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14/12/70          COMPILED BY XALE MK.  4B

'DUMP ON' (ED)
'RUN'
'LEADERS'
'OVERLAY PROGRAM' (TEMP)
'BLOCKSIZES'
'OVERLAY' (1) 3:4:12:16:17;
'BEGIN'     'COMMENT'GRAHAM WHITEHEAD  FOURIER/ HANNING SPECTRUM ANALYSIS;
      'COMMENT'   INSERT PARAMETER CARDS
                   AUTO:= 1 OR 0
                   CROSS:= 1 OR 0
              SU:= TAPE SPEED UP RATIO
                     N := NUMBER OF CORRELATION POINTS
                     T := TIME DELAY INCREMENT
              AUTONORM  := R(INFINITY)
              CROSSNORM := RXY(INFINITY)
                   TD:= TIME DELAY FOR RXY(T);
                 'REAL' A,D,NORM,RX0,C,PIE,AUTONORM,CROSSNORM ,T,TD,P,
OCK      1
              B,Q;
                 'INTEGER' K,N,J,F,H,Z,E,SU,S1,S2,G,Y;
                 'BOOLEAN' AUTO,CROSS;
AUTO:='TRUE';
CROSS:='TRUE';
           N:=256;
              SU:=4;
               Z:=1;
           T:=0,02;
                 PIE := 3.14159265358979323846;
              P:=0;
   'BEGIN'   'REAL''ARRAY' RX[1:N],RXY[1:N],MCOS[1:N],MSIN[1:N],
OCK      2
                 NSIN[1:N], NCOS[1:N],M[1:N];
'PROCEDURE' PART 1;
OCK      3
'BEGIN'
    L1:          'FOR' F:=1'STEP'1'UNTIL'N'DO'
'BEGIN'
                 MCOS[F]:=0.0;
                 MSIN[F]:=0.0;
'END';
                 SELECT INPUT(3);
              TD:=READ;
              Q:=READ;
       'COMMENT'PRESENT AUTO DATA FIRST THEN CROSS;
                 'FOR'F:=1'STEP'1'UNTIL'N'DO'  RX[F]:=0.0;
                 A:= READ;
                 'FOR' F:=1'STEP'1'UNTIL' N 'DO'
'BEGIN'
                 A:= READ;
                 D:= READ;
                 RX[F]:= RX[F] + D;
'END';
           A:=READ;
           A:=READ;
                 A:=READ;
                 'FOR'F:=1'STEP'1'UNTIL' N 'DO' RXY[F]:=0.0;
                 'FOR' F:=1'STEP'1'UNTIL' N 'DO'
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'BEGIN'
            A:= READ;
            D:= READ;
            RXY[F]:= RXY[F] + D ;
'END';
        B:=0.0;
        'FOR' F:=1 'STEP' 1 'UNTIL' 100 'DO'
'BEGIN'
        'IF' RX[F]> B 'THEN'
'BEGIN'
        B:=RX[F];
        Y:=F;
'END'
'END';
        RX0:=RX[Y];
          A:=0.0;
        'FOR' F:=1 'STEP' 1 'UNTIL'100'DO'
'BEGIN'
        'IF' ABS(RXY[F])> A 'THEN'
'BEGIN'
         A:=ABS(RXY[F]);
        Z:= F;
'END'
'END';
        CROSSNORM:=0.0;
        AUTONORM:=0.0;
        'IF' Z>Y 'THEN'
'BEGIN'
        'FOR' F:=(Y+P)'STEP'1'UNTIL'N-(Z-Y)'DO'AUTONORM:=
        AUTONORM+RX[F];
        'FOR'F:=(Z+P)'STEP'1'UNTIL'N'DO' CROSSNORM
        := CROSSNORM + RXY[F];
        AUTONORM:=AUTONORM/(N-(Z+P-1));
        CROSSNORM:=CROSSNORM/(N-(Z+P-1));
        G:=(N-Z)+1;
'END'
        'ELSE'
'BEGIN'
        'FOR'F:=(Y+P)'STEP'1'UNTIL'N'DO' AUTONORM :=
        AUTONORM + RX[F];
        'FOR'F:=(Z+P)'STEP'1'UNTIL'N-(Y-Z)'DO' CROSSNORM
        :=CROSSNORM + RXY[F];
        AUTONORM:=AUTONORM/(N-(Y+P-1));
        CROSSNORM:=CROSSNORM/(N-(Y+P-1));
        G:=(N-Y)+1;
'END';
        'IF' (A- CROSSNORM)>(B- AUTONORM) 'THEN'
'BEGIN'
            'FOR' F:=1'STEP' 1'UNTIL' N 'DO'
'BEGIN'
        RX[F]:=(RX[F]- AUTONORM)/(A- CROSSNORM);
        RXY[F]:=(RXY[F] - CROSSNORM)/(A- CROSSNORM);
'END'
'END'
        'ELSE'
'BEGIN'
            'FOR' F:=1'STEP'1'UNTIL'N 'DO'
'BEGIN'
        RX[F]:=(RX[F]- AUTONORM)/( B- AUTONORM);
        RXY[F]:=(RXY[F]- CROSSNORM)/( B- AUTONORM);
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'END'
'END';
              'FOR' F:=1'STEP' 1 'UNTIL' G 'DO'
              M[F]:=RX[(F+Y)-1];
              E:=ENTIER(G*Q);
                 H:= 0;
'END' OF PART 1 PROCEDURE;
'PROCEDURE' PART 2 (LAB); 'LABEL' LAB;
CK     4
'BEGIN'
   RESTART:        H:=H+1;
      'COMMENT' THE NEXT SEGMENT CALCULATES THE FOURIER COEFFICIENTS
                 AT SPECIAL FREQUENCIES  (2J+1)*2PIE/NT THE RESULT IS
                 DEPOSITED IN ARRAYS  MSIN AND MCOS;
'BEGIN'
                 J:= -1;
   RECYCLE:      J:= J+1;
                 K:=0.0;
     LOOP :      K:=K+1;
              A:=(J/2)*(K-1)/(N-1);
                 A:=A-ENTIER(A);
                 'IF' A>0.0 'AND' A <0.25 'THEN'
'BEGIN'
                 MSIN[J+1]:=MSIN[J+1]+M[K]*SIN(2*PIE*A);
                 MCOS[J+1]:=MCOS[J+1]+M[K]*COS(2*PIE*A);
'END'
               'ELSE''IF' A>0.25'AND' A <0.5 'THEN'
'BEGIN'
                 MSIN[J+1]:= MSIN[J+1]+ M[K]*SIN(PIE-2*PIE*A);
              MCOS[J+1]:=MCOS[J+1]+M[K]*(-1)*COS(PIE-2*PIE*A);
'END'
               'ELSE''IF' A>0.5 'AND' A<0.75 'THEN'
'BEGIN'
                 MSIN[J+1]:= MSIN[J+1] +M[K]*SIN((-1)*(2*PIE*A -PIE));
                 MCOS[J+1]:= MCOS[J+1] +M[K]*COS((-1)*(2*PIE*A -PIE));
'END'
               'ELSE''IF' A>0.75 'AND' A < 1.0 'THEN'
'BEGIN'
                 MSIN[J+1]:= MSIN[J+1]+M[K]*SIN((-1)*(2*PIE-2*PIE*A));
                 MCOS[J+1]:= MCOS[J+1]+M[K]*COS(       (2*PIE-2*PIE*A));
'END'
              'ELSE''IF' A=0.0 'OR' A=1.0 'THEN'
                 MCOS[J+1]:= MCOS[J+1] +M[K]
'ELSE''IF' A=0.5 'THEN' MCOS[J+1]:=MCOS[J+1]+M[K]*(-1)
   'ELSE''IF' A=0.25 'THEN' MSIN[J+1]:=MSIN[J+1]+M[K]
      'ELSE''IF'A=0.75'THEN' MSIN[J+1]:= MSIN[J+1]+M[K]*(-1);
'END';
                 'IF' K = G 'THEN''GOTO' L6 'ELSE''GOTO' LOOP;
        L6:      'IF' J=(  E -1) 'THEN''GOTO'L7 'ELSE''GOTO'RECYCLE ;
   L7:        MSIN[1]:=0.5*MSIN[1]+0.5*MSIN[2];
   MCOS[1]:=0.5*MCOS[1]+0.5*MCOS[2];
              'FOR' J:=2 'STEP' 1 'UNTIL' (E-1) 'DO'
'BEGIN'
   MSIN[J]:=0.25*MSIN[J-1]+0.5*MSIN[J]+0.25*MSIN[J+1];
   MCOS[J]:=0.25*MCOS[J-1]+0.5*MCOS[J]+0.25*MCOS[J-1];
'END';
   MSIN[E]:=0.5*MSIN[E-1]+0.5*MSIN[E];
   MCOS[E]:=0.5*MCOS[E-1]+0.5*MCOS[E];
                 'IF' H =1 'AND' AUTO 'THEN''GOTO' L8 'ELSE''IF' H=1
                 'AND''NOT'AUTO'THEN''GOTO' L9 'ELSE''GOTO'L10;
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        L8:        'FOR' F:=1 'STEP'1'UNTIL' N 'DO'
'BEGIN'
                      NSIN[F] := MSIN[F];
                      NCOS[F] := MCOS[F];
'END';
               'FOR' F:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
               MSIN[F]:=0.0;
               MCOS[F]:=0.0;
'END';
                   'IF' AUTO 'AND' CROSS 'THEN''GOTO'  L11  'ELSE''GOTO'
                   L10;
        L11:       'FOR' F:=1'STEP' 1 'UNTIL' G 'DO'
                   M[F] := RXY[(F*2)-1];
                  'GOTO'RESTART;
        L9 :       'GOTO'  L13;
        L10:'COMMENT' THE FOLLOWING PROCEEDURE  PLOTS THE AUTOCORRELOGRA
                      AND THE POWER SPECTRAL DENSITY;
'END' OF PART 2;
'BEGIN'
                   'REAL' MAX,POWER;
CK      5
                   'INTEGER' LITA;
               'REAL' 'ARRAY' AB[1:N],BC[1:N] ;
               'INTEGER' 'ARRAY' CD[1:17],DE[1:3],EF[1:22],FG[1:13],
               GH[1:13],HJ[1:24],JK[1:28],KL,MN,TU,VW[1:4],LM[1:5],
               NO,OP,PQ,QR[1:7],RS[1:20],ST[1:25],UV[1:9];
'BEGIN'
          'PROCEDURE' OPENPLOT;'EXTERNAL';
CK      6
CK      7
          'PROCEDURE' CLOSEPLOT;'EXTERNAL';
CK      8
          'PROCEDURE' HGPLOT (X,Y,IC,L);'VALUE'X,Y,IC,L;'INTEGER'IC,L;
CK      9
                      'REAL'X,Y;'EXTERNAL';
          'PROCEDURE' STRARR(A,N,S);'ARRAY'A;'INTEGER' N;
CK     10
          'STRING' S; 'EXTERNAL';
           'PROCEDURE'  HGPAXISV (X,Y,BCD,NC,S,THETA,XMIN,DX,GAP,NH);
CK     11
           'VALUE'X,Y,NC,S,THETA,XMIN,DX,GAP,NH;'INTEGER'NC,NH;'ARRAY'
           BCD;'REAL'X,Y,S,THETA,XMIN,DX,GAP;'EXTERNAL';
           'PROCEDURE' HGPLINE (X,Y,N,K);'VALUE'N,K; 'ARRAY'X,Y;
CK     12
          'INTEGER' N,K;'EXTERNAL';
           'PROCEDURE'  HGPNUMBER (X,Y,HT,FL,THETA,I,IP,IQ);'VALUE'X,Y,
CK     13
           HT,FL,THETA,I,IP,IQ;'INTEGER'I,IP,IQ;'REAL'X,Y,HT,FL,THETA;
           'EXTERNAL';
           'PROCEDURE'  HGPSYMBL (X,Y,HT,BCD,THETA,N);'VALUE'X,Y,HT,
CK     14
           THETA,N;'INTEGER'N;'ARRAY'BCD;'REAL'X,Y,HT,THETA;'EXTERNAL';
'PROCEDURE' PART 3;
CK     15
'BEGIN'
                    OPENPLOT ;
                   HGPLOT (-3.0,10.5,0,4);
                   MAX:=0.0;
                  'FOR' J:=1'STEP'1'UNTIL' E     'DO'
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'BEGIN'
'IF'ABS( NCOS[J ])> MAX'THEN' MAX:=ABS( NCOS[J])'ELSE'MAX:=MAX;
OUTPUT(NCOS[J]);
'END';
            CROSSNORM:=MAX;
NEWLINE(1);
OUTPUT(MAX);
    'COMMENT' THE NEXT SEGMENT COMPUTES THE X AND Y COORDINATES TO
              PLOT A HISTOGRAM;
            A:=(2*PIE)/((G-1)*T*SU);
            D:=((ENTIER(((PIE/T)/SU)/8)+1)*8)*Q;
               'FOR' J:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
            AB[1]:=(J-1)*(A/2)*4.5/D;
            AB[2]:=AB[1];
            AB[3]:=AB[1];
            BC[1]:=0.0;
               BC[2] :=ABS( NCOS[J])*2.5/MAX;
            BC[3]:=0.0;
            HGPLINE (AB,BC,3,+1);
OUTPUT(NSIN[J]);
'END';
            'FOR' J:=1'STEP'1'UNTIL' E 'DO'
'BEGIN'
            AB[J]:=(J-1)*(A/2)*4.5/D;
            BC[J]:=ABS(NCOS[J])*2.5/MAX;
'END';
            HGPLINE(AB,BC,E,+1);
               STRARR(CD,LITA,'('FREQUENCY%RAD/SEC')');
        HGPAXISV (0,0,0.0,CD,-17,4.5,0,0,0.0,D/10,0,45,-2);
               STRARR(DE,LITA,'('%%%')');
               HGPAXISV (0.0,0.0,DE,0,3.5,90.0,0.0,0.5,1.25,+2);
               STRARR(EF,LITA,'('POWER%SPECTRAL%DENSITY')');
               HGPSYMBL (-1.0,0.5,0.125,EF,90.0,22);
            C:=(4.5*2*PIE)/D;
            STRARR(EF,LITA,'('CYCLES/SECOND')');
            HGPAXISV (0.0,-0.5,EF,-13,4.5,0.0,0.0,1.0,C,-1);
               STRARR(FG,LITA,'('CM.CM/RAD/SEC')');
               HGPSYMBL (-0.625,1.0,0.1,FG,90,0,13);
               STRARR(GH,LITA,'('G.D.WHITEHEAD')');
               HGPSYMBL (3.5,8.875,0.1,GH,0.0,13);
               STRARR(HJ,LITA,'('FOURIER/HANNING%TRANSFORM')');
   HGPAXISV (0.0,6.0,DE,-0,4.5,0.0,0.0,(T*(N-1)*SU)/5,0.9,+1);
               HGPSYMBL (-1.25,8.875,0.2,HJ,0,0,25);
               STRARR(JK,LITA,'('AUTOCORRELATION%COEFFICIENT')');
               HGPSYMBL (-1.0,4.75,0.125,JK,90.0,27);
               HGPAXISV (0.0,4.25,DE,0,3.5,90.0,-1,0.5,0.875,+1);
HGPAXISV(0.0,6.0,DE,-0,4.5,0.0,0.0,(T*(N-1)*SU)/5,0.9,+1);
               STRARR(KL,LITA,'('TIME')');
               HGPSYMBL (4.5,6.125,0.1,KL,0.0,4);
               STRARR(LM,LITA,'('DELAY')');
               HGPSYMBL (4.5,5.75,0.1,LM,0.0,5);
               STRARR(MN,LITA,'('SECS')');
               HGPSYMBL (4.5,5.5,0.1,MN,0.0,4);
               HGPLOT (0.0,0.0,3,0);
               'FOR' F:=1'STEP'1'UNTIL'N'DO'
            M[F]:=(F/N*4.5)-(4.5/N);
               'FOR' F:=1 'STEP' 1 'UNTIL' N 'DO'
            RX[F]:=1.75*RX[F];
            'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
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            RXY[F]:=1.75*RXY[F];
                HGPLOT (0.0,-6.0,0,4);
            HGPLINE (M,RX,N,+1);
                STRARR(NO,LITA,'('R(0)%%=')');
                HGPSYMBL (2.875,1.65,0.1,NO,0.0,7);
                STRARR(OP,LITA,'('R(INF)=')');
                HGPSYMBL (2.875,1.5,0.1,OP,0.0,7);
                HGPNUMBER (3.375,1.65,0.1,RX0,0.0,1.0,2,6);
                HGPNUMBER (3.375,1.50,0.1,AUTONORM,0.0,1.0,2,6);
                STRARR(PQ,LITA,'('POWER=')');
                POWER:= 0.0;
               'FOR' F:=1'STEP'1'UNTIL' E     'DO'
                POWER:= POWER + NCOS[F] ;
                HGPSYMBL (2.875,-2.625,0.1,PQ,0.0,6);
                HGPNUMBER (3.125,-2.625,0.1,POWER,0.0,1.0,2,6);
                STRARR(QR,LITA,'('%%MAX=')');
                HGPSYMBL (2.875,-2.475,0.1,QR,0.0,6);
                HGPNUMBER (3.125,-2.475,0.1,MAX,0.0,1.0,2,6);
                AB[1] := 5.25;
                AB[2] := 5.25;
                AB[3] := -1.75;
                AB[4] := -1.75;
                AB[5] :=  5.25;
                BC[1] := -7.25;
                BC[2] := +3.25;
                BC[3] := +3.25;
                BC[4] := -7.25;
                BC[5] := -7.25;
                HGPLINE (AB,BC,5,+1);
                HGPLOT (0.0,0.0,3,0);
     L11:     HGPLOT (-10.75,0.0,0,4);
            HGPLINE (M,RXY,N,+1);
            HGPAXISV (0.0,-1.75,DE,0,3.5,90.0,-1,0.5,0.875,+2);
  HGPAXISV (0.0,0.0,DE,-0,4.5,0.0,0.0,(T*(N-1)*SU)/5,0.9,+1);
            STRARR(JK,LITA,'('CROSSCORRELATION%COEFFICIENT')');
            HGPSYMBL (-1.0,-1.375,0.125,JK,90,0,28);
            HGPSYMBL (4.625,0.125,0.1,KL,0.0,4);
            HGPSYMBL (4.625,0.0,0.1,LM,0.0,5);
             HGPSYMBL (4.625,-0.125,0.1,MN,0.0,4);
                MAX:=0.0;
                'FOR' F:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
'IF'ABS( MSIN[F]) > MAX 'THEN' MAX :=ABS(MSIN[F] )'ELSE' MAX:=MAX;
'END';
            'FOR' F:=1'STEP'1'UNTIL' E'DO'
'BEGIN'
            'IF'ABS(MCOS[F])>MAX 'THEN' MAX:=ABS(MCOS[F]) 'ELSE' MAX
            :=MAX;
'END';
                'FOR' F:=1 'STEP'1'UNTIL' E     'DO'
'BEGIN'
            RX[F]:=(MCOS[F]/MAX)*2.5;
            RXY[F]:=(MSIN[F]/MAX)*2.5;
'END';
            HGPLOT (0.0,0.0,3,0);
            HGPLOT (-1.75,4.625,0,4);
            HGPLINE (RX,RXY,E,+1);
                HGPAXISV (-2.5,0.0,DE,0,5.0,0.0,-1.0,0.2,0.5,1);
                HGPAXISV (0.0,-2.5,DE,0,5.0,90.0,-1.0,0.2,0.5,1);
                HGPAXISV (0.0,-2.5,DE,0,5.0,
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            STRARR(UV,LITA,'('IMAGINARY')');
            HGPSYMBL (2.75,0.0,0.125,TU,0.0,4);
            HGPSYMBL (-0.5,2.7,0.125,UV,0.0,9);
            STRARR(RS,LITA,'('FOURIER/HANNING%CROSS')');
            STRARR(ST,LITA,'('SPECTRAL%DENSITY%ANALYSIS')');
        HGPLOT (0,0,0,0,3,0);
        HGPLOT (0.0,-1.875,0,4);
            HGPSYMBL (1.5,5.5,0.1  ,GH,0.0,13);
            HGPSYMBL (-2.875,5.5,0.125,RS,0.0,21);
            HGPSYMBL (-2.875,5.25,0.125,ST,0.0,25);
            STRARR(VW,LITA,'('MAX=')');
            HGPSYMBL (-2.875,5.0,0.1,VW,0.0,4);
            HGPNUMBER (-2.275,5.0,0.1,MAX,0.0,1,0,6);
            AB[1] := -3.5;
            AB[2] := -3.5;
            AB[3] :=  3.5;
            AB[4] :=  3.5;
            AB[5] := -3.5;
            BC[1] :=  6.0;
            BC[2] := -4.5;
            BC[3] := -4.5;
            BC[4] :=  6.0;
            BC[5] :=  6.0;
            HGPLINE (AB,BC,5,+1);
'END' OF PART 3 ;
'PROCEDURE' PART 4;
CK    16
'BEGIN'
'COMMENT' THE NEXT SEGMENT PLOTS THE CO AND QUAD SPECTRUM;
        HGPLOT (0.0,0.0,3,0);
        HGPLOT (-10.0,0.0,0,4);
            HGPSYMBL (-2.875,5.5,0.125,RS,0.0,21);
            HGPSYMBL (-2.875,5.25,0.125,ST,0.0,25);
            HGPSYMBL (1.5,5.5,0.1  ,GH,0.0,13);
            HGPLINE (AB,BC,5,+1);
        HGPLOT (0,0,0.0,3,0);
        HGPLOT (1.75,2.625,0,4);
            'FOR' J:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
        AB[1]:=(J-1)*(A/2)*4.5/D;
        AB[2]:=AB[1];
        AB[3]:=AB[1];
        BC[1]:=0.0;
        BC[2]:=MSIN[J]*2.5/MAX;
        BC[3]:=0.0;
        HGPLINE (AB,BC,3,+1);
'END';
        'FOR' J:=1'STEP'1'UNTIL' E 'DO'
'BEGIN'
        AB[J]:=(J-1)*(A/2)*4.5/D;
        BC[J]:=MSIN[J]*2.5/MAX;
'END';
        HGPLINE(AB,BC,E,+1);
        AB[1]:=0.0;
        AB[2]:=4.5;
        BC[1]:=0.0;
        BC[2]:=0.0;
        HGPLINE(AB,BC,2,+1);
    HGPAXISV (U,0,0,0,CD,-17,4.5,0.0,0.0,D/10,0.45,-2);
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            HGPAXISV (0.0,-1.25,DE,0,3.75,90.0,-0.5,0.5,1.25,+2);
            STRARR(JK,LITA,'('QUAD%SPECTRUM')');
            HGPSYMBL (-1.0,-0.25,0.125,JK,90.0,13);
            HGPSYMBL (-0.625,-0.25,0.1,FG,90.0,13);
            HGPLOT (0.0,0.0,3,0);
            HGPLOT (0.0,-4.375,0,4);
                'FOR' J:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
            AB[1]:=(J-1)*(A/2)*4.5/D;
            AB[2]:=AB[1];
            AB[3]:=AB[1]:
            BC[1]:=0.0;
            BC[2]:=MCOS[J]*2.5/MAX;
            BC[3]:=0.0;
            HGPLINE (AB,BC,3,+1);
'END';
            'FOR' J:=1'STEP'1'UNTIL' E 'DO'
'BEGIN'
            AB[J]:=(J-1)*(A/2)*4.5/D;
            BC[J]:=MCOS[J]*2.5/MAX;
'END';
            HGPLINE(AB,BC,E,+1);
            AB[1]:=0.0;
            AB[2]:=4.5;
            BC[1]:=0.0;
            BC[2]:=0.0;
            HGPLINE(AB,BC,2,+1);
      HGPAXISV (0.0,0.0,CD,-17,4.5,0.0,0.0,D/10,0.45,-2);
            HGPAXISV (0.0,-1.25,EF,13,4.5,0.0,0.0,1.0,C,-1);
            HGPAXISV (0.0,-1.25,DE,0,3.75,90.0,-0.5,0.5,1.25,+2);
            STRARR(JK,LITA,'('CO%SPECTRUM')');
            HGPSYMBL (-1.0,0.0,0.125,JK,90.0,11);
            HGPSYMBL (-0.625,-0.25,0.1,FG,90.0,13);
'END' OF PART 4;
'PROCEDURE' PART 5;
CK    17
'BEGIN'
'COMMENT' THE NEXT SEGMENT COMPUTES AND PLOTS THE GAIN & PHASE;
            'FOR' F:=1'STEP' 1 'UNTIL' E 'DO'
            RX[F]:=20.0*0.43429448*(LN(SQRT((MSIN[F]*MSIN[F])
            +(MCOS[F]*MCOS[F]))/
            SQRT((NSIN[F]*NSIN[F])+(NCOS[F]*NCOS[F]))));
            'FOR' F:=1'STEP'1'UNTIL'E'DO'
            RX[F]:=RX[F]*1.875/20;
            HGPLOT (0.0,0.0,3,0);
            HGPLOT (10.0,14.25,0,4);
                'FOR' J:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
            AB[1]:=(J-1)*(A/2)*4.5/D;
            AB[2]:=AB[1];
            AB[3]:=AB[1];
            BC[1]:=0.0;
            BC[2]:=RX[J];
            BC[3]:=0.0;
            HGPLINE (AB,BC,3,+1);
'END';
            'FOR' J:=1'STEP'1'UNTIL' E 'DO'
            AB[J]:=(J-1)*(A/2)*4.5/D;
            HGPLINE(AB,RX,E,+1);
```

```
HGPAXISV (0,0,0.0,CD,-17,4.5,0.0,0.0,D/10,0.45,-2);
        HGPAXISV (0.0,-1.875,EF,13,4.5,0.0,0.0,1.0,C,-1);
        HGPAXISV (0.0,-1.875,DE,0,3.75,90.0,-20,10,0.9375,-2);
        STRARK(JK,LITA,'('GAIN-DECIBELS')');
        HGPSYMBL (-1.0,-0.875,0.125,JK,90.0,13);
        HGPLOT (0.0,0.0,3,0);
        HGPLOT (0,0,4.375,0,4);
        HGPNUMBER (2.25,1.1,0.1,TD,0.0,0,2,3);
        STRARK(JK,LITA,'('M%SECS.')');
        HGPSYMBL (3.0,1.1,0.1,JK,0.0,7);
        'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
        RX[F]:=-(180*TD*F)/(T*(G-1)*SU);
        'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
        S1:=SIGN(MCOS[F]);
        S2:=SIGN(MSIN[F]);
        T:=ABS(ARCTAN(MSIN[F]/MCOS[F])*57.29578);
        'IF' S1=1'AND'S2=-1'AND'F<17'THEN' RX[F]:=RX[F]+T
   'ELSE' 'IF' S1=0 'AND' S2=0'THEN' RX[F]:=RX[F]+0.0
        'ELSE''IF'S1=1'AND'S2=1'THEN' RX[F]:=RX[F]-T
        'ELSE''IF'S1=0'AND'S2=1 'THEN' RX[F]:=RX[F]-90
        'ELSE' 'IF' S1=0 'AND' S2=-1'THEN'RX[F]:=RX[F]-270
        'ELSE''IF'S1=1'AND'S2=0'THEN' RX[F]:=RX[F]+0.0
        'ELSE''IF'S1=-1'AND'S2=0'THEN' RX[F]:=RX[F]-180
       'ELSE''IF'S1 =-1'AND'S2=1'THEN' RX[F]:=RX[F]-(180-T)
        'ELSE''IF'S1=1'AND'S2=-1'THEN' RX[F]:=RX[F]-(360-T)
        'ELSE''IF'S1=-1'AND'S2=-1'THEN' RX[F]:=RX[F]-(180+T);
'END';
        'FOR' F:=1'STEP'1'UNTIL'E'DO'
'BEGIN'
        S1:=SIGN(NCOS[F]);
        S2:=SIGN(NSIN[F]);
        T:=ABS(ARCTAN(NSIN[F]/NCOS[F])*57.29578);
        'IF' S1=1'AND'S2=-1'AND'F<17'THEN' RX[F]:=RX[F]-T
   'ELSE' 'IF' S1=0 'AND' S2=0'THEN' RX[F]:=RX[F]+0.0
        'ELSE''IF'S1=1'AND'S2=1'THEN' RX[F]:=RX[F]+T
        'ELSE''IF'S1=0'AND'S2=1 'THEN' RX[F]:=RX[F]+90
        'ELSE' 'IF' S1=0 'AND' S2=-1'THEN'RX[F]:=RX[F]+270
        'ELSE''IF'S1=1'AND'S2=0'THEN' RX[F]:=RX[F]+0.0
        'ELSE''IF'S1=-1'AND'S2=0'THEN' RX[F]:=RX[F]+180
       'ELSE''IF'S1 =-1'AND'S2=1'THEN' RX[F]:=RX[F]+(180-T)
        'ELSE''IF'S1=1'AND'S2=-1'THEN' RX[F]:=RX[F]+(360-T)
        'ELSE''IF'S1=-1'AND'S2=-1'THEN' RX[F]:=RX[F]+(180+T);
        'IF' RX[F]<-360'THEN'RX[F]:=-360;
'END';
'FOR'F:=1'STEP'1'UNTIL'E'DO' RX[F]:=RX[F]*1.875/360;
        'FOR' F:=1 'STEP' 1 'UNTIL' E'DO'
        RXY[F]:=F*A*2.25/D;
        HGPLINE (RXY,RX,E,+1);
   HGPAXISV (0,0,0.0,CD,-17,4.5,0.0,0.0,D/10,0.45,-2);
        HGPAXISV (0.0,0.0,DF,-0,4.5,0.0,0.0,D/10,0.45,-1);
        HGPAXISV (0.0,-1.875,EF,13,4.5,0.0,0.0,1.0,C,-1);
        HGPAXISV (0.0,-1.875,DE,0,3.75,90.0,-360,180,0.9375,-2);
         HGPAXISV (0.0,-1.875,DE,0,3.75,90.0,-360,180,0.9375,-2);
        STRARK(JK,LITA,'('PHASE%ANGLE%-DEGREES')');
        HGPSYMBL (-1,0,-1.0,0.125,JK,90.0,20);
        STRARK(JK,LITA,'('AGGREGATE%TIME%DELAY')');
        HGPSYMBL (2.25,1.25,0.1,JK,0.0,20);
        HGPLOT (0,0,0.0,3,0);
        HGPLOT (-1.75,-1.75,0,4);
```

```
               HGPSYMBL (-2.875,5.25,0.125,ST,0.0,25);
               HGPSYMBL (1,5,5.5,0.1  ,GH,0.0,13);
               AB[1] := -3.5;
               AB[2] := -3.5;
               AB[3] :=  3.5;
               AB[4] :=  3.5;
               AB[5] := -3.5;
          BC[1]:=6.0;
               BC[2] := -4.5;
               BC[3] := -4.5;
               BC[4] :=  6.0;
               BC[5] :=  6.0;
               HGPLINE (AB,BC,5,+1);
      L12:      CLOSEPLOT;
SELECT OUTPUT(4);
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
          PRINT(NCOS[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(NSIN[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(MCOS[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(MSIN[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
'END' OF PART 5;
PART 1;
PART 2 (L13);
PART 3;
PART 4;
PART 5;
'END';
'END';
'END';     L13: 'END';
```

12

```
BLOCK     8 -      0 WORDS
BLOCK     9 -      0 WORDS
BLOCK    10 -      0 WORDS
BLOCK    11 -      0 WORDS
BLOCK    12 -      0 WORDS
BLOCK    13 -      0 WORDS
BLOCK    14 -      0 WORDS
BLOCK     6 -     41 WORDS
BLOCK     5 -    176 WORDS
BLOCK     2 -     90 WORDS
BLOCK     1 -     22 WORDS
BLOCK     3 -    835 WORDS
BLOCK     4 -   1094 WORDS
BLOCK    15 -   2505 WORDS
BLOCK    16 -   1065 WORDS
BLOCK    17 -   2663 WORDS
NO OF BUCKETS USED   209
COMPILED   #TEMP          EC

10/46/59  0#INFP   #UA68
10/47/03  0#INFP    DLTD    OK
10/47/03  0#INFP    CLKD     0
10/47/06  0#XALE   #INFP
10/48/37  0#XALE    DISP    COMPILED  #TEMP
10/48/38  0#XALE    DLTD    FI #XPCK
10/48/38  0#XALE    CLKD     10
10/48/41  0#XPCK   #XALE
10/50/10  0#XPCK    DLTD    FI #TEMP
10/50/10  0#XPCK    CLKD     23
10/50/15  0#TEMP   #XPCK
10/54/12  0#TEMP    HALT    77
10/54/14  0#TEMP    MT32   2   *00055555    SCRATCH TAPE    0   *00000000
10/54/21  0#TEMP    MT32   2   *00055555    PLOTTERFILE     0   *00000000
10/58/04  0#TEMP    CLOSED  MT32
10/58/04  0#TEMP    HALT    END OF GRAPH
10/58/08  0#TEMP    HALT    AH
10/58/08  0#TEMP    CLKD    216


   JOB HANNING,ASSTF215,WHITEHEAD,STREAM 3      RUN AT ASTON UNIV.
   BULLETIN HANNING ,3,300
   VOLUME 5000
   ALGOLUB
   ****

   JOB COST NO        178                 DATE         14/12/70
   JOB NAME         HANNING               START TIME   10/46/49
   USER NAME        WHITEHEAD             END TIME     10/58/13
   PERIPHERALS USED; 32

  TOTAL MILL TIME     271
  INPUT RECORDS      1622
  OUTPUT RECORDS     1034
```

FOURIER MILNER POWER & CROSS POWER SPECTRAL DENSITY PROGRAM.

```
14/12/70          COMPILED BY XALE MK. 4B

'DUMP ON' (ED)
'RUN'
'LEADERS'
                  'LIBRARY'(ED,SUBGROUPGRAH)
'OVERLAY PROGRAM' (TEMP)
'BLOCKSIZES'
'OVERLAY' (1) 3:4:15:16:17;
'BEGIN'     'COMMENT'GRAHAM WHITEHEAD  FOURIER/MILNER SPECTRUM ANALYSIS;  MAL
      'COMMENT'  INSERT PARAMETER CARDS
                  AUTO:= 1 OR 0
                  CROSS:= 1 OR 0
            SU:= TAPE SPEED UP RATIO
                  N := NUMBER OF CORRELATION POINTS
                  T := TIME DELAY INCREMENT
            AUTONORM  := R(INFINITY)
            CROSSNORM := RXY(INFINITY)
                  TD:= TIME DELAY FOR RXY(T);
               'REAL' A,D,NORM,RX0,C,PIE,AUTONORM,CROSSNORM ,T,TD,P;
CK      1
               'INTEGER' K,N,J,F,H,Z,E,SU,S1,S2,G;
               'BOOLEAN' AUTO,CROSS;
AUTO:='TRUE';
CROSS:='TRUE';
            N:=256;
            SU:=4;
            T:=0.02;
             Z:=1;
P:=42;
               PIE := 3.14159265358979323846;
  'BEGIN'    'REAL''ARRAY' RX[1:N],RXY[1:N],MCOS[1:N],MSIN[1:N],
CK      2
               NSIN[1:N], NCOS[1:N],M[1:N];
'PROCEDURE' PART 1;
CK      3
'BEGIN'
    L1:        'FOR' F:=1'STEP'1'UNTIL'N'DO'
'BEGIN'
               MCOS[F]:=0.0;
               MSIN[F]:=0.0;
'END';
               SELECT INPUT(3);                                F
            TD:=READ;
       'COMMENT'PRESENT AUTO DATA FIRST THEN CROSS;
               'FOR'F:=1'STEP'1'UNTIL'N'DO'  RX[F]:=0.0;
               A:= READ;
               'FOR' F:=1'STEP'1'UNTIL' N 'DO'
'BEGIN'
               A:= READ;
               D:= READ;
               RX[F]:= RX[F] + D;
'END';
          A:=READ;
          A:=READ;
```

```
                    A:= READ;
                    D:= READ;
                    RXY[F]:= RXY[F] + D ;
'END';
                       A:=0.0;
                 'FOR' F:=P 'STEP' 1 'UNTIL' N 'DO' A:=A+RX[F];
                AUTONORM:=A/(N-(P-1));
                A:=0.0;
                'FOR' F:=P 'STEP' 1 'UNTIL' N 'DO' A:=A+RXY[F];
CROSSNORM:=A/(N-(P-1));
        'COMMENT'NORM IS THE VALUE R(0)- R(INF) THAT IS USED TO NORMAL
                    RXY AND RX0 IS THE VALUE OF R(0) ;
                    NORM := RX[1]-AUTONORM;
                    RX0  := RX[1];
                    'FOR' F:=1'STEP'1'UNTIL'N 'DO'
'BEGIN'
                    RX[F] :=(RX[F] -AUTONORM)/(RX0  -AUTONORM);
                    RXY[F]:=(RXY[F]-CROSSNORM)/NORM;
'END';
                  A:=0.0;
                'FOR' F:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
                'IF' ABS(RXY[F])> A 'THEN'
'BEGIN'
                 A:=ABS(RXY[F]);
                Z:= F;
'END'
'END';
                E:=ENTIER(((N-Z)+1)/4);
                G:=(N-Z)+1;
                    'FOR' F:=1'STEP' 1'UNTIL' N 'DO'
                    M[F] := RX[F];
          'COMMENT' THE NEXT SEGMENT CALCULATES THE MILNER MODULATORS
                    RSINX/X  THE CORRELATION NUMBERS ARE HELD IN
                    ARRAY  M[ ];
                    H:= 0;
'END' OF PART 1 PROCEDURE;
'PROCEDURE' PART 2 (LAB); 'LABEL' LAB;
CK     4
'BEGIN'
    RESTART:        H:=H+1;
                    'FOR' K:=1 'STEP' 1'UNTIL' (G-1)'DO'
'BEGIN'
                C:=K/(G-1);
                    'IF' C>0.0 'AND' C<0.25 'THEN'
                    M[K+1]:= M[K+1]*SIN(2*PIE*(K/(G-1)))/(K*T) 'ELSE''IF'
                    C>0.25 'AND' C<0.5 'THEN'
    M[K+1]:= M[K+1]*SIN(PIE-2*PIE*K/(G-1))/(K*T) 'ELSE''IF'
                    C>0.5 'AND' C<0.75 'THEN'
       M[K+1]:= M[K+1]*SIN((-1)*(2*PIE*(K/(G-1))-PIE))/(K*T) 'ELSE'
                    'IF' C>0.75 'AND'C<1.0 'THEN'
       M[K+1]:= M[K+1]*SIN((-1)*(2*PIE - 2*PIE*(K/(G-1))))/(K*T)
                    'ELSE''IF' C= 0.25 'THEN'  M[K+1]:= M[K+1]*(1/(K*T))
                    'ELSE''IF' C= 0.5 'THEN'   M[K+1]:=0.0
                    'ELSE''IF' C=0.75'THEN'   M[K+1]:= M[K+1]*(-1/(K*T))
                    'ELSE''IF' C=1.0 'THEN'  M[K+1]:= 0.0;
OUTPUT(M[K+1]);
'END';
```

3

```
'COMMENT' THE NEXT SEGMENT CALCULATES THE FOURIER COEFFICIENTS
          AT SPECIAL FREQUENCIES  (2J+1)*2PIE/NT THE RESULT IS
          DEPOSITED IN ARRAYS  MSIN AND MCOS;
'BEGIN'
                J:= -1;
   RECYCLE:     J:= J+1;
                K:=0.0;
     LOOP :     K:=K+1;
                A:=((2*J)+1)*(K-1)/(G-1);
                A:=A-ENTIER(A);
                'IF' A>0.0 'AND' A <0.25 'THEN'
'BEGIN'
                MSIN[J+1]:=MSIN[J+1]+M[K]*SIN(2*PIE*A);
                MCOS[J+1]:=MCOS[J+1]+M[K]*COS(2*PIE*A);
'END'
                'ELSE''IF' A>0.25'AND' A <0.5 'THEN'
'BEGIN'
                MSIN[J+1]:= MSIN[J+1]+ M[K]*SIN(PIE-2*PIE*A);
             MCOS[J+1]:=MCOS[J+1]+M[K]*(-1)*COS(PIE-2*PIE*A);
'END'
                'ELSE''IF' A>0.5 'AND' A<0.75 'THEN'
'BEGIN'
                MSIN[J+1]:= MSIN[J+1] +M[K]*SIN((-1)*(2*PIE*A -PIE));
                MCOS[J+1]:= MCOS[J+1] +M[K]*COS((-1)*(2*PIE*A -PIE));
'END'
                'ELSE''IF' A>0.75 'AND' A < 1.0 'THEN'
'BEGIN'
                MSIN[J+1]:= MSIN[J+1]+M[K]*SIN((-1)*(2*PIE-2*PIE*A));
                MCOS[J+1]:= MCOS[J+1]+M[K]*COS(     (2*PIE-2*PIE*A));
'END'
             'ELSE''IF' A=0.0 'OR' A=1.0 'THEN'
                MCOS[J+1]:= MCOS[J+1] +M[K]
'ELSE''IF' A=0.5 'THEN' MCOS[J+1]:=MCOS[J+1]+M[K]*(-1)
   'ELSE''IF' A=0.25 'THEN' MSIN[J+1]:=MSIN[J+1]+M[K]
      'ELSE''IF'A=0.75'THEN' MSIN[J+1]:= MSIN[J+1]+M[K]*(-1);
'END';
                'IF' K = G 'THEN''GOTO' L6 'ELSE''GOTO' LOOP;
       L6:      'IF' J=(  E -1) 'THEN''GOTO'L7 'ELSE''GOTO'RECYCLE ;
       L7:      'IF' H =1 'AND' AUTO 'THEN''GOTO' L8 'ELSE''IF' H=1
                'AND''NOT'AUTO'THEN''GOTO' L9 'ELSE''GOTO'L10;
       L8:      'FOR' F:=1 'STEP'1'UNTIL' N 'DO'
'BEGIN'
                  NSIN[F] := MSIN[F];
                  NCOS[F] := MCOS[F];
'END';
             'FOR' F:=1 'STEP' 1 'UNTIL' N 'DO'
'BEGIN'
             MSIN[F]:=0.0;
             MCOS[F]:=0.0;
'END';
                'IF' AUTO 'AND' CROSS 'THEN''GOTO'  L11  'ELSE''GOTO'
                L10;
       L11:     'FOR' F:=1'STEP' 1 'UNTIL' G 'DO'
                M[F] := RXY[(F*2)-1];
                'GOTO'RESTART;
       L9 :     'GOTO'  L13;
       L10:'COMMENT' THE FOLLOWING PROCEDURE  PLOTS THE AUTOCORRELOGRA
```

```
'BEGIN'

                    'REAL' MAX,POWER;
CK        5
                    'INTEGER' LITA;
              'REAL' 'ARRAY' AB[1:5],BC[1:5] ;
              'INTEGER' 'ARRAY' CD[1:17],DE[1:3],EF[1:22],FG[1:13],
              GH[1:13],HJ[1:24],JK[1:28],KL,MN,TU,VW[1:4],LM[1:5],
              NO,OP,PQ,QR[1:7],RS[1:20],ST[1:25],UV[1:9];
'BEGIN'
          'PROCEDURE' OPENPLOT;'EXTERNAL';
CK        6
CK        7
          'PROCEDURE' CLOSEPLOT;'EXTERNAL';
CK        8
          'PROCEDURE' HGPLOTT(X,Y,IC,L);'VALUE'X,Y,IC,L;'INTEGER'IC,L;
CK        9
                        'REAL'X,Y;'EXTERNAL';
          'PROCEDURE' STRARR(A,N,S);'ARRAY'A;'INTEGER' N;
CK       10
          'STRING' S; 'EXTERNAL';
           'PROCEDURE'  HGPAXISVT(X,Y,BCD,NC,S,THETA,XMIN,DX,GAP,NH);
CK       11
           'VALUE'X,Y,NC,S,THETA,XMIN,DX,GAP,NH;'INTEGER'NC,NH;'ARRAY'
           BCD;'REAL'X,Y,S,THETA,XMIN,DX,GAP;'EXTERNAL';
           'PROCEDURE' HGPLINET(X,Y,N,K);'VALUE'N,K; 'ARRAY'X,Y;
CK       12
          'INTEGER' N,K;'EXTERNAL';
           'PROCEDURE'  HGPNUMBERT(X,Y,HT,FL,THETA,I,IP,IQ);'VALUE'X,Y,
CK       13
           HT,FL,THETA,I,IP,IQ;'INTEGER'I,IP,IQ;'REAL'X,Y,HT,FL,THETA;
           'EXTERNAL';
           'PROCEDURE'  HGPSYMBLT(X,Y,HT,BCD,THETA,N);'VALUE'X,Y,HT,
CK       14
           THETA,N;'INTEGER'N;'ARRAY'BCD;'REAL'X,Y,HT,THETA;'EXTERNAL';
'PROCEDURE' PART 3;
CK       15
'BEGIN'
                     OPENPLOT ;
                    HGPLOTT(-3,0,10,5,0,4);
                    MAX:=0.0;
                   'FOR' J:=1'STEP'1'UNTIL' E     'DO'
'BEGIN'
 'IF'ABS( NCOS[J ])> MAX'THEN' MAX:=ABS( NCOS[J])'ELSE'MAX:=MAX;
OUTPUT(NCOS[J]);
'END';
               CROSSNORM:=MAX;
NEWLINE(1);
OUTPUT(MAX);
      'COMMENT' THE NEXT SEGMENT COMPUTES THE X AND Y COORDINATES TO
                PLOT A HISTOGRAM;
               A:=(A*PIE)/(G*T*SU);
              D:=(ENTIER(((PIE/T)/SU)/8)+1)*8;
                  'FOR' J:=1 'STEP' 1 'UNTIL' E      'DO'
'BEGIN'
                AB[1]:=(((2*(J-1)+1)*A)-A)*4.5/D;
              AB[3]:=(((2*(J-1)+1)*A)+A)*4.5/D;
                  AB[2] := AB[1];
```

```
                BC[4] := 0.0;
                HGPLINET(AB,BC,4,+1);
OUTPUT(NSIN[J]);
'END';
                STRARR(CD,LITA,'('FREQUENCY%RAD/SEC')');
         HGPAXISVT(0.0,0.0,CD,-17,4.5,0.0,0.0,D/8.0,5625,-2);
                STRARR(DE,LITA,'('%%%')');
                HGPAXISVT(0.0,0.0,DE,0,3.5,90.0,0.0,0.5,1.25,+2);
                STRARR(EF,LITA,'('POWER%SPECTRAL%DENSITY')');
                HGPSYMBLT(-1.0,0.5,0.125,EF,90.0,22);
            C:=(4.5*2*PIE)/D;
            STRARR(EF,LITA,'('CYCLES/SECOND')');
            HGPAXISVT(0.0,-0.5,EF,-13,4.5,0.0,0.0,1.0,C,-1);
                STRARR(FG,LITA,'('CM.CM/RAD/SEC')');
                HGPSYMBLT(-0.625,1.0,0.1,FG,90.0,13);
                STRARR(GH,LITA,'('G.D.WHITEHEAD')');
                HGPSYMBLT(3.5,8.875,0.1,GH,0.0,13);
                STRARR(HJ,LITA,'('FOURIER/MILNER%TRANSFORM')');
                HGPSYMBLT(-1.25,8.875,0.2,HJ,0.0,24);
                STRARR(JK,LITA,'('AUTOCORRELATION%COEFFICIENT')');
                HGPSYMBLT(-1.0,4.75,0.125,JK,90.0,27);
                HGPAXISVT(0.0,4.25,DE,0,3.5,90.0,-1,0.5,0.875,+1);
HGPAXISVT(0.0,6.0,DE,-0,4.5,0.0,0.0,(T*(N-1)*SU)/5,0.9,+1);
                STRARR(KL,LITA,'('TIME')');
                HGPSYMBLT(4.5,6.125,0.1,KL,0.0,4);
                STRARR(LM,LITA,'('DELAY')');
                HGPSYMBLT(4.5,5.75,0.1,LM,0.0,5);
                STRARR(MN,LITA,'('SECS')');
                HGPSYMBLT(4.5,5.5,0.1,MN,0.0,4);
                HGPLOTT(0.0,0.0,3,0);
                'FOR' F:=1'STEP'1'UNTIL'N'DO'
            M[F]:=(F/N*4.5)-(4.5/N);
                'FOR' F:=1 'STEP' 1 'UNTIL' N 'DO'
            RX[F]:=1.75*RX[F];
                HGPLOTT(0.0,-6.0,0,4);
            HGPLINET(M,RX,N,+1);
            'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
            RXY[F]:=1.75*RXY[F];
                STRARR(NO,LITA,'('R(0)%%=')');
                HGPSYMBLT(2.875,1.65,0.1,NO,0.0,7);
                STRARR(OP,LITA,'('R(INF)=')');
                HGPSYMBLT(2.875,1.5,0.1,OP,0.0,7);
                HGPNUMBERT(3.375,1.65,0.1,RX0,0.0,1.0,2,6);
                HGPNUMBERT(3.375,1.50,0.1,AUTONORM,0.0,1.0,2,6);
                STRARR(PQ,LITA,'('POWER=')');
                POWER:= 0.0;
              'FOR' F:=1'STEP'1'UNTIL' E      'DO'
                POWER:= POWER + NCOS[F] ;
                HGPSYMBLT(2.875,-2.625,0.1,PQ,0.0,6);
                HGPNUMBERT(3.125,-2.625,0.1,POWER,0.0,1.0,2,6);
                STRARR(QR,LITA,'('%%MAX=')');
                HGPSYMBLT(2.875,-2.475,0.1,QR,0.0,6);
                HGPNUMBERT(3.125,-2.475,0.1,MAX,0.0,1.0,2,6);
                AB[1] := 5.25;
                AB[2] := 5.25;
                AB[3] := -1.75;


                AB[4] := -1.75;
```

```
                    BC[3] := +3.25;
                    BC[4] := -7.25;
                    BC[5] := -7.25;
                    HGPLINET(AB,BC,5,+1);
                    HGPLOTT(0,0,0.0,3,0);
     L11:       HGPLOTT(-10.75,0.0,0,4);
               HGPLINET(M,RXY,N,+1);
               HGPAXISVT(0,0,-1.75,DE,0,3.5,90.0,-1,0.5,0.875,+2);
HGPAXISVT(0.0,0.0,DE,-0,4.5,0.0,0.0,(T*(N-1)*SU)/5,0.9,+1);
               STRARR(JK,LITA,'('CROSSCORRELATION%COEFFICIENT')');
               HGPSYMBLT(-1.0,-1.375,0.125,JK,90.0,28);
               HGPSYMBLT(4.625,0.125,0.1,KL,0.0,4);
               HGPSYMBLT(4.625,0.0,0.1,LM,0.0,5);
                HGPSYMBLT(4.625,-0.125,0.1,MN,0.0,4);
                    MAX:=0.0;
                    'FOR' F:=1 'STEP' 1 'UNTIL' E      'DO'
'BEGIN'
'IF'ABS( MSIN[F]) > MAX 'THEN' MAX :=ABS(MSIN[F] )'ELSE' MAX:=MAX;
'END';
               'FOR' F:=1'STEP'1'UNTIL' E'DO'
'BEGIN'
               'IF'ABS(MCOS[F])>MAX 'THEN' MAX:=ABS(MCOS[F]) 'ELSE' MAX
               :=MAX;
'END';
                    'FOR' F:=1 'STEP'1'UNTIL' E      'DO'
'BEGIN'
               RX[F]:=(MCOS[F]/MAX)*2.5;
               RXY[F]:=(MSIN[F]/MAX)*2.5;
'END';
               HGPLOTT(0.0,0.0,3,0);
               HGPLOTT(-1.75,4.625,0,4);
               HGPLINET(RX,RXY,E,+1);
                    HGPAXISVT(-2.5,0.0,DE,0,5.0,0.0,-1,0,0.2,0.5,1);
                    HGPAXISVT(0.0,-2.5,DE,0,5.0,90.0,-1.0,0.2,0.5,1);
                    STRARR(TU,LITA,'('REAL')');
                    STRARR(UV,LITA,'('IMAGINARY')');
                    HGPSYMBLT(2.75,0.0,0.125,TU,0.0,4);
                    HGPSYMBLT(-0.5,2.7,0.125,UV,0.0,9);
                    STRARR(RS,LITA,'('FOURIER/MILNER%CROSS')');
                    STRARR(ST,LITA,'('SPECTRAL%DENSITY%ANALYSIS')');
               HGPLOTT(0.0,0.0,3,0);
               HGPLOTT(0.0,-1.875,0,4);
                    HGPSYMBLT(1.5,5.5,0.1   ,GH,0.0,13);
                    HGPSYMBLT(-2.875,5.5,0.125,RS,0.0,20);
                    HGPSYMBLT(-2.875,5.25,0.125,ST,0.0,25);
                    STRARR(VW,LITA,'('MAX=')');
                    HGPSYMBLT(-2.875,5.0,0.1,VW,0.0,4);
                    HGPNUMBERT(-2.275,5.0,0.1,MAX,0.0,1,0,6);
                    AB[1] := -3.5;
                    AB[2] := -3.5;
                    AB[3] :=  3.5;
                    AB[4] :=  3.5;
                    AB[5] := -3.5;
                    BC[1] :=  6.0;
                    BC[2] := -4.5;
                    BC[3] := -4.5;
                    BC[4] :=  6.0;
                    BC[5] :=  6.0;
                    HGPLINET(AB,BC,5,+1);
'END' OF PART 3 ;
```

```
'PROCEDURE' PART 4;
CK    16
'BEGIN'
'COMMENT' THE NEXT SEGMENT PLOTS THE CO AND QUAD SPECTRUM;
              HGPLOTT(0.0,0.0,3,0);
              HGPLOTT(-10.0,0.0,0,4);
                  HGPSYMBLT(-2.875,5.5,0.125,RS,0.0,20);
                  HGPSYMBLT(-2.875,5.25,0.125,ST,0.0,25);
                  HGPSYMBLT(1.5,5.5,0.1  ,GH,0.0,13);
                  HGPLINET(AB,BC,5,+1);
              HGPLOTT(0.0,0.0,3,0);
              HGPLOTT(1.75,2.625,0,4);
                  'FOR' J:=1 'STEP' 1 'UNTIL' E      'DO'
'BEGIN'
                AB[1]:=(((2*(J-1)+1)*A)-A)*4.5/D;
              AB[3]:=(((2*(J-1)+1)*A)+A)*4.5/D;
                  AB[2] := AB[1];
                  AB[4] := AB[3];
                  BC[1] := 0.0;
              BC[2]:=MSIN[J]*2.5/MAX;
                  BC[3] := BC[2];
                  BC[4] := 0.0;
                  HGPLINET(AB,BC,4,+1);
'END';
         --> HGPAXISVT(0.0,-1.25,CD,-17,4.5,0.0,0.0,D/8,0.5625,-2);
              HGPAXISVT(0.0,-1.25,EF,13,4.5,0.0,0.0,1.0,C,-1);
              HGPAXISVT(0.0,-1.25,DE,0,3.75,90.0,-0.5,0.5,1.25,+2);
              STRARK(JK,LITA,'('QUAD%SPECTRUM')');
              HGPSYMBLT(-1.0,-0.25,0.125,JK,90.0,13);
              HGPSYMBLT(-0.625,-0.25,0.1,FG,90.0,13);
              HGPLOTT(0.0,0.0,3,0);
              HGPLOTT(0.0,-4.375,0,4);
                  'FOR' J:=1 'STEP' 1 'UNTIL' E      'DO'
'BEGIN'
                AB[1]:=(((2*(J-1)+1)*A)-A)*4.5/D;
              AB[3]:=(((2*(J-1)+1)*A)+A)*4.5/D;
                  AB[2] := AB[1];
                  AB[4] := AB[3];
                  BC[1] := 0.0;
              BC[2]:=MCOS[J]*2.5/MAX;
                  BC[3] := BC[2];
                  BC[4] := 0.0;
                  HGPLINET(AB,BC,4,+1);
'END';
              HGPAXISVT(0.0,-1.25,CD,-17,4.5,0.0,0.0,D/8,0.5625,-2);
              HGPAXISVT(0.0,-1.25,EF,13,4.5,0.0,0.0,1.0,C,-1);
              HGPAXISVT(0.0,-1.25,DE,0,3.75,90.0,-0.5,0.5,1.25,+2);
              STRARK(JK,LITA,'('CO%SPECTRUM')');
              HGPSYMBLT(-1.0,0.0,0.125,JK,90.0,11);
              HGPSYMBLT(-0.625,-0.25,0.1,FG,90.0,13);
'END' OF PART 4;
'PROCEDURE' PART 5;
CK    17
'BEGIN'
'COMMENT' THE NEXT SEGMENT COMPUTES AND PLOTS THE GAIN & PHASE;
              'FOR' F:=1'STEP' 1 'UNTIL' E 'DO'
              RX[F]:=20.0*0.43429448*(LN(SQRT((MSIN[F]*MSIN[F])
              +(MCOS[F]*MCOS[F]))/
              SQRT((NSIN[F]*NSIN[F])+(NCOS[F]*NCOS[F]))));
              'FOR' F:=1'STEP'1'UNTIL'E'DO'
```

```
            RX[F]:=RX[F]*1,875/20;
            HGPLOTT(0,0,0.0,3,0);
            HGPLOTT(10,0,14.25,0,4);
                'FOR' J:=1 'STEP' 1 'UNTIL' E     'DO'
'BEGIN'
              AB[1]:=(((2*(J-1)+1)*A)-A)*4.5/D;
            AB[3]:=(((2*(J-1)+1)*A)+A)*4.5/D;
                AB[2] := AB[1];
                AB[4] := AB[3];
                BC[1] := 0.0;
            BC[2]:=RX[J];
                BC[3] := BC[2];
                BC[4] := 0.0;
                HGPLINET(AB,BC,4,+1);
'END';
            HGPAXISVT(0.0,0.0,DF,-0,4.5,0.0,0.0,D/8,0.5625,-1);
            HGPAXISVT(0.0,-1.875,CD,-17,4.5,0.0,0.0,D/8,0.5625,-2);
            HGPAXISVT(0.0,-1.875,EF,13,4.5,0.0,0.0,1.0,C,-1);
            HGPAXISVT(0.0,-1.875,DE,0,3.75,90.0,-20,10,0.9375,-2);
            STRARR(JK,LITA,'('GAIN-DECIBELS')');
            HGPSYMBLT(-1.0,-0.875,0.125,JK,90.0,13);
            HGPLOTT(0.0,0.0,3,0);
            HGPLOTT(0.0,4.375,0,4);
            HGPNUMBERT(2.25,1.1,0.1,TD,0.0,0,2,3);
            STRARR(JK,LITA,'('M%SECS.')');
            HGPSYMBLT(3.0,1.1,0.1,JK,0.0,7);
            'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
            RX[F]:=-(360*TD)/((6*T*SU)/(2*(F-1)+1));
            'FOR' F:=1 'STEP' 1 'UNTIL' E'DO'
'BEGIN'
            S1:=SIGN(MCOS[F]);
            S2:=SIGN(MSIN[F]);
            T:=ABS(ARCTAN(MSIN[F]/MCOS[F])*57.29578);
            'IF' S1=1'AND'S2=-1'AND'F<17'THEN' RX[F]:=RX[F]+T
     'ELSE' 'IF' S1=0 'AND' S2=0'THEN' RX[F]:=RX[F]+0.0
            'ELSE''IF'S1=1'AND'S2=1'THEN' RX[F]:=RX[F]-T
            'ELSE''IF'S1=0'AND'S2=1 'THEN' RX[F]:=RX[F]-90
            'ELSE''IF'S1=0'AND'S2=-1'THEN'RX[F]:=RX[F]-270
            'ELSE''IF'S1=1'AND'S2=0'THEN' RX[F]:=RX[F]+0.0
            'ELSE''IF'S1=-1'AND'S2=0'THEN' RX[F]:=RX[F]-180
            'ELSE''IF'S1 =-1'AND'S2=1'THEN' RX[F]:=RX[F]-(180-T)
            'ELSE''IF'S1=1'AND'S2=-1'THEN' RX[F]:=RX[F]-(360-T)
            'ELSE''IF'S1=-1'AND'S2=-1'THEN' RX[F]:=RX[F]-(180+T);
            'IF' RX[F]<-360'THEN'RX[F]:=-360;
'END';
'FOR'F:=1'STEP'1'UNTIL'E'DO' RX[F]:=RX[F]*1.875/360;
             'FOR' F:=0'STEP'1'UNTIL'(E-1)'DO'
           RXY[F+1]:=(((2*F)+1)*A)*4.5/D;
           HGPLINET(RXY,RX,E,+1);
           HGPAXISVT(0.0,0.0,DF,-0,4.5,0.0,0.0,D/8,0.5625,-1);
           HGPAXISVT(0.0,-1.875,CD,-17,4.5,0.0,0.0,D/8,0.5625,-2);
           HGPAXISVT(0.0,-1.875,EF,13,4.5,0.0,0.0,1.0,C,-1);
            HGPAXISVT(0.0,-1.875,DE,0,3.75,90.0,-360,180,0.9375,-2);
           STRARR(JK,LITA,'('PHASE%ANGLE%-DEGREES')');
           HGPSYMBLT(-1.0,-1.0,0.125,JK,90.0,20);
           STRARR(JK,LITA,'('AGGREGATE%TIME%DELAY')');
           HGPSYMBLT(2.25,1.25,0.1,JK,0.0,20);
           HGPLOTT(0.0,0.0,3,0);
           HGPLOTT(-1.75,-1.75,0,4);
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                        HGPSYMBLT(-2,875,5.25,0.125,ST,0,0,25);
                        HGPSYMBLT(1.5,5.5,0.1   ,GH,0,0,13);
                        AB[1]  := -3.5;
                        AB[2]  := -3.5;
                        AB[3]  :=  3.5;
                        AB[4]  :=  3.5;
                        AB[5]  := -3.5;
                    BC[1]:=6.0;
                        BC[2]  := -4.5;
                        BC[3]  := -4.5;
                        BC[4]  :=  6.0;
                        BC[5]  :=  6.0;
                        HGPLINET(AB,BC,5,+1);
          L12:           CLOSEPLOT;
SELECT OUTPUT(4);
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
                  PRINT(NCOS[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(NSIN[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(MCOS[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
NEWLINE(1);
'FOR' F:=1 'STEP' 1 'UNTIL' E 'DO'
'BEGIN'
PRINT(MSIN[F],2,5);
NEWLINE(1);
'END';
RUNOUT;
RUNOUT;
'END' OF PART 5;
PART 1;
PART 2 (L13);
PART 3;
PART 4;
PART 5;
'END';
'END';

          L13: 'END';
'END';
 0 WORDS
```

```
JOB TWOFIVE,ASSTF215,WHITHEAD,STREAM 3        RUN AT ASTON UNIV.
BULLETIN TWOFIVE ,3,300
VOLUME 5000
ALGOLUB
****
```

```
JOB COST NP          202                 DATE          14/12/70
JOB NAME             TWOFIVE             START TIME    11/31/27
USER NAME            WHITHEAD            END TIME      11/44/26
PERIPHERALS USED: 30

TOTAL MILL TIME      274
INPUT RECORDS       1561
OUTPUT RECORDS      1489
MAX. CORE SIZE     16000
```

```
25/02/71            COMPILED BY XALT MK.   3

'SEND TO' ( ED,ICLA-DEFAULT(0),,PROGRAM)
'WORK' (ED,WORK FILE    (0))
'BEGIN'
            'REAL' A,D,NORM,RXO,C,PIE,AUTONORM,CROSSNORM ,T,TD,P;
            'INTEGER' K,N,J,F,H,Z,E,SU,S1,S2,G;
        T:=0,02;
            PIE := 3.14159265358979323846;
         SU:=4;
            SELECT INPUT(3);
         N:=READ;
         H:=READ;
         C:=READ;
C:=60,0;
         G:=N*H;
   'BEGIN'    'REAL''ARRAY' RX[1:G],RXY[1:G],M[1:G];
'BEGIN'
            'REAL' MAX,POWER;
            'INTEGER' LITA;
         'REAL' 'ARRAY' AB[1:5],BC[1:5] ;
         'INTEGER' 'ARRAY' CD[1:17],DE[1:3],FF[1:22],FG[1:13],
         GH[1:13],HJ[1:24],JK[1:28],KL,MN,TU,VW[1:4],LM[1:5],
         NO,OP,PQ,QR[1:7],RS[1:20],ST[1:25],UV[1:9];
            'FOR' J:=1'STEP'1'UNTIL' H 'DO' CD[J]:= READ;
'BEGIN'
      'PROCEDURE' OPENPLOT;'EXTERNAL';
      'PROCEDURE' CLOSEPLOT;'EXTERNAL';
      'PROCEDURE' HGPLOT (X,Y,IC,L);'VALUE'X,Y,IC,L;'INTEGER'IC,L;
                'REAL'X,Y;'EXTERNAL';
      'PROCEDURE' STRARR(A,N,S);'ARRAY'A;'INTEGER' N;
      'STRING' S; 'EXTERNAL';
       'PROCEDURE'  HGPAXISV (X,Y,BCD,NC,S,THETA,XMIN,DX,GAP,NH);
       'VALUE'X,Y,NC,S,THETA,XMIN,DX,GAP,NH;'INTEGER'NC,NH;'ARRAY'
       BCD;'REAL'X,Y,S,THETA,XMIN,DX,GAP;'EXTERNAL';
       'PROCEDURE' HGPLINE (X,Y,N,K);'VALUE'N,K; 'ARRAY'X,Y;
      'INTEGER' N,K;'EXTERNAL';
       'PROCEDURE'  HGPNUMBER (X,Y,HT,FL,THETA,I,IP,IQ);'VALUE'X,Y,
       HT,FL,THETA,I,IP,IQ;'INTEGER'I,IP,IQ;'REAL'X,Y,HT,FL,THETA;
       'EXTERNAL';
       'PROCEDURE'  HGPSYMBL (X,Y,HT,BCD,THETA,N);'VALUE'X,Y,HT,
       THETA,N;'INTEGER'N;'ARRAY'BCD;'REAL'X,Y,HT,THETA;'EXTERNAL';
         'FOR' F:=1'STEP'1'UNTIL'G'DO'
         RX[F]:=READ;
         'FOR' J:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
         'FOR' F:=1'STEP'1'UNTIL' N 'DO'
         RX[F+(N*(J-1))]:= RX[F+(N*(J-1))]*CD[J];
'END';
         A:=0,0;
         'FOR' F:=1'STEP'1'UNTIL'G'DO'
'BEGIN'
         'IF' ABS(RX[F])> A 'THEN' A:= ABS(RX[F]);
'END';
         'FOR' F:=1'STEP'1'UNTIL'G'DO'
         RX[F]:=RX[F]*1,66/A;
   AUTONORM:=(4,3/(H-1));
    NORM:=(4,3/(H-1))/(SIN(C/57,29578)/COS(C/57,29578));
```

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```
            HGPLOT(-6.0,12.0,0,4);
            'FOR' J:=1'STEP'1'UNTIL' N'DO'
'BEGIN'
            Z:=0.0;
             'FOR' F:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
             M[F]:=AUTONORM*(F-1)+1.66-(ABS(RX[F+Z+J-1]));
            Z:=Z+N-1;
'END';
             'FOR' F:=1'STEP'1'UNTIL' H 'DO'
             RXY[F]:=((F-1)*NORM)+  ((J-1)*(3.0/(N-1)));
            HGPLINE(RXY,M,H,+1);
'END';
             'FOR' J:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
             'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
              M[F]:=AUTONORM*(J-1)+1.66-(ABS(RX[F+(N*(J-1))]));
             'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
             RXY[F]:=(F-1)*3.0/(N-1)+NORM*(J-1);
             HGPLINE(RXY,M,N,+1);
'END';
             AB[1]:=0.0;
            AB[2]:=0.0;
            AB[3]:=3.0;
            AB[4]:=4.3/(SIN(C/57.29578)/(COS(C/57.29578)))+3.0;
            BC[1]:=1.66;
BC[2]:=0.0;
BC[3]:=0.0;
BC[4]:=4.3;
                HGPLINE (AB,BC,4,+1);
            D:=(ENTIER(((PIE/T)/SU)/8)+1)*8;
            A:=(3.0*2*PIE)/D;
                STRARR(CD,LITA,'('FREQUENCY%RAD/SEC')');
        HGPAXISV (0.0,0.0,CD,-17,3.0,0.0,0.0,D/8.0,3750,-1);
                STRARR(EF,LITA,'('POWER%SPECTRAL%DENSITY')');
            HGPSYMBL (-0.625,0.5,0.1,EF,90.0,22);
            STRARR(EF,LITA,'('CYCLES/SECOND')');
            HGPAXISV (0.0,-0.5,EF,-13,4.5,0.0,0.0,1.0,A,-1);
                STRARR(GH,LITA,'('G.D.WHITEHEAD')');
HGPSYMBL(3.9,7.95,0.1,GH,0.0,13);
AB[1]:=-1.25;
AB[2]:=5.65;
AB[3]:=5.65;
AB[4]:=-1.25;
AB[5]:=-1.25;
BC[1]:=-2.0;
BC[2]:=-2.0;
BC[3]:=8.45;
BC[4]:=8.45;
BC[5]:=-2.0;
            HGPLINE(AB,BC,5,+1);
AB[1]:=-1.9;
AB[2]:=6.25;
AB[3]:=6.25;
AB[4]:=-1.9;
AB[5]:=-1.9;
BC[1]:=-2.65;
BC[2]:=-2.65;
BC[3
```

```
BC[5]:=-2,65;
                HGPLINE(AB,BC,5,+1);
                HGPLOT(0,0;0.0;3,0);
                HGPLOT(-12.5,0,0,0,4);
                'FOR' J:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
                'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
                M[F]:=ABS(RX[F+(N*(J-1))])+AUTONORM*(J-1);
                'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
                RXY[F]:=(F-1)*3.0/(N-1)+NORM*(J-1);
                HGPLINE(RXY,M,N,+1);
'END';
               'FOR' J:=1'STEP'1'UNTIL' N'DO'
'BEGIN'
               Z:=0,0;
                'FOR' F:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
                M[F]:=AUTONORM*(F-1)+      (ABS(RX[F+Z+J-1]));
               Z:=Z+N-1;
'END';
                'FOR' F:=1'STEP'1'UNTIL' H 'DO'
                RXY[F]:=((F-1)*NORM)+  ((J-1)*(3.0/(N-1)));
               HGPLINE(RXY,M,H,+1);
'END';
                 AB[1]:=0,0;
                AB[2]:=0,0;
                AB[3]:=3,0;
                AB[4]:=4,3/(SIN(C/57,29578)/(COS(C/57,29578)))+3,0;
                BC[1]:=1,66;
BC[2]:=0,0;
BC[3]:=0,0;
BC[4]:=4,3;
                     HGPLINE (AB,BC,4,+1);
                     STRARR(CD,LITA,'('FREQUENCY%RAD/SEC')');
         HGPAXISV (0,0,0,0,CD,-17,3,0,0,0,0,0,D/8,0,3750,-1);
                     STRARR(EF,LITA,'('POWER%SPECTRAL%DENSITY')');
               HGPSYMBL (-0,625,0.5,0.1,EF,90.0,22);
               STRARR(EF,LITA,'('CYCLES/SECOND')');
               HGPAXISV (0,0,-0,5,FF,-13,4.5,0,0,0.0,1,0,A,-1);
                     STRARR(GH,LITA,'('G,D,WHITEHEAD')');
HGPSYMBL(3,9,7,95,0,1,GH,0,0,13);
AB[1]:=-1,25;
AB[2]:=5,65;
AB[3]:=5,65;
AB[4]:=-1,25;
AB[5]:=-1,25;
BC[1]:=-2,0;
BC[2]:=-2,0;
BC[3]:=8,45;
BC[4]:=8,45;
BC[5]:=-2,0;
               HGPLINE(AB,BC,5,+1);
AB[1]:=-1,9;
AB[2]:=6,25;
AB[3]:=6,25;
AB[4]:=-1,9;
AB[5]:=-1,9;
BC[1]:=-2,65;
BC[2]:=-2,65;
```

```
BC[4]:=9,1;
BC[5]:=-2,65;
                HGPLINE(AB,BC,5,+1);
                HGPLOT(0,0,0,0,3,0);
                HGPLOT(0,0,13,0,0,4);
                'FOR' J:=1'STEP'1'UNTIL' H 'DO'
'BEGIN'
                'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
                M[F]:=ABS(RX[F+(N*(J-1))])+AUTONORM*(J-1);
                'FOR' F:=1'STEP' 1 'UNTIL' N 'DO'
                RXY[F]:=(F-1)*3,0/(N-1)+NORM*(J-1);
                HGPLINE(RXY,M,N,+1);
'END';
                 AB[1]:=0,0;
                AB[2]:=0,0;
                AB[3]:=3,0;
                AB[4]:=4,3/(SIN(C/57,29578)/(COS(C/57,29578)))+3,0;
                BC[1]:=1,66;
BC[2]:=0,0;
BC[3]:=0,0;
BC[4]:=4,3;
                   HGPLINE (AB,BC,4,+1);
AB[1]:=-1,25;
AB[2]:=5,65;
AB[3]:=5,65;
AB[4]:=-1,25;
AB[5]:=-1,25;
BC[1]:=-2,0;
BC[2]:=-2,0;
BC[3]:=8,45;
BC[4]:=8,45;
BC[5]:=-2,0;
                HGPLINE(AB,BC,5,+1);
AB[1]:=-1,9;
AB[2]:=6,25;
AB[3]:=6,25;
AB[4]:=-1,9;
AB[5]:=-1,9;
BC[1]:=-2,65;
BC[2]:=-2,65;
BC[3]:=9,1;
BC[4]:=9,1;
BC[5]:=-2,65;
                HGPLINE(AB,BC,5,+1);
                CLOSEPLOT;
'END';
'END';
'END';
'END';
    LENGTH  2233
ED   47
        EC
```

```
JOB DISPLAY,ASSTF215,WHITHEAD,STREAM 3          RUN AT ASTON UNIV.
BULLETIN DISPLAY ,3,300
ALGOL ,,,,[BINARYSTORE F215ISD,AXXX]
STOREDPROG F215ISD
****
```

```
JOB COST NP            196             DATE           25/02/71
JOB NAME            DISPLAY            START TIME     19/33/44
USER NAME           WHITHEAD           END TIME       19/46/38
PERIPHERALS USED;

TOTAL MILL TIME      120
INPUT RECORDS        689
OUTPUT RECORDS       308
MAX. CORE SIZE     18752
```

PHASE PLANE ANALYSIS PROGRAM

```
16/07/70            COMPILED BY XALE MK.  4B

'DUMP ON' (ED,PROGRAM TEMP(6),,NEWFILE)
'RUN'
'LIBRARY' (ED,SUBGROUPSRA3)
 'LIBRARY'(ED,SUBGROUPGRAH)
         'INPUT'3=TRO
'OUTPUT' 0=LPO
'BEGIN' 'REAL' A,B,C,D,VEL1,VEL2,VEL3,ACCEL1,ACCEL2,AMPAV1,AMPAV2,
         AMPAV3,AMDIF1,AMDIF2,ANGLE1,ANGLE2,ANGDIFF,ANGVEL,SAMPINT,
     VOLTRANGE,ADDRESS,HISTOGRAM,W,X,FORG1,DYNAMRANGE;
       'INTEGER'SAMPLESIZE,I,J,K,L,COUNTER,E,F,G,H,R,S,N,P,Q,Y,Z,LITA,
          LITB,LITC,LITD,ONE,SGN1,SGN2,SGN3,SGN4,SGN5,SIGN1,SIGN2,
     ALPHA,QUAD1,QUAD2,QUAD3,QUAD4 ;
       'BOOLEAN' BOOL1,BOOL2,BOOL3,BOOL4,BOOL5;
          A:=B:=C:=D:=ADDRESS:= HISTOGRAM:=0;
         SELECT OUTPUT (0);
          SELECT INPUT (3);
     SAMPLESIZE := READ;
              I:= READ;
              J:= READ;
              K:= READ;
              L:= READ;
       SAMPINT  := READ;
      VOLTRANGE := READ;
      DYNAMRANGE:= READ;
        QUAD3   := READ;
           G:=2*J+1;
'BEGIN'        'INTEGER''ARRAY' AMP[I:J],DEG[0:360],ANG[K:L],EF[1:20]
           ,FG[1:14],GH,HJ,JK[1:2],BCD[1:4],LM[1:14],MN,NO[1:10],
           OR[1:7],RS[1:11],UV[1:25],ABC[1:17],DEF[1:29],XYZA
[1:22],XYZC[1:9],KL[1:16];
           'REAL''ARRAY' PMA[1:SAMPLESIZE],LEVGNA[1:SAMPLESIZE],
AB,BC,CD,DE,UVW,XYZ,ST,TU[1:5],GHI[1:12],OP,PQ[1:361],
           XY,YZ,JKL,MNO,PQR,STU[1:6];
    'FOR' Y:=I'STEP'1'UNTIL'J'DO' AMP[Y]:=0;
    'FOR' Y:=1'STEP'1'UNTIL' SAMPLESIZE 'DO' PMA[Y]:=0;
    'FOR' Z:=0'STEP'1'UNTIL' 360 'DO' DEG[Z]:=0;
    'FOR' Z:=K'STEP'1'UNTIL' L 'DO' ANG[Z]:=0;
    'FOR' Z:=1'STEP'1'UNTIL'SAMPLESIZE'DO'LEVGNA[Z]:= 0;
   SELECT INPUT (5);
F:=READ;
    A:= READ;
F:=READ;
    B:= READ;
F:=READ;
    C:= READ;
        'FOR' W:= A,B,C'DO'
'BEGIN'  HISTOGRAM := (W/VOLTRANGE)*J;
         HISTOGRAM := ENTIER(HISTOGRAM);
        'IF' HISTOGRAM > J-1'THEN'F:=J'ELSE''IF'HISTOGRAM < I+1'THEN'
         F:=I'ELSE'F:= HISTOGRAM ;
'END'; AMP[F]:=AMP[F]+1;

         N:= 3 ;
         VEL1:= (A-B);
```

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```
        AMPAV2:= (B+C)/2;
        AMDIF1:= AMPAV1-AMPAV2;
        SGN1 := SIGN(ACCEL1);
        SGN2 := SIGN(AMDIF1);
       'IF' SGN1=0 'AND'SGN2=0'THEN''GOTO'L2'ELSE''IF'SGN1=0'THEN'
        'GOTO'L3'ELSE''IF'SGN2=0'THEN''GOTO'L4;
        'IF' SGN1=1'AND'SGN2=1'THEN'QUAD1:=1'ELSE''IF'SGN1=1'AND'
        SGN2=-1'THEN'QUAD1:=2'ELSE''IF' SGN1=-1'AND'SGN2=-1'THEN'
        QUAD1:=3 'ELSE''IF' SGN1=-1'AND'SGN2=1'THEN'QUAD1:=4;
        ANGLE1 := ABS(ARCTAN(ACCEL1/AMDIF1)* 57.29578);
       'IF' QUAD1=2 'THEN' ANGLE1:= 180-ANGLE1 'ELSE''IF'QUAD1=3
       'THEN' ANGLE1:= 180+ANGLE1 'ELSE''IF'QUAD1=4 'THEN'ANGLE1:=
        360 - ANGLE1 'ELSE' ANGLE1 := ANGLE1 ;
       'GOTO' L5;
    L2:QUAD1:=1;
        ANGLE1:=0;
       'GOTO' L5;
    L3:'IF' SGN2=  -1'THEN''BEGIN' ANGLE1:= 180; QUAD1:=2;'END'
  'ELSE'
       'IF' SGN2=+1'THEN''BEGIN'ANGLE1:=0;QUAD1:=1;'END';
       'GOTO' L5;
    L4: 'IF' SGN1=-1'THEN''BEGIN' ANGLE1:=270; QUAD1:=3;'END''ELSE'
        'IF' SGN1=+1 'THEN''BEGIN' ANGLE1:=90; QUAD1:=1; 'END';
        'GOTO' L5;
    L5: F:= ENTIER(ANGLE1);
                DEG[F]:=DEG[F]+1;
        A:= B;
        B:= C;
  RECYCLE:  SELECT INPUT (3) ;
F:=READ;
        C:= READ;
        VEL1:= (A-B);
        VEL2:= (B-C);
        ACCEL2:= VEL1-VEL2;
        AMPAV1:= (A+B)/2;
        AMPAV2:= (B+C)/2;
        AMDIF2:= AMPAV1-AMPAV2;
        SGN3 := SIGN(ACCEL2);
        SGN4 := SIGN(AMDIF2);
       'IF' SGN3=0'AND'SGN4=0'THEN''GOTO'L6'ELSE''IF'SGN3=0 'THEN'
        'GOTO'L7 'ELSE''IF'SGN4=0'THEN''GOTO'L8;
       'IF' SGN3=1'AND' SGN4=1 'THEN' QUAD2:=1'ELSE''IF' SGN3=1'AND'
        SGN4=-1 'THEN' QUAD2:=2 'ELSE''IF'SGN3=-1 'AND' SGN4=-1
  'THEN' QUAD2:=3'ELSE''IF'SGN3=-1'AND'SGN4 =+1 'THEN'
        QUAD2 :=4;
        ANGLE2 := ABS(ARCTAN(ACCEL2/AMDIF2)*57.29578);
       'IF' QUAD2 =2 'THEN' ANGLE2:= 180-ANGLE2 'ELSE''IF'QUAD2=3
       'THEN' ANGLE2 := 180+ANGLE2 'ELSE''IF' QUAD2    =4'THEN'
        ANGLE2 :=360-ANGLE2 'ELSE'    ANGLE2 := ANGLE2 := ANGLE2 ;
       'GOTO' L9;
    L6: QUAD2:=1;
         ANGLE2:=0;
        'GOTO'L9;
    L7:'IF' SGN4=-1 'THEN''BEGIN'ANGLE2:= 180;QUAD2:=2;'END'
        'ELSE''IF' SGN4=1 'AND'QUAD1=1 'OR' QUAD1=2 'THEN''BEGIN'
        ANGLE2:=0; QUAD2:=1;'END' 'ELSE''IF' SGN4=1 'AND'QUAD1=3'OR'
        QUAD1=4 'THEN''BEGIN' ANGLE2 := 360; QUAD2 :=4; 'END';
```

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```
            'GOTO'L9;
L9:'IF'QUAD1=1'AND'QUAD2=4'OR'QUAD1=4'AND'QUAD2=1
            'THEN''GOTO'L10'ELSE''GOTO'L11;
L10:'IF'QUAD1=4'THEN'ANGDIFF:=((360-ANGLE1)-ANGLE2)
'ELSE''IF'QUAD2=4'THEN'ANGDIFF:=(ANGLE1-(360-ANGLE2));
'GOTO'L12;
        L11: ANGDIFF := ANGLE1-ANGLE2;
       L12: ANGVEL := (ANGDIFF/SAMPINT);
            HISTOGRAM := (C/VOLTRANGE)*J;
            HISTOGRAM := ENTIER(HISTOGRAM);
           'IF' HISTOGRAM > J-1'THEN' F:=J 'ELSE''IF' HISTOGRAM < I+1
            'THEN' F:=I  'ELSE' F:= HISTOGRAM ;
            AMP[F]:= AMP[F]+1;
            ADDRESS:=(ANGVEL/720)*L;
            ADDRESS := ENTIER(ADDRESS);
     'IF' ADDRESS > L-1'THEN' G:=L'ELSE''IF'ADDRESS <K+1 'THEN'
            G:=K 'ELSE' G:= ADDRESS;
            ANG[G]:= ANG[G]+1;
            Z := F;
            S := G;
            F:= ENTIER(ANGLE2);
            DEG[F] := DEG[F]+1;
            N := N+1 ;
            PMA[N-3]:=  (C/VOLTRANGE)* 3,0;
            LEVGNA[N-3] := (ANGDIFF/720)* 3,0;
           'IF' N  'LE'  SAMPLESIZE 'THEN''GOTO' REARRANGE 'ELSE''GOTO'
            FINISH;
  REARRANGE: A:=B;
             B:=C;
            QUAD1:=QUAD2;
            ANGLE1:= ANGLE2;
           'GOTO' RECYCLE;
   FINISH;
           'FOR' F:=0'STEP'1'UNTIL' 360'DO'
           'BEGIN'    OP[F+1]:=(DEG[F]/(N-2))* 6,0;
                      PO[F+1]:= -3,0 +(F*6/360);
     'END';
            G:= (L*2)+1;
           'FOR' F:= K'STEP'1'UNTIL'L'DO'
           'BEGIN'  P:= ABS( L + F );
            XY[P+1] := (ANG[F]/(N-3))* 6,0;
OUTPUT(ANG[F]);
            YZ[P+1]:= -3,0 +ABS(P*(3/K));
     'END';
NEWLINE(1);
           'FOR' F:= I'STEP'1'UNTIL' J 'DO'
    'BEGIN'    P:= ABS((J)+(F));
           JKL[P+1]:= (AMP[F]/N)* 6,0;
OUTPUT(AMP[F]);
           MNO[P+1]:= -3,0+(ABS(P*(3/J)));
     'END';
'BEGIN'
           'PROCEDURE' OPENPLOT;'EXTERNAL';
           'PROCEDURE' CLOSEPLOT;'EXTERNAL';
           'PROCEDURE' HGPLOTT(X,Y,IC,L);'VALUE'X,Y,IC,L;'INTEGER'IC,L;
                    'REAL'X,Y;'EXTERNAL';
```

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```
        BCD;'REAL'X,Y,S,THETA,XMIN,DX,GAP;'EXTERNAL';
        'PROCEDURE' HGPLINET(X,Y,N,K);'VALUE'N,K; 'ARRAY'X,Y;
        'INTEGER' N,K;'EXTERNAL';
        'PROCEDURE'  HGPNUMBERT(X,Y,HT,FL,THETA,I,IP,IQ);'VALUE'X,Y,
        HT,FL,THETA,I,IP,IQ;'INTEGER'I,IP,IQ;'REAL'X,Y,HT,FL,THETA;
        'EXTERNAL';
        'PROCEDURE'  HGPSYMBLT(X,Y,HT,BCD,THETA,N);'VALUE'X,Y,HT,
        THETA,N;'INTEGER'N;'ARRAY'BCD;'REAL'X,Y,HT,THETA;'EXTERNAL';
        OPENPLOT;
'BEGIN' HGPLOTT(-5.0,8.5,0,4);
                                  'COMMENT' SERIES ANGAMP PLOT;
        HGPLINET(PMA,LEVGNA,SAMPLESIZE-3,+1);
        STRARR(BCD,LITA,'('%%%')');
        HGPAXISVT(-3.0,-3.0,BCD,0,6.0,0.0,-10,1.0,0.3,-1);
        HGPAXISVT(0.0,-3.0,BCD,3,6.0,90.0,-10,1.0,0.3,-1);
        AB[1] := -3.0;
        AB[2] :=  3.0;
        AB[3] :=  3.0;
        AB[4] := -3.0;
        AB[5] := -3.0;
        BC[1] := -3.0;
        BC[2] := -3.0;
        BC[3] :=  3.0;
        BC[4] :=  3.0;
        BC[5] := -3.0;
        HGPLINET(AB,BC,5,+1);
        CD[1] := -4.125;
        CD[2] :=  4.125;
        CD[3] :=  4.125;
        CD[4] := -4.125;
        CD[5] := -4.125;
        DE[1] := -5.5;
        DE[2] := -5.5;
        DE[3] :=6.25;
        DE[4]:= 6.25;
        DE[5]:= -5.5;
        HGPLINET(CD,DE,5,+1);
        STRARR(EF,LITA,'('ANGAMP% DISTRIBUTION')');
        HGPSYMBLT(-3.0,5.125,0.25,EF,0.0,19);
        STRARR(FG,LITA,'('G.D.WHITEHEAD')');
        HGPSYMBLT(2.5,5.625,0.125,FG,0.0,13);
        STRARR(GH,LITA,'('N=')');
        HGPSYMBLT(2.5,4.5,0.125,GH,0.0,2);
        HGPNUMBERT(2.75,4.5,0.125,SAMPLESIZE,0.0,1,4,0);
        STRARR(HJ,LITA,'('T=')');
        HGPSYMBLT(2.5,4.25,0.125,HJ,0.0,2);
        STRARR(JK,LITA,'('V=')');
        HGPSYMBLT(2.5,4.0,0.125,JK,0.0,2);
        HGPNUMBERT(2.75,4.0,0.125,VOLTRANGE,0.0,1,5,0);
        STRARR(KL,LITA,'('ANGULAR%VELOCITY')');
        HGPSYMBLT(-0.75,3.5,0.125,KL,0.0,16);
        STRARR(LM,LITA,'('DEG % SEC % * 36/T')');
        HGPSYMBLT(-0.75,3.25,0.125,LM,0.0,13);
        STRARR(MN,LITA,'('AMPLITUDE')');
        HGPSYMBLT(3.125,0.125,0.125,MN,0.0,9);
        STRARR(NO,LITA,'('VOLTS*V/10')');
```

```
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
        WRITETEXT('(''('C')' SUCCESS1')');
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
        'COMMENT'                        DEGREE PLOT ;
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
'BEGIN'
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
        HGPLOTT(-10.0,2.0,0,4);
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
            HGPLINET(PQ,OP,361,+1);
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
         STRARR(QR,LITA,'('DEGREES')');
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
         HGPAXISVT(-3,0,0.0,QR,7,6.0,0.0,0.0,45,0.75,-1);
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
         STRARR(RS,LITA,'('PROBABILITY')');
            F:=F+1;
            PRINT(F,2,0);
            NEWLINE(1);
         HGPAXISVT(0.0,0.0,RS,11,6.0,90.0,0.0,1.0,0.6,-1);
            F:=F+1;
PRINT(F,2,0);
          NEWLINE(1);
         ST[1] := -4.125;
            F:=F+1;
          NEWLINE(1);
PRINT(F,2,0);
         ST[2] :=  4.125;
            F:=F+1;
            NEWLINE(1);
PRINT(F,2,0);
         ST[3] :=  4.125;
            F:=F+1;
PRINT(F,2,0);
          NEWLINE(1);
         ST[4] := -4.125;
            F:=F+1;
PRINT(F,2,0);
          NEWLINE(1);
         ST[5] := -4.125;
            F:=F+1;
```

```
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           TU[2] := 2.5;
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           TU[3] :=  9.25;
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           TU[4] :=  9.25;
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           TU[5] := -2.5;
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           HGPLINET(ST,TU,5,+1);
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           STRARR(UV,LITA,'('ANGLES % IN% THE% PHASE% PLANE')');
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           HGPSYMBLT(-3.0,8.25,0.25,UV,0.0,25);
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
           HGPSYMBLT(2.5,8.75,0.125,FG,0.0,13);
            F:=F+1;
PRINT(F,2,0);
            NEWLINE(1);
          HGPLOTT(0.0,0.0,3,0);
            F:=F+1;
PRINT(F,2,0);
'END';
          WRITETEXT('('('('C')' SUCCESS1')')');
          'COMMENT' ANGULAR VELOCITY DISTRIBUTIONS;
'BEGIN';
          HGPLOTT(-10.0,0.0,0,4);
F:=F+1;
NEWLINE(1);
PRINT(F,2,0);
          HGPLINET(YZ,XY,G,+1);
F:=F+1;
NEWLINE(1);
PRINT(F,2,0);
          STRARP(ABC,LITA,'('DEGREES% PER%SEC*T ')');
          HGPAXISVT(-3.0,0.0,ABC,17,6.0,0.0,-720,180,0.75,-1);
          HGPAXISVT(0.0,0.0,RS,11,6.0,90.0,0.0,1,0.6,-1);
          STRARR(DEF,LITA,'('ANGULAR % VELOCITY % DISTRIBUTION')');
          HGPSYMBLT(-3.0,8.25,0.25,DEF,0.0,29);
          HGPSYMBLT(2.5,8.75,0.125,FG,0.0,13);
          HGPLINET(ST,TU,5,+1);
```

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```
        'COMMENT' AMPLITUDE DISTRIBUTION;
'BEGIN';
        HGPLOTT(+20,0,15.5,0,4);
F:=F+1;
NEWLINE(1);
PRINT(F,2,0);
        HGPLINET(MNO,JKL,6,+1);
F:=F+1;
NEWLINE(1);
PRINT(F,2,0);
        HGPAXISVT(-3.0,0.0,BCD,0,6.0,0.0,-10,1,0.3,-1);
        HGPAXISVT(0.0,0.0,RS,11,6.0,90.0,0.0,1,0.6,-1);
        STRARR(XYZA,LITA,'('AMPLITUDE % DISTRIBUTION')');
        HGPSYMBLT(-3.0,8.25,0.25,XYZA,0.0,22);
        HGPSYMBLT(2.5,8.75,0.125,FG,0.0,13);
        STRARR(XYZC,LITA,'('AMPLITUDE')');
        HGPSYMBLT(-0.75,-0.6,0.125,XYZC,0.0,9);
        HGPLINET(ST,TU,5,+1);
       HGPLOTT(0,0,0.0,3,0);
'END';
        CLOSEPLOT;
'END';
'END';
'END';
```

```
JOB PHASEPLANE,ASSTF215,WHITEHEAD,STREAM 4  RUN BY GEORGE 2/MK8C
DISPLAY '  4PHASEPLANE  '
LOG
MAXTIME 600
VOLUME 4000
BULLETIN
! PROGRAMLOAD INFO
ALGOLUB ZPHASEPLANE,600,SECS,3000,LINES
! PROGRAMLOAD XALE,COMP
! CONSOLIDATE
! RUN
****
```

ACCOUNT CODE	ASSTF215	DATE	16/07/70
JOB NAME	PHASEPLANE	START TIME	21/11/51
USER NAME	WHITEHEAD,ST	END TIME	21/16/16

PERIPHERALS USED: 32

TOTAL MILL TIME	78
INPUT RECORDS	913
OUTPUT RECORDS	613

Simulation Program.

```
PHASE PLANE REPRESENTATION OF A WAVEFORM'

BEGIN   REAL A,B,C'

INTEGER I,J,K,L,M,N,CHARAC'

        SWITCH S:= RECYCLE,POTS'

         SETORIGIN(550,6,6,1)'

        AXES(5,5,10,10,10,10)'

        PLOTTER(5,1)'

        FOR I:=-50 STEP 5 UNTIL 50 DO

BEGIN   CHARAC:=(I/5)'

        MOVEPEN(I-2,3)'

        PRINT DIGITS(2), CHARAC'

  END'

        PLOTTER(5,3)'

        FOR I:=-100 STEP 5 UNTIL 100 DO

BEGIN   CHARAC:=(I/5)'

        MOVEPEN(-3,I)'

        PRINT DIGITS(2), CHARAC '

END'

        MOVEPEN(-50,120)'

         PRINT PLOTTER(10,1), £   G.D.WHITEHEAD   PHASE PLANE PLOT ?'

        MOVEPEN(-40,6)'

        PRINT PLOTTER(5,1), £ AMPLITUDE ?'

        MOVEPEN(-6,80)'

        PRINT PLOTTER(5,3), £ SLOPE ?'


READ N,A,B'

        M:=2'

         C:=(A-B)'
```

```
RECYCLE : A:=B'

                B:=0'

                READ B'

                C:=A-B '

                DRAWLINE(B,C)'

                M:= M+ 1'

                IF M LESSEQ N THEN GOTO RECYCLE ELSE GOTO POTS'

      POTS : PRINT £ END ?'

END'

END'
```

A P P E N D I X C

ESTIMATE UNCERTAINTY IN WAVEFORM ANALYSIS

In the practical analysis of continuous random variables or random processes, it is necessary to know how to control or specify the measurement accuracy of the analysis data that has been generated. If the measurement accuracy is not known the analysis data reduces to qualitative data.

Basically there are two sources of measurement error in random analysis which must be treated separately, for the purposes of error or uncertainty analysis. When the uncertainty analysis is complete and the analysis data collected the two separated uncertainties or errors may be compounded into a single statement of accuracy.

Firstly there is the basic measurement or calibration error. A statement of this error describes the accuracy with which the equivalent electrical voltage or waveform represents the actual quantity measured. Basic measurement and calibration error may be controlled and measured through the proper choice of instrumentation and calibration procedures. However, it is not the purpose of this appendix to discuss instrumentation and calibration, no further comment will therefore be made on this topic. Some details of the basic measurement accuracy of random data analysis is given in Table II of reference 2.

Secondly, and equally important, there is always associated with the analysis of random raw data (whether in the Amp, time or frequency domain) a statistical or estimate uncertainty which is conceptually different to basic measurement and calibration uncertainty. To understand statistical uncertainty it is essential to think in terms of populations and samples taken from populations.

With deterministic vibration the amplitude time history record will repeat itself exactly if the population of deterministic vibration from which it came is sampled at different times. If the population of random vibration is sampled at different times each sample will be unique and is never likely to be exactly the same as any other sample. Since different samples from the same population can never be exactly the same, it is reasonable to suggest that an average description of each sample would also differ from sample to sample. Further, if the average value of two very long samples were compared we would expect the differences to be less than if we were to compare an average description for the population or universe of discourse, with the average description of a long sample and then a short sample, we would expect to find that the difference was less in the case of the long sample. This expected difference between the population or true values and the sample or estimated values is what is referred to as statistical measurement uncertainty.

These concepts will now be expanded into the theory of statistical estimates. Terms such as bias error, variance error, standard error and confidence limits, which are the language of statistical uncertainty will also be explained. An attempt will also be made to locate the frontier of knowledge in uncertainty analysis based upon current research work.

STATISTICAL ESTIMATION THEORY

If $\{x(\alpha,t)\}$ represents a set of estimates where the range of t and α are $-\infty < t < \infty$, $1 \leq \alpha \leq \infty$. If U is a true average value of the random process $\{x(\alpha,t)\}$ such as the mean value, autocorrelation value or power spectral density value. Suppose that $\hat{U}(\alpha,T)$ is an estimate of U obtained by time averaging a single sample function (for a description of the structure of random processes see reference 1) over a finite time period T. If the ensemble of time average values is $\{\hat{U}(\alpha,T)\}$, if this ensemble is averaged this average value should equal the true value U without error. A set of estimates having this property is said to be unbiased, i.e. a set of estimates $\{\hat{U}(\alpha,T)\}$ is said to be unbiased if independent of T. The expected value is the true value.
i.e.

$$\underset{\alpha}{E}\left\{\hat{U}(\alpha,T)\right\} = U$$

When this happens $\hat{U}(\alpha,T)$ is said to be an unbiased estimate of U.

It should be realised, however, that the mere fact that this ensemble average $\underset{\alpha}{E}\{\hat{U}(\alpha,T)\}$ for fixed T, is unbiased does not imply that any one of the sample functions $\hat{U}(\alpha,T)$ will have a value close to the true value. Wide deviations from the true value might be observed. To describe these deviations the mean square value is used.
i.e.

$$\underset{\alpha}{E}\left\{\hat{U}(\alpha,T) - U\right\}^2$$

If this mean square error approaches zero as $T \to \infty$ or becomes large, then any sample function estimate $\hat{U}(\alpha,T)$ would tend to closely approximate its true value U. Any set of estimates having this property are known as consistent estimates.

i.e. A set of estimates is consistent if

$$\lim_{T \to \infty} \underset{\alpha}{E} \left\{ \hat{u}(\alpha, T) - u \right\}^2 = 0$$

when this happens $\hat{u}(\alpha, T)$ is a consistent estimate.

Digressing for a moment, note that the mean square value, variance value and mean value are related by the following:

Mean Square Value = Variance + Square of

Value Mean Value

i.e. $\bar{x}^2 = \sigma_x^2 + [\bar{x}]^2$

In these terms the mean square error is the sum of variance error and the bias or mean value error squared, thus:

$$\underset{\alpha}{E} \left\{ \hat{u}(\alpha, T) - u \right\}^2 = \sigma^2 \left\{ \hat{u}(\alpha, T) \right\} + \left[b\left\{ \hat{u}(\alpha, T) \right\} \right]^2$$

where $\sigma^2 \left\{ \hat{u}(\alpha, T) \right\} = \underset{\alpha}{E} \left[\hat{u}(\alpha, T) - \underset{\alpha}{E} \left\{ \hat{u}(\alpha, T) \right\} \right]^2$

$$= \underset{\alpha}{E} \left\{ \hat{u}^2(\alpha, T) \right\} - \left[\underset{\alpha}{E} \left\{ \hat{u}(\alpha, T) \right\} \right]^2$$

and $b\left\{ \hat{u}(\alpha, T) \right\} = \left[\underset{\alpha}{E} \left\{ \hat{u}(\alpha, T) \right\} - u \right]$

In practical terms it is the job of the Test Engineer and Data Analyst to ensure that this bias and variance error will approach zero as T Expressions for the bias and variance error in practical analysis of random data are given shortly.

A third property that is desirable of estimates is that they should be efficient. Estimate efficiency is difficult to define mathematically and if defined rarely leads to a clearer understanding of the term. In words, however, an efficient estimate may be thought of as an estimate chosen from a population of many, where all estimates have the same mean or expected value. The most efficient estimate being the one having the least variance and the least efficient estimate being the estimate with the greatest variance.

Some examples of efficient estimates are the arithmetic mean of a sample made up of N observations

i.e. $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ is an efficient estimation of the population mean μ. The sample variance S^2.

i.e. $S^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$

is an efficient estimation of the population variance S^2.

Note, however, that whilst the expected value of the sample mean value

$$E[\bar{x}] = \mu$$

indicates that the sample mean value is an unbiased estimate of μ (since by definition the expected value is the true value) the sample variance, S^2, is a biased estimate of the population variance σ^2 because

$$E[S^2] = \left(\frac{N-1}{N}\right)\sigma^2$$

To remove the bias of the sample variance, it may be corrected as follows:

$$E\left[\left(\frac{N}{N-1}\right)S^2\right] = E\left[\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2\right] = \sigma^2$$

(A proof of this relationship is given by Weatherburr (1961) on page 131. From this expression, note that the bias in estimating the variance is small for large samples but large for small samples. Correction of Variance bias is therefore usually reserved for small samples (say 30 or below).

MEASUREMENT UNCERTAINTY OF ESTIMATES IN TERMS OF STANDARD ERROR AND CONFIDENCE INTERVAL

The variance in a set of estimates has already been defined as the second moment of the estimate values about the estimate mean value.

i.e.

$$S^2 = \underset{\alpha}{E}\left[\hat{u}(\alpha, T) - \underset{\alpha}{E}\left\{\hat{u}(\alpha, T)\right\}\right]^2$$

or for simplicity $\quad S^2 = E\left[(x_i - \bar{x})\right]^2$

where x_i are the individual estimates and $\bar{x}$ the estimate mean value. It is usual when dealing with samples and populations to denote the sample or estimate variance as S^2 and the population or true value as σ^2.

Note that for a set of estimates having a zero mean value the variance is identical to the mean square value.

i.e.

$$E\left[x_i^2\right] = S^2, \quad \text{for } \bar{x} = 0.$$

Again it is usual to call the sample an estimate, mean value $\bar{x}$ and the true value μ.

The standard deviation of a set of estimates is the positive square root of its variance, hence, for a set of estimates:

$$\text{Standard Deviation} = \sqrt{S^2} = S$$

For the population from which the sample came and for which S is an estimate, the standard deviation is σ.

The standard deviation provides only a simple description of the variation in a set of estimates. A better description of variation in a set of estimates is obtained by studying the distribution of estimates. If the distribution of estimate values happens to follow a NORMAL distribution certain conclusions may be drawn from this distribution by studying the standard deviation in the estimates.

If the estimates $\hat{u}(\alpha,T)$ of the true value U follow a normal distribution then it can be said that about two thirds of the estimates, $\hat{u}(\alpha,T)$ will have values lying within $\pm$ S of U. More precisely we can expect to find U lying within the intervals $\hat{u}(\alpha,T) \pm S$, $\hat{u}(\alpha,T) \pm 2S$ and $\hat{u}(\alpha,T) \pm 3S$ for about 68.27%, 95.45% and 99.73% of the number of estimates respectively. (Note that the number of estimates should be greater than 30 in order to make the assumption of normality reasonbly valid.)

In the Analysis of Uncertainty to follow use will be made of a measure called the Standard Error or Normalised Standard Deviation. Standard Error carries exactly the same interpretation as standard deviation, as given above in conjunction with estimate distributions. It is the dimensionless equivalent of standard deviation.

At this point it would be valid to ask what has statistical estimation theory to do with waveform analysis. The answer is nothing if you are ~~not~~ thinking of the waveform as a single time history record and not as a sample from an everlasting population. What must be done to ensure that estimation theory has a place in your thought is to construct a Random Process of the type shown in Figure 4, Reference 1 . This may be done by conducting a single experiment and acquiring a long time history record of the variable during this experiment. In this case the random process would be made artifically by dividing the long record into a large number of smaller records each of equal length and placing them side by side. Alternatively a more realistic random process may be constructed by repeating an experiment a large number of times under the same controlled conditions (Reference 1, page 156 explains how a Random Process arises). With a Random Process in mind it should be possible to grasp the importance of estimates, uncertainty and sets, if the Random Process is now thought of as a sample from a population. The population existing not only now but well into the future.

Whilst Random Processes provide a foundation for generating predictive analysis data it must be admitted that a process or ensemble is not always a necessity. In particular in diagnostic analysis and most descriptive analysis sufficiently accurate estimates of the chosen descriptive functions can be extracted from sufficiently long single samples. Since these later modes of analysis probably form 90% of all analyses carried out in waveform analysis more emphasis will now be placed upon the uncertainty in estimating population values from single samples. It must be clearly understood however, that no statistical statements in the true sense can be made based on a sample size of one.

The preceding notes should provide a reasonable background for what is to follow. If they have not been understood the reader is strongly recommended to read Spiegel (1961), Chapter 9 and Weatherburn (1961), Chapters 3 and 6.

STATISTICAL DEGREE OF FREEDOM

Statistical degree of freedom is defined now because it will be used extensively throughout the following treatments. It represents the equivalent number of events or maximum number of individual events contained in a continuous random variable. It is calculated from the equivalent ideal bandwidth B_N c/s of the signal and the length of the record. Since a continuous random variable may be completely reconstructed from digital samples taken at $1/2B_N$ seconds apart the Statistical degree of freedom of a waveform of length T seconds is given by

$$n = 2BT.$$

The equivalent ideal bandwidth B_N and equivalent noise bandwidth of a waveform are defined in Reference 2, page 33.

of τ_W by some distribution not yet known.

The number of statistical degrees of freedom for each measurement of τ_W is given by :

$$n = 2B_N \tau_W$$

Since $\tau_W = WT\hat{\rho}(x)$ from above

$$n = 2WTB_N \hat{\rho}(x)$$

B_N is the equivalent ideal noise bandwidth of the time history record.

The mean square standard error is here defined for the measurement $\hat{\rho}(x)$ as the ratio of the variance in the measurement $\sigma^2 \hat{\rho}(x)$ (for zero mean data) to the square of the measurement value $\hat{\rho}(x)$ i.e.

$$\epsilon^2 = \frac{\sigma^2 \hat{\rho}(x)}{\hat{\rho}^2(x)}$$

In terms of the true population variance, $\sigma^2 \hat{\rho}(x)$ the mean square error is given by:

$$\epsilon^2 = \frac{\sigma^2 \hat{\rho}(x)}{n\hat{\rho}^2(x)} \quad \text{since} \quad \sigma^2 \hat{\rho}(x) = \frac{\sigma^2 \rho(x)}{n}$$

Assuming $\sigma^2 \rho(x) \doteqdot \hat{\rho}^2(x)$ we now obtain

$$\epsilon^2 = \frac{1}{n} = \frac{1}{2WB_NT\hat{p}(x)}$$

or

$$\epsilon = \left(\frac{1}{2WB_NT\hat{p}(x)}\right)^{1/2}$$

This expression gives the most conservative estimate of Standard Error. It was stated that the measurement of t_W will be distributed in some manner about the true value of t_W. This is now equivalent to saying that $\hat{p}(x)$ will be distributed in some manner about $p(x)$. It might be assumed that $\hat{p}(x)$ is normally distributed with a standard error of 1ϵ. This would only be true if the original random variable was also normally distributed. However since the original random variable was time, i.e. t_W in $\hat{p}(x) = 1/TW\int_0^T t_W dt$ and since time cannot physically take on negative values t_W could not have been normally distributed. To the rescue comes the Central Limit Theorem (see Bendat, 1958, page 96). This states, in essence, that the distribution of $\hat{p}(x)$ will approach normality about $p(x)$ as the number of degrees of freedom becomes large. This is true regardless of the distribution of the original random variable. In terms of the standard error, if ϵ is less than unity, then $\hat{p}(x)$ may be considered to be normally distributed about a mean of $p(x)$ with normalised standard deviation ϵ.

When this occurs specific confidence statements can be associated with ϵ using a normal distribution table. Note that the standard error of the measurement becomes large as $\hat{p}(x)$ becomes small. This imposes a great restriction upon the generality of the above uncertainty. Realising this, Bolt, Beranek and Newman Inc. conducted an experimental programme to try to establish a more general relationship. Their empirical expression for ϵ will be given shortly.

To determine if the above expression for ϵ gives a valid estimate of a Normal distribution, standard error ϵ should not be allowed to exceed 0.3. This discussion may be clarified by the following example.

Assume we have an analogue amplitude probability analyser and a time history record of say random vibration. Let the length of the record be 10 seconds. Assume also that this sample record has a uniform power spectrum with a sharp cut off at 500 c/s, by a low pass filter. That is, the Equivalent Ideal Noise bandwidth $B_{\sim}$ is 500 c/s. Let the analogue analyser have an amplitude window equal to $^1/$10th of the rms voltage of the sample record. If the rms voltage is one volt, then the window is 0.1 volts. Note that the rms voltage is also the standard deviation of the record, which is thus unity. If the centre of the amplitude window is now set to some voltage x_1 and the time t_{ω} is measured that the signal spends in the window, if this is divided by T and W an estimate $\hat{p}(x)$ is obtained. If the centre voltage is now moved 0.1 volts a second probability density estimate will be obtained. The procedure is repeated until the amplitude range of interest has been covered: say $\pm 4\sigma = \pm 4$ volts. From Table V of Reference 2 the minimum analysis time is given as $t = Ta \times A/W = 10.8/0.1 = 800$ seconds. The amplitude range could be swept or stepped. If swept the scan rate should be no greater than $W/Ta = 0.1/10 = 0.01$ volts per second.

The standard error of the estimates will be given by:

$$\epsilon = \left[\frac{1}{2(500)(10)(0.1)\hat{p}(x)} \right]^{1/2}$$

$$\therefore \quad \epsilon = \left(\frac{0.001}{\hat{p}(x)} \right)^{1/2}$$

i.e. the standard error at any centre voltage is a function of the probability density measured at that voltage. For the purpose of the example and simplicity, assume that the measured probability density function followed a normal distribution. Suppose that the measurements at principle points on the normal distribution and their ϵ values were as follows. (See Figure)

Centre voltage of window	Measured value for p(x)	
0	0.4	0.05
±1	0.24	0.064
±2	0.054	0.14
±3	0.0044	0.48
±4	0.0001	3.2

Consider a 68% confidence interval, i.e.

$$\text{Prob}\left[p(x) - \epsilon\hat{p}(x) \leq \hat{p}(x) < p(x) + \epsilon\hat{p}(x) \right] = 0.68$$

In words the probability that the estimated value $\hat{p}(x)$ will fall within the range of $\pm\,\epsilon$ of the true value is 0.68.

The 68% confidence interval for the true probability density at zero volts is the range of $\hat{p}(x) \pm \epsilon \ \hat{p}(x)$ between 0.4 - 5% of 0.4 and 0.4 + 5% of 0.4, i.e. from 0.38 to 0.42. At one volt we can be 68% confident that p(x) is between 0.22 and 0.26. At two volts for a 68% confidence interval p(x) is between 0.046 and 0.062. At three or four volts the error has become too large (greater than 0.3) to allow an assumption that the distribution of p(x) is normal. At four volts ϵ indicates a 68% confidence range of 1 - 0.0002 to + 0.0004 for p(x). Since p(x) cannot be negative this latter statement has no meaning. Therefore at three and four volts the statistical accuracy is not known. The three and four volt centre voltages in the example provide an opportunity for specifying a statistical accuracy. Since the existing 10 seconds of raw data have yielded statistical uncertainty so wide as to be indefinable we will reverse the procedure and find the length of record based upon an acceptable uncertainty (say 0.3 = ϵ).

At three volts $\hat{p}(x)$ is 0.0044. Therefore if ϵ is established as 0.3

$$0.3 = \left[\frac{1}{2(500)(0.1)(0.0044)(T)}\right]^{1/2}$$

solving for T yields : T = 25.2 seconds. At four volts $\hat{p}(x)$ is 0.0001

$$\therefore \quad 0.3 = \left[\frac{1}{2(500)(0.1)(0.001)(T)}\right]^{1/2}$$

and T is 1110 seconds = 18.5 mins.

This concludes the example.

Bolt Beranek and Newman (1960) have derived an expression for standard error based upon threshold crossing theory. Their expression (for a guide to the derivation see Rice, 1944) given below, makes the assumption that the random waveform has a normal amplitude probability density function.

$$\epsilon = \left[\frac{1}{2.89 B_N T W \hat{\rho}(x)}\right]^{1/2}$$

When this relationship was checked experimentally using a random noise generator with a sharp frequency cut off, it was found that the results led to the empirical relationship.

$$\epsilon = \left[\frac{1}{\pi^3 B_N T W \hat{\rho}(x)}\right]^{1/2}$$

Thus the experimental derivation indicated that the standard error was about half that predicted by their theoretical formulae and less than half that predicted by the previously derived expression.

Note that each expression is identical except for the coefficients which were 0.707, 0.588 and 0.254.

Bendat and Piersol (1965) have suggested a more general relationship for which is based upon the spectral characteristics of the data and upon the way in which the data was analysed. Their general relationship is given by:

$$\epsilon = \left[\frac{C_o}{X T W \hat{\rho}(x)}\right]^{1/2}$$

where $X = B_N$ or $\overline{\nu_o}$

and C_o is a coefficient dependent upon the frequency composition and way in which the estimate $\hat{p}(x)$ is obtained.

$\bar{\nu}_o$ is the expected number of zero crossings per second.

In the previous expressions for ϵ the frequency bandwidth of the signal was assumed to range from zero to some upper frequency f_B the bandwidth from zero to f_B being the equivalent noise bandwith B_N c/s. Bendat and Piersol have found experimentally that for signals having a relatively wide bandwidth, X, in the above expression for ϵ, is $\bar{\nu}_o$. They have evaluated the coefficient C_o for this case as 0.26, i.e.

$$\epsilon^2 = \frac{0.068}{\bar{\nu}_o T W \hat{p}(x)}$$

$$\text{or} \quad \epsilon = \left[\frac{0.068}{\bar{\nu}_o T W \hat{p}(x)}\right]^{1/2}$$

For the case of signals with relatively narrow bandwidths X = B_N in the above expression. The coefficient for this case was found to be 0.17, i.e.

$$\epsilon^2 = \frac{0.028}{B_N T W \hat{p}(x)}$$

$$\text{or} \quad \epsilon = \frac{0.17}{\left(B_N T W \hat{p}(x)\right)^{1/2}}$$

It is thought that the first expression applies when the ratio $B_N / \bar{\nu}_o$ is greater than a third. This applies to low frequency signals where the bandwidth is narrow but the expected number of zero crossings is small. The second expression with a coefficient of 0.17 is thought to apply to signals having a ratio which is smaller than a third. This happens when a narrow bandwidth signal is centred on a high frequency.

In human factors research it is unlikely that frequencies will be encountered much above 1000 c/s. The range of frequency of interest is thus 0-1000 c/s. Since 1000 c/s may be regarded as a fairly low frequency it can be seen that in human factors research, waveforms most closely approximate to the range of frequency specified by the expression

$$\epsilon = \frac{0.26}{\left(\bar{\nu}_0 T w \hat{p}(x)\right)^{\frac{1}{2}}}$$

The value of $\bar{\nu}_0$ for signals having a uniform power spectrum over the bandwidth B_N c/s which extends to zero frequency is given by,

$$\bar{\nu}_0 = 1.15\, B_N$$

If the power spectrum has a sharp peak at some centre frequency fc then,

$$\bar{\nu}_0 = 2 f_c$$

If the power spectrum is uniform between fa and fb

$$\bar{\nu}_0 = \frac{2}{\sqrt{3}} \left\{ f_a^2 + f_a f_b + f_b^2 \right\}^{\frac{1}{2}}$$

The generally expected number of zero crossings for any signal may be deduced from the power spectrum of the signal thus,

$$\bar{\nu}_0 = 2 \left\{ \frac{\int_0^\infty f^2 G_x(f)\, df}{\int_0^\infty G_x(f)\, df} \right\}^{\frac{1}{2}}$$

For most physical analyses of amplitude probability density, if the uncertainty criterion is ϵ = 0.1 or greater, then it will be reasonably accurate to use a coefficient value somewhere between 0.26 and 0.17, say 0.2. Bandwidth rather than expected zero crossings may then be used as the frequency description. It is therefore suggested that for most human factors research the expression:-

$$\epsilon = \frac{0.2}{\left(B_N T W \hat{p}(x)\right)^{1/2}}$$

will be adequate. A plot of ϵ with Co = 0.2 is given in Figure versus the product B_N T W for various values of $\hat{p}(x)$.

By now it will be apparent that some a priori knowledge of the waveform noise bandwidth and the amplitude probability density values of the waveform are needed to specify a value for ϵ. Since this is not possible, for the most part, it is thus essential to pre-analyse the waveform in the amplitude domain to ascertain the set of values $\hat{p}(x)$. Additionally the equivalent noise bandwidth must also be known.

MEAN SQUARE UNCERTAINTY

So far, what has been considered is only the variance term (normalised) of the mean square estimate error. This was mentioned during discussion on statistical estimation theory. The mean square error is given by:

M.S. Error = Variance + $(\text{Bias})^2$

Error (Error)

$= \epsilon^2 + (\text{Bias})^2$

The bias error may be approximated by:

$$b \approx \frac{W^2}{24} \hat{p}(\ddot{x})$$

$\hat{p}(\ddot{x})$ being the second derivative of the amplitude probability density estimate (the derivation of the bias error in amplitude estimates is similar to that for power spectral density estimates, a derivation for which will be given later).

Thus the total mean square error is given by:

$$\text{M.S. Error} = \frac{0.04}{B_N T W \hat{p}(x)} + \left[\frac{W^2}{24} \hat{p}(\ddot{x}) \right]^2$$

i.e.

$$E\left[\hat{p}(x) - p(x)\right]^2 = \frac{0.04}{B_N T W \hat{p}(x)} + \frac{W^4}{576}\left[\hat{p}(\ddot{x})\right]^2$$

CONCLUSIONS REGARDING MEASUREMENT UNCERTAINTY IN AMPLITUDE PROBABILITY DENSITY ESTIMATES

A general expression for the variance term given as the standard error is not yet either theoretically established or empirically suggested. The general expression will be a complicated function of frequency bandwidth and expected number of zero crossings per second.

The best approximates, theoretically derived and experimentally substantiated is that given by Bendat and Prensel as

$$E = 0.2 \qquad \epsilon = \frac{0.2}{B_N T W \hat{p}(x)}$$

A suggested criterion for ϵ based upon values used by NASA is $\epsilon = 0.01$. This gives 68% confidence that the true value p(x) lies within ± 1% of the measured value.

To specify the sample length of record needed to maintain the measurement uncertainty ϵ for a given amplitude window and frequency bandwidth B_N c/s it is necessary to know the value of the measured amplitude probability density $\hat{p}(x)$. Hence, since $\hat{p}(x)$ is an unknown during the experimental planning stage, the required record length cannot be predicted without carrying out some analysis. The mutual interaction of ϵ and T can therefore be removed only by a process of iterative measurement and analysis. The contribution made to the mean square error by the bias error will be very small if W is small. In comparison with the variance error it will be negligible and may therefore be neglected. In general terms the bias error may be thought of as that due to an amplitude window of finite value and the variance as that due to a sample record of finite length.

MEASUREMENT UNCERTAINTY IN SECOND ORDER OR JOINT AMPLITUDE PROBABILITY DENSITY ESTIMATES

No published experimental work on the measurement uncertainty in joint APD estimates is known to the author.

Example: Physical values of the 1st order amplitude probability density function.

Often it helps to know the range of physical values that a measuring instrument should read for a given input. This is basically the calibration problem but some numerical values often help fix some of the ideas as well. The example chosen is an analysis of the amplitude probability density of the output from a random noise generator. The frequency bandwidth of the random noise generator was 100 c/s to 8000 c/s the r.m.s. level of the output was 1.00 volt and the amplitude window was 0.100 volt. Figure shows the amplitude probability density for the random noise plotted against voltage amplitude (also in this case, standard deviation).

The analyser was a Bruel and Kjaen Model 160 and the noise generator a General Radio Co. Model 1390A. Fitted to this measured curve is a true normal distribution. The measured distribution is shown to have a very slight positive skew but is otherwise truly normal. The physical values of $\hat{p}(x)$ for this data have a maximum of 0.4 and tails off to zero at about $3\frac{1}{2}\,\sigma$. The measurement uncertainty at any point on the curve is given by:

$$\epsilon = 0.2/\left(W\,B_n\,T\,\hat{p}(x)\right)^{\frac{1}{2}}$$

where $W = 0.1$ volt

$T = 4.6$ seconds = Averaging Time

$B_n = 7900$ c/s

$$\epsilon = \frac{0.0042}{\hat{p}(x)}$$

For example the interval in which we are 68% confident that the true value will lie at a value $\hat{p}(x) = 0.4$ is

$$\epsilon = \frac{0.0042}{0.4} = 0.00665$$

i.e. the true value at $\hat{p}(x) = 0.4$ lies in the range $\hat{p}(x) = 0.39734$ to 0.40665. At $\hat{p}(x) = 0.05$

$$\varepsilon = 0.01875$$

∴ the range in which the true value will lie with 68% confidence is $\hat{p}(x) = 0.04999064$ to 0.05000936. This concludes the example.

MEASUREMENT UNCERTAINTY IN FREQUENCY ANALYSIS

Power spectral density - weakly stationary

An estimate of the true mean square spectral density function $G_x(f)$ is given by:

$$\hat{G}_x(f) = \frac{1}{BT}\int_0^T x_B^2(t)\,dt$$

where $x_B^2(t)$ is the square of the amplitude time history record passed by the narrow bandpass filter of B c/s.

The mean square measurement error of such an estimate is compounded from a bias error and variance error thus.

Mean Square Error = Variance Error + (Bias Error)^2

i.e.

$$E\left[\hat{G}_x(f) - G_x(f)\right]^2 = E\left[\hat{G}_x(f) - E\hat{G}_x(f)\right]^2 + \left[E(\hat{G}_x(f) - G_x(f))\right]^2$$

BIAS ERROR

It can be shown that $\hat{G}_x(f)$ provides an asymptotically unbiased estimate of $G_x(f)$ as $T \to \infty$ provided that as $T \to \infty$ the bandwidth $B \to 0$. This condition on B as a function of T is usually assumed, although of course, not true. By definition the bias error is given by:

$$b\left[\hat{G}_x(f)\right] = E\left[\hat{G}_x(f)\right] - G_x(f)$$

At this point it is necessary to indulge in deep mathematics to obtain the following result. The result for the bias analysis is expressed in the important asymptotic formula which assumes that the autocorrelation function $R_x(\tau)$ of the signal, $\tau R_x(\tau)$ and $\tau^2 R_x(\tau)$ are absolutely integrable over the range $-\infty < \tau < \infty$.

Thus $$\lim_{T \to \infty} B^{-2} \left| b\left[\hat{G}_x(f)\right]\right| \doteq \frac{1}{24} \left| G_x(\ddot{f})\right|$$

$G_x(\ddot{f})$ is the second derivative of $G_x(f)$ with respect to f. In terms of the equivalent autocorrelation function $G_x(\ddot{f})$ is given by Bendat (1958) as

$$G_x(\ddot{f}) = -4\pi^2 \int_0^{\infty} \tau^2 R_x(\tau) e^{-j2\pi f \tau} d\tau$$

From the above term for bias it follows that the bias error approaches zero as $B \to 0$ and the record length $T \to \infty$.

Making the assumption that T is large leads not unreasonably to the final result:

$$\left| b\left[\hat{G}_x(f)\right]\right| \doteq \frac{B^2}{24}\left| G_x(\ddot{f})\right|$$

VARIANCE ERROR

$$\sigma^2\left[\hat{G}_x(f)\right] = E\left[\hat{G}_x(f) - E(\hat{G}_x(f)\right]^2$$

After a lengthy derivation it has been shown that as

$$\lim_{T \to \infty} BT\sigma^2\left[\hat{G}_x(f)\right] \doteq G_x^2(f), \text{ for } f \neq 0$$
$$\doteq 2G_x^2(f), \text{ for } f = 0$$

If T is large

$$\sigma^2\left[\hat{G}_x(f)\right] \doteq \frac{G_x^2(f)}{BT}, \quad \text{for } f \neq 0$$

$$\doteq \frac{2G_x^2(f)}{BT}, \quad \text{for } f = 0$$

This equation indicates that the variance approaches zero as $T \to \infty$ provided that $BT \to \infty$.

MEAN SQUARE ERROR

$$E\left[\hat{G}_x(f) - G_x(f)\right]^2 = \sigma^2\left[\hat{G}_x(f)\right] + \left[b(\hat{G}_x(f)\right]^2$$

$$= \frac{\hat{G}_x^2(f)}{BT} + \left[\frac{B^2}{24}\left|G_x(\ddot{f})\right|\right]^2$$

From the variance the standard error may be defined as the ratio of the variance in the estimate to the square of the true value.

Hence

$$\epsilon^2 = \frac{\sigma^2\left[\hat{G}_x(f)\right]}{G_x^2(f)} = \frac{1}{BT}\frac{G_x^2(f)}{G_x^2(f)}$$

$$\therefore \quad \epsilon^2 = \frac{1}{BT}$$

An alternative derivation of the variance term comes from the statistical degree of freedom of the signal for which $\hat{G}_x(f)$ is an estimate.

The equivalent number of events is contained in a continuous white noise signal of length T secs and bandwidth B c/s is given by

$n = 2\,BT$

If the signal x(t) is assumed to have a normal amplitude probability density function, the sampling distribution for an estimate $\hat{G}_x(f)$ will be:

$$\hat{G}_x(f) \sim \frac{G_x(f)\chi^2_{n-1}}{(n-1)}$$

$\sim$ means distributed as and χ^2 is a chi-square distribution with $(n - 1)$ degrees of freedom.

The variance in $\hat{G}_x(f)$ is given by

$$\text{Variance } \hat{G}_x(f) = \text{Var} \left[\frac{G_x(f)\chi^2}{n-1} \right]$$

$$= \left[\frac{G_x(f)}{n-1} \right]^2 \text{Var } \chi^2$$

The variance of a χ^2 distribution is equal to twice the degree of freedom hence:

$$\text{Variance } \chi^2 = 2(n-1)$$

$$\therefore \text{Variance } \hat{G}_x(f) = \frac{2G_x^2(f)(n-1)}{(n-1)^2} = \frac{2G_x^2(f)}{(n-1)}$$

substituting $n = 2BT$ into the variance expression derives

$$\text{Variance } \left[\hat{G}_x(f) \right] = \frac{2G_x^2(f)}{2BT-1}$$

with

$$\epsilon^2 = \frac{\text{Var.}\left[\hat{G}_x(f)\right]}{G_x^2(f)}$$

$$\epsilon^2 = \frac{1}{BT-1} \doteqdot \frac{1}{BT} \quad \text{for } BT >> 1$$

This concludes the derivation of uncertainty for the power spectral density. In the expression for mean square error the bias term will be insignificant except for cases where the derivative on slope of the power spectrum is changing rapidly with the bandwidth B. For most cases the measurement uncertainty may be taken as the variance ter. Figure shows ϵ plotted against the BT product.

MEASUREMENT UNCERTAINTY OF JOINT POWER SPECTRAL DENSITY FUNCTION

An estimate of the joint or cross power spectral density function for a pair of records is given by

$$\hat{G}_{xy}(f) = \hat{C}_{xy}(f) - j\,\hat{Q}_{xy}(f)$$

$$\hat{C}_{xy}(f) = \frac{1}{BT}\int_0^T x_B(t)\, y_B(t)\, dt$$

$$\hat{Q}_{xy}(f) = \frac{1}{BT}\int_0^T x_B(t)\, y_B(t)\, dt$$

Physically, the cross spectral density function must be measured in two parts. The first part given by $\hat{C}_{xy}(f)$ is the Co-Spectrum. The second part given by $\hat{Q}_{xy}(f)$ is the Quadrature Spectrum. To form the cross spectrum the two components are subtracted "out of phase". To compute the

Co and Quad spectrum of $\hat{G}xy(f)$ the signals x(t) and y(t) must both be weakly stationary.

The measurement uncertainty for $\hat{G}xy(f)$ is obtained in the same way as the measure of $\hat{G}xy(f)$. That is by subtracting the measurement uncertainty for the components $\hat{C}_{xy}(f)$ and $\hat{Q}_{xy}(f)$ "out of phase". The measurement uncertainty for the components $\hat{C}_{xy}(f)$ and $\hat{Q}_{xy}(f)$ is the same as for the singular power spectral density function except for a multiplication between $x_B(t)$ and $y_B(t)$. This is equivalent to squaring of $x_B(t)$ in $\hat{G}_x(f)$.

$\tilde{x}_B(t)$ in $\hat{Q}xy(f)$ indicates that $x_B(t)$ has been phase lagged 90° relative to $y_B(t)$.

In terms of mean square error, the bias and variance errors for the components are given by:

Bias Error

$$b\left[\hat{C}_{xy}(f)\right] \leq \frac{B^2}{24}\left|\hat{G}_{xy}(\overset{\infty}{f})\right|$$

$$b\left[\hat{Q}_{xy}(f)\right] \leq \frac{B^2}{24}\left|G_{xy}(\overset{\infty}{f})\right|$$

As for the ordinary singular power spectral density function:

$$\left|G_{xy}(\overset{\infty}{f})\right| = -4\pi^2\int_0^\infty \tau^2 R_{xy}(\tau)\, e^{-2\pi f\tau}\, d\tau$$

Variance Error

Variance $\left[\hat{C}_{xy}(f)\right] \leq \dfrac{\hat{G}_x(f)\,\hat{G}_y(f)}{BT}$

Variance $\left[\hat{Q}_{xy}(f)\right] \leq \dfrac{\hat{G}_x(f)\,\hat{G}_y(f)}{BT}$

In terms of standard error, which is defined for the cross spectral components as

$$\epsilon^2 = \frac{\sigma^2\,\hat{C}_{xy}(f)}{\hat{G}_x(f)\,\hat{G}_y(f)} \div \frac{1}{BT}$$

and

$$\epsilon^2 = \frac{\sigma^2\,\hat{Q}_{xy}(f)}{\hat{G}_x(f)\,\hat{G}_y(f)} \div \frac{1}{BT}$$

Mean Square Error

$$E\left[\hat{G}_{xy}(f) - G_{xy}(f)\right]^2 = E\left[\left(\hat{C}_{xy}(f) - j\,\hat{Q}_{xy}(f)\right) - \left(C_{xy}(f) - j\,Q_{xy}(f)\right)\right]^2$$

In this form the mean square error in a cross spectral density estimate is very difficult to evaluate. The treatment is beyond the scope of this note.

SPECTRAL BANDWIDTH FOR ORDINARY POWER SPECTRUM

The mean square error for the ordinary singular power spectrum was found to be:

$$E\left[\hat{G}_x(f) - G_x(f)\right]^2 = \frac{G_x^2(f)}{BT} + \left[\frac{B^2}{24}\left|\overset{\circ\circ}{G_x}(f)\right|\right]^2$$

If this expression is now normalised, as before to the mean square standard error ε^2, ε^2 is given by

$$\varepsilon^2\left[\hat{G}_x(f)\right] = \frac{E\left[\hat{G}_x(f) - G_x(f)\right]^2}{G_x^2(f)}$$

$$\therefore \quad \varepsilon^2 \doteq \frac{1}{BT} + \frac{B^4}{576}\left|\frac{\overset{\circ\circ}{G_x}(f)}{G_x(f)}\right|^2$$

If $\lambda(f)$ is defined such that,

$$\lambda(f) = \left|\frac{G_x(f)}{\overset{\circ\circ}{G_x}(f)}\right|^{1/2}$$

then:

$$\varepsilon^2 \doteqdot \frac{1}{BT} + \frac{B^4}{576}\left[\frac{1}{\lambda(f)}\right]^4$$

$\lambda(f)$ is known as the spectral bandwidth of the signal $x(t)$. $\lambda(f)$ has units of frequency, i.e. c/s ε^2 is usually written

$$\varepsilon^2 \doteqdot \frac{1}{BT} + \frac{1}{576}\left[\frac{B}{\lambda(f)}\right]^4$$

In physical terms the spectral bandwidth may be regarded as the frequency bandwidth of the narrowest peak in the power spectral density curve $\hat{G}x(f)$ $\lambda(f)$ is closely related to the resolution needs in power spectral density analysis. By resolution is meant the ability of an analysis to properly separate adjacent peaks in the power spectrum. If these spectral peaks are one spectral bandwidth apart, then it seems reasonable to select a discriminating filter whose bandwidth B c/s is one half of the spectral bandwidth, i.e.

$$B \leq \lambda(f)/2$$

If B is chosen in this way, the analysis of the data will ensure, with a low probability of error, that if Gx(f) has two peaks $\lambda(f)$ apart that these two peaks can be resolved by taking measurements of Gx(f) at intervals B c/s apart.

Although the spectral bandwidth is really a matter for resolution to be covered in Analysis Specification, it has been introduced here because it will influence the choice of filter bandwidth. Since filter bandwidth for a given record length will control the measurement uncertainty in the analysis, the presence of $\lambda(f)$ here is valid. The following example may help clarify these ideas.

Assume we have a sample record of random vibration which we wish to describe in the frequency domain. If this record is weakly stationary the power spectral density function Gx(f) may be employed. Suppose that it is desired to estimate Gx(f) with a standard error not exceeding the criterion $\epsilon = 0.01$. That is we want to be 68% confident that the true value Gx(f) for the population $\{x(\alpha, t)\}$ from which the sample record came, lies within ± 1% of the estimated value.

To control ϵ we have only to treat B, T and $\lambda(f)$ since the total mean square error (ϵ^2) was given by:

$$\epsilon^2 \doteq \frac{1}{BT} + \frac{1}{576}\left[\frac{B}{\lambda(f)}\right]^4$$

However, whilst B and T may be specified to control ϵ, $\lambda(f)$ is an unknown at the raw data stage. In order to gain some knowledge of $\lambda(f)$ the procedure is to diagnose the raw data by a pilot analysis. To conduct a pilot analysis it is helpful to have some a priori knowledge of the raw data sample. If for instance the raw vibration sample was representative of the whole body vibration of an automobile, we could make a shrewd guess that its power spectrum would have at least two peaks. One peak would occur round about the natural frequency of the body on its road springs and a second around the natural frequency of the unsprung mass on the tyre springs. Suppose the former peak was of order 5 c/s in width, centred around 2 c/s, and the latter 4 c/s centred on 12 c/s. We would chose for the pilot analysis a filter bandwidth of 1 c/s.

The pilot analysis might have revealed a peak in the power spectrum of width 2 c/s. In this case we would maintain the pilot bandwidth of 1 c/s as given by

$$B \leq \frac{\lambda(f)}{2}$$

When B satisfied $B \leq \lambda(f)/2$, the second term in ϵ^2 becomes negligible and ϵ^2 reduces to $1/BT$. Therefore to maintain an E of 0.01 a raw data sample must be acquired of length

$$T = 1/B\epsilon^2 = \frac{1}{1.(0.01)^2}$$

$$T = 10^4 \text{ seconds}$$

$$T = 166.6 \text{ minutes}$$

To illustrate the importance of the relationship $B \leq \lambda(f)/2$ consider the case where the filter bandwidth in the above example was 4 c/s instead of 1 c/s, i.e. B = $2\lambda(f)$. If the same record length is used namely 10^4 seconds the mean square standard error becomes:

$$\epsilon^2 = \frac{1}{BT} + \frac{1}{576}\left[\frac{B}{\lambda(f)}\right]^4$$

$$= \frac{1}{4.10^4} + \frac{1}{576}\left[2\right]^4$$

$$= 0.25.10^{-4} + 16/576$$

$$= 25.10^{-6} + 0.0277$$

Taking the positive square root:

$$\epsilon = 0.005 + 0.167$$

$$\epsilon \approx 0.172 = 17.2\%$$

Notice how the bias error as given by the second term has swamped the variance error. The measurement uncertainty has been increased from 1% to 17.2%. It is clear from this example how important it is to maintain $B \leq \lambda(f)/2$ if this is possible. This concludes the example.

CONFIDENCE INTERVAL

The measurement uncertainty in a power spectral density estimate, may alternatively be expressed by a confidence interval. The confidence interval gives the range over which the true value, Gx(f) is known to lie with a given level of confidence. If it is assumed that the distribution of estimated values about the true value follows Chi Square (χ^2) (This also assumes that the original raw data $x(t)$ followed a normal distribution). Then the ratio of the estimated value $\hat{G}x(f)$, to the true value Gx(f), for a given confidence interval will vary with the degree of freedom of the analysis. Figure illustrates this description of measurement uncertainty. For example if the length of record is 10 seconds and the filter bandwidth

3.5 c/s the degree of freedom is 70 from

$$n = 2BT$$

The 90% confidence limits for a power spectral density estimate with a degree of freedom of 70 is given by:

$$Prob\left[\hat{G}_x(f) - 0.75\,\hat{G}_x(f) \leq G_x(f) < \hat{G}_x(f) + 1.37\,\hat{G}_x(f)\right] = 0.9$$

From Figure If $\hat{G}x(f)$ is 2.0 volts^2/ c/s then Gx(f) lies in the range 1.5 to 2.73 volts^2/ c/s with 90% confidence.

MEASUREMENT UNCERTAINTY IN FREQUENCY RESPONSE ESTIMATES

For a constant parameter linear system having a weakly stationary excitation x(t) and response y(t) and estimate of the frequency response of the system is given by:

$$\hat{H}(f) = \frac{G_{xy}(f)}{\hat{G}_x(f)}$$

$\hat{G}xy(f)$ is the estimated cross power spectral density function between the input and output and $\hat{G}x(f)$ is the estimated power spectral density function of the input. $\hat{H}$ (f) is the estimated frequency response function which carries a gain factor $|\hat{H}(f)|$ and phase factor $\hat{\phi}(f)$ thus:

$$\hat{H}(f) = |\hat{H}(f)|\,e^{j\hat{\phi}(f)}$$

The measurement uncertainty in estimating $\hat{H}(f)$ has been studied by Goodman

(1957). Goodman has derived an expression for the measurement uncertainty in the gain and phase factors of $\hat{H}(f)$ based upon the Coherence function $\gamma^2_{xy}(f)$.

where

$$\gamma^2_{xy}(f) = \frac{|G_{xy}(f)|^2}{G_x(f)G_y(f)}$$

Goodman's derivation is given by:

$$P\left[\left|\frac{\hat{H}(f) - H(f)}{H(f)}\right| < \sin\epsilon \text{ and } \left|\hat{\phi}(f) - \phi(f)\right| < \epsilon\right]$$

$$\doteq 1 - \left[\frac{1 - \gamma^2_{xy}(f)}{1 - \gamma^2_{xy}(f)\cos^2\epsilon}\right]^n$$

n is the degree of freedom in the measurement given by

$n = 2BT$ (B is the filter bandwidth).

and E is the standard error in estimating the gain or phase factors.

Goodman's equation is shown plotted in Figure . The three sets of curves are plotted for P = 0.9, 0.85 and 0.8 with ϵ = 0.005, 0.10 and 0.15. P and ϵ give the probability of maintaining a desired measurement accuracy for a given degree of freedom in the determination of $\gamma^2_{xy}(f)$. For small ϵ, $\sin\epsilon = \epsilon$, therefore the plotted curves are valid for both the gain and phase factors. For the gain factor the curves represent a measurement uncertainty of 5, 10 and 15%. For the phase factor they represent a measurement uncertainty of 0.05, 0.1 and 0.15 radians.

If it is desired to measure the frequency response function of a linear system with a known accuracy, iterative measures must be used. All that was said about spectral bandwidth etc. when dealing with the ordinary power spectrum applies to the frequency response function. This is because the measurement of $\hat{H}(f)$ involves $\hat{G}_x(f)$.

To iterate a frequency response function to a given level of measurement accuracy first measure the coherence function for a chosen length of data sample. Next enter the ordinate of Figure at the measured value of $\gamma^2_{xy}(f)$ and follow up to the chosen combination of P and ϵ . Then read off the number of degrees of freedom and calculate a new record length for a given bandwidth. From the new length of data sample estimate the coherence function a second time repeating the procedures until the desired measurement accuracy is realised.

CONCLUSIONS REGARDING THE MEASUREMENT UNCERTAINTY IN FREQUENCY ANALYSIS

For ordinary power spectral density analysis the mean square standard error is given by:

$$\epsilon^2 = \frac{1}{BT} + \frac{1}{576}\left[\frac{B}{\lambda(f)}\right]^4$$

$\lambda(f)$ is the spectral bandwidth of the power spectrum on the bandwidth of the narrowest peak than can be resolved. If possible $\lambda(f)$ should be determined prior to settling the analysis data, i.e. $\hat{G}_x(f)$ It is suggested that $\lambda(f)$ be determined by an iterative procedure. When is known, if possible, the filter bandwidth should maintain the relationship.

$$B \leq \frac{\lambda(f)}{2}$$

Ideally (see Table V, Ref. 3) B should be chosen such that $B \leq \lambda(f)/4$ When B maintains this relationship with $\lambda(f)$ the mean square standard error may be taken as the variance term and the bias terms neglected. If B does not maintain the above relationship with $\lambda(f)$ the bias error must be included.

The mean square error in estimating the cross power spectral density function is known only for the components $\hat{C}_{xy}(f)$ and $\hat{Q}_{xy}(f)$. The mean square standard error for the components is similar to that for the ordinary power spectral density function, namely

$$\epsilon^2\left[\hat{C}_{xy}(f) \text{ and } \hat{Q}_{xy}(f)\right] \doteq \frac{1}{BT}$$

For a frequency response function estimate the measurement uncertainty is given by Goodman in terms of phase factor, gain factor, degree of freedom and coherence. Since the frequency response function contains both the ordinary power spectrum and cross spectrum functions all remarks relating to the ordinary and cross spectrums apply to the frequency response function. In order to measure the frequency response function with a known accuracy the coherence function must be determined iteratively.

MEASUREMENT UNCERTAINTY IN TIME ANALYSIS

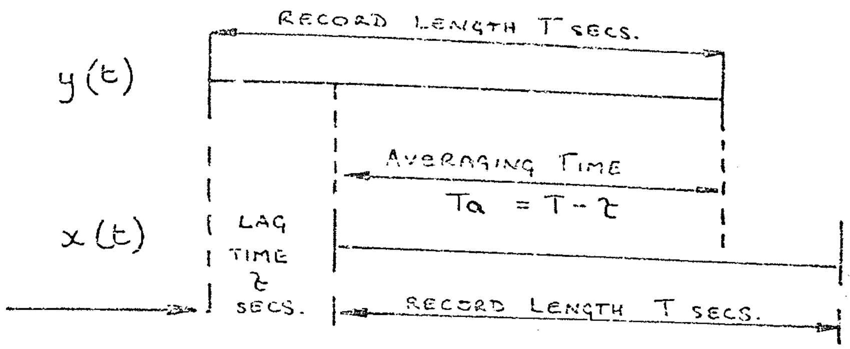
Weakly Stationary - Cross Correlation Function

An estimate of the cross correlation function $R_{xy}(\tau)$ is given by:

$$\hat{R}_{xy}(\tau) = \begin{cases} \dfrac{1}{T-\tau}\displaystyle\int_0^{T-\tau} x(t)y(t+\tau)\,dt, & 0 \leq \tau \leq T. \\ \dfrac{1}{T-|\tau|}\displaystyle\int^{T} x(t)y(t+\tau)\,dt, & -T \leq \tau \leq 0. \end{cases}$$

Note that the integral, for positive τ extends from 0 to $T - \tau$ i.e. the integration time is $T - \tau$ and not T which is the record or sample length of vibrations. The diagram below illustrates this point.

Figure



For most practical purposes, if T is long compared to τ then a "second class" estimate of $R_{xy}(\tau)$ may be used as given by

$$\hat{R}_{xy}(\tau) \doteq \frac{1}{T}\int_0^T x(t)\,y(t+\tau)\,dt$$

note that $\hat{R}_{xy}(\tau) \neq \hat{R}_{xy}(-\tau)$. From now on only a positive lag time will be considered. The measurement uncertainty being the same for both positive and negative lag.

MEAN SQUARE MEASUREMENT UNCERTAINTY

The mean square error in an estimate $\hat{R}_{xy}(\tau)$ is given by

$$E\left[\hat{R}_{xy}(\tau) - R_{xy}(\tau)\right]^2 = E\left[\hat{R}_{xy}(\tau) - E\left(\hat{R}_{xy}(\tau)\right)\right]^2$$

$$+ \left[E\left(\hat{R}_{xy}(\tau)\right) - R_{xy}(\tau)\right]^2$$

BIAS ERROR

The second term in the expression for mean square error is the square of the bias error. For a cross correlation estimate the bias error is zero. This is proved below.

$$E\left[\hat{R}_{xy}(\tau)\right] = E\left[\frac{1}{T}\int_0^T x(t)y(t+\tau)\,dt\right]$$

$$= \frac{1}{T}\int_0^T E\left[x(t)y(t+\tau)\right]dt$$

$$= \frac{1}{T}\int_0^T R_{xy}(\tau)\,dt$$

Since the expected value of an expectancy is the expected value itself (the terms, integral or average, may replace the expectancy term in continuous data)

$$E\left[\hat{R}_{xy}(\tau)\right] = R_{xy}(\tau)$$

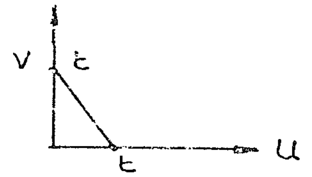
Therefore, by definition if the expected value of an estimate is the true value, the estimate is unbiased.

VARIANCE ERROR

The variance error is the first term in the expression for mean square error. Since, however, the bias error is zero, the variance error is also the mean square error in this case.

$$\therefore\; E\left[\hat{R}_{xy}(\tau) - R_{xy}(\tau)\right]^2 = E\left[\hat{R}^2_{xy}(\tau)\right] - R^2_{xy}(\tau)$$

Placing t on a (U, V) plane i.e.



we get

$$= \frac{1}{T^2} \int_0^T \int_0^T \left\{ E\left[x(U) y(U+\tau) x(V) y(V+\tau) \right] - R_{xy}^2(\tau) \right\} dU\,dV$$

If the processes $\{x(\alpha, t)\}$ and $\{y(\alpha, t)\}$ from which the sample functions $x(t)$ and $y(t)$ are derived, are normally distributed, the expectancy bracket may be re-written thus,

$$E\left[x(U) y(U+\tau) x(V) y(V+\tau) \right] = R_{xy}^2(\tau) + R_x(V-U)$$

$$\times\ R_y(V-U) + R_{xy}(V+U+\tau) R_{yx}(V-U+\tau)$$

Hence the mean square error becomes:

$$E\left[\hat{R}_{xy}(\tau) - R_{xy}(\tau) \right]^2 = \frac{1}{T^2} \int_0^T \int_0^T \left[R_x(V-U) R_y(V-U) + R_{xy}(V-U+\tau) \right.$$

$$\left. \times\ R_{yx}(V-U+\tau) \right] dU\,dV$$

If $\xi = V-U$ and $d\xi = dU$ if the order of integration between ξ and U are reversed the above expression becomes:

$$E\left[\hat{R}_{xy}(\tau) - R_{xy}(\tau) \right]^2 = \frac{1}{T} \int_{-T}^{T} \left(1 - \frac{|\xi|}{T} \right) \left[R_x(\xi) R_y(\xi) \right.$$

$$\left. +\ R_{xy}(\xi+\tau) R_{yx}(\xi+\tau) \right] d\xi$$

To check that this estimate $\hat{R}_{xy}(\tau)$ is a consistent estimate of $R_{xy}(\tau)$ i.e. that the mean square error approaches zero as $T \to \infty$, integrate the above expression from $-\infty$ to $+\infty$. This is difficult mathematically, however, when done it proves that $\hat{R}_{xy}(\tau)$ is a consistent estimate of $R_{xy}(\tau)$.

Weakly Stationary - Autocorrelation Function

The autocorrelation function is a special case of the cross correlation function and occurs when $y(t) = x(t)$. An estimate of the true autocorrelation function is given by

$$\hat{R}_x(\tau) = \frac{1}{T}\int_0^T x(t)x(t+\tau)\,dt.$$

The measurement uncertainty associated with $\hat{R}_x(\tau)$ has been studied in depth by Bendat (1958), Chapter 7. Bendat has found that at $\tau = 0$ the mean square standard error is given by

$$\epsilon^2 = \epsilon^2 \hat{R}_x(0) = \frac{1}{B_N T}$$

The autocorrelation function maintains this measurement uncertainty fairly closely in the small lag region round about $\tau = 0$.

For large lag times a more complicated expression applies for ϵ^2 However, the more complicated expression gives a smaller value for the mean square standard error than does $\epsilon^2 = 1/B_N T$. Therefore it would be conservative to use the expression based upon small lag times for large lag times.

CONCLUSIONS REGARDING THE MEASUREMENT UNCERTAINTY IN CORRELATION ANALYSIS

The bias error in auto and cross correlation estimates is zero. The mean square error is therefore made up of the variance term only. Auto and cross correlation estimates are consistent.

The mean square error for cross correlation estimates is complicated at all values of τ because of the transform of the variable t into U and V. In this form it is not readily applicable to physical problems. A conservative estimate for the mean square error in cross correlation estimates would be given by the expression for autocorrelation error.

The measurement uncertainty in autocorrelation estimates may be conservatively taken as

$$\epsilon^2 = 1/B_N T$$

where B_N is the equivalent ideal noise bandwidth of the signal and T is the averaging time. This relationship is more accurate at small values of lag, (τ).

Note that the measurement uncertainty in correlation analysis is the same as that in frequency analysis. Do not confuse, however, the B's in the expressions for ϵ^2. It is not surprising that the expressions are similar for the frequency and time analyses. The reason being that for weakly stationary records, they are directly related by the Wiener-Khinchine equations (see Ref. 2).

MEASUREMENT RELIABILITY

Measurement reliability is a statement of confidence with which future measurements will not exceed measurements made in the past. Measurement reliability is specified for analysis data in terms of confidence limits. The confidence limit gives a measure of the range over which the true or population value will lie with a known probability. These ideas have already been introduced.

If the reliability of the analysis data has been established, the analysis data may be used to predict the magnitude of statistical properties (analysis data) that future random vibration will not exceed. Bear in mind, however, that any statement of non-occurence will eventually be exceeded.

The actual confidence interval chosen to specify a confidence limit will depend upon the importance with which the statement of non-occurrence must be maintained. Figure will give some idea of the confidence levels that may be chosen. The example is taken from N.A.S.A. data on vibration of a Saturn Launch vehicle. Each point on the power spectrum represents the average over an ensemble of size 20.

MEASUREMENT UNCERTAINTY IN ENSEMBLE AND NON STATIONARY ANALYSIS

Mention is here made of these uncertainties, although it is not proposed to cover them in any depth.

For Ensemble Amplitudes and Time Analysis the expressions will be similar to those for single time records. Substitute in the single time record expressions for uncertainty, the expression 2/N for the product BT.

Ensemble frequency analysis is more difficult to implement than amplitude or time analysis. The frequency filtering operation is the difficulty that arises. One approach would be to filter the analogue time signal and then digitise and square the output. An ensemble of filtered squared digital values would arise from the collection of records. An ensemble of ensembles would be generated whose size would be the number of frequency steps needed to cover the frequency range. The measurement uncertainty for this sort of ensemble frequency analysis has not been studied. However, an alternative method which yields similar results is to perform an ordinary power spectral density analysis for each time history record. This would generate a set of values $\{\hat{G}_x(f)\}$ which could then be averaged. The difference between the two types of analysis is that the first analysis makes no assumptions about stationarity and could yield a power spectrum at different times (i.e. a time varying or non stationary frequency analysis). The second analysis involves the stationary assumption. Therefore, for the second case the measurement uncertainty is given by $\epsilon^2 = 1/BT$ for each estimate $\hat{G}_x(f)$. For the collection of estimates $\{\hat{G}_x(f)\}$ (in accordance with statement made earlier in connection with statistical estimation theory) if $\hat{G}_x(f)$ is an unbiased estimate of $G_x(f)$, the true value, the average $\underset{\alpha}{\epsilon}\left[\hat{G}_x(f)\right]$ should give $G_x(f)$ without error, if the size of the ensemble is large, i.e.

The measurement uncertainty in non-stationary analyis is not yet fully understood. For the time varying non-stationary analysis the error expressions for stationary time signals should provide a reasonable approximation in most cases. As with stationary analysis, non-stationary analysis will have a compound error made up of the bias error due to finite amplitude and frequency filters and a variance error due to a finite number of ensemble members. (It has been assumed here that all non-stationary analysis will be carried out on an ensemble of records.)

APPENDIX D

DERIVATION OF THE SPECTRAL RATIOS FOR THE HUMAN OPERATOR & MAN/MACHINE DESCRIBING FUNCTIONS

Y_p in the derivation below is the Human Operator Transfer Function which is refferred to as the Describing Function because the Function is based upon Statistical rather than Deterministic measures.

Y_c in the equations is the Controlled Element Describing Function and the product Y_pY_c is the combined Man/Machine Open Loop Describing Function.

The symbols used are:

i = Input Command Signal

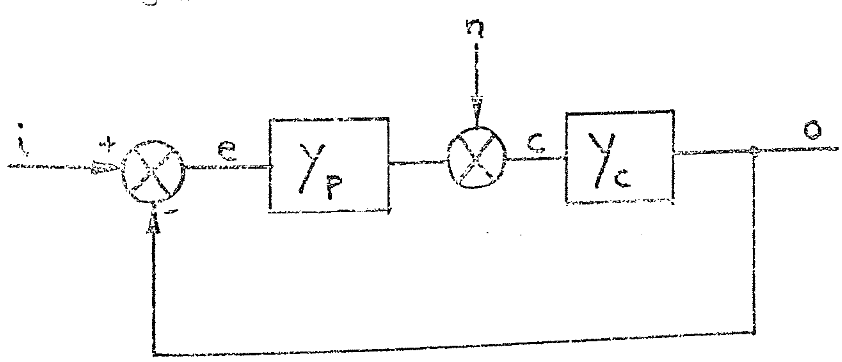
e = Error as seen by the Human Operator

c = The Human Operators Control Movement

o = Output or State of the Controlled Element

n = Random Noise injected into the system by the Operator

The Control System Block diagram for the Man/Machine configuration in Compensatory Tracking is shown below:



Now: $e = i - o$, $i = e + o$ & $o = i - e$. ——————(1)

& $c = (i - o)\left\{Y_p\right\} + n.$

$$\text{also: } o = c\{Y_p\}$$

$$\therefore \quad c = (i - c\{Y_c\}).\{Y_p\} + n$$

$$c = i.Y_p - c.Y_cY_p + n$$

$$c + c.Y_cY_p = i.Y_p + n$$

$$c(1 + Y_cY_p) = i.Y_p + n$$

$$\therefore \quad c = \left\{\frac{Y_p}{1 + Y_cY_p}\right\} i + \frac{n}{1 + Y_cY_p} \qquad (2)$$

$$\text{also: } ic = \left\{\frac{Y_p}{1 + Y_cY_p}\right\} ii + \frac{ni}{1 + Y_cY_p}$$

$$\text{or: } G_{ic} = \left\{\frac{Y_p}{1 + Y_cY_p}\right\} G_{ii} + G_{ni}\left\{\frac{1}{1 + Y_cY_p}\right\}$$

If the Input and Remnant are uncorrelated then:

$$G_{ni}\left\{\frac{1}{1 + Y_cY_p}\right\} = 0$$

$$\text{and: } \frac{G_{ic}}{G_{ii}} = \frac{Y_p}{1 + Y_cY_p} \qquad (3)$$

This equatio is sometimes used to define the Human Operator Transfer Function. Also:

$$e = \frac{i}{1 + Y_cY_p} + \left\{\frac{Y_c}{1 + Y_cY_p}\right\} n$$

$$\text{and } G_{ie} = G_{ii}\left\{\frac{1}{1 + Y_cY_p}\right\} + G_{in}\left\{\frac{Y_c}{1 + Y_cY_p}\right\}$$

Again if $G_{in} = 0$

$$\text{then } \frac{G_{ie}}{G_{ii}} = \frac{1}{1 + Y_cY_p} \qquad (4)$$

From this Equa.

$$Y_pY_c = \left\{\frac{G_{ii}}{G_{ie}}\right\} - 1 \qquad (5)$$

This is the Combined Man/Machine Describing Function.

Dividing (3) by (4) gives (ignoring the noise term);

$$Y_p = \frac{\dfrac{Y_p}{1 + Y_p Y_c}}{\dfrac{1}{1 + Y_p Y_c}} = \frac{G_{ic}}{G_{ie}} \qquad (6)$$

This eqution is considered to provide the best estimate of Y_p

From these ratio's one concludes that;

FOR CLOSED LOOP ANALYSIS; G_{ic}/G_{ii} gives the closed loop describing function $Y_p / 1 + Y_p Y_c$

G_{ie}/G_{ii} gives the ratio $1 / 1 + Y_p Y_c$ which if inverted and unity subtracted gives $Y_p Y_c$.

G_{ic}/G_{ie} computes Y_p the Human Operator describing Function relative to the Input.

FOR OPEN LOOP ANALYSIS G_{ec}/G_{ee} gives Y_p without reference to the input signal.

Note that: In the above G_{ic}/G_{ie} cannot be easily be implemented on computer without first modifying the program to calculate the maximun value of G_{ic} for normalisation purposes. All the other ratio's which use the Auto Spectra as the denominator will normalise.

REFERENCES. Part I. - Waveform Analysis.

ANDREWS H.C. (1969) Decision making and pattern recognition of multi-dimensional data. IEEE Conferences record. 69C58-MMS.

ANDRONOV A.A., PONTYRAGIN L.S. and VITT A.A. (1933). On the statistical investigation og dynamical systems. J. Exp. Theor. Phys. Vol.3(3). 165

ANSTEY N.A. (1966). Wiggles. Data Correlation Division, Seismograph Service (England) Ltd..

ARIARATNAM S.T. (1960) Random vibration of non-linear suspensions. J.Mech. Eng. Science. Vol.2 no.3.

BARRETT J.F. (1958). Problems arising in the analysis of randomly disturbed non-linear control systems. PhD. Thesis. Univ. of Cambridge.

BARTLETT M.S. (1950). Periodogram analysis and continuous spectra. Biometrika, 37, p.1-16

BARTLETT M.S. (1955). An introduction to stöchastic processes. Cambridge Univ. Press, Cambridge, England.

BATCHELOR B.G. (1968). Learning machines for pattern recognition. PhD. thesis. N.P.L. and Plessey Co. Ltd..

BENDAT J.S. (1958). Principles and applications of random noise theory. New York, Wiley.

BENDAT J.S., ENOCHSON L.D., KLEIN G.H. and PIERSON A.G. (1962). Advanced concepts of stochastic processes and statistics for flight vehicle vibration estimation and measurement. Wright-Patterson Air Force Base Ohio, USA. ASD-TDR-62-973

BENDAT J.S. and PIERSOL A.G. (1966). Measurement and analysis of random data. New York, Wiley.

BLACKMAN R.B. (1965). Data smoothing and prediction. Addison Wesley.

BLACKMAN R.B. and TUKEY J.W. (1958). The measurement of power spectra from the point of view of communications engineering. Dover.

CHUANG K. and KAZDA F. (1959). A study of non-linear systems with random inputs. Amer. Inst. Elec. Engineers. 'Applications and Industry' No.42, p.100.

CRANDALL S.H., CAUGHEY T.K. and LYON R.H. (1963). Vibrations in non-linear structures. J. Acoustic Soc. of Amer. Vol.35, no.11, p.1683-1721.

DEUTSCH R. (1966). Non-linear Transformations of random processes. Prentice-Hall Applied Mathematics Series. Prentice-Hall International Inc.

DOOB J.L. (1953). Stochastic processes. New York, Wiley.

FENWICK and FENTON (1971). On the use of autoregression to determine the impulse response function of the brain from an analysis of the background EEG. Paper presented to the Quantification group of the EEG Society and to a meeting on Biological Signal Processing. Univ. College, London / IEE.

FOKKER A. (1913). Dissertation, Leiden.

GARNER W.R. (1962). Uncertainty and structure as psychological concepts. New York, Wiley.

HAAR A. (1944). "Zur theorie der orthogonalen funktionen-systems". Inaugral dissertation, Maths. Annals. Vol.69, 146-152.

KHINCHINE A. (1934). Korrelations theorie der stationaren stochastischen. prozesse. Maths. Annals. 109. p.604-605.

LEE Y.W. (1960). Statistical theory of communication. Wiley.

McKAY D. (1950). The quantal aspects of scientific information. Phil, Mag. 41, p.289-311.

MILSUM J.H. (1966). Biological control systems analysis. Electronic Sciences series. McGraw-Hill Book Company.

OLIVER R.E. and WIN T.Y. (1958). Sled track interaction and a rapid method for track alignment measurement. Aeronautical Engineering Research Inc.

PEARSON K. (1894). Contributions to the mathematical theory of evolution. Phil. Trans. Roy. Soc. A. Vol. 185, p.71.

PEARSON K. (1901). On lines and planes of closest fit to systems of points in space. Phil. mag. series 6, vol.2, p.559.

PEARSON K. (1901). On the systemmatic fitting of curves to observations and measurements. Biometrika vol.1, p.265 and vol.2, p.1.

PLANK M. (1927). Berl. Ber. p.324.

RAO C.R. (1964). The use and interpretation of principal component analyses in applied research. Sankhya, India Journal Stat. Ser.A 26, p.329-359.

RICE S.O. (1944). Mathematical analysis of random noise. Bell System Tech. J. 23, 282-332 (1944); 23, 46-156 (1955).

SHANNON C.E. and WEAVER W. (1949). The mathematical theory of communication. Univ. of Illinois Press.

SMOLUCHOWSKI M.V. (1916) Physics, Zeits, 17, 557.

SPIEGEL M.R. (1961). Theory and problems of statistics. Schaum's Outline Series. Schaum Publishing Co., New York.

WALSH J.L. (1923). A closed set of normal orthogonal functions. Am. J. Math., Vol. 45, 5-24.

WALTER W.G. (1959). Intrinsic rhythms of the brain, In Handbook of Physiology Sec 1, Neurophysiology Vol.1. Magoun, ed., ch.11. Am. Phy. Soc. Washington

WANG M.C. and UHLENBECK G.E. (1945). On the theory of Brownian motion. I and II. Reviews of Modern Physics. Vol. 17, no. 2 and 3.

WEATHERBURN C.E. (1961). A first course in mathematical statistics. Univ. Press. Cambridge.

WHECHEL J.E. and GUINN D.F. (1968). The fast fourier-hadamard transform and its use in signal representation and classification. Eascon. 1968, Record

WEINER N. (1930). Generalised harmonic analysis. Acta Mathematica 55, p.117-258.

WEINER N. (1933). Plancherel's theorem. The Fourier integral and certain of its applications. Dover Publication. New York.

WEINER N. (1948). Cybernetics. M.I.T. Press. Cambridge, U.S.A.

WEINER N. (1949). Extrapolation, interpolation and smoothing of stationary time series with engineering applications. J. Wiley. New York.

WEINER N. (1958). Non-linear problems in random theory. The Technology Press. M.I.T. and Wiley, New York.

REFERENCES. ADAPTIVE TASK TECHNIQUE.

BAHRICK H.P., FITTS P.M. and BRIGGS G.E. (1957). Learning curves - facts or artifacts ? Psychol.Bull., 54,256-268.

BAHRICK H.P., NOBEL M.A. and FITTS P.M. (1954). Extra-task performance as a measure of learning a primary task. Journal of Experimental Psychology, 48, 298-302.

BIRMINGHAM H.P. (1959). The instantaneous measurement of human bandwidth Naval Research Laboratory, Washington D.C. Paper presented at the 8th. Annual Conference on Manual Control, Dunlap and Associates, Stamford, Conn..

BIRMINGHAM H.P., CHERNIKOFF R. and ZIEGLER P.N. (1962). The design and use of 'equalisation' teaching machines, Naval Research Laboratory, Washington D.C. Paper presented at the IRE International Congress on Human Factors in Electronics, Long Beach, Calif..

BOWEN H.M., HALE A. and KELLEY C.R. (1962). Tracking Training V.: Field study of the effectiveness of the general vehicular research tool, NAVTRADEVCEN TR 955-1.

BRIGGS G.E. (1961). On the scheduling of training conditions for the acquisition and transfer of perceptual-motor skills. Fort Washington : Naval Training Device Center, Contract N61339-836. Technical Report NAVTRADEVCEN 836-1

BRIGGS G.E. (1962). Research on augmented feedback and the acquisition and transfer of skill. NAVTRADEVCEN TR 836-3.

CARO P.R.Jr. (1969). Adaptive training - an application to flight simulation. Human Factors 11(6), 569-576.

CHERNIKOFF R. (1962). Human equalisation : concept machine and measu[illegible] ... Proceedings of ONR symposium on complex vehicular controls. RAF. Test Pilot School, Farnborough, England. Douglas Aircraft Co..

CLAUSEN J.T., CURTIN J.G. and EGLER J.F. (1968). Concept formulation report, synthetic training system Device 2B24. Orlando, Fla., Naval Training Device Center, Contract N61339-68-C-0108, Technical Report NAVTRADEVCEN -68-C-0108-1 (AD 673982).

COBURN E. (1968). Some initial considerations on adaptive performance measures, Royal Aircraft Establishment, Farnborough, England Tech. Memo EP 403.

CONNELLY E., SCHULER A. and KNOOP Patricia A. (1969). Study of adaptive mathematical models for deriving automated pilot performance measurement techniques. Wright-Patterson Air Force Base, Ohio. Air Force Human Resources Laboratory, Contract F33615-68-C-1278, Tech. Rpt. AFHRL-TR-69-7.

CONNELLY E.M. and SCHULER A.R. (1969). A theory of adaptive man-machine systems applied to automated training. International symposium on man-machine systems, Cambridge, England. IEEE conference record no.69C58-mms..

ELLIS N.C., LOWES A.L., MATHENY W.G..and NORMAN D.A. (1971) The feasibility of using an adaptive technique in flight simulator training. Ergonomics Vol.14, No.3, 381-390.

FACONTI V., MORTIMER C.P.L. and SIMPSON D.W. (1970). Automated instruction and performance monitoring in flight simulator training. Wright-Patterson Air Force, Ohio : Air Force Human Resources Laboratory, Contract F33615-69-C-1159. Tech. Rpt. AFHRL-TR-69-29.

FLEXMAN R.E., JAMESON W.P., WALSH J.M. and COHEN E. (1968). Synthetic flight training system (SFTS) concept formulation report. Orlando Fla.: Naval Training Device Center, TR NAVTRADEVCEN 1948.

GAINES B.R. (1965). Automated feedback-trainers for perceptual-motor skills. Tech. Rpt.1 (1966(1)). The evaluation and control of human skill. IRE Conf. proc., Colloquium on Human Operator Studies.

GAINES B.R. (1967). Teaching machines for perceptual-motor skills. In D.Unwin and J.Leedham (Eds.). Aspects of educational technology : The proceedings of the programmed learning conference, Methuen, London, England 337-358.

GAINES B.R. and ANDREAE J.H. (1966). A learning machine in the context of the general control problem. Proc. 3rd. Int. Cong.

GAINES B.R. (1967). Automated feedback trainers for perceptual-motor skills. M.o.D. Contract : "Servomechanisms in operator training for tracking tasks".

GUILFORD J.P. (1965). Fundamental statistics in psychology and education New York; M^c^Graw Hill.

HOLDINGS D.H. (1962). Transfer between difficult and easy tasks. Brit. J. Psychol. 53(4) 397-407.

HUDSON E.M. (1962). An adaptive tracking simulator. Otis Elevator Co.. Brooklyn N.Y. Paper presented at the IRE First International Congress on Human Factors in Electronics. Long Beach, Calif.

HUDSON E.M. (1964). Adaptive training and non-verbal behaviour. Fort Washington N.Y. : Naval Training Device Center, Contract N61339-1395. Tech. Rpt. 1395-1.

HUDSON E.M. (1967). Adaptive training device. U.S.Patent 3311995. In U.S. Commerce Department Patent Office, Official Gazette, Washington D.C. Government Printing Office, p.41.

HUDSON E.M. (1969). Adaptive techniques in multiparameter problems. Human Factors, 11, 561-568.

JAMESON W.P., WALSH J.M., FLEXMAN R.E. and COHEN E. (1968) Synthetic flight training system (SFTS). Concept formulation report. Orlando Fla.. Naval Training Device Center, Contract N61339-68-C-0106 Tech. Rpt. NAVTRADENCEN 68-C-0106-1.

JEX H.R., McDONNELL J.D. and PHATAK A.V. (1966). A "critical" tracking task for manual control research. IEEE Transactions on Human Factors in Electronics HFE-7(4), 138-145

KAPPAUF W.E. (1969) Use of an om-line computer for psychophysical testing with the up-and-down methods. American Psychology, 24(3) 207-211

KELLEY C.R. (1950). The critical flicker frequency in peripheral vision Columbus, Ohio : Ohio State University, unpublished M.A. thesis.

KELLEY C.R., BOWEN H.M., ELY J.H. and ANDREAS J.L. (1960). Tracking training : III Transfer of training, Fort Washington N.Y. : U.S. Naval Training Device Center Tech. Rpt. 1908 -00-3.

KELLEY C.R. (1961). A study of performance decrement in simulated complex task situations and simulated environments. Appendix B : Description of electronic display techniques. Stamford, Conn..: Public Service Research Inc.. rpt. to U.S.Dept. of Health, Education and Welfare.

KELLEY C.R. (1961). Self-adjusting vehicle simulators. In C.R.Kelley (Ed.) The predictor instrument : Final report and summary of project activities during 1961. Arlington, Va.: Office of Naval Research. Contract No. 2822(00). Tech. Rpt. Jan. 1962. Also in C.R.Kelley (Ed.) Adaptive simulation. Arlington Va.. Office of Naval Research Contract No. 4986(00). Final Rpt. August 1966 (AD-637658).

KELLEY C.R. (1965). Testing the effects of extended space missions on piloting skill. In E.Burgess (Ed.) AAS Science and Technology series Vol.5 P.Horowitz (Ed.) Physiological and performance determinants of manual space system design. Baltimore Md. American Astronautical Society 165-186

KELLEY C.R., MITCHELL M.B. and STRUDWICK P.H. (1965). The MOL tracking task : Development of a test of tracking skill for inclusion in the general performance battery of the Air Force Manned Orbital Lab.. Prelim. Tech. Rpt. Santa Monica, Calif.. Dunlap and Associates Inc..

KELLEY C.R. (1966). Design applications of adaptive (self-adjusting) simulators. Second Annual NASA Conference on Manual Control, Cambridge, Mass. Feb 28-March 2. Washington D.C.: Scientific and Technical Information Division, National Aeronautic and Space Administration NASA SP-128.

KELLEY C.R. (1966) Adaptive simulation (Office of Naval Research. Tech. Rep. No.4986(00)). Santa Monica, Calif.:Dunlap and Associates. Inc.

KELLEY C.R., WARGO M.J., MITCHELL M.B. and PROSIN D.J. (1966). In-space measurement of piloting skill : The NOL tracking task, Santa Monica, Calif. : Douglas Aircraft Co..

KELLEY C.R. and WARGO M.J. (1967). Cross-adaptive operator loading tasks Human Factors 9(5) 395-404. Also in Adaptive Simulation Santa Monica Calif.: Dunlap and Associates Inc.

KELLEY C.R. and PROSIN D.J. (1967). Frequency adaptive tracking systems In Further Research with Adaptive tasks. Santa Monica, Calif. : Dunlap and Associates Inc. Final rpt. to Office of Naval Research.

KELLEY C.R. (1967). Further research with adaptive tasks. Arlington, Va.: Office of Naval Research, Contract No.4986(00). Final Rpt. AD-657-343.

KELLEY C.R. (1967). The measurement of tracking proficiency. In Further Research with Adaptive tasks. Santa Monica, Calif. : Dunlap and Associates Inc. Final rt. to Office of Namal Research.

KELLEY C.R. (1967). Research applications of adaptive tasks. In Further Research with Adaptive tasks. Santa Monica, Calif.: Dunlap and Assoc. Final [illegible] Also presented as a paper at the

International Ergonomics Conference, Birmingham, England. Sept.1967

KELLEY C.R. and PROSIN D.J. (1968). Adaptive performance measurment. Santa Monica, Calif.:Dunlap and Associates Inc.. Tech. rpt. to Office of Naval Research.

KELLEY C.R. and WARGO M.J. (1968). Adaptive techniques for synthetic flight training systems (U.S.Naval Training Device Center,Tech.rpt. NAVTRADEVCEN 68-C-0136-1) Santa Monica Calif.: Dunlap and Assoc. Inc.

KELLEY C.R. and PROSIN D.J. (1969). Adaptive techniques in measuring complex performance. Arlington, Va. Office of Naval Research Contract No. 4986(00). Final rpt. AD-694-509.

KELLEY C.R. (1968). Engineering models of the human operator. Chap.13 Manual and Automotive Control. New York : John Wiley and Sons.

KELLEY C.R. (1969). What is adaptive training ? Human Factors 11(6).547-556

KELLEY C.R. (1969). The measurement of tracking proficiency. Human Factors 11(1) 43-64.

KELLEY C.R. (1969). Adaptive and automated research techniques from engineering psychology. American Psychologist. 24, 293-297.

KELLEY C.R. and KELLEY Erica J. (1970). A manual for adaptive techniques Arlington Va.:Office of Naval Research, Contract No. 4986(00). Final rpt. AD-711-985.

KNOOP Patricia A. (1966). Programming techniques for the automatic monitoring of human performance. Wright-Patterson Air Force Base, Ohio. : Aerospace Research Lab. Project 6114, Task 611412, Tech. rpt. AMRL -TR-66-16.

KNOOP Patricia A. (1967). Automated aids for flight simulator instructors USAF Instructor's Journal 4(4). 61-64.

KNOOP Patricia A. (1968). Development and evaluation of a digital computer program for automatic human performance monitoring in flight simulator training. Wright-Patterson Air Force Base, Ohio. Aerospace Medical Research Lab. Project 6114. Task 611412 Tech.rpt. AMRL -TR-67-97.

KNOWLES W.B. (1963). Operator loading tasks. Human Factors 5, 155-161.

KOPSTEIN F.F. and SEIDEL R.J. (1970) The computer as an adaptive instructional decision maker. Alexandria, Va.: Human Resources Research Organisation. Contract DA11C 19-70-C-0012. HUMRRO Professional Paper-1-70 AD-703-597.

LEVITT H. (1970). Transformed up-down methods in psychophysics. J. Acou. Soc. Amer. Vol. 49 No.2 part 2. p.467-474.

LEWIS B.N. and PASK G. (1965). The theory and practice of adaptive teaching systems. In R.Glaser (Ed.) Teaching Machines and Programmed Learning Vol.II Data and Directions Washington D.C. : National Education Association. 213-266.

LOWES A.L., ELLIS N.C., NORMAN D.A. and MATHENY W.G. (1968). Improving piloting skills in turbulent air using a self-adaptive technique for a digital operational flight trainer. Orlando, Fla : Naval Tr Training Device Center. Contract N61339-67-C-0034. Tech. Rpt. NAVTRADEVCEN 67-C-0034-2 (AD-675-805).

MATHENY W.G. (1969). The effective time constant - a new technique for adaptive training. Human Factors 11, 557-560.

MATHENY W.G. and NORMAN D.A. (1968). The effective time constant in tracking behaviour. Orlando, Fla.: Naval Training Device Center Contract N61339-67-C-0034. Tech. Rpt. NAVTRADEVCEN 67-C-0034-3.

McGRATH J.J. and HARRIS D.H. (1971). Eds. Adaptive Training. Aviation Research Monographs. Vol.1,No.2. Aviation Research Laboratory. Inst. of Aviation. University of Illinois at Urbana, Champain, U.S.A.

MIRABELLA A. and LAMB J.C. (1966). Computer based adaptive training applied to symbolic displays. Perceptual and Motor Skills 23,647-661

MELCHING W.H. (1968). A concept of the role of man in automated systems. Alexandria, Va.: Human Resources Research Office. Professional paper14-68

PASK G. (1960). The teaching machine as a control mechanism. Trans. Soc. Instr. Tech.12(2) 72-82.

PASK G. (1960). Adaptive teaching with adaptive machines. In A.A.Lumsdaine and R.Glaser (Eds.)Teaching machines and programmed learning. A source book. Washington D.C. National Education Association 349-366.

PASK G. (1960). Electronic keyboard teaching machines. In A.A.Lumsdaine and R.Glaser (Eds.) Teaching machines and programmed learning. A source book. Washington D.C. : National Education Association. 336-348

PASK G. (1961). The cybernetics of evolutionary processes and of self organising systems. System Research Ltd. Richmond, Surrey, England. Paper delivered at the 3rd. Congress of the International Assoc. of Cybernetics. Namur, Belgium.

PASK G. and Von FOERSTER H.(1961). A predictive model for self-organising systems. Cybernetics 4, 20-55.

PASK G. (1962). The logic and behaviour of self-organising systems as illustrated by the interaction between man and adaptive machines System Research Ltd.. Richmond, Surrey, England. Paper delivered to the International Symposium on Information Theory. Brussels, Belgium

PASK G. and LEWIS B.N. (1962). An adaptive automation for teaching small groups. Perceptual and Motor Skills. 14, 183-188.

PASK G. (1965). Teaching as a control engineering process. Control and Automation progress. Jan.-April 1965. 9(79) 11-82

PASK G. (1967). The adaptively controlled instruction of a transformational skill. Programmed Learning 74-86.

POULTON E.C. (1965) On increasing the sensitivity of measures of performance. Ergonomics Vol.8 No.1 p.69.

RATH G. (1967). The development of computer-assisted instruction. In IEEE Transactions on Human Factors in Electronics, 8(2) 60-64

REGAN J.J. (1969). (Ed.) Adaptive Training - special issue of Human Factors 11, 545.

SENDERS J.W. (1961). Adaptive teaching machines in programmed learning and computer based instruction. Wiley

STEWART W.A. (1970). Rand symposium on pilot training and the pilot career : Recollections of the chairman. Santa Monica, Calif. :Rand Corporation. Report RM-6282-1-PR.

STOLUROW L.M. (1968) Some factors in the design of systems for computer-assisted instructions. Arlington, Va. : Office of Naval Research Contract No. NOO014-67-A-0298-0003. Report 7.

TAYLOR M. (1969). on "Post and experiments using adaptive proceedures. N.A.T.O. Advanced Studies Institute in on-line computing in the behavioural sciences. University of Sheffield, England.

WALSH J.M. and FLEXMAN R.E. (1970). Preliminary adaptive training design report for synthetic flight training system. Device 2B24, Orlando, Fla. : Naval Training Device Center. Contract N61339-69-C-0200. Tech. rpt. 69-C-0200-15.

REFERENCES - Effects of Vibration on Manual Control.

ALLEN G.R. (1971). Human reaction to vibration. Royal Aircraft Establishment, Farnborough, Hampshire, England. Paper presented to Institute of Environmental Sciences, U.S.A.

ASHE W.F. et al. (1961). Tracking performance during low frequency vibration. Aerospace medicine Vol 32 No.9

BEAUPEURT J.E. (1962). A study of the effects of vibration on human performance. Naval Research Review. Aug.12-17. Boeing Airplane Co. Wichita, Kansas.

BEAUPEURT J.E., PARKS D.L. and SNYDER F.W. ( ). Human capabilities in a vibration environment. Boeing Tech, Rpt. No.D3-3512-1.Cont.NONR2994(00)

BERGERON H.P. and ADAMS J.J. (1964). Measured transfer functions of pilots during two-axis tasks with motion. Langley Station,Va.: NASA Rep. NASA-TN-D-2177 Contract N64-17157.

BESCO R.O. (1961). The effect of cockpit vertical accelerations on a simple piloted tracking task. Report NA-61-47. North American Aviation Inc. Ohio.

BROADBENT D.E. (1971). Decision and stress. Academic Press, London.

BROWN B.P., JOHNSON H.I. and MUNGALL R.G. (1960). Simulator motion effect on a pilots ability to perform a precise longitudinal flying task. NASA Tech. Rep. D-367 NASA Washington D.C.

BROWN J.L., KUEHNEL H., NICHOLSON F.T. and FUTTERWIET A. (1960). Comparison of tracking performance in the TV-2 aircraft and the ACL computer/AMAL Human centrifuge simulator of this aircraft. Rep. NADC-MA-6016/NADC-AC-6008. U.S.Naval Air Dev. Cen., Pa..

BUCKHOUT R. (1964). Effect of whole body vibration on Human performance. Hu...

BUCKHOUT R., SHERMAN H., GOLDSMITH C.T. and VITALE P.A. (1963). The effect of variations in motion fidelity during training on simulated low altitude flight. USAF Aerospace Med. Res. Labs. AMRL-TDR-63-108.

CAIGER B. (1966). Some problems in control arising from operational experiences with jet transports. National Research Council, Canada Rpt. NAE MISC 41.

CATTERSON A.D., HOOVER G.D. and ASHE W.F. (1962). Human psychomotor performance during prolonged vertical vibration. Aerospace Med. 33(5) 598-602.

CHANEY R.E. and PARKS D.L. (1964). Visual-motor performance during whole-body vibration. The Boeing Co., Wichita, Kansas. Human Factors Tech, Repm D3-3512-5

CHANEY R.E. and PARKS D.L. (1064). Tracking performance during whole-body vibration. The Boeing Co., Wichita, Kansas : Human Factors Tech.Rpt. D3-3512-6.

CHILES W.D. and CUSTER Carolyn L. (1963). Summaries of research on the human performance effects of vibration. USAF Aerospace Med. Res. Lab AMRL Memo P-60.

CLARK C.C. (1962). Human control performance and tolerance under severe complex waveform vibration with a prelimimary historical review of flight simulation. The Martin Co., Baltimore : Engineering Rep.ER 12406

CLARKE N.P. et al. (1965). Preliminary study of dial reading performance during sustained acceleration and vibration. USAF Aerospace Med. Res. Lab. Tech. Rpt. AMRL-TR-65-110.

CLEMENS D.E. (1964). The effect of vibration on certain psychomotor responses. Lobbock : Texas Tech. College, Central Library.

COERMANN ... performance under

vibrational stress. Human Factors 4, No,45. 315-324.

CROOK M.N., HOFFMAN C. and WESSEL N.Y. (1947). Effect of vibration, illumination and type size on legibility of numerals. ATI-6672 USAF AMC., Wright Field Memo.

CROOK M.N. et al. (1947). Effect of vibration on legibility of tabular numerical material. Exps. 1-4 Rept. No.1 and Exps. 5-7 Rept. No.4 Aero. Med. Lab. Wright Field Dayton.

CROOK M.N. et al. (1948) Studies of the effect of typographical spacing on the legibility of numerals under vibration. USAF Air Material Command : Memo Rpt. MCREXD-694-IQ.

CROOK M.N. et al. (1949). Effect of vibration on legibility of tabular numerical material. Exps. 8&9 Rept. No.5 USAF Air. Mat. Comm. Wright Patterson Air Base.

CROOK M.N. et al. (1950). Effect on amplitude of apparent vibration, brightness and type size on numeral reading. Tech. Rep. 6246 USAF Air Mat, Comm. Wright-Patterson Air Base.

CROSSMAN E.R.F.W. (1960). The information capacity of the human motor system in pursuit tracking. Quart. J. Exp. Psychol. 19609 12, 1-16.

DEAN R.D., FARRELL R.J. and HITT J.D. (1969). Effect of vibration on the operation of decimal input devices. Human Factors 11(3) 257-271

DEAN R.D., M^c^GLOTHLEN C.L. and MONROE J.L. (1964). Performance and Physiological effects of CH-46A noise and vibration. The Boeing Co, Seattle. Doc. D2-90583.

DEAN R.D., M^c^GLOTHLEN L. and MONROE J.L. (1965). Performance and physiological effects of 6 hour exposure to helicopter noise and vibration. Paper to 36th. Ann. meet. Aerospace Med. Assoc.

DENNIS J.P. (1963). A survey of experimental data on the effects of ... object upon visual performance. Occ. Psych. 37,4, 277-282.

DENNIS J.P. (1963). Some comparisons between the effects of whole body vibration and those of vibrating the visual object upon visual performance. Unpub. thesis Univ. Bristol Library.

DENNIS J.P. (1960). The effect of whole body vibration on a visual performance task, Clothing and Equipment Physiological Research Estab. Farnborough. Rpt. R.: CEPRE 104.

DENNIS J.P. (1965). Some effects of vibration upon visual performance. J. Appl. psycho. 49, 245-252.

DENNIS J.P. (1965). The effect of whole body vibration on a visual performance task. Ergonomics 8, 193-205.

DRAZIN D.H. (1961). Effects of vibration on dial legibility. Paper read at B. Ass. Advancement of Science meeting at Norwich.

DUDEK R.A. and CLEMENS D.E. (1965). Effect of vibration on certain psychomotor responses. J.Eng.Psychology. 4, 126-143.

FORBES A.R. (1960). The effects of whole body vibration on compensatory tracking performance. RAF Institute of Aviation Medicine, Farnborough. Rpt. 144.

FRASER T.M., HOOVER G.N. and ASHE W.F. (1961). Tracking performance during low frequency vibration. Aerospace Med. 32 (9), 829-835.

GRETHER W.F. (1971). Vibration and human performance. Human Factors 13(3) 203-216.

GUIGNARD J.C. (1959). An effect of whole body vibration on vision. Royal Air Force Inst. Av. Med.. Farnborough. Scientific Memo. 521.

HARRIS C.S., CHILES W.D. and TOUCHSTONE R.M. (1964). Human performance as a function of intensity of vibration at 5c/s, Wright-Patterson AFB : 6570th, AMRL Rpt. AMRL-TDR-64-83. Project 1710, Task 171002, AD [illegible]

HARRIS C.S. and SHOENBERGER R.W. (1964). Human performance during vibration. Office of Naval Res. Eng. Psycho. branch AD 602813,38-47

HARRIS C.S. and SHOENBERGER R.W. (1965). Human performance during vibration. Wright-Patterson AFB : Aerospace M.R.Labs. AMRL-TR-65-204 Project AF-1710. Task 171002, AD-624,196.

HARRIS C.S. and SHOENBERGER R.W. (1965). The effects of vibration on human performance. Paper 23 to 22nd. mtg. AGARD Space Med. Panel, Munich, Sept.1965. In AGARD Collected papers, no.2, 307-326, Paris. NATO/AGARD.

HARRIS C.S. and SHOENBERGER R.W. (1966). Effects of frequency of vibration on human performance. J. Eng. Psycho. 5,1-15, also identified as AMRL-TR-66-152.

HARWOOD K.G. and LOVESEY E.J. (1966). Preliminary tests on the effects of low frequency whole body vibration on aircrew performance. RAE, Farnborough, Tech.Memo 363.

HARWOOD K.G. et al. (1968). The effects of low frequency vibration on pilot performance in a simulated supersonic transport aircraft take-off and flight. RAE., Farnborough Tech. Memo EP 392.

HARWOOD K.G. and LOVESEY E.J. (1966). An attempt to assess the effects of helmet weight on navigators efficiency in a simulated terrain following sortie. RAE., Farnborough, Tech.Memo ME 361.

HOLLAND C.L. (1967). Performance effects of long-term random vertical vibration. Human Factors 9, 93-103.

HORNICK R.J. (1961). The relative effects of noise and vibration upon simple reaction time. Bostrom Research Labs., Milwaukee Rpt. 132.

HORNICK R.J. et al.(1961) The effects of low frequency high amplitude, whole body, longitudinal and transverse vibration upon Human performance Bostrom Research Labs., Milwaukee Final Rpt. (ordanance project

HORNICK R.J. (1962). Effects of whole body vibration in three directions upon human performance. J.Eng.Psycho. 1,3, 93-107.

HORNICK R.J. and LEFRITZ N.M. (1966). A study and review of human response to prolonged random vibration. Human Factors 8, 481-492.

HUDDLESTON H.F. (1964). Human performance and behaviour in vertical sinusoidal vibration RAF.Inst.Av.Med. Rpt. R303.

HURT G.J. (1963). Rough air effects on crew performance during a simulated low-altitude high speed surveillance mission. NASA Tech. Note D-1924.

JOHNSTON W.L. (1972). Body orientation under vertical sinusoidal vibration. Human Factors 14 (4) 349-356.

KAMINSKY G. (1965). Realistic investigations of the effects of vibrations of the hand-arm-body system on manual dexterity. Proc.2nd.Int. Congress on Ergonomics. Taylor and Francis, London.

KOTTENHOFF H. and HOFFMAN H. (1962). Performance and emotional behaviour during vibration stress. (in German). Acta Psychol. 20(4) 320-356

LANGE K.O. and COERMANN R.R. (1962). Visual acuity under vibration. Human Factors 4,5, 291-300.

LARUE M.A. (1965). The effects of vibration on accuracy of a positioning task. J.Environ. science 8 (4) 33-35.

LEWIS D. (1943). The effect of noise and vibration on certain psychomotor responses. Res. at State Univ. Iowa by Civil Aeronautics Administration PB 50288. Res.Rep. 8, 107.

LOEB M. (1955). A further investigation of the influence of whole body vibration and noise on tremor and visual acuity. U.S.Army Med. Res. Lab. Fort Knox : Rpt. 165.

LOVESEY E.J. (1968). The influence of a restraining harness upon human ... performance under single and multi-axis heave and sway vibration. RAE., Farnborough Tech.Memo. EP 397.

LOVESEY E.J. (1970). The multi-axis vibration environment and man. Appl. Ergonomics Vol. 1 No. 5. p.134-139.

MAGID E.B. et al. (1962). Physiological and mechanical responses of the human to longitudinal whole-body vibration as determined by subjective responses. MRL-TDR-62-66.

MAGID E.B. et al. (1960). Human tolerance to whole body sinusoidal vibration. Aerospace Med.. 31(11) 915-924.

MAGID E.B. and COERMANN R.R. (1960). The reaction of the human body to extreme vibrations. Paper to Inst.Environ.Sciences Conf. on Hyper-environments Space Frontier, Los Angeles.

MOZELL M.M. and WHITE D.C. (1958). Behavioural effects of whole body vibration. J.Av.Med. Vol.29 NADC-MA-5802.

O'BRIANT C.R. and OHLBAUM M.K. (1969). Visual acuity decrements associated with whole body $\pm g_z$ vibration stress. USAF Aerospace Med.Res.Lab. Tech.Rpt. AMRL-TR-69-90 also in Aerospace Med. 41(1) 79-82

PARKS D.L. (1961). A comparison of sinusoidal and random vibration effects on human performance. The Boeing Co. Wichita, Kansas : Doc. D3-3512-2.

PARKS D.L. and HICKEY L.F. (1965). Simulation test of the effects of extended exposure to low altitude flight turbulance on flight crew operating with continuous heavy work loads. Part 11 Boeing docDC/2137/ITN.

PROVINS K.A. (1966). Environmental heat, body temperature and behaviour an hypothesis. Australian J. Psychol. 18, 118-129.

RISER G.T. (1961). Exploratory Investigation of the effects of low frequency random amplitude vibration on human performance. Boeing Co.,Kansas. Doc. D3-3328.

RUBENSTEIN L. et al.(1967). Some effects of Y-axis vibration on visual acuity. NASA-CR-96466.

SCHMITZ M.A. (1958). The effect of low-frequency, high amplitude whole-body vertical vibration on human performance. Boston Res. Lab.Proj.128

SCHOHAN B., RAWSON H.E. and SOLIDAY S.M. (1965). Pilot and observer performance in a simulated low altitude high speed flight. Human Factors 7(3) 257-265.

SHOENBERGER R.W. (1967). Effects of vibration on complex psychomotor performance. Aerospace Med. 38, 1264-1269. also USAF AMRL-TR-67-40.

SHURMER C.R. (1967). A review of the effects of low frequency vibration on man and on his tracking performance. BAC.(operating) Ltd.,Bristol Human Factors Study note series 4 no.7.

SHURMER C.R. and SILVERTHORN D.G. (1967). Effects on tracking performance of vibration in each and in combination of the heave, sway and roll axes. BAC.(operating) Ltd., Bristol. Human Factors Study note series4,12

SIMONS A.K. and SCHMITZ M.A. (1958). The effects of low frequency high amplitude vibration on human performance Rep.1 Contract DA-49-007-MD-797. Bostrom Res.Lab., Milwaukee.

SIMONS A.K. and HORNICK R.J.(1961). Effects of the vibration environment in mobile systems on human performance. Bostrom Res,Labs.,Milwaukee Rpt. 137.

SINGLETON W.T. (1970). The measurement of man at work with particular reference to arousal. In Singleton W.T., Fox J.G. and Whitfield D. (Eds.). The measurement of man at work. Taylor and Francis, London.

SNYDER F.W. (1962). Effects of low frequency vertical vibration on human performance. Paper read at Aerospace.Med. Ass. meeting.

SNYDER F.W. (1965). Vibration and vision. In C.A.Baker(Ed.). Visual ... Pergamon Press. 183-201.

SOLIDAY M. and SCHOHAN B. (1964). A simulator investigation of pilot performance during periods of low altitude, high speed flight. NASA CR-63

SOLIDAY S.M. and SCHOHAN B. (1965). Task loading of pilots in simulated low altitude, high speed flight. Human Factors 7(1) 1, 45-53.

STEVENS S.S. (1941). Effects of noise and vibration in psychomotor efficiency. Harvard Univ. ATI-274-27428.

STEVENS S.S. (1941). Effect of vibration on visual acuity. In Waring C.E. Transmission and reception of rounds under combat conditions.

TAUB H.A. (1964). The effects of vibration on dial reading performance. USAF Aerospace Med.Res.Labs.Tech.Doc.Rpt. AMRL-TDR-64-70.

TEARE R.J. and PARKS D.L. (1963). Visual performance during whole body vibration Boeing Co., Wichita, Kansas D3-3512-4.

TEMPLE W.E. et al. (1964). Man's short time tolerance to sinusoidal vibration. Aerospace Med. 35(10) 923-930.

TINKER M.A. (1948). Effects of vibration upon reading. Am. J. Psycho. 61, 386-390.

TORLE G. (1965). Tracking performance under acceleration,-effects of control dynamics. Ergonomics 8(4) 481-486.

WEISZ A.Z., ALLEN R.W. and GODDARD C.J. (1965). Human performance under random and sinusoidal vibration. USAF Aerospace Med.Res.Labs. Tech.Rpt. AMRL-TR-65-209.

WEISZ A.Z., ALLEN R.W. and GODDARD C.J. (1966). An evaluation of three types of hand controllers under random vertical vibration. NASA SP-128

WYON D. (1970). Studies of children under imposed noise and heat stress. Ergonomics 13, 598-612.

REFERENCES For Sections 2.4. - 2.11. & General Introduction.

ASHLEY C. (1969). Simulation of the vehicle ride environment in the laboratory. Transpo 69 Symposium. Soc. Environmental Engineers, London.

JEX H.R., M^cDONNEL J.D. and PHATAK A.V. (1966). A Critical control task for manual control research. Systems Technology Inc. NASA CR 616.

M^cRUER D. and WEIR D.H. (1969). Theory of manual vehicular control. Ergonomics Vol,12 no.4 p.599-633

SMITH O.J.M. (1962). Non-linear computations in the human controller. IRE Trans. Bio-Med. Electron. BME-9(2)

WHITEHEAD G.D. (1967). Human factor problems in the design of vibration isolation systems. A.P. Report 21. Applied Psychology Dept. Univ. Aston Birmingham.

WILDE R.W. and WESTCOTT J.H. (1963). The characterisation of the human operator engaged in a tracking task. Automatica 1(1).

YOUNG L.R. and MEIRY J.L. (1965). Manual control of an unstable system with visual and motion cues. Dept. Aeronautics and Astronautics Dept. Aeronautics and Astronautics, M.I.T., Cambridge, U.S.A.

YOUNG L.R. (1969). On Adaptive Manual Control. Ergonomics Vol.12 no.4.

PUBLICATIONS BY THE AUTHOR.

Ref.1. (1968) Human Factor Problems in the Design of Vibration Isolation Systems. A.P. Report 21. Applied Psychology Dept., University of Aston in Birmingham, Gosta Green, Birmingham 4.

Ref.2. (1967) The Description of Continuous Random Variables. A.P.Report 11. Applied Psychology Dept., University of Aston in Birmingham, Gosta Green, Birmingham 4. Also a series of articles with the same title in Automotive Design Engineering, 1967.

(1965) The Application of Statistics to the Motor Vehicle Ride Comfort Problem. in Advances in Automobile Engineering, Part II edited by G.H.Tidbury, Pergamon Press.

(1965) A Review of data analysis systems. Advanced School of Automobile Engineering, Cranfield, Bedford. A.S.A.E. Memo no.76.

(1968) System Ergonomics - the Human Factor in System Design. Paper presented at the Symposium, Industry, Environment Design, Helsinki, Finland.

REFERENCES

ALLEN.G.R (1971). Human reaction to vibration. Royal Aircraft Establishment, Farnborough,Hampshire,England. Paper presented to Institute of Environmental Sciences,U.S.A.

ANDREWS H.C. (1969). Decision making and pattern recognition of multi-dimensional data. IEEE Conferences record. 69C58-MMS.

ANDRONOV A.A., PONTRYAGIN L.S.and VITT A.A. (1933). On the statistical investigation of dynamical systems. J.Exp.Theor. Phys. Vol 3(3),165

ANSTEY N.A. (1966). Wiggles. Data Correlation Division, Seismograph Service (England) Ltd.

ARIARATNAM S.T. (1960). Random Vibration of non-linear suspensions. J.Mech.Eng. Science. Vol.2,No.3.

ASHE W.F. et al. (1961). Tracking performance during low frequency vibration. Aerospace Medicine Vol 32 No.9

ASHLEY C. (1969). Simulation of the vehicle ride environment in the laboratory.Transpo 69 Symposium. Soc. Environmental Engineers, London.

BAHRICK H.P., FITTS P.M. and BRIGGS G.E. (1957). Learning curves - facts or artifacts? Psychol. Bull., 54,256-268

BAHRICK H.P.,NOBEL M.A. and FITTS P.M. (1954). Extra-task performance as a measure of learning a primary task. Journal of Experimental Psychology,48,298-302.

BARRETT J.F. (1958). Problems arising in the analysis of randomly disturbed non linear control systems. PhD. Thesis Univ. of Cambridge, England.

BARTLETT SIR FREDERICK (1951).The Effects of Flying Upon Human Perf-ormance. Air Ministry Flying Personnel Research Committee: Rpt-FPRC 765.

BARTLETT M.S. (1950). Periodogram analysis and continuous spectra. Biometrika,37,p 1-16.

BARTLETT M.S. (1955). An introduction to Stochastic Processes. Cambridge University Press,Cambridge, England.

BATCHELOR B.G. (1968). Learning Machines for Pattern Recognition. PhD. Thesis University of Southampton, England.

BEAUPEURT J.E. (1962). Astudy of the effects of vibration on human performance. Naval Research Review. Aug. 12-17. Boeing Airplane

BEAUPEURT J.E., PARKS D.L. and SNYDER F.W.(1962). Human Capabilities in a vibration environment. Boeing Tech. Report No.D3-3512-1

BENDAT J.S. (1958). Principles and Applications of Random Noise Theory. Wiley, New York.

BENDAT J.S., ENOCHSON L.D., KLEIN G.H. and PIERSON A.G. (1962). Advanced concepts of stochastic processes and statistics for flight vehicle vibration estimation and measurement. Wright- Patterson Air Force Base, Ohio, U.S.A. ASD?TDR?62-973.

BENDAT J.S. and PIERSON A.G. (1966). Measurement and Analysis of Random data. Wiley, New York.

BERGERON H.P. and ADAMS J.J. (1964). Measured transfer functions of pilots during two axis tasks with motion. Langley Station, Va.: NASA-TN-D-2177 Contract N64-17157.

BESCO R.O. (1961). The effects of cockpit vertical accelerations on a simple piloted tracking task. Report NA-61-47. North American Aviation Inc.Ohio.

BILLINGS C.E. (1968). Studies of pilot performance: 1 Theoretical considerations. Aerospace Med.39,17-19.

BILLINGS C.E., EGGSPUEHLER J.J., GERKE R.J., CHASE R.C. (1968). Studies of pilot performance. 2:Evaluation of performance during low altitude flight in helicoptors. Ibid,39,19-31.

BIRMINGHAM H.P. (1959). The instantaneous measurement of human band-width. Naval res. lab, Washington D.C. Paper presented at the 8th Annual Conf. on Manual Control, Dunlap and Associates, Stamford, Conn. U.S.A.

BIRMINGHAM H.P., CHERNIKOFF R. and ZIEGLER P.N. (1962). The design and use of 'equalisation' teaching machines, Naval Res. Laboratories Washington D.C. Paper presented at the IRE International Congress on Human Factors in Electronics, Long Beach, Calif., U.S.A.

BLACKMAN R.B. (1965). Data smoothing and prediction. Addison Wesley.

BLACKMAN R.B. and TUKEY J.W. (1958).The measurement of power spectra from the point of view of communications engineering. Dover.

BOWEN H.M., HALE A. and KELLEY C.R. (1962). Tracking Training V.: Field study of the effectiveness of the general vehicular research tool, NAVTRADEVCAN TR 955-1.

BRIGGS G.E. (1961). On the scheduling of training conditions for the aquisition and transfer of perceptual-motor skills. Fort Washington:Naval Training Device Centre, Contract N61339-836 TechRep.

BRIGGS G.E. (1962). Research on augmented feedback and the aquisition and transfer of skill. NAVTRADEVCEN 836-3.

BROADBENT D.E. (1971). Decision and Stress.Academic Press, London.

BROWN B.P.,JOHNSON H.I. andMUNGALL R.G. (1960). Simulator motion effect on a pilots ability to perform a precise longitudinal flying task. NASA Tech. Rep. D-367 NASA Washington D.C.

BROWN J.L.,KUEHNEL H.,NICHOLSON F.T. and FUTTERWIET A. (1960). Comparison of tracking performance in the TV-2 aircraft and the ACL computer/AMAL Human centifuge simulator of this aircraft. Rep. NADC-MA-6016/NADC-AC-6008. U.S.Naval Air Dev. Cen., Pa.

BROWN W.K.,ROGGE J.D.,MEYER J.F.,BUCKLEY C.J.,BROWN C.A. (1969). Aeromedical aspects of the first non stop transatlantic helicoptor flight: 2 Heart rate and ECG changes. Aerospace Med., 40(7),714-717

BUCK L. (1968). Experiments on railway vigilance devices. Ergonomics, 11(6),557-564.

BUCKHOUT R. (1964). Effect of whole body vibration on human performance. Human Factors 6(2),153-157.

BUCKHOUT R., SHERMAN H., GOLDSMITH C.T. and VITALE P.A. (1963). The effect of variations in motion fidelity during training on simulated low altitude flight. USAF Aerospace Med. Res. Labs. AMRL-TDR-63-108.

CAIGER B. (1966). Some problems in control arising from operational experiences with jet transports. National Research Council,Canada. Report NAE Misc 41.

CARO P.R. (1969). Adaptive training - An application to flight simulation. Human Factors 11(6),569-576.

CATTERSON A.D., HOOVER G.D. and ASHE W.F. (1962). Human psychomotor performance during prolonged vertical vibration. Aerospace Med. 33(5),598-602.

CHANEY R.E. and PARKS D.L. (1964). Visual- motor performance during whole-body vibration. The Boeing Co., Wichita, Kansas. Human Factors Tech Report D3-3512-5.

CHANEY R.E. and PARKS D.L. (1064). Tracking performance during whole body vibration. The Boeing Co., Wichita, Kansas: Human Factors Tech. Report D3-3512-6.

CHERNIKOFF R. (1962). Human equalisation: Concept machine and measurement.Proceedings of ONR symposium on complex vehicular controls RAF Test Pilot School, Farnboruogh, England. also Douglas Aircraft

CHILES W.D. and CUSTER Carolyn L. (1963). Summaries of research on the human performance effects of vibration. USAF Aerospace Med. Res. Lab AMRL Memo P-60.

CHUANG K. and KAZDA F. (1959). A study of non-linear systems with random inputs. Amer. Inst. Elec. Engineers. 'Applications and Industry' No. 42. p.100.

CLARK C.C. (1962). Human control performance and tolerance under severe complex waveform vibration with a preliminary historical review of flight simulation. The Martin Co., Baltimore : Engineering Rep. ER 12406.

CLARKE N.P. et al. (1965). Preliminary study of dial reading performance during sustained acceleration and vibration. USAF Aerospace Med. Res. Lab. Tech. Rpt. AMRL-TR-65-110.

CLAUSEN J.T., CURTIN J.G. and EGLER J.F. (1968). Concept formulation report, synthetic training system Device 2B24. Orlando, Fla., Naval Training Device Center, Contract N61339-68-C-0108, Technical Report NAVTRADEVCEN-68-C-0108-1 (AD 673982).

CLEMENS D.E. (1964). The effect of vibration on certain psychomotor responses. Lobbock : Texas Tech. College, Central Library.

COBURN B. (1968). Some initial consideration on adaptive performance measures. Royal Aircraft Establishment, Farnborough, England Tech. Memo EP 403.

COERMANN P.R., MAGID E.B. and LANGE H.O. Human performance under vibrational stress. Human Factors 4, No45. 315-324.

CONNELLY E., SCHULER A. and KNOOP Patricia A. (1969). Study of adaptive mathematical models for deriving automated pilot performance measurement techniques. Wright-Patterson Air Force Base, Ohio. Air Force Human Resources Laboratory, Contract F331615-68-C-1278, Tech. Rpt. AFHRL-TR-69-7.

CONNELLY E.M. and SCHULER A.R. (1969). A theory of adaptive man-machine systems applied to automated training. International symposium on man-machine systems, Cambridge, England. IEEE conference record no. 69C58-mms.,

CRANDALL S.H., CAUGHEY T.K. and LYON R.H. (1963). Vibrations in non-linear structures. J. Acoustic Soc. of Amer. Vol. 35, No 11, p. 1683-1721.

CROOK M.N., HOFFMAN C. and WESSEL N.Y. (1947). Effect of vibration, illumination and type size on legibility of numerals. ATI-6672 USAG AMC., Wright Field Memo.

...tion on legibility of tabular

CROOK M.N. et al. (1948) Studies of the effect of typographical spacing on the legibility of numerals under vibration. USAF Air Material Command : Memo Rpt. MCREXD-694-IQ.

CROOK M.N. et al. (1949) Effect of vibration on legibility of tabular numerical material. Exps. 8 & 9 Rept. No, 5 USAF Air. Mat. Comm. Wright Patterson Air Base.

CROSSMANN E.R.F.W. (1960). The information capacity of the human motor system in pursuit tracking. Quart. J. Exp. Psychol. 19609 12, 1-16.

DEAN R.D., FARRELL R.J. and HITT J.D. (1969). Effect of vibration on the operation of decimal input devices. Human Factors 11(3) 257-271.

DEAN R.D., McGLOTHLEN C.L. and MONROE J.L. (1964). Performance and Physiological effects of CH-46A noise and vibration. The Boeing Co., Seattle. Doc. D2-90583.

DEAN R.D., McGLOTHLEN L. and MONROE J.L. (1965). Performance and physiological effects of 6 hour exposure to helicopter noise and vibration. Paper to 36th Ann. meeting Aerospace Med. Assoc.

DENNIS J.P. (1963). A survey of experimental data on the effects of whole body vibration and of vibration of the visual object upon visual performance. OCC. Psych. 37,4, 277-282.

DENNIS J.P. (1963). Some comparisons between the effects of the whole body vibration and those of vibrating the visual object upon visual performance. Unpub. thesis Univ. Bristol Library.

DENNIS J.P. (1960). The effect of whole body vibration on a visual performance task. Clothing and Equipment Physiological Research Estab. Farnborough. Rpt. R.: CEPRE 104.

DENNIS J.P. (1965). Some effects of vibration upon visual performance. J. Appl. psycho. 49, 245-252.

DENNIS J.P. (1965). The effect of whole body vibration on a visual performance task. Ergonomics 8, 193-205.

DEUTSCH R. (1966). Non-linear Transofmrations of random processes. Prentice Hall Applied Mathematics Series. Prentice-Hall International Inc.

DOOB J.L. (1953). Stochastic processes. New York. Wiley.

DRAZIN D.H. (1961). Effects of vibration on dial legibility. Paper read at B. Ass. Advancement of Science meeting at Norwich.

DRINKWATER B.L. (1967). Performance of Civil Aviation Pilots under conditions of sensory input overload. Aerospace Med. 38, 164-168.

DUDEK R.A. and CLEMENS D.E. (1965). Effect of vibration on certain psychomotor responses. J. Eng. Psychology, 4, 126-143.

ELLIS N.C., LOWES A.L. MATHENY W.G. and NORMAN D.A. (1971). The feasibility of using an adaptive technique in flight simulator training. Ergonomics Vol. 14, No. 3 381-390.

EMA T, TSYKAHARA T. (1968). Human responses and controllability limits as measured from fixed base flight simulator tests. Memoirs of the Defence Academy, Japan VII (4), 1627-1644.

FACONTI V., MORTIMER C.P.L. and SIMPSON D.W. (1970). Automated instruction and performance monitoring in flight simulator training. Wright-Patterson Air Force, Ohio : Air Force Human Resources Laboratory, Contract F33615-69-C-1159. Tech. Rpt. AFHRL-TR-69-29.

FENWICK and FENTON (1971). On the use of autoregression to determine the impulse response function of the brain from an analysis of the background EEG. Paper presented to the Quantification group of the EEG Society and to a meeting on Biological Signal Processing. Univ. College, London/IEE.

FLEXMAN R.E., JAMESON. W.P., WALSH and COHEN E. (1968). Synthetic flight training system (SFTS) concept forulation report. Orlando Fla.: Naval Training Center, TR NAVTRADEVCEN 1948.

FOKKER A. (1913). Dissertation, Leiden.

FORBES, A.R. (1960). The effects of whole body vibration on compensatory tracking performance. RAF Institute of Aviation Medicine, Farnborough, Rpt, 144.

FRASER T.M. (1964) Aspects of the Human response to High Speed Low Level Flight. Aerospace Med. 35(4), 365-370.

FRASER T.M., HOOVER G.N. and ASHE W.F. (1961). Tracking performance
during low frequency vibration. Aerospace Med. 32(9), 829-835.

GAINES B.R. (1965). Automated feedback-trainers for perceptual-motor
skills. Tech. Rpt 1 (1966(1)). The evaluation and control of human
skill. IRE Conf. proc., Colloquim on Human Operator Studies.

GAINES B.R. (1967). Teaching machines for perceptual-motor skills. In
D. Unwin and J. Leedham (Eds). Aspects of educational technology :
The proceedings of the programmed learning conference, Methuen,
London, England 337-358.

GAINES B.R. and ANDREAE J.H. (1966). A learning machine in the context
Proc. 3rd Int. Cong.
of the general control problem. ... trainers for perceptual-motor

GARNER W.R. (1962). Uncertainty and structure as psychological concepts. New York, Wiley.

GRETHER W.F. (1971). Vibration and human performance. Human Factors 13(3) 203-216.

GUIGNARD J.C. (1959). An effect of whole body vibration on vision. Royal Air Force Inst. Av. Med., Farnborough. Scientific Memo. 521.

GUILFORD J.P. (1965). Fundamental statistics in psychology and education New York; McGraw Hill.

HAAR A. (1944). "Zur theorie der orthogonalen funktionen-systems". Inaugral dissertation, Maths. Annals. Vol 69, 146-152.

HARRIS C.S., CHILES W.D. and TOUCHSTONE R.M. (1964). Human performance as a function of intensity of vibration at 5c/s. Wright-Patterson AFB : 6570th. AMRL Rpt. AMRL-TDR-64-83. Project 1710, Task 171002, AD - 607,760.

HARRIS C.S. and SHOENBERGER R.W. (1964). Human performance during vibration. Office of Naval Res. Eng. Psycho. branch AD 602813,38-47.

HARRIS C.S. and SHOENBERGER R.W. (1965). Human performance during vibration. Wright-Patterson AFB: Aerospace M.R. Labs. AMRL-TR-65-204 Project AF-1710. Task 171002, AD-624,196.

HARRIS C.S. and SHOENBERGER R.W. (1965). The effects of vibration on human performance. Paper 23 to 22nd. mtg. AGARD Space Med. Panel, Munich, Sept 1965. In AGARD Collected papers, No 2 307-326. Paris, NATO/AGARD.

HARRIS C.S. and SHOENBERGER R.W. (1966). Effects of frequency of vibration on human performance. J. Eng. Psycho. 5, 1-15, also identified as AMRL-TR-66-152.

HARWOOD K.G. and LOVESEY E.J. (1966). Preliminary tests on the effects of low frequency whole body vibration on aircrew performance, RAE, Farnborough, Tech. Memo 363.

HARWOOD K.G. et al (1968). The effects of low frequency vibration on pilot performance in a simulated supersonic transport aircraft take-off and flight. RAE., Farnborough Tech. Memo EP 392.

HARWOOD K.G. and LOVESEY E.J. (1966). An attempt to assess the effects of helmet weight on navigators efficiency in a simulated terrain following sortie. RAE., Farnborough, Tech. Memo ME 361.

HOLLAND C.L. (1967). Performance effects of long-term random vertical vibration. Human Factors 9, 93-103.

HORNICK R.J. (1961). The relative effects of noise and vibration upon
... Labs., Milwaukee Rpt 132.

HORNICK R.J. et al (1961) The effects of low frequency high amplitude, whole body, longitudinal and transverse vibration upon Human performance Bostrom Research Labs., Milwaukee Final Rpt. (ordance project TEI-1000) AD 630012.

HORNICK R.J. (1962). Effects of whole body vibration in three directions upon human performance. J. Eng. Psycho. 1, 3, 93-107.

HORNICK R.J. and LEFRITZ N.M. (1966). A study and review of human response to prolonged random vibration. Human Factors 8, 481-492.

HOLDINGS D.H. (1962). Transfer between difficult and easy tasks. Brit. J Psychol. 53(4) 397-407.

HUDDLESTON H.F. (1964). Human performance and behaviour in vertical sinusoidal vibration RAF. Inst. Av. Med. Rpt R303.

HUDSON E.M. (1962) An adaptive tracking simulator. Otis Elevator Co., Brooklyn N.Y. Paper presented at the IRE First International Congress on Human Factors in Electronics. Long Beach, Calif.

HUDSON E.M. (1964). Adaptive training and non-verbal behaviour. Fort Washington N.Y. : Naval Training Device Center, Contract N61339-1395. Tech. Rpt. 1395-1.

HUDSON E.M. (1967). Adaptive training device. U.S. Patent 3311995. In U.S. Commerce Department Patent Office, Official Gazette, Washington D.C. Government Printing Office, p 41.

HUDSON E.M. (1969). Adaptive techniques in multiparameter problems. Human Factors 11, 561-568.

HURT G.J. (1963). Rough air effects on crew performance during a simulated low-altitude high speed surveillance mission. NASA Tech. Note D-1924.

JAMESON W.P., WALSH J.M., FLEXMAN R.E. and COHEN E (1968). Synthetic flight training system (SFTS). Concept formulation report. Orlando Fla., Naval Training Device Center, Contract N61339-68-C-0106. Tech Rpt. NAVTRADENCEN 68-C-0106-1.

JEX H.R. McDONNELL J.D. and PHATAK A.V. (1966). A "critical" tracking task for manual control research. IEEE Transactions on Human Factors in Electronics HFE-7(4), 138-145 Also NASA CR616.

JOHNSTON W.L. (1972). Body orientation under vertical sinusoidal vibration. Human Factors 14 (4) 349-356.

KAEHLER R.C. (1959). Human Pilot performance during boost and atmosphere re-entry. Aerospace Med. 30, 491-486.

KAMINSKY G. (1965). Realistic investigations of the effects of vibrations of the hand-arm-body system on manual dexterity. Proc. 2nd. Int. Congress on Ergonomics. Taylor and Francis, London.

KAPPAUF W.E. (1969) Use of an on-line computer for psychophysical testing with the up-and-down methods. American Psychology. 24(3) 207-211.

KELLEY C.R. (1950). The critical flicker frequency in peripheral vision Colombus, Ohio : Ohio State University, unpublished M.A. thesis.

KELLEY C.R., BOWEN, H.M., ELY J.H. and ANDREAS J.L. (1960). Tracking training : III Transfer of training, Fort Washington N.Y. : U.S. Naval Training Device Center Tech. Rpt. 1908-00-3.

KELLEY C.R. (1961). A study of performance decrement in simulated complex task situations and simulated environments. Appendix B : Description of electronic display techniques. Stamford, Conn., : Public Service Research In., Rpt. to U.S. Department of Health, Education and Welfare.

KELLEY C.R. (1961). Self-adjusting vehicle simulators. In C.R. Kelley (Ed). The predictor instrument :Final report and summary of project activities during 1961. Arlington, V.A. : Office of Naval Research. Contract No. 2822(00). Tech. Rpt. Jan. 1962. Also in C.R. Kelley (Ed) Adaptive simulation. Arlington Va., Office of Naval Research Contract No. 4986(00). Final Rpt. August 1966 (AD-637658).

KELLEY C.R. (1965). Testing the effects of extended space missions on piloting skill. In E. Burgess (Ed) AAS Science and Technology series Vol. 5 P. Horowitz (Ed). Physiological and performance determinants of manual space system design. Baltimore Md. American Astronautical Society 167-186.

KELLEY C.R., MITCHELL M.B. and STRUDWICK P.H. (1965). The MOL tracking task : Development of a test of tracking skill for inclusion in the general performance battery of the Air Force Manned Orbital Lab., Prelin., Tech., Rpt.Santa Monica, Calif., Dunlap and Associates Inc.,

KELLEY C.R. (1966), Design applications of adaptive (self-adjusting) simulators. Second Annual NASA Conference on Manual Control, Cambridge, Mass. Feb 28 - March 2. Washington D.C.: Scientific and Technical Information Division, National Aeronautic and Space Administration NASA SP-128.

KELLEY C.R. (1966) Adaptive simulation (Office of Naval Research. Tech. ... Calif., : Dunlap and Associates. Inc., KELLEY C.R., WARGO M.J., MITCHELL M.B. and PROSIN D.J. (1966). In-space measurement of piloting skill : The NOL tracking task, Santa Monica, Calif : Douglas Aircraft Co.,

KELLEY C.R. and WARGO M.J. (1967). Cross-adaptive operator loading tasks Human Factors 9(5) 395-404. Also in Adaptive Simulation Santa Monica Calif : Dunlap and Associates Inc.

KELLEY C.R. and PROSIN D.J. (1967). Frequency adaptive tracking systems In Further Research with Adaptive tasks. Santa Monica, Calif : Dunlap and Associates Inc. Final rpt. to Office of Naval Research

KELLEY C.R. (1967). Further research with adaptive tasks. Arlington, Va: Office of Naval Research, Contract No. 4986(00). Final Rpt. AD-657-343.

KELLEY C.R. (1967). The measurement of tracking proficiency. In Further Research with Adaptive tasks. Santa Monica, Calif. : Dunlap and Associates Inc. Final rpt. to Office of Naval Research.

KELLEY C.R. (1967). Research applications of adaptive tasks. In Further Research with Adaptive tasks. Santa Monica, Calif.: Dunlap and Associates. Final rpt. Office of Naval Research. Also presented as a paper at the International Ergonomics Conference, Birmingham, England. Sept 1967.

KELLEY C.R. and PROSIN D.J. (1968). Adaptive performance measurement. Santa Monica, Calif. : Dunlap and Associates Inc., Tech Rpt. to Office of Naval Research.

KELLEY C.R. and WARGO M.J. (1968). Adaptive techniques for synthetic flight training systems (U.S. Naval Training Device Center, Tech, rpt. NAVTRADEVCEN 68-C-0136-1) Santa Monica Calif: Dunlap and Assoc. Inc.

KELLEY C.R. and PROSIN D.J. (1969). Adaptive techniques in measuring complex performance. Arlington, Va. Office of Naval Research Contract No 4986(00). Final rpt. AD-694-509.

KELLEY C.R. (1968). Engineering models of the human operator. Chap 13 Manual and Automotive Control. New York : John Wiley and Sons.

KELLEY C.R. (1969). What is adaptive training ? Human Factors 11(6) 547-556.

KELLEY C.R. (1969). The measurement of tracking proficiency. Human Factors 11(1) 43-64.

KELLEY C.R. (1969). Adaptive and automated research techniques from engineering psychology. American Psychologist. 24, 293-297.

KELLEY C.R. and KELLEY Erica J (1970). A manual for adaptive techniques Arlington Va.: Office of Naval Research, Contract No 4986(00). Final rpt. AD-711-985

KHINCHINE A (1934). Korrelations theorie der stationaren stochastischen. prozesse. Maths. Annals. 109. p.604-605.

KNOOP Patricia A (1966). Programming techniques for the automatic monitoring of human performance. Wright-Patterson Air Force Base, Ohio. : Aerospace Research Lab. Project 6114, Task 611412, Tech rpt. AMRL -TR-66-16.

KNOOP Patricia A (1967). Automated aids forflight simulator instructors USAF Instructor's Journal 4(4) 61-64.

KNOOP Patricia A. (1968). Development and evaluation of a digital computer program for automatic human performance monitoring in flight simulator training. Wright-Patterson Air Force Base, Ohio. Aerospace Medical Research Lab. Project 6114. Task 611412 Tech. rpt. AMRL-TR-67-97.

KNOWLES W.B. (1963). Operator loading tasks. Human Factors 5, 155-161.

KOPSTEIN F.F. and SEIDEL R.J. (1970) The computer as an adaptive instructional decision maker. Alexandria, va: Human Resources Research Organisation. Contract DA 11C 19-70-C-0012. HUMRRO Professional Paper 1-70 AD 703-597.

KOTTENHOFF H. and HOFFMAN H (1962). Performance and emotional behaviour during vibration stress (in German). Acta. Psychol. 20(4) 320-356

LANGE K.O. and COERMANN R.R. (1962). Visual acuity under vibration. Human Factors 4, 5, 291-300.

LARUE M.A. (1965). The effects of vibration on accuracy of a positioning task. J. Environ. science 8 (4) 33-35.

LEE Y.W. (1960). Statistical theory of communication. Wiley.

LEVITT H. (1970). Transformed up-down methods in psychophysics. J. Acou. Soc. Amer. Vol. 49 No. 2 part 2 p. 467-474.

LEWIS D (1943). The effect of noise and vibration on certain psychomotor responses. Res. at State Univ. Iowa by Civil Aeronautics Administration PB 50188. Res. Rep. 8, 107.

LEWIS R.E.F. (1961). Pilot performance during low speed, low level navigations. Defence Research Medical Labs, Canada : Report 248-1.

LEWIS B.N. and PASK G (1965). The theory and practice of adaptive teaching systems. In R. Glaser (Ed). Teaching Machines and programmed [illegible] D.C.: National

LEOB M. (1955). A further investigation of the influence of whole body vibration and noise on tremor and visual acuity. U.S. Army Med. Res. Lab. Fort Knox : Rpt 165.

LOWES A.L., ELLIS N.C., NORMAN D.A. and MATHENY W.G. (1968). Improving piloting skills in turbulent air using a self-adaptive technique for a digital operational flight trainer. Orlando, Fla. : Naval Training Device Center. Contract N61339-67-C-0034. Tech. Rpt. NAVTRADEVCEN 67-C-0034-2 (AD-675-805).

LOVESEY E.J. (1968). The influence of a restraining harness upon human comfort and tracking performance under single and multi-axis heave and sway vibration. RAE., Farnborough Tech. Memo. EP 397.

LOVESEY E.J. (1970). The multi-axis vibration environment and man. Appl. Ergonomics Vol. 1 No. 5. p. 134-139.

MAGID E.B. et at (1962). Physiological and mechanical responses of the human to longitudinal whole-body vibration as determined by subjective responses. MRL-TDR-62-66.

MAGID E.B. et al (1960). Human tolerance to whole body sinusoidal vibration. Aerospace Med., 31(11) 915-924.

MAGID E.B. and COERMANN R.R. (1960). The reaction of the human body to extreme vibrations. Paper to Inst. Environ. Sciences Conf. on Hyper-environments Space Frontier, Los Angeles.

MATHENY W.G. (1969). The effect time constant - a new techniqe for adaptive training. Human Factors 11, 557-560.

MATHENY W.G. and NORMAN D.A. (1968). The effective time constant in tracking behaviour. Orlando, Fla : Naval Training Device Center Contract N61339-67-C-0034. Tech Rpt. NAVTRADEVCEN 67-C-0034-3.

MELCHING W.H. (1968). A concept of the role of man in automated systems. Alexandria, Va., : Human Resources Research Office. Professional paper 16-68.

McGRATH J.J. and HARRIS D.H. (1971). Eds. Adaptive Training. Aviation Research Monographs. Vol. 1, No. 2. Aviation Research Laboratory Inst. of Aviation, University of Illinois at Urbana, Champain, USA.

McKENZIE J.M., FIORICA V. (1967). Stress responses or pilots to severe weather flying. Aerospace Med. 38, 576-590.

McKAY D. (1950). The quantal aspects of scientific information. Phil. Mag. 41, p 289-311.

McRUER D and WEIR D.H. (1969). Theory of manual vehicular control. Ergonomics Vol. 12 No. 4 p599-633.

MILSUM J.H. (1966). Biological control systems analysis. Electronic Sciences series. McGraw-Hill Book Company.

MIRABELLA A and LAMB J.C. (1966). Computer based adaptive training applied to symbolic displays. Perceptual and Motor Skills 23,647-661

MOZELL M.M. and WHITE D.C. (1958). Behavioural effects of whole body vibration. J. Av. Med. Vol 29 NADC-MA-5802.

O'BRIANT C.R. and OHLBAUM M.K. (1969). Visual acuity decrements associated with whole body $\pm$ g_3 vibration stress. USAF Aerospace Med. Res. Lab. Tech. Rpt. AMRL-TR-69-90 also in Aerospace Med 41(1) 79-82.

OLIVER R.E. and WIN T.Y. (1958). Sled track interaction and a rapid method for track alignment measurement. Aeronautical Engineering Research Inc. Rep. 114. part 2.

PARKS D.L. (1961). A comparison of sinusoidal and random vibration effects on numan performance. The Boeing Company Wichita, Kansas : Doc. D3-3512-2.

PARKS D.L. and HICKEY L.F. (1965). Simulation test of the effects of extended exposure to low altitude flight turbulance on flight crew operating with continuous heavy work loads. Part 11 Boeing doc DC/2137/ITN.

PASK G. (1960). The teaching machine as a control mechanism. Trans. Soc. Instr. Tech. 12 (2) 72-82.

PASK G. (1960). Adaptive teaching with adaptive machines. In A.A. Lumsdaine and R. Glaser (Eds) Teaching machines and programmed learning. A source book. Washington D.C. National Education Association 349-366.

PASK G (1960). Electronic keyboard teaching machines. In A.A, Lumsdaine and R. Glaser (Eds) Teaching machines and programmed learning. A source book. Washington D.C. : National Education Association. 336-348.

PASK G. (1961). The cybernetics of evolutionary processes and of self organising systems. System Research Ltd., Richmond, Surrey, England. Paper delivered at the 3rd Congress of the International Assoc. of Cybernetics. Namur, Belgium.

PASK G. and Von FOERSTER H. (1961). A predictive model for self-organising systems. Cybernetics 4, 20-55.

PASK G. (1962) The Logic and behaviour of self-organising systems, as ... adaptive machines

PASK G and LEWIS B.N. (1962). An adaptive automation for teaching small groups. Perceptual and Motor Skills 14, 183-188.

PASK G. (1965). Teaching as a control engineering process. Control and automation progress. Jan- April 1965. 9(79) 11-82).

PASK G (1967). The adaptively controlled instruction of a transformational skill, Programmed Learning 74-86.

PEARSON K. (1894). Contributions to the mathematical theory of evolution. Phil. Trans. Roy Soc. A. Vol. 185, p 71.

PEARSON K. (1901). On lines and planes of closest fit to systems of points in space. Phil. mag. series 6, vol. 2, p 559.

PEARSON K. (1901). On the systemmatic fitting of curves to observations and measurements. Biometrika vol. 1, p. 265 and vol 2, p. 1.

PLANK M (1927). Berl. Ber. p. 324.

POULTON E.G. (1965). On increasing the sensitivity of measures of performance. Ergonomics Vol. 8 No. 1 p. 69.

PROVINS K.A. (1966). Environmental heat, body temperature and behaviour an hypothesis. Australian J, Psychol. 18, 118-129.

RAO C.R. (1964). The use and interpretation of principal component analyses in applied research. Sankhya, India Journal Stat. Ser. A 26. P. 329-359.

RATH G. (1967). The development of computer-assisted instruction. In IEEE Transactions on Human Factors in Electronics, 8 (2) 60-64

REGAN J.J. (1969). (Ed) Adaptive Training - special issue of Human Factors 11, 545.

RICE S.O. (1944). Mathematical analysis of random noise. Bell System Tech. J. 23, 282-332 (1944); 23, 46-156 (1955).

RISER G.T. (1961). Exploratory Investigation of the effects of low frequency random amplitude vibration on human performance. Boeing Co., Kansas. Doc. D3-3328.

RUBENSTEIN L. et al (1967). Some effects of Y-axis vibration on visual acuity. NASA-CR-96466.

SADOFF M, BRAY R.S., ANDREWS W.H. (1966). Summary of NASA Research on jet transport control problems in severe turbulence. J. Aircraft 3, 193-200.

SADOFF M. DOLKAS C.B. (1967). Acceleration stress effects on pilot performance and dynamic response. IEEE trans human factors in electronics. HFE8(2) 103-112.

SCHMITZ M.A. (1958). The effect of low-frequency, high amplitude whole- ... Boston Res. Lab Proj. 128.

SCHOHAN B., RAWSON H.E. and SOLIDAY S.M. (1965). Pilot and observer performance in a simulated low altitude high speed flight. Human Factors 7(3) 257-265.

SENDERS J.W. (1961). Adaptive teaching machines in programmed learning and computer based instruction. Wiley

SHANNON C.E. and WEAVER W. (1949). The mathematical theory of communication. Univ. of Illinois Press.

SHOENBERGER R.W. (1967). Effects of vibration on complex psychomotor performance. Aerospace Med. 38, 1264-1269, also USAF AMRL-TR-67-40.

SHURMER C.R. (1967). A review of the effects of low frequency vibration on man and on his tracking performance. BAC (operating) Ltd., Bristol Human Factors Study note series 4 no. 7.

SCHURMER C.R. and SILVERTHRON D.G. (1967). Effects on tracking performance of vibration in each and in combination of the heave, sway and roll axes. BAC (operating) Ltd., Bristol. Human Factors Study note series 4.12.

SIMONS A.K. and SCHMITZ M.A. (1958). The effects of low frequency high amplitude vibration on human performance Rep. 1 Contract DA-49-007-MD-797. Bostrom Res. Lab., Milwaukee.

SIMONS A.K. and HORNICK R.J. (1961). Effects of the vibration environment in mobile systems on human performance. Bostrom Res. Labs. Milwaukee Rpt 137.

SINGLETON W.T. (1970). The measurement ofman at work wity particular reference to arousal. In Singleton W.T., Fox J.G and Whitfield D. (Eds). The measurement of man at work. Taylor and Francis, London.

SMITH O.J.M. (1962). Non-linear computations in the human controller. IRE Trans. Bio-Med. Electron. BME-9(2).

SMOLUCHOWSKI M.V. (1916) Physics, Zeits, 17,557.

SNYDER F.W. (1962). Effects of low frequency vertical vibration on human performance. Paper read at Aerospace. Med. Ass. meeting.

SNYDER F.W. (1965). Vibration and vision. In C.A. Baker (Ed). Visual capabilities in the space environment. Pergamon Press 183-201.

SOLIDAY M and SCHOHAN B. (1964). A simulator investigation of pilot performance during periods of low altitude, high speed flight. NASA CR-63.

SOLIDAY S.M. and SCHOHAN B. (1965). Task loading of pilots in simulated low altitude, high speed flight. Human Factors 7(1) 1, 45-53.

SPIEGEL M.R. (1961). Theory and problems of statistics. Schaum's Outline Series. Schaum Publishing Co., New York.

STEVENS S.S. (1941). Effects of noise and vibration in psychomotor efficiency. Harvard Univ. ATI-274-27428.

STEVENS S.S. (1941). Effect of vibration on visual acuity. In Waring C.E. Transmission and reception of rounds under combat conditions.

STEWART W.A. (1970). Rand symposium on pilot training and the pilot career : Recollections of the chairman. Santa Monica, Calif. : Rand Corporation. Report RM-6282-1-PR.

STOLUROW L.M. (1968). Some factors in the design of systems for computer assisted instructions. Arlington, Va. : Office of Naval Research Contract No. N00014-67-A-0298-0003. Report 7.

TAUB H.A. (1964). The effects of vibration on dial reading performance. USAF Aerospace Med. Res. Labs. Tech. Doc. Rpt. AMRL-TDR-64-70.

TAYLOR M. (1969) On "Pest and experiments using adaptive procedures." NATO Advanced Studies Institute in on-line computing in the behavioural sciences. University of Sheffield, England.

TEARE R.J. and PARKS D.L. (1963). Visual performance during whole body vibration Boeing Co., Wichita, Kansas D3-3512-4.

TEMPLE W.E. et al (1964). Man's short time tolerance to sinusoidal vibration. Aerospace Med 35 (10) 923-930.

TINKER M.A. (1948). Effects of vibration upon reading Am. J. Psycho. 61, 386-390.

TORLE G (1965). Tracking performance under acceleration, - effects of control dynamics. Ergonomics 8 (4) 481-486.

WALSH J.M. and FLEXMAN R.E. (1970) Preliminary adaptive training design report for synthetic flight training system. Device 2B24, Orlando, Fla. : Naval Training Device Center. Contract N61339-69-C-0200. Tech Rpt 69-C-0200-15.

WALSH J.L. (1923). A closed set of normal orthogonal functions. An. J. Math., Vol. 45,5-24.

WALTER W.G. (1959). Intrinsic rhythms of the brain. In Handbook of Physiology Sec 1, Neurophysiology Vol 1. Magoun, ed., ch. 11. Am. Phy. Soc Washington.

WANG. M.C. and UHLENBECK G.E. (1945). On the theory of Browian motion. I and II. Reviews of Modern Physics. Vol 17, no. 2 and 3.

WEATHERBURN C.E. (1961). A first course in mathematical statistics. Univ. Press.Cambridge.

WEINDER N. (1930). Generalised harmonic analysis. Acta Mathematica 55, p.117-258

... integral and

WEINER N. (1948). Cybernetics. M.I.T. Press. Cambridge, U.S.A.

WEINER N. (1949). Extrapolation, interpolation and smoothing of stationary time series with engineering applications. J. Wiley. New York.

WEINER N (1958). Non-linear problems in random theory. The Technology press. M.I.T. and Wiley, New York.

WEISZ A.Z., ALLEN R.W. and GODDARD C.J. (1965). Human performance under random and sinusoidal vibration. USAF Aerospace Med. Res. Labs. Tech. Rpt AMRL-TR-65-209.

WEISZ A.Z., ALLEN R.W. and GODDARD C.J. (1966). An evaluation of three types of hand controllers under random vertical vibration. NASA SP-128

WHECHEL J.E. and GUINN D.F. (1968). The fast fourier-hadanard transform and its use in signal representation and classification. Eascon. 1968, Record 561-573.

WHITEHEAD G.D. (1968 Human Factor Problems in the Design of Vibration Ref. 1 Isolation Systems A.P. Report 21. Applied Psychology Dept., University of Aston in Birmingham, Gosta Green, Birmingham 4.

WHITEHEAD G.D. (1967) The Description of Continuous Random Variables. Ref 2 A.P. Report 11. Applied Psychology Dept., University of Aston in Birmingham, Gosta Green, Birmingham 4. Also a series of articles with the same title in automotive design engineering, 1967.

WHITEHEAD G.D. (1965) The Application of Statistics to the Motor Vehicle Ride Comford Problem, in Advances in Automobile Engineering, Part II edited by G.H. Tidbury, Pergamon Press.

WHITEHEAD G.D. (1965). A review of data analysis systems, Advanced School of Automobile Engineering, Cranfield, Bedford, A.S.A.E. Memo No 76.

WHITEHEAD G.D. (1968). System Ergonomics - the Human Factor in System Design. Paper presented at the Symposium, Industry, Environment Design, Helsinki, Finland.

WILDE R.W. and WESTCOTT J.H. (1963). The characterisation of the human operator engaged in a tracking task. Automica 1(1).

WYON D (1970). Studies of Children under imposed noise and heat stress. Ergonomics 13, 598-612.

YOUNG J.W., BARKER L.E. (1963). Moving cockpit simulator study of piloted entries into the earth's atmosphere for a capsule type vehicle at parabolic velocity. NASA Tech Note D-1797.

YOUNG L.R. and WEIRY J.L. (1965). Manual control of an unstable system with visual and motion cues. Dept. Aeronautics and Astronautics Dept. Aeronautics and Astronautics, M.I.T. Cambridge, U.S.A.

YOUNG L.R. (1969). On Adaptive Manual Control. Ergonomics Vol. 12 no.4.

WOOD (See McGrath and Harris (1971)).