	0.50810	0.52875	0.54730	0.56543	0.58178
3	3	0.87900	0.88937	0.89810	0.90556	0.91200	0.91761	0.92256	0.92694	0.93036	0.93438	0.93755
4	1	0.05579	0.07252	0.09025	0.10859	0.12725	0.14600	0.16466	0.18309	0.20120	0.21892	0.23619
4	2	0.23860	0.27186	0.30303	0.33218	0.35938	0.38476	0.40842	0.43051	0.45115	0.47045	0.48853
4	3	0.54051	0.57160	0.59837	0.62296	0.64438	0.66354	0.68077	0.69634	0.71047	0.72338	0.73519
4	4	1.10485	1.09466	1.08664	1.07972	1.07382	1.06873	1.06430	1.06041	1.05696	1.05388	1.05112
5	1	0.04463	0.05921	0.07493	0.09146	0.10850	0.12582	0.14322	0.16057	0.17774	0.19466	0.21126
5	2	0.18465	0.21531	0.24470	0.27269	0.29921	0.32427	0.34792	0.37021	0.39122	0.41103	0.42971
5	3	0.39541	0.43021	0.46154	0.48982	0.51544	0.53872	0.55996	0.57939	0.59723	0.61365	0.62882
5	4	0.71296	0.73523	0.75432	0.77085	0.78532	0.79807	0.80940	0.81953	0.82865	0.83688	0.84437
5	5	1.29018	1.26064	1.23654	1.21651	1.19960	1.18513	1.17262	1.16169	1.15205	1.14350	1.13586
6	1	0.03719	0.05016	0.06437	0.07949	0.09525	0.11142	0.12780	0.14424	0.16062	0.17685	0.19285
6	2	0.15066	0.17897	0.20656	0.23321	0.25876	0.28317	0.30640	0.32848	0.34944	0.36932	0.38818
6	3	0.31286	0.34772	0.37972	0.40908	0.43605	0.46086	0.48372	0.50483	0.52437	0.54250	0.55934
6	4	0.53547	0.56676	0.59422	0.61849	0.64009	0.65941	0.67680	0.69252	0.70680	0.71983	0.73176
6	5	0.86169	0.87343	0.88334	0.89181	0.89913	0.90553	0.91116	0.91616	0.92063	0.92464	0.92827
6	6	1.44700	1.39920	1.36058	1.32873	1.30203	1.27932	1.25977	1.24278	1.22786	1.21467	1.20291
7	1	0.03188	0.04360	0.05661	0.07061	0.08532	0.10054	0.11606	0.13173	0.14744	0.16307	0.17854
7	2	0.12731	0.15355	0.17949	0.20485	0.22941	0.25307	0.27576	0.29747	0.31820	0.33797	0.35681
7	3	0.25919	0.29304	0.32460	0.35395	0.38121	0.40652	0.43005	0.45193	0.47232	0.49134	0.50911
7	4	0.43093	0.46520	0.49563	0.52333	0.54810	0.57052	0.59088	0.60946	0.62646	0.64207	0.65645
7	5	0.66018	0.68558	0.70749	0.72657	0.74334	0.75819	0.77142	0.78328	0.79399	0.80368	0.81252
7	6	0.99193	0.99260	0.99327	0.99378	0.99423	0.99461	0.99495	0.99524	0.99551	0.99574	0.99596
7	7	1.58275	1.51807	1.46617	1.42364	1.38817	1.35815	1.33241	1.31011	1.29059	1.27338	1.25809
8	1	0.02789	0.03962	0.05065	0.06371	0.07756	0.09197	0.10677	0.12178	0.13689	0.15200	0.16701
8	2	0.11023	0.13470	0.15919	0.18336	0.20698	0.22990	0.25202	0.27331	0.29373	0.31329	0.33201
8	3	0.22139	0.25392	0.28464	0.31353	0.34061	0.36597	0.38969	0.41191	0.43271	0.45222	0.47052
8	4	0.36129	0.39633	0.42811	0.45698	0.48327	0.50727	0.52925	0.54944	0.56802	0.58519	0.60108
8	5	0.53844	0.56962	0.59697	0.62113	0.64263	0.66185	0.67915	0.69478	0.70898	0.72193	0.73379
8	6	0.77196	0.79036	0.80600	0.81949	0.83122	0.84153	0.85065	0.85879	0.86603	0.87266	0.87862
8	7	1.10756	1.09732	1.08886	1.08175	1.07570	1.07048	1.06593	1.06193	1.05840	1.05524	1.05241
8	8	1.70246	1.62207	1.55799	1.50575	1.46236	1.42578	1.39451	1.36750	1.34393	1.32318	1.30478
9	1	0.02479	0.03470	0.04591	0.05819	0.07130	0.08503	0.09919	0.11363	0.12822	0.14286	0.15746
9	2	0.09720	0.12014	0.14335	0.16645	0.18919	0.21140	0.23297	0.25381	0.27390	0.29322	0.31177
9	3	0.19328	0.22443	0.25419	0.28243	0.30913	0.33429	0.35799	0.38029	0.40127	0.42103	0.43964
9	4	0.31136	0.34620	0.37819	0.40757	0.43455	0.45938	0.48226	0.50340	0.52297	0.54112	0.55799
9	5	0.45564	0.48959	0.51961	0.54645	0.57055	0.59230	0.61201	0.62995	0.64633	0.66135	0.67516
9	6	0.63651	0.66319	0.68628	0.70645	0.72420	0.73995	0.75401	0.76664	0.77804	0.78339	0.79782
9	7	0.87304	0.88363	0.89302	0.90082	0.90757	0.91346	0.91864	0.92324	0.92734	0.93103	0.93436
9	8	1.21143	1.19049	1.17332	1.15898	1.14683	1.13641	1.12736	1.11944	1.11244	1.10622	1.10065
9	9	1.80942	1.71446	1.63914	1.57800	1.52741	1.48489	1.44865	1.41740	1.39025	1.36630	1.34515
10	1	0.02231	0.03153	0.04206	0.05366	0.06613	0.07926	0.09237	0.10660	0.12093	0.13516	0.14938
10	2	0.08693	0.10855	0.13061	0.15275	0.17469	0.19624	0.21726	0.23763	0.25743	0.27649	0.29484
10	3	0.17154	0.20136	0.23013	0.25766	0.28387	0.30873	0.33226	0.35451	0.37553	0.39540	0.41418
10	4	0.27370	0.30792	0.33968	0.36909	0.39633	0.42155	0.44494	0.46665	0.48683	0.50563	0.52317
10	5	0.39575	0.43055	0.46167	0.49015	0.51576	0.53904	0.56026	0.57968	0.59751	0.61393	0.62909
10	6	0.54324	0.57422	0.60139	0.62538	0.64671	0.66577	0.68292	0.69841	0.71248	0.72530	0.73705
10	7	0.72633	0.74710	0.76613	0.78199	0.79585	0.80806	0.81839	0.82857	0.83722	0.84514	0.85228
10	8	0.96512	0.96824	0.97085	0.97306	0.97496	0.97661	0.97806	0.97933	0.98047	0.98149	0.98241
10	9	1.30566	1.27440	1.24891	1.22774	1.20968	1.19461	1.18140	1.16987	1.15972	1.15071	1.14266
10	10	1.90609	1.79753	1.71160	1.64246	1.58527	1.53731	1.49655	1.46147	1.43099	1.40425	1.38061

30% LIMITS

n	r	$-\beta_w-$ 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	0.35667	0.39172	0.42354	0.45247	0.47884	0.50294	0.52501	0.54529	0.56398	0.58124	0.59722
2	1	0.17834	0.20860	0.23770	0.26548	0.29186	0.31683	0.34043	0.36271	0.38373	0.40357	0.42230
2	2	0.79346	0.81052	0.82465	0.83697	0.84763	0.85707	0.86537	0.87276	0.87939	0.88536	0.89076
3	1	0.11839	0.14429	0.16955	0.19435	0.21847	0.24179	0.26422	0.28574	0.30633	0.32601	0.34481
3	2	0.45139	0.48524	0.51538	0.54234	0.56657	0.58844	0.60827	0.62632	0.64281	0.65794	0.67186
3	3	1.10695	1.09677	1.08836	1.08129	1.07527	1.07008	1.06556	1.06159	1.05807	1.05493	1.05211
4	1	0.08917	0.11128	0.13341	0.15577	0.17789	0.19959	0.22074	0.24126	0.26109	0.28021	0.29861
4	2	0.31798	0.35289	0.38489	0.41422	0.44114	0.46587	0.48865	0.50968	0.52912	0.54715	0.56390
4	3	0.67648	0.70075	0.72201	0.74033	0.75640	0.77061	0.78326	0.79460	0.80482	0.81407	0.82248
4	4	1.34739	1.31136	1.28207	1.25780	1.23736	1.21991	1.20485	1.19172	1.18015	1.16992	1.16077
5	1	0.07133	0.09069	0.11077	0.13120	0.15168	0.17200	0.19201	0.21158	0.23065	0.24916	0.26709
5	2	0.24593	0.27938	0.31071	0.33994	0.36717	0.39254	0.41616	0.43819	0.45874	0.47795	0.49592
5	3	0.49400	0.52671	0.55561	0.58131	0.60427	0.62491	0.64354	0.66045	0.67585	0.68993	0.70285
5	4	0.86273	0.87439	0.88422	0.89263	0.89990	0.90625	0.91185	0.91681	0.92124	0.92523	0.92883
5	5	1.54179	1.48229	1.43446	1.39520	1.36240	1.33460	1.31074	1.29004	1.27192	1.25592	1.24169
6	1	0.05945	0.07654	0.09516	0.11403	0.13316	0.15232	0.17133	0.19006	0.20843	0.22636	0.24382
6	2	0.20065	0.23220	0.26224	0.29068	0.31750	0.34274	0.36646	0.38875	0.40970	0.42940	0.44794
6	3	0.39056	0.42541	0.45682	0.48519	0.51092	0.53431	0.55566	0.57520	0.59315	0.60968	0.62495
6	4	0.64637	0.67253	0.69514	0.71486	0.73220	0.74758	0.76129	0.77361	0.78472	0.79479	0.80397
6	5	1.02066	1.01876	1.01719	1.01585	1.01471	1.01373	1.01286	1.01210	1.01142	1.01082	1.01028
6	6	1.70479	1.62409	1.55976	1.50733	1.46379	1.42707	1.39571	1.36860	1.34495	1.32413	1.30567
7	1	0.05095	0.06679	0.08368	0.10128	0.11928	0.13744	0.15559	0.17359	0.19132	0.20872	0.22573
7	2	0.16951	0.19919	0.22786	0.25531	0.28146	0.30629	0.32980	0.35203	0.37306	0.39293	0.41171
7	3	0.32343	0.35838	0.39038	0.41967	0.44653	0.47118	0.49387	0.51480	0.53414	0.55207	0.56871
7	4	0.51960	0.55146	0.57950	0.60434	0.62648	0.64632	0.66419	0.68037	0.69508	0.70852	0.72083
7	5	0.77968	0.79752	0.81270	0.82577	0.83714	0.84712	0.85595	0.86381	0.87087	0.87723	0.88299
7	6	1.15746	1.14218	1.12959	1.11906	1.11010	1.10240	1.09570	1.08983	1.08463	1.08000	1.07585
7	7	1.84505	1.74512	1.66599	1.60185	1.54884	1.50431	1.46641	1.43376	1.40534	1.38040	1.35833
8	1	0.04458	0.05915	0.07487	0.09139	0.10843	0.12573	0.14313	0.16047	0.17764	0.19456	0.21115
8	2	0.14676	0.17473	0.20207	0.22852	0.25393	0.27823	0.30139	0.32342	0.34435	0.36423	0.38309
8	3	0.27619	0.31047	0.34225	0.37168	0.39390	0.42411	0.44746	0.46914	0.48929	0.50805	0.52554
8	4	0.43537	0.46956	0.50009	0.52747	0.55213	0.57443	0.59469	0.61315	0.63004	0.64554	0.65983
8	5	0.63502	0.66178	0.68494	0.70517	0.72299	0.73879	0.75291	0.76558	0.77703	0.78741	0.79688
8	6	0.89752	0.90666	0.91409	0.92043	0.92590	0.93066	0.93485	0.93856	0.94187	0.94485	0.94753
8	7	1.27902	1.24993	1.22682	1.20768	1.19151	1.17767	1.16570	1.15523	1.14601	1.13782	1.13050
8	8	1.96812	1.85063	1.75810	1.68342	1.62195	1.57049	1.52660	1.48926	1.45667	1.42812	1.40290
9	1	0.03963	0.05315	0.06787	0.08348	0.09968	0.11624	0.13297	0.14973	0.16639	0.18286	0.19907
9	2	0.12941	0.15564	0.18195	0.20744	0.23210	0.25584	0.27859	0.30034	0.32110	0.34088	0.35973
9	3	0.24109	0.27437	0.30560	0.33477	0.36199	0.38736	0.41102	0.43309	0.45370	0.47297	0.49101
9	4	0.37505	0.41003	0.44165	0.47031	0.49634	0.52007	0.54176	0.56165	0.57994	0.59681	0.61242
9	5	0.53716	0.56840	0.59580	0.62001	0.64155	0.66081	0.67815	0.69332	0.70805	0.72104	0.73292
9	6	0.73906	0.75966	0.77726	0.79248	0.80575	0.81744	0.82780	0.83705	0.84537	0.85287	0.85969
9	7	1.00375	1.00341	1.00313	1.00289	1.00268	1.00250	1.00234	1.00221	1.00208	1.00197	1.00188
9	8	1.38573	1.34524	1.31240	1.28524	1.26241	1.24295	1.22617	1.21155	1.19870	1.18732	1.17717
9	9	2.07774	1.94410	1.83933	1.75510	1.68597	1.62827	1.57941	1.53751	1.50120	1.46944	1.44144
10	1	0.03567	0.04829	0.06217	0.07698	0.09245	0.10835	0.12450	0.14073	0.15693	0.17300	0.18886
10	2	0.11573	0.14079	0.16578	0.19036	0.21430	0.23746	0.25981	0.28125	0.30173	0.32142	0.34019
10	3	0.21395	0.24615	0.27665	0.30539	0.33239	0.35772	0.38146	0.40371	0.42457	0.44415	0.46255
10	4	0.32962	0.36481	0.39659	0.42583	0.45261	0.47717	0.49975	0.52057	0.53979	0.55760	0.57412
10	5	0.46614	0.49965	0.52938	0.55592	0.57973	0.60119	0.62062	0.63828	0.65440	0.66917	0.68275
10	6	0.63017	0.65719	0.68059	0.70103	0.71905	0.73503	0.74931	0.76214	0.77373	0.78425	0.79383
10	7	0.83363	0.84753	0.85930	0.86938	0.87811	0.88576	0.89250	0.89849	0.90335	0.90867	0.91303
10	8	1.09971	1.09025	1.08243	1.07585	1.07025	1.06542	1.06121	1.05750	1.05422	1.05130	1.04867
10	9	1.48304	1.42035	1.38476	1.35412	1.32511	1.30048	1.27930	1.26089	1.24476	1.23050	1.21780
10	10	2.17655	2.02796	1.91194	1.81896	1.74280	1.67950	1.62594	1.58011	1.54046	1.50582	1.47531

40% LIMITS						-βw-						
n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	0.51033	0.54299	0.57134	0.59648	0.61890	0.63902	0.65716	0.67359	0.68854	0.70220	0.71472
2	1	0.25541	0.28915	0.32065	0.34997	0.37723	0.40256	0.42611	0.44804	0.46848	0.48755	0.50538
2	2	1.00091	1.00083	1.00076	1.00070	1.00065	1.00061	1.00057	1.00054	1.00051	1.00048	1.00046
3	1	0.17028	0.20001	0.22871	0.25620	0.28237	0.30721	0.33073	0.35297	0.37399	0.39386	0.41264
3	2	0.56727	0.59728	0.62349	0.64656	0.66702	0.68527	0.70165	0.71643	0.72982	0.74202	0.75318
3	3	1.33487	1.30027	1.27213	1.24879	1.22913	1.21234	1.19784	1.18519	1.17405	1.16418	1.15536
4	1	0.12771	0.15393	0.17996	0.20534	0.22992	0.25360	0.27630	0.29802	0.31875	0.33852	0.35736
4	2	0.39923	0.43399	0.46525	0.49346	0.51899	0.54219	0.56334	0.58268	0.60043	0.61677	0.63185
4	3	0.81081	0.82641	0.83965	0.85101	0.86088	0.86952	0.87715	0.88394	0.89002	0.89549	0.90045
4	4	1.58607	1.52093	1.46870	1.42591	1.39023	1.36003	1.33414	1.31170	1.29208	1.27477	1.25939
5	1	0.10217	0.12571	0.14942	0.17295	0.19605	0.21854	0.24033	0.26136	0.28159	0.30101	0.31963
5	2	0.30860	0.34347	0.37546	0.40485	0.43186	0.45672	0.47965	0.50083	0.52045	0.53865	0.55557
5	3	0.59105	0.61999	0.64519	0.66731	0.68687	0.70429	0.71989	0.73394	0.74666	0.75823	0.76880
5	4	1.00790	1.00718	1.00658	1.00607	1.00564	1.00526	1.00493	1.00464	1.00433	1.00415	1.00394
5	5	1.78709	1.69521	1.62227	1.56300	1.51393	1.47264	1.43745	1.40709	1.38064	1.35740	1.33682
6	1	0.08514	0.10651	0.12336	0.15032	0.17211	0.19353	0.21445	0.23478	0.25446	0.27347	0.29178
6	2	0.25178	0.28541	0.31685	0.34614	0.37338	0.39873	0.42232	0.44428	0.46476	0.48389	0.50178
6	3	0.46694	0.50041	0.53013	0.55665	0.58044	0.60187	0.62128	0.63892	0.65502	0.66977	0.68333
6	4	0.75336	0.77300	0.78977	0.80424	0.81685	0.82794	0.83777	0.84654	0.85441	0.86152	0.86796
6	5	1.17298	1.15609	1.14220	1.13058	1.12071	1.11223	1.10486	1.09839	1.09268	1.08760	1.08304
6	6	1.95457	1.83904	1.74800	1.67450	1.61396	1.56327	1.52022	1.48322	1.45109	1.42293	1.39806
7	1	0.07298	0.09258	0.11289	0.13351	0.15416	0.17463	0.19475	0.21443	0.23358	0.25216	0.27014
7	2	0.21260	0.24481	0.27527	0.30399	0.33098	0.35630	0.38004	0.40229	0.42317	0.44276	0.46117
7	3	0.38652	0.42141	0.45288	0.48133	0.50714	0.53062	0.55206	0.57169	0.58973	0.60635	0.62171
7	4	0.60495	0.63323	0.65780	0.67934	0.69837	0.71528	0.73042	0.74404	0.75636	0.76756	0.77778
7	5	0.89351	0.90271	0.91044	0.91703	0.92273	0.92769	0.93205	0.93591	0.93936	0.94246	0.94526
7	6	1.31486	1.28254	1.25622	1.23437	1.21594	1.20020	1.18659	1.17470	1.16424	1.15496	1.14667
7	7	2.09807	1.96139	1.85431	1.76829	1.69774	1.63888	1.58905	1.54634	1.50935	1.47699	1.44847
8	1	0.06385	0.08200	0.10100	0.12048	0.14014	0.15976	0.17916	0.19823	0.21688	0.23504	0.25269
8	2	0.18412	0.21474	0.24411	0.27208	0.29859	0.32364	0.34729	0.36958	0.39059	0.41040	0.42909
8	3	0.32999	0.36499	0.39697	0.42620	0.45297	0.47753	0.50011	0.52091	0.54013	0.55793	0.57445
8	4	0.50659	0.53889	0.56738	0.59267	0.61523	0.63548	0.65374	0.67030	0.68536	0.69912	0.71175
8	5	0.72677	0.74616	0.76647	0.78231	0.79615	0.80835	0.81917	0.82883	0.83752	0.84538	0.85251
8	6	1.01669	1.01516	1.01389	1.01281	1.01189	1.01109	1.01040	1.00978	1.00924	1.00875	1.00831
8	7	1.43920	1.39235	1.35447	1.32322	1.29701	1.27472	1.25553	1.23883	1.22418	1.21122	1.19967
8	8	2.22353	2.06773	1.94631	1.84912	1.76968	1.70361	1.64782	1.60011	1.55886	1.52286	1.49117
9	1	0.05676	0.07367	0.09156	0.11004	0.12883	0.14769	0.16644	0.18496	0.20314	0.22092	0.23824
9	2	0.16234	0.19152	0.21980	0.24697	0.27291	0.29759	0.32101	0.34320	0.36421	0.38409	0.40292
9	3	0.28800	0.32251	0.35440	0.38384	0.41101	0.43611	0.45933	0.48084	0.50080	0.51937	0.53666
9	4	0.43625	0.47042	0.50093	0.52829	0.55293	0.57520	0.59543	0.61387	0.63074	0.64623	0.66049
9	5	0.61434	0.64216	0.66630	0.68744	0.70609	0.72267	0.73749	0.75081	0.76286	0.77381	0.78380
9	6	0.83563	0.84933	0.86102	0.87099	0.87962	0.88718	0.89384	0.89976	0.90506	0.90982	0.91413
9	7	1.12643	1.11435	1.10434	1.09594	1.08879	1.08264	1.07728	1.07257	1.06840	1.06469	1.06136
9	8	1.54966	1.48933	1.44071	1.40081	1.36746	1.33925	1.31502	1.29401	1.27561	1.25937	1.24493
9	9	2.33512	2.16135	2.02733	1.92006	1.83264	1.76011	1.69900	1.64684	1.60133	1.56260	1.52811
10	1	0.05106	0.06694	0.08386	0.10148	0.11949	0.13767	0.15534	0.17384	0.19159	0.20900	0.22601
10	2	0.14518	0.17302	0.20026	0.22663	0.25196	0.27623	0.29936	0.32137	0.34229	0.36216	0.38102
10	3	0.25556	0.28930	0.32080	0.35012	0.37736	0.40271	0.42637	0.44819	0.46863	0.48770	0.50553
10	4	0.38331	0.41622	0.44973	0.47825	0.50412	0.52767	0.54913	0.56889	0.58700	0.60369	0.61912
10	5	0.53235	0.56433	0.59179	0.61616	0.63755	0.65726	0.67477	0.69052	0.70468	0.71797	0.72996
10	6	0.71169	0.73422	0.75337	0.76996	0.78443	0.79727	0.80864	0.81881	0.82795	0.83622	0.84373
10	7	0.93396	0.93978	0.94465	0.94880	0.95237	0.95547	0.95820	0.96061	0.96275	0.96468	0.96642
10	8	1.22549	1.20304	1.18465	1.16931	1.15633	1.14519	1.13552	1.12706	1.11959	1.11296	1.10702
10	9	1.64952	1.57615	1.51751	1.46959	1.42973	1.39606	1.36725	1.34232	1.32055	1.30136	1.28433
10	10	2.43547	2.24615	2.09968	1.98322	1.88856	1.81013	1.74427	1.68811	1.63971	1.59759	1.56060

60% LIMITS

n	r	β_w 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	0.91629	0.92360	0.92974	0.93496	0.93947	0.94338	0.94683	0.94986	0.95259	0.95503	0.95723
2	1	0.45315	0.49184	0.52180	0.54857	0.57261	0.59430	0.61394	0.63182	0.64814	0.66310	0.67686
2	2	1.48986	1.43683	1.39408	1.35891	1.32946	1.30446	1.28297	1.26430	1.24794	1.23347	1.22060
3	1	0.30543	0.34020	0.37219	0.40159	0.42863	0.45353	0.47651	0.49775	0.51742	0.53567	0.55266
3	2	0.83716	0.85061	0.86234	0.87223	0.88078	0.88827	0.89487	0.90074	0.90598	0.91070	0.91497
3	3	1.85427	1.75305	1.67293	1.60800	1.55436	1.50932	1.47098	1.43797	1.40924	1.38402	1.36172
4	1	0.22907	0.26191	0.29285	0.32186	0.34901	0.37438	0.39809	0.42026	0.44099	0.46041	0.47862
4	2	0.58789	0.61697	0.64231	0.66456	0.68424	0.70177	0.71748	0.73163	0.74444	0.75609	0.76674
4	3	1.11119	1.10059	1.09184	1.08448	1.07822	1.07282	1.06812	1.06398	1.06032	1.05706	1.05413
4	4	2.12119	1.98103	1.87133	1.78326	1.71109	1.65090	1.59997	1.55635	1.51857	1.48554	1.45643
5	1	0.18326	0.21332	0.24316	0.27110	0.29759	0.32263	0.34627	0.36856	0.38957	0.40939	0.42809
5	2	0.45411	0.48739	0.51796	0.54485	0.56900	0.59080	0.61055	0.62853	0.64496	0.66002	0.67387
5	3	0.80637	0.82276	0.83625	0.84783	0.85789	0.86670	0.87448	0.88141	0.88761	0.89320	0.89826
5	4	1.32588	1.29231	1.26499	1.24232	1.22322	1.20690	1.19279	1.18049	1.16966	1.16005	1.15147
5	5	2.33181	2.15907	2.02494	1.91797	1.83079	1.75845	1.69750	1.64547	1.60057	1.56143	1.52703
6	1	0.15272	0.18117	0.20888	0.23562	0.26125	0.28571	0.30898	0.33108	0.35205	0.37193	0.39079
6	2	0.37026	0.40526	0.43694	0.46567	0.49180	0.51563	0.53742	0.55742	0.57581	0.59279	0.60849
6	3	0.63636	0.66305	0.68615	0.70632	0.72408	0.73983	0.75390	0.76653	0.77794	0.78829	0.79772
6	4	0.98596	0.98723	0.98829	0.98918	0.98995	0.99062	0.99120	0.99172	0.99218	0.99259	0.99296
6	5	1.50245	1.44737	1.40369	1.36773	1.33748	1.31180	1.28974	1.27058	1.25378	1.23895	1.22575
6	6	2.50575	2.30500	2.15305	2.02710	1.92733	1.84484	1.77556	1.71660	1.66584	1.62169	1.58296
7	1	0.13090	0.15748	0.18370	0.20928	0.23401	0.25780	0.28060	0.30238	0.32315	0.34295	0.36180
7	2	0.31268	0.34753	0.37953	0.40889	0.43587	0.46068	0.48354	0.50466	0.52420	0.54233	0.55917
7	3	0.52630	0.55793	0.58573	0.61033	0.63224	0.65186	0.66953	0.68552	0.70005	0.71332	0.72547
7	4	0.78989	0.80701	0.82156	0.83407	0.84495	0.85450	0.86294	0.87045	0.87719	0.88326	0.88876
7	5	1.13761	1.12435	1.11342	1.10426	1.09646	1.08975	1.08391	1.07879	1.07425	1.07021	1.06659
7	6	1.65244	1.57869	1.51975	1.47160	1.43154	1.39771	1.36876	1.34372	1.32185	1.30258	1.28547
7	7	2.65390	2.42857	2.25547	2.11868	2.00805	1.91686	1.84047	1.77560	1.71985	1.67147	1.62908
8	1	0.11454	0.13947	0.16436	0.18885	0.21272	0.23585	0.25813	0.27954	0.30005	0.31967	0.33843
8	2	0.27065	0.30479	0.33652	0.36592	0.39317	0.41841	0.44183	0.46358	0.48381	0.50265	0.52024
8	3	0.44909	0.48299	0.51319	0.54021	0.56450	0.58644	0.60633	0.62444	0.64099	0.65617	0.67014
8	4	0.66062	0.68599	0.70788	0.72694	0.74369	0.75852	0.77174	0.78359	0.79423	0.80397	0.81279
8	5	0.92263	0.92946	0.93514	0.93998	0.94414	0.94777	0.95095	0.95377	0.95623	0.95853	0.96056
8	6	1.26915	1.24175	1.21972	1.20123	1.18560	1.17222	1.16063	1.15051	1.14158	1.13365	1.12657
8	7	1.78281	1.69152	1.61903	1.56012	1.51134	1.47029	1.43530	1.40511	1.37931	1.35569	1.33522
8	8	2.78293	2.53567	2.34649	2.19748	2.07731	1.97849	1.89589	1.82588	1.76582	1.71375	1.66821
9	1	0.10181	0.12531	0.14899	0.17249	0.19556	0.21804	0.23981	0.26082	0.28104	0.30046	0.31908
9	2	0.23861	0.27181	0.30298	0.33212	0.35933	0.38470	0.40837	0.43046	0.45110	0.47040	0.48848
9	3	0.39182	0.42665	0.45804	0.48639	0.51209	0.53545	0.55677	0.57628	0.59420	0.61071	0.62595
9	4	0.56246	0.59341	0.62457	0.64760	0.66801	0.68622	0.70256	0.71730	0.73067	0.74284	0.75396
9	5	0.77371	0.79662	0.81135	0.82498	0.83639	0.84641	0.85528	0.86318	0.87027	0.87665	0.88244
9	6	1.03975	1.03607	1.03302	1.03044	1.02824	1.02633	1.02466	1.02320	1.02189	1.02073	1.01968
9	7	1.38533	1.34468	1.31208	1.28495	1.26214	1.24271	1.22594	1.21134	1.19851	1.18714	1.17700
9	8	1.89812	1.79070	1.70583	1.63718	1.58053	1.53303	1.49263	1.45787	1.42766	1.40116	1.37772
9	9	2.89720	2.63015	2.42651	2.26656	2.13788	2.03229	1.94418	1.86961	1.80574	1.75044	1.70212
10	1	0.09163	0.11387	0.13647	0.15906	0.18136	0.20325	0.22453	0.24515	0.26506	0.28425	0.30270
10	2	0.21337	0.24554	0.27602	0.30475	0.33175	0.35707	0.38031	0.40306	0.42393	0.44352	0.46192
10	3	0.34759	0.38284	0.41453	0.44359	0.47010	0.49437	0.51662	0.53706	0.55596	0.57340	0.58957
10	4	0.49922	0.53176	0.56050	0.58602	0.60883	0.62930	0.64779	0.66454	0.67980	0.69375	0.70655
10	5	0.67475	0.69932	0.72048	0.73888	0.75502	0.76930	0.78201	0.79341	0.80367	0.81297	0.82143
10	6	0.88414	0.89409	0.90247	0.90963	0.91580	0.92119	0.92592	0.93015	0.93338	0.93725	0.94029
10	7	1.14446	1.13051	1.11901	1.10937	1.10116	1.09412	1.08799	1.06261	1.07784	1.07360	1.06979
10	8	1.48937	1.43640	1.39370	1.35656	1.32915	1.30417	1.28270	1.26406	1.24771	1.23326	1.22040
10	9	2.00149	1.87914	1.76291	1.70524	1.64155	1.58319	1.54293	1.50407	1.47034	1.44031	1.41474
10	10	2.99974	2.71464	2.49767	2.32803	2.19167	2.07997	1.98691	1.90026	1.84097	1.73278	1.73198

70% LIMITS

n	r	$-\beta_w-$ 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	1.20397	1.18383	1.16729	1.15349	1.14173	1.13173	1.12301	1.11538	1.10863	1.10263	1.09726
2	1	0.60199	0.63041	0.65512	0.67679	0.69592	0.71295	0.72818	0.74190	0.75431	0.76558	0.77588
2	2	1.81192	1.71661	1.64103	1.57968	1.52392	1.48625	1.44990	1.41356	1.39127	1.36730	1.34608
3	1	0.40132	0.43606	0.46728	0.49545	0.52093	0.54408	0.56518	0.58447	0.60217	0.61846	0.63350
3	2	1.01264	1.01149	1.01053	1.00971	1.00901	1.00841	1.00788	1.00742	1.00700	1.00663	1.00630
3	3	2.18840	2.03801	1.92061	1.82657	1.74964	1.68559	1.63147	1.58517	1.54511	1.51013	1.47932
4	1	0.30099	0.33571	0.36767	0.39709	0.42417	0.44913	0.47217	0.49348	0.51323	0.53156	0.54863
4	2	0.71010	0.73255	0.75179	0.76848	0.78307	0.79594	0.80737	0.81760	0.82680	0.83512	0.84267
4	3	1.30054	1.26964	1.24481	1.22402	1.20647	1.19147	1.17850	1.16716	1.15718	1.14833	1.14041
4	4	2.46148	2.26795	2.11334	1.99949	1.90294	1.82304	1.75589	1.69869	1.64942	1.60654	1.56891
5	1	0.24079	0.27407	0.30529	0.33446	0.36163	0.38705	0.41070	0.43278	0.45339	0.47266	0.49071
5	2	0.54820	0.57899	0.60596	0.62977	0.65092	0.66982	0.68681	0.70215	0.71606	0.72878	0.74040
5	3	0.94207	0.94720	0.95149	0.95514	0.95827	0.96100	0.96339	0.96551	0.96739	0.96908	0.97061
5	4	1.52314	1.46597	1.41978	1.38219	1.35060	1.32381	1.30080	1.28034	1.26334	1.24790	1.23415
5	5	2.67582	2.44600	2.27099	2.13213	2.01989	1.92740	1.84995	1.78421	1.72773	1.67872	1.63579
6	1	0.20066	0.23221	0.26225	0.29069	0.31751	0.34275	0.36647	0.38876	0.40971	0.42941	0.44795
6	2	0.44685	0.48030	0.51105	0.53813	0.56249	0.58448	0.60443	0.62260	0.63921	0.65444	0.66847
6	3	0.74222	0.76261	0.78002	0.79507	0.80820	0.81976	0.83001	0.83915	0.84737	0.85479	0.86152
6	4	1.12910	1.11671	1.10643	1.09790	1.09060	1.08431	1.07884	1.07404	1.06978	1.06599	1.06259
6	5	1.70483	1.62412	1.55979	1.50736	1.46382	1.42710	1.39573	1.36862	1.34497	1.32415	1.30569
6	6	2.85227	2.59304	2.39511	2.23947	2.11415	2.01122	1.92528	1.85250	1.79012	1.73610	1.68887
7	1	0.17200	0.20184	0.23064	0.25819	0.28441	0.30928	0.33231	0.35506	0.37609	0.39595	0.41472
7	2	0.37729	0.41225	0.44324	0.47246	0.49845	0.52214	0.54378	0.56562	0.58186	0.59868	0.61424
7	3	0.61351	0.64133	0.66556	0.68673	0.70542	0.72202	0.73687	0.75022	0.76230	0.77327	0.78327
7	4	0.90327	0.91167	0.91672	0.92473	0.92991	0.93443	0.93840	0.94191	0.94505	0.94787	0.95041
7	5	1.28612	1.25704	1.23330	1.21357	1.19690	1.18265	1.17031	1.15954	1.15004	1.14161	1.13407
7	6	1.85841	1.75660	1.67604	1.61076	1.55684	1.51157	1.47304	1.43985	1.41099	1.38565	1.36323
7	7	3.00221	2.71667	2.49953	2.32950	2.19295	2.08110	1.98793	1.90918	1.84181	1.78355	1.73269
8	1	0.15050	0.17877	0.20635	0.23299	0.25853	0.28293	0.30616	0.32824	0.34920	0.36908	0.38794
8	2	0.32655	0.36152	0.39351	0.42278	0.44959	0.47420	0.49684	0.51771	0.53699	0.55486	0.57144
8	3	0.52334	0.55507	0.58297	0.60768	0.62969	0.64941	0.66717	0.68324	0.69736	0.71120	0.72342
8	4	0.75487	0.77441	0.79109	0.80548	0.81302	0.82905	0.83882	0.84754	0.85536	0.86242	0.86883
8	5	1.04136	1.03753	1.03435	1.03166	1.02937	1.02738	1.02565	1.02412	1.02277	1.02156	1.02047
8	6	1.42157	1.37683	1.34063	1.31073	1.28564	1.26429	1.24589	1.22983	1.21583	1.20339	1.19230
8	7	1.99145	1.87056	1.77544	1.69875	1.63565	1.58287	1.53808	1.49962	1.46624	1.43700	1.41119
8	8	3.13258	2.82371	2.58971	2.40693	2.26056	2.14093	2.04145	1.95752	1.88582	1.82390	1.76991
9	1	0.13377	0.16062	0.18706	0.21280	0.23767	0.26157	0.28444	0.30627	0.32708	0.34689	0.36575
9	2	0.28787	0.32237	0.35427	0.38370	0.41088	0.43598	0.45920	0.48071	0.50067	0.51924	0.53654
9	3	0.45650	0.49023	0.52023	0.54705	0.57114	0.59287	0.61256	0.63048	0.64684	0.66185	0.67565
9	4	0.64925	0.67525	0.69771	0.71730	0.73453	0.74979	0.76341	0.77563	0.78665	0.79665	0.80576
9	5	0.87302	0.88846	0.89726	0.90478	0.91127	0.91693	0.92191	0.92633	0.93028	0.93382	0.93703
9	6	1.16236	1.14657	1.13357	1.12269	1.11345	1.10550	1.09859	1.09253	1.08719	1.08240	1.07813
9	7	1.54071	1.48135	1.43362	1.39445	1.36172	1.33398	1.31017	1.28951	1.27142	1.25546	1.24126
9	8	2.10881	1.97051	1.86222	1.77525	1.70394	1.64446	1.59413	1.55099	1.51363	1.48097	1.45217
9	9	3.24790	2.91806	2.66592	2.47481	2.31970	2.19315	2.08810	1.99960	1.92408	1.85894	1.80219
10	1	0.12040	0.14595	0.17134	0.19624	0.22044	0.24332	0.26631	0.28786	0.30848	0.32818	0.34698
10	2	0.25741	0.29121	0.32274	0.35207	0.37933	0.40465	0.42819	0.45010	0.47051	0.48955	0.50735
10	3	0.40492	0.43960	0.47076	0.49885	0.52426	0.54732	0.56833	0.58754	0.60516	0.62137	0.63633
10	4	0.56999	0.59928	0.62598	0.64895	0.66930	0.68746	0.70375	0.71845	0.73177	0.74389	0.75498
10	5	0.76036	0.77953	0.79563	0.80998	0.82226	0.83306	0.84263	0.85116	0.85881	0.86572	0.87198
10	6	0.97730	0.98345	0.98940	0.99022	0.99091	0.99151	0.99204	0.99251	0.99292	0.99330	0.99363
10	7	1.27011	1.24240	1.22349	1.20193	1.18624	1.17231	1.16116	1.15102	1.14206	1.13410	1.12699
10	8	1.64709	1.57414	1.51564	1.46793	1.42823	1.39469	1.36599	1.34116	1.31946	1.30035	1.28339
10	9	2.21331	2.05951	1.93917	1.84286	1.76412	1.69861	1.64323	1.59546	1.55505	1.51933	1.48789
10	10	3.35130	3.00288	2.73954	2.53519	2.37220	2.23945	2.12940	2.03680	1.95737	1.88985	1.83065

80% LIMITS

n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
						-β_W-						
1	1	1.60944	1.54129	1.48672	1.44205	1.40483	1.37336	1.34639	1.32304	1.30262	1.28462	1.26864
2	1	0.80472	0.82077	0.83439	0.84609	0.85625	0.86516	0.87303	0.88003	0.88630	0.89195	0.89706
2	2	2.24835	2.06870	1.96436	1.86494	1.78374	1.71623	1.65926	1.61057	1.56849	1.53176	1.49945
3	1	0.53648	0.56773	0.59515	0.61939	0.64095	0.66024	0.67760	0.69329	0.70754	0.72054	0.73245
3	2	1.24778	1.22292	1.20259	1.18564	1.17131	1.15902	1.14838	1.13908	1.13086	1.12357	1.11704
3	3	2.63551	2.41326	2.24244	2.10738	1.99810	1.90799	1.83249	1.76835	1.71322	1.66536	1.62343
4	1	0.40236	0.43708	0.46829	0.49643	0.52189	0.54502	0.56609	0.58536	0.60303	0.61930	0.63432
4	2	0.87336	0.88418	0.89329	0.90108	0.90781	0.91368	0.91885	0.92344	0.92753	0.93121	0.93454
4	3	1.54967	1.48918	1.44057	1.40068	1.36737	1.33914	1.31492	1.29392	1.27553	1.25929	1.24486
4	4	2.91400	2.64401	2.43823	2.27667	2.14673	2.04014	1.95122	1.87598	1.81155	1.75577	1.70704
5	1	0.32169	0.35683	0.38862	0.41813	0.44500	0.46968	0.49240	0.51335	0.53272	0.55068	0.56735
5	2	0.67372	0.69835	0.71956	0.73801	0.75420	0.76852	0.78127	0.79270	0.80299	0.81232	0.82081
5	3	1.11903	1.10764	1.09825	1.09036	1.08364	1.07785	1.07282	1.06839	1.06447	1.06098	1.05784
5	4	1.78017	1.68925	1.61703	1.55834	1.50974	1.46884	1.43397	1.40388	1.37767	1.35464	1.33423
5	5	3.13161	2.82291	2.58905	2.40635	2.26006	2.14048	2.04106	1.95717	1.88550	1.82360	1.76964
6	1	0.26824	0.30233	0.33402	0.36341	0.39066	0.41593	0.43937	0.46114	0.48141	0.50029	0.51792
6	2	0.54896	0.57972	0.60666	0.63044	0.65156	0.67044	0.68740	0.70273	0.71664	0.72931	0.74092
6	3	0.88043	0.89068	0.89931	0.90669	0.91305	0.91861	0.92349	0.92783	0.93170	0.93517	0.93831
6	4	1.31435	1.28209	1.25581	1.23400	1.21561	1.19989	1.18630	1.17444	1.16399	1.15473	1.14645
6	5	1.96697	1.84964	1.75724	1.68266	1.62127	1.56987	1.52624	1.48875	1.45620	1.42768	1.40249
6	6	3.31024	2.96892	2.71154	2.51126	2.35141	2.22112	2.11306	2.02209	1.94451	1.87763	1.81941
7	1	0.22992	0.26279	0.29375	0.32278	0.34993	0.37530	0.39901	0.42117	0.44190	0.46130	0.47950
7	2	0.46340	0.49697	0.52678	0.55341	0.57730	0.59883	0.61834	0.63607	0.65226	0.66710	0.68074
7	3	0.72724	0.74860	0.76688	0.78270	0.79652	0.80869	0.81950	0.82915	0.83783	0.84566	0.85278
7	4	1.04955	1.04495	1.04113	1.03791	1.03515	1.03277	1.03069	1.02886	1.02723	1.02578	1.02448
7	5	1.47698	1.42553	1.38403	1.34986	1.32124	1.29693	1.27602	1.25786	1.24193	1.22785	1.21531
7	6	2.12411	1.98351	1.87347	1.78515	1.71277	1.65241	1.60135	1.55763	1.51973	1.48662	1.45743
7	7	3.46175	3.09221	2.81457	2.59922	2.42779	2.28839	2.17300	2.07602	1.99346	1.92238	1.86058
8	1	0.20118	0.23275	0.26282	0.29127	0.31810	0.34334	0.36706	0.38935	0.41030	0.43000	0.44853
8	2	0.40102	0.43576	0.46699	0.49516	0.52065	0.54381	0.56491	0.58421	0.60192	0.61822	0.63326
8	3	0.62003	0.64761	0.67149	0.69238	0.71080	0.72716	0.74179	0.75494	0.76682	0.77761	0.78745
8	4	0.87624	0.88663	0.89575	0.90337	0.90995	0.91569	0.92075	0.92523	0.92923	0.93283	0.93608
8	5	1.19323	1.17426	1.15865	1.14560	1.13453	1.12502	1.11677	1.10954	1.10315	1.09746	1.09237
8	6	1.61648	1.54742	1.49214	1.44690	1.40922	1.37736	1.35007	1.32644	1.30579	1.28758	1.27141
8	7	2.25979	2.09836	1.97268	1.87223	1.79022	1.72205	1.66453	1.61538	1.57291	1.53586	1.50326
8	8	3.59330	3.19885	2.90343	2.67487	2.49334	2.34600	2.22425	2.12207	2.03520	1.96049	1.89560
9	1	0.17883	0.20912	0.23825	0.26604	0.29243	0.31741	0.34101	0.36329	0.38431	0.40415	0.42288
9	2	0.35349	0.38854	0.42039	0.44937	0.47579	0.49994	0.52208	0.54243	0.56118	0.57850	0.59455
9	3	0.54074	0.57182	0.59909	0.62317	0.64458	0.66373	0.68095	0.69652	0.71066	0.72355	0.73535
9	4	0.75319	0.77285	0.78962	0.80410	0.81672	0.82782	0.83765	0.84643	0.85430	0.86141	0.86786
9	5	1.00489	1.00444	1.00407	1.00376	1.00349	1.00325	1.00305	1.00287	1.00271	1.00257	1.00244
9	6	1.31646	1.28574	1.25909	1.23697	1.21832	1.20239	1.18862	1.17660	1.16602	1.15663	1.14824
9	7	1.73969	1.65343	1.58557	1.53034	1.48453	1.44594	1.41299	1.38455	1.35974	1.33793	1.31860
9	8	2.37918	2.19890	2.05915	1.94787	1.85728	1.78218	1.71896	1.66505	1.61855	1.57804	1.54246
9	9	3.70954	3.29272	2.98149	2.74118	2.55068	2.39633	2.26895	2.16219	2.07152	1.99362	1.92602
10	1	0.16094	0.19002	0.21822	0.24533	0.27123	0.29583	0.31923	0.34146	0.36246	0.38235	0.40118
10	2	0.31607	0.35096	0.38295	0.41230	0.43924	0.46400	0.48681	0.50787	0.52735	0.54541	0.56220
10	3	0.47955	0.51258	0.54204	0.56818	0.59159	0.61266	0.63171	0.64901	0.66479	0.67923	0.69249
10	4	0.66098	0.68654	0.70821	0.72725	0.74399	0.75880	0.77200	0.78384	0.79452	0.80420	0.81301
10	5	0.86956	0.88068	0.89006	0.89807	0.90499	0.91103	0.91635	0.92107	0.92529	0.92908	0.93250
10	6	1.11332	1.10701	1.09767	1.08983	1.08315	1.07740	1.07239	1.06799	1.06410	1.06062	1.05751
10	7	1.42943	1.38376	1.34681	1.31632	1.29073	1.26895	1.25020	1.23389	1.21956	1.20689	1.19559
10	8	1.84747	1.74721	1.66782	1.60347	1.55029	1.50563	1.46761	1.43486	1.40637	1.38135	1.35922
10	9	2.48579	2.28800	2.13576	2.01467	1.91635	1.83503	1.76671	1.70854	1.65845	1.61488	1.57664
10	10	3.81366	3.37670	3.05107	2.80019	2.60162	2.44096	2.30855	2.19769	2.10362	2.02298	1.95236

90% LIMITS		-βw-										
n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	2.30259	2.13445	2.00376	1.89945	1.81437	1.74372	1.68417	1.63331	1.58939	1.55110	1.51743
2	1	1.15129	1.13664	1.12457	1.11446	1.10587	1.09848	1.09205	1.08640	1.08141	1.07697	1.07298
2	2	2.96974	2.68995	2.47703	2.31009	2.17599	2.06607	1.97446	1.89701	1.83072	1.77337	1.72329
3	1	0.76753	0.78621	0.80213	0.81585	0.82780	0.83829	0.84759	0.85587	0.86330	0.87001	0.87609
3	2	1.63066	1.55976	1.50306	1.45665	1.41804	1.38540	1.35746	1.33328	1.31214	1.29351	1.27697
3	3	3.36649	3.01475	2.74988	2.54402	2.37988	2.24622	2.13543	2.04223	1.96280	1.89436	1.83480
4	1	0.57565	0.60528	0.63115	0.65389	0.67404	0.69200	0.70810	0.72263	0.73579	0.74777	0.75871
4	2	1.13800	1.12470	1.11374	1.10455	1.09673	1.09000	1.08415	1.07901	1.07446	1.07040	1.06677
4	3	1.94800	1.83342	1.74311	1.67017	1.61009	1.55976	1.51702	1.48029	1.44838	1.42041	1.39571
4	4	3.64980	3.24455	2.94143	2.70717	2.52128	2.37053	2.24604	2.14164	2.05292	1.97666	1.91045
5	1	0.46052	0.49415	0.52405	0.55075	0.57473	0.59634	0.61593	0.63374	0.65000	0.66491	0.67861
5	2	0.87681	0.88735	0.89623	0.90382	0.91037	0.91609	0.92112	0.92558	0.92957	0.93315	0.93638
5	3	1.39964	1.35768	1.32352	1.29529	1.27157	1.25137	1.23395	1.21879	1.20547	1.19367	1.18315
5	4	2.18716	2.03696	1.91970	1.82577	1.74893	1.68495	1.63089	1.58464	1.54462	1.50968	1.47891
5	5	3.87032	3.42222	3.08280	2.83213	2.62917	2.46508	2.32993	2.21684	2.12093	2.03864	1.96731
6	1	0.38376	0.41867	0.45018	0.47868	0.50455	0.52809	0.54959	0.56929	0.58739	0.60407	0.61949
6	2	0.71400	0.73620	0.75523	0.77172	0.78613	0.79885	0.81014	0.82024	0.82932	0.83752	0.84498
6	3	1.09903	1.08964	1.08187	1.07534	1.06977	1.06498	1.06079	1.05712	1.05386	1.05095	1.04835
6	4	1.60490	1.53735	1.48323	1.43892	1.40200	1.37077	1.34402	1.32085	1.30058	1.28272	1.26685
6	5	2.37952	2.19919	2.05940	1.94808	1.85746	1.78235	1.71912	1.66519	1.61868	1.57816	1.54257
6	6	4.05089	3.56713	3.20843	2.93323	2.71622	2.54117	2.39728	2.27710	2.17535	2.08816	2.01268
7	1	0.32894	0.36393	0.39591	0.42516	0.45194	0.47652	0.49911	0.51994	0.53918	0.55700	0.57353
7	2	0.60251	0.63091	0.65560	0.67724	0.69636	0.71336	0.72858	0.74228	0.75467	0.76594	0.77622
7	3	0.90679	0.91469	0.92169	0.92749	0.93249	0.93685	0.94063	0.94407	0.94709	0.94980	0.95225
7	4	1.27797	1.24979	1.22678	1.20764	1.19148	1.17764	1.16567	1.15521	1.14598	1.13779	1.13047
7	5	1.77409	1.68399	1.61242	1.55424	1.50605	1.46549	1.43090	1.40106	1.37505	1.35219	1.33195
7	6	2.54054	2.33408	2.17490	2.04872	1.94641	1.86188	1.79093	1.73058	1.67865	1.63350	1.59391
7	7	4.20379	3.68932	3.30903	3.01803	2.78906	2.60472	2.45344	2.32728	2.22059	2.12923	2.05032
8	1	0.28782	0.32233	0.35422	0.38366	0.41083	0.43593	0.45915	0.48066	0.50063	0.51919	0.53649
8	2	0.52129	0.55309	0.58108	0.60585	0.62793	0.64772	0.66554	0.68167	0.69634	0.70973	0.72200
8	3	0.77266	0.79099	0.80659	0.82004	0.83175	0.84202	0.85112	0.85923	0.86650	0.87306	0.87901
8	4	1.06530	1.05919	1.05413	1.04986	1.04622	1.04308	1.04033	1.03791	1.03577	1.03386	1.03214
8	5	1.42353	1.38295	1.34609	1.31566	1.29013	1.26841	1.24970	1.23342	1.21913	1.20648	1.19521
8	6	1.91831	1.80800	1.72094	1.65055	1.59252	1.54388	1.50253	1.46698	1.43608	1.40898	1.38503
8	7	2.67908	2.44951	2.27329	2.13413	2.02164	1.92896	1.85136	1.78549	1.72890	1.67979	1.63679
8	8	4.33639	3.79496	3.39578	3.09100	2.85162	2.65921	2.50153	2.37018	2.25923	2.16437	2.08240
9	1	0.25584	0.28960	0.32110	0.35042	0.37768	0.40301	0.42656	0.44849	0.46892	0.48799	0.50581
9	2	0.45944	0.49310	0.52303	0.54976	0.57377	0.59541	0.61502	0.63286	0.64916	0.66409	0.67782
9	3	0.67350	0.69814	0.71937	0.73783	0.75402	0.76835	0.78111	0.79254	0.80285	0.81218	0.82067
9	4	0.91464	0.92228	0.92851	0.93383	0.93841	0.94239	0.94589	0.94899	0.95176	0.95424	0.95647
9	5	1.20075	1.18094	1.16469	1.15111	1.13960	1.12971	1.12113	1.11362	1.10698	1.10108	1.09579
9	6	1.55376	1.49711	1.44760	1.40699	1.37309	1.34437	1.31974	1.29837	1.27968	1.26317	1.24850
9	7	2.04410	1.91546	1.81447	1.73319	1.66643	1.61065	1.56338	1.52232	1.48765	1.45637	1.42972
9	8	2.80067	2.55037	2.35895	2.20825	2.08676	1.98690	1.90344	1.83272	1.77206	1.71950	1.67352
9	9	4.45344	3.88797	3.47200	3.15498	2.90639	2.70665	2.54352	2.40761	2.29291	2.19492	2.11032
10	1	0.23026	0.26315	0.29411	0.32314	0.35030	0.37567	0.39933	0.42153	0.44226	0.46166	0.47985
10	2	0.41075	0.44536	0.47641	0.50437	0.52964	0.55257	0.57344	0.59251	0.60979	0.62607	0.64090
10	3	0.59712	0.62577	0.65070	0.67257	0.69190	0.70910	0.72450	0.73836	0.75091	0.76232	0.77273
10	4	0.80236	0.81853	0.83235	0.84419	0.85446	0.86347	0.87143	0.87851	0.88486	0.89057	0.89575
10	5	1.03785	1.03435	1.03144	1.02899	1.02689	1.02506	1.02349	1.02209	1.02085	1.01975	1.01875
10	6	1.31932	1.28649	1.25977	1.23759	1.21888	1.20291	1.18910	1.17705	1.16643	1.15702	1.14861
10	7	1.67364	1.59709	1.53598	1.48610	1.44464	1.40964	1.37972	1.35394	1.33124	1.31135	1.29369
10	8	2.15567	2.01086	1.89604	1.80552	1.73091	1.66374	1.61618	1.57113	1.53223	1.49820	1.46822
10	9	2.90903	2.63991	2.43476	2.27368	2.14412	2.03782	1.94914	1.87410	1.80933	1.75420	1.70559
10	10	4.55322	3.97104	3.53994	3.21192	2.95507	2.74914	2.58075	2.44077	2.32272	2.22195	2.13500

-βw-

95% LIMITS n	r	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
1	1	2.99573	2.71134	2.49539	2.32563	2.18957	2.07811	1.98525	1.90676	1.83960	1.78152	1.73082
2	1	1.49787	1.44385	1.40032	1.36452	1.33456	1.30913	1.28727	1.26829	1.25166	1.23696	1.22387
2	2	3.67614	3.26582	2.95910	2.72218	2.53426	2.38193	2.25616	2.15072	2.06114	1.98415	1.91733
3	1	0.99858	0.99871	0.99881	0.99891	0.99898	0.99905	0.99911	0.99916	0.99921	0.99925	0.99929
3	2	1.99989	1.87777	1.78171	1.70429	1.64061	1.58734	1.54216	1.50336	1.46969	1.44020	1.41417
3	3	4.07734	3.58830	3.22533	2.94796	2.72883	2.55222	2.40705	2.28584	2.18323	2.09532	2.01924
4	1	0.74393	0.76882	0.78590	0.80060	0.81342	0.82470	0.83469	0.84361	0.85162	0.85885	0.86541
4	2	1.39189	1.35067	1.31726	1.28963	1.26641	1.24663	1.22957	1.21471	1.20166	1.19010	1.17978
4	3	2.32676	2.15482	2.02128	1.91477	1.82795	1.75591	1.69520	1.64337	1.59864	1.55965	1.52537
4	4	4.36269	3.81604	3.41307	3.10552	2.86406	2.67003	2.51107	2.37869	2.26689	2.17132	2.08875
5	1	0.59915	0.62771	0.65255	0.67433	0.69358	0.71070	0.72604	0.73984	0.75233	0.76368	0.77405
5	2	1.07122	1.06454	1.05900	1.05434	1.05037	1.04693	1.04393	1.04130	1.03896	1.03687	1.03500
5	3	1.66466	1.58930	1.52911	1.47996	1.43909	1.40459	1.37508	1.34956	1.32727	1.30764	1.29022
5	4	2.57124	2.35970	2.19678	2.06773	1.96318	1.87685	1.80443	1.74285	1.68989	1.64386	1.60351
5	5	4.53476	3.99206	3.55711	3.22630	2.96735	2.75980	2.59014	2.44912	2.33023	2.22875	2.14120
6	1	0.49929	0.53183	0.56057	0.58609	0.60889	0.62936	0.64784	0.66460	0.67986	0.69381	0.70660
6	2	0.87160	0.88274	0.89197	0.89985	0.90665	0.91260	0.91783	0.92247	0.92661	0.93034	0.93370
6	3	1.30439	1.27326	1.24788	1.22680	1.20902	1.19382	1.18067	1.16919	1.15908	1.15011	1.14210
6	4	1.87626	1.77194	1.68945	1.62266	1.56751	1.52123	1.48187	1.44798	1.41850	1.39264	1.36977
6	5	2.76701	2.52248	2.33530	2.18780	2.06881	1.97094	1.88911	1.81973	1.76020	1.70859	1.66343
6	6	4.76523	4.13545	3.67405	3.32409	3.05078	2.83215	2.65374	2.50568	2.38102	2.27475	2.18317
7	1	0.42796	0.46229	0.49299	0.52055	0.54540	0.56790	0.58834	0.60699	0.62406	0.63974	0.65419
7	2	0.73543	0.75627	0.77408	0.78948	0.80292	0.81476	0.82526	0.83463	0.84306	0.85067	0.85757
7	3	1.07511	1.06805	1.06221	1.05729	1.05309	1.04946	1.04630	1.04352	1.04105	1.03885	1.03687
7	4	1.49023	1.43715	1.39437	1.35916	1.32969	1.30467	1.28317	1.26448	1.24811	1.23363	1.22075
7	5	2.04983	1.92035	1.81372	1.73693	1.66977	1.61366	1.56612	1.52533	1.48997	1.45902	1.43172
7	6	2.93040	2.65754	2.44966	2.28652	2.15536	2.04779	1.95808	1.88219	1.81721	1.76097	1.71184
7	7	4.91977	4.25638	3.77242	3.40616	3.12066	2.89265	2.70686	2.55286	2.42334	2.31303	2.21805
8	1	0.37447	0.40944	0.44107	0.46974	0.49579	0.51953	0.54123	0.56113	0.57944	0.59632	0.61194
8	2	0.63616	0.66246	0.68597	0.70615	0.72392	0.73968	0.75375	0.76639	0.77780	0.78816	0.79760
8	3	0.91551	0.92259	0.92908	0.93435	0.93890	0.94285	0.94633	0.94940	0.95214	0.95461	0.95683
8	4	1.24050	1.21643	1.19673	1.18031	1.16642	1.15451	1.14419	1.13516	1.12719	1.12011	1.11378
8	5	1.64557	1.57272	1.51448	1.46689	1.42729	1.39383	1.36520	1.34043	1.31879	1.29972	1.28280
8	6	2.19721	2.04547	1.92705	1.83222	1.75466	1.69011	1.63557	1.58892	1.54856	1.51333	1.48230
8	7	3.07069	2.77295	2.54700	2.37026	2.22857	2.11263	2.01615	1.93468	1.86503	1.80484	1.75234
8	8	5.05234	4.36092	3.85727	3.47681	3.18072	2.94458	2.75239	2.59325	2.45954	2.34575	2.24785
9	1	0.33286	0.36787	0.39984	0.42905	0.45578	0.48029	0.50282	0.52357	0.54274	0.56048	0.57694
9	2	0.56060	0.59089	0.61737	0.64071	0.66141	0.67989	0.69648	0.71146	0.72504	0.73742	0.74873
9	3	0.79771	0.81427	0.82833	0.84042	0.85092	0.86013	0.86827	0.87551	0.88200	0.88785	0.89315
9	4	1.06438	1.05836	1.05337	1.04917	1.04557	1.04247	1.03977	1.03738	1.03527	1.03338	1.03169
9	5	1.38084	1.34092	1.30854	1.28175	1.25922	1.24002	1.22346	1.20903	1.19635	1.18511	1.17509
9	6	1.77933	1.68852	1.61640	1.55778	1.50923	1.46838	1.43354	1.40349	1.37731	1.35430	1.33392
9	7	2.32537	2.15365	2.02028	1.91389	1.82718	1.75521	1.69456	1.64280	1.59811	1.55916	1.52492
9	8	3.19362	2.87368	2.63170	2.44292	2.29193	2.16865	2.06622	1.97987	1.90615	1.84252	1.78707
9	9	5.17027	4.45296	3.93183	3.53880	3.23335	2.99003	2.79219	2.62853	2.49113	2.37429	2.27382
10	1	0.29957	0.33427	0.36623	0.39565	0.42274	0.44772	0.47078	0.49211	0.51188	0.53024	0.54733
10	2	0.50114	0.53363	0.56230	0.58776	0.61050	0.63092	0.64935	0.66605	0.68126	0.69516	0.70792
10	3	0.70705	0.72962	0.74910	0.76593	0.78060	0.79365	0.80520	0.81553	0.82482	0.83322	0.84086
10	4	0.93299	0.93889	0.94384	0.94804	0.95166	0.95481	0.95758	0.96002	0.96220	0.96415	0.96591
10	5	1.19225	1.17334	1.15782	1.14484	1.13383	1.12438	1.11617	1.10898	1.10262	1.09697	1.09190
10	6	1.50709	1.44343	1.40439	1.36318	1.33788	1.31217	1.29003	1.27089	1.25403	1.23923	1.22601
10	7	1.89693	1.78966	1.70494	1.63639	1.57982	1.53233	1.49205	1.45734	1.42716	1.40069	1.37729
10	8	2.43331	2.24895	2.10208	1.98532	1.89041	1.81184	1.74577	1.68946	1.64096	1.59874	1.56167
10	9	3.30303	2.96305	2.70662	2.50706	2.34775	2.21790	2.11019	2.01950	1.94216	1.87548	1.81743
10	10	5.27534	4.53515	3.99630	3.59400	3.28015	3.03040	2.82753	2.65983	2.51913	2.39956	2.29681

5% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-βw-						
1	1	0.24308	0.25922	0.27489	0.29008	0.30481	0.31906	0.33285	0.34618	0.35908	0.37155	0.41745
2	1	0.17474	0.18916	0.20336	0.21732	0.23100	0.24439	0.25749	0.27027	0.28274	0.29490	0.34046
2	2	0.51962	0.53551	0.55025	0.56412	0.57718	0.58951	0.60117	0.61219	0.62264	0.63255	0.66757
3	1	0.14406	0.15732	0.17050	0.18354	0.19642	0.20910	0.22158	0.23383	0.24585	0.25762	0.30219
3	2	0.39927	0.41628	0.43245	0.44782	0.46245	0.47637	0.48964	0.50228	0.51435	0.52588	0.56718
3	3	0.69053	0.70226	0.71313	0.72325	0.73268	0.74150	0.74976	0.75751	0.76480	0.77167	0.79556
4	1	0.12562	0.13804	0.15045	0.16280	0.17507	0.18720	0.19919	0.21100	0.22263	0.23406	0.27767
4	2	0.33833	0.35541	0.37176	0.38741	0.40239	0.41673	0.43046	0.44361	0.45622	0.46831	0.51203
4	3	0.55081	0.56594	0.58012	0.59344	0.60595	0.61774	0.62886	0.63937	0.64930	0.65872	0.69188
4	4	0.80873	0.81657	0.82380	0.83048	0.83667	0.84243	0.84780	0.85281	0.85751	0.86191	0.87711
5	1	0.11295	0.12472	0.13654	0.14835	0.16012	0.17180	0.18339	0.19484	0.20614	0.21729	0.26003
5	2	0.29951	0.31639	0.33262	0.34823	0.36324	0.37766	0.39153	0.40487	0.41769	0.43002	0.47491
5	3	0.47539	0.49174	0.50715	0.52170	0.53546	0.54848	0.56081	0.57252	0.58364	0.59421	0.63173
5	4	0.66119	0.67374	0.68541	0.69628	0.70643	0.71594	0.72486	0.73323	0.74112	0.74856	0.77450
5	5	0.89754	0.90196	0.90601	0.90975	0.91320	0.91639	0.91936	0.92212	0.92470	0.92712	0.93541
6	1	0.10356	0.11480	0.12613	0.13750	0.14886	0.16017	0.17141	0.18256	0.19358	0.20447	0.24646
6	2	0.27192	0.28350	0.30452	0.31999	0.33491	0.34930	0.36318	0.37656	0.38945	0.40189	0.44739
6	3	0.42553	0.44238	0.45835	0.47349	0.48786	0.50152	0.51450	0.52686	0.53863	0.54986	0.58994
6	4	0.57825	0.59282	0.60645	0.61923	0.63121	0.64248	0.65310	0.66311	0.67257	0.68152	0.71297
6	5	0.74677	0.75675	0.76598	0.77453	0.78249	0.78991	0.79684	0.80333	0.80942	0.81514	0.83498
6	6	0.96793	0.96937	0.97068	0.97189	0.97300	0.97402	0.97497	0.97585	0.97668	0.97744	0.98007
7	1	0.09623	0.10704	0.11796	0.12894	0.13995	0.15095	0.16190	0.17278	0.18356	0.19423	0.23553
7	2	0.25097	0.26724	0.28303	0.29831	0.31310	0.32740	0.34122	0.35458	0.36749	0.37996	0.42577
7	3	0.38920	0.40625	0.42248	0.43792	0.45263	0.46664	0.48000	0.49275	0.50492	0.51656	0.55830
7	4	0.52197	0.53763	0.55233	0.56616	0.57920	0.59149	0.60311	0.61410	0.62451	0.63439	0.66928
7	5	0.65967	0.67226	0.68397	0.69488	0.70507	0.71461	0.72356	0.73197	0.73989	0.74736	0.77340
7	6	0.81607	0.82365	0.83062	0.83707	0.84305	0.84860	0.85378	0.85861	0.86314	0.86738	0.88202
7	7	1.02586	1.02467	1.02359	1.02259	1.02168	1.02084	1.02006	1.01933	1.01866	1.01803	1.01590
8	1	0.09030	0.10073	0.11130	0.12197	0.13267	0.14339	0.15409	0.16473	0.17530	0.18578	0.22646
8	2	0.23434	0.25032	0.26586	0.28094	0.29558	0.30977	0.32351	0.33681	0.34969	0.36216	0.40812
8	3	0.36113	0.37824	0.39457	0.41016	0.42505	0.43927	0.45285	0.46585	0.47828	0.49019	0.53308
8	4	0.48023	0.49651	0.51186	0.52634	0.54003	0.55298	0.56525	0.57688	0.58793	0.59843	0.63569
8	5	0.59946	0.61356	0.62673	0.63905	0.65060	0.66145	0.67165	0.68127	0.69034	0.69892	0.72901
8	6	0.72659	0.73722	0.74706	0.75619	0.76469	0.77262	0.78004	0.78699	0.79352	0.79966	0.82097
8	7	0.87396	0.87932	0.88425	0.88880	0.89300	0.89690	0.90052	0.90389	0.90705	0.91000	0.92016
8	8	1.07484	1.07132	1.06812	1.06519	1.06250	1.06003	1.05774	1.05562	1.05365	1.05182	1.04559
9	1	0.08538	0.09548	0.10575	0.11612	0.12657	0.13704	0.14751	0.15795	0.16832	0.17862	0.21875
9	2	0.22073	0.23642	0.25172	0.26661	0.28108	0.29515	0.30879	0.32203	0.33486	0.34729	0.39331
9	3	0.33856	0.35565	0.37200	0.38765	0.40262	0.41696	0.43069	0.44384	0.45645	0.46854	0.51225
9	4	0.44755	0.46420	0.47996	0.49486	0.50898	0.52238	0.53509	0.54718	0.55867	0.56962	0.60861
9	5	0.55418	0.56925	0.58336	0.59661	0.60907	0.62079	0.63185	0.64230	0.65218	0.66154	0.69449
9	6	0.66387	0.67635	0.68794	0.69875	0.70884	0.71829	0.72714	0.73546	0.74330	0.75068	0.77644
9	7	0.78313	0.79138	0.79996	0.80743	0.81437	0.82083	0.82685	0.83249	0.83777	0.84272	0.85986
9	8	0.92344	0.92679	0.92986	0.93268	0.93529	0.93770	0.93993	0.94201	0.94396	0.94577	0.95200
9	9	1.11712	1.11151	1.10641	1.10176	1.09750	1.09358	1.08996	1.08661	1.08350	1.08061	1.07080
10	1	0.08116	0.09098	0.10097	0.11110	0.12130	0.13155	0.14182	0.15206	0.16227	0.17241	0.21202
10	2	0.20930	0.22472	0.23979	0.25449	0.26881	0.28274	0.29629	0.30944	0.32221	0.33461	0.38061
10	3	0.31989	0.33690	0.35322	0.36888	0.38389	0.39829	0.41210	0.42536	0.43808	0.45030	0.49462
10	4	0.42100	0.43789	0.45389	0.46908	0.48350	0.49721	0.51024	0.52265	0.53448	0.54575	0.58606
10	5	0.51835	0.53406	0.54883	0.56272	0.57581	0.58817	0.59984	0.61089	0.62136	0.63130	0.66640
10	6	0.61619	0.62990	0.64269	0.65463	0.66582	0.67632	0.68619	0.69548	0.70424	0.71252	0.74151
10	7	0.71877	0.72964	0.73971	0.74906	0.75776	0.76589	0.77350	0.78062	0.78732	0.79362	0.81550
10	8	0.83190	0.83889	0.84532	0.85126	0.85676	0.86187	0.86663	0.87107	0.87523	0.87912	0.89255
10	9	0.96652	0.96802	0.96939	0.97064	0.97180	0.97287	0.97386	0.97479	0.97564	0.97645	0.97919
10	10	1.15421	1.14671	1.13991	1.13370	1.12803	1.12281	1.11801	1.11356	1.10944	1.10560	1.09262

10% LIMITS

-βw-

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
1	1	0.34246	0.35955	0.37590	0.39155	0.40651	0.42083	0.43454	0.44767	0.46025	0.47231	0.51588
2	1	0.24613	0.26238	0.27810	0.29333	0.30808	0.32235	0.33615	0.34950	0.36240	0.37487	0.42074
2	2	0.63091	0.64426	0.65569	0.66830	0.67916	0.68934	0.69891	0.70791	0.71639	0.72440	0.75240
3	1	0.20296	0.21822	0.23315	0.24773	0.26195	0.27580	0.28928	0.30238	0.31511	0.32748	0.37344
3	2	0.48405	0.50028	0.51558	0.53001	0.54364	0.55653	0.56874	0.58032	0.59131	0.60176	0.63881
3	3	0.79881	0.80700	0.81456	0.82155	0.82804	0.83407	0.83969	0.84495	0.84987	0.85449	0.87045
4	1	0.17698	0.19147	0.20574	0.21975	0.23348	0.24691	0.26004	0.27286	0.28535	0.29754	0.34314
4	2	0.41005	0.42701	0.44311	0.45839	0.47292	0.48674	0.49989	0.51243	0.52438	0.53578	0.57660
4	3	0.63580	0.64902	0.66134	0.67283	0.68358	0.69365	0.70311	0.71202	0.72040	0.72832	0.75600
4	4	0.91316	0.91694	0.92040	0.92359	0.92653	0.92925	0.93178	0.93413	0.93633	0.93839	0.94543
5	1	0.15913	0.17300	0.18671	0.20024	0.21354	0.22661	0.23941	0.25196	0.26422	0.27621	0.32135
5	2	0.36297	0.38008	0.39641	0.41199	0.42686	0.44107	0.45465	0.46763	0.48005	0.49193	0.53475
5	3	0.54840	0.56358	0.57781	0.59117	0.60373	0.61556	0.62672	0.63727	0.64725	0.65670	0.69001
5	4	0.74442	0.75447	0.76377	0.77240	0.78042	0.78789	0.79488	0.80143	0.80757	0.81334	0.83335
5	5	0.99850	0.99856	0.99863	0.99868	0.99874	0.99878	0.99883	0.99887	0.99891	0.99895	0.99907
6	1	0.14590	0.15924	0.17248	0.18559	0.19852	0.21126	0.22378	0.23607	0.24812	0.25992	0.30457
6	2	0.32951	0.34656	0.36290	0.37856	0.39356	0.40793	0.42170	0.43491	0.44758	0.45973	0.50375
6	3	0.49074	0.50688	0.52208	0.53641	0.54994	0.56274	0.57485	0.58633	0.59722	0.60757	0.64425
6	4	0.65044	0.66328	0.67522	0.68636	0.69677	0.70652	0.71567	0.72428	0.73238	0.74002	0.76670
6	5	0.82798	0.83511	0.84168	0.84775	0.85337	0.85859	0.86345	0.86799	0.87224	0.87622	0.88995
6	6	1.06593	1.06264	1.06003	1.05746	1.05509	1.05292	1.05091	1.04905	1.04732	1.04571	1.04022
7	1	0.13558	0.14847	0.16130	0.17404	0.18665	0.19910	0.21136	0.22343	0.23528	0.24690	0.29107
7	2	0.30411	0.32102	0.33727	0.35290	0.36791	0.38234	0.39620	0.40952	0.42232	0.43463	0.47940
7	3	0.44878	0.46542	0.48116	0.49605	0.51016	0.52354	0.53624	0.54831	0.55979	0.57072	0.60965
7	4	0.58689	0.60128	0.61473	0.62732	0.63913	0.65023	0.66068	0.67053	0.67983	0.68864	0.71953
7	5	0.73055	0.74105	0.75077	0.75979	0.76819	0.77602	0.78334	0.79020	0.79665	0.80271	0.82373
7	6	0.89534	0.89985	0.90399	0.90780	0.91132	0.91458	0.91761	0.92043	0.92307	0.92553	0.93400
7	7	1.12132	1.11550	1.11021	1.10539	1.10096	1.09690	1.09315	1.08968	1.08645	1.08346	1.07329
8	1	0.12722	0.13972	0.15221	0.16462	0.17694	0.18913	0.20116	0.21302	0.22469	0.23615	0.27986
8	2	0.28396	0.30068	0.31681	0.33235	0.34732	0.36174	0.37562	0.38899	0.40187	0.41427	0.45952
8	3	0.41638	0.43329	0.44934	0.46457	0.47904	0.49279	0.50588	0.51834	0.53022	0.54155	0.58207
8	4	0.53983	0.55517	0.56956	0.58307	0.59579	0.60778	0.61909	0.62978	0.63991	0.64950	0.68333
8	5	0.66351	0.67599	0.68760	0.69842	0.70852	0.71797	0.72683	0.73516	0.74300	0.75040	0.77618
8	6	0.79611	0.80440	0.81205	0.81912	0.82569	0.83179	0.83749	0.84281	0.84779	0.85247	0.86863
8	7	0.95147	0.95362	0.95559	0.95740	0.95907	0.96061	0.96204	0.96338	0.96462	0.96577	0.96974
8	8	1.16812	1.15990	1.15244	1.14565	1.13943	1.13373	1.12847	1.12361	1.11910	1.11491	1.10074
9	1	0.12028	0.13244	0.14461	0.15674	0.16880	0.18076	0.19258	0.20425	0.21575	0.22706	0.27033
9	2	0.26745	0.28398	0.29995	0.31539	0.33029	0.34466	0.35853	0.37191	0.38481	0.39726	0.44284
9	3	0.39034	0.40739	0.42361	0.43905	0.45374	0.46775	0.48110	0.49383	0.50600	0.51762	0.55931
9	4	0.50302	0.51898	0.53399	0.54813	0.56147	0.57408	0.58600	0.59729	0.60800	0.61817	0.65416
9	5	0.61319	0.62698	0.63983	0.65185	0.66310	0.67367	0.68359	0.69294	0.70176	0.71010	0.73928
9	6	0.72690	0.73751	0.74734	0.75647	0.76496	0.77288	0.78029	0.78724	0.79376	0.79989	0.82118
9	7	0.85133	0.85758	0.86333	0.86863	0.87354	0.87809	0.88233	0.88628	0.88998	0.89344	0.90537
9	8	0.99938	0.99940	0.99943	0.99945	0.99946	0.99950	0.99951	0.99953	0.99955	0.99956	0.99961
9	9	1.20850	1.19814	1.18876	1.18023	1.17243	1.16528	1.15869	1.15262	1.14698	1.14175	1.12408
10	1	0.11440	0.12625	0.13613	0.15001	0.16183	0.17358	0.18521	0.19670	0.20805	0.21923	0.26208
10	2	0.25361	0.26993	0.28574	0.30105	0.31586	0.33018	0.34401	0.35737	0.37028	0.38275	0.42853
10	3	0.36880	0.38591	0.40222	0.41777	0.43262	0.44679	0.46032	0.47325	0.48562	0.49745	0.54004
10	4	0.47314	0.48951	0.50495	0.51953	0.53332	0.54637	0.55874	0.57048	0.58163	0.59223	0.62988
10	5	0.57344	0.58812	0.60185	0.61472	0.62680	0.63816	0.64887	0.65897	0.66851	0.67755	0.70930
10	6	0.67443	0.68662	0.69793	0.70847	0.71830	0.72750	0.73613	0.74422	0.75184	0.75903	0.78405
10	7	0.78077	0.78960	0.79775	0.80530	0.81230	0.81882	0.82491	0.83060	0.83593	0.84094	0.85825
10	8	0.89886	0.90323	0.90724	0.91092	0.91433	0.91749	0.92042	0.92315	0.92570	0.92808	0.93627
10	9	1.04104	1.03914	1.03740	1.03582	1.03436	1.03302	1.03177	1.03062	1.02955	1.02855	1.02515
10	10	1.24392	1.23164	1.22053	1.21044	1.20123	1.19279	1.18502	1.17786	1.17123	1.16508	1.14432

20% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-βw-						
1	1	0.48956	0.50571	0.52093	0.53527	0.54882	0.56164	0.57377	0.58526	0.59618	0.60654	0.64329
2	1	0.35193	0.36904	0.38538	0.40100	0.41593	0.43020	0.44386	0.45692	0.46943	0.48141	0.52465
2	2	0.77957	0.78844	0.79663	0.80422	0.81126	0.81781	0.82392	0.82964	0.83500	0.84004	0.85744
3	1	0.29014	0.30692	0.32310	0.33867	0.35366	0.36808	0.38196	0.39532	0.40818	0.42055	0.46567
3	2	0.59698	0.61115	0.62437	0.63675	0.64835	0.65924	0.66950	0.67916	0.68828	0.69691	0.72715
3	3	0.94044	0.94306	0.94547	0.94768	0.94972	0.95161	0.95336	0.95499	0.95650	0.95792	0.96278
4	1	0.25299	0.26930	0.28511	0.30041	0.31522	0.32953	0.34336	0.35672	0.36963	0.38210	0.42789
4	2	0.50548	0.52140	0.53638	0.55048	0.56378	0.57635	0.58823	0.59948	0.61016	0.62029	0.65614
4	3	0.74604	0.75604	0.76529	0.77387	0.78184	0.78928	0.79623	0.80273	0.80884	0.81458	0.83447
4	4	1.04863	1.04637	1.04431	1.04242	1.04069	1.03910	1.03762	1.03625	1.03498	1.03380	1.02976
5	1	0.22749	0.24333	0.25875	0.27374	0.28830	0.30243	0.31612	0.32940	0.34225	0.35471	0.40071
5	2	0.44735	0.46401	0.47976	0.49467	0.50880	0.52219	0.53491	0.54700	0.55850	0.56945	0.60845
5	3	0.64286	0.65591	0.66804	0.67937	0.68995	0.69987	0.70918	0.71794	0.72619	0.73398	0.76118
5	4	0.85120	0.85745	0.86321	0.86851	0.87342	0.87798	0.88222	0.88618	0.88988	0.89335	0.90528
5	5	1.12899	1.12278	1.11714	1.11200	1.10729	1.10296	1.09896	1.09526	1.09183	1.08864	1.07782
6	1	0.20857	0.22397	0.23903	0.25372	0.26802	0.28195	0.29548	0.30863	0.32140	0.33379	0.37979
6	2	0.40607	0.42305	0.43918	0.45449	0.46906	0.48292	0.49611	0.50869	0.52068	0.53214	0.57313
6	3	0.57503	0.58968	0.60333	0.61621	0.62826	0.63959	0.65027	0.66034	0.66986	0.67887	0.71052
6	4	0.74272	0.75233	0.76218	0.77086	0.77892	0.78644	0.79347	0.80005	0.80623	0.81204	0.83218
6	5	0.93157	0.93457	0.93733	0.93986	0.94219	0.94435	0.94636	0.94822	0.94996	0.95159	0.95716
6	6	1.19238	1.18288	1.17427	1.16644	1.15928	1.15271	1.14665	1.14106	1.13588	1.13107	1.11480
7	1	0.19381	0.20882	0.22353	0.23793	0.25200	0.26572	0.27908	0.29210	0.30476	0.31708	0.36295
7	2	0.37475	0.39185	0.40814	0.42367	0.43847	0.45260	0.46609	0.47897	0.49128	0.50306	0.54541
7	3	0.52574	0.54133	0.55597	0.56974	0.58271	0.59494	0.60649	0.61742	0.62777	0.63759	0.67226
7	4	0.66974	0.68206	0.69350	0.70415	0.71410	0.72341	0.73214	0.74034	0.74805	0.75533	0.78068
7	5	0.82059	0.82800	0.83482	0.84112	0.84696	0.85239	0.85745	0.86217	0.86659	0.87074	0.88503
7	6	0.99615	0.99632	0.99648	0.99663	0.99676	0.99689	0.99700	0.99711	0.99721	0.99730	0.99762
7	7	1.24441	1.23210	1.22097	1.21086	1.20162	1.19317	1.18539	1.17821	1.17156	1.16540	1.14460
8	1	0.18187	0.19652	0.21092	0.22506	0.23889	0.25241	0.26562	0.27850	0.29105	0.30327	0.34897
8	2	0.34991	0.36701	0.38336	0.39899	0.41392	0.42821	0.44187	0.45495	0.46748	0.47948	0.52279
8	3	0.48772	0.50390	0.51914	0.53352	0.54710	0.55994	0.57209	0.58362	0.59456	0.60495	0.64180
8	4	0.61583	0.62955	0.64234	0.65430	0.66549	0.67600	0.68587	0.69517	0.70394	0.71223	0.74124
8	5	0.74468	0.75473	0.76402	0.77264	0.78065	0.78812	0.79510	0.80164	0.80777	0.81354	0.83353
8	6	0.88405	0.88902	0.89358	0.89778	0.90166	0.90526	0.90860	0.91171	0.91462	0.91735	0.92671
8	7	1.04985	1.04753	1.04542	1.04348	1.04171	1.04007	1.03856	1.03716	1.03585	1.03464	1.03050
8	8	1.28836	1.27361	1.26028	1.24819	1.23717	1.22709	1.21782	1.20928	1.20138	1.19406	1.16940
9	1	0.17195	0.18628	0.20040	0.21428	0.22790	0.24123	0.25428	0.26702	0.27946	0.29160	0.33709
9	2	0.32956	0.34662	0.36296	0.37861	0.39361	0.40798	0.42176	0.43497	0.44763	0.45979	0.50380
9	3	0.45718	0.47374	0.48938	0.50417	0.51817	0.53144	0.54403	0.55599	0.56736	0.57818	0.61667
9	4	0.57371	0.58839	0.60211	0.61497	0.62705	0.63841	0.64911	0.65920	0.66875	0.67777	0.70951
9	5	0.68791	0.69971	0.71065	0.72084	0.73034	0.73922	0.74754	0.75535	0.76269	0.76961	0.79369
9	6	0.80644	0.81437	0.82167	0.82842	0.83468	0.84051	0.84593	0.85100	0.85575	0.86020	0.87558
9	7	0.93739	0.94015	0.94267	0.94500	0.94714	0.94912	0.95096	0.95266	0.95426	0.95575	0.96085
9	8	1.09564	1.09110	1.08697	1.08320	1.07974	1.07656	1.07362	1.07090	1.06838	1.06602	1.05803
9	9	1.32629	1.30937	1.29412	1.28029	1.26770	1.25619	1.24562	1.23589	1.22689	1.21856	1.19055
10	1	0.16354	0.17756	0.19142	0.20507	0.21849	0.23165	0.24455	0.25716	0.26949	0.28153	0.32681
10	2	0.31250	0.32946	0.34576	0.36140	0.37642	0.39083	0.40467	0.41796	0.43072	0.44299	0.48752
10	3	0.43193	0.44873	0.46464	0.47972	0.49402	0.50761	0.52051	0.53280	0.54449	0.55564	0.59541
10	4	0.53956	0.55490	0.56930	0.58282	0.59554	0.60753	0.61885	0.62955	0.63967	0.64927	0.68311
10	5	0.64313	0.65616	0.66829	0.67961	0.69019	0.70011	0.70941	0.71816	0.72641	0.73419	0.76137
10	6	0.74783	0.75773	0.76697	0.77549	0.78342	0.79081	0.79771	0.80418	0.81025	0.81595	0.83571
10	7	0.85379	0.86475	0.87023	0.87529	0.87996	0.88430	0.88834	0.89210	0.89562	0.89892	0.91026
10	8	0.98324	0.98399	0.98468	0.98532	0.98590	0.98644	0.98694	0.98740	0.98783	0.98824	0.98961
10	9	1.13543	1.12889	1.12296	1.11755	1.11259	1.10803	1.10383	1.09994	1.09634	1.09298	1.08161
10	10	1.35957	1.34072	1.32374	1.30836	1.29437	1.28159	1.26986	1.25907	1.24911	1.23988	1.20891

30% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-β_w-						
1	1	0.61206	0.62587	0.63876	0.65080	0.66208	0.67266	0.68261	0.69199	0.70083	0.70918	0.73844
2	1	0.44000	0.45673	0.47256	0.48755	0.50176	0.51525	0.52806	0.54024	0.55183	0.56288	0.60225
2	2	0.89568	0.90018	0.90431	0.90810	0.91161	0.91486	0.91788	0.92070	0.92332	0.92578	0.93422
3	1	0.36274	0.37985	0.39618	0.41176	0.42664	0.44085	0.45443	0.46741	0.47983	0.49172	0.53455
3	2	0.68470	0.69659	0.70763	0.71790	0.72748	0.73644	0.74483	0.75271	0.76012	0.76710	0.79140
3	3	1.04957	1.04727	1.04517	1.04324	1.04148	1.03985	1.03835	1.03695	1.03566	1.03445	1.03033
4	1	0.31630	0.33329	0.34960	0.36525	0.38026	0.39467	0.40850	0.42177	0.43451	0.44676	0.49118
4	2	0.57949	0.59404	0.60765	0.62039	0.63235	0.64360	0.65419	0.66418	0.67362	0.68255	0.71392
4	3	0.83017	0.83723	0.84372	0.84971	0.85527	0.86042	0.86523	0.86971	0.87391	0.87784	0.89140
4	4	1.15256	1.14515	1.13842	1.13229	1.12667	1.12152	1.11676	1.11237	1.10829	1.10450	1.09166
5	1	0.28442	0.30114	0.31728	0.33282	0.34779	0.36221	0.37609	0.38946	0.40233	0.41473	0.45998
5	2	0.51276	0.52857	0.54342	0.55741	0.57060	0.58304	0.59481	0.60595	0.61651	0.62653	0.66196
5	3	0.71475	0.72575	0.73593	0.74539	0.75421	0.76243	0.77013	0.77735	0.78413	0.79051	0.81268
5	4	0.93210	0.93509	0.93782	0.94033	0.94265	0.94479	0.94678	0.94863	0.95036	0.95197	0.95750
5	5	1.22896	1.21749	1.20712	1.19769	1.18908	1.18118	1.17392	1.16722	1.16101	1.15525	1.13580
6	1	0.26076	0.27719	0.29310	0.30847	0.32333	0.33768	0.35154	0.36491	0.37782	0.39028	0.43596
6	2	0.46540	0.48187	0.49741	0.51210	0.52599	0.53915	0.55163	0.56347	0.57473	0.58544	0.62350
6	3	0.63910	0.65224	0.66447	0.67588	0.68656	0.69656	0.70595	0.71478	0.72311	0.73097	0.75842
6	4	0.81237	0.82008	0.82718	0.83375	0.83983	0.84549	0.85076	0.85569	0.86030	0.86463	0.87955
6	5	1.00978	1.00934	1.00893	1.00856	1.00821	1.00790	1.00760	1.00733	1.00708	1.00684	1.00603
6	6	1.28920	1.27440	1.26103	1.24890	1.23785	1.22773	1.21844	1.20987	1.20195	1.19460	1.16987
7	1	0.24231	0.25844	0.27410	0.28928	0.30400	0.31824	0.33203	0.34536	0.35826	0.37073	0.41664
7	2	0.42949	0.44631	0.46224	0.47734	0.49167	0.50528	0.51822	0.53053	0.54226	0.55343	0.59332
7	3	0.58420	0.59865	0.61216	0.62480	0.63667	0.64782	0.65832	0.66823	0.67758	0.68643	0.71750
7	4	0.73216	0.74261	0.75228	0.76125	0.76960	0.77739	0.78468	0.79150	0.79791	0.80394	0.82485
7	5	0.88824	0.89304	0.89744	0.90150	0.90524	0.90872	0.91194	0.91495	0.91776	0.92039	0.92942
7	6	1.07212	1.06873	1.06564	1.06282	1.06024	1.05785	1.05565	1.05361	1.05172	1.04995	1.04395
7	7	1.33866	1.32103	1.30514	1.29073	1.27762	1.26564	1.25465	1.24452	1.23517	1.22650	1.19740
8	1	0.22738	0.24322	0.25864	0.27363	0.28819	0.30231	0.31601	0.32923	0.34214	0.35459	0.40059
8	2	0.40100	0.41801	0.43416	0.44952	0.46413	0.47804	0.49129	0.50392	0.51597	0.52747	0.56870
8	3	0.54189	0.55719	0.57154	0.58502	0.59770	0.60965	0.62093	0.63159	0.64168	0.65124	0.68494
8	4	0.67302	0.68524	0.69660	0.70717	0.71704	0.72627	0.73493	0.74306	0.75070	0.75791	0.78304
8	5	0.80554	0.81350	0.82083	0.82761	0.83390	0.83975	0.84520	0.85029	0.85506	0.85953	0.87498
8	6	0.94997	0.95219	0.95422	0.95608	0.95780	0.95939	0.96086	0.96223	0.96351	0.96471	0.96879
8	7	1.12391	1.11796	1.11255	1.10762	1.10310	1.09895	1.09511	1.09156	1.08827	1.08521	1.07482
8	8	1.38046	1.36038	1.34230	1.32593	1.31105	1.29747	1.28501	1.27356	1.26298	1.25319	1.22035
9	1	0.21498	0.23054	0.24572	0.26052	0.27492	0.28892	0.30252	0.31572	0.32852	0.34094	0.38695
9	2	0.37768	0.39477	0.41105	0.42656	0.44135	0.45545	0.46892	0.48177	0.49406	0.50581	0.54804
9	3	0.50792	0.52331	0.53874	0.55281	0.56607	0.57860	0.59044	0.60166	0.61229	0.62238	0.65809
9	4	0.62688	0.64033	0.65287	0.66457	0.67552	0.68579	0.69544	0.70452	0.71308	0.72116	0.74943
9	5	0.74385	0.75392	0.76324	0.77188	0.77992	0.78741	0.79441	0.80097	0.80712	0.81290	0.83296
9	6	0.86590	0.87159	0.87681	0.88163	0.88608	0.89021	0.89405	0.89764	0.90099	0.90412	0.91491
9	7	1.00179	1.00170	1.00163	1.00156	1.00150	1.00144	1.00139	1.00134	1.00129	1.00125	1.00110
9	8	1.16806	1.15984	1.15239	1.14560	1.13939	1.13368	1.12843	1.12357	1.11906	1.11488	1.10070
9	9	1.41656	1.39431	1.37430	1.35622	1.33979	1.32480	1.31107	1.29845	1.28681	1.27604	1.23996
10	1	0.20446	0.21976	0.23472	0.24933	0.26358	0.27745	0.29094	0.30406	0.31680	0.32917	0.37514
10	2	0.35811	0.37523	0.39156	0.40716	0.42206	0.43630	0.44991	0.46293	0.47539	0.48732	0.53033
10	3	0.47984	0.49613	0.51148	0.52597	0.53966	0.55262	0.56489	0.57653	0.58759	0.59809	0.63538
10	4	0.58950	0.60383	0.61722	0.62975	0.64151	0.65256	0.66296	0.67276	0.68202	0.69077	0.72150
10	5	0.69527	0.70635	0.71759	0.72758	0.73690	0.74560	0.75375	0.76140	0.76859	0.77536	0.79892
10	6	0.80261	0.81067	0.81810	0.82498	0.83135	0.83728	0.84280	0.84797	0.85280	0.85734	0.87301
10	7	0.91700	0.92061	0.92393	0.92698	0.92980	0.93240	0.93482	0.93708	0.93918	0.94115	0.94789
10	8	1.04630	1.04415	1.04219	1.04040	1.03875	1.03723	1.03583	1.03453	1.03332	1.03219	1.02835
10	9	1.20643	1.19618	1.18690	1.17846	1.17074	1.16367	1.15715	1.15114	1.14556	1.14038	1.12290
10	10	1.44825	1.42407	1.40235	1.38273	1.36492	1.34869	1.33333	1.32018	1.30759	1.29595	1.25702

40% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-βw-						
1	1	0.72624	0.73688	0.74673	0.75587	0.76438	0.77232	0.77975	0.78670	0.79324	0.79939	0.82073
2	1	0.52208	0.53773	0.55243	0.56626	0.57929	0.59158	0.60320	0.61419	0.62460	0.63447	0.66936
2	2	1.00043	1.00041	1.00040	1.00038	1.00036	1.00035	1.00034	1.00033	1.00031	1.00030	1.00027
3	1	0.43041	0.44722	0.46315	0.47824	0.49256	0.50616	0.51909	0.53139	0.54310	0.55426	0.59411
3	2	0.76341	0.77284	0.78154	0.78961	0.79711	0.80409	0.81061	0.81671	0.82243	0.82781	0.84642
3	3	1.14745	1.14029	1.13380	1.12789	1.12247	1.11749	1.11291	1.10866	1.10473	1.10106	1.08866
4	1	0.37531	0.39240	0.40869	0.42422	0.43902	0.45314	0.46662	0.47950	0.49181	0.50358	0.54591
4	2	0.64582	0.65878	0.67084	0.68210	0.69261	0.70247	0.71172	0.72041	0.72861	0.73634	0.76333
4	3	0.90495	0.90907	0.91255	0.91632	0.91953	0.92250	0.92526	0.92783	0.93023	0.93248	0.94018
4	4	1.24564	1.23326	1.22207	1.21190	1.20262	1.19412	1.18630	1.17908	1.17240	1.16620	1.14530
5	1	0.33747	0.35455	0.37090	0.38655	0.40153	0.41587	0.42961	0.44277	0.45539	0.46748	0.51123
5	2	0.57134	0.58606	0.59984	0.61275	0.62487	0.63628	0.64702	0.65716	0.66674	0.67581	0.70770
5	3	0.77848	0.78739	0.79562	0.80324	0.81031	0.81689	0.82303	0.82878	0.33416	0.83922	0.85670
5	4	1.00375	1.00358	1.00343	1.00328	1.00315	1.00303	1.00292	1.00281	1.00272	1.00263	1.00232
5	5	1.31847	1.30200	1.28715	1.27368	1.26142	1.25020	1.23990	1.23042	1.22165	1.21353	1.18621
6	1	0.30941	0.32636	0.34264	0.35828	0.37329	0.38771	0.40156	0.41486	0.42764	0.43992	0.48454
6	2	0.51853	0.53424	0.54900	0.56289	0.57598	0.58833	0.60001	0.61105	0.62152	0.63145	0.66654
6	3	0.69583	0.70740	0.71812	0.72810	0.73740	0.74609	0.75423	0.76186	0.76904	0.77580	0.79932
6	4	0.87383	0.37921	0.88414	0.88869	0.89289	0.89679	0.90042	0.90380	0.90696	0.90991	0.92008
6	5	1.07893	1.07521	1.07183	1.06874	1.06590	1.06329	1.06087	1.05863	1.05656	1.05462	1.04804
6	6	1.37593	1.35611	1.33827	1.32212	1.30744	1.29402	1.28173	1.27042	1.25998	1.25031	1.21788
7	1	0.28751	0.30427	0.32043	0.33599	0.35097	0.36539	0.37927	0.39264	0.40550	0.41789	0.46306
7	2	0.47848	0.49479	0.51016	0.52467	0.53838	0.55135	0.56365	0.57531	0.58638	0.59691	0.63426
7	3	0.63594	0.64916	0.66147	0.67296	0.68371	0.69378	0.70324	0.71214	0.72052	0.72844	0.75610
7	4	0.78715	0.79576	0.80370	0.81105	0.81787	0.82422	0.83015	0.83568	0.84087	0.84574	0.86258
7	5	0.94780	0.95011	0.95223	0.95417	0.95596	0.95762	0.95916	0.96059	0.96192	0.96316	0.96743
7	6	1.13922	1.13249	1.12638	1.12081	1.11571	1.11102	1.10670	1.10270	1.09899	1.09553	1.08384
7	7	1.42314	1.40050	1.38013	1.36173	1.34502	1.32977	1.31581	1.30297	1.29114	1.28019	1.24352
8	1	0.26980	0.28635	0.30235	0.31780	0.33272	0.34710	0.36097	0.37435	0.38725	0.39969	0.44523
8	2	0.44673	0.46340	0.47916	0.49407	0.50821	0.52161	0.53434	0.54643	0.55794	0.56890	0.60793
8	3	0.58982	0.60414	0.61752	0.63005	0.64180	0.65284	0.66324	0.67303	0.68229	0.69104	0.72174
8	4	0.72337	0.73409	0.74403	0.75325	0.76184	0.76985	0.77734	0.78437	0.79096	0.79717	0.81872
8	5	0.85901	0.86496	0.87044	0.87548	0.88015	0.88449	0.88852	0.89227	0.89579	0.89908	0.91040
8	6	1.00791	1.00755	1.00722	1.00692	1.00664	1.00639	1.00615	1.00593	1.00572	1.00553	1.00488
8	7	1.18931	1.17998	1.17152	1.16382	1.15678	1.15031	1.14436	1.13887	1.13377	1.12904	1.11303
8	8	1.46307	1.43798	1.41544	1.39510	1.37664	1.35982	1.34443	1.33029	1.31727	1.30522	1.26495
9	1	0.25508	0.27143	0.28726	0.30258	0.31740	0.33173	0.34556	0.35893	0.37184	0.38431	0.43007
9	2	0.42074	0.43763	0.45364	0.46883	0.48325	0.49696	0.51000	0.52241	0.53424	0.54552	0.58583
9	3	0.55280	0.56790	0.58204	0.59532	0.60780	0.61955	0.63063	0.64110	0.65101	0.66039	0.69343
9	4	0.67367	0.68537	0.69721	0.70776	0.71762	0.72683	0.73547	0.74359	0.75122	0.75842	0.78350
9	5	0.79294	0.80135	0.80910	0.81627	0.82293	0.82912	0.83489	0.84029	0.34535	0.85010	0.86650
9	6	0.91805	0.92162	0.92490	0.92791	0.93069	0.93327	0.93566	0.93788	0.93996	0.94190	0.94856
9	7	1.05335	1.05563	1.05315	1.05088	1.04879	1.04687	1.04510	1.04345	1.04192	1.04050	1.03565
9	8	1.23201	1.22038	1.20986	1.20030	1.19156	1.18356	1.17619	1.16939	1.16310	1.15726	1.13754
9	9	1.49756	1.47032	1.44569	1.42384	1.40386	1.38566	1.36902	1.35375	1.33969	1.32669	1.28329
10	1	0.24260	0.25873	0.27440	0.28959	0.30430	0.31855	0.33234	0.34568	0.35857	0.37104	0.41695
10	2	0.39894	0.41596	0.43213	0.44750	0.46213	0.47605	0.48932	0.50197	0.51405	0.52558	0.56689
10	3	0.52222	0.53787	0.55257	0.56640	0.57942	0.59171	0.60332	0.61431	0.62472	0.63459	0.66947
10	4	0.63342	0.64670	0.65907	0.67062	0.68143	0.69155	0.70106	0.71001	0.71845	0.72641	0.75425
10	5	0.74099	0.75115	0.76056	0.76928	0.77739	0.78496	0.79203	0.79865	0.30487	0.81071	0.83098
10	6	0.85059	0.85687	0.86264	0.86797	0.87290	0.87748	0.88173	0.88571	0.38942	0.89290	0.90488
10	7	0.96799	0.96942	0.97073	0.97193	0.97304	0.97406	0.97501	0.97589	0.97672	0.97748	0.98011
10	8	1.10167	1.09663	1.09243	1.08842	1.08474	1.08135	1.07822	1.07532	1.07263	1.07013	1.06163
10	9	1.26912	1.25545	1.24309	1.23187	1.22164	1.21227	1.20366	1.19571	1.18837	1.18155	1.15859
10	10	1.52787	1.49872	1.47258	1.44903	1.42769	1.40827	1.39053	1.37425	1.35927	1.34543	1.29927

60% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
1	1	0.95923	0.96104	0.96270	0.96423	0.96564	0.96694	0.96814	0.96926	0.97030	0.97128	0.97462
2	1	0.68956	0.70131	0.71221	0.72236	0.73182	0.74066	0.74894	0.75671	0.76402	0.77091	0.79487
2	2	1.20907	1.19868	1.18927	1.18071	1.17289	1.16572	1.15912	1.15302	1.14738	1.14213	1.12441
3	1	0.56349	0.58327	0.59710	0.61007	0.62225	0.63371	0.64451	0.65470	0.66433	0.67345	0.70551
3	2	0.91885	0.92239	0.92564	0.92863	0.93138	0.93393	0.93630	0.93850	0.94056	0.94248	0.94907
3	3	1.34184	1.32403	1.30797	1.29342	1.28017	1.26807	1.25697	1.24674	1.23730	1.22854	1.19915
4	1	0.49571	0.51177	0.52690	0.54116	0.55461	0.56733	0.57937	0.59077	0.60159	0.61187	0.64827
4	2	0.77650	0.78548	0.79377	0.80144	0.80857	0.81520	0.82140	0.82719	0.83262	0.83772	0.85535
4	3	1.05149	1.04909	1.04691	1.04491	1.04308	1.04138	1.03982	1.03837	1.03703	1.03577	1.03150
4	4	1.43059	1.40749	1.38673	1.36797	1.35093	1.33539	1.32116	1.30808	1.29603	1.28487	1.24754
5	1	0.44574	0.46241	0.47818	0.49311	0.50725	0.52067	0.53341	0.54552	0.55704	0.56801	0.60709
5	2	0.68666	0.69849	0.70948	0.71970	0.72923	0.73814	0.74649	0.75432	0.76169	0.76863	0.79280
5	3	0.90286	0.90706	0.91092	0.91447	0.91774	0.92078	0.92360	0.92622	0.92867	0.93097	0.93883
5	4	1.14376	1.13680	1.13048	1.12472	1.11944	1.11460	1.11013	1.10599	1.10216	1.09859	1.08650
5	5	1.49655	1.46938	1.44500	1.42300	1.40306	1.38491	1.36830	1.35307	1.33903	1.32606	1.28276
6	1	0.40867	0.42563	0.44174	0.45704	0.47158	0.48541	0.49858	0.51113	0.52309	0.53452	0.57539
6	2	0.62305	0.63660	0.64922	0.66101	0.67205	0.68240	0.69213	0.70129	0.70992	0.71807	0.74660
6	3	0.80635	0.81428	0.82159	0.82834	0.83460	0.84043	0.84586	0.85093	0.85568	0.86014	0.87552
6	4	0.99329	0.99360	0.99387	0.99413	0.99436	0.99458	0.99478	0.99496	0.99514	0.99530	0.99585
6	5	1.21392	1.20327	1.19363	1.18486	1.17685	1.16950	1.16274	1.15649	1.15071	1.14534	1.12720
6	6	1.54871	1.51822	1.49091	1.46630	1.44403	1.42376	1.40525	1.38828	1.37267	1.35825	1.31019
7	1	0.37975	0.39683	0.41310	0.42860	0.44338	0.45747	0.47091	0.48375	0.49602	0.50774	0.54989
7	2	0.57487	0.58952	0.60322	0.61606	0.62811	0.63945	0.65013	0.66020	0.66972	0.67873	0.71039
7	3	0.73664	0.74695	0.75648	0.76533	0.77356	0.78124	0.78841	0.79514	0.80145	0.80738	0.82796
7	4	0.89376	0.89834	0.90253	0.90640	0.90997	0.91328	0.91635	0.91921	0.92189	0.92439	0.93298
7	5	1.06332	1.06035	1.05766	1.05519	1.05292	1.05084	1.04891	1.04712	1.04546	1.04391	1.03865
7	6	1.27019	1.25646	1.24405	1.23278	1.22250	1.21309	1.20445	1.19647	1.18909	1.18225	1.15919
7	7	1.59166	1.55839	1.52861	1.50182	1.47759	1.45557	1.43547	1.41706	1.40013	1.38451	1.33251
8	1	0.35635	0.37346	0.38980	0.40541	0.42032	0.43457	0.44819	0.46122	0.47369	0.48564	0.52871
8	2	0.53668	0.55208	0.56653	0.58010	0.59287	0.60492	0.61628	0.62703	0.63720	0.64685	0.68086
8	3	0.68304	0.69498	0.70606	0.71637	0.72600	0.73499	0.74342	0.75133	0.75878	0.76579	0.79022
8	4	0.82065	0.82825	0.83506	0.84136	0.84719	0.85261	0.85766	0.86238	0.86679	0.87093	0.88521
8	5	0.96241	0.96408	0.96562	0.96703	0.96833	0.96952	0.97064	0.97167	0.97263	0.97353	0.97661
8	6	1.12019	1.11443	1.10919	1.10441	1.10003	1.09601	1.09229	1.08885	1.08566	1.08269	1.07262
8	7	1.31697	1.30059	1.28581	1.27241	1.26021	1.24905	1.23880	1.22936	1.22064	1.21256	1.18537
8	8	1.62805	1.59238	1.56049	1.53182	1.50592	1.48239	1.46093	1.44129	1.42323	1.40659	1.35125
9	1	0.33691	0.35399	0.37034	0.38599	0.40097	0.41532	0.42906	0.44222	0.45484	0.46694	0.51071
9	2	0.50543	0.52135	0.53633	0.55043	0.56374	0.57630	0.58819	0.59944	0.61011	0.62025	0.65610
9	3	0.64007	0.65319	0.66540	0.67679	0.68744	0.69742	0.70679	0.71560	0.72391	0.73175	0.75913
9	4	0.76417	0.77357	0.78225	0.79030	0.79777	0.80473	0.81124	0.81732	0.82302	0.82838	0.84694
9	5	0.88771	0.89253	0.89696	0.90103	0.90479	0.90828	0.91152	0.91454	0.91737	0.92001	0.92908
9	6	1.01874	1.01788	1.01709	1.01638	1.01572	1.01511	1.01454	1.01402	1.01353	1.01308	1.01153
9	7	1.16790	1.15969	1.15224	1.14546	1.13926	1.13356	1.12831	1.12345	1.11895	1.11477	1.10061
9	8	1.35686	1.33817	1.32133	1.30608	1.29220	1.27952	1.26789	1.25719	1.24731	1.23815	1.20742
9	9	1.65955	1.62177	1.58803	1.55773	1.53035	1.50551	1.48287	1.46215	1.44312	1.42558	1.36734
10	1	0.32043	0.33744	0.35376	0.36941	0.38443	0.39882	0.41264	0.42589	0.43861	0.45083	0.49513
10	2	0.47923	0.49552	0.51088	0.52538	0.53908	0.55204	0.56433	0.57598	0.58704	0.59755	0.63487
10	3	0.60459	0.61858	0.63163	0.64384	0.65528	0.66602	0.67612	0.68564	0.69462	0.70311	0.73286
10	4	0.71834	0.72922	0.73930	0.74867	0.75738	0.76552	0.77314	0.78027	0.78698	0.79329	0.81520
10	5	0.82916	0.83625	0.84278	0.84881	0.85439	0.85958	0.86441	0.86892	0.87314	0.87710	0.89073
10	6	0.94305	0.94557	0.94787	0.94999	0.95194	0.95374	0.95542	0.95697	0.95843	0.95978	0.96443
10	7	1.06636	1.06325	1.06042	1.05783	1.05546	1.05327	1.05124	1.04937	1.04763	1.04600	1.04048
10	8	1.20588	1.19850	1.18910	1.18055	1.17274	1.16557	1.15898	1.15289	1.14724	1.14200	1.12430
10	9	1.39156	1.37042	1.35215	1.33525	1.31990	1.30589	1.29304	1.28123	1.27033	1.26023	1.22640
10	10	1.68726	1.64762	1.61223	1.58047	1.55179	1.52579	1.50210	1.48043	1.46053	1.44221	1.38140

The column values 2.10–3.40 are values of $-\beta_W-$.

70% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-βw-						
1	1	1.09242	1.08804	1.08405	1.08041	1.07708	1.07401	1.07117	1.06854	1.06610	1.06383	1.05611
2	1	0.78531	0.79398	0.80199	0.80940	0.81627	0.82267	0.82864	0.83422	0.83945	0.84436	0.86134
2	2	1.32716	1.31020	1.29489	1.28103	1.26840	1.25685	1.24626	1.23650	1.22748	1.21912	1.19103
3	1	0.64742	0.66034	0.67237	0.68358	0.69406	0.70388	0.71309	0.72176	0.72992	0.73762	0.76451
3	2	1.00600	1.00573	1.00548	1.00525	1.00504	1.00484	1.00466	1.00450	1.00434	1.00420	1.00370
3	3	1.45199	1.42759	1.40566	1.38586	1.36789	1.35151	1.33651	1.32274	1.31004	1.29830	1.25903
4	1	0.56454	0.57940	0.59332	0.60636	0.61862	0.63015	0.64102	0.65128	0.66099	0.67017	0.70248
4	2	0.84957	0.85589	0.86170	0.86706	0.87202	0.87663	0.88091	0.88491	0.88865	0.89215	0.90421
4	3	1.13330	1.12687	1.12104	1.11571	1.11084	1.10635	1.10222	1.09840	1.09485	1.09154	1.08035
4	4	1.53562	1.50597	1.47940	1.45545	1.43377	1.41403	1.39601	1.37947	1.36425	1.35020	1.30334
5	1	0.50763	0.52352	0.53846	0.55253	0.56579	0.57832	0.59017	0.60139	0.61203	0.62213	0.65786
5	2	0.75108	0.76091	0.77001	0.77844	0.78627	0.79358	0.80040	0.80679	0.81279	0.81842	0.83795
5	3	0.97199	0.97324	0.97439	0.97544	0.97641	0.97731	0.97814	0.97891	0.97963	0.98031	0.98260
5	4	1.22185	1.21077	1.20075	1.19163	1.18330	1.17567	1.16864	1.16215	1.15615	1.15057	1.13174
5	5	1.59791	1.56422	1.53409	1.50698	1.48246	1.46018	1.43985	1.42123	1.40410	1.38831	1.33574
6	1	0.46542	0.48188	0.49742	0.51211	0.52600	0.53916	0.55164	0.56348	0.57474	0.58545	0.62351
6	2	0.68141	0.69340	0.70452	0.71488	0.72454	0.73358	0.74204	0.74999	0.75747	0.76452	0.78905
6	3	0.86766	0.87327	0.87843	0.88319	0.88759	0.89167	0.89546	0.89900	0.90231	0.90541	0.91605
6	4	1.05952	1.05674	1.05421	1.05189	1.04977	1.04781	1.04600	1.04432	1.04276	1.04130	1.03636
6	5	1.28921	1.27441	1.26105	1.24892	1.23786	1.22774	1.21845	1.20988	1.20196	1.19461	1.16988
6	6	1.64724	1.61029	1.57728	1.54761	1.52081	1.49649	1.47431	1.45401	1.43536	1.41818	1.36107
7	1	0.43248	0.44927	0.46518	0.48025	0.49455	0.50812	0.52103	0.53330	0.54499	0.55613	0.59587
7	2	0.62866	0.64207	0.65455	0.66622	0.67713	0.68736	0.69697	0.70601	0.71454	0.72259	0.75075
7	3	0.79244	0.80086	0.80863	0.81582	0.82249	0.82869	0.83448	0.83989	0.84496	0.84972	0.86615
7	4	0.95271	0.95481	0.95673	0.95850	0.96013	0.96163	0.96302	0.96432	0.96553	0.96666	0.97052
7	5	1.12730	1.12118	1.11561	1.11054	1.10589	1.10162	1.09768	1.09403	1.09065	1.08750	1.07682
7	6	1.34327	1.32537	1.30924	1.29462	1.28132	1.26916	1.25800	1.24773	1.23825	1.22946	1.19994
7	7	1.68792	1.64823	1.61281	1.58101	1.55230	1.52627	1.50255	1.48086	1.46095	1.44260	1.38173
8	1	0.40583	0.42281	0.43894	0.45426	0.46882	0.48269	0.49588	0.50846	0.52046	0.53192	0.57292
8	2	0.58687	0.60126	0.61471	0.62730	0.63911	0.65021	0.66066	0.67051	0.67982	0.68862	0.71952
8	3	0.73466	0.74503	0.75462	0.76353	0.77181	0.77954	0.78676	0.79353	0.79988	0.80586	0.82659
8	4	0.87467	0.88001	0.88491	0.88943	0.89361	0.89748	0.90109	0.90444	0.90758	0.91052	0.92062
8	5	1.01948	1.01859	1.01778	1.01703	1.01634	1.01571	1.01512	1.01458	1.01407	1.01360	1.01199
8	6	1.18235	1.17338	1.16526	1.15785	1.15109	1.14487	1.13915	1.13386	1.12896	1.12441	1.10900
8	7	1.38823	1.36768	1.34919	1.33246	1.31725	1.30336	1.29063	1.27893	1.26812	1.25812	1.22459
8	8	1.72244	1.68039	1.64289	1.60926	1.57892	1.55143	1.52640	1.50352	1.48252	1.46319	1.39912
9	1	0.38370	0.40077	0.41703	0.43250	0.44725	0.46131	0.47472	0.48752	0.49975	0.51144	0.55342
9	2	0.55268	0.56778	0.58193	0.59520	0.60769	0.61944	0.63053	0.64100	0.65090	0.66029	0.69333
9	3	0.68833	0.70016	0.71110	0.72127	0.73076	0.73963	0.74794	0.75574	0.76307	0.76998	0.79403
9	4	0.81409	0.82174	0.82878	0.83529	0.84133	0.84693	0.85216	0.85704	0.86162	0.86590	0.88070
9	5	0.93993	0.94253	0.94501	0.94724	0.94930	0.95120	0.95296	0.95460	0.95613	0.95756	0.96246
9	6	1.07427	1.07078	1.06760	1.06469	1.06203	1.05957	1.05730	1.05520	1.05325	1.05143	1.04524
9	7	1.22855	1.21711	1.20675	1.19734	1.18875	1.18087	1.17362	1.16693	1.16073	1.15498	1.13557
9	8	1.42660	1.40375	1.38320	1.36463	1.34777	1.33238	1.31830	1.30535	1.29341	1.28237	1.24539
9	9	1.75235	1.70823	1.66892	1.63368	1.60192	1.57315	1.54697	1.52305	1.50112	1.48093	1.41407
10	1	0.36492	0.38203	0.39835	0.41393	0.42879	0.44299	0.45655	0.46951	0.48192	0.49379	0.53653
10	2	0.52401	0.53963	0.55430	0.56810	0.58110	0.59336	0.60494	0.61590	0.62628	0.63612	0.67089
10	3	0.65018	0.66302	0.67498	0.68612	0.69654	0.70630	0.71545	0.72406	0.73216	0.73981	0.76651
10	4	0.76515	0.77452	0.78317	0.79119	0.79863	0.80557	0.81205	0.81811	0.82379	0.82913	0.84761
10	5	0.87769	0.88291	0.88770	0.89212	0.89620	0.89999	0.90351	0.90679	0.90985	0.91272	0.92258
10	6	0.99393	0.99421	0.99446	0.99469	0.99490	0.99510	0.99528	0.99545	0.99560	0.99575	0.99625
10	7	1.12059	1.11461	1.10955	1.10476	1.10036	1.09632	1.09260	1.08915	1.08594	1.08296	1.07286
10	8	1.26823	1.25401	1.24229	1.23111	1.22092	1.21158	1.20300	1.19508	1.18776	1.18097	1.15808
10	9	1.46000	1.43510	1.41273	1.39254	1.37422	1.35752	1.34224	1.32820	1.31527	1.30331	1.26331
10	10	1.77869	1.73274	1.69132	1.65516	1.62213	1.59223	1.56503	1.54020	1.51743	1.49648	1.42717

80% LIMITS

n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
						-βw-						
1	1	1.25434	1.24149	1.22987	1.21931	1.20968	1.20085	1.19274	1.18526	1.17833	1.17190	1.15023
2	1	0.90171	0.90596	0.90966	0.91345	0.91676	0.91983	0.92268	0.92534	0.92782	0.93014	0.93810
2	2	1.47080	1.44524	1.42223	1.40156	1.38276	1.36563	1.34996	1.33557	1.32231	1.31005	1.26908
3	1	0.74339	0.75348	0.76231	0.77146	0.77951	0.78701	0.79403	0.80059	0.80676	0.81255	0.83264
3	2	1.11117	1.10586	1.10103	1.09662	1.09259	1.08887	1.08544	1.08227	1.07932	1.07658	1.06727
3	3	1.58640	1.55347	1.52400	1.49748	1.47349	1.45168	1.43178	1.41354	1.39677	1.38130	1.32979
4	1	0.64822	0.66112	0.67312	0.68431	0.69478	0.70458	0.71377	0.72242	0.73057	0.73825	0.76509
4	2	0.93755	0.94031	0.94283	0.94514	0.94728	0.94925	0.95109	0.95279	0.95438	0.95587	0.96096
4	3	1.23194	1.22032	1.20980	1.20024	1.19151	1.18350	1.17614	1.16935	1.16305	1.15721	1.13750
4	4	1.66412	1.62604	1.59203	1.56148	1.53390	1.50886	1.48605	1.46517	1.44600	1.42833	1.36967
5	1	0.58287	0.59735	0.61088	0.62356	0.63545	0.64663	0.65716	0.66708	0.67646	0.68533	0.71649
5	2	0.82856	0.83567	0.84222	0.84827	0.85387	0.85907	0.86392	0.86845	0.87268	0.87665	0.89033
5	3	1.05501	1.05245	1.05011	1.04797	1.04601	1.04420	1.04253	1.04098	1.03954	1.03820	1.03363
5	4	1.31604	1.29971	1.28498	1.27163	1.25946	1.24834	1.23812	1.22871	1.22002	1.21196	1.18486
5	5	1.72218	1.68015	1.64267	1.60905	1.57873	1.55124	1.52622	1.50335	1.48236	1.46304	1.39899
6	1	0.53440	0.54984	0.56433	0.57794	0.59076	0.60284	0.61425	0.62503	0.63524	0.64492	0.67908
6	2	0.75157	0.76139	0.77047	0.77889	0.78671	0.79400	0.80082	0.80719	0.81318	0.81880	0.83829
6	3	0.94116	0.94376	0.94614	0.94832	0.95034	0.95220	0.95393	0.95554	0.95704	0.95844	0.96324
6	4	1.13901	1.13229	1.12619	1.12063	1.11554	1.11086	1.10654	1.10255	1.09884	1.09539	1.08371
6	5	1.38008	1.36002	1.34195	1.32561	1.31075	1.29718	1.28474	1.27329	1.26273	1.25295	1.22014
6	6	1.76828	1.72306	1.68277	1.64668	1.61415	1.58470	1.55790	1.53343	1.51099	1.49034	1.42200
7	1	0.49658	0.51263	0.52775	0.54199	0.55543	0.56814	0.58016	0.59155	0.60236	0.61262	0.64898
7	2	0.69332	0.70496	0.71576	0.72580	0.73516	0.74391	0.75211	0.75980	0.76703	0.77384	0.79754
7	3	0.85927	0.86522	0.87068	0.87572	0.88038	0.88471	0.88873	0.89248	0.89599	0.89927	0.91058
7	4	1.02330	1.02223	1.02125	1.02036	1.01953	1.01878	1.01807	1.01742	1.01682	1.01625	1.01433
7	5	1.20408	1.19396	1.18479	1.17645	1.16883	1.16183	1.15540	1.14945	1.14394	1.13883	1.12154
7	6	1.43152	1.40837	1.38756	1.36875	1.35167	1.33610	1.32183	1.30873	1.29664	1.28546	1.24804
7	7	1.80637	1.75847	1.71584	1.67767	1.64330	1.61221	1.58394	1.55814	1.53449	1.51274	1.44084
8	1	0.46599	0.48244	0.49798	0.51266	0.52654	0.53969	0.55216	0.56400	0.57525	0.58595	0.62398
8	2	0.64719	0.66012	0.67215	0.68337	0.69385	0.70368	0.71289	0.72156	0.72973	0.73743	0.76434
8	3	0.79646	0.80474	0.81238	0.81944	0.82600	0.83209	0.83778	0.84309	0.84807	0.85274	0.86887
8	4	0.93903	0.94172	0.94418	0.94644	0.94853	0.95046	0.95225	0.95391	0.95547	0.95692	0.96189
8	5	1.08779	1.08363	1.07986	1.07640	1.07324	1.07033	1.06763	1.06514	1.06283	1.06067	1.05335
8	6	1.25695	1.24396	1.23220	1.22153	1.21179	1.20287	1.19467	1.18711	1.18011	1.17361	1.15171
8	7	1.47436	1.44857	1.42542	1.40452	1.38557	1.36830	1.35250	1.33799	1.32462	1.31227	1.27098
8	8	1.83874	1.78853	1.74389	1.70394	1.66800	1.63550	1.60597	1.57903	1.55435	1.53167	1.45673
9	1	0.44057	0.45729	0.47312	0.48810	0.50231	0.51579	0.52859	0.54077	0.55235	0.56339	0.60274
9	2	0.60946	0.62333	0.63627	0.64837	0.65971	0.67035	0.68035	0.68978	0.69867	0.70707	0.73650
9	3	0.74619	0.75619	0.76543	0.77401	0.78198	0.78941	0.79635	0.80286	0.80896	0.81470	0.83458
9	4	0.87374	0.87912	0.88405	0.88861	0.89281	0.89672	0.90034	0.90373	0.90689	0.90985	0.92001
9	5	1.00232	1.00222	1.00212	1.00203	1.00195	1.00188	1.00181	1.00174	1.00168	1.00163	1.00143
9	6	1.14071	1.13390	1.12773	1.12209	1.11693	1.11219	1.10782	1.10378	1.10003	1.09653	1.08471
9	7	1.30134	1.28566	1.27138	1.25920	1.24764	1.23707	1.22736	1.21841	1.21014	1.20247	1.17667
9	8	1.51095	1.48237	1.45769	1.43498	1.41439	1.39566	1.37853	1.36282	1.34835	1.33498	1.29037
9	9	1.86683	1.81460	1.76820	1.72670	1.68938	1.65565	1.62502	1.59709	1.57151	1.54801	1.47044
10	1	0.41901	0.43591	0.45193	0.46714	0.48158	0.49531	0.50837	0.52080	0.53265	0.54395	0.58434
10	2	0.57783	0.59241	0.60605	0.61883	0.63083	0.64211	0.65273	0.66275	0.67222	0.68118	0.71265
10	3	0.70472	0.71602	0.72649	0.73623	0.74530	0.75378	0.76171	0.76915	0.77614	0.78273	0.80561
10	4	0.82106	0.82845	0.83526	0.84155	0.84738	0.85279	0.85784	0.86255	0.86696	0.87109	0.88535
10	5	0.93561	0.93845	0.94104	0.94343	0.94563	0.94766	0.94955	0.95131	0.95295	0.95448	0.95973
10	6	1.05469	1.05215	1.04982	1.04770	1.04575	1.04395	1.04229	1.04075	1.03931	1.03798	1.03344
10	7	1.18547	1.17633	1.16806	1.16052	1.15363	1.14731	1.14148	1.13610	1.13111	1.12648	1.11080
10	8	1.33950	1.32132	1.30588	1.29144	1.27830	1.26628	1.25526	1.24511	1.23573	1.22704	1.19786
10	9	1.54282	1.51271	1.48573	1.46143	1.43941	1.41939	1.40110	1.38432	1.36888	1.35463	1.30711
10	10	1.89160	1.83756	1.78961	1.74673	1.70819	1.67337	1.64177	1.61295	1.58658	1.56236	1.48246

90% LIMITS

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n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
1	1	1.48759	1.46098	1.43709	1.41554	1.39600	1.37820	1.36193	1.34698	1.33322	1.32050	1.27801
2	1	1.06939	1.06613	1.06317	1.06046	1.05797	1.05568	1.05357	1.05160	1.04978	1.04808	1.04231
2	2	1.67921	1.64011	1.60520	1.57336	1.54557	1.51990	1.49651	1.47512	1.45548	1.43738	1.37732
3	1	0.88162	0.88669	0.89134	0.89562	0.89958	0.90324	0.90666	0.90983	0.91280	0.91558	0.92513
3	2	1.26219	1.24890	1.23689	1.22598	1.21603	1.20692	1.19854	1.19081	1.18367	1.17703	1.15468
3	3	1.78253	1.73630	1.69515	1.65828	1.62507	1.59500	1.56766	1.54269	1.51980	1.49874	1.42907
4	1	0.76876	0.77800	0.78654	0.79445	0.80179	0.80863	0.81502	0.82100	0.82660	0.83186	0.85008
4	2	1.06349	1.06052	1.05781	1.05534	1.05307	1.05098	1.04904	1.04725	1.04558	1.04403	1.03875
4	3	1.37372	1.35404	1.33631	1.32027	1.30568	1.29235	1.28013	1.26889	1.25851	1.24891	1.21667
4	4	1.85245	1.80126	1.75576	1.71506	1.67845	1.64535	1.61528	1.58785	1.56274	1.53965	1.46343
5	1	0.69126	0.70296	0.71381	0.72391	0.73333	0.74213	0.75037	0.75811	0.76538	0.77223	0.79608
5	2	0.93932	0.94199	0.94444	0.94669	0.94877	0.95069	0.95247	0.95413	0.95568	0.95712	0.96207
5	3	1.17371	1.16520	1.15748	1.15045	1.14401	1.13811	1.13267	1.12764	1.12298	1.11865	1.10399
5	4	1.45160	1.42722	1.40532	1.38553	1.36758	1.35121	1.33623	1.32247	1.30979	1.29806	1.25882
5	5	1.90493	1.84994	1.80112	1.75750	1.71830	1.68289	1.65076	1.62147	1.59467	1.57006	1.48890
6	1	0.63373	0.64705	0.65941	0.67096	0.68175	0.69187	0.70137	0.71032	0.71874	0.72670	0.75451
6	2	0.85179	0.85802	0.86375	0.86904	0.87393	0.87847	0.88270	0.88664	0.89033	0.89378	0.90567
6	3	1.04599	1.04336	1.04191	1.04013	1.03849	1.03699	1.03559	1.03430	1.03310	1.03198	1.02816
6	4	1.25266	1.23990	1.22836	1.21788	1.20831	1.19955	1.19149	1.18406	1.17718	1.17080	1.14928
6	5	1.51106	1.48297	1.45773	1.43506	1.41448	1.39574	1.37861	1.36289	1.34842	1.33505	1.29042
6	6	1.94675	1.88863	1.83718	1.79121	1.74993	1.71267	1.67888	1.64810	1.61995	1.59411	1.50901
7	1	0.58892	0.60326	0.61667	0.62921	0.64098	0.65204	0.66245	0.67227	0.68154	0.69030	0.72107
7	2	0.73564	0.79430	0.80229	0.80969	0.81656	0.82295	0.82891	0.83448	0.83970	0.84461	0.86156
7	3	0.95447	0.95650	0.95835	0.96005	0.96162	0.96306	0.96441	0.96566	0.96682	0.96791	0.97163
7	4	1.12389	1.11794	1.11253	1.10760	1.10308	1.09893	1.09510	1.09155	1.08826	1.08519	1.07481
7	5	1.31389	1.29769	1.28307	1.26981	1.25774	1.24669	1.23655	1.22721	1.21858	1.21057	1.18366
7	6	1.55891	1.52777	1.49988	1.47475	1.45201	1.43133	1.41245	1.39514	1.37921	1.36451	1.31552
7	7	1.98140	1.92076	1.86702	1.81907	1.77606	1.73725	1.70207	1.67005	1.64077	1.61391	1.52554
8	1	0.55264	0.56774	0.58189	0.59516	0.60765	0.61940	0.63049	0.64096	0.65087	0.66025	0.69330
8	2	0.73329	0.74370	0.75334	0.76228	0.77060	0.77837	0.78562	0.79242	0.79881	0.80481	0.82563
8	3	0.88442	0.88937	0.89392	0.89811	0.90198	0.90556	0.90889	0.91200	0.91490	0.91762	0.92695
8	4	1.03058	1.02917	1.02789	1.02671	1.02563	1.02463	1.02371	1.02285	1.02205	1.02131	1.01878
8	5	1.18510	1.17599	1.16773	1.16021	1.15333	1.14702	1.14121	1.13584	1.13086	1.12624	1.11059
8	6	1.36371	1.34462	1.32742	1.31185	1.29768	1.28474	1.27287	1.26195	1.25187	1.24253	1.21119
8	7	1.59883	1.56509	1.53490	1.50774	1.48318	1.46086	1.44050	1.42184	1.40469	1.38887	1.33622
8	8	2.01092	1.94807	1.89240	1.84277	1.79825	1.75812	1.72176	1.68867	1.65844	1.63071	1.53954
9	1	0.52250	0.53814	0.55284	0.56666	0.57968	0.59197	0.60357	0.61456	0.62496	0.63483	0.66969
9	2	0.69049	0.70221	0.71309	0.72321	0.73264	0.74146	0.74972	0.75747	0.76476	0.77163	0.79553
9	3	0.82843	0.83555	0.84210	0.84815	0.85376	0.85897	0.86382	0.86835	0.87258	0.87656	0.89025
9	4	0.95850	0.96035	0.96204	0.96359	0.96503	0.96635	0.96757	0.96871	0.96978	0.97077	0.97416
9	5	1.09102	1.08671	1.08279	1.07921	1.07592	1.07290	1.07011	1.06752	1.06512	1.06288	1.05528
9	6	1.23538	1.22357	1.21288	1.20316	1.19430	1.18617	1.17869	1.17179	1.16540	1.15947	1.13946
9	7	1.40559	1.38400	1.36458	1.34702	1.33107	1.31651	1.30317	1.29090	1.27959	1.26911	1.23402
9	8	1.63299	1.59699	1.56481	1.53589	1.50975	1.48602	1.46438	1.44456	1.42636	1.40957	1.35378
9	9	2.03659	1.97179	1.91444	1.86333	1.81752	1.77623	1.73883	1.70481	1.67374	1.64525	1.55165
10	1	0.49693	0.51298	0.52808	0.54232	0.55576	0.56846	0.58048	0.59186	0.60266	0.61292	0.64926
10	2	0.65462	0.66735	0.67919	0.69023	0.70054	0.71019	0.71925	0.72777	0.73579	0.74335	0.76974
10	3	0.78223	0.79106	0.79916	0.80666	0.81362	0.82010	0.82615	0.83180	0.83710	0.84208	0.85928
10	4	0.90045	0.90476	0.90870	0.91233	0.91569	0.91880	0.92168	0.92437	0.92688	0.92923	0.93729
10	5	1.01785	1.01703	1.01628	1.01560	1.01497	1.01439	1.01385	1.01336	1.01289	1.01246	1.01099
10	6	1.14106	1.13424	1.12804	1.12239	1.11722	1.11247	1.10809	1.10403	1.10027	1.09677	1.08492
10	7	1.27793	1.26376	1.25096	1.23935	1.22875	1.21906	1.21015	1.20193	1.19433	1.18728	1.16355
10	8	1.44161	1.41734	1.39648	1.37719	1.35967	1.34370	1.32907	1.31564	1.30325	1.29180	1.25347
10	9	1.66277	1.62478	1.59085	1.56037	1.53285	1.50787	1.48511	1.46428	1.44515	1.42752	1.36898
10	10	2.05926	1.99275	1.93389	1.88147	1.83450	1.79218	1.75387	1.71903	1.68722	1.65805	1.56230

95% LIMITS

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n	r	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.40
1	1	1.68619	1.64662	1.61130	1.57958	1.55096	1.52500	1.50135	1.47972	1.45986	1.44157	1.38085
2	1	1.21216	1.20160	1.19204	1.18335	1.17541	1.16813	1.16142	1.15523	1.14950	1.14417	1.12618
2	2	1.85881	1.80716	1.76126	1.72020	1.68328	1.64990	1.61959	1.59194	1.56662	1.54335	1.46653
3	1	0.99932	0.99935	0.99938	0.99941	0.99943	0.99945	0.99947	0.99949	0.99951	0.99953	0.99958
3	2	1.39103	1.37032	1.35167	1.33481	1.31948	1.30548	1.29266	1.28086	1.26997	1.25990	1.22611
3	3	1.95279	1.89428	1.84239	1.79607	1.75449	1.71696	1.68293	1.65193	1.62358	1.59757	1.51190
4	1	0.87139	0.87686	0.88188	0.88651	0.89079	0.89476	0.89846	0.90190	0.90512	0.90813	0.91848
4	2	1.17053	1.16218	1.15461	1.14772	1.14141	1.13562	1.13028	1.12535	1.12078	1.11653	1.10214
4	3	1.49501	1.46793	1.44363	1.42172	1.40185	1.38375	1.36721	1.35202	1.33803	1.32511	1.28194
4	4	2.01676	1.95347	1.89742	1.84745	1.80264	1.76225	1.72565	1.69235	1.66193	1.63402	1.54230
5	1	0.78354	0.79228	0.80034	0.80780	0.81473	0.82118	0.82719	0.83281	0.83808	0.84303	0.86014
5	2	1.03330	1.03176	1.03036	1.02908	1.02790	1.02681	1.02581	1.02487	1.02401	1.02320	1.02044
5	3	1.27466	1.26067	1.24804	1.23657	1.22611	1.21654	1.20774	1.19962	1.19212	1.18515	1.16170
5	4	1.56736	1.53613	1.50773	1.48215	1.45901	1.43796	1.41875	1.40113	1.38493	1.36998	1.32017
5	5	2.06496	1.99801	1.93878	1.88603	1.83877	1.79619	1.75765	1.72260	1.69060	1.66126	1.56497
6	1	0.71639	0.72927	0.73935	0.74871	0.75743	0.76556	0.77318	0.78031	0.78702	0.79332	0.81523
6	2	0.93676	0.93954	0.94210	0.94444	0.94660	0.94860	0.95046	0.95218	0.95379	0.95530	0.96045
6	3	1.13490	1.12839	1.12248	1.11709	1.11215	1.10761	1.10343	1.09955	1.09596	1.09262	1.08129
6	4	1.34940	1.33114	1.31469	1.29979	1.28623	1.27383	1.26247	1.25200	1.24234	1.23338	1.20332
6	5	1.62361	1.58823	1.55660	1.52817	1.50246	1.47912	1.45783	1.43834	1.42042	1.40390	1.34897
6	6	2.10349	2.03358	1.97178	1.91678	1.86754	1.82321	1.78310	1.74665	1.71338	1.68290	1.58294
7	1	0.66754	0.67992	0.69142	0.70213	0.71213	0.72149	0.73027	0.73851	0.74627	0.75359	0.77909
7	2	0.86387	0.86964	0.87493	0.87982	0.88434	0.88853	0.89243	0.89606	0.89946	0.90264	0.91358
7	3	1.03509	1.03347	1.03199	1.03063	1.02939	1.02825	1.02719	1.02620	1.02529	1.02443	1.02153
7	4	1.20921	1.19881	1.18940	1.18083	1.17301	1.16583	1.15922	1.15312	1.14747	1.14222	1.12449
7	5	1.40746	1.38577	1.36625	1.34860	1.33256	1.31793	1.30452	1.29219	1.28082	1.27030	1.23504
7	6	1.66358	1.63020	1.59592	1.56514	1.53734	1.51212	1.48914	1.46811	1.44880	1.43101	1.37193
7	7	2.13549	2.06310	1.99915	1.94227	1.89138	1.84558	1.80416	1.76654	1.73221	1.70078	1.59777
8	1	0.62642	0.63958	0.65242	0.66413	0.67510	0.68537	0.69503	0.70412	0.71269	0.72078	0.74909
8	2	0.80623	0.81416	0.82148	0.82823	0.83450	0.84033	0.84576	0.85084	0.85559	0.86005	0.87544
8	3	0.95884	0.96067	0.96235	0.96389	0.96531	0.96662	0.96784	0.96897	0.97002	0.97101	0.97437
8	4	1.10808	1.10292	1.09823	1.09395	1.09003	1.08642	1.08309	1.08001	1.07715	1.07448	1.06544
8	5	1.26767	1.25408	1.24130	1.23064	1.22047	1.21115	1.20259	1.19469	1.18739	1.18061	1.15777
8	6	1.45478	1.43020	1.40812	1.38818	1.37009	1.35360	1.33850	1.32464	1.31186	1.30004	1.26052
8	7	1.70615	1.66522	1.62870	1.59593	1.56637	1.53957	1.51516	1.49284	1.47235	1.45349	1.39093
8	8	2.16280	2.08828	2.02248	1.96399	1.91168	1.86462	1.82209	1.78346	1.74823	1.71598	1.61036
9	1	0.59225	0.60652	0.61985	0.63233	0.64403	0.65502	0.66536	0.67512	0.68432	0.69303	0.72358
9	2	0.75912	0.76869	0.77753	0.78573	0.79335	0.80044	0.80707	0.81327	0.81909	0.82455	0.84348
9	3	0.89797	0.90237	0.90641	0.91013	0.91356	0.91674	0.91970	0.92245	0.92503	0.92743	0.93569
9	4	1.03016	1.02877	1.02750	1.02634	1.02527	1.02429	1.02338	1.02253	1.02175	1.02102	1.01852
9	5	1.16610	1.15798	1.15062	1.14391	1.13778	1.13214	1.12695	1.12215	1.11770	1.11356	1.09956
9	6	1.31574	1.29943	1.28472	1.27138	1.25922	1.24811	1.23791	1.22851	1.21982	1.21177	1.18469
9	7	1.49458	1.46753	1.44326	1.42136	1.40151	1.38344	1.36690	1.35173	1.33776	1.32484	1.28172
9	8	1.73834	1.69519	1.65674	1.62225	1.59116	1.56299	1.53735	1.51391	1.49242	1.47263	1.40708
9	9	2.18659	2.11020	2.04279	1.98288	1.92932	1.88117	1.83766	1.79815	1.76213	1.72917	1.62128
10	1	0.56327	0.57816	0.59210	0.60517	0.61745	0.62901	0.63990	0.65018	0.65991	0.66912	0.70150
10	2	0.71966	0.73050	0.74054	0.74987	0.75855	0.76666	0.77424	0.78135	0.78802	0.79431	0.81612
10	3	0.84783	0.85421	0.86009	0.86551	0.87052	0.87518	0.87951	0.88355	0.88733	0.89087	0.90307
10	4	0.96751	0.96896	0.97029	0.97151	0.97264	0.97368	0.97464	0.97553	0.97637	0.97714	0.97981
10	5	1.08734	1.08321	1.07945	1.07602	1.07287	1.06997	1.06729	1.06482	1.06251	1.06037	1.05308
10	6	1.21417	1.20351	1.19385	1.18507	1.17705	1.16969	1.16292	1.15667	1.15088	1.14550	1.12734
10	7	1.35646	1.33779	1.32097	1.30573	1.29188	1.27921	1.26760	1.25691	1.24704	1.23790	1.20720
10	8	1.52387	1.49965	1.47346	1.44985	1.42847	1.40901	1.39123	1.37492	1.35991	1.34604	1.29980
10	9	1.76645	1.72135	1.68118	1.64518	1.61274	1.58337	1.55665	1.53224	1.50986	1.48926	1.42109
10	10	2.20764	2.12959	2.06073	1.99958	1.94491	1.89578	1.85140	1.81112	1.77440	1.74081	1.63090

APPENDIX B1

Maintenance Strategies for A Fleet of Office Photocopying Machines

Errata:

Page	Line	Printed As	Should be
6A/2/4	33	$j \neq 1$	$j \neq i$
6A/2/5	19	mutually exclusive	independent

MAINTENANCE STRATEGIES FOR A FLEET OF OFFICE PHOTOCOPYING MACHINES

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The paper presents an analysis and correlation of the reliability and causes of failure of a general purpose photocopying machine. Fleets of 10 and 8 such machines, in a non-commercial zone and a commercial zone were analysed separately, using the Weibull distribution to correlate the failure data. The shape and characteristic age parameters were found to be widely different for each machine and reasons for these differences are discussed.

Three components were found to be the most frequent cause of failure and an opportunistic preventive replacement strategy has been shown to be cost-effective.

INTRODUCTION

The photocopier on which this study was made, is a dry copier. The ink is a thermoplastic powder (toner) which is blended with a mixture of iron filings (MIX). The image to be copied is transferred as magnetic lines to a film strip called the organic photo condenser (OPC). The copy paper and OPC pass brushes which apply the toner and mix the magnetised regions of the paper. This is then fused to the paper as it passes through rollers. One of these rollers is heated; it is made of silicon rubber and is impregnated with a lubricant to prevent the toner sticking to it. The heated roller is designated LSR.

The principal mode of failure of the machine is that it gives unsatisfactory copies. However, toward the end of its useful life, the lubricant on the LSR becomes depleted and toner adheres to it. This can lead to a fuser jam, in which the copy paper gets stuck to the LSR and becomes rolled up in the fuser unit. The cause of failure of the machine can be due to the failure of any component; there are 95 causes listed on the service engineer's record card.

The company who market these machines, and the users, have a common aim of maximising machine availability. The company also requires to maximise the profitability of its machines, which are hired to the users on a basis of fixed plus sliding charge per page copied. The company's operating costs include replacement of parts and materials plus service engineer's costs and overheads. The machines are required by the users at random intervals during office hours.

The survey started as an analysis, on behalf of a user, of the causes of failures of a single machine, with the objective of increasing its availability. The down-time for this machine, after each failure was between 4 and 48 hours, including non-office hours when the machine was not available for repair. Analysis of the failures for the two years since the machine was installed was based on the Weibull distribution (1) and found to have a shape parameter of 0.94. However, the parts most often causing the failure or replaced as a preventive replacement action were the LSR, the OPC and the MIX. The shape parameters for these items were found to be 2.8, 2.0 and 1.28, respectively. Clearly, these items were failing due to a combination of wearout and random causes. Discussion with the company and the service engineers indicated that the performance of similar machines elsewhere varied according to the pattern of usage and furthermore, the engineers were expected to decide whether to carry out preventive replacement of the LSR, OPC and MIX. In order to study the effective usage patterns and to devise optimal strategies for the service engineers, it was decided to analyse the data from 10 machines in a non-commercial research establishment which forms a zone served by a single engineer. These users copy a variety of items, ranging from typescript to half-tone prints and the machines are available to all members of the establishment. On the other hand, commercial users principally require to copy letters, which have less than 10% of the page covered with print; furthermore, they tend to designate a limited number of machine operators to carry out this work. Eight machines in a commercial zone were analysed; they are all served by one engineer.

FAILURE ANALYSIS

The shape parameter β and the characteristic age η were found by means of a computer programme which fits the failure data to the Weibull cumulative distribution function, "see Equation (1)"

$$F(t) = 1 - \exp - \left(t/\eta\right)^{\beta} \tag{1}$$

Eqn. (1) can be transformed into a linear equation as follows, from which the parameters β and η were found by least squares.

$$\ln \ln (1-F(t))^{-1} = \beta \ln t - \beta \ln \eta \tag{2}$$

The programme also computes the mean life μ and the standard deviation σ from Equation (3) and (4), in which gamma functions are evaluated by the approximation shown in Equation (5).

$$\mu = \eta \, \Gamma(1 + 1/\beta) \tag{3}$$

$$\sigma^2 = \eta^2 \, \Gamma(1 + 2/\beta) - \mu^2 \tag{4}$$

$$\Gamma(x) = x^x \exp(-x) \left\{\frac{2\pi}{x}\left(1 + \frac{1}{12x} + \frac{1}{288x^2}\right)\right\}^{0.5} \tag{5}$$

These data were found for the eighteen machines, for all failures, for the failure/replacement of LSR, OPC and MIX and for fuser jams. The results are shown separately for the non-commercial and for the commercial zones in Tables 1 and 2. Tables 3 and 4 show the considerable range in β values for the various machines. Thus maintenance decisions which are based upon the probability of failure, should be computed for separate machines and for the separate parts which are the principal causes of machine failure.

OPTIMAL PREVENTIVE REPLACEMENT AGE OF COMPONENTS

Two objective functions can be considered: maximum availability and minimum cost. The strategy would be to replace a component either when it had failed or when it had reached an optimal age, whichever was the sooner. When responding to a machine failure call, the engineer would replace only the component which had caused the machine to fail.

The following sources of data were used in this analysis.

. From the maintenance section log books: Date and time when call was received from the user, engineer's arrival time at site and his departure time, meter reading (cumulative number of pages copied), failed parts, cause of failure if diagnosed, any additional parts replaced on a preventive basis.

. From the engineer's estimates, the following repair times were obtained:

Replacement of LSR = 45 minutes
Changing MIX = 20 minutes
Advancing OPC = 5 minutes
Changing OPC after 12 advances = 25 minutes
Travelling to site = 30 minutes

. From the dates and meter readings in the engineer's log books, the average rate of copying is 500 pages per day.

. Scaled cost data supplied by the company: LSR = 42.48, MIX = 11.00, OPC = 192.00 for 12 advances, engineer's time = 20 per hour, revenue loss due to waiting time after machine failure = 10.06.

The availability of a component which is replaced at intervals of t_i or sooner if failed, is given in Equation (6). The model is explained by Jardine (2).

$$A(t_i) = \frac{t_i\ R(t_i) + M(t_i)\ F(t_i)}{(t_i + T_i)\ R(t_i) + (M(t_i) + T_r)\ F(t_i)} \tag{6}$$

$R(t_i)$ and $F(t_i)$ are the probability of survival and of failure during the campaign interval t_i. T_i is the time required to replace the component plus travelling time to site. T_r is the total time taken to repair the machine after failure of a component; it includes waiting time for an engineer to become available, travelling time and replacement time.

$M(t_i)$ is the expected length of a cycle which terminates with a failure.

$$M(t_i) = \frac{1}{F(t_i)} \int_0^{t_i} tf(t)dt \tag{7}$$

For the Weibull distribution

$$f(t) = \frac{\beta}{\eta}\left[\frac{t}{\eta}\right]^{\beta-1} \exp - \left[\frac{t}{\eta}\right]^{\beta} \tag{8}$$

From the maintenance section log books, the response time between receiving a call and the engineer getting to the machine was correlated by the Weibull distribution with the following parameters: $\beta = 1.693$, $\eta = 6$ hours, $\mu = 5.365$ hours and $\sigma = 3.255$ hours.

From the engineer's log books the time taken to overhaul the machine and replace the LSR, the MIX and advance the OPC plus 30 minutes estimated travelling time is 3 hours. This value was taken for T_i when considering complete preventive overhaul of the machine; it is somewhat longer than the sum of the estimated times for each component plus travelling time.

Table 5 shows that there are optimal replacement ages for the three major components and for the machine as a whole; however the improvement in availability is insignificant.

The cost per unit time for preventive replacement strategy is given in Equation (9) which is similar to Jardine (2).

$$C(t_i) = \frac{C_p\ R(t_i) + C_f\ F(t_i)}{(t_i + T_i)\ R(t_i) + (M(t_i) + T_r)\ F(t_i)} \tag{9}$$

C_p = price of component + engineer's time

$C_f = C_p$ + value of production lost during waiting time

+ Penalty cost

The penalty cost might be a refund which the company makes to the user for the time that the machine is in the failed state. Table 6 lists these cost data without a penalty cost.

The analysis using the failure data for all 10 machines in the non-commercial zone, showed no savings due to preventive replacement of any of the components; in the absence of a penalty cost.

OPPORTUNISTIC PREVENTIVE REPLACEMENT

This is a strategy which the engineers have been applying for some time. It involves the decision whether or not to replace a component which has not failed, at the same time as failed components are being replaced. Thus the engineers only attend the machine when it has already failed. The engineer's decisions will be optimal if they always minimise cost.

Denoting i as the component which has failed and j as the good component which is replaced at the same time as i, we have the following cost matrix; where j = 4 means all three components are replaced.

$$[C] = \begin{array}{c|cccc} i \; j \rightarrow & 1 & 2 & 3 & 4 \\ 1 & c_{11} & c_{12} & c_{13} & c_{14} \\ 2 & c_{21} & c_{22} & c_{23} & c_{24} \\ 3 & c_{31} & c_{32} & c_{33} & c_{34} \end{array} \qquad (10)$$

The cost elements are made up of the following costs:

c_{ii} = cost of failed part i
+ engineer's costs (working + travelling time)
+ lost production cost

$c_{ij(i \neq j)} = c_{ii}$
+ cost of j x Proportion of useful life lost for j.
- (lost production costs due to failure of j + engineer's cost for another travel) x Prob. (j will fail before another opportunity)

Useful Life Lost Through Opportunistic Replacement

It has been mentioned above that if any opportunistic replacement of component j is made when i has failed, some useful life of the used component j is lost. The proportion of the useful life of this component which is lost, can be determined from the mean value of the conditional probability density function $f_{cj}(t)$ of component j, which is given by Green and Bourne (3). This is the mean of the failure distribution truncated at the present age T_j of component j. Denoting this mean by μ_j,

$$\mu_j = \frac{1}{R(T_j)} \int_{T_j}^{\infty} t f(t) dt \qquad (11)$$

which on simplification becomes

$$\mu_j = T_j + \frac{1}{R(T_j)} \int_{T_j}^{\infty} R(t) dt \qquad (12)$$

if the mean is measured from the last replacement of the j component.

Now, the proportion of useful life lost for component j, is

$$\frac{\mu_j - T_j}{\mu_j} \qquad (13)$$

Probability of a Good Component Failing Before the Next Replacement Opportunity

When a component i has failed, it has provided an opportunity of considering the replacement of component k as well. However, if component k is not replaced at this opportunity and if it fails before any other component, this will incur a penalty cost or lost production cost. The probability of this event is the failure probability of component k before the failure of new component i and the other non-failed components j; where j≠1 and j≠k.

The probability that k will survive until the next failure of i is denoted by R_{ki}. This can be evaluated by Equation (14) given in Green and Bourne (3) for the special case where the age of k is zero.

$$R_{ki} = \int_0^{\infty} f_k(x)\{\int_0^{x} f_i(y)dy\}dx = \int_0^{\infty} f_k(x)\ F_i(x)dx \qquad (14)$$

In practice, the component k must have aged to T_k. So, the survival probability of component k will be given by

$$R_{ki} = \int_0^{\infty} \frac{f_k(x + T_k)}{R(T_k)} F_i(x)dx \qquad (15)$$

The term $f_k(x + T_k)/R(T_k)$ represents the conditional probability density function for component k which has survived up to age T_k.

The survival probability of component k with respect to other unfailed compnents j can be determined in a similar way. The relationship is given by Equation (16).

$$R_{kj} = \int_0^{\infty} \frac{f_k\ (x + T_k)}{R(T_k)} \{ \frac{F_j(x + T_j) - F_j(T_j)}{R(T_j)} \} dx, \quad j = 1,2,3,\ j \neq k,\ j \neq i \qquad (16)$$

where T_j is the age of component j.

The respective failure probabilities of component k with respect to components i and j are

$F_{ki} = 1 - R_{ki}$, and

$F_{kj} = 1 - R_{kj}$

The probability that k will fail before any other component can be evaluated as follows:

Let, Event A = Component k fails before component i.
Event B = Component k fails before a component j.

Since these events are mutually exclusive, the probability of both events occuring is

$$P(AB) = P(A) \times P(B) = F_{ki} \times F_{kj} \qquad (17)$$

For all values of j excluding i and k,

$$P(AB) = F_{ki} \times \prod_j F_{kj} \qquad (18)$$

P(AB) is the probability that if k is not replaced, it will fail before any of the other components i and j. In considering the various preventive replacement decisions, P(AB) is evaluated for all values of k excluding i.

A programme has been written which computes and prints the cost matrix for given sets of ages of the three components at the time of failure of one of them. The minimum cost from the elemets of the appropriate row is printed as well as the instruction of which components to replace. The engineer can override this decision rule if he considers the cost benefit to be insignificant; he may decide that it costs very little more to replace more or fewer components than the rule indicates. Of course, he must replace the failed part.

Table 7 is an example of the computer print-out. It is based on the failure data for all 10 machines in the non-commercial zone. Because the decisions are based upon the probability of failure of the components and because the failure parameters for each machine are so different (see Table 3 for example), the computer programme is being used for each machine separately.

DISCUSSION

When the shape parameter β is unity, the Weibull distribution corresponds to the negative exponential distribution. This latter distribution applies when failures arrive at random intervals. The more that wearout contributes to the total failures, the higher will be the value of β. Indeed, when β reaches about three the probability density function is similar to the normal distribution and wearout should be the dominant cause of failure.

Conversely, when β is less than unity, the Weibull distribution is similar to the hyper-exponential distribution and Sherwin (4) states that failures may be due to incompetent and/or incomplete maintenance and/or the use of incorrect spares (4). Table 3 shows a wide spread of values of β which are difficult to explain, except in terms of the effect which users have on their machine's reliability and replacement record. All these machines are serviced by the same engineer using standard replacement parts; thus the low values of β for machine number 2 can not be explained by incompetent maintenance or the use of incorrect spares.

It must be remembered that the engineer has been using a strategy of opportunistic preventive replacement. These replacements have been considered as failures in analysing the data. It is to be expected, when the machine characteristic can be strongly affected by the user, the values of β will be low as the engineer strives to prolong the useful life of the components but at the same time to reduce the number of break-down calls.

Machine number 4 in the non-commercial zone is equally interesting. It has high values of β indicating control of random cuases of failure; it also has high values of η. Now, the components are replaced with standard parts having an inherent characteristic life; so where the values of η and β are higher than for the other machines one can conclude that the absence of confusing random failures has enabled the service engineer to find the optimal replacement intervals for machine number 4. It is hoped that by continually updating the correlation of failure/replacement data of each machine and by using its cost matrix, the service engineers will be able to improve the cost effectiveness of their opportunistic preventive replacement decisions.

This study has demonstrated the advantage of analysing the failure data of component parts and of individual machines. For example, in Tables 1 and 2 the values of β for all failures is less than any of the values of β for the three principal causes of machine failure. Conversely, the values of β for all failures in Table 3 and 4 are widely distributed for the individual machines which are classified in Tables 1 and 2, respectively. The classification into zones, in order to compare commercial with non-commercial users and one engineer's skill with another's has not produced significantly differing data. The reasons for the random variations in performance of identical machines in different locations requires further investigation.

CONCLUSIONS

. There is considerable difference in the failure characteristics of identical machines in different locations.
. Preventive replacement of parts at fixed intervals does not yield a significant increase in machine availability nor is it cost-effective unless there is a penalty on the company for the duration when the machine is in the failed state.
. An opportunistic preventive replacement policy has been shown to be cost-effective when based on the failure data for the fleet of machines in the non-commercial zone. This policy would be even more cost-effective if the decision tables are drawn up for each machine, based on its failure characteristics.

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3. Green, A.E., and A.J. Bourne, 1972, "Reliability Technology", Chaps. 7 and 10, Wiley-Interscience, London.

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TABLE 1 - Correlation Parameters for Failure Data of the 10 Machines in the Non-commercial Zone

	β	η(pages)	μ(pages)	σ(pages)
LSR	2.198	13160	11664	5584
OPC	1.571	9720	8735	5675
MIX	1.907	19944	17708	9642
Fuser Jams	1.264	12014	11166	8884
All failures	1.243	5474	5108	4131

TABLE 2 - Correlation Parameters for Failure Data of the 8 Machines in the Commercial Zone

	β	η(pages)	μ(pages)	σ(pages)
LSR	2.565	10637	9453	3935
OPC	2.152	8608	7630	3722
MIX	1.755	22858	20367	11956
All failures	1.194	5580	5258	4417

TABLE 3 - Range of Values of the Shape Parameter β for the 10 Machines in the Non-commercial Zone

Machine No.	LSR	OPC	MIX	Fuser Jam	All Failures
1	2.794	1.994	1.281	1.780	0.941
2	1.142	0.797	1.264	0.752	0.871
3	1.358	1.093	3.082	1.667	1.119
4	3.95	2.156	1.733	1.114	2.433
5	2.134	1.267	-	1.156	1.459
6	2.48	1.499	1.529	0.864	1.225
7	1.591	1.226	-	0.973	1.000
8	2.570	2.070	1.526	1.488	1.441
9	1.650	1.870	-	1.526	2.004
10	2.130	1.280	1.606	1.200	1.870

TABLE 4 - Range of Values of the Shape Parameter β for the 8 Machines in the Commercial Zone

Machine No.	LSR	OPC	MIX	All Failures
1	3.430	1.925	1.026	1.874
2	2.193	1.440	1.432	1.066
3	3.448	2.600	1.824	1.579
4	2.263	1.469	1.890	1.513
5	2.454	2.000	1.845	1.574
6	2.720	1.900	1.664	0.719
7	1.860	1.953	-	0.710
8	1.963	2.392	-	1.292

TABLE 5 - Effect of Preventive Replacement on Availability (for all 10 machines in the non-commercial zone)

Component	Optimal replacement age (pages)	Maximum Availability (%)	Availability for replacement only after failure (%)
LSR	4000	97.19	96.08
OPC	6500	97.81	96.72
MIX	8500	97.68	97.09
Machine Overhaul	24500	91.59	90.82

TABLE 6 - Cost Input Data (scaled monetary units)

Component	LSR	OPC	MIX
C_p	67.48	28.36	27.67
C_f with no penalty costs	77.54	38.42	37.73

TABLE 7 - Example of Cost Matrix for Opportunistic Preventive Replacement

Comp. Failed	Age at Failure	Replacement Costs for Components				* Replace		
		OPC	LSR	MIX	ALL	OPC	LSR	MIX
OPC	5000	38.67	55.80	44.66	57.45	*		
LSR	5000	73.96	77.79	83.67	78.87	*	*	
MIX	5000	35.81	56.27	37.97	52.09	*		*
OPC	5000	38.67	48.55	41.54	46.90	*		
LSR	7500	75.24	77.79	79.96	76.06	*	*	
MIX	10000	37.13	48.17	37.97	45.94	*		*
OPC	5000	38.67	42.28	41.96	41.44	*		
LSR	10000	75.24	77.79	79.96	76.06	*	*	
MIX	10000	38.24	41.99	37.97	40.26			*
OPC	7500	38.67	41.20	45.33	44.35	*		
LSR	10000	70.62	77.79	83.97	75.53	*	*	
MIX	5000	34.95	42.91	37.97	38.40	*		*
OPC	7500	38.67	42.28	41.96	41.44	*		
LSR	10000	71.88	77.79	80.40	72.70	*	*	
MIX	10000	34.95	42.91	37.97	38.40	*		*
OPC	10000	38.67	55.80	44.66	57.45	*		
LSR	5000	68.25	77.79	84.20	73.16	*	*	
MIX	5000	30.08	57.73	37.97	46.37	*		*
OPC	12500	38.67	42.28	41.96	41.44	*		
LSR	10000	67.23	77.79	81.12	68.05	*	*	
MIX	10000	30.32	44.56	37.97	33.77	*		*
OPC	5000	38.67	26.21	43.47	27.04	*	*	
LSR	20000	75.24	77.79	79.96	76.06	*	*	
MIX	10000	41.19	28.17	37.97	23.95	*	*	*
OPC	5000	38.67	27.38	39.76	23.07	*	*	*
LSR	20000	77.15	77.79	74.23	74.44		*	*
MIX	20000	41.19	28.17	37.97	23.95	*	*	*
OPC	12500	38.67	27.38	39.76	23.07	*	*	*
LSR	20000	69.06	77.79	76.07	63.92	*	*	*
MIX	20000	33.64	30.81	37.97	26.08	*	*	*

APPENDIX B2

List of OPPORTUNIST Programme

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      MASTER OPPORTUNIST
C****************************************************************************************
C*    MAIN SYMBOLS                                                                      *
C*    ------------                                                                      *
C*    FS   =FAILURE PROBABILITY OF AN UNFAILED COMPONENT BEFORE A FAILED ITEM           *
C*    ILFA =IDENTIFICATION NUMBERS OF FAILED COMPONENTS                                 *
C*    IMFA =SAME AS ILFA                                                                *
C*    IOR  =IDENTIFICATION NUMBERS OF OPPORTUNISTICALLY REPLACED ITEMS                  *
C*    IR   =INDEX USED FOR THE AGE OF THE COMPONENT                                     *
C*    IULF =IDENTIFICATION NUMBERS OF UNFAILED COMPONENTS                               *
C*    IUMF ==SAME AS IULF                                                               *
C*    LFA  =NUMBER OF FAILED COMPONENTS                                                 *
C*    LUF  =NUMBEER OF UNFAILED COMPONENTS                                              *
C*    MFA  =SAME AS LFA                                                                 *
C*    MUF  =SAME AS LUF                                                                 *
C*    NAL  =LOWER VALUE OF THE TIME RANGE OVER WHICH CALCULATION IS TO BE DONE          *
C*    NAN  =INCREMENTAL VALUE OF THE TIME RANGE                                         *
C*    NAU  =UPPER VALUE OF THE TIME RANGE                                               *
C*    P(J) =PRICE OF THE JTH COMPONENT                                                  *
C*    PC   =PENALTY COST IF ANY                                                         *
C*    PE   =ENGINEER'S COST / HOUR                                                      *
C*    PF   =OVERALL FAILURE PROBABILITY OF AN UNFAILED ITEM BEFORE OTHER ITEMS          *
C*    RSS  =SURVIVAL PROBABILITY, 1-FS                                                  *
C*    RTT  =SAME AS RSS                                                                 *
C*    TT   =TIME TO TRAVEL TO THE SITE                                                  *
C****************************************************************************************
      DIMENSION P(3),NAL(3),NAU(3),NAN(3),IULF(3),ILFA(3),ILA(3),SRC(3,2
     150),LN(3),MN(3),RRL(3,250),BBL(3,250),FFL(3,250),NNF(3),FS(3,3),IO
     2R(3),IR(3),PF(3)
      DIMENSION IUMF(3),IMFA(3)
C
C*    DEFINING GAIN FUNCTION FOR DETERMINING OPPORTUNISTIC MAINTENANCE PLAN
      CGN1(PEE,PCP,TTT,PFF,PP,SRR)=(((PEE*TTT)+PCP)*PFF)-PP*SRR
      WRITE(2,550)
  550 FORMAT(1H1,5(/),10X,'MAINTENANCE POLICY BY OPPORTUNISTIC MODEL',/,
     110X,45('-'),//,1 X,'COMPONENT AGE',/,13X,'1',3X,'2',3X,'3',7X,'DEC
     2ISION RULES',/,10X,50('-'))
      READ(1,530) PE,TT,(P(J),J=1,3),PC
  530 FORMAT(6F0.0)
      CALL FIRST(NAL,NAU,NAN,LUF,IULF,LFA,ILFA,LA,ILA,SRC,RRL,BBL,FFL,
     1NNF)
      DO 400 I=1,LUF
  400 LN(IULF(I))=1
      MUF=LUF
      MFA=LFA
      DO 411 I=1,LUF
  411 IUMF(I)=IULF(I)
      DO 412 J=1,LFA
  412 IMFA(J)=ILFA(J)
C*    DO LOOPS FOR UNFAILED COMPONENTS
      DO 701 I1=NAL(1),NAU(1),NAN(1)
      MN(1)=((I1-NAL(1))/NAN(1))+1
      IFP=0
```

```
      DO 702 I2=NAL(2),NAU(2),NAN(2)
      MN(2)=((I2-NAL(2))/NAN(2))+1
      IR(3)=1
      IR(1)=I1
      IR(2)=I2
      NPUF=LUF
      IDR=LFA
C
C*    FINDING FAILURE PROBABILITY OF AN UNFAILED COMPONENT BEFORE A FAILED ITEM
      DO 401 J2=1,LUF
      IJ=IULF(J2)
      IF(MN(IJ)-LN(IJ)) 500,501,500
  501 DO 402 J3=1,LFA
      IK=ILFA(J3)
      CALL PROB1(IJ,IK,RRL,BBL,FFL,NNF,IR,RSS)
      FS(IJ,IK)=1.0-RSS
  402 CONTINUE
  500 CONTINUE
  401 CONTINUE
C
      NFF=LUF-1
      DO 403 J4=1,NFF
      IJJ=IULF(J4)
      J5=J4+1
      DO 404 J6=J5,LUF
      IKK=IULF(J6)
      IF(MN(IKK)-LN(IKK)) 504,505,504
  505 CALL PROB2(IJJ,IKK,RRL,BBL,FFL,NNF,IR,RTT)
      FS(IJJ,IKK)=1.0-RTT
      FS(IKK,IJJ)=RTT
  504 CONTINUE
  404 CONTINUE
  403 CONTINUE
  524 DO 508 I=1,LUF
      ID=IULF(I)
      PF(ID)=1.0
      DO 509 J=1,LA
      IE=ILA(J)
      IF(ID.EQ.IE) GO TO 509
      PF(ID)=PF(ID)*FS(ID,IE)
  509 CONTINUE
  508 CONTINUE
      NJJ=0
      DO 510 I=1,LUF
      ID=IULF(I)
      ISG=MN(ID)
      Y=CGN1(PE,PC,TT,PF(ID),P(ID),SRC(ID,ISG))
      IF(Y) 511,511,512
  512 NJJ=NJJ+1
      IOR(NJJ)=ID
  511 CONTINUE
  510 CONTINUE
```

```
C*    SCREENING OUT OPPORTUNISTICALLY MAINTAINED COMPONENTS FROM OTHER UNFAILED
C*    ITEMS
      ND=LUF-NJJ
      IF(NJJ) 513,513,514
  514 IF(ND) 502,502,515
  515 JK=0
      JX=0
      JS=0
  518 JX=JX+1
  517 JS=JS+1
      IF(IOR(JX).EQ.IULF(JS)) GO TO 516
      JK=JK+1
      IULF(JK)=IULF(JS)
      IF(JS-LUF) 517,520,520
  516 IF(JX-NJJ) 518,519,519
  519 IF(JS-LUF) 521,520,520
  521 JS=JS+1
      JK=JK+1
      IULF(JK)=IULF(JS)
      GO TO 519
  520 LUF=ND
C*    PUTTING OPPORTUNISTICALLY REPLACED COMPONENTS INTO FAILED COMPONENTS'
C*    CLASS
      DO 525 J9=1,NJJ
      J10=LFA+J9
  525 ILFA(J10)=IOR(J9)
      LFA=LFA+NJJ
      IDR=LFA
C*    RE-EVALUATION TO CHECK IF ANY OTHER UNFAILED COMPONENT CAN BE REPLACED
C*    AT THIS OPPORTUNITY
      DO 522 J7=1,LUF
      IJ=IULF(J7)
      DO 523 J8=1,NJJ
      IK=IOR(J8)
      CALL PROB1(IJ,IK,RRL,BBL,FFL,NNF,IR,RSS)
      FS(IJ,IK)=1.0-RSS
  523 CONTINUE
  522 CONTINUE
      GO TO 524
  502 DO 503 L9=1,NJJ
      L10=LFA+L9
  503 ILFA(L10)=IOR(L9)
      LFA=LFA+NJJ
      IDR=LFA
  513 CONTINUE
      IF(IDR.EQ.IFP) GO TO 526
      WRITE(2,527) (IR(I),I=1,3),(ILFA(M),M=1,LFA)
  527 FORMAT(10X,3I4,7X,'REPLACE COMPONENTS',3I4)
      IFP=IDR
  526 CONTINUE
      DO 405 I=1,MUF
  405 LN(IUMF(I))=MN(IUMF(I))+1
      DO 413 I=1,MUF
  413 IULF(I)=IUMF(I)
```

```
      DO 414 J=1,MFA
  414 ILFA(J)=IMFA(J)
      LUF=MUF
      LFA=MFA
  702 CONTINUE
      LN(2)=1
  701 CONTINUE
      STOP
      END
C*    SUBROUTINE FIRST READS NUMBER OF ITEMS, NUMBER OF FAILED ITEMS, IDENTIFIC-
C*    ATION NUMBERS, ETC. IT CALCULATES FOR EACH UNFAILED ITEM THE PROPORTION OF
C*    LIFE THAT CAN BE LOST THROUGH EARLY RETIREMENT, HAVING SURVIVED UPTO
C*    AN AGE
      SUBROUTINE FIRST(IAL,IAU,IAN,NUF,IUNF,NFA,IFA,NA,IA,SR,RLL,BFL,FL,
     1NF)
C****************************************************************************
C*    MAIN SYMBOLS                                                          *
C*    ------------                                                          *
C*    BFF  =CUMULATIVE DISTRIBUTION FUNCTION FOR AN ITEM                    *
C*    BFL  =SAME AS BFF                                                     *
C*    BT   =SHAPE PARAMETER OF AN ITEM'S FAULURE DISTRIBUTION               *
C*    BV   =SAME AS BT                                                      *
C*    ET   =SCALE PARAMETER OF AN ITEM'S FAILURE DISTRIBUTION               *
C*    EV   =SAME AS ET                                                      *
C*    FF   =PROBABILITY DENSITY FUNCTION FOR AN ITEM                        *
C*    FL   =SAME AS FF                                                      *
C*    IA   =IDENTIFICATION NUMBERS OF ITEMS                                 *
C*    IAL  =LOWER VALUE OF THE TIME RANGE                                   *
C*    IAN  =INCREMENTAL VALUE OF THE TIME RANGE                             *
C*    IAU  =UPPER VALUE OF THE TIME RANGE                                   *
C*    IFA  =IDENTIFICATION NUMBERS OF FAILED ITEMS                          *
C*    IUNF =IDENTIFICATION NUMBERS OF UNFAILED ITEMS                        *
C*    NA   =NUMBER OF ITEMS                                                 *
C*    NAP  =PREVENTIVE REPLACEMENT AGE OF AN ITEM; WHEN THERE IS NO SUCH AGE *
C*          NAP=0                                                           *
C*    NF   =TIME AT WHICH FAILURE PROBABILITY OF AN ITEM IS 98'/.           *
C*    NB   =SAME AS NF                                                      *
C*    NFA  =NUMBER OF FAILED ITEMS                                          *
C*    NUF  =NUMBER OF UNFAILED ITEMS                                        *
C*    RLF  =RELIABILITY FUNCTION FOR AN ITEM                                *
C*    RLL  =SAME AS RLF                                                     *
C*    SR   =PROPORTION OF LIFE THAT WILL BE LOST THROUGH EARLY RETIREMENT   *
C*    XF1  =CONDITIONAL MEAN TIME TO FAILURE
C****************************************************************************
      DIMENSION IA(3),IFA(3),IUNF(3),BT(3),ET(3),RLF(250),BFF(250),FF(25
     10),RLL(3,250),FL(3,250),BFL(3,250),IAL(3),IAU(3),IAN(3),NF(3),QS(
     2250),SR(3,250),NAP(3)
      READ(1,201) NA
  201 FORMAT(I0)
      READ(1,203) (IA(I),I=1,NA)
  203 FORMAT(15I0)
      READ(1,201) NFA
      IF(NFA.EQ.0) GO TO 209
```

```
      READ(1,208) (IFA(JJ),JJ=1,NFA)
  209 CONTINUE
      NUF=NA-NFA
      READ(1,208) (IUNF(J1),J1=1,NUF)
C*    READING DISTRIBUTION PARAMETERS FOR ALL ITEMS
      DO 203 I=1,NA
      IL=IA(I)
      READ(1,204) BT(IL),ET(IL)
  204 FORMAT(2F0.0)
      BV=BT(IL)
      EV=ET(IL)
      CALL ZK1(BV,EV,RLF,BFF,NB,FF)
      NF(IL)=NB
      DO 205 IP=1,NB
      RLL(IL,IP)=RLF(IP)
      FL(IL,IP)=FF(IP)
  205 BFL(IL,IP)=BFF(IP)
  203 CONTINUE
      READ(1,208) (NAP(I),I=1,NA)
C*    READING TIME RANGES OVER WHICH CALCULATIONS WILL BE CARRIED OUT
      DO 206 J=1,NA
      IM=IA(J)
      READ(1,207) IAL(IM),IAU(IM),IAN(IM)
  207 FORMAT(3I0)
C*    SETTING THE UPPER VALUE OF TIME RANGE ATLEAST 4 TIME UNITS LOWER IF NF
C*    IS LESS THAN IAU
      IF(IAU(IM).GT.NF(IM)) IAU(IM)=NF(IM)-IAN(IM)*4
  206 CONTINUE
C*    CALCULATION OF THE PROPORTION OF LIFE THAT WILL BE LOST THROUGH EARLY
C*    RETIREMENT, WHEN THERE IS NO PREVENTIVE REPLACEMENT AGE
      DO 210 K1=1,NUF
      IL=IUNF(K1)
      IF(NAP(IL).NE.0) GO TO 320
      DO 700 IA1=IAL(IL),IAU(IL),IAN(IL)
      NL3=(NF(IL)-(IA1-1))
      DO 380 KK1=1,NL3
      IZ1=KK1+(IA1-1)
  380 GS(KK1)=RLL(IL,IZ1)
      HS=FLOAT(NL3)/2.0
      IHS=INT(HS)
      IF(HS-FLOAT(IHS)) 382,383,382
  382 CALL SIMPSON(NL3,GS,SA)
      GO TO 381
  383 CALL TRAPSIM(NL3,GS,SA)
  381 CONTINUE
      XF1=(SA/RLL(IL,IA1))+FLOAT(IA1)
      ISN=((IA1-IAL(IL))/IAN(IL))+1
      SR(IL,ISN)=(XF1-FLOAT(IA1))/XF1
  700 CONTINUE
      GO TO 210
C*    CALCULATION OF PROPORTION OF LIFE TO BE LOST WHEN THERE IS A PREVENTIVE
C*    REPLACEMENT AGE
  320 DO 321 IA1=IAL(IL),IAU(IL),IAN(IL)
```

```
      ISN=((IA1-IAL(IL))/IAN(IL))+1
      NL3=(NAP(IL)-(IA1-1))
      IF(NL3.EQ.1) GO TO 822
      IF(NL3.EQ.2) GO TO 823
      DO 824 KK=1,NL3
      IZ=KK+(IA1-1)
  824 QS(KK)=RLL(IL,IZ)
      HS=FLOAT(NL3)/2.0
      IHS=INT(HS)
      IF(HS-FLOAT(IHS)) 825,826,825
  825 CALL SIMPSON(NL3,QS,SA)
      GO TO 827
  826 CALL TRAPSIM(NL3,QS,SA)
      GO TO 827
  823 SA=(RLL(IL,NAP(IL))+RLL(IL,IA1))/2.0
  827 SR(IL,ISN)=(((FLOAT(IA1)*RLL(IL,IA1))-(FLOAT(NAP(IL))*RLL(IL,NAP(I
     1L))))+SA)/(RLL(IL,IA1)-RLL(IL,NAP(IL)))
      GO TO 821
  822 SR(IL,ISN)=0.0
  821 CONTINUE
  210 CONTINUE
      DO 828 I=1,NA
      IL=IUNF(I)
      IF(NAP(IL).NE.0) NF(IL)=NAP(IL)
  828 CONTINUE
      RETURN
      END
C*
C*    SUBROUTINE PROB1 CALCULATES THE FAILURE PROBABILITY OF AN UNFAILED ITEM
C*    BEFORE A FAILED ITEM
C*
      SUBROUTINE PROB1(IIJ,IIK,RSL,BSL,FSL,NSF,IRR,RS)
      DIMENSION NSF(3),IRR(3),SL(250),BSL(3,250),RSL(3,250),FSL(3,250)
      NL2=NSF(IIJ)-(IRR(IIJ)-1)
      IF(NSF(IIK)-NL2) 300,300,301
  301 NLV=NL2
      DO 306 KX=1,NLV
      ITX=KX+(IRR(IIJ)-1)
  306 SL(KX)=BSL(IIK,KX)*FSL(IIJ,ITX)/RSL(IIJ,IRR(IIJ))
      HN=FLOAT(NLV)/2.0
      IHN=INT(HN)
      IF(HN-FLOAT(IHN)) 303,304,303
  303 CALL SIMPSON(NLV,SL,RS)
      GO TO 302
  304 CALL TRAPSIM(NLV,SL,RS)
      GO TO 302
  300 NLV=NSF(IIK)
      DO 305 KX=1,NLV
      ITX=KX+(IRR(IIJ)-1)
  305 SL(KX)=FSL(IIK,KX)*(BSL(IIJ,ITX)-BSL(IIJ,IRR(IIJ)))/RSL(IIJ,IRR(II
     1J))
      HN=FLOAT(NLV)/2.
      IHN=INT(HN)
```

```
      IF(HN-FLOAT(IHN)) 371,372,371
  371 CALL SIMPSON(NLV,SL,RX)
      GO TO 373
  372 CALL TRAPSIM(NLV,SL,RX)
  373 RS=1.0-RX
  302 CONTINUE
      RETURN
      END
C*
C*    SUBROUTINE PROB2 FOR CALCULATING FAILURE PROBABILITY OF AN UNFAILED ITEM
C*    BEFORE ANOTHER UNFAILED ITEM
C*
      SUBROUTINE PROB2(LJJ,LKK,RTL,BTL,FTL,NTF,ITR,RT)
      DIMENSION NTF(3),ITR(3),SM(250),BTL(3,250),RTL(3,250),FTL(3,250)
      NL=NTF(LJJ)-(ITR(LJJ)-1)
      NM=NTF(LKK)-(ITR(LKK)-1)
      IF(NL-NM) 306,306,307
  306 NLX=NL
      DO 507 KY=1,NLX
      ITY=KY+(ITR(LJJ)-1)
      ITZ=KY+(ITR(LKK)-1)
  507 SM(KY)=((BTL(LKK,ITZ)-BTL(LKK,ITR(LKK)))/RTL(LKK,ITR(LKK)))
     1*(FTL(LJJ,ITY)/RTL(LJJ,ITR(LJJ)))
      HM=FLOAT(NLX)/2.0
      IHM=INT(HM)
      IF(HM-FLOAT(IHM)) 309,310,309
  309 CALL SIMPSON(NLX,SM,RT)
      GO TO 311
  310 CALL TRAPSIM(NLX,SM,RT)
      GO TO 311
  307 NLX=NM
      DO 374 KY=1,NLX
      ITY=KY+(ITR(LJJ)-1)
      ITZ=KY+(ITR(LKK)-1)
  374 SM(KY)=((BTL(LJJ,ITY)-BTL(LJJ,ITR(LJJ)))/RTL(LJJ,ITR(LJJ)))
     1*(FTL(LKK,ITZ)/RTL(LKK,ITR(LKK)))
      HM=FLOAT(NLX)/2.
      IHM=INT(HM)
      IF(HM-FLOAT(IHM)) 375,376,375
  375 CALL SIMPSON(NLX,SM,RY)
      GO TO 308
  376 CALL TRAPSIM(NLX,SM,RY)
  308 RT=1.0-RY
  311 CONTINUE
      RETURN
      END
C*
C*    SUBROUTINE ZK1 CALCULATES PROBABILITY DENSITY, CUMULATIVE DISTRIBUTION
C*    AND RELIABILITY FUNCTIONS UNTIL FAILURE PROBABILITY OF AN ITEM IS
C*    GREATER THAN 99%.
C*
      SUBROUTINE ZK1(B,TE,RL,BF,NPP,F)
      DIMENSION EX(250),BF(250),RL(250),F(250)
```

```
      N=50
      DO 69 KK=1,N
      EX(KK)=(FLOAT(KK)/TE)**5
   69 BF(KK)=1.0-EXP(-EX(KK))
      NPP=N+1
   72 EX(NPP)=(FLOAT(NPP)/TE)**5
      BF(NPP)=1.0-EXP(-EX  (NPP))
      IF(BF(NPP)-.98) 71,71,70
   71 NPP=NPP+1
      GO TO 72
   70 CONTINUE
      DO 82 LI=1,NPP
   82 RL(LI)=1.0-BF(LI)
      F(1)=BF(1)
      DO 75 KJ=2,NPP
   75 F(KJ)=BF(KJ)-BF(KJ-1)
      RETURN
      END
C*
C*    SUBROUTINE SIMPSON IS FOR INTEGRATION
C*
      SUBROUTINE SIMPSON(KOP,ZH,ZR)
      DIMENSION ZH(250)
      LM1=KOP-1
      SUM1=0.0
      DO 801 IN=2,LM1,2
  801 SUM1=ZH(IN)+SUM1
      LM2=KOP-2
      SUM2=0.0
      IF(LM2-3) 803,804,804
  804 DO 802 IR=3,LM2,2
  802 SUM2=ZH(IR)+SUM2
  803 CONTINUE
      ZR=(ZH(1)+(4.0*SUM1)+(2.0*SUM2)+ZH(KOP))/3.0
      RETURN
      END
C*
C*    SUBROUTINE TRAPSIM IS FOR INTEGRATION
C*
      SUBROUTINE TRAPSIM(KN,ZS,ZT)
      DIMENSION ZS(250)
      LM3=KN-1
      SUM3=0.0
      DO 221 IM=3,LM3,2
  221 SUM3=ZS(IM)+SUM3
      LM4=KN-2
      SUM4=0.0
      IF(LM4-4) 222,223,223
  223 DO 224 IQ=4,LM4,2
  224 SUM4=ZS(IQ)+SUM4
  222 ZT=((ZS(2)+ZS(1))/2.0)+((ZS(2)+(4.0*SUM3)+(2.0*SUM4)+ZS(KN))/3.0)
      RETURN
      END
      FINISH
****
```

APPENDIX C1

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OPTIMAL TEST INTERVALS FOR SAFETY TRIP SYSTEMS

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APPENDIX D1

List of PREMAIN Programme

```
      MASTER PREMAIN
      FINDING PREVENTIVE REPLACEMENT AGE
C
C*****************************************************************************
C*    MAIN SYMBOLS                                                           *
C*    ------------                                                           *
C*    B     =SHAPE PARAMETER OF THE FAILURE DISTRIBUTION                     *
C*    BF    =CUMULATIVE DISTRIBUTION FUNCTION OF A COMPONENT                 *
C*    CF    =COST OF A FAILURE MAINTENANCE                                   *
C*    CORF  =TOTAL COST OF FAILURE MAINTENANCE / UNIT TIME                   *
C*    COST  =TOTAL COST OF PREVENTIVE MAINTENANCE/ UNIT TIME                 *
C*    CP    =COST OF A PREVENTIVE MAINTENANCE                                *
C*    CRL   =INTEGRAL VALUE OF THE RELIABILITY FUNCTION                      *
C*    ETA   =SCALE PARAMETER OF THE FAILURE DISTRIBUTION                     *
C*    F     =PROBABILITY DENSITY FUNCTION OF A COMPONENT                     *
C*    FMT   =MEAN TIME TO FAILURE OF A COMPONENT                             *
C*    HEAD  =HEADING OF THE PROBLEM OR IDENTIFICATION REMARK ABOUT THE ITEM  *
C*    ITP   =PREVENTIVE MAINTENANCE AGE                                      *
C*    N     =PRE-ASSIGNED TIME UNITS UPTO WHICH DENSITY AND RELIABILITY FUNCTI- *
C*           ONS ARE CALCULATED IN THE FIRST INSTANCE                        *
C*    NC    =TIME UNIT WHEN THE PROBABILITY OF FAILURE OF A COMPONENT IS     *
C*           ATLEAST 99.5'/.                                                 *
C*    NX    =NUMBER OF COMPONENTS FOR WHICH PREVENTIVE MAINTENANCE AGE IS TO *
C*           BE FOUND                                                        *
C*    RA    =RATIO OF CF TO CP                                               *
C*    RL    =RELIABILITY FUNCTION OF A COMPONENT                             *
C*    SVGA  =PERCENTAGE SAVINS FROM THE CHOSEN MAINTENANCE ACTION            *
C*    TF    =TIME REQUIRED TO EFFECT A FAILURE MAINTENANCE                   *
C*    TI    =TIME REQUIRED TO EFFECT A PREVENTIVE MAINTENANCE                *
C*    WFR   =HAZARD RATE OF A COMPONENT                                      *
C*    YY    =AN APPROXIMATE RELATIONSHIP FOR EVALUATING GAMMA FUNCTION       *
C*****************************************************************************
      DIMENSION BF(200),CRL(200),F(200),RL(200),EX(200),COST(200),WFR(
     1200)
      COMMON BF,CRL,F,WFR,RL,EX,COST,B,ETA,N,FMT,NC
      INTEGER HEAD(20)
      DO 580 NX=1,3
      READ(1,299) HEAD
  299 FORMAT(90A2)
      WRITE(2,130) HEAD
  130 FORMAT(1H,5X,90A2)
      WRITE(2,132)
  132 FORMAT(6X,20('-'),/)
      READ(1,579) N,B,ETA,TI,TF,CP,CF
  579 FORMAT(I5,6F0.0)
      CALL ZK1
      WRITE(2,150)
  150 FORMAT(6X,17H RELATIVE COST OF,8X,8H OPTIMAL,7X,11H SAVINGS IN,4X,
     19H COST PER,/,3X,20H FAILURE REPLACEMENT,8X,4H AGE,8X,11H PERCENTA
     2GE,4X,10H UNIT TIME,/)
      RA=CF/CP
      IT=1
      COST(IT)=(RL(IT)+(RA*BF(IT)))/(CRL(IT)+(TI*RL(IT))+(TF*BF(IT)))
      COPA=COST(IT)
```

```
      IT=2
   32 COST(IT)=(RL(IT)+(RA*BF(IT)))/(CRL(IT)+(TI*RL(IT))+(TF*BF(IT)))
      IF(COST(IT)-COPA) 30,31,31
   30 COPA=COST(IT)
      IF(IT-NC) 180,181,181
  180 IT=IT+1
      GO TO 32
   31 ITP=IT-1
      SVGA=100.0*(1.0-((COPA*FMT)/RA))
      IF(SVGA) 34,34,35
   35 CONTINUE
      WRITE(2,151) RA,ITP,SVGA,COPA
  151 FORMAT(13X,F4.1,17X,I2,12X,F4.1,13X,F4.2)
      GO TO 580
   34 CONTINUE
  181 CORF=RA/FMT
      COPA=CORF
      WRITE(2,152) RA,COPA
  152 FORMAT(1X,T14,F4.1,12X,'REPLACE ON FAILURE ONLY',T66,F4.2)
  580 CONTINUE
      STOP
      END
C     SUBROUTINE FOR WEIBULL ANALYSIS
      SUBROUTINE ZK1
      DIMENSION BF(200),CRL(200),F(200),RL(200),EX(200),COST(200),WFR(
     1200)
      COMMON BF,CRL,F,WFR,RL,EX,COST,B,ETA,N,FMT,NC
      FMT=YY(1.0+(1.0/B))*ETA
C     FINDING WEIBULL PROBABILITIES
      DO 69 KK=1,N
      EX(KK)=(FLOAT(KK)/ETA)**B
   69 BF(KK)=1.0-EXP(-EX(KK))
      NC=NC+1
   72 EX(NC)=(FLOAT(NC)/ETA)**B
      BF(NC)=1.0-EXP(-EX(NC))
      IF(BF(NC)-0.995) 71,71,70
   71 NC=NC+1
      GO TO 72
   70 CONTINUE
      DO 82 LI=1,NC
   82 RL(LI)=1.0-BF(LI)
      F(1)=BF(1)
      DO 75 KJ=2,NC
   75 F(KJ)=BF(KJ)-BF(KJ-1)
C     FINDING HAZARD RATE
      DO 81 LL=1,NC
   81 WFR(LL)=F(LL)/RL(LL)
      CRL(1)=(1.0+RL(1))/2.0
      DO 83 LJ=2,NC
   83 CRL(LJ)=CRL(LJ-1)+((RL(LJ-1)+RL(LJ))/2.0)
      RETURN
      END
```

APPENDIX D2

List of STRATEGY Programme

```
      MASTER STRATEGY
C****************************************************************************
C*    MAIN SYMBOLS                                                          *
C*    ------------                                                          *
C*    CH   =ESTIMATED COST OF A HAZARD                                      *
C*     CM   =LOST PRODUCTION COST/ HOUR FOR THE MAIN PLANT                  *
C*    CP   =LOST PRODUCTION COST / HOUR FOR THE GENERATOR PLANT             *
C*    CS   =COST OF A START - UP                                            *
C*    IA   =IDENTIFICATION NUMBER OF COMPONENTS IN EACH CLASS               *
C*    IFA  =IDENTIFICATION NUMBER OF FAILED COMPONENTS IN EACH CLASS        *
C*    IUNF =IDENTIFICATION NUMBER OF UNFAILED COMPONENTS IN EACH CLASS      *
C*    NA   =NUMBER OF COMPONENTS IN EACH CLASS                              *
C*    NCL  =NUMBER OF CLASSES                                               *
C*    NFA  =NUMBER OF FAILED COMPONENTS IN EACH CLASS                       *
C*    NUF  =NUMBER OF UNFAILED COMPONENTS IN EACH CLASS                     *
C*    TCS  =TIME IN HOURS FOR WHICH MAIN PLANT CAN RUN WITHOUT STOPPAGE     *
C*    TCS3 =MAIN PLANT SHUTDOWN TIME IN HOURS FROM CLASS3                   *
C*    TCS4 =MAIN PLANT SHUTDOWN TIME IN HOURS FROM CLASS4                   *
C*    TS   =GENERATOR PLANT SHUTDOWN TIME DUE TO FAILED ITEMS ONLY          *
C*    TS3  =GENERATOR PLANT SHUTDOWN TIME FROM CLASS3                       *
C*    TS4  =GENERATOR PLANT SHUTDOWN TIME FROM CLASS4                       *
C*    TXZ  =MAIN PLANT SHUTDOWN TIME FROM CLASS2                            *
C*    TZC  =MAIN PLANT SHUTDOWN TIME FROM CLASS1                            *
C*    TZS  =GENERATOR PLANT SHUTDOWN TIME FROM CLASS1                       *
C*    TZZ  =GENERATOR PLANT SHUTDOWN TIME FROM CLASS2                       *
C****************************************************************************
      DIMENSION NA(5),IA(5,10),NFA(5),IFA(5,10),NUF(5),IUNF(5,10)
C*
C*    READING NUMBER OF CLASSES AND IDENTIFICATION NUMBERS OF ALL ITEMS
      READ(1,200) NCL
  200 FORMAT(I0)
      READ(1,201) (NA(K),K=1,NCL)
  201 FORMAT(5I0)
      DO 202 K=1,NCL
      READ(1,208) (IA(K,I),I=1,NA(K))
  208 FORMAT(10I0)
  202 CONTINUE
      READ(1,201) (NFA(K),K=1,NCL)
      DO 209 KK=1,NCL
      IF(NFA(KK).EQ.0) GO TO 209
      READ(1,208) (IFA(KK,JJ),JJ=1,NFA(KK))
  209 CONTINUE
C*
C*    SCREENING OUT THE UNFAILED COMPONENTS IN EACH CLASS
      CALL SCREEN(NCL,NA,IA,NFA,IFA,NUF,IUNF)
C*    READING TIMES OF SHUTDOWN FOR GENERATOR AND MAIN PLANTS, STORAGE CAPACITY
C*    AND VARIOUS COSTS
      READ(1,252) TS,TCS,CP,CM,CS,CH
  252 FORMAT(6F0.0)
C*    EXECUTING OPPORTUNITY PROGRAMMES FOR DIFFERENT CLASSES OF ITEMS
      EXECUTING OPPORTUNITY PROGRAMMES FOR,IFA,NUF,IUNF,TZS,TZC,CP,CM,CS)
      CALL CLASS1(TS,TCS,NA,IA,NFA,IFA,NUF,IUNF,TZZ,TXZ,CP,CM,CS,TCS)
      CALL CLASS2(TZS,TZC,NFA,IFA,NUF,IUNF,TS3,TCS3,CP,CM,CS,CH,TCS)
      CALL CLASS3(TZZ,TXZ,NFA,IFA,NUF,IUNF,TS
```

```
      CALL CLASS4(TS3,TCS3,NFA,IFA,NUF,IUNF,TS4,TCS4,CP,CM,CS,CH,TCS)
      WRITE(2,253) TS4,TCS4
  253 FORMAT(5X,'REACTOR SYSTEM BEING SHUTDOWN FOR',F5.1,'HOURS',/,5X,
     1'MAIN PLANT BEING SHUTDOWN FOR',F5.1,'HOURS')
      STOP
      END
C*
C*    SUBROUTINE SCREEN FOR SEPARATING FAILED AND UNFAILED ITEMS
      SUBROUTINE SCREEN(ICL,KA,IKA,KFA,IKFA,KUF,IUKF)
      DIMENSION KA(5),IKA(5,10),KFA(5),IKFA(5,10),KUF(5),IUKF(5,10)
      DO 241 KCL=1,ICL
      KUF(KCL)=KA(KCL)-KFA(KCL)
      IF(KFA(KCL).EQ.0) GO TO 254
      JKK=0
      JXX=0
      JSS=0
  248 JXX=JXX+1
  247 JSS=JSS+1
      IF(IKFA(KCL,JXX).EQ.IKA(KCL,JSS)) GO TO 246
      JKK=JKK+1
      IUKF(KCL,JKK)=IKA(KCL,JSS)
      IF(JSS-KA(KCL)) 247,250,250
  246 IF(JXX-KUF(KCL)) 248,249,249
  249 IF(JSS-KA(KCL)) 251,250,250
  251 JSS=JSS+1
      JKK=JKK+1
      IUKF(KCL,JKK)=IKA(KCL,JSS)
      GO TO 249
  254 DO 255 J=1,KUF(KCL)
  255 IUKF(KCL,J)=IKA(KCL,J)
  250 CONTINUE
  241 CONTINUE
      RETURN
      END
C*
C*    SUBROUTINE CLASS1 FOR DETERMINING OPPORTUNISTIC MAINTENANCE PLANS FOR
C*    CLASS1 EQUIPMENTS
      SUBROUTINE CLASS1(TSS,TCSS,LA,ILA,LFA,ILFA,LUF,IULF,TSZ,TCZ,CP1,
     1CM1,CS1)
C***********************************************************************
C*    MAIN SYMBOLS                                                     *
C*    ------------                                                     *
C*    BFF   =CUMULATIVE DISTRIBUTION FUNCTION FOR AN ITEM              *
C*    BFL   =SAME AS BFF                                               *
C*    BV    =SHAPE PARAMETER OF THE WEIBULL FAILURE DISTRIBUTION FOR AN ITEM *
C*    CR    =COST OF REPAIR / REPLACEMENT OF AN ITEM                   *
C*    EV    =SCALE PARAMETER                                           *
C*    FF    =PROBABILITY DENSITY FUNCTION OF THE FAILURE TIMES OF AN ITEM *
C*    FL    =SAME AS FF                                                *
C*    FS(I,J)=PROBABILITY OF I FAILING BEFORE J                        *
C*    IOR   =IDENTIFICATION NUMBER OF OPPORTUNISTICALLY MAINTAINED ITEMS *
C*    NAP   =PREVENTIVE REPLACEMENT / MAINTENANCE AGE OF AN ITEM ( IF THERE IS *
C*          NO SUCH AGE, NAP=0 )
```

```
C*    NAU   =PRESENT AGE OF AN ITEM ( FOR FAILED ITEM, IT IS PUT AS 1 )
C*    NB    =TIME WHEN THE FAILURE PROBABILITY OF AN ITEM IS ATLEAST 98/        *
C*    PF(J)=PROBABILITY THAT J WILL BE REPLACED ALONE AT A LATER DATE          *
C*    RLF   =RELIABILITY FUNCTION OF AN ITEM                                    *
C*    RLL   =SAME AS RLF                                                        *
C*    RSS   =PROBABILITY OF SURVIVAL , (1 - FS )                                *
C*    RTT   =SAME AS RSS                                                        *
C*    TCZ   =MODIFIED MAIN PLANT SHUTDOWN TIME                                  *
C*    TMS   =MAIN PLANT SHUTDOWN TIME AT THE BEGINNING                          *
C*    TR    =TIME OF REPAIR / REPLACEMENT OF AN ITEM                            *
C*    TSZ   =MODIFIED GENERATOR PLANT SHUTDOWN TIME                             *
C*    TX    =EXTRA TIME FOR WHICH GENERATOR WILL BE SHUTDOWN, IF AN UNFAILED    *
C*           COMPONENT IS REPLACED                                              *
C*    TY    =EXTRA TIME FOR WHICH MAIN PLANT WILL BE SHUTDOWN                   *
C*    TZ    =TIME FOR WHICH MAIN PLANT WILL BE SHUTDOWN IF AN UNFAILED ITEM     *
C*           IS REPLACED / REPAIRED ALONE AT A LATER DATE                       *
C*****************************************************************************
      DIMENSION CR(10),TR(10),NAU(10),NAP(10),LA(5),ILA(5,10),NF(10),
     1LFA(5),ILFA(5,10),RLL(10,300),FL(10,300),BFL(10,300),RLF(300),FF(
     2300),BFF(300),GS(300),SR(10),LUF(5),IULF(5,10),FS(10,10),PF(10),
     3IOR(10)
C*    DEFINING THE GAIN FUNCTION FOR UNFAILED ITEMS OF CLASS1
      CGN1(CRR,SRR,CPP,TRR,CSS,PFF,TXX,TYY,TZZ,CMP)=(((CPP*TRR)+(CMP*TZZ
     1)+CSS)*PFF)-(CRR*SRR)-(CPP*TXX)-(CMP*TYY)
C*    READING RELEVANT COSTS AND TIMES FOR UNFAILED ITEMS
      READ(1,530) (CR(IULF(1,I)),I=1,LUF(1))
      READ(1,530) (TR(IULF(1,I)),I=1,LUF(1))
  530 FORMAT(10F0.0)
C*    READING PRESENT AGE AND PREVENTIVE MAINTENANCE AGE OF THE ITEMS
      READ(1,532) (NAU(ILA(1,I)),I=1,LA(1))
      READ(1,532) (NAP(ILA(1,I)),I=1,LA(1))
  532 FORMAT(10I0)
C*    READING DISTRIBUTION PARAMETERS AND THEN CALCULATING RELIABILITY AND
C*    DENSITY FUNCTIONS
      DO 203 I=1,LA(1)
      IL=ILA(1,I)
      READ(1,204) BV,EV
  204 FORMAT(2F0.0)
      CALL ZK1(BV,EV,RLF,BFF,NB,FF)
      NF(IL)=NB
      DO 205 IP=1,NB
      RLL(IL,IP)=RLF(IP)
      FL(IL,IP)=FF(IP)
  205 BFL(IL,IP)=BFF(IP)
  203 CONTINUE
C*    CALCULATING PROPORTION OF USEFUL LIFE THAT WILL BE LOST THROUGH EARLY
C*    RETIREMENT
      DO 210 K1=1,LUF(1)
      IL=IULF(1,K1)
      IF(NAP(IL)) 384,384,385
  384 IA1=NAU(IL)
      NL3=(NF(IL)-(IA1-1))
      DO 380 KK1=1,NL3
```

```
      IZ1=KK1+(IA1-1)
  380 QS(KK1)=RLL(IL,IZ1)
      HS=FLOAT(NL3)/2.0
      IHS=INT(HS)
      IF(HS-FLOAT(IHS)) 382,383,382
  382 CALL SIMPSON(NL3,QS,SA)
      GO TO 381
  383 CALL TRAPSIM(NL3,QS,SA)
  381 CONTINUE
      XF1=(SA/RLL(IL,IA1))+FLOAT(IA1)
      SR(IL)=(XF1-FLOAT(IA1))/XF1
      GO TO 210
  385 IA1=NAU(IL)
      NL3=(NAP(IL)-(IA1-1))
      IF (NL3.EQ.1) GO TO 822
      IF (NL3.EQ.2) GO TO 823
      DO 824 KK=1,NL3
      IZ=KK+(IA1-1)
  824 QS(KK)=RLL(IL,IZ)
      HS=FLOAT(NL3)/2.0
      IHS=INT(HS)
      IF(HS-FLOAT(IHS)) 825,826,825
  825 CALL SIMPSON (NL3,QS,SA)
      GO TO 827
  826 CALL TRAPSIM (NL3,QS,SA)
      GO TO 827
  823 SA=(RLL(IL,NAP(IL))+RLL(IL,IA1))/2.0
  827 SR(IL)=(((FLOAT(IA1)*RLL(IL,IA1))-(FLOAT(NAP(IL))*RLL(IL,NAP(IL)))
     1)+SA)/(RLL(IL,IA1)-RLL(IL,NAP(IL)))
      GO TO 821
  822 SR(IL)=0.0
  821 CONTINUE
  210 CONTINUE
C*    CALCULATION OF THE PROBABILITY OF NOT GETTING ANOTHER OPPORTUNITY FOR
C*    MAINTAINING EACH OF THE UNFAILED COMPONENTS IN RELATION TO FAILED ITEMS
      DO 401 J2=1,LUF(1)
      IJ=IULF(1,J2)
      DO 402 J3=1,LFA(1)
      IK=ILFA(1,J3)
      CALL PROB1(IJ,IK,RLL,BFL,FL,NF,NAU,RSS)
      FS(IJ,IK)=1.0-RSS
  402 CONTINUE
  401 CONTINUE
C*    CALCULATION OF THE PROBABILITY OF NOT GETTING ANOTHER OPPORTUNITY IN
C*    RELATION TO THE OTHER UNFAILED COMPONENTS
      NFF=LUF(1)-1
      DO 403 J4=1,NFF
      IJJ=IULF(1,J4)
      J5=J4+1
      DO 404 J6=J5,LUF(1)
      IKK=IULF(1,J6)
      CALL PROB2(IJJ,IKK,RLL,BFL,FL,NF,NAU,RTT)
      FS(IJJ,IKK)=1.0-RTT
```

```
      FS(IKK,IJJ)=RTT
  404 CONTINUE
  403 CONTINUE
C*    CALCULATION OF THE OVERALL PROBABILITY OF NOT GETTING ANOTHER OPPORTUNITY
C*    OF GROUP MAINTENANCE
  524 DO 508 I=1,LUF(1)
      ID=IULF(1,I)
      PF(ID)=1.0
      DO 509 J=1,LA(1)
      IE=ILA(1,J)
      IF(ID.EQ.IE) GO TO 509
      PF(ID)=PF(ID)*FS(ID,IE)
  509 CONTINUE
  508 CONTINUE
      NJJ=0
      TSF=TSS
      TCSF=TCSS
      IF(TSF-TCSF) 900,900,901
  900 TMS=0.0
      GO TO 902
  901 TMS=TSF-TCSF
  902 CONTINUE
      TMSS=TMS
C*    FINDING IF ANY UNFAILED COMPONENT CAN BE MAINTAINED AT THIS OPPORTUNITY
      DO 510 I=1,LUF(1)
      ID=IULF(1,I)
      IF(TR(ID)-TSS) 551,551,552
  551 TX=0.0
      TY=0.0
      GO TO 555
  552 TX=TR(ID)-TSS
      TSS=TR(ID)
      IF(TMS) 553,553,554
  554 TY=TX
      GO TO 555
  553 IF(TSS-TCSF) 556,556,559
  556 TY=0.0
      GO TO 555
  559 TY=TSS-TCSF
      TMS=TY
  555 CONTINUE
      IF(TR(ID)-TCSF) 892,892,893
  892 TZ=0.0
      GO TO 894
  893 TZ=TR(ID)-TCSF
  894 CONTINUE
      Y1=CGN1(CR(ID),SF(ID),CP1,TR(ID),CS1,PF(ID),TX,TY,TZ,CM1)
      IF(Y1) 511,511,512
  512 NJJ=NJJ+1
      IOR(NJJ)=ID
      GO TO 510
  511 CONTINUE
      IF(NJJ) 557,557,558
```

```
  557 TSS=TSF
      TMS=TMSS
  558 CONTINUE
  510 CONTINUE
C*    SCREENING OUT COMPONENTS WHICH CAN BE MAINTAINED OPPORTUNISTICALLY
      ND=LUF(1)-NJJ
      IF(NJJ) 513,513,514
  514 IF(ND) 502,502,515
  515 JK=0
      JX=0
      JS=0
  518 JX=JX+1
  517 JS=JS+1
      IF(IOR(JX).EQ.IULF(1,JS)) GO TO 516
      JK=JK+1
      IULF(1,JK)=IULF(1,JS)
      IF(JS-LUF(1)) 517,520,520
  516 IF(JX-NJJ) 518,519,519
  519 IF(JS-LUF(1)) 521,520,520
  521 JS=JS+1
      JK=JK+1
      IULF(1,JK)=IULF(1,JS)
      GO TO 519
  520 LUF(1)=ND
C*    PUTTING OPPORTUNISTICALLY MAINTAINED COMPONENT INTO FAILED COMPONENTS
C*    CATEGORY FOR FURTHER EVALUATION
      DO 525 J9=1,NJJ
      J10=LFA(1)+J9
  525 ILFA(1,J10)=IOR(J9)
      LFA(1)=LFA(1)+NJJ
C*    RE-EVALUATING IF ANY OTHER UNFAILED COMPONENT CAN BE MAINTAINED
      DO 522 J7=1,LUF(1)
      IJ=IULF(1,J7)
      DO 523 J8=1,NJJ
      IK=IOR(J8)
      CALL PROB1(IJ,IK,RLL,BFL,FL,NF,NAU,RSS)
      FS(IJ,IK)=1.0-RSS
  523 CONTINUE
  522 CONTINUE
      GO TO 524
  502 DO 503 L9=1,NJJ
      L10=LFA(1)+L9
  503 ILFA(1,L10)=IOR(L9)
      LFA(1)=LFA(1)+NJJ
  513 CONTINUE
      TSZ=TSS
      TCZ=TMSS
      IF(LFA(1)) 857,857,858
  858 WRITE(2,527) (ILFA(1,M),M=1,LFA(1))
  527 FORMAT(5X,'REPLACE/REPAIR COMPONENTS FROM CLASS1',10I4)
  857 CONTINUE
      RETURN
      END
```

```
C*
C*    SUBROUTINE CLASS2 FOR DETERMINING OPPORTUNISTIC MAINTENANCE PLANS FOR
C*    COMPONENTS OF CLASS2 CATEGORY
      SUBROUTINE CLASS2(TFS,TSC,MFA,IMFA,MUF,IUMF,TS2,TCS2,CP2,CM2,CS2,R
     1CS)
C***************************************************************************
C*    MAIN SYMBOLS                                                         *
C*    ------------                                                         *
C*    CR2   =COST OF REPAIR / REPLACEMENT FOR EACH ITEM                     *
C*    FR    =FAILURE RATE / YEAR FOR EACH ITEM                              *
C*    MJJ   =NUMBER OF ITEMS BEING OPPORTUNISTICALLY TESTED AND REPAIRED /  *
C*           REPLACED ON FAILURE                                           *
C*    MOR   =IDENTIFICATION NUMBERS OF UNFAILED ITEMS WHICH WILL BE OPPORTUNIST-*
C*           ICALLY MAINTAINED                                             *
C*    PP    =PROBABILITY OF FAILURE DURING THE TIME SINCE LAST TEST         *
C*    PT    =PROBABILITY OF FAILURE DURING THE TEST INTERVAL               *
C*    TCS2  =MODIFIED TIME OF MAIN PLANT SHUTDOWN                          *
C*    TI    =TEST INTERVAL IN DAYS FOR EACH ITEM                           *
C*    TIP   =TIME SINCE LAST TEST FOR EACH ITEM                            *
C*    TR2   =TIME NEEDED FOR REPAIR / REPLACEMENT IN HOURS                 *
C*    TS2   =MODIFIED TIME OF GENERATOR SHUTDOWN                           *
C*    TX2   =EXTRA TIME IN HOURS FOR WHICH GENERATOR WILL BE SHUTDOWN      *
C*    TY2   =EXTRA TIME IN HOURS FOR WHICH MAIN PLANT WILL BE SHUTDOWN     *
C*    TZ2   =TIME FOR WHICH MAIN PLANT WILL BE SHUTDOWN IF THE UNFAILED ITEM IS *
C*           TO BE REPLACED ALONE AT A LATER DATE                          *
C***************************************************************************
      DIMENSION MFA(5),IMFA(5,10),MUF(5),IUMF(5,10),CR2(10),TR2(10),TI(
     110),TIP(10),FR(10),PT(10),PP(10),MOR(10)
C*
C*    DEFINING THE GAIN FUNCTION FOR CLASS2 ITEMS
      CGN2(CRX,CPX,TRX,CMPX,TZX,CSX,TXY,TYX,PFW,PFP)=((CRX+(CPX*TRX)+(
     1CMPX*TZX)+CSX)*PFW)-(CRX+(CPX*TXY)+(CMPX*TYX))*PFP
C*    READING VARIOUS COSTS AND TIMES FOR DETERMINING MAINTENANCE ACTIONS
      READ(1,570)  (CR2(IUMF(2,I)), I=1,MUF(2))
      READ(1,570)  (TR2(IUMF(2,I)), I=1,MUF(2))
      READ(1,570)  (TI(IUMF(2,I)), I=1,MUF(2))
      READ(1,570)  (TIP(IUMF(2,I)), I=1,MUF(2))
      READ(1,570)  (FR(IUMF(2,I)), I=1,MUF(2))
  570 FORMAT(10F8.0)
C*
C*    CALCULATING VARIOUS PROBABILITIES
      DO 571 I=1,MUF(2)
      ID=IUMF(2,I)
      PT(ID)=1.0-EXP(-(FR(ID)* TI(ID)/365.0))
  571 PP(ID)=1.0-EXP(-(FR(ID)*TIP(ID)/365.0))
      MJJ=0
      TSSF=TFS
      TCSSF=TSC
      DO 572 I=1,MUF(2)
      ID=IUMF(2,I)
      IF(TR2(ID)-TFS) 573,573,574
  573 TX2=0.0
      TY2=0.0
```

```
      GO TO 575
  574 TX2=TR2(ID)-TFS
      TFS=TR2(ID)
      IF(TSC) 576,576,577
  577 TY2=TX2
      GO TO 575
  576 IF(TFS-RCS) 590,590,591
  590 TY2=0.0
      GO TO 575
  591 TY2=TFS-RCS
      TSC=TY2
  575 CONTINUE
      IF(TR2(ID)-RCS) 579,579,580
  579 TZ2=0.0
      GO TO 581
  580 TZ2=TR2(ID)-RCS
  581 CONTINUE
      Y2=CGN2(CR2(ID),CP2,TR2(ID),CM2,TZ2,CS2,TX2,TY2,PT(ID),PP(ID))
      IF(Y2) 582,582,583
  583 MJJ=MJJ+1
      MOR(MJJ)=ID
      GO TO 572
  582 CONTINUE
      IF(MJJ) 584,584,585
  584 TFS=TSSF
      TSC=TCSSF
  585 CONTINUE
  572 CONTINUE
      IF(MJJ) 586,586,587
  587 DO 596 M9=1,MJJ
      M10=MFA(2)+M9
  596 IMFA(2,M10)=MOR(M9)
      MFA(2)=MFA(2)+MJJ
  586 CONTINUE
      TS2=TFS
      TCS2=TSC
      IF(MFA(2)) 588,588,589
  589 WRITE(2,597) (IMFA(2,I),I=1,MFA(2))
  597 FORMAT(5X,'TEST AND ON FAILURE REPLACE/ REPAIR COMPONENTS FROM CLA
     1SS2',10I4)
  588 CONTINUE
      RETURN
      END
C*
C*    SUBROUTINE CLASS3 FOR DETERMINING OPPORTUNISTIC MAINTEANCE PLANS FOR
C*    CLASS3 EQUIPMENTS
      SUBROUTINE CLASS (TSP,TCSP,JFA,IJFA,JUF,IUJF,TSF3,TCSP3,CP3,CMP3,
     1CS3,CH3,TCS3)
C***********************************************************************
C*                                                                     *
C*    MAIN SYMBOLS                                                     *
C*    ------------                                                     *
C*    CR3  =COST OF REPAIR / REPLACEMENT FOR EACH ITEM                 *
C*    DR3  =DEMAND RATE / YEAR FOR EACH ITEM
```

```
C*    FR3   =FAILURE RATE / YEAR FOR EACH ITEM
C*    JJJ   =NUMBER OF ITEMS BEING OPPURTUNISTICALLY MAINTAINED                     *
C*    JOR   =IDENTIFICATION NUMBERS OF OPPORTUNISTICALLY MAINTAINED ITEMS           *
C*    PH    =PROBABILITY OF HAZARD DURING REPAIR / REPLACEMENT                      *
C*    PP3   =PROBABILITY OF FAILURE DURING THE INTERVAL SINCE LAST TEST             *
C*    PS    =PROBABILITY OF SUTDOWN DUE TO FAILURE OF AN ITEM                       *
C*    PT3   =PROBABILITY OF FAILURE DURING THE TEST INTERVAL                        *
C*    TCSP3=MODIFIED TIME OF MAIN PLANT SHUTDOWN                                    *
C*    TI3   =TEST INTERVAL IN DAYS FOR EACH ITEM                                    *
C*    TIP3 ==TIME SINCE LAST TEST FOR EACH ITEM                                     *
C*    TR3   = TIME NEEDED FOR REPAIR / REPLACEMENT FOR EACH ITEM                    *
C*    TSF3 =MODIFIED TIME FOR GENERATOR SHUTDOWN                                    *
C*    TX3   =EXTRA TIME FOR WHICH GENERATOR WILL BE SHUTDOWN                        *
C*    TY3   =EXTRA TIME FOR WHICH MAIN PLANT WILL BE SHUTDOWN                       *
C*    TZ3   =TIME OF MAIN PLANT SHUTDOWN IF AN UNFAILED ITEM IS TO BE REPAIRED /*
C*           REPLACED ALONE AT A LATER DATE                                         *
C*******************************************************************************
      DIMENSION JFA(5),IJFA(5,10),JUF(5),IUJF(5,10),CR3(15),TR3(15),TI3(
     115),TIP3(15),FR3(15),DR3(15),PS(15),PT3(15),PP3(15),PH (15),JOR(10
     2)
C*    DEFINING GAIN FUNCTION FOR DETERMINING OPPORTUNISTIC MAINTENANCE ACTIONS
      CGN3(CRX3,CPX3,CMPX3,TZX3,CSX3,PW3,PPS,PS3,CHX3,TRX3,TXY3,TYX3,
     1PHX3)=(((CPX3*TRX3)+(CMPX3*TZX3)+CSX3)*PW3*PS3)+(CRX3*PW3)+(CHX3*
     2PHX3*(1.0-PS3))-(CRX3+(CPX3*TXY3)+(CMPX3*TYX3))*PPS
C*    READING REPAIR / REPLACEMENT COSTS , TIMES OF REPAIR / REPLACEMENT,TEST
C*    INTERVALS AND DEMAND RATES
      READ(1,600)  (CR3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (TR3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (TI3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (TIP3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (FR3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (DR3(IUJF(3,I)),I=1,JUF(3))
      READ(1,600)  (PS(IUJF(3,I)),I=1,JUF(3))
  600 FORMAT(10F0.0)
      DO 601 I=1,JUF(3)
      ID=IUJF(3,I)
      PT3(ID)=1.0-EXP(-(FR3(ID)*TI3(ID)/365.0))
      PP3(ID)=1.0-EXP(-(FR3(ID)*TIP3(ID)/365.0))
  601 PH(ID)=1.0-EXP(-(DR3(ID)*TR3(ID)/8760.0))
      JJJ=0
      TSPP=TSP
      TCSPP=TCSP
      DO 602 I=1,JUF(3)
      ID=IUJF(3,I)
      IF(TR3(ID)-TSP) 603,603,604
  603 TX3=0.0
      TY3=0.0
      GO TO 605
  604 TX3=TR3(ID)-TSP
      TSP=TR3(ID)
      IF(TCSP) 605,606,607
  607 TY3=TX3
      GO TO 605
```

```
 606 IF(TSP-TCSO) 621,621,622
 621 TY3=0.0
     GO TO 605
 622 TY3=TSP-TCSO
     TCSP=TY3
 605 CONTINUE
     IF(TR3(ID)-TCSO) 609,609,610
 609 TZ3=0.0
     GO TO 611
 610 TZ3=TR3(ID)-TCSO
 611 CONTINUE
     Y3=CGN3(CR3(ID),CP3,CMP3,TZ3,CS3,PT3(ID),PP3(ID),PS(ID),CH3,TR3(
    1ID),TX3,TY3,FH(ID))
     IF(Y3) 612,612,613
 613 JJJ=JJJ+1
     JOR(JJJ)=ID
     GO TO 602
 612 CONTINUE
     IF(JJJ) 614,614,615
 614 TSP=TSPP
     TCSP=TCSPP
 615 CONTINUE
 602 CONTINUE
     IF(JJJ) 616,616,617
 617 DO 626 L9=1,JJJ
     L10=JFA(3)+L9
 626 IJFA(3,L10)=JOR(L9)
     JFA(3)=JFA(3)+JJJ
 616 CONTINUE
     TSF3=TSP
     TCSP3=TCSP
     IF(JFA(3)) 618,618,619
 619 WRITE(2,627) (IJFA(3,I),I=1,JFA(3))
 627 FORMAT(5X,'TEST AND ON FAILURE REPLACE/ REPAIR COMPONENTS FROM CLA
    1SS3',10I4)
 618 CONTINUE
     RETURN
     END
C*
C*   SUBROUTINE CLASS4 FOR DETERMINING OPPORTUNISTIC MAINTENANCE PLAN FOR
C*   CLASS4 EQUIPMENTS
     SUBROUTINE CLASS4(TSQ,TCSQ,JJFA,IJJFA,JJUF,IUJJF,TSQ4,TCSQ4,CP4,CM
    1P4,CS4,CH4,TCSI)
C****************************************************************************
C*   MAIN SYMBOLS                                                           *
C*   ------------                                                           *
C*                                                                          *
C*   CR4   =COST OF REPAIR / REPLACEMENT                                    *
C*   DR4   =DEMAND RATE / YEAR                                              *
C*   FP4   =FAILURE RATE / YEAR                                             *
C*   KJJ   =NUMBER OF ITEMS BEING OPPORTUNISTICALLY MAINTAINED              *
C*   KOR   =IDENTIFICATION NUMBERS OF OPPORTUNISTICALLY MAINTAINED ITEMS    *
C*   PA4   =PROBABILITY OF ALTERNATIVE SAFETY SYSTEM FAILURE                *
C*   PH4   =PROBABILITY OF HAZARD DURING REPAIR / REPLACEMENT               *
```

```
C*   PP4  =PROBABILITY OF FAILURE SINCE LAST TEST                                      *
C*   PS4  =PROBABILITY OF SHUTDOWN DUE TO FAILURE                                      *
C*   PT4  =PROBABILITY OF FAILURE DURING TEST INTERVAL                                 *
C*   TCSQ4=MODIFIED TIME OF MAIN PLANT SHUTDOWN                                        *
C*   TI4  =TEST INTERVAL IN DAYS                                                       *
C*   TIP4 =TIME SINCE LAST TEST                                                        *
C*   TR4  =TIME OF REPAIR / REPLACEMENT                                                *
C*   TSQ4 =MODIFIED TIME OF GENERATOR SHUTDOWN TIME                                    *
C*   TX4  =EXTRA TIME OF GENERATOR SHUTDOWN                                            *
C*   TY4  =EXTRA TIME OF MAIN PLANT SHUTDOWN                                           *
C*   TZ4  =TIME OF MAIN PLANT SUTDOWN WHEN AN UNFAILED ITEM IS REPAIRED /              *
C*          REPLACED ALONE                                                             *
C*************************************************************************************
      DIMENSION JJFA(5),IJJFA(5,10),JJUF(5),IUJJF(5,10),CR4(15),TR4(15),
     1TI4(15),TIP4(15),FR4(15),DR4(15),PS4(15),PT4(15),PP4(15),PH4(15),
     2KOR(10),PA4(15)
C*    DEFINING GAIN FUNCTION FOR CLASS4 ITEMS
      CGN4(CRX4,CPX4,CMPX4,CSX4,CHX4,PW4,PSX4,TZX4,TRX4,PPX,PAX4,TXY4,TY
     1X4,PHX4)=(CRX4*PW4)+((CPX4*TRX4+CMPX4*TZX4+CSX4)*PW4*PSX4)+(CHX4*
     2PHX4*(1.0-PSX4)*PAX4)-(CRX4+(CPX4*TXY4)+(CMPX4*TYX4))*PPX
C*    READING FIGURES OF COSTS, TIMES, ETC.
      READ(1,630) (CR4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (TR4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (TI4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (TIP4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (FR4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (DR4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (PS4(IUJJF(4,I)),I=1,JJUF(4))
      READ(1,630) (PA4(IUJJF(4,I)),I=1,JJUF(4))
  630 FORMAT(10F0.0)
      DO 631 I=1,JJUF(4)
      ID=IUJJF(4,I)
      PT4(ID)=1.0-EXP(-(FR4(ID)*TI4(ID)/365.0))
      PP4(ID)=1.0-EXP(-(FR4(ID)*TIP4(ID)/365.0))
  631 PH4(ID)=1.0-EXP(-(DR4(ID)*TR4(ID)/8760.0))
      KJJ=0
      TSQQ=TSQ
      TCSQQ=TCSQ
      DO 632 I=1,JJUF(4)
      ID=IUJJF(4,I)
      IF(TR4(ID)-TSQ) 633,633,634
  633 TX4=0.0
      TY4=0.0
      GO TO 635
  634 TX4=TR4(ID)-TSQ
      TSQ=TR4(ID)
      IF(TCSQ) 636,636,637
  637 TY4=TX4
      GO TO 635
  636 IF(TSQ-TCSI) 661,661,662
  661 TY4=0.0
      GO TO 635
  662 TY4=TSQ-TCSI
```

```
      TCSQ=TY4
  635 CONTINUE
      IF(TR4(ID)-TCSI) 639,639,640
  639 TZ4=0.0
      GO TO 641
  640 TZ4=TR4(ID)-TCSI
  641 CONTINUE
      Y4=CGN4(CR4(ID),CP4,CMP4,CS4,CH4,PT4(ID),PS4(ID),TZ4,TR4(ID),PP4(
     1ID),PA4(ID),TX4,TY4,PH4(ID))
      IF(Y4) 642,642,643
  643 KJJ=KJJ+1
      KOR(KJJ)=ID
      GO TO 632
  642 CONTINUE
      IF(KJJ) 644,644,645
  644 TSQ=TSGQ
      TCSQ=TCSGQ
  645 CONTINUE
  632 CONTINUE
      IF(KJJ) 646,646,647
  647 DO 651 K9=1,KJJ
      K10=JJFA(4)+K9
  651 IJJFA(4,K10)=KOR(K9)
      JJFA(4)=JJFA(4)+KJJ
  646 CONTINUE
      TSQ4=TSQ
      TCSQ4=TCSQ
      IF(JJFA(4)) 648,648,649
  649 WRITE(2,652) (IJJFA(4,I),I=1,JJFA(4))
  652 FORMAT(5X,'TEST AND ON FAILURE REPLACE / REPAIR COMPONENTS FROM CL
     1ASS4',10I4)
  648 CONTINUE
      RETURN
      END
C*    SUBROUTINE PROB1 FOR CALCULATING FAILURE PROBABILITY OF AN UNFAILED ITEM
C*    BEFORE A FAILED ITEM WHICH IS BEING REPAIRED / REPLACED AT THE PRESENT
C*    OPPORTUNITY
      SUBROUTINE PROB1(IIJ,IIK,RSL,BSL,FSL,NSF,IRR,RS)
      DIMENSION NSF(10),IRR(10),SL(300),BSL(10,300),RSL(10,300),FSL(10,
     1300)
      NL2=NSF(IIJ)-IRR(IIJ)
      IF(NSF(IIK)-NL2) 300,300,301
  301 NLV=NL2
      DO 306 KX=1,NLV
      ITX=KX+IRR(IIJ)
  306 SL(KX)=BSL(IIK,KX)*FSL(IIJ,ITX)/RSL(IIJ,IRR(IIJ))
      HN=FLOAT(NLV)/2.0
      IHN=INT(HN)
      IF(HN-FLOAT(IHN)) 303,304,303
  303 CALL SIMPSON(NLV,SL,RS)
      GO TO 302
  304 CALL TRAPSIM(NLV,SL,RS)
      GO TO 302
```

```
  303 NLV=NSF(IIK)
      DO 305 KX=1,NLV
      ITX=KX+IRR(IIJ)
  305 SL(KX)=FSL(IIK,KX)*(BSL(IIJ,ITX)-BSL(IIJ,IRR(IIJ)))/RSL(IIJ,IRR(II
     1J))
      HN=FLOAT(NLV)/2.0
      IHN=INT(HN)
      IF(HN-FLOAT(IHN)) 371,372,371
  371 CALL SIMPSON(NLV,SL,RX)
      GO TO 373
  372 CALL TRAPSIM(NLV,SL,RX)
  373 RS=1.0-RX
  302 CONTINUE
      RETURN
      END
C*    SUBROUTINE PROB2 FOR CALCULATING FAILURE PROBABILITY OF AN UNFAILED ITEM
C*    BEFORE ANOTHER UNFAILED ITEM
      SUBROUTINE PROB2(LJJ,LKK,RTL,BTL,FTL,NTF,ITR,RT)
      DIMENSION NTF(10),ITR(10),SM(300),BTL(10,300),RTL(10,300),FTL(10,
     1300)
      NL=NTF(LJJ)-ITR(LJJ)
      NM=NTF(LKK)-ITR(LKK)
      IF(NL-NM) 306,306,307
  306 NLX=NL
      DO 507 KY=1,NLX
      ITY=KY+ITR(LJJ)
      ITZ=KY+ITR(LKK)
  507 SM(KY)=((BTL(LKK,ITZ)-BTL(LKK,ITR(LKK)))/RTL(LKK,ITR(LKK)))
     1*(FTL(LJJ,ITY)/RTL(LJJ,ITR(LJJ)))
      HM=FLOAT(NLX)/2.0
      IHM=INT(HM)
      IF(HM-FLOAT(IHM)) 309,310,309
  309 CALL SIMPSON(NLX,SM,RT)
      GO TO 311
  310 CALL TRAPSIM(NLX,SM,RT)
      GO TO 311
  307 NLX=NM
      DO 374 KY=1,NLX
      ITY=KY+ITR(LJJ)
      ITZ=KY+ITR(LKK)
  374 SM(KY)=((BTL(LJJ,ITY)-BTL(LJJ,ITR(LJJ)))/RTL(LJJ,ITR(LJJ)))
     1*(FTL(LKK,ITZ)/RTL(LKK,ITR(LKK)))
      HM=FLOAT(NLY)/2.0
      IHM=INT(HM)
      IF(HM-FLOAT(IHM)) 375,376,375
  375 CALL SIMPSON(NLX,SM,RY)
      GO TO 308
  376 CALL TRAPSIM(NLX,SM,RY)
  308 RT=1.0-RY
  311 CONTINUE
      RETURN
      END
```

```
C*     SUBROUTINE ZK1 CALCULATES PROBABILITY DENSITY, CUMULATIVE DISTRIBUTION
C*     AND RELIABILITY FUNCTIONS UNTIL FAILURE PROBABILITY OF AN ITEM IS
C*     GREATER THAN 93°/.
       SUBROUTINE ZK1(B,TE,RL,BF,NPP,F)
       DIMENSION EX(300),BF(300),RL(300),F(300)
       N=50
       DO 69 KK=1,N
       EX(KK)=(FLOAT(KK)/TE)**B
    69 BF(KK)=1.0-EXP(-EX(KK))
       NPP=N+1
    72 EX(NPP)=(FLOAT(NPP)/TE)**B
       BF(NPP)=1.0-EXP(-EX  (NPP))
       IF(BF(NPP)-.93) 71,71,70
    71 NPP=NPP+1
       GO TO 72
    70 CONTINUE
       DO 82 LI=1,NPP
    82 RL(LI)=1.0-BF(LI)
       F(1)=BF(1)
       DO 75 KJ=2,NPP
    75 F(KJ)=BF(KJ)-BF(KJ-1)
       RETURN
       END
C*     SUBROUTINE SIMPSON IS FOR INTEGRATION
       SUBROUTINE SIMPSON(KOP,ZH,ZR)
       DIMENSION ZH(300)
       LM1=KOP-1
       SUM1=0.0
       DO 801 IN=2,LM1,2
   801 SUM1=ZH(IN)+SUM1
       LM2=KOP-2
       SUM2=0.0
       IF(LM2-3) 803,804,804
   804 DO 802 IR=3,LM2,2
   802 SUM2=ZH(IR)+SUM2
   803 CONTINUE
       ZR=(ZH(1)+(4.0*SUM1)+(2.0*SUM2)+ZH(KOP))/3.0
       RETURN
       END
C*     SUBROUTINE TRAPSIM IS FOR INTEGRATION
       SUBROUTINE TRAPSIM(KN,ZS,ZT)
       DIMENSION ZS(300)
       LM3=KN-1
       SUM3=0.0
       DO 221 IM=3,LM3,2
   221 SUM3=ZS(IM)+SUM3
       LM4=KN-2
       SUM4=0.0
       IF(LM4-4) 222,223,223
   223 DO 224 IQ=4,LM4,2
   224 SUM4=ZS(IQ)+SUM4
   222 ZT=((ZS(2)+ZS(1))/2.0)+((ZS(2)+(4.0*SUM3)+(2.0*SUM4)+ZS(KN))/3.0)
       RETURN
       END
       FINISH
****
```

NOTATIONS

CHAPTER 2

A = Constant used in sequential testing

B = " " " " "

$A(n,r,i)$ = Weighting factor used in the BLIE method, for ith failure when r failures have been observed out of n items

$C(n,r,i)$ = ---- ditto ----

$C_{r,n}$ = Constant used in the GLUE method

d = Discriminiation ratio

$h(\ )$ = Hazard rate function

$H(\ )$ = Cumulative hazard function

H_0, H_1 = Null and alternative hypotheses for statistical hypothesis testing

$k_{r,n}$ = Constant used in the GLUE method

k_n = " " " " " "

r = Number of observed failures

r^* = Euler's constant

$t_{i:n}$ = ith failure time out of n items

T = Cumulative operating time including failures and successes

CHAPTER 3

A = Constant used in SEQLIM method

B = " " " " "

b_ε = 100ε per cent percentage point of the beta distribution

d	=	Discrimination ratio
$t_\varepsilon(r)$	=	100ε per cent probability limit for the rth failure
v_r	=	Acceptance point of a scale parameter (used in SEQLIM) with r observed failures
ε	=	Percentage point

CHAPTER 4

A	=	Time rate of amortization
$C(\ )$	=	Total expected cost over a cycle
$C(t_p)$	=	Total expected cost/unit time when t_p is the preventive replacement age (preventive replacement models)
C_f	=	Cost of a failure replacement
C_i	=	Cost of replacing part i alone
C_o	=	Cost of replacing part o alone
C_{oi}	=	Cost of replacing part o and i together
C_p	=	Cost of a preventive replacement
d_{M+1}	=	Likelihood that part o is not replaced until its preventive replacement age (Jorgenson's model)
$D(t_p)$	=	Total downtime/unit time when t_p is the preventive replacement age (Preventive replacement model)
$f_k(\ )$	=	Probability density function of the kth component
$F_i(\ )$	=	Cumulative distribution function of the ith component
F_{ki}	=	Probability that component k will fail before

$G(\ )$ = Expected on-stream time over a cycle

$H(t_p)$ = Expected number of failures in interval $(0, t_p)$

L = Length of the cycle (Jorgenson's model)

$M(t_p)$ = Expected length of a failure cycle (Preventive replacement model)

N_k = Preventive replacement age for component k (Opportunistic model)

q_{oi} = Probability that the cycle ends with a replacement of parts o and i together (Jorgenson's model)

$R(t_p)$ = Reliability function upto time t_p

R_{ki} = Probability that component k will not fail before component i

T_f = Downtime for a failure replacement

T_k = Present age of component k

t_p = Preventive replacement age (Preventive replacement model)

T_p = Downtime for a preventive replacement

μ = Mean time to failure

ρ = Relative cost of a failure replacement $(=C_f/C_p)$

CHAPTER 5

C_c = Annual cost of a single channel trip system

C_H = Average cost of a hazard

C_s = Cost of a spurious trip

C_T = Total cost

d = Demand rate/unit time

D = Total downtime

e = Probability of human error per operation, in ... time model; otherwise exponential .

fdt	=	Fractional dead time
h	=	Probability of hazard
I	=	Probable number of inspections as random variable
n	=	Number of operations during an inspection or trip test
q	=	Probability of an item remaining in good state during an inspection interval
r	=	Average hazard rate over an inspection interval or probability of an item passing from good to failed state during an inspection interval
r_t	=	Hourly cost of an inspection
S	=	Spurious failure rate of a safety component
t'	=	Time taken to perform an inspection
t_c, t_c^*	=	Inspection or optimal inspection interval by the cost approach
t_i, t_i^*	=	Inspection or optimal inspection interval by the reliability approach
T_i	=	Age of a component at ith inspection or ith inspection time since the start
t_r, t_r^*	=	Operating time or optimal operating time during an inspection, by reliability approach. In the fractional dead time model, inspection interval.
t_{rc}, t_{rc}^*	=	Operating time or optimal operating time during an inspection, by cost approach
x_i	=	ith inspection time (Munford-Sahani model)
τ	=	Mean downtime due to equipment failure

C_s	=	Mass of silt per unit volume of cooling water (kg/m^3)
D	=	Inner diameter of the tubes of a condenser (m)
d_s	=	Diameter of silt particles (m)
f	=	Fanning friction factor
I	=	Inhibition factor (K)
K	=	Thermal conductivity of silt deposit (W/mK)
k	=	Mass transfer coefficient of silt to the deposit layer (m/s)
p_a	=	Parts per million (W/W) of algae in cooling water
R	=	Fouling resistance inside the tubes (m^2K/W)
R_∞	=	Ultimate fouling resistance (m^2K/W)
R_e	=	Reynolds number
T	=	Mean temperature of cooling water in a condenser (K)
t	=	Time elapsed since last cleaning operation (days)
U	=	Overall heat transfer coefficient based on the inside area of tubes (subscripts are condenser numbers) (W/m^2K)
V	=	Superficial velocity of water in the tubes (m/s)
δ	=	Thickness of silt deposit (m)
δ_∞	=	Ultimate thickness of silt deposit (m)
ε_w	=	Roughness of wall (m)
ε_a	=	Roughness of algae growing on wall (m)
μ_d	=	Cohesion of silt deposit (N_s/m^2)
ρ_m	=	Density of silt and water mixture flowing in the tubes

ρ_s = Bulk density of silt deposit (kg/m^3)

τ = Shear stress (N/m^2)

∂ = Partial derivative (subscript D: for deposition)

CHAPTER 8

$C(i,i)$ = Total cost of replacement /repair for ith component alone when i has failed or reached its preventive replacement age, if any

C_H = Estimated cost of a hazard

C_p = Lost production cost/hour, for generator system

C_{mp} = Lost production cost/hour, for the main plant

$C_{r(i)}$ = Cost of replacing/repairing failed component i (including engineer's cost)

$D(j)$ = Demand rate/hour on the jth component of the safety system

$G(\ )$ = Gain or cost advantage function for replacing an unfailed component with a failed component

$P(j)$ = Probability of not getting another opportunity for replacement/repair of the jth component, if the present opportunity created by component i is not taken

$P_{af(j)}$ = Probability of alternative safety system failure during the replacement/repair of the jth component

$P_{li(j)}$ = Probability of failure of the jth component since last test (component j is tested periodically)

$P_{s(i)}$ = Probability of generator plant shutdown due to ith component failure (either zero or unity)

$p_{t(j)}$ = Probability of jth component failure during the test interval

$s(j)$ = Proportion of useful life that will be lost for the jth component through early retirement, if the opportunity created by the ith component is taken

$t_{r(i)}$ = Time in hours, of replacing/repairing ith component

t_s = Time in hours, for which main plant can be operatred at full capacity

$t_{x(j)}$ = Extra time in hours, for which the generator plant will be shutdown if the replacement/ repair time for the jth component is greater than that of component i which has caused the shutdown

$t_{y(j)}$ = Extra time in hours, for which the main plant will be shutdown, if jth component is replaced/repaired

$t_{z(j)}$ = Time in hours, for which the main plant will be shutdown if jth component alone needs to be replaced/repaired at some later date

COMMON NOTATIONS

$E(\ )$ = Expected value of a parameter

$f(\ )$ = Probability density function of the time to failure

$F(\ )$ = Cumulative distribution function

$\ln$ = Natural lorarithm

$P(\ )$ = Probability function

t = Time

α = Producer's risk

β = Customer's risk

β_W = Shape parameter of the Weibull distribution

η = Scale parameter of the Weibull distribution

λ = Failure rate/unit time

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