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Sustainability in the evaluation of bus rapid transportation projects considering both managers and passengers perspectives: a triple-level efficiency evaluation approach

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Abstract

The significant positive and negatives effects of transportation systems (TSs) on the sustainability of cities and human life draw much attention from both researchers and managers. Constructing bus rapid transit (BRT) networks, or adding new lines to the existing ones, is one of the cheapest and easiest solutions to improve the performance of the urban transportation network (UTN). Often, large number of candidate projects (BRT lines) renders the execution of all these projects impossible due to technical and financial limitations. Hence, evaluating the candidate projects and developing the best plan for constructing a BRT network is an important issue requiring a complex decision-making process. In this study, a multi-period triple-level sustainable BRT network design model has been proposed using data envelopment analysis, game theory, Malmquist Index (MI) and considering all sustainability dimensions including environment, economic and society. Both managers' and passengers' perspectives have been considered in the modeling. A procedure based on a genetic algorithm (GA) has been developed to solve the presented triple-level model. Finally, the model has been applied to a real-world case study of evaluating and selecting the BRT projects in the city of Isfahan, and the results have been analyzed.

Keywords: Data envelopment analysis; Game theory; Transportation; Sustainability; Bus rapid transit.

1. Introduction

Urban Transportation Systems (TS) are one of the most important socio-economic systems. An inefficient urban TS will increase the levels of energy consumption, travel time, travel costs, traffic congestion, air pollution, etc., and can even lead to life-threatening situations for passengers and drivers (Mahmoudi et al., 2019c). Recently, introducing sustainability and sustainable development issues has drawn the attention of policymakers and researchers to evaluate the performance of urban TSs (Hahn et al., 2017). The objectives of such assessments include identifying the weaknesses of the existing systems and proposing solutions for the improvement of their performance, thereby achieving a sustainable situation. The proposed short-term, mid-term and long-term solutions entail either traffic regulatory projects or else operational and infrastructural projects.

Expanding public urban TSs in most developed and developing cities is one of the high priorities of transportation targets. For a number of years, urban public bus TSs, in particular bus rapid transit (BRT) networks, have attracted the attention of policymakers and local governments. This could be attributed both to the lower construction costs and the shorter construction period. Currently, an approximate number of 33.5 million passengers in more than 169 cities in the world enjoy BRT services each day, where the total of covered length is more than 5020 Km (GBD, 2018).

Implementing a public TS, in particular BRT routes, strongly depends on the culture and political backgrounds, as well as the technical, geographical and financial limitations in the related city or region (Filipe and Macário, 2014). Appling any policy or constructing any project such as creating a BRT network or adding a BRT line to an existing network could have significant effects on the performance of the affected TS. These effects can be short- or long-term and positive or negative in nature. Hence, considering the crucial role of public transportation systems in the social, economic and environmental dimensions of human life, each candidate policy or project must be fundamentally analyzed prior to execution in the existing TSs.

The problem of evaluating all possible candidate policies/projects in a TS and selecting the most efficient one is known as the network design problem (NDP). In particular, the BRT network design problem (BRTNDP) is a challenging subject, whether it is the first-time creation of an urban BRT network or developing an existing BRT network by adding new lines or applying new regulations. There are often a considerable number of candidate projects or policies related to a BRT network in a city. Based on the technical, financial and other limitations, applying or constructing all of the candidates is impossible. On the other hand, the plurality of beneficiaries (users, managers, local governments, etc.), objectives and evaluation criteria increases the complexity of the decision making process. Therefore, researchers have proposed different methods to study BRTNDP. To maximize the total population covered, Laporte et al. (2002) developed a heuristic method to find the optimal location of BRT stations. Fan and Machemehl (2011) considered the spatial equity as the objective function in their proposed bi-level model for public transportation NDP. Considering the infrastructure costs, an integrated reliability approach was presented by Caicedo et al. (2012) to design a BRT system. Mechanical performance of asphalt, life-cycle of existing or new pavements, size of buses, frequency of the routes and passenger demands are considered in their study. Kermanshahi et al. (2015) proposed a mixed-integer mathematical model and developed a solution approach to survey BRTNDP, maximizing the total population coverage as the objective function. Considering various evaluation criteria, An et al. (2008) presented different single objective bi-level models for the BRT route design problem, followed by a hybrid heuristic algorithm to solve the presented models. Walteros et al. (2013) developed a mixed-integer linear (MIP) model and a hybrid genetic algorithm to solve the route design problems and frequencies. The total sum of passenger and operation costs was considered as the objective function. Based on a set of predefined corridors and a finite number of candidate routes, Schmid (2014) proposed a large neighborhood search and a linear programming hybrid algorithm to study BRTNDP, where the total travel time was considered as the objective function. Scheduling new infrastructure projects related to BRT systems in the existing urban transportation network was analyzed by Mulley and Tsai (2016). Using a multi-level model considering property, neighborhood, and accessibility as the evaluation criteria, they surveyed how new infrastructures affect land values and housing prices. The effects of BRT projects on land use and land price was also studied by Rodríguez and Mojica (2009), where a before and after hedonic model was used.

Recently, many researchers have used Data Envelopment Analysis (DEA) (Emrouznejad and Yang, 2018) as a non-parametric mathematical method to survey different issues related to transportation systems (Chen et al., 2019; Cui and Li, 2018; Su and Rogers, 2012). As shown by Mahmoudi et al. (2019b), transportation is the fourth major application in DEA literature, but the number of studies that have applied DEA for BRT systems, even in recent years, is significantly low. Sheth et al. (2007) evaluated the performance of bus routes in the urban public transportation networks from users' and providers' perspectives, using a network DEA model. Garcia Sanchez (2009) used DEA to survey the scale and technical efficiency of urban bus transportation systems in 24 Spanish cities. Caulfield et al. (2013) applied DEA to analyze different candidate public transportation projects in Dublin, Ireland, including the Dublin area rapid transit, a BRT route, a metro line, and a tram line. The results showed the BRT to be a superior option. Xin et al. (2014) combined DEA with other mathematical models and proposed a triple-level approach to evaluate the efficiency of multi-modal urban public TSs. The understudy network was served by BRT, conventional public bus, tram, and subway. Using game theory, DEA and considering operational and spatial inputs, Rezaee et al. (2016) presented an approach for evaluating the performance of urban public bus transportation lines. Kathuria et al. (2017) used the super efficiency DEA model to evaluate 12 routes in the BRT system of Ahmedabad, India. They studied the efficiency of a set of routes in five aspects, including comfort and safety efficiency, route design efficiency, schedule design efficiency, service, and delivery efficiency, and cost efficiency. Finally, based on obtained efficiency scores and using the analytical hierarchy process, they ranked the routes. Merkert et al. (2017) applied a bootstrap DEA model to analyze the performance and revenue of 58 BRT systems around the world. Their results indicated that BRT systems in developing countries are more successful with respect to revenue generation.

To develop the best constructing plan, a multi-period triple-level model is presented in this paper, which serves to evaluate the candidate BRT projects by using DEA, game theory and Malmquist Index (MI). Not only have all sustainability dimensions been included in the proposed model, but also both the managers' and passengers' perspectives have been considered. Based on the structure of the problem, the lower level model is required to assign the traffic flow to the network while the second level will analyze the changes in the performance of the network according to a selected scenario during the time. In addition, due to technical and financial constraints and finally the objective function value for making the final decision, a set of possible solutions will be obtained by the upper-level model. Hence, having three types of decision/analysis structure including traffic assignment, performance analysis during the time and feasibility analysis makes a need for a triple-level structure model. Table 1 shows the main features of this study and its superiority to the previous research. To the best of authors' knowledge, this is the first study using DEA, game theory and multi-period triple-level programming for evaluating, selecting and scheduling of BRT projects. The main features of this study are as follows:

1) Evaluating, selecting and scheduling of BRT projects are considered simultaneously.
2) Both of the technical limitations and financial limitations are considered in the modeling process.
3) Time dimensions including planning horizon and evaluation horizon are considered to enable better analysis.
4) All sustainability dimensions have been included to evaluate the candidate projects. In addition, to analyze the performance of the system, both managers' and passengers' perspectives have been considered.
5) An integrated DEA, game theory and triple-level efficiency based approach has been developed to survey the BRTNDP.
6) Nash bargaining game model (NBGM) and MI have been especially applied in this study.

Table 1. A comparison between previous articles and current study.

Ref.	**Bus transportation system**	**BRT Network design**	**Triple-level programming**	**Genetic algorithm**	**Scheduling BRTPs**	**Multi-period (time depended)**	**Green issues**	**All sustainability aspects**	**Different perspectives**	**DEA**	**Network DEA**	**Malmquist Index**	**Game theory**
Laporte et al. (2002)	√	√											
Sheth et al. (2007)	√	√							√	√	√		
An et al. (2008)	√	√							√				
Fan and Machemehl (2011)	√	√		√									
Caicedo et al. (2012)	√	√											
Caulfield et al. (2013)	√	√								√			
Walteros et al. (2013)	√	√		√					√				
Schmid (2014)	√	√											
Xin et al. (2014)	√	√	√						√	√			
Kermanshahi et al. (2015)	√	√											
Rezaee et al. (2016)	√									√			√
Kathuria et al. (2017)	√									√			
Merkert et al. (2017)	√									√			
Current study	√	√	√	√	√	√	√	√	√	√	√	√	√

The rest of this paper has been organized as follows: The problem formulation is presented in section 2. The considered inputs and outputs as the evaluation criteria have been introduced in section 3. A solution procedure based on a genetic algorithm is developed in section 4. To show the applicability of the proposed algorithm, some numerical examples have been solved and analyzed in section 5. A real case study of evaluating and selecting of BRT projects in Isfahan is discussed in

Section 6, and the results have been analyzed. Finally, the conclusions of the study and future research directions have been provided in Section 7.

2. Problem formulation

In this section, first, CCR (Charles, Cooper, Rhodes) DEA model and Malmquist Index (MI) for evaluating the performance of decision making units (DMUs) are introduced briefly. Then, as the cooperative game model used in this study, NBGM is discussed. Finally, the problem definition is presented and a triple-level model is developed for the problem formulation.

2.1. CCR DEA model

CCR is the first DEA model proposed by Charnes et al. (1978). Using this model, the performance of a set of DMUs that use multiple inputs to produce multiple outputs can be evaluated. Suppose there are *n* different DMUs, where each DMU_j uses *m* different inputs (x_{ij}, $i = 1,...,m$) to produce *s* different outputs (y_{rj}, $r = 1,...,s$). Charnes et al. (1978) proposed the following model to obtain the relative efficiency score of DMU_O:

$$\begin{aligned}
& Max\ \theta(x_o, y_o) = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \\
& \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,...,n \\
& \quad \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \quad u_r \ge 0, \quad r = 1,...,s, \\
& \quad v_i \ge 0, \quad i = 1,....,m
\end{aligned} \tag{1}$$

where v_i and u_r are the variables of model (1) that shows the relative importance of the inputs and outputs, respectively. The optimal value of the objective function (θ^*) shows the relative efficiency score of DMU_O. This model must be executed for each DMU, separately.

2.2. Malmquist Index (MI)

Malmquist Index (MI) or Malmquist productivity index is developed by Färe et al. (1992) to investigate the efficiency changes over time. Especially when the production technologies of DMUs have been changed, as a DEA based method, MI has proven itself as an efficient tool to study efficiency changes (Chen and Ali, 2004).

Suppose that related data for a set of specific DMUs in periods *t* (x_{ij}^t, y_{ij}^t) and *t+1* $(x_{ij}^{t+1}, y_{ij}^{t+1})$ are available. To obtain MI score, first using model (1) the single period efficiency measures for each DMU in periods *t* (name $\theta^t(x_o^t, y_o^t)$) and *t+1* (name $\theta^{t+1}(x_o^{t+1}, y_o^{t+1})$) must be calculated. Then two mixed-period efficiency measures for each DMU must be calculated. To obtain mixed-measures, $\theta^t(x_o^{t+1}, y_o^{t+1})$ and $\theta^{t+1}(x_o^t, y_o^t)$, model (2) and (3) must be run (Färe et al., 1992):

$$
\begin{aligned}
&Max\ \theta^{t}(x_o^{t+1}, y_o^{t+1}) = \sum_{r=1}^{s} u_r y_{ro}^{t+1} \\
&s.t. \\
&\quad \sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{t} \le 0, \qquad j = 1,\dots,n,\ j \ne o \\
&\quad \sum_{r=1}^{s} u_r y_{ro}^{t+1} - \sum_{i=1}^{m} v_i x_{io}^{t+1} \le 0, \qquad j = o \\
&\quad \sum_{i=1}^{m} v_i x_{io}^{t+1} = 1, \\
&\quad u_r \ge 0, \quad r = 1,\dots,s, \\
&\quad v_i \ge 0, \quad i = 1,\dots,m
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&Max\ \theta^{t+1}(x_o^{t}, y_o^{t}) = \sum_{r=1}^{s} u_r y_{ro}^{t} \\
&s.t. \\
&\quad \sum_{r=1}^{s} u_r y_{rj}^{t+1} - \sum_{i=1}^{m} v_i x_{ij}^{t+1} \le 0, \qquad j = 1,\dots,n,\ j \ne o \\
&\quad \sum_{r=1}^{s} u_r y_{ro}^{t} - \sum_{i=1}^{m} v_i x_{io}^{t} \le 0, \qquad j = o \\
&\quad \sum_{i=1}^{m} v_i x_{io}^{t} = 1, \\
&\quad u_r \ge 0, \quad r = 1,\dots,s, \\
&\quad v_i \ge 0, \quad i = 1,\dots,m
\end{aligned} \tag{3}
$$

To survey the efficiency changes among periods *t* and *t+1*, Färe et al. (1992) suggested MI as follows:

$$
M_0 = \sqrt{\frac{\theta^{t}(x_o^{t+1}, y_o^{t+1})}{\theta^{t}(x_o^{t}, y_o^{t})} \times \frac{\theta^{t+1}(x_o^{t+1}, y_o^{t+1})}{\theta^{t+1}(x_o^{t}, y_o^{t})}} \tag{4}
$$

$M_O \ge 1$ shows efficiency growth for DMU_O, where $M_O \le 1$ indicates efficiency loss. $M_O = 1$ means no change in efficiency of DMU_O during period *t* to *t+1*.

2.3. Nash bargaining game model (NBGM)

As a cooperative game model, for the first time, two-person NBGM was presented by Nash Jr (1950), then Harsanyi (1963) developed NBGM for *n-person* games. The main idea of NBGM model is dividing benefits or utilities between a set of players according to their competition power. The *n-person* NBGM is as follows (Harsanyi, 1963):

$$
\begin{aligned}
&Max \prod_{i=1}^{n} (u_i(x) - \partial_i) \\
&s.t \\
&\qquad u_i(x) \ge \partial_i \qquad \forall i \in \{1,\dots,n\} \\
&\qquad x \in S
\end{aligned} \tag{5}
$$

In model (5), as the variable of the model, x shows the strategies of players in the game theory context and S is the feasible set of the model. $u_i(x)$ is the benefit/utility of player i when chooses strategy x. ∂_i $(i \in \{1,\ldots,n\})$ are positive parameters of the model, called breakdown points. The breakdown point is a value of utility that a player is considered as a minimum acceptable level of utility for itself (Mahmoudi et al., 2019a; Mahmoudi et al., 2018). Indeed, the utility of the player must be at least as large as its breakdown point, otherwise, the player will withdraw from the game (Mahmoudi et al., 2014). There are different suggested methods to obtain breakdown points (Du et al., 2011; Tavana et al., 2018). In this study, it is assumed that the breakdown points are managerial parameters and will be determined by the TS managers. Cooperative games and DEA have been used in several studies (Li et al., 2019; Mahmoudi, 2019).

2.4. Problem definition

Managers of urban transportation networks (UTNs) always try to improve the performance of their network by applying new policies or constructing new projects. In any UTN, usually, the number of candidate policies/projects is significantly large. The decision-making process about candidate projects is complicated because of financial and technical limitations, the plurality of objectives and beneficiaries, and the positive/negative effects of each project. As a public transportation project, developing a new BRT network or adding some new lines to an existing network, are important and popular projects, especially in the congested cities. Assume that the network managers have decided to add new BRT lines to existing BRT network (in this study, the problem is adding a set of new BRT lines to existing BRT network, but all explanations and presented models are completely true and useable for constructing a new BRT network). Two periods are considered for constructing and evaluating these projects: A T-year planning time horizon and a T'-year evaluation horizon ($T \le T'$). A planning horizon is a time slot considered to start and finish all selected construction activities, while an evaluation horizon is a period, usually longer than planning horizon, to evaluate short- and long-term effects of new projects. Considering all constraints, objectives and evaluation criteria, the manager should decide which BRT line/lines must be added to the network and when to do it. Assume n is the number of candidate lines, without considering the limitations, the total number of scenarios for selecting and scheduling new BRT lines is $(T+1)^n$. A scenario shows selected projects and the period that each project starts to operate. It is clear that there are many possible scenarios and manager must select the best one. Indeed, the problem is evaluating Possible Candidate Scenarios (PCS) and selecting the optimal solution(s).

Consider each PCS as a DMU which has T' sub-DMUs. The efficiency of the DMU will be obtained based on the performance of the sub-DMUs. Indeed, sub-DMUs are the UTN under the selected scenario in a specific period; also, a DMU is the UTN during the evaluation horizon and under the selected construction scenario. To analyze the performance of the system in each period, both managers' and passengers' perspectives are considered. This is an important issue considered by this study, because TSs are socio-economic systems and any applied policy by managers should improve the level of provided services and users' satisfaction. Therefore, users' perspective should be considered in addition to managers' one in order to have a true efficiency score of a TS. Figure 1 shows the considered structure for evaluating the PCSs.

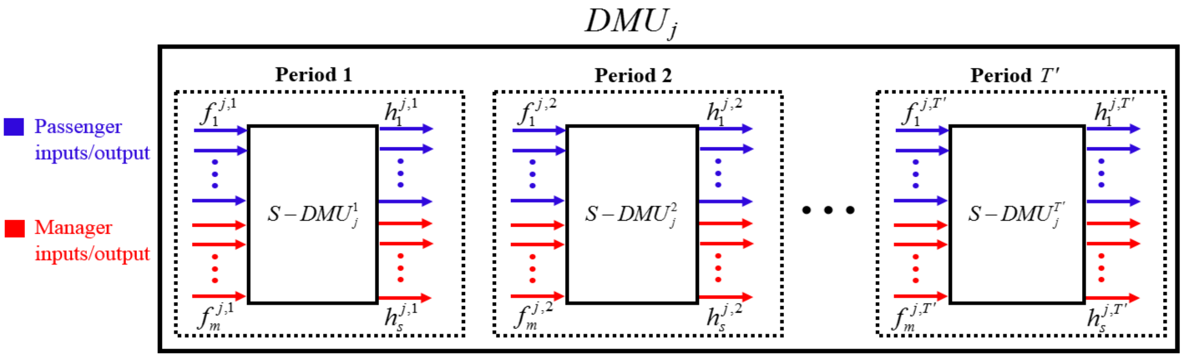


Figure 1. Considered structure and inputs/outputs for evaluating the candidate PCSs (DMUs).

where $f_i^{j,t}$ $(i=1,\dots,m)$ and $h_l^{j,t}$ $(l=1,\dots,s)$ are inputs and outputs of *sub-DMU$_j$* in period t, respectively.

For evaluating the performance of a network and planning for constructions, both long-term and short-term effects must be considered. Therefore, the performance of the network must be evaluated in each period. Evaluating network in each period can control shocks in the network. The shock here means the undesirable and unexpected big changes in the performance of the network in the current period compared to the previous one. In this study, shocks in each period have been controlled with breakdown points, by considering all sustainability dimensions and using the proposed model especially based on the application of NBGM. A specific construction scenario will be a candidate if and only if the efficiency of the network under this scenario in each period is bigger than the breakdown point related to that period. This feature of the proposed model (controlling the shocks) is an important advantage of this study.

Therefore, considering all sustainability dimensions, both manager and passenger perspectives, and shocks, this study aims to develop an efficiency-based approach to evaluate PCS. The following assumptions are considered in the problem formulation.

- The travel demand between each origin-destination (O-D) is known in each year.
- The available budget is considered as the financial limitation. It is assumed that the available budget for each year is fixed and known, and cannot be carried over or postponed from one period to the next one.
- In real-world projects, usually, completion of a project lasts more than a period. The maximum possible proportions of adding/constructing BRT lines are considered as the technical constraints. Transportation projects are always assumed large scale time consuming and costly projects that technically need to be completed step by step during the time. It means to do the next step of the project, the previous must be already done. However, the proposed model is

flexible and in the case, that project is small enough to be completed in one period, this assumption can be ignored by setting the control parameter to 1.

- For route selection behavior, the deterministic user equilibrium assignment method has been considered.
- All costs related to the problem are fixed and known.
- A BRT line can't be utilized until being completed. All stations in a specific line should be constructed and useable.
- Constructing a BRT line will start if and only if it's completed within the planning horizon. All construction activities must be done in the planning horizon.
- Discontinuity in construction activities is not allowed. Therefore, a minimum amount of budget in each period must be allocated to construct incomplete projects. This assumption is considered according to social sustainability (e.g. users' satisfaction). Residents and beneficiaries like to see developments in the started public projects. People will consider any stop for a period as a problem in completing the started project in the network. Particularly, it should be noted that urban transportation systems and the related projects affect the daily life in the city.
- All existing or newly constructed BRT lines will be operated at least until the end of the evaluation horizon. Therefore, closing an open line is not allowed. Closing a newly conducted project by the government, especially the expensive socio-economic projects, that mostly are paid by the taxes, will be assumed as a failure for the managers by the society. In addition, wasting money and making a disturbance in daily activities in the network by constructing and closing such large projects will lead to dissatisfaction among the residents.

2.5. Proposed triple-level model

In this section, due to the problem definition and considered assumptions in section 2.4, a new triple-level programming model based on user equilibrium assignment, efficiency, NBGM, and MI has been proposed for sustainable BRTNDP. Using the presented model, according to the financial/technical limitation, evaluation criteria (inputs/outputs in DEA concept) and sustainability targets, the candidate BRT projects will be evaluated and the best one(s) will be selected. Also, a schedule for constructing the selected projects will be provided. The notations are as follows:

Sets:

N	the set of nodes
O	the set of origin
D	the set of destinations
W	the set of O-D pairs
A_1	the set of all existing links and BRT lines in the network
A_2	the set of all candidate BRT lines
A	$A = A_1 \cup A_2$

Parameters:

X	the under evaluation PCS which contains the selected BRT lines and the periods that each line starts to utilize
x_{ij}^{th}	Link (or a BRT line) $(i,j) \in A$ in the network in period *t* and scenario *h*
T	number of periods in the planning horizon
T'	number of periods in the evaluation horizon
C_a	the total net present construction cost of BRT line $a \in A_2$
α_a	an approximate parameter related to the technical limitation, which shows the maximum possible proportion of constructing of link *a* in each period
B^t	the available budget for period *t*
a	a candidate BRT line, *a* is used to show each $(i,j) \in A_2$ in briefly form
$d_{r,s}^{th}$	a parameter which shows the demand value between $(r,s) \in W$ in period *t* and scenario *h*
f_a	service frequency associated with $a \in A_2$
$w_{i,s}^{th}$	waiting time in node $i \in N$ for destination $s \in D$ (in person minute)
M	a large positive number
ε	a small positive number

Variables:

φ_k	the relative importance of inputs, $k \in \{1,...,m\}$
$u_{k'}$	the relative importance of outputs, $k' \in \{1,...,s\}$
x_a^{th}	binary decision variable related to BRT line $a \in A_2$ and PCS *h*, which $x_a^{th} = 1$ if line a is utilized in period *t*
y_a^{th}	continuous decision variable which shows the constructed proportion of BRT line *a* in PCS *h*
G_a^h	a binary decision variable related to line $a \in A_2$ and scenario *h*, $G_{ij} = 1$ if constructing of line *a* is started in any period, otherwise $G_{ij} = 0$
v_{ij}^{th}	the flow volume on link $(i,j) \in A$ in period *t* and scenario *h*
$v_{i,j,s}^{th}$	the flow volume on link $(i,j) \in A$ with destination $s \in D$ in period *t* and scenario *h*
v_h^t	the flow vector of the network in period *t* and scenario *h*

Functions:

$\tau_{ij}^h(v_{ij}^{th})$	the travel cost on link $(i,j) \in A$ in period *t* and scenario *h*, which is considered as a function of v_{ij}^{th}, travel time is often considered as a cost.

Assume there are *n* PCSs, the proposed model to evaluate the DMU_O (the PCS_O) is presented as follows:

$$Max \prod_{i=1}^{n} (M_o^t(X) - \partial^t) \tag{6}$$

$s.t.$

$$M_o^t(X) \geq \partial^t \qquad t = 1,...,T' \tag{7}$$

$$\sum_{a \in A_2} C_a y_a^{th} \leq B^t \qquad t = 1,...,T, h = 1,...,n \tag{8}$$

$$\sum_{p=1}^{t} y_a^{ph} \geq x_a^{th} \qquad t=1,...,T',\ h=1,...,n,\ a \in A_2 \qquad (9)$$

$$x_a^{th} \geq (\sum_{p=1}^{t} y_a^{ph} \text{ -}1)M+1 \qquad t=1,...,T', h=1,...,n,\ a \in A_2 \qquad (10)$$

$$y_a^{th} \geq (G_a^h - \sum_{p=1}^{t-1} y_a^{ph})\varepsilon \qquad t=2,...,T', h=1,...,n,\ a \in A_2 \qquad (11)$$

$$G_a^h \geq \sum_{p=1}^{t} y_a^{ph} \qquad t=1,...,T, h=1,...,n,\ a \in A_2 \qquad (12)$$

$$\sum_{p=1}^{T} y_a^{ph} = G_a^h \qquad h=1,...,n,\ a \in A_2 \qquad (13)$$

$$\sum_{p=T+1}^{T'} y_a^{ph} = 0 \qquad h=1,...,n,\ a \in A_2 \qquad (14)$$

$$y_a^{th} \leq \alpha_a \qquad t=1,...,T', h=1,...,n,\ a \in A_2 \qquad (15)$$

$$y_a^{th} \geq 0 \qquad t=1,...,T', h=1,...,n,\ a \in A_2 \qquad (16)$$

$$x_a^{th} \in \{0,1\} \qquad t=1,...,T', h=1,..,n,\ a \in A_2 \qquad (17)$$

$$G_a^h \in \{0,1\} \qquad h=1,...,n,\ a \in A_2 \qquad (18)$$

where M_o^t is the MI score of *DMUo* in period *t*, which will be calculated using eq. (4) in the second-level model. In eq. (4), for example, $\theta_o^t(f_k^{o,t}, h_{k'}^{o,t})$ is the optimal objective value of the following model:

$$Max\ \theta_o^t(f_k^{o,t}(v_o^t), h_{k'}^{o,t}(v_o^t)) = \sum_{k'=1}^{s} u_{k'} h_{k'}^{o,t}(v_o^t) \qquad (19)$$

s.t.

$$\sum_{k'=1}^{s} u_{k'} h_{k'}^{j,t}(v_o^t) - \sum_{k=1}^{m} \varphi_k f_k^{j,t}(v_o^t) \leq 0, \qquad j=1,...,n \qquad (20)$$

$$\sum_{k=1}^{m} \varphi_k f_k^{o,t}(v_o^t) = 1, \qquad (21)$$

$$u_{k'} \geq 0, \quad k'=1,...,s, \qquad (22)$$

$$\varphi_k \geq 0, \quad k=1,....,m \qquad (23)$$

In the second-level models, v_h^t is the solution of the deterministic user equilibrium in the network. In the lower-level model, to find v_h^t values, the problem of private and public traffic assignment must be solved. Proposed model by Spiess (1993) can be used to solve the public traffic assignment problem as follows:

$$Min \sum_{(i,j)\in A} \int_0^{v_{ij}^{th}} \tau_{ij}^h(\theta)d\theta + \sum_{i\in N}\sum_{s\in D} w_{i,s}^{th} \qquad (24)$$

s.t.

$$\sum_{j:(i,j)\in A} v_{i,j,s}^{th} - \sum_{j:(j,i)\in A} v_{j,i,s}^{th} = d_{i,s}^{th} \qquad \forall i \in N,\ \forall s \in D \qquad (25)$$

$$v_{ij}^{th} = \sum_{s\in D} v_{i,j,s}^{th} \qquad \forall (i,j)\in A \tag{26}$$

$$v_{i,j,s}^{th} \le (f_{ij} \sum_{s\in D} w_{i,s}^{th}) x_{ij}^{th} \qquad \forall (i,j)\in A, s\in D \tag{27}$$

$$v_{i,j,s}^{th} \ge 0 \qquad \forall (i,j)\in A, \quad \forall s\in D \tag{28}$$

Also, the following model can be applied for private traffic assignment:

$$Min \sum_{(i,j)\in A} \int_0^{v_{ij}^{th}} \tau_{ij}^{h}(\theta)d\theta \tag{29}$$

s.t.

$$\sum_{j:(i,j)\in A} v_{i,j,s}^{th} - \sum_{j:(j,i)\in A} v_{j,i,s}^{th} = d_{i,s}^{th} \qquad \forall i\in N, \quad \forall s\in D \tag{30}$$

$$v_{ij}^{th} = \sum_{s\in D} v_{i,j,s}^{th} \qquad \forall (i,j)\in A \tag{31}$$

$$v_{ij}^{th} \le M x_{ij}^{th}, \qquad \forall (i,j)\in A_2 \tag{32}$$

$$v_{i,j,s}^{th} \ge 0 \qquad \forall (i,j)\in A, \quad \forall s\in D \tag{33}$$

The upper-level of the proposed model can be found in Eqs. (6)-(18). In the upper-level model, based on the MI score of the network for each period and under each PCS, decision-makers obtain the best scenario. According to flow volume in the network, first inputs and outputs will be calculated and then MI scores for each PCS will be obtained in the second-level. Finally, the lower-level of the proposed model is Eqs. (24)-(28). In the lower-level model, deterministic user equilibrium for route selection behavior will be obtained as the equilibrium reactions of the network users.

Eq. (6) is the objective function of the upper-level model which shows the score of PCS_O in Nash bargaining game concept. The shocks in the urban network will be controlled by Eq. (7). This constraint is related to NBGM and due to this constraint MI for each period must be bigger than the breakdown points. Eq. (8) shows budget limitation in each period. Once a BRT line is completed, it must be utilized at least until the end of the evaluation horizon (eqs. (9) and (10)). Eqs. (11)-(13) indicate some important assumptions: discontinuity in construction activities is not allowed and a BRT line will not be utilized unless its construction process is completed within the planning horizon. Any construction activity in the evaluation horizon is unauthorized and it can be seen in Eq. (14). Eq. (15) is the technical limitation related to each candidate BRT line. Condition of the variables of the upper-level model is presented in Eqs. (16)-(18). The upper-level of the proposed model is a mixed-integer programming model.

At the second-level, MI for each period of each PCS will be obtained. For this purpose and based on eq. (4), CCR model must be solved four times for each period in each PSC. Eqs. (19)-(23) shows an example of a customized CCR model that must be solved in the second-level.

By assigning users demands based on deterministic user equilibrium, in the lower-level model traffic flows in the network in period t and under PCS_h will be obtained. In eq. (24), the objective function is minimizing travel costs in the network. Eq. (25) satisfies the travel demands of all users and balances receive and delivery in each node. Eq. (26) is related to total flow on a link. Eq. (27) shows a technical fact: flow on the non-constructed BRT lines is unauthorized. Also, this equation shows the maximum possible flow volume in each boarding links based on the waiting time and service frequency. According to eq. (28), flow volume in each link of the network cannot be a negative number. Eqs. (29)-(33), are completely similar to eqs. (24)-(28), just waiting time is not considered in the objective function of the private traffic assignment model.

A leader-follower structure is considered to solve the proposed triple-level model. First, the lower level models will be solved for each evaluation period for all PCS to obtain flow volumes on the network. Based on the results of lower-level models, the MI scores will be calculated at the second-level. Considering the obtained MI scores in the second-level, the best PCS(s) can be identified by solving the upper-level model.

3. Inputs and outputs

In this section, all inputs and outputs are described. It should be noted that some of the considered input/output are directly related to BRT network performance and some of them are related to the whole network. The undesirable inputs/outputs are converted to desirable form using a method proposed by Seiford and Zhu (2002).

3.1. Inputs

3.1.1. Construction cost

Total construction costs in a period under PCS_O (TCC_O^t) is considered as an input for that period. Based on the financial limitation, the total construction costs in each period is less than or equal to the budget of that period.

$$TCC_O^t = \sum_{a \in A_2} C_a y_a^{th} \qquad t = 1, \ldots, T,\ O = 1, \ldots, n \tag{29}$$

3.1.2. Bus-kilometer and Bus-hours

Operation cost is one of the most considered inputs in BRT and DEA studies (Hawas et al., 2012). Different measures can be used to show operation costs. In this study, two inputs are considered to cover operation costs. The first one is Bus-Kilometer (BK), which shows the total kilometers traveled by the buses in the BRT network. Another considered input is Bus-Hours (BH) showing the total service time (hours) by buses in the BRT network.

3.1.3. Total number of seats

The total number of seats (TNS) is another considered criterion in evaluating the performance of urban bus TSs and BRT lines (Daraio et al., 2016; Odeck, 2008). In this study, the TNS is considered as an input obtained by summing the number of the seats of the active buses in the whole BRT network.

3.1.4. Total number of stations

The total number of stations (TNST) can be seen as a cost criterion. It is expected that the more stations constructed, the more passengers served (Hahn et al., 2013). TNST is assumed as input.

3.2. Outputs

3.2.1. Emission

As an environmental criterion, the value of CO emission released by all transportation modes in UTNs always is a popular and important criterion to analyze the sustainability of TSs (Mahmoudi et al., 2019b). Although there is a range of pollutants, considering only CO is a rational assumption, since reducing one pollutant will lead to the reduction of other pollutants levels (Alexopoulos et al., 1993). CO is mostly considered as an output for evaluating the performance of TSs. As it is mentioned by Yin and Lawphongpanich (2006), the value of released pollution depends on the link flow, which can be calculated as follows:

$$P_{ij}^{h}(v_{ij}^{th}) = 0.2038 \times \tau_{ij}^{h}(v_{ij}^{th}) \times e^{0.7962(L_{ij} / \tau_{ij}^{h}(v_{ij}^{th}))} \tag{30}$$

$$TPR = \sum_{(i,j)\in A} v_{ij}^{th} P_{ij}^{h}(v_{ij}^{th}) \tag{31}$$

L_{ij} shows the length of link $(i, j) \in A$. $P_{ij}(v_{ij}^{th})$ is the CO emission released on this link in period t and PCS_h.

3.2.2. Total travel time

Bureau of Public Roads equation is a common formula used in the literature to obtain the travel cost and total travel time (TTT) in the UTNs as follows (Hosseininasab and Shetab-Boushehri, 2015; Hosseininasab et al., 2018):

$$\tau_{ij}^{h}(v_{ij}^{th}) = a_{ij} + b_{ij} \times (v_{ij}^{th})^{4} \tag{35}$$

$$TTT = \sum_{(i,j)\in A} v_{ij}^{th} \tau_{ij}^{h}(v_{ij}^{th}) \tag{36}$$

where a_{ij} and b_{ij} ($(i, j) \in A$) are the free-flow time and congestion parameter, respectively. Delay and waiting time at the signalized and unsignalized intersections, are considered during the solving of the lower-level model. Such as the emission, the total travel time in each period is considered as an undesirable output.

3.2.3. Spatial equity

Spatial equity (SE) is a social criterion to analyze the performance of TSs from the passengers' perspectives (Camporeale et al., 2017; Chen and Yang, 2004; Thomopoulos et al., 2009). Constructing new transportation infrastructure and applying new policies will affect SE in the network. Therefore, SE is considered as an output. Different approaches are proposed to calculate SE. To obtain the value of SE in a UTN, in this study, new measure is defined based on the accessibility of a region to the public transportation facilities. The proposed methods include some steps as follows:

Step 1. Show access matrix in period k by ψ_k. In matrix ψ_k let *array(i,j)=1* if passengers in i^{th} origin have access to the public transportation to go j^{th} destination, otherwise consider *array(i,j)=0*.

Step 2. Obtain the access value of period k (AV_k) as follows:

$$AV_k = sum(arreay(i,j)), \qquad array(i,j) \in \psi_k \tag{37}$$

Step 3. Obtain SE value of the network in period k as follows:

$$SE_k = \frac{AV_K}{|O|(|O|-1)} \tag{38}$$

where $|O|$ is the total number of the exiting origin nodes in the network. As it is clear from eq. (38), the large SE_k values show more covered origin nodes by the public transportation network; In particular, $SE_k = 1$ means all origins are serviceable by the public transportation system.

3.2.4. Operational benefits

Benefits in the most DEA related studies are considered as outputs (Daraio et al., 2016). In this study, operational benefits (OB), including revenue from the passenger transportation and other marginal income such as advertising, is a desirable output.

3.2.5. Passenger volume

Passenger volume (PV) is a crucial criterion in the evaluating public TSs. There is a significant number of studies that have considered PV as an output (Daraio et al., 2016; Li, 2013; Odeck, 2008; Sheth et al., 2007). It is clear that the more transported passenger, the more efficient performance. Therefore, PV is a desirable output. It should be noted that here the passenger means the people who have used the public transportation services.

3.2.6. Passenger-kilometer

The total kilometers traveled by the passengers, called passenger-kilometer (PK), is an important feature of a BRT network which depends on the constructed projects. PK is considered as an output.

4. Solution approach

Based on the structure of the triple-level models and network design problems, presenting an exact algorithm to solve these types of models seems to be impossible. Metaheuristic algorithms are popular methods to deal with these models. In this section, an evolutionary metaheuristic algorithm is developed based on the concept of the genetic algorithm (GA) to solve the proposed model.

4.1. Complexity

Several reasons are causing the complexity of the proposed model including:

- **Triple-level structure.** Based on Ben-Ayed and Blair (1990), the bi-level models are strongly NP-hard problems. Also, even a small scale bi-level NDP model is extremely difficult to solve (Hosseininasab et al., 2018). The triple-level models are more complicated than the bi-level ones, especially in the large scale real UTNs.

- **Planning and evaluation horizon.** The proposed triple-level model is a multi-period model. In each period the lower-level models will be run, then in the second-level, the CCR model will be solved four times. Finally, the upper-level model will be run for each PCS to obtain efficiency scores of the DMUs. Therefore, multi-period models are more complicated and time-consuming than single-period NDP models.
- **Large scale network.** An urban transportation network usually is a large scale network. Therefore, the number of candidate projects and solutions is significantly large.

4.2. General solving procedure

The general solving procedure of the proposed triple-level model is inspired by Stackelberg leader-follower structure in the game theory concept. First, feasibility of any X (PCS) will be checked in eqs. (8)-(18). For this purpose, the phase-I of two-phase simplex method will be used (Bazaraa et al., 2011). Eqs (8)-(18) are in the linear form. For each feasible X, the lower-level model will be run. In this study, frank wolf algorithm and optimal strategy are used to solve the lower-level model for private and public traffic assignments in the network, respectively. Based on the flow volume in the network, the value of inputs and outputs will be obtained. Note that some of the inputs and outputs are the features of X, and will not be obtained in lower-level model (for example TNS or TNST). Considering the values of inputs and outputs, in the second level, the MI score for each period of X will be obtained. Finally using eqs. (6) and (7), all PCSs will be ranked and the best PCS(s) will be identified.

4.3. An exact algorithm based on full enumeration

Following the proposed solving procedure in section 4.2, a full enumeration algorithm can be developed as follows:

Step 1. Check the feasibility of X.

Step 2. If X is feasible, solve the lower-level models for this scenario.

Step 3. Calculate the inputs and outputs values in each period of X.

Step 4. Based on the optimal values of the lower-level model and using CCR model and eq. (4), obtain the MI score of each period of X.

Step 5. Sort all PCS(s) using eqs. (6) and (7) and find the optimal solution(s).

4.4. The proposed solution approach

The number of candidate projects is $(T+1)^n$. This number is a very large value in real cases. Therefore, applying an exact algorithm such as full enumeration method to solve the large scale BRTNDPs will be a time-consuming procedure. The metaheuristic approaches are popular algorithms to deal with these issues (Poorzahedy and Rouhani, 2007). This section provides a GA based evolutionary algorithm to obtain a near to optimal solution(s) in an acceptable time. The steps of the proposed algorithm are:

Step 1. Create a construction scenario (*X*).

Step 2. Use the phase-I of the two-phase simplex method to check the feasibility of *X*.

Step 3. For each feasible X, estimate the private and public demand matrices in each period.

Step 4. Use EMME software for the traffic assignment in the network.

Step 5. Calculate the inputs/outputs values related to *X*.

Step 6. Obtain the MI score of each period of *X*.

Step 7. Based on NBGM in the upper-level model rank the feasible solutions and find the best one(s).

The proposed procedure is an evolutionary algorithm and will be repeated until meeting one of the stopping conditions. The flowchart of the suggested algorithm is shown in Figure 2.

EMME is a powerful transportation software that has been used in many populated cities all around the world. In the large scale networks, the transportation experts trust the results of EMME (EMME, 2017). In this study, EMME is used to traffic assignment in the network. The existing network, newly added lines, demand matrices, mode choice, flows and delays in each period will be considered in EMME.

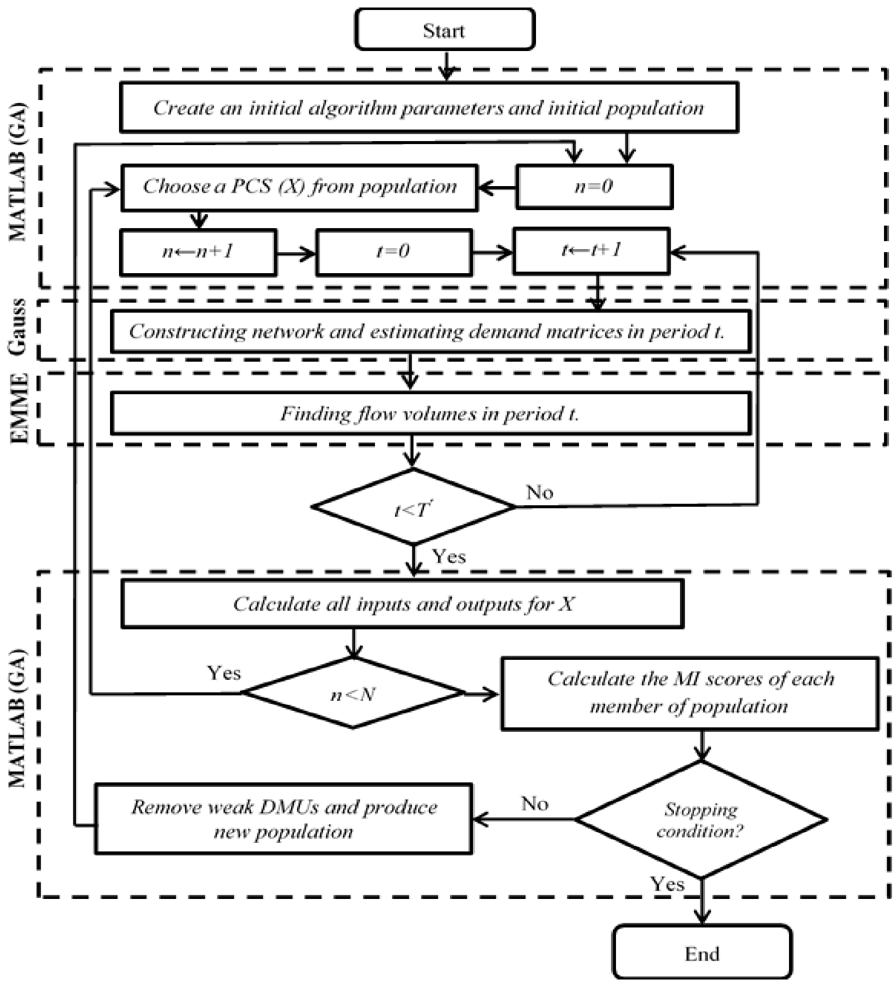


Figure 2. Flowchart of proposed evolutionary approach (N is the population size).

4.5. Genetic algorithm

Solution representation is the first and important step in each metaheuristic algorithm. A good solution representation can affect the running time significantly. In this study, a solution will be represented with a $1 \times n$ matrix. Figure 3 shows an example of the suggested solution representation for $n=6$ and $T=5$. For example, based on this representation, project 1 will not be constructed and project 6 will be operated in period 1. Each solution is assumed as a chromosome for the GA algorithm and each gene (cell) shows the period in which a project starts to operate. To run the GA some assumptions are set as follows:

- First, an initial population will be generated randomly, then, the feasibility of each X will be checked and infeasible ones will be removed.
- The optimal value of the objective function in the upper-level model is considered as the fitness value.
- In each iteration, the parents will be selected based on the roulette-wheel method.
- Crossover operators including single-point, double-point, and uniform, are used.
- Newly born children must be prevented from converging to a local solution. For this purpose, the uniform mutation operator is used and an integer number like as i ($0 \le i \le T$) will be replaced in the randomly selected cell.
- Any generation must not be worse than their predecessors. Therefore, if there is any offspring better than any current individual, it should be inserted into the population and the worst individuals must be eliminated.
- In all iteration the population size is fixed.
- The algorithm will be stopped when meeting at least one of the two specific conditions: (a) a special number of iteration (b) a fixed number of iteration without improvement in the optimal solution.
- Based on the network specifications, Taguchi method (Phadke, 1995) will be used for setting the parameters of the algorithm.

1	*2*	*3*	*4*	*5*	*6*
0	3	4	2	5	1

Figure 3. An example of solution representation in the proposed algorithm

5. An illustrative application

In this section to verify the validity of the developed algorithm, a numerical example is presented and the results have been analyzed. To this end, the UTN of Sioux Falls city with seven candidate BRT lines is considered (Figure 4). In the transportation studies, the UTN of Sioux Falls city is always assumed as a well-known urban medium size test network. This network contains 24 nodes, 76 links, and 528 O-D pairs. All assumptions about this network are in accordance with Leblanc (1975). Just to make the network more congested, the demands between all O-D pairs are considered 4 times bigger than Leblanc (1975). Also, the headway for all BRT lines is considered equal to 2 minutes. Based on the different values of the budge limitation, construction cost and number of candidate BRT projects, 27 random instances have been generated and solved. $T=5$, $T'=7$ and $d_i=0.6\ (\forall i=1,\ldots,T')$ is considered for all instances. For this example, TCC, TNST, BK, and BH are considered as inputs and emission, SE, TTT, PV and PK are considered as outputs. Taguchi method is applied to set the algorithm's parameters. The suggested algorithm is coded and run on a PC with a 2.26 gigahertz Corei7 CPU and Windows Seven using 4 GB of RAM.

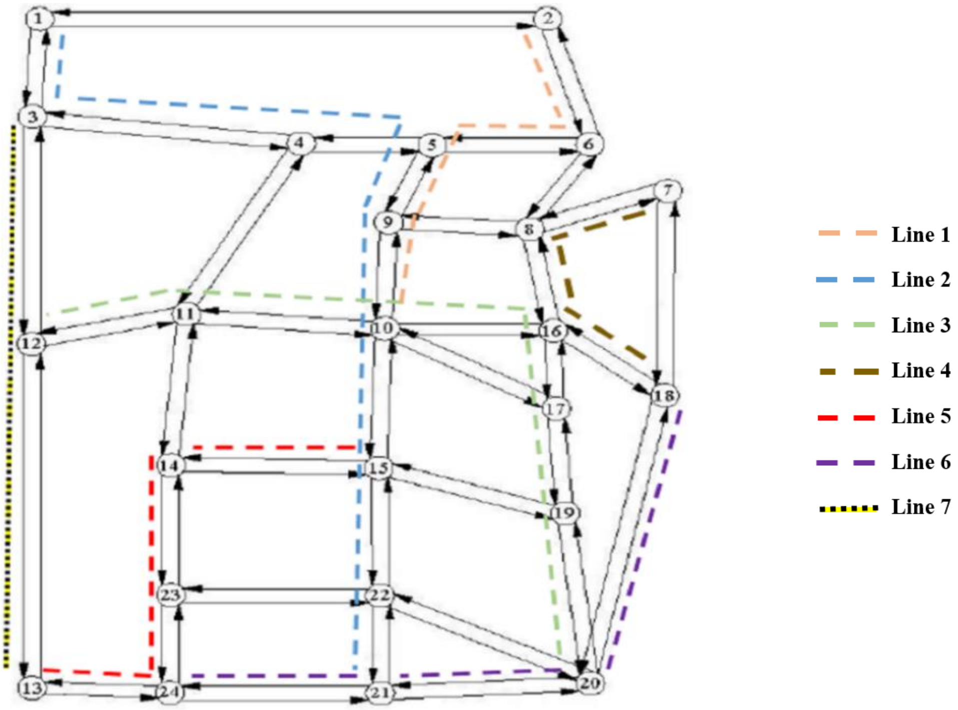


Figure 4. Sioux Falls network and candidate BRT lines.

The suggested algorithm is a stochastic search algorithm; hence, each generated instance is solved 5 times, then the average results are reported. The test instances and full enumeration results are presented in Table 2.

Table 2. Test instances and full enumeration results.

No.	$\lvert A_2 \rvert$	$(\sum_{t=1}^{T} B_t)/(\sum_{a\in A_2} C_a)$	Number of PCS	Execution time (Second)	Optimal Solution(s)
1	3	0.3	17	1517.7689	[0,0,3]
2	3	0.3	11	700.4837	[2,0,4]
3	3	0.3	27	2219.3481	[1,4,0]
4	3	0.5	29	2984.0900	[4,0,1]
5	3	0.5	22	2080.5839	[2,0,4]
6	3	0.5	29	2474.4321	[1,0,3]
7	3	0.7	35	3190.7301	[0,2,4]
8	3	0.7	31	3122.0181	[0,3,5]
9	3	0.7	48	4505.6685	[2,0,4]
10	5	0.3	99	7321.7653	[0,3,0,0,2]
11	5	0.3	131	13484.5858	[5,0,4,1,0]
12	5	0.3	307	34885.0807	[2,0,1,3,0]
13	5	0.5	671	69365.8504	[3,0,3,0,1]
14	5	0.5	998	85306.5564	[1,4,0,2,0]
15	5	0.5	718	71916.5057	[1,2,5,0,0]
16	5	0.7	1111	119072.6968	[1,0,3,2,0]
17	5	0.7	786	85696.2239	[0,5,3,0,2]
18	5	0.7	1095	106999.8151	[0,2,4,5,1]
19	7	0.3	3803	468946.8772	[1,0,4,0,0,0,4]

20	7	0.3	2473	229525.5785	[0,0,3,0,1,5,5]
21	7	0.3	4435	--------------	--------------
22	7	0.5	17192	--------------	--------------
23	7	0.5	19941	--------------	--------------
24	7	0.5	23345	--------------	--------------
25	7	0.7	37831	--------------	--------------
26	7	0.7	41582	--------------	--------------
27	7	0.7	41288	--------------	--------------

The parameters of each instance and the results of the proposed algorithm are reported in Table 3.

Table 3. Parameters and results of the proposed algorithm.

No.	Population size	Number of generation	Mutation rate	Prob. of single point crossover	Prob. of double point crossover	Prob. of uniform point crossover	Mutation prob.
1	16	15	1	0.1	0.2	0.7	0.2
2	16	15	1	0.1	0.2	0.7	0.2
3	16	15	1	0.1	0.2	0.7	0.2
4	16	15	1	0.1	0.2	0.7	0.2
5	16	15	1	0.1	0.2	0.7	0.2
6	16	15	1	0.1	0.2	0.7	0.2
7	16	15	1	0.1	0.2	0.7	0.2
8	16	15	1	0.1	0.2	0.7	0.2
9	16	15	1	0.1	0.2	0.7	0.2
10	20	25	1	0.0	0.0	1.00	0.1
11	20	25	1	0.0	0.0	1.00	0.1
12	20	25	1	0.0	0.0	1.00	0.1
13	20	25	1	0.0	0.0	1.00	0.1
14	20	25	1	0.0	0.0	1.00	0.1
15	20	25	1	0.0	0.0	1.00	0.1
16	20	25	1	0.0	0.0	1.00	0.1
17	20	25	1	0.0	0.0	1.00	0.1
18	20	25	1	0.0	0.0	1.00	0.1
19	35	45	1	0.05	0.1	0.85	0.1
20	35	45	1	0.05	0.1	0.85	0.1
21	35	45	1	0.05	0.1	0.85	0.1
22	35	45	1	0.05	0.1	0.85	0.1
23	35	45	1	0.05	0.1	0.85	0.1
24	35	45	1	0.05	0.1	0.85	0.1
25	35	45	1	0.05	0.1	0.85	0.1
26	35	45	1	0.05	0.1	0.85	0.1
27	35	45	1	0.05	0.1	0.85	0.1

No.	Average Execution time (second)	Time ratio	Frequency of finding the optimal solution	Average error	Relative standard deviation
1	817.6833	2.2275	5	0.0000	0.0000
2	2842.7144	1.8729	5	0.0000	0.0000
3	4164.8286	1.8766	5	0.0000	0.0000
4	5835.2304	1.9554	5	0.0000	0.0000
5	4673.2575	2.2461	5	0.0000	0.0000
6	5318.4244	2.1493	5	0.0000	0.0000
7	7251.5268	2.2726	5	0.0000	0.0000
8	6475.6958	2.0742	5	0.0000	0.0000
9	7902.5316	1.7539	5	0.0000	0.0000
10	10893.3526	1.4878	5	0.0000	0.0000
11	18469.3586	1.3696	5	0.0000	0.0000
12	32566.5720	0.9335	5	0.0000	0.0000
13	30527.1193	0.4401	2	0.0012	0.0023
14	24930.9259	0.2922	3	0.0002	0.0005
15	31916.6983	0.4438	2	0.0019	0.0007
16	31840.9410	0.2674	3	0.0002	0.0004
17	33086.0966	0.3861	2	0.0004	0.0002
18	27739.1145	0.2592	4	0.0002	0.0004
19	54808.3378	0.1168	1	0.0034	0.0004
20	43399.1273	0.1891	1	0.0022	0.0006
21	48315.8564	--------	1*	0.0023#	0.0017#
22	53070.2565	--------	2*	0.0005#	0.0005#
23	49046.3666	--------	1*	0.0004#	0.0007#
24	62387.6499	--------	1*	0.0004#	0.0002#
25	53240.4412	--------	2*	0.0001#	0.0000#

26	57465.7001	--------	1*	0.0015#	0.0008#
27	28454.5423	--------	1*	0.0005#	0.0005#
Ave.				0.0006	0.0004

Note. For all instances: crossover rate=0.6, elitism rate=0.8, and the algorithm have been stopped after maximum 5 days for the large scale instances or 10 iterations without a change in the optimal solution. "*" and "#" shows the frequency of finding the best solution in five executions and average error/relative standard deviation from the best solution found, respectively.

Most studies consider "average error in the objective function value" as a criterion to show how efficient is their proposed heuristic or metaheuristic algorithms. However, this cannot be a good measure to evaluate the performance of the developed algorithm in this study since the objective function in the upper-level model strictly depends on the under evaluation set of DMUs. Because in each iteration the number of DMUs is different from the other iterations and the exact methods, comparing the objective function values and average error are not meaningful. Hence, a different measure is introduced to evaluate the quality of the obtained solutions. The main idea behind this method is evaluating the exact solution and obtained solutions in a randomly created population. The suggested method is as follows:

Step 1. Create *N* feasible solutions (PCSs) at random.

Step 2. Consider the random solutions, exact solution(s) and obtained solution(s) by the proposed algorithm as a set of DMUs.

Step 3. Run the exact algorithm to evaluate the members of this set.

Step 4. Based on the objective value of the upper-level model, calculate the average error and Relative Standard Deviation (RSD). Low average error and RSD values mean the proposed algorithm has an acceptable performance.

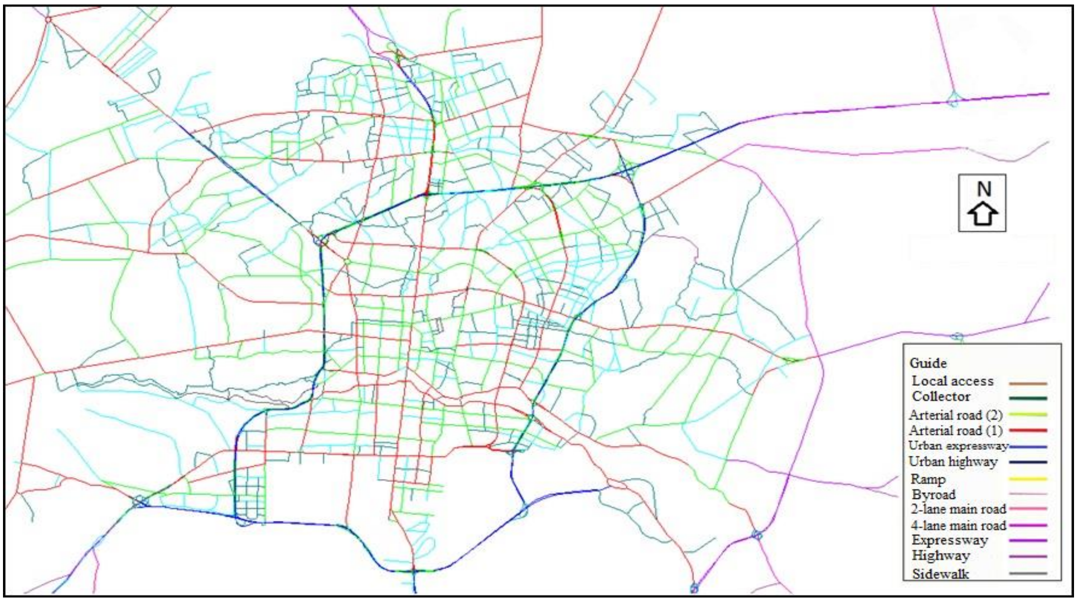
Comparing the results presented in Table 2 and Table 3, the performance of the developed metaheuristic algorithm can be analyzed in terms of solution quality and execution time. Each instance has been solved 5 times and the optimal solutions for instances 1-12 have been found in all of the executions. For large scale instances, the proposed algorithm is able to find the best solution at least one time. Average error equal to 0.0006 and average RSD equal to 0.0004 indicate that the proposed metaheuristic algorithm has found a near to optimal solutions in all executions. Therefore, the quality of the obtained solutions by the proposed algorithm is satisfying. The average execution time of the proposed algorithm to run time of full enumeration ratio, is defined as "time ratio". As it is clear from Table 3 time ratio for the large scale instances is significantly a small value. In particular, while the largest instances solved by the full enumeration approach is instance 19 (with 3083 feasible solutions and about 130 hours execution time), the proposed algorithm just took approximately 16 hours to solve instance 26 with 41582 feasible solutions. Based on the presented explanations, all results show that the performance of the proposed algorithm in large-scale instances is acceptable in both solution quality and execution time perspectives.

6. Case study: A real application

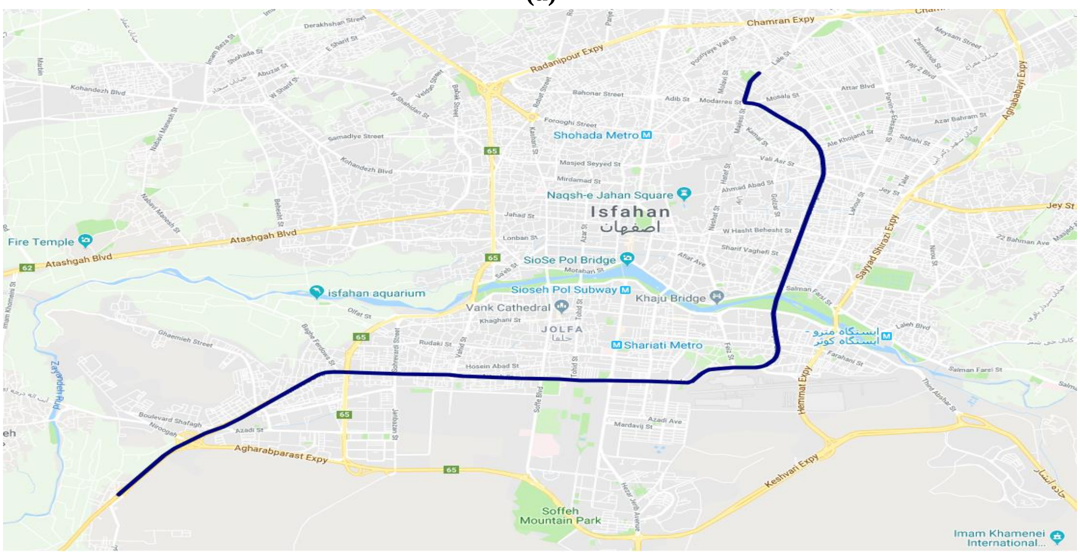
In this section, first, some information about the urban transportation network, especially the BRT network, in the city of Isfahan is given. Then the proposed approach is applied to evaluate the candidate projects and results are analyzed in the last subsection. The correlation issue between the candidate inputs/outputs has been considered before selecting the final list. Hence, the correlation between the selected inputs/outputs are small enough to consider them separately. It should be noted that some of the considered criteria are related to the BRT network and others are related to the whole transportation network. For example, emission here is the total released emission in the network and it has been considered to analyze the effect of adding BRT system on the performance of the whole network. In addition, since this is a real case study conducted on an on-going project, all criteria have been verified by the managers of the transportation ministry in Isfahan.

6.1. The BRT network of Isfahan

Located in the central area of Iran, Isfahan is one of the most important and populated cities of this country with a population of over 1,900,000 people. The urban network of Isfahan contains 2144 nodes, 3145 links and 186 traffic zones, and the total length of urban streets is 2226 kilometers. The BRT network of Isfahan includes 1 corridor and 32 stations and 17 km is the total length of the network, where about 135000 passengers travel in BRT network each day (GBD, 2018). Figure 5 shows the urban transportation network and the existing BRT network of Isfahan.



(a)



(b)

Figure 5. (a) Urban transportation network of Isfahan (b) Existing BRT network in Isfahan.

The municipality of Isfahan is planning to operate new BRT lines and there are 12 candidate lines. The geographical locations of candidate BRT lines are shown in Figure 6.

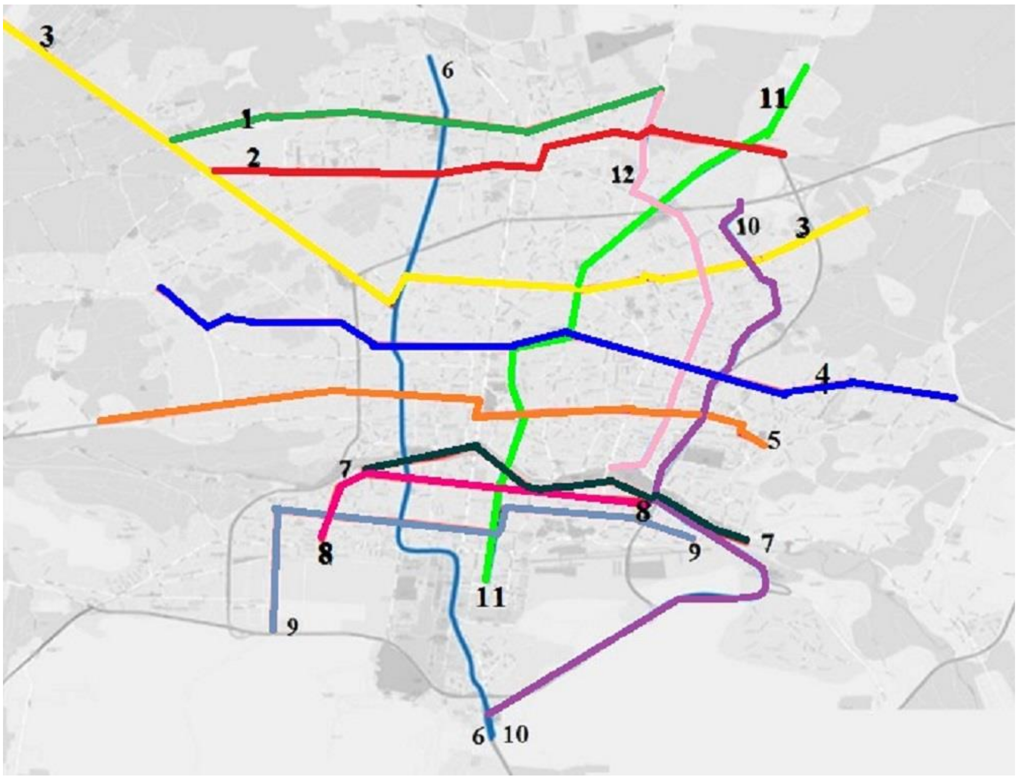


Figure 6. the geographical location of candidate BRT lines

Also, the related data to candidate projects are presented in Table 4. A three-year time-period (2019-2021) as the planning horizon and a six-year time-period (2019-2024) as the evaluation horizon ($T=3,\ T'=6$) are considered. 120 billion Rials is the available budget in each period of the planning horizon. In addition, based on the technical reasons, some specific projects cannot be operated at the same time and at most one of them can be selected. These projects are: lines 1 and 2, lines 4 and 5, lines 7 and 8, and lines 10 and 12.

The parameters of the algorithm are set similar to instances 25-27. It should be noted that the demand matrices in the evaluation horizon are considered based on the travel demand in the peak time in the morning. Based on the available data, EMME software is used to estimate flow volume and assign traffic in UTN of Isfahan from 2019 to 2026.

Table 4. Data of candidate BRT projects.

No.	Length (km)	Needed cost to construct (billion Rial)	Needed cost to fleet preparation (billion Rial)	Total needed budget	Minimum time needed to construct (year)
1	9.6	3.36	22.5	25.86	1
2	11.6	4.06	25.2	29.26	2
3	23.3	8.155	45.9	54.055	3
4	16	5.6	57.6	63.095	2
5	15.7	5.495	57.6	63.2	2
6	16.9	5.915	60.3	66.215	2
7	8.6	3.01	34.2	37.21	1
8	8.9	3.115	36.9	40.015	1

9	11.5	4.025	43.2	47.225	2
10	15.7	5.495	34.2	39.695	3
11	13.9	4.865	31.5	36.365	2
12	11	3.85	43.2	47.05	2
Sum				549.245	

6.2. Results and discussion

The transportation network of Isfahan is a very large scale network; therefore, the full enumeration isn't able to solve this problem in a reasonable time. Table 5 shows the results of the proposed algorithm for the problem of evaluating, selecting and scheduling the BRT projects in Isfahan. Top 3 PCSs in the last population and the values of outputs in the last evaluation horizon ($t=6$) related to each solution have been reported in Table 5.

Table 5. Results for top 3 selected solution by proposed metaheuristic algorithm.

Rank	Solution	Objective function	Emission	SE	TTT
1	[0,2,3,2,0,0,1,0,0,3,2,0]	0.03268	-0.01781	0.21308	0.05394
2	[1,0,3,2,0,0,1,0,0,3,2,0]	0.03261	-0.01758	0.21174	0.4914
3	[0,3,3,2,0,0,0,0,0,3,2,0]	0.03194	-0.01737	0.22153	0.05531
Rank	**TCC**	**BK**	**BH**	**TNS**	**TNST**
1	259.68	4396.249	12984.170	11430	240
2	256.28	4346.134	12811.500	11295	231
3	222.47	3782.554	12070.249	9720	218
Rank	**OB**	**PV**	**PK**		
1	235126090	24880.87	88876.15		
2	245868630	26742.09	87942.55		
3	224917110	24220.73	86495.13		

6.3. Results analyzes

The selected BRT lines are:

Line 2. Crossing the congested regions and streets, this line serves the passengers in areas 7, 12 and 14. The passengers in these areas can use BRT services to access line 1 of the metro.

Line 3. This line connects the north, northwest and east regions of Isfahan and provides services for the citizens in areas 2, 7, 8, 10, 11 and 12. In particular, lines 3 can be very helpful to reduce the traffic volume in Emam Street and Emam bridge, as two important links in the network.

Line 4. In addition to serving the passengers in areas 1, 2, 3, 4, 10 and 11, this line connects the near small suburban cities such as Khorasghan to the downtown of Isfahan. Therefore, this line can help the people living suburbs and working in the downtown or northwest of Isfahan.

Line 7. Zayandehroad river and the historic bridges are the most important tourist attractions in Isfahan. Also, a lot of medical centers are located near the bridges. Therefore, the passengers in the streets around these locations are always facing traffic jams. Line 7 provides a public transportation service to the east-west travels along the river and can improve the traffic jams, remarkably.

Line 10. This line serves the passengers in one of the congested suburbs of Isfahan, Keshveri town. Also, the citizens in areas 4, 6 and 15 can enjoy the services of line 10 in their home-work travels during rush hours.

Line 11. Covering some travel demands in areas 3, 6, 10 and 14, and crossing the congested regions in the downtown, line 11 makes the north-south travels easier.

6.4. Budget allocation

Another important output of the proposed model is the budget allocation for a specific PCS. It should be noted that the key required information to budget allocation are selected projects and the period that they will start to operate. After identifying best PCS(s) infinite number of budget allocation patterns can be designed for this solution(s). An example of the budget allocation for the top identified PCS is presented in Table 6.

Table 6. Budget allocation for top identified PCS.

	Planning period		
Projects	1	2	3
1	0	0	0
2	14.630	14.630	0
3	18.018	18.018	18.019
4	21.032	21.032	18.031
5	0	0	0
6	0	0	0
7	37.210	0	0
8	0	0	0
9	0	0	0
10	13.232	13.232	13.231
11	12.122	12.122	12.121
12	0	0	0
Sum.	116.244	79.034	61.402

7. Conclusions and direction for future studies

The development of an efficient and sustainable urban transportation network has become one of the challenging issues for both local and national governments. Within this context, the development of public transportation infrastructures and increasing the accessibility of public TSs, in particular the BRT network, for the citizens are often deemed as popular and efficient strategies. To fulfill such goals, managers of the urban TSs must continuously analyze the different policies and scenarios to improve the performance of the existing public TSs. To design an efficient and sustainable BRT network, the number of the candidate projects/scenarios is often considerable. In particular, the necessity of considering different evaluation criteria, limitations and beneficiaries, renders such processes more complicated. Therefore, the decision-makers often face a complicated problem when evaluating candidate projects and selecting superior ones. This paper introduces a model that can facilitate such a process. Herein, by considering both passengers' and managers' perspectives and incorporating game theory, DEA and MI, a multi-period triple-level approach has been developed to deal with sustainable BRT network design problems. Further, a GA based evolutionary algorithm has been presented to solve the proposed approach in an acceptable time and obtain a near to optimal solution. Some numerical instances have been solved to analyze the applicability of the proposed algorithm. The results clearly show that this algorithm has excellent performance in both solution quality and execution time. Finally, the developed approach has been applied to the case of the BRT network of Isfahan and results have been analyzed, which confirm that under the selected scenario, the sustainability of the network will be improved.

In this study, it is assumed that all parameters and variables are in the deterministic form, whereas in some real problems they are in the probabilistic or stochastic form. Proposing an efficiency-based approach by considering the probabilistic or stochastic variables/parameters can be interesting future research. Nash bargaining game model as a cooperative game model and CCR model as a DEA model have been used in this study. Researchers can use other game theory or DEA models and compare their results with those presented herein. The objective of this study is to design an efficient urban transportation network during and end of the evaluation period by improving the outputs of the system. Hence, in the transportation systems, both managers and users mostly evaluate the performance of the network based on the outputs because outputs are more touchable. The output-oriented model has been considered in the modeling process in this study. In addition, since this study has been conducted on a real case study, to make a decision about the models, the authors conducted regular meetings with transportation managers of Isfahan. The main concerns of the managers were related to outputs of the system. This was another motivation to apply the output-oriented model. For the other cases and future studies, researchers can apply input-oriented or non-oriented DEA models and compare the results. Because of the nature of the problem in this study and since DMUs are almost same size, the Constant Return to Scale (CRS) assumption has been considered. Reserachers interested can apply this to Variable Return to Scale assumption, the modeling is almost similar but needs to add the convexity assumption. Although the proposed approach has been applied to the BRT network design problem, it will be interesting but challenging to customize this model for evaluating other transportation projects. Finally, as a metaheuristic algorithm, GA has been used for solving the proposed approach. Developing different metaheuristic algorithms to solve the proposed triple-level model can be another research direction.

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