To:-

MY FATHER

MATHEMATICAL METHODS IN POWER
SYSTEM STABILITY STUDIES

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Thesis submitted for the degree of
Doctor of Philosophy of the University of Aston in Birmingham

December 1971

S U M M A R Y

In this thesis various mathematical methods of studying the transient and dynamic stability of practical power systems are presented. Certain long established methods are reviewed and refinements of some proposed. New methods are presented which remove some of the difficulties encountered in applying the powerful stability theories based on the concepts of Liapunov.

Chapter 1 is concerned with numerical solution of the transient stability problem. Following a review and comparison of synchronous machine models the superiority of a particular model from the point of view of combined computing time and accuracy is demonstrated. A digital computer program incorporating all the synchronous machine models discussed, and an induction machine model, is described and results of a practical multi-machine transient stability study are presented.

Chapter 2 reviews certain concepts and theorems due to Liapunov. In Chapter 3 transient stability regions of single, two and multi-machine systems are investigated through the use of energy type Liapunov functions. The treatment removes several mathematical difficulties encountered in earlier applications of the method. In Chapter 4 a simple criterion for the steady state stability of a multi-machine system is developed and compared with established criteria and a state space approach.

In Chapters 5, 6 and 7 dynamic stability and small signal dynamic response are studied through a state space representation of the system. In Chapter 5 the state space equations are derived for single machine systems. An example is provided in which the dynamic stability limit curves are plotted for various synchronous machine representations. In Chapter 6 the state space approach is extended to multi-machine systems. To draw conclusions concerning dynamic stability or dynamic response the system eigenvalues must be properly interpreted, and a discussion

concerning correct interpretation is included. Chapter 7 presents a discussion of the optimisation of power system small signal performance through the use of Liapunov functions.

ACKNOWLEDGEMENTS

The author wishes to thank particularly the Nelson Research Laboratories of the English Electric Company Ltd. for making it possible to pursue this work while in their employment. He is grateful to his supervisors Dr.J.C.W. Corcoran of the English Electric Company Ltd., Dr.N.R. Tomlinson formerly of the University of Aston in Birmingham and Mr. N. Kerruish of the University of Aston in Birmingham for their encouragement and help during the work. He would also like to thank the Central Electricity Generating Board for allowing time for the writing up of the thesis while engaged in one of their projects.

C O N T E N T S

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LIST OF PRINCIPAL SYMBOLS

Only the principal symbols are listed below. Other symbols and all the matrices are defined as they are introduced in the text.

v_d, v_q	=	stator voltages in d and q axes respectively.
v_{fd}	=	field voltage
v_f	=	field voltage referred to the armature circuit
i_d, i_q	=	stator currents in d and q axes respectively
i_{fd}, i_{1d}, i_{1q}	=	currents in field and d and q axis damper windings respectively
r_a	=	stator resistance in d and q axis windings.
r_{fd}, r_{1d}, r_{1q}	=	resistances in field and d and q axis damper windings respectively
Ψ_d, Ψ_q,	=	stator flux linkages in d and q axes respectively
Ψ_{fd}, Ψ_{1d}, Ψ_{1q}	=	flux linkages in field and d and q axis damper windings respectively
x_{ffd}, x_{11d}, x_{11q}	=	total reactances of field and d and q axis damper windings respectively.
x_{ad}, x_{aq}	=	mutual reactances between any pair of d axis windings and between the q axis windings respectively
x_a	=	stator leakage reactance.
x_f, x_{1d}, x_{1q}	=	leakage reactances of field and d and q axis damper windings respectively.
x_d, x_q	=	synchronous reactances in d and q axes respectively.
x'_d	=	d axis transient reactance
x_d'', x_q''	=	d and q axis subtransient reactances respectively
e_q'	=	q axis voltage behind transient impedance
e_d'', e_q''	=	d and q axis voltages behind subtransient impedances respectively
T_{do}', T_d'	=	d axis transient open and short circuit time constants respectively (sec.)
T_{do}'', T_d''	=	d axis subtransient open and short circuit time constants respectively (sec.)

T_{qo}'', T_q''	= q axis subtransient open and short circuit time constants respectively (sec.)
P_e	= electrical terminal power
T_e	= air gap torque
T_m, P_m	= mechanical torque and power input to rotor respectively
K_d	= damping torque coefficient
H	= inertia constant, kWs/kVA
M	= $H/\ \pi f_o$
ω_o, f_o	= rated frequency (rad./s and Hz respectively)
ω	= instantaneous angular velocity of rotor (rad./s)
δ	= rotor angle (rad.)
V_b, V_t	= voltages at infinite busbar and machine terminals respectively
R_t, X_t	= total resistance and reactance, respectively, between generator terminals and infinite busbar
V	= Liapunov function
ℓ	= constant to determine the extent of asymptotic stability
λ_i	= eigenvalues, i = 1, 2, ...n
Δ	= prefix to denote a small change about the initial operating point
d, q	= subscripts to denote machine reference frame
D, Q	= subscripts to denote network reference frame
p, $\cdot$	= differential operator d/dt
t	= time (sec.)

The constants τ, G and μ with appropriate subscripts denote control system time constants and stage gains. K with appropriate subscripts represents constant bias.

INTRODUCTION

The importance of the knowledge of stability properties of electrical power systems has grown steadily since the early days of development of power system interconnection and long distance transmission. In the early years of power system design, stability was not as great a problem as it became a few decades later. In those days, because of the low loadings and low machine and system impedances, power systems tended to possess inherent stability. With the introduction of automatic voltage regulators to control generator terminal voltage, it was possible to increase generator reactances and design of generators of high rating became more practicable and economical. Also, with the development of methods to control voltages at intermediate feeder points, transmission lines of higher impedance could be designed. This increase in generator and transmission line impedances, and the necessity to utilise transmission lines to their maximum capacity in order to reduce costs to a minimum, inevitably resulted in decrease in the inherent stability. With the growth of industrial complexes heavily dependent on electricity, continuity of electricity supply became extremely important. Recognising the hazards caused by loss of stability - the operational mode in which machines in a power system are running out of step with each other - stability consideration, which ensures that the various machines in a power system will maintain synchronism through normal and abnormal conditions, has occupied a leading place in power system design.

Theories have been developed and used since the early days of the realisation of the problem. Basically an electrical power system is a dynamic system with rotating electrical machines, static electrical networks and other apparatus of certain characteristics. Among these synchronous machines play the most important part in system stability. Therefore they deserve special attention. Like any other dynamic system, a power system can be described mathematically by a set of differential and algebraic equations and a knowledge of the behaviour of the system can be obtained from a solution of these equations. However, these equations are non-linear and formal solution is not practicable. Alternative ways have to be sought. The degree of complexity by which power system apparatus has been represented in a stability study and the methods of solution employed at a particular time has largely depended on the precision required by the particular study and the means of solution available. In early days, because of the higher stability margin normally allowable, it was not necessary to obtain results of high accuracy. A good deal of allowance on the optimistic side could be accepted. Also none of the modern computing facilities was available and formal solution of the equations being impracticable, longhand calculation using a suitable numerical method of solving non-linear differential and algebraic equations was the only way of handling the situation. Obviously this was not an easy task. However, as time passed, various means were developed to ease the problem.

The various developments can be broadly listed as

a) A.C. Network analyser

b) Differential analyser

c) Analogue computer

d) Digital computer

e) Hybrid methods

f) Micro-machines and networks

g) Direct methods including the methods of Liapunov

It may be noted that the necessity of formulating the problem in more detail and getting a more accurate solution has developed almost concurrently with the development of the means of achieving this.

At the present time a comprehensive study on a large power system may require representation of a network of a few hundred busbars and a similar number of machines. Most practical studies, however, demand an accurate representation of only part of a system and the number of busbars and machines can be reduced to reasonable limits. Since present-day power systems have inherently low stability limits and modern excitation and prime mover control equipment has an important effect on the behaviour of machines in an interconnected system, it is essential to represent this equipment in sufficient detail, and thus the problem of studying a system even with a few machines can be formidable. However, once the problem has been formulated in terms of mathematical equations it becomes essentially analytical and the digital computer is ideally suited to its solution. A modern large computer can handle a system of considerable size.

In studying stability of a system we are concerned with two significant problems to be analysed - steady state and transient stability. Steady state stability is the term used in connection with studies of the behaviour of system machines under slow change of system conditions; these occur in any power system under normal operating conditions. The term transient stability is used in connection with studies of the behaviour of system machines after an abrupt large disturbance or a series of disturbances, e.g. a heavy load change or a system fault. The question of steady state stability can again depend on whether the system machines are equipped with conventional slow acting automatic voltage regulators - whose effects on stability properties are little different from those of manual regulators - or modern fast acting ones. In the simple form of a power system comprising a synchronous machine connected to an infinite bus, the steady state stability limit is reached when the rotor angle is somewhere near 90°, when the machine is fitted with a slow acting voltage regulator. With a fast acting voltage regulator, stable operation with a rotor angle as high as 140° can be obtained. This artificial form of stability is termed dynamic stability. Nowadays, since virtually all machines in a power system are fitted with some form of fast acting voltage regulator, there is really no need to distinguish between the terms.

Since a power system can be described completely by a set of differential and algebraic equations, studying the stability of a power system is equivalent to studying the stability properties of the set of mathematical equations describing the system with appropriate constraints imposed upon it. A power system network is usually satisfactorily represented, for stability study purposes, by a set of algebraic equations, the number of equations depending on the size of the system. The various machines in a system, on the other hand, must be represented by a set of ordinary differential and algebraic equations, and the number and complexity of the equations depend not only on the size of the system but also on the degree of detail required in describing the machine behaviour. If the system is operating near the stability limit and the accompanying control equipment has marked influence on the machine operation, the study will demand machine and control equipment representation in much more detail than otherwise. However, the question arises as to how far one should go into details of representation. A balance must be obtained between the amount of effort involved in obtaining the solution, the accuracy required, and the availability of accurate data.

As the equations describing the system are non-linear, formal solution becomes impracticable and some sort of numerical or other techniques must be employed to obtain a solution and then establish stability or instability from it. Alternatively, some special technique may be used to check for stability or instability without having to solve the equations. Also when using numerical techniques to solve the equations the task becomes tremendous because of the size and complexity of the problem. Methods of improved efficiency may, however, be devised to ease the situation. It is the object of the present thesis to investigate some of the mathematical methods available and already used in solving power system stability problems and introduce new improvements from the point of view of both time saving and accuracy. It is also the object to record some new techniques to solve certain problems, a rigorous solution of which has rarely been attempted previously because of the extreme complexity.

Throughout the thesis stress is laid on the efficient use of a digital computer which has been accepted as the ideal for solving power system problems. Theories and equations have been developed up to the point where a digital computer can be used most efficiently to obtain the final answer. In the first chapter a discussion and review of various synchronous machine representations is given and merits and demerits of each are pointed out. This is followed by a description of a multi-machine transient stability program incorporating all the machine models discussed. Standard numerical technique for solving ordinary non-linear differential equations on a computer is used.

Although the digital approach to the solution of power system equations has been widely accepted as an ideal one, studying a system of considerable size can be very time-consuming and overall accuracy may be questionable. It would be useful to be able to use some sort of direct method to establish stability or instability without going into detailed solution. Liapunov's theorems on stability and instability offer such a possibility. In the next three chapters an attempt is made to explore this.

With the growth in use of fast acting automatic voltage regulators employing wide varieties of possible feedbacks, it has been necessary to investigate their effects thoroughly to ensure that their presence does not impair stability during normal steady state operation. This is especially true in an interconnected system where machines are fitted with different types of complex A.V.R.s and prime mover controls. Unless the control parameters are properly selected these may transform a stable system into an unstable one from the steady state stability point of view, although they may apparently be beneficial for improvement of transient stability. Thus the question of dynamic stability has become an increasingly important topic for investigation in recent years. In studying dynamic stability in a multi-machine system a state space approach has certain advantages over other methods and this forms the subject matter for chapters 5 and 6.

Finally, a short chapter is included to discuss optimisation study in a multi-machine system.

CHAPTER 1.

NUMERICAL SOLUTION OF THE TRANSIENT STABILITY PROBLEM

1.1 INTRODUCTION

In any power system, stability is mainly concerned with the dynamics of synchronous machines. Naturally, therefore, representation of synchronous machines has received a great deal of attention in the process of development of power system stability theory. In early stability studies, synchronous machines were represented exclusively by constant voltage behind either synchronous or transient reactance depending on whether steady state or transient stability was being studied. With the development of synchronous machine theory and increase in complexity of power system networks, it has become possible as well as desirable to employ more detailed and accurate representation of synchronous machines in stability studies. By synchronous machine representation here is meant the representation of the various magnetic circuits including effects of saturation, as well as associated excitation and prime mover control equipment. Although with the use of large modern computers it is possible to represent a synchronous machine with its control equipment in great detail, it may, from the point of view of total computing time, accuracy in available data and overall accuracy in the computation itself in a multi-machine study, be desirable to introduce simplifications of various kinds but taking care not to sacrifice so much that the overall picture of the system behaviour is impaired. It is therefore important to investigate thoroughly the various simplifications possible and how they compare with more detailed models in terms of accuracy and computing time. A number of synchronous machine representations suitable for power system stability studies are discussed, and comparisons made between them, in the next few sections. Results of field tests described in a later section are used as a basis for comparisons.

1.2 DESCRIPTION OF VARIOUS SYNCHRONOUS MACHINE REPRESENTATIONS

In the following descriptions the synchronous machine representations are numbered Model 1, 2, etc., based on the machine equations. When any particular model, for instance Model 1, uses alternative control equipment it is numbered Model 1A, 1B, etc.

1.2.1 Model 1

This model is based on Park's generalised two axis flux linkage and voltage equations[1-6] which are obtained by applying a certain axis transformation to the actual 3-phase equations of the idealized synchronous machine. Actually the transformation resolves the stator quantities into components along the pole and interpole axes, known as direct and quadrature axes, and identified by the subscripts d and q respectively, and also a component relating to the zero-sequence effects. However, only the modes of operation which do not require zero-sequence components are considered. In the general model, paths for current flow in the rotor iron, and in the damper windings if present, are represented by a number of additional rotor windings on both the direct and quadrature axes. However, for simplicity, only one additional rotor winding on each axis is considered. Although it is only an approximation most of the known synchronous machine behaviour can be explained on this basis. The equations, in per unit form, are given by:-

Flux linkage equations:

$$\Psi_{fd} = x_{ffd}\, i_{fd} - x_{ad}\, i_d + x_{ad}\, i_{1d} \qquad 1.1$$

$$\Psi_d = x_{ad}\, i_{fd} - x_d\, i_d + x_{ad}\, i_{1d} \qquad 1.2$$

$$\Psi_{1d} = x_{ad}\, i_{fd} - x_{ad}\, i_d + x_{11d}\, i_{1d} \qquad 1.3$$

$$\Psi_q = - x_q\, i_q + x_{aq}\, i_{1q} \qquad 1.4$$

$$\Psi_{1q} = - x_{aq}\, i_q + x_{11q}\, i_{1q} \qquad 1.5$$

In this and all other models throughout the thesis, positive armature current corresponds to generator action.

The voltage equations are:

$$v_{fd} = \frac{1}{\omega_o} p\,\Psi_{fd} + r_{fd}\, i_{fd} \qquad 1.6$$

$$v_d = \frac{1}{\omega_o} p\,\Psi_d - \frac{\omega}{\omega_o}\Psi_q - r_a i_d \qquad 1.7$$

$$0 = \frac{1}{\omega_o} p\,\Psi_{1d} + r_{1d}\, i_{1d} \qquad 1.8$$

$$v_q = \frac{1}{\omega_o} p\,\Psi_q + \frac{\omega}{\omega_o}\Psi_d - r_a i_q \qquad 1.9$$

$$0 = \frac{1}{\omega_o} p\,\Psi_{1q} + r_{1q} i_{1q} \qquad 1.10$$

The air gap torque is given by

$$T_e = \Psi_d\, i_q - \Psi_q\, i_d \qquad 1.11$$

For this and subsequent models the equation of motion of the rotor is given by

$$M\, p^2\,\delta = T_m - T_e - K_d \frac{p\delta}{\omega_o} \qquad 1.12$$

and the machine terminal voltage by

$$V_t = \sqrt{(v_d^{\,2} + v_q^{\,2})} \qquad 1.13$$

In equation 1.12 T_m is the machine input torque and K_d is a coefficient to account for mechanical damping. Other damping terms can be included if desired. However, in this particular machine representation inclusion of other damping terms is not necessary, since most of them have been accounted for fairly satisfactorily in the machine equations. In the subsequent representations which are derived after applying certain simplification to the above set of equations it may be necessary to include other damping torques in a special manner.

1.2.2 Model 2

The above equations can be combined into operational forms eliminating the rotor currents and after suitable algebraic manipulations, as shown in appendix A 1.1, the following equations are obtained:

$$pe_d'' = \left\{ (x_q - x_q'')\, i_q - e_d'' \right\} / T_{qo}'' \qquad 1.14$$

$$pe_q' = \left\{ v_f - (x_d - x_d'')\, i_d - e_q'' \right\} / T_{do}' \qquad 1.15$$

$$pe_q'' = \left[(e_q' - (x_d' - x_d'')\, i_d - e_q'' - \left\{ T_1 (e_q'' - x_d'' i_d) + T_2\, x_d i_d - T_{kd}\, v_f \right\} / T_{do}' \right] / T_{do}'' \qquad 1.16$$

and

$$v_d = \frac{1}{\omega_o}\, p\, (e_q'' - x_d'' i_d) + \frac{\omega}{\omega_o}\, (e_d'' + x_q'' i_q) - r_a i_d \qquad 1.17$$

$$v_q = \frac{1}{\omega_o}\, p\, (-e_d'' - x_q'' i_q) + \frac{\omega}{\omega_o}\, (e_q'' - x_d'' i_d) - r_a i_q \qquad 1.18$$

In the above two equations $\frac{1}{\omega_o}\, p\, i_d$ and $\frac{1}{\omega_o}\, p\, i_q$ terms are usually negligible compared with the others, and so the equations reduce to[7]

$$\frac{1}{\omega_o}\, p\, e_q'' + \frac{\omega}{\omega_o}\, e_d'' = v_d + r_a i_d - \frac{\omega}{\omega_o}\, x_q'' i_q \qquad 1.19$$

$$-\frac{1}{\omega_o}\, p\, e_d'' + \frac{\omega}{\omega_o}\, e_q'' = v_q + r_a i_q + \frac{\omega}{\omega_o}\, x_d'' i_d \qquad 1.20$$

Equations 1.14 to 1.16 and 1.19 and 1.20 can be written in a slightly simplified form[8] after neglecting the less significant terms.

$$p\, e_d'' = \left\{ (x_q - x_q'')\, i_q - e_d'' \right\} / T_{qo}'' \qquad 1.21$$

$$p\, e_q' = \left\{ v_f - (x_d - x_d')\, i_d - e_q' \right\} / T_{do}' \qquad 1.22$$

$$p\, e_q'' = \left\{ e_q' - (x_d' - x_d'')\, i_d - e_q'' \right\} / T_{do}'' \qquad 1.23$$

and

$$e_d'' = v_d + r_a\, i_d - x_q''\, i_q \qquad 1.24$$

$$e_q'' = v_q + r_a\, i_q + x_d''\, i_d \qquad 1.25$$

The air gap torque for the above model is given by

$$T_e = e_d''\, i_d + e_q''\, i_q - (x_d'' - x_q'')\, i_d\, i_q \qquad 1.26$$

1.2.3 Model 3

In model 2, effects of damper windings on both direct and quadrature axes are considered. Since the influence on machine performance of damper windings on the direct axis is much less than that of damper windings on the quadrature axis, a further simplification can be made by neglecting the effect of d-axis dampers. On this basis the following set of equations is obtained[9,10]:

$$p\, e_d'' = \left\{ (x_q - x_q'')\, i_q - e_d'' \right\} / T_{qo}'' \qquad 1.27$$

$$p\, e_q' = \left\{ v_f - (x_d - x_d')\, i_d - e_q' \right\} / T_{do}' \qquad 1.28$$

$$e_d'' = v_d + r_a\, i_d - x_q''\, i_q \qquad 1.29$$

$$e_q' = v_q + r_a\, i_q + x_d'\, i_d \qquad 1.30$$

The air gap torque is given by

$$T_e = e_d''\, i_d + e_q'\, i_q \cdot (x_d' - x_q'')\, i_d\, i_q \qquad 1.31$$

2.1.4 Model 4

A still further simplification can be achieved by neglecting the effects of all damper windings and the following set of equations[12-19] follows:

$$p\, e_q' = \left\{ v_f - (x_d - x_d')\, i_d - e_q' \right\} / T_{do}' \qquad 1.32$$

$$0 = v_d + r_a\, i_d - x_q\, i_q \qquad 1.33$$

$$e_q' = v_q + r_a\, i_q + x_d'\, i_d \qquad 1.34$$

The air gap torque is given by

$$T_e = e_q'\, i_q - (x_d' - x_q)\, i_d\, i_q \qquad 1.35$$

Formulae are available to account for damping torques, if desired, in an approximate way[3, 13, 20].

1.2.5 Model 5

This is the classical representation of a synchronous machine by a voltage constant in magnitude behind transient reactance[12, 13].

The equations are

$$0 = v_d + r_a\, i_d - x_d'\, i_q \qquad 1.36$$

$$e_q' = v_q + r_a\, i_q + x_d'\, i_d \qquad 1.37$$

Air gap torque $T_e = e_q'\, i_q$ 1.38

The justification in using this model is that in many cases during the transient period of interest this voltage behind transient reactance remains constant, at least approximately, due to the action of automatic voltage regulator which tends to cancel the effect of flux decay.

The assumption that the phase angle of the constant voltage is the same as the mechanical position of the rotor is not correct. This is, however, justified in multi-machine stability studies by the fact that as far as the variations of relative rotor angles are concerned this model and the more detailed ones often exhibit very similar patterns.

In stability studies quite often machines away from the centre of disturbance can be adequately represented by this model and a saving in computing time can be achieved due to the reduction in the number of equations.

1.3 SATURATION

Saturation can be accounted for in a manner based on the method described in reference 5. The basic assumptions made are:

a) The iron saturation is not influenced by the leakage fluxes and is determined solely by the mutual flux.

b) The leakage reactances are unaffected by saturation, thus the only change is in the mutual reactances.

c) Direct and quadrature axis mutual reactances are equally affected due to saturation.

d) A measure of saturation can be obtained from a knowledge of the air gap voltage.

The method can be applied directly when using machine model 1. At any stage x_{ad} and x_{aq} are expressed as k x_{ad1} and k x_{aq1}, where x_{ad1} and x_{aq1} are the unsaturated values of x_{ad} and x_{aq} respectively and k is a non-linear function of m.m.f. given by

$$k = f\left\{\sqrt{x_{ad1}^2 \, (i_{fd} + i_{1d} - i_d)^2 + x_{aq1}^2 \, (i_{1q} - i_q)^2}\right\} \qquad 1.39$$

The open circuit saturation curve can be conveniently used for the purpose of calculating k.

When applying this method to machine models 2 - 4, the machine differential equations have to be rewritten, expressing the reactances and time constants in terms of mutual and leakage reactances. At any instant saturated values of the mutual reactances, which can be calculated from the current value of the saturation factor k, have to be used in the equations. The saturation factor k can be found from a knowledge of the air gap voltage as found from armature leakage reactance and terminal voltage and current conditions, and the open circuit saturation curve.

The author has shown previously[21], that saturation has very little effect in influencing transient stability. Very little difference was observed in the machine behaviour between the case when saturation was considered in full detail using the method described above and the case when calculations were performed using machine constants measured at full terminal voltage and considering saturation only to calculate initial steady state values of the machine quantities using appropriate saturated values of the mutual reactances. For this reason saturation has not been considered in developing the multi-machine transient stability program incorporating various synchronous machine models, described in a later section. Saturation can, however, be included easily in the program if desired.

1.4 DESCRIPTION OF THE TEST SYSTEM

Tests carried out at Goldington power station in May 1960 and reported in reference 5 are now used as a basis for comparison between the different synchronous machine representations described in section 1.2. The system is shown schematically in Fig. 1.1. It consists of a single synchronous generator with its excitation and prime mover control systems and a transmission line connecting the generator through a step-up transformer to a system of infinite capacity. The reason for selecting this example is that it represents a typical system, the test results are widely known, and the full amount of data is available to enable investigation of any of the machine representations discussed. To complete the mathematical description of the system it is also necessary to formulate the equations describing the control systems and the transmission line.

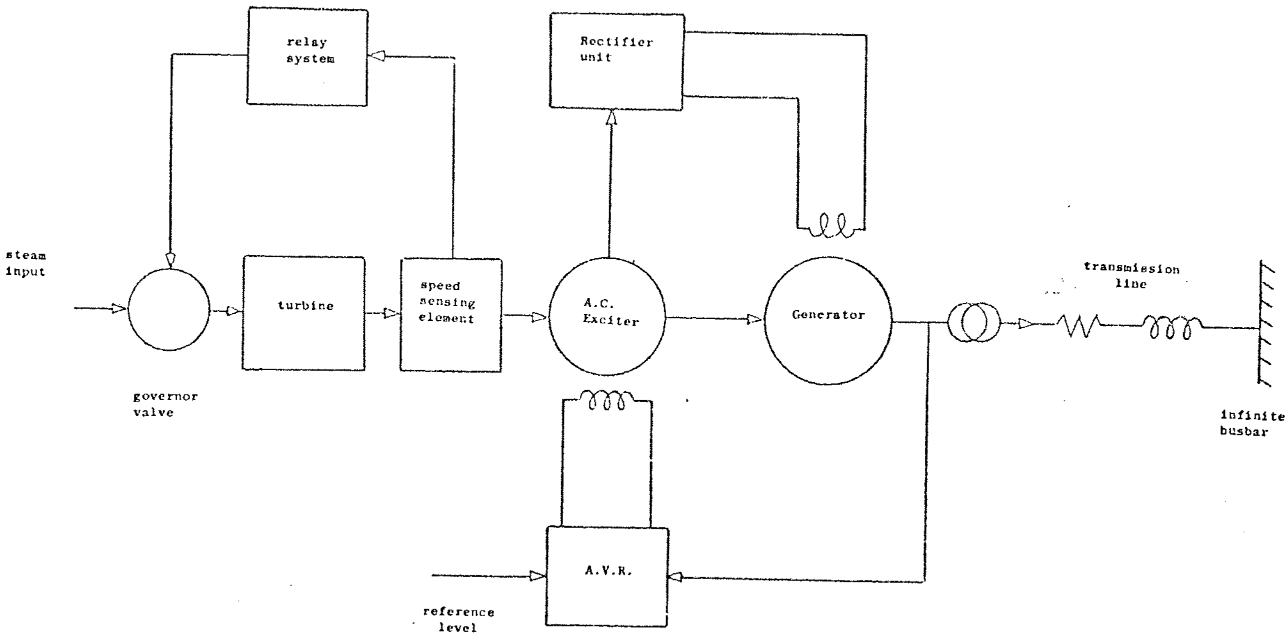


Fig. 1.1 Schematic diagram of the single-machine system studied

1.4.1 Excitation Control System

A block diagram of the excitation system is shown in Fig. 1.2. This is virtually taken from reference 5. The excitation of the main field system is supplied from an A.C. exciter through rectifiers, the exciter in turn being supplied from the voltage-regulator system of amplifiers and stabilizing feedback loops.

The main assumptions made in the formation of a set of equations for the control system are that:

a) The two stages of the magnetic amplification and the exciter may each be represented by a simple time lag.

b) Saturation of the magnetic amplifiers may be represented by limits on their output voltage.

c) The characteristics of the rectifiers are linear and affect the operation of the generator only in exceptional circumstances; e.g. when the rotor current is forced to zero.

d) The parameters of the control system remain unaltered throughout the transient period of interest.

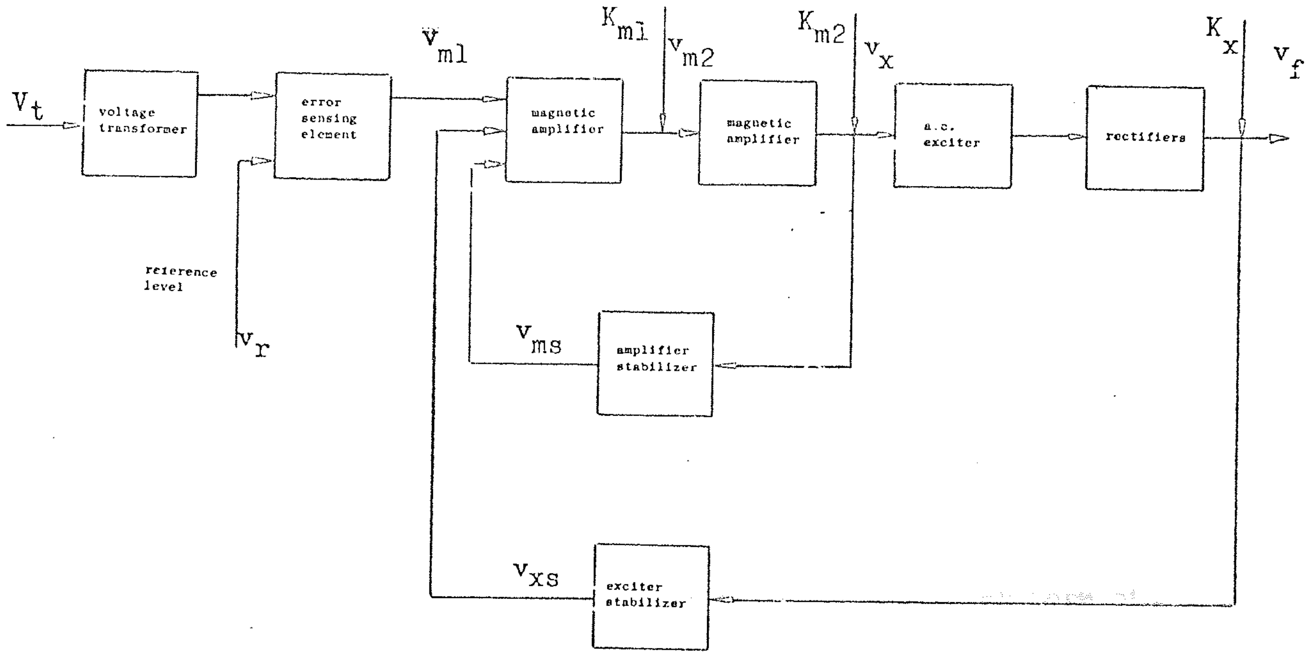


Fig. 1.2 Block diagram of excitation control system

The equations of the elements of the voltage regulator (in volts, amperes and seconds) are:

voltage sensitive circuit

$$v_{m1} = -G_{vs}\,(V_t - v_r) \qquad 1.40$$

First magnetic amplifier

$$v_{m2} = \frac{G_{m1}}{1 + \tau_{m1}p}\,(v_{m1} + v_{xs} + v_{ms}) + K_{m1} \qquad 1.41$$

$$v_{m2\ min} \leq v_{m2} \leq v_{m2\ max} \qquad 1.42$$

Second magnetic amplifier

$$v_x = \frac{G_{m2}}{1 + \tau_{m2}p}\,v_{m2} + K_{m2} \qquad 1.43$$

$$v_{x\ min} \leq v_x \leq v_{x\ max} \qquad 1.44$$

Exciter

$$v_f = \frac{G_x}{1 + \tau_x p} v_x + K_x \qquad 1.45$$

Amplifier stabilizer

$$v_{ms} = - \frac{G_{ms} \tau_{ms} p}{1 + \tau_{ms} p} v_x \qquad 1.46$$

Exciter stabilizer

$$v_{xs} = - \frac{G_{xs} \tau_{xs} p}{1 + \tau_{xs} p} v_f \qquad 1.47$$

Since the above equations are expressed in standard units of volts, amperes and seconds, two scaling factors are used to connect these equations with the machine equations which are given in per unit form.

The equations just considered give a fairly detailed representation of the excitation control system. It involves five first order differential equations and two constraints to represent amplifier saturation. In large power system studies it is often desirable to use a simpler form of excitation control representation which resembles the operation of the detailed one in so far as the variation of generator field voltage is concerned. A simple form of excitation control system, which uses one forward and one feedback control loop, is shown in Fig. 1.3. This form of excitation control has been used extensively in power system stability studies. The equations are given by

Exciter $$v_f = \frac{\mu}{1 + \tau_e p} (v_r - v_s - V_t) \qquad 1.48$$

Stabilizer $$v_s = \frac{\mu_s p}{1 + \tau_s p} v_f \qquad 1.49$$

An attempt was made to select appropriate values of the parameters for this control system to achieve an overall behaviour similar to that of the more detailed one. For this, both control systems were assumed to be connected to unloaded generators and responses in the field and terminal voltages were calculated after applying a step change in the reference level. It was possible to find suitable parameters fairly easily to

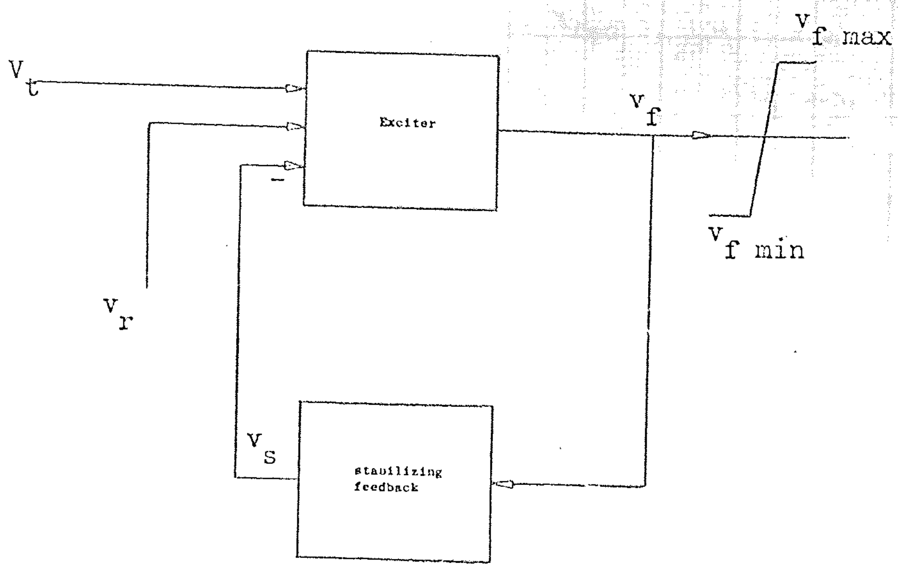


Fig. 1.3 Block diagram of simplified form of excitation control system

achieve desired results. A cut and try process was used. An examination of how the various blocks in the detailed model could be combined to reduce it to the simplified model provided a good guide for the parameter selection. The responses of the two systems are shown in Fig. 1.4. It may be seen that the field voltage response for the detailed representation is oscillatory for the initial period, the frequency of oscillations being about 5 Hz. This oscillatory behaviour cannot be expected from the simplified representation if the same overall gain is to be maintained.

Therefore parameters were selected to give a response passing through the mean of the oscillatory response of the detailed representation. These two alternative representations gave almost identical results in the transient and dynamic stability studies reported later.

1.4.2 The Governor and Turbine

A block diagram for the governor and turbine fitted to the test set is shown in Fig. 1.5 (taken from reference 5). This governor is a standard oil servo type, actuated by either the speed of the set or the speeder-gear

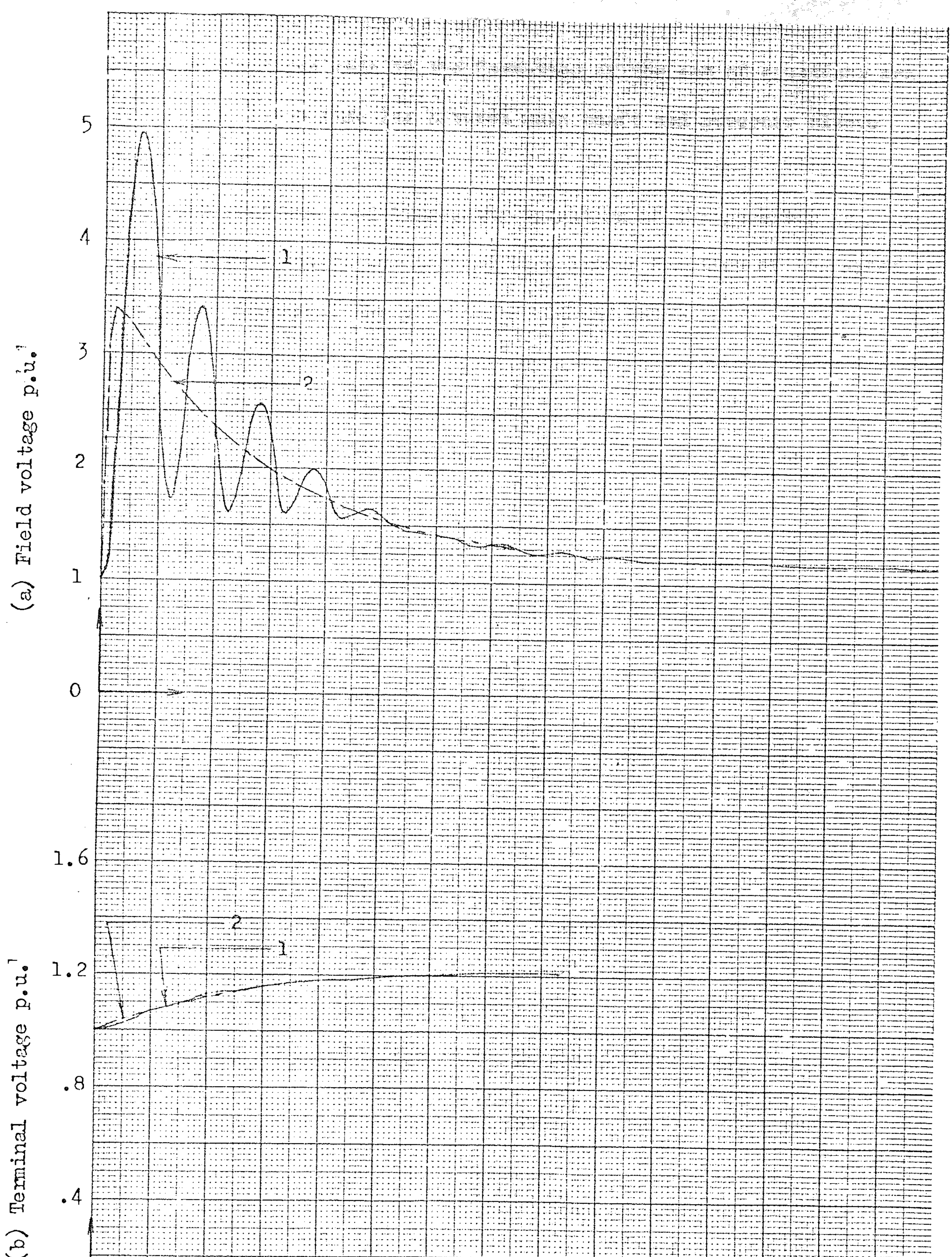


Fig. 1.4 Transient response of A.V.R. following a step change in reference voltage (20% change)

1) —— for detailed representation

2) - - - for simplified representation

motor. The assumptions made in the formation of the set of equations are

a) There is no time lag between main shaft and governor sleeve movement.

b) Under steady conditions, the operation of the governor is such that the steam admitted to the turbine is a linear function of the speed of the turbine throughout the governor's working range.

c) The pilot valve, relay valve and turbine-entrained steam may each be represented by a simple time lag.

d) The boiler may be represented by a source of steam at constant pressure and temperature.

e) The efficiency of the turbine does not change over the small range of speed in which it is normally required to operate.

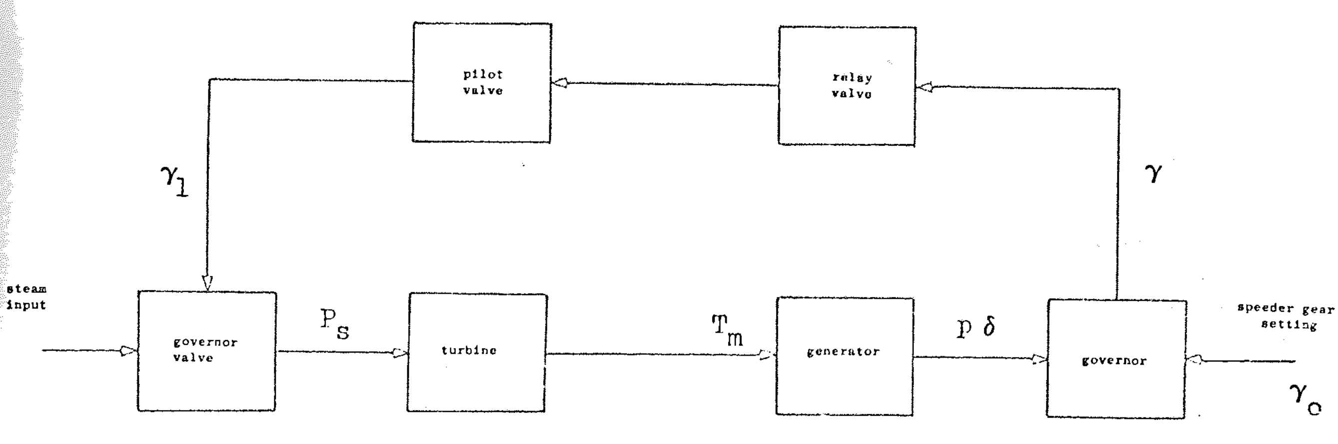


Fig. 1.5 Block diagram of governing system

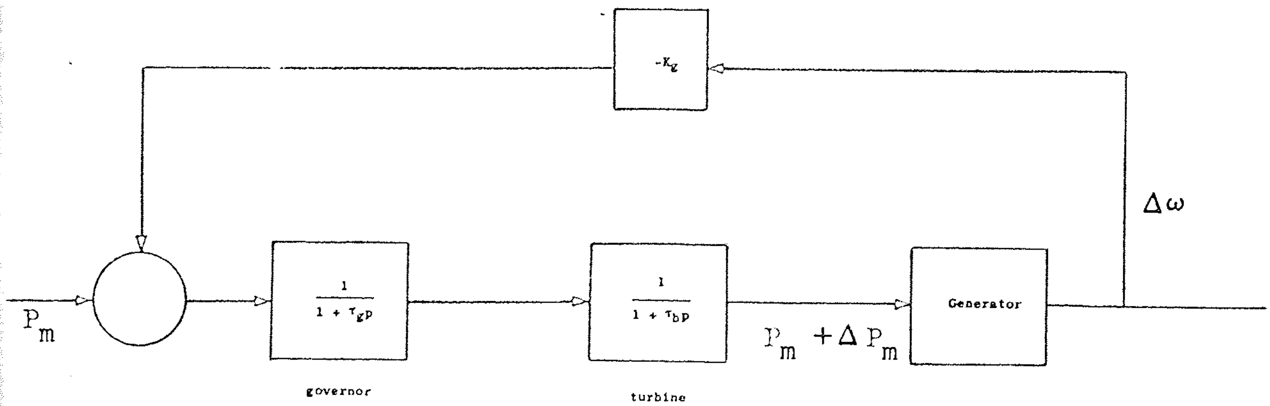


Fig. 1.6 Block diagram of simplified governing system

On this basis, the sleeve movement Υ is given by

$$\Upsilon = \Upsilon_o - G_1 \, p\delta \qquad 1.50$$

$$0 \leq \Upsilon \leq 1 \qquad 1.51$$

where Υ_o is proportional to the speeder-gear setting.

The governor-valve position Υ_1 is related to Υ by

$$\Upsilon_1 = \frac{G_2}{(1 + \tau_1 p)(1 + \tau_2 p)} \Upsilon + K_2 \qquad 1.52$$

G_2 and K_2 are constants associated with this representation of the relays.

The steam power P_s at the turbine input is given by $P_s = G_3 \Upsilon_1$

The power output from the turbine or the power input to the generator rotor, P_m, is given by

$$P_m = \frac{1}{1 + \tau_3 p} P_s \qquad 1.55$$

and the torque input to the rotor, T_m, is

$$T_m = P_m \left(1 - \frac{p\delta}{\omega_o}\right) \qquad 1.56$$

For many purposes the above governor simulation can be simplified into the following form, using an appropriate ceiling for the change in generator power input.

$$\Delta P_m = \frac{-K_g}{(1 + \tau_g p)(1 + \tau_h p)} \frac{p\delta}{\omega_o} \qquad 1.57$$

where ΔP_m is the change in generator power input, K_g is the total loop gain of the governing system, τ_g is the equivalent time constant of governor valve, pilot valve and relay valve and τ_h is the turbine delay. The arrangement is shown schematically in Fig. 1.6.

For certain purposes this can be further simplified by combining τ_g and τ_h into one time constant.

All the above governor simulations represent satisfactorily the operations of 2-cylinder type turbines not including reheating or secondary governing. In turbines that include steam reheating between the high-pressure and the intermediate-pressure cylinders, some account of this, and also the effect of a secondary governing system where this is present, has to be taken in studies where the prime mover contribution to the transient response is significant.

1.4.3 Transmission System

In formulating the equations for the transmission system between the generator terminals and the infinite busbar the following assumptions are made:

a) Transformer magnetising and line-charging currents can be neglected since in the present case they are small.

b) The transformer and the transmission line connecting the generator to the rest of the system can be represented by lumped series inductance and resistance.

The equations, in the d- q axes of the generator are then

$$v_d = V_b \sin \delta + \frac{X_t}{\omega_o} p\, i_d + R_t\, i_d - \frac{\omega}{\omega_o} X_t\, i_q \qquad 1.58$$

$$v_q = V_b \cos \delta + \frac{X_t}{\omega_o} p\, i_q + R_t\, i_q + \frac{\omega}{\omega_o} X_t\, i_d \qquad 1.59$$

1.5 COMPARISON OF COMPUTED AND TEST RESULTS

In this section a comparison is made between the test results and the results obtained by employing various synchronous machine models discussed earlier. For most of the models, while computing the transient behaviour following a 3-phase fault, both the detailed as well as simplified representations of control systems have been used. Any particular machine model has then been referred to by a letter A or B after its number depending on whether detailed or simplified control system representation has been considered. To compare the accuracies and advantages of different machine models, test results of both a symmetrical 3-phase fault test and a load rejection test have been used.

In the 3 phase fault test, the fault was applied at the high-voltage terminals of the step-up transformer with the generator at rated load and power factor. The fault duration time was .38 second.

The load rejection test was carried out at full load and rated power factor - the conditions were slightly different from that in the 3-phase fault test (see table 1.5). The load rejection was initiated by opening the high voltage circuit breakers.

In the computations, the pi_d and pi_q terms in the transmission line equations have been neglected. This is a valid procedure in most transient stability studies since the predominating periodic times of the machine swings are large compared with the short durations of the high frequency transients to which these terms give rise in the transmission network. The increase in accuracy obtained by including these terms is, only in rare cases, sufficient to justify the much more involved, and lengthy, computational process required. Physically, their neglect is equivalent to the assumption that the static network passes from one steady state to another continuously throughout the period involved in machine swings.

1.5.1 Principal Data for Goldington Turbogenerator Unit

The principal data for the generator and transmission system are listed in Table 1.1. All values quoted are in p.u. on machine rating unless otherwise stated.

Table 1.1

GENERATOR AND TRANSMISSION SYSTEM DATA

Generator rating	37.5 MVA; 30 MW
	11.8 kV; 3000 r.p.m.
x_d	2 (unsaturated)
	1.68 (full load)
x_q	1.526 (full load)
x_d'	.27 (unsaturated)

x_d''	.17 (unsaturated)
x_q''	.18 (unsaturated)
x_{ad}	1.86 (unsaturated)
	1.54 (full load)
x_{aq}	1.674 (unsaturated)
	1.386 (full load)
x_a, x_f	.14 (unsaturated)
x_{1d}, x_{1q}	.04 (unsaturated)
r_a	.002
r_{1d}, r_{1q}	.00318
H	5.3 kWs/kVA
M	.0338
ω_o	314.159 rad/s
T_{do}'	5.96 sec
T_{do}''	.17 sec
T_{qo}''	1.6 sec
Transformer reactance	.354 on 100 MVA
Transformer resistance	.0135 on 100 MVA
Transmission system reactance	.1265 on 100 MVA
Transmission system resistance	.046 on 100 MVA

Parameters of the excitation control system as used on the test generator are listed in Table 1.2.

Table 1.2

PARAMETERS OF EXCITATION CONTROL SYSTEM

K_1	=	11800 V	K_{m2}	=	-130 V
K_2	=	106 V	$v_{x\ min}$	=	0 V
G_{vs}	=	.00159	$v_{x\ max}$	=	227 V
G_{m1}	=	52			

τ_{m1}	=	.044 sec	G_x	=	3.06
K_{m1}	=	65.4 V	τ_x	=	.2 sec
$v_{m2\ min}$	=	1.8 V	K_x	=	-15 V
$v_{m2\ max}$	=	51.6 V	G_{ms}	=	.00525
G_{m2}	=	12.2	τ_{ms}	=	.1 sec
τ_{m2}	=	.1 sec	G_{xs}	=	.0139
			τ_{xs}	=	2 sec

Parameters of the simplified form of excitation control system are listed in Table 1.3.

Table 1.3

Parameters of the simplified excitation control system

μ	=	364	τ_e	=	.35 sec
μ_s	=	.15	τ_s	=	2 sec

Parameters of the governor and turbine are listed in Table 1.4.

Table 1.4

Parameters of governor and turbine

G_1	=	.108	K_2	=	-.267
G_2	=	1.33	G_3	=	1.42
τ_1	=	.1 sec	τ_3	=	.49 sec
τ_2	=	.188 sec			

Table 1.5

Initial operating conditions for the 3-phase fault and load rejection tests

	3-phase fault		Load rejection	
Load	30	MW	30	MW
	22.5	MVAR	25.2	MVAR
Field voltage	250	V	255	V
Field current	380	A	390	A

Stator voltage	13.05 kV	12.1 kV
Rotor angle	36^{o}	39^{o}

The infinite busbar voltage was adjusted, during computation, to give the correct generator terminal conditions before each test.

Although for the purpose of comparison between different synchronous machine models the system used was a single synchronous generator connected through a transmission line to an infinite busbar, the study was carried out using a multi-machine transient stability program incorporating all the models. The present single machine-infinite busbar system is treated as a special case of a general multi-machine system. The working of this program is discussed in some detail in the next section. In the remaining part of the present section the computer results of the 3-phase fault and load rejection tests are discussed.

1.5.2 Model 1

Computer results for the 3-phase fault using Model 1 and full control system representations are shown in Figs. 1.7 and 1.9 - 1.12. Curve 2 in Fig. 1.7 shows a plot of the rotor angle versus time. The solid line curve 1 is a plot of the rotor angle as recorded during the test. Although the agreement during the first swing is fairly close, the peak of the computed curve is slightly below that of the test curve. This shows that the machine simulation provided more initial braking torque than was actually present in the test. This may be attributed to the incorrect values of damper winding resistances used in the model. During subsequent swings, however, there is marked difference between the test and computed results. Although the frequency of rotor oscillations seems to be the same in both cases, the calculated curve lies slightly above the test curve on the whole, and it appears to be less damped. This behaviour will be noticeable with all the machine models, being more prominent with the

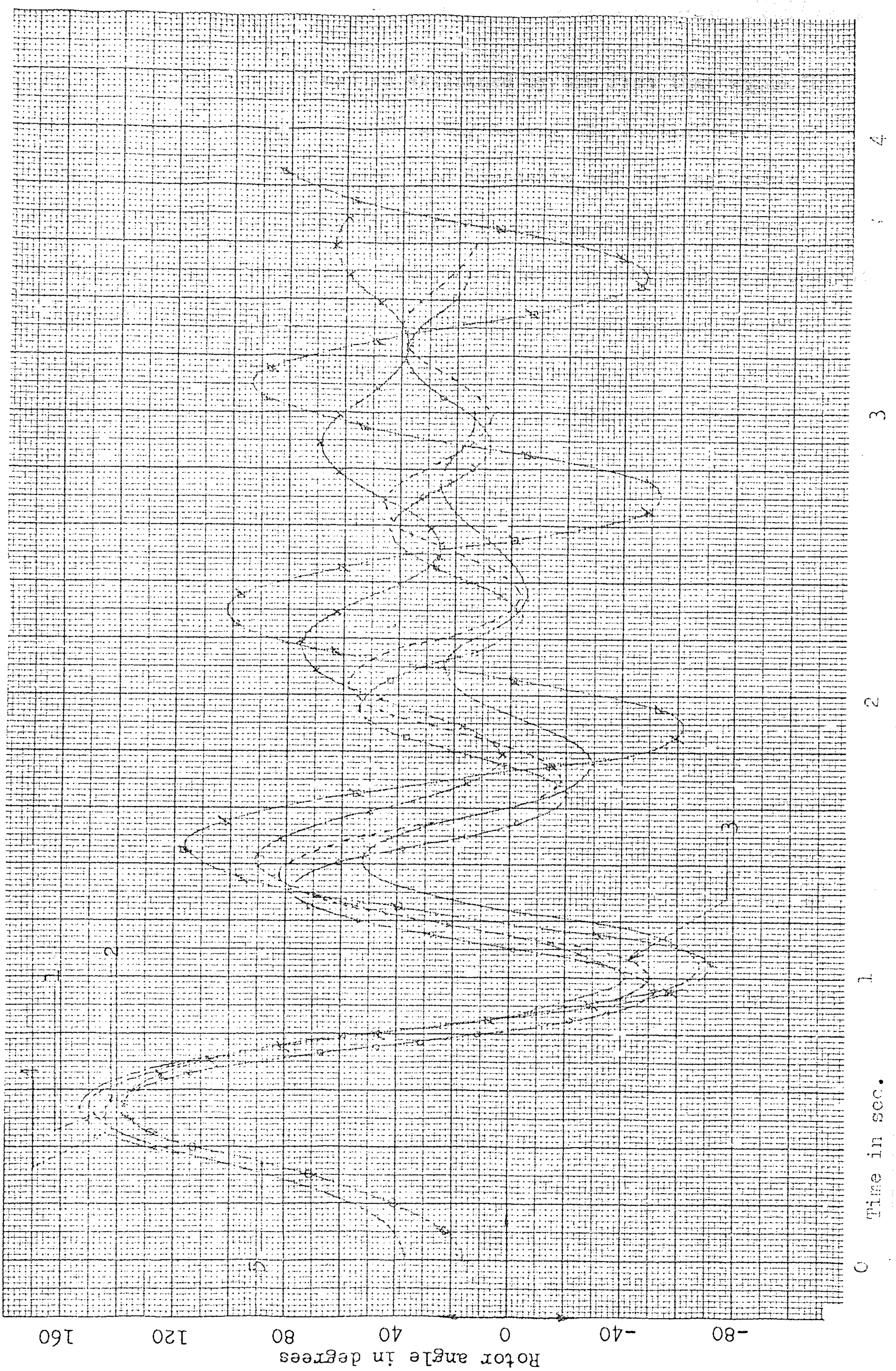


Fig. 1.7 Rotor angle transients following a three phase fault at the H.V. terminals of the step-up transformer - Generator at rated load and p.f. - fault duration .38 sec.

1) — test result; (2) ⊖⊖⊖ computed result using model 1; 3) ....computed result using model 2A; (4) ××× computed result using model 2B; (5) ⫽⫽⫽ computed result using model 5.

simpler representations. It can be explained by the fact that while the governor does not contribute significantly during the initial period of the transients because of its large time constants and the negligible rotor speed rise, it plays an important role during subsequent swings. In the present simulation a single time constant has been used to represent valve closing and opening action. In practice, however, these two are not the same. Valve closing is usually much faster than valve opening. Naturally some discrepancies are expected in the subsequent rotor angle swings.

Plot of field voltage versus time is shown in curve 2 of Fig. 1.9c. Curve 1 shows the test result. There is a great disagreement between the two. This shows that the excitation control simulation was considerably in error. However, the simulation is good enough to give fairly satisfactory agreement between the test and computed results for the more important machine quantities e.g. rotor angle, terminal voltage, field current etc.

Due to the inclusion of $p\Psi_d$ and $p\Psi_q$ terms in this particular machine model the solution for the machine voltages, currents etc. exhibit oscillations at power frequency immediately after any disturbance. The time of decay of the oscillations depends on the proximity of the disturbance to the machine terminals. The high frequency oscillation is seen in the plots of machine terminal voltage, field current and output power in Fig. 1.10 a and b, and Fig. 1.11 respectively. Due to the high machine inertia the rotor angle cannot follow this high frequency oscillation and therefore takes the mean. Test results for variations of terminal voltage and field current with time are shown by curve 1 in Fig. 1.9 a and b. The instrument used in the test recorded the mean values of the quantities. It may be seen that in Fig. 1.10 a and b, if a line is drawn through the mean of the oscillations, quite good agreement is obtained with the test result.

The advantage of representing machines by this model in transient stability studies is that it accounts automatically for the initial braking torque - caused by the induced direct current in the armature, which

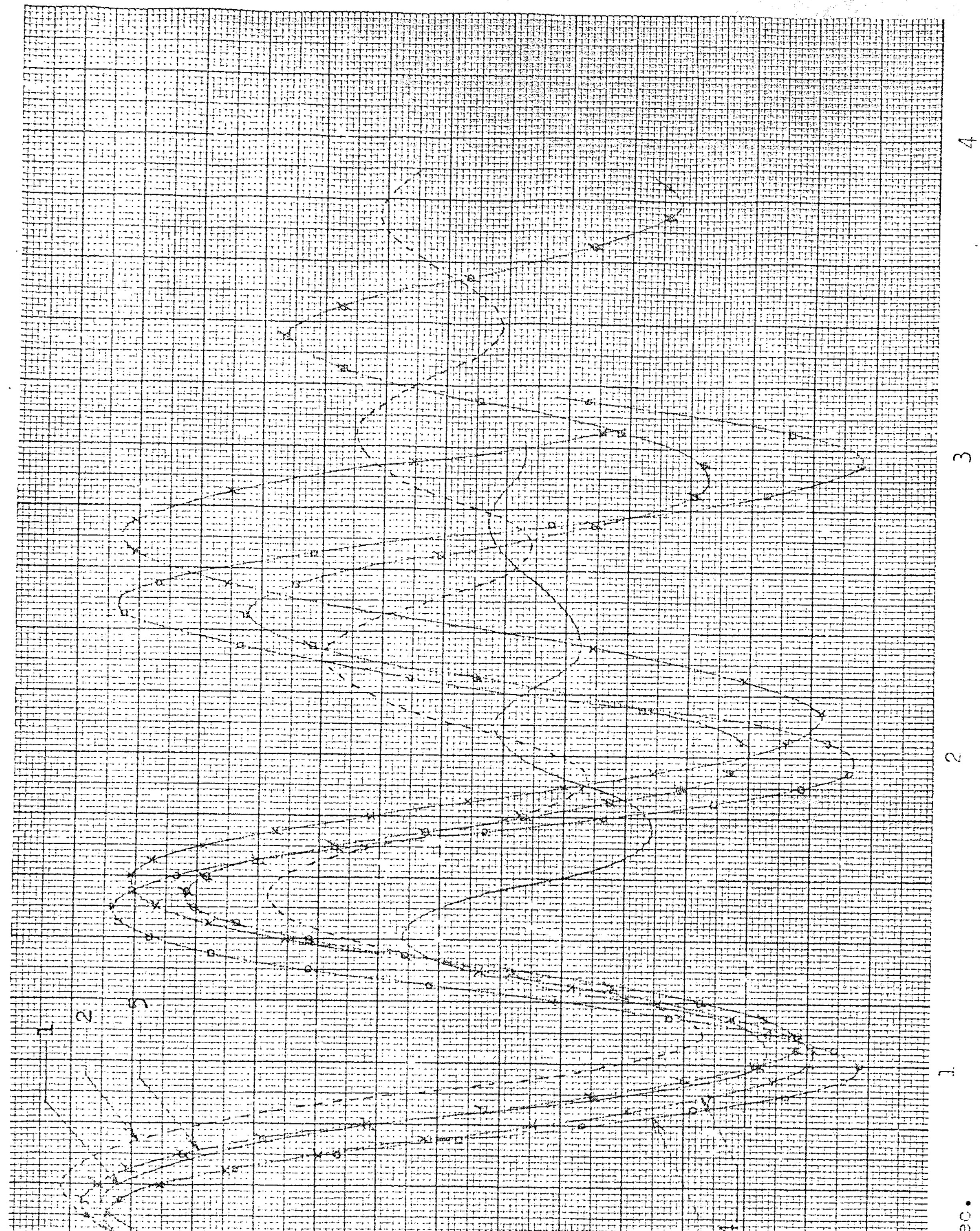


Fig. 1.8 Rotor angle transients following a three phase fault at the H.V. terminals of the step-up transformer - Generator at rated load and p.f. - fault duration .38 sec.

1) —— test result; (2) ---- computed result using model 3; 3) ⊖⊖⊖ computed result using model 4A; (4) ××× computed result using model 4B; (5) ⊬⊬ computed result using model 4A with const. input power.

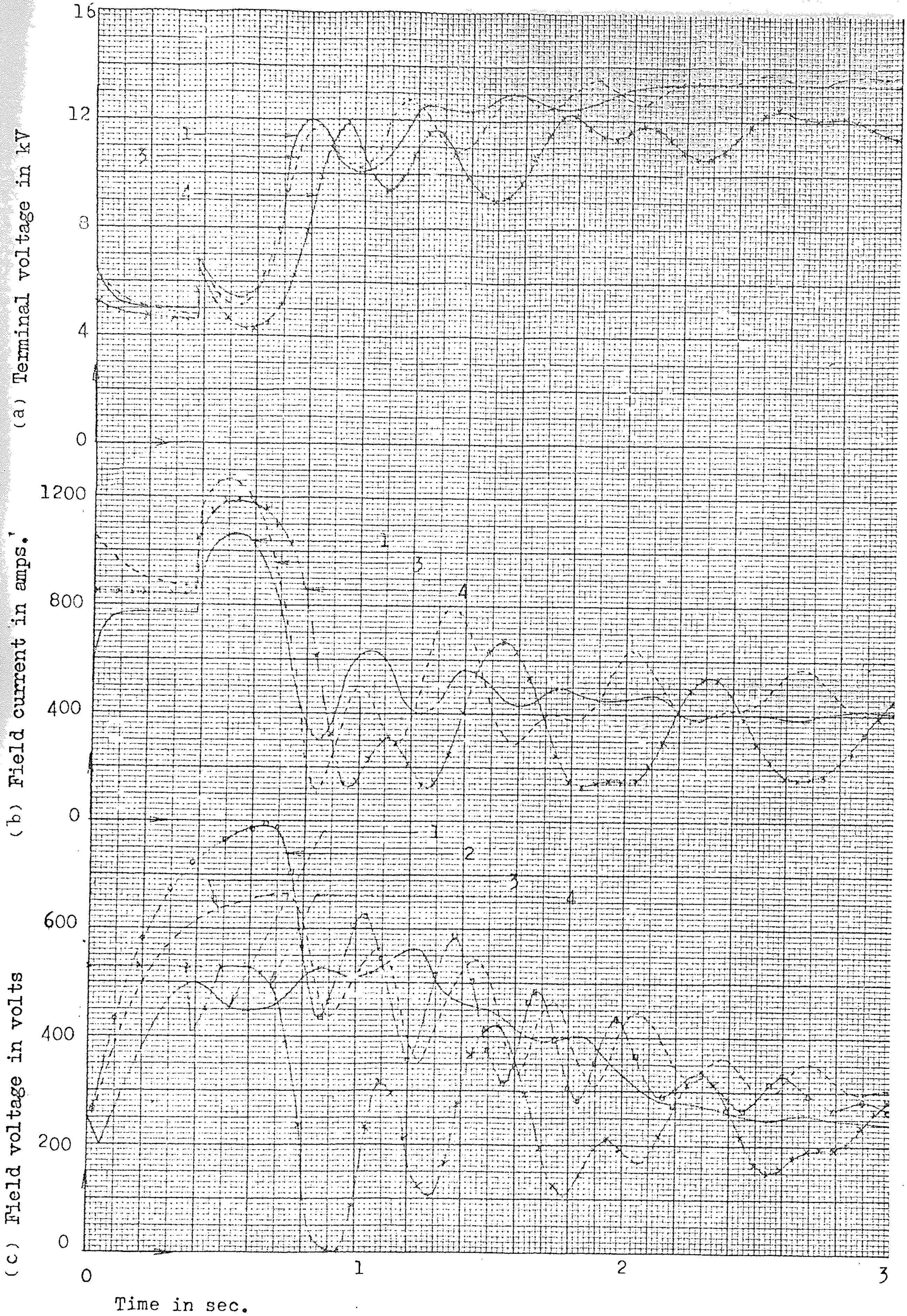


Fig. 1.9 Plots of machine terminal voltage, field current and field voltage following a three phase fault - duration .38 sec.
1)—— test result; (2)-o-o-o computed result using model 1;
3)- - - - computed result using model 2; (4)-x-x-x computed result using model 3.

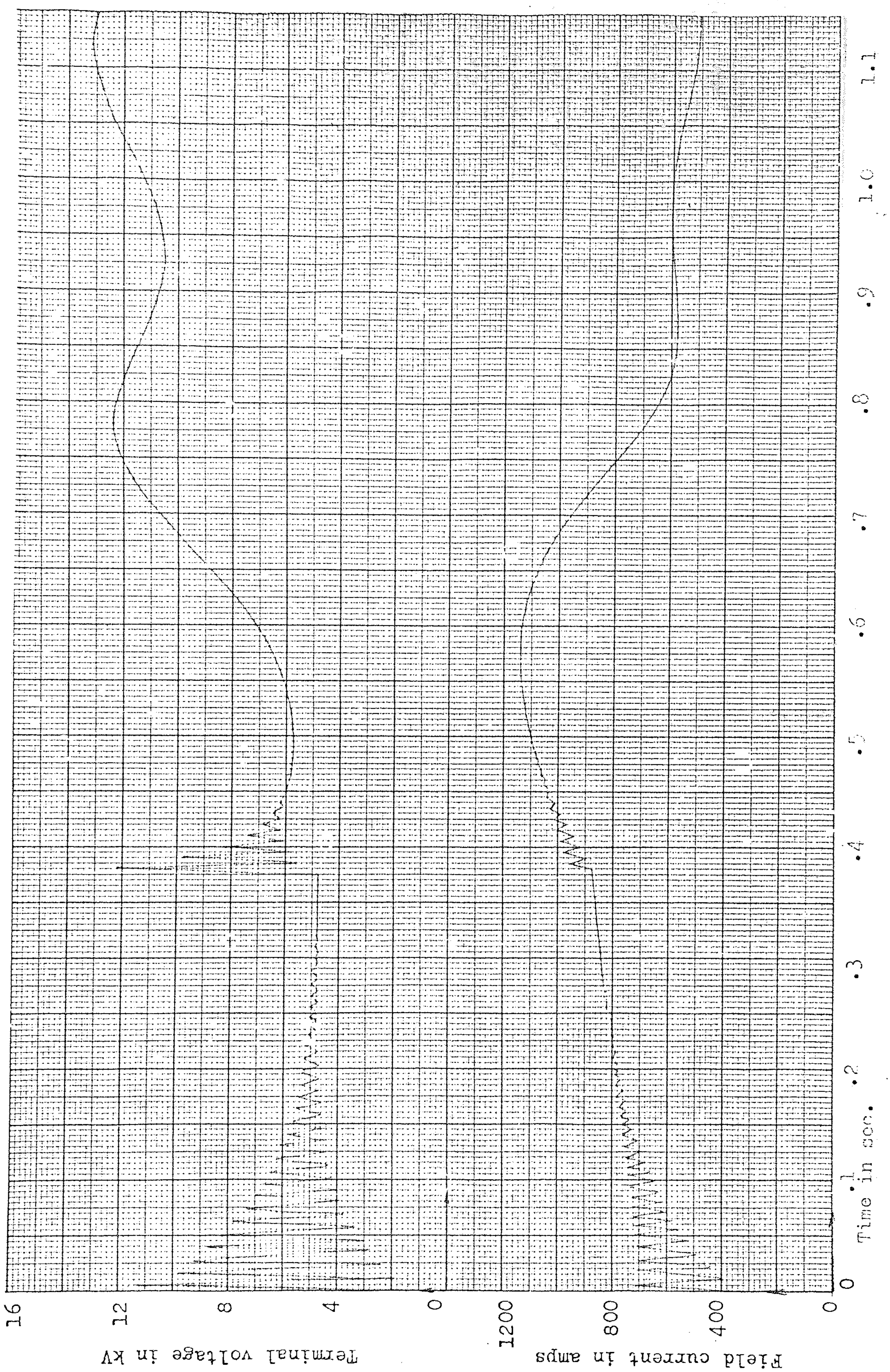


Fig. 1.10 Plots of machine terminal voltage and field current following a three phase fault - duration .38 sec.; results computed using model 1.

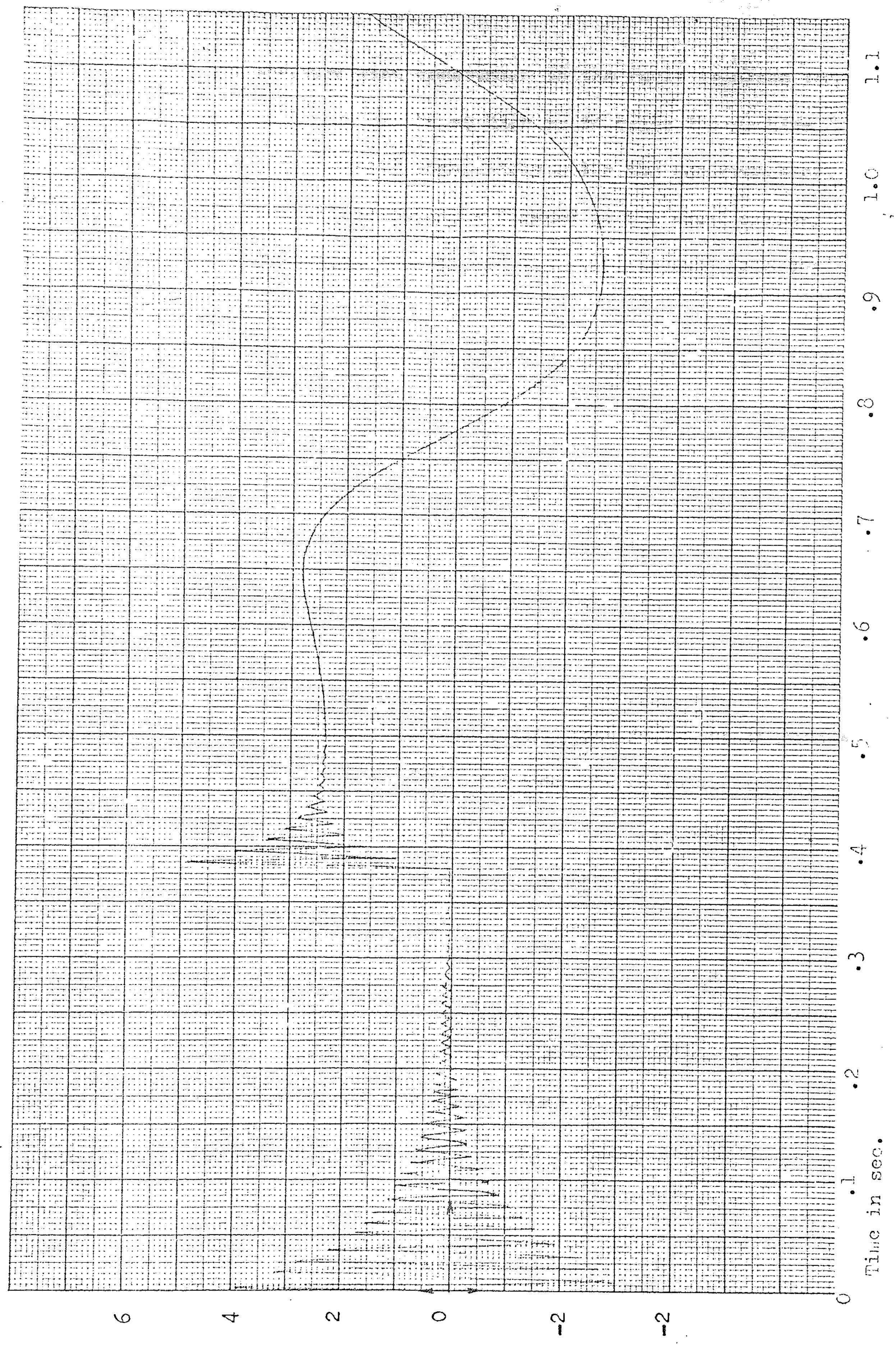


Fig. 1.11 Plot of machine output power following a three phase fault - duration .38 sec.; results computed using model 1.

decays quickly with the armature time constant. If the fault is close to the machine terminals this torque can be so high as to cause the machine to swing backwards for a brief period immediately after the occurrence of the fault before the machine starts swinging forward in the usual manner. This phenomenon is not noticeable, however, if the fault is away from the machine.

This model offers another advantage in multi-machine studies and this will be pointed out in the next section.

The main disadvantage of using this model is the necessity of using a very small time step in the numerical integration. The largest time step permissible was found to be of the order of one millisecond. Therefore general use of this model in multi-machine studies may be prohibitive.

The present model was found unsuitable for use in predicting load rejection results. This is because when there is an open circuit at the machine terminal the fluxes Ψ_d and Ψ_q do not remain continuous and therefore the differential equations involving these as integrable variable becomes invalid. The remaining machine models, however, use voltages proportional to the flux linkages in the rotor windings as integrable variables and therefore do not suffer from this drawback.

1.5.3 Model 2

In using this model to compute the machine quantities following a three phase fault both detailed and simplified forms of control system representations were considered. The swing curves obtained are shown in Fig. 1.7. Curve 3 corresponds to detailed control representation and curve 4 to the simplified one. It can be seen that in curve 3 the first swing compares very well with the test curve 1. The same type of discrepancies as found with model 1 are seen present in this model also and this can be explained as before. There is close agreement between the results obtained from the two models. Between curves 3 and 4 there is close agreement for the first two swings; thereafter they differ appreciably.

This is because the simplified governor representation did not match properly the more detailed one. This can be seen from the plot of turbine output power versus time as shown in Fig. 1.12bcurve 1 and 2. However, the agreement in the first two swings shows that it will be permissible to use simplified excitation control representation in many studies. Plots of terminal voltage, field current and field voltage with time for machine model 2A are shown in Fig. 1.9 a (curve 3), b (curve 3) and c (curve 3) respectively. While the agreement between the test and computed results in terminal voltage plot is fairly close this is not so in the cases of field current and voltage. This can be attributed to the same reason as before. Plots of output power versus time for models 2A and 2B are shown in Fig. 1.12 a, curves 1 and 2 respectively.

Model 2A was used to compute results of full load rejection. These are shown in Fig. 1.13, a, b and c. In these, terminal voltage, field current and field voltage have been plotted against time. In all these figures curve 2 corresponds to the results obtained from this model. Curve 1 shows the test result.

The advantage of using model 2 compared to model 1 lies in the fact that this permits a much larger time step to be used in the numerical integration process. An initial step length of 5 ms which was increased to 10-15 ms as computation progressed was found to give results of acceptable accuracy.

The other advantage of using this model in large multi-machine studies will be discussed in the next section.

1.5.4 Model 3

This model is the simplified version of model 2 with effect of damper winding on the direct axis neglected. Therefore this model can be expected to give results very similar to those using model 2 but on the slightly pessimistic side as far as stability is conerned. Swing curves obtained

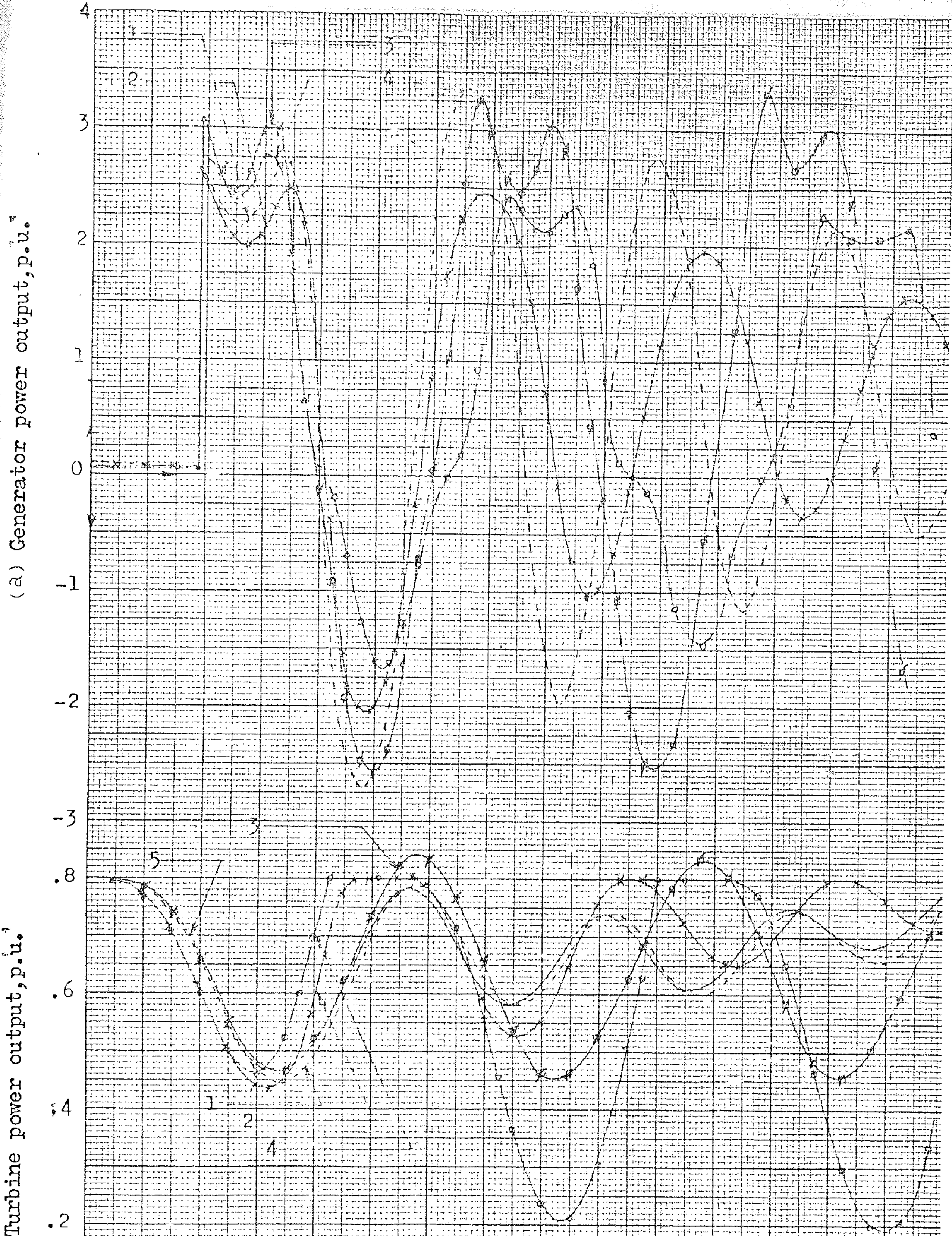


Fig. 1.12 Plots of machine output power and turbine output power following a three phase fault - duration .38 sec.
1)----computed result using model 2A; (2)×××computed result using model 2B; (3)computed result using model 4A; 4)ooo computed result using model 4B; (5)—— computed result using model 1.

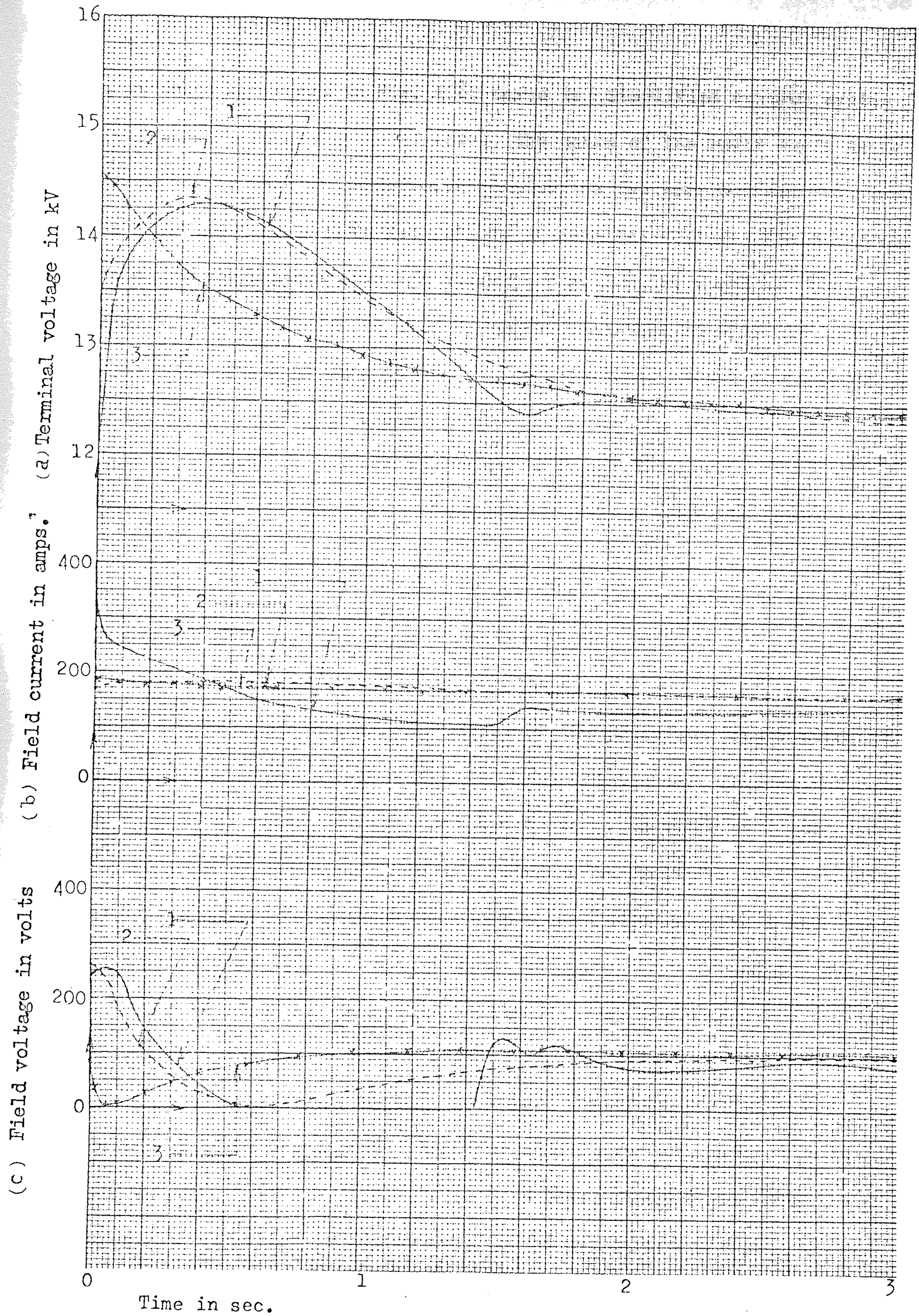


Fig. 1.13 Plots of machine terminal voltage, field current and field voltage following a 30 MW load rejection
1) —— test result; (2)--- -computed result using model 2;
3) ×-×-× computed result using model 3.

from this model are shown in Fig. 1.8, curve 2. Simplified control system representations were used. As expected, the peaks of the rotor angle swing are placed higher than those corresponding to the two previous models. The oscillations are also less damped. Plots of terminal voltage, field current and field voltage against time are shown in Fig. 1.9, a, b and c. The agreement with test results is slightly worse than that in model 2.

This model was also used to study load rejection. Results are shown as curve 3 in Fig. 1.13, a, b and c. The discrepancies are as expected.

With this model it is possible to use a larger integration step than with model 2 since the small direct axis subtransient time constant is absent from the equations.

1.5.5 Model 4

In this model damper windings on both direct and quadrature axes are neglected. To simulate the damping effect of the additional rotor windings the damping term in the equation of motion was considered. In previous calculations K_d in equation 1.12 was taken as zero. A value of 2 was given to this while using model 4. Transient rotor angle plots for both detailed and simplified control system representation are shown as curve 3 and 4 respectively in Fig. 1.8. It may be seen that apart from agreement with test result in the first swing the result is highly unsatisfactory. The amplitude of oscillation tends to remain unchanged. It was thought in both cases governor similation was responsible for this. Therefore another set of results was obtained keeping the generator input power constant. The result is shown as curve 5 of Fig. 1.8. It is seen that although the first swing increases slightly there is improvement in the subsequent swings. It is evident, however, that this model is so far the least satisfactory.

1.5.6 Model 5

This is the conventional network analyser representation of synchronous machine in power system stability studies. The rotor angle transient calculated using the model is shown as curve 5 in Fig. 1.7. Because of the special representation initial angle (which is not the true rotor angle) is much lower than that for other models. Swings are also much less damped. Evidently it is not very meaningful to make direct comparison between this curve and that obtained from any other model. The results from this model have to be interpreted differently.

1.6 MULTIMACHINE STUDIES

In this section an examination is made of the use of various synchronous machine models discussed earlier, in relation to multi-machine studies. It will be observed that all the machine models offer certain advantages and disadvantages as far as accuracy of solution and computational time are concerned. Machine models 2, 3 and 4 involve transient or subtransient saliency and this introduces complication in solving network and machine algebraic equations during integration steps. An iterative method is described which reduces the computing time to a great extent. Later in the section a multi-machine transient stability program is described which incorporates all the synchronous machine models discussed and an induction machine model. Finally some results of a typical multi-machine study are presented.

1.6.1 Use of Model 1 in Multi-machine Studies

It is convenient to rewrite equation 1.6 in the form

$$v_f = T_{do}' \frac{x_{ad}}{x_{ffd}} p\Psi_{fd} + x_{ad} i_{fd} \qquad 1.60$$

where $v_f = v_{fd} \dfrac{x_{ad}}{r_{fd}}$, is the field voltage referred to the armature circuit and $T_{do}' = \dfrac{x_{ffd}}{\omega_o r_{fd}}$, is the open circuit transient time constant.

For each machine, substituting the expressions for the flux linkages, given by equations 1.1 to 1.5, into the voltage equations 1.6 to 1.10 and rearranging, the following matrix equation is obtained.

$$\begin{bmatrix} v_f \\ v_d \\ 0 \\ v_q \\ 0 \end{bmatrix} = \begin{bmatrix} x_{ad} & & & & \\ & -r_a & & \frac{\omega}{\omega_o} x_q & -\frac{\omega}{\omega_o} x_{aq} \\ & & r_{1d} & & \\ \frac{\omega}{\omega_o} x_{ad} & \frac{-\omega}{\omega_o} x_d & \frac{\omega}{\omega_o} x_{ad} & -r_a & \\ & & & & r_{1q} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_d \\ i_{1d} \\ i_q \\ i_{1q} \end{bmatrix} +$$

$$\begin{bmatrix} T_{do}'\, x_{ad} & -T_{do}' \frac{x^2_{ad}}{x_{ffd}} & T_{do}' \frac{x^2_{ad}}{x_{ffd}} & & \\ \frac{x_{ad}}{\omega_o} & \frac{-x_d}{\omega_o} & \frac{x_{ad}}{\omega_o} & & \\ \frac{x_{ad}}{\omega_o} & \frac{-x_{ad}}{\omega_o} & \frac{x_{11d}}{\omega_o} & & \\ & & & -\frac{x_q}{\omega_o} & \frac{x_{aq}}{\omega_o} \\ & & & -\frac{x_{aq}}{\omega_o} & \frac{x_{11q}}{\omega_o} \end{bmatrix} \begin{bmatrix} p\, i_{fd} \\ p\, i_d \\ p\, i_{1d} \\ p\, i_q \\ p\, i_{1q} \end{bmatrix} \qquad 1.61$$

or in short $$v = Z_1\, i + Z_2\, pi \qquad 1.62$$

This gives $$pi = Z_2^{-1}\, v - Z_2^{-1}\, Z_1\, i \qquad 1.63$$

writing equation 1.12 in first order form

$$p\delta = y \qquad 1.64$$

$$py = (T_m - T_e - K_d \frac{y}{\omega_o}) \;/\; M \qquad 1.65$$

where $$T_e = (x_{ad}\, i_{fd} - x_d\, i_d + x_{ad}\, i_{1d})i_q - (-x_q\, i_q + x_{aq}\, i_{1q})i_d \qquad 1.66$$

Similarly the equations representing the control systems can be written down as first order differential equations along with the appropriate constraints, after suitable rearrangement.

The calculation starts with known initial values of the variables on the right hand side of the differential equations 1.63 to 1.65. During forward integration new values of the variables on the left hand side of the equations are calculated. Machine stator currents i_d and i_q are then used to calculate v_d and v_q from the network equations as explained later in this section. All the new values of the variables on the right hand side are known and thus the calculation can proceed repetitively.

It is, however, more straightforward to deal with the differential equations 1.6 to 1.10 in terms of flux linkages as they are. Rewriting them in the form suitable for numerical solution

$$p\Psi_{fd} = (v_f - x_{ad}\, i_{fd}) \,/\, (T_{do}'\ \frac{x_{ad}}{x_{ffd}}) \qquad 1.67$$

$$p\Psi_d = \omega_o\, v_d + \omega_o\, r_a\, i_d + \omega\Psi_q \qquad 1.68$$

$$p\Psi_{1d} = -\,\omega_o\, r_{1d}\, i_{1d} \qquad 1.69$$

$$p\Psi_q = \omega_o\, v_q + \omega_o\, r_a\, i_q - \omega\Psi_d \qquad 1.70$$

$$p\Psi_{1q} = -\omega_o\, r_{1q}\, i_{1q} \qquad 1.71$$

it can be seen that new values of flux linkages will be obtained during forward integration. From these, machine currents can be calculated from equations 1.1 to 1.5.

A four stage Runge-Kutta method has been used in solving these and other differential equations discussed in this thesis. The process requires calculating the variables four times during each step.

In multi-machine calculations some transformations of voltages and currents are necessary. This is because all the network quantities are expressed with reference to a set of two synchronously rotating axes separated by 90^o, called the network reference frame, whereas the machine quantities are expressed with reference to axes fixed on the individual rotor. The relationship between the axis voltages in machine and network reference frames is given by, from Fig. 1.14.

$$\begin{bmatrix} v_D \\ v_Q \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad 1.72$$

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} v_D \\ v_Q \end{bmatrix} \qquad 1.73$$

Similar relations exist for the currents.

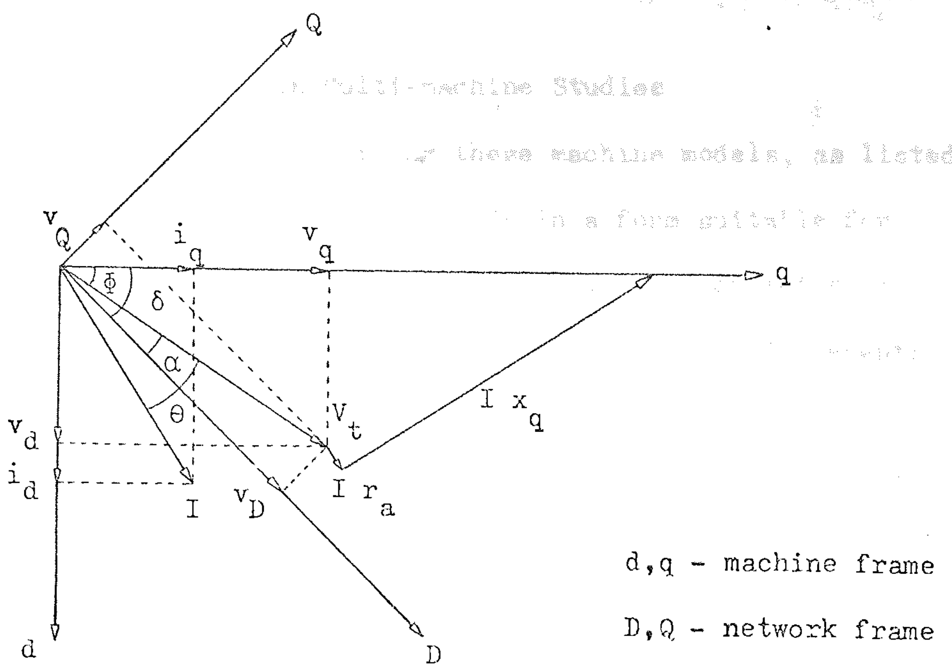


Fig. 1.14 Steady state vector diagram of synchronous machine

If it is assumed that the network can be represented by lumped resistances and reactances and that it has been reduced retaining only the nodes at which machines are connected, the relationship between voltages and currents in network reference frame is given by

$$I = YV \qquad 1.74$$

Where Y is the complex network nodal admittance matrix, and I and V are the vectors of complex current and voltage respectively in network frame.

Therefore, the process involved in calculating v_d and v_q is to transform individual machine currents into network currents and use equation 1.74 to calculate network voltages by direct multiplication of Y^{-1} and I. The network voltages can then be transformed into machine voltages using the transformation given by equation 1.73.

Thus it is seen that the calculations involved are straightforward and this provides an advantage in using this machine model, since the whole set of calculations has to be repeated four times for each integration step. However, the very small time step necessary puts a limitation in its use in large multi-machine studies.

1.6.2 Use of Models 2, 3 and 4 in Multi-machine Studies

The differential equations describing these machine models, as listed in sections 1.2.2, 1.2.3 and 1.2.4 are already in a form suitable for digital computer solution. As with model 1, the equation of motion and the control system equations are written down as first order differential equations.

The machine differential equations for these models employ, as integrable variables, voltages proportional to rotor winding flux linkages. These have been identified as d and q axis voltages behind transient or subtransient reactances. The relationships between these voltages and machine d - q terminal voltages are given by equations 1.24 and 1.25 for model 2, equations 1.29 and 1.30 for model 3 and equations 1.33 and 1.34 for model 4. In matrix form these can be expressed as

$$e = Z\,i + v \qquad 1.75$$

where the vectors i and v are given by

$\begin{bmatrix} i_d \\ i_q \end{bmatrix}$ and $\begin{bmatrix} v_d \\ v_q \end{bmatrix}$ respectively. The vector e and the matrix Z are given by

$\begin{bmatrix} e_d'' \\ e_q'' \end{bmatrix}$ and $\begin{bmatrix} r_a & -x_q'' \\ x_d'' & r_a \end{bmatrix}$ for machine model 2;

those for machine models 3 and 4 are given by

$\begin{bmatrix} e_d'' \\ e_q' \end{bmatrix}$, $\begin{bmatrix} r_a & -x_q'' \\ x_d' & r_a \end{bmatrix}$ and

$\begin{bmatrix} 0 \\ e_q' \end{bmatrix}$, $\begin{bmatrix} r_a & -x_q \\ x_d' & r_a \end{bmatrix}$ respectively

When a number of machines is present in a system, equation 1.75 will still apply. However, the vectors i and v will now be given by

$$\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad \text{respectively}$$

and the vector e and the matrix Z by , using for example machine model 2,

$$\begin{bmatrix} e_d''1 \\ e_q''1 \\ e_d''2 \\ e_q''2 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} r_{a1} & -x_q''1 & & & \\ x_d''1 & r_{a1} & & & \\ & & r_{a2} & -x_q''2 & \\ & & x_d''2 & r_{a2} & \\ & & & & \ddots \end{bmatrix}$$

The current and voltage vectors, i and v, in machine reference frame are related to those in the network reference frame, from equation 1.72 and 1.73, by i = TI and v = TV where

$$T = \begin{bmatrix} \sin\delta_1 & -\cos\delta_1 & & & \\ \cos\delta_1 & \sin\delta_1 & & & \\ & & \sin\delta_2 & -\cos\delta_2 & \\ & & \cos\delta_2 & \sin\delta_2 & \\ & & & & \ddots \end{bmatrix} \qquad 1.76$$

The equation 1.74 (I = YV) giving the relationship between voltages and currents in network reference frame is now written in real form, separating the real and imaginary parts from the set of complex equations[22]. The matrix Y will now become

$$\begin{bmatrix} g_{11} & -b_{11} & g_{12} & -b_{12} & \cdots \\ b_{11} & g_{11} & b_{12} & g_{12} & \cdots \\ g_{21} & -b_{21} & g_{22} & -b_{22} & \cdots \\ b_{21} & g_{21} & b_{22} & g_{22} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

and the vectors I and V will become

$$\begin{bmatrix} i_{D1} \\ i_{Q1} \\ i_{D2} \\ i_{Q2} \\ \vdots \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} v_{D1} \\ v_{Q1} \\ v_{D2} \\ v_{Q2} \\ \vdots \end{bmatrix} \quad \text{respectively}$$

Combining this network equation with equations 1.75 and 1.76 the following equation is obtained

$$i = T\,(Y^{-1} + T^{-1}ZT)^{-1}\,T^{-1}\,e \qquad 1.77$$

The procedure in multi-machine calculations will thus be to transform the voltage vector e, obtained at any stage during forward integration, from machine to network reference frame, then to multiply it by the matrix $(Y^{-1} + T^{-1}ZT)^{-1}$ to obtain the currents in the network frame and finally to transform these into the machine frames. Calculation for the next stage can then begin.

Because of the asymetric nature of the Z matrix, which is due to transient or subtransient saliency in the machines, the matrix $(Y^{-1} + T^{-1}ZT)^{-1}$ has to be calculated at every stage and this process is very time consuming. Thus the advantage obtained in being able to use a larger time step when using machine models 2, 3 or 4 as compared with model 1, is to some extent offset by a considerable increase in computation time for each integration step if calculations are performed in this way.

While using model 2 in a multi-machine study an approximation can be applied[7, 8] and as a result a great saving in computer time can be achieved. In this particular model the Z matrix contains machine subtransient reactances. In many synchronous machines these two reactances are very nearly equal. If the two are taken as equal, it may be seen that the matrix $T^{-1}ZT$ simply becomes Z. Thus the matrix $(Y^{-1} + T^{-1}ZT)^{-1}$ becomes equal to the matrix $(Y^{-1} + Z)^{-1}$ and for a particular network matrix this has to be calculated only once during the step by step integration.

1.6.3 An Iterative Method of Calculation when Transient or Subtransient Saliency is Present.

It has been shown in the previous section that due to the presence of transient or subtransient saliency, direct solution of the algebraic equations, which is required at every stage of the step by step integration, becomes extremely time consuming. While it is possible to apply approximation to remove subtransient saliency in machine model 2, it is not so with machine models 3 and 4. An iterative method is now described, which removes this difficulty.

Consider machine model 3. Rewriting the machine voltage equations 1.29 and 1.30

$$e_d'' = v_d + r_a\, i_d - x_q''\, i_q \qquad 1.29$$

$$e_q' = v_q + r_a\, i_q + x_d'\, i_d \qquad 1.30$$

it may be seen that the reactances x_d' and x_q'' are involved and these are usually widely different. A voltage on the direct axis is defined as

$$e_{ed} = v_d + r_a\, i_d - x_d'\, i_q \qquad 1.78$$

which is related to e_d'' by

$$e_{ed} = e_d'' + (x_q'' - x_d')\, i_q \qquad 1.79$$

Equations 1.30 and 1.78 can be arranged in matrix form as

$$\begin{bmatrix} e_{ed} \\ e_q' \end{bmatrix} = \begin{bmatrix} r_a & -x_d' \\ x_d' & r_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad 1.80$$

Thus the saliency is removed from the Z matrix and hence $T^{-1}ZT = Z$. However, the new defined voltage e_{ed} is undetermined and this is calculated in an iterative way.

The whole process in a multi-machine system can be summarised as follows: The matrix $(Y^{-1} + Z)^{-1}$ is formed once for all for a particular network condition. At each integration stage e_d'' and e_q' are known for all machines. e_{ed} is determined for each machine from equation 1.79 using the latest value of i_q. This voltage and e_q' are used in the voltage vector in equation 1.77. New values of i_q and e_{ed} are calculated for each machine in turn from equations 1.77 and 1.79 respectively. Immediately following the calculation of i_q for a particular machine the voltage vector in equation 1.77 is updated by replacing the old value of e_{ed} for that machine by the new value calculated from equation 1.79. The whole process is repeated until the absolute value of the difference between two consecutive values of e_{ed} for each machine becomes less than a specified amount, say.001 p.u. The machine currents now calculated can be used for the remaining calculations for this stage and the beginning of the next stage. The process can thus continue throughout the step-by-step integration.

Using the defined voltage e_{ed}, the air gap torque is given, assuming negligible speed change, by

$$T_e = e_{ed}\, i_d + e_q'\, i_q \qquad 1.81$$

Although the convergence is reasonably fast except immediately following a major disturbance, it can be made faster by using some acceleration technique. If during the iterations the following expression is used to calculate e_{ed} instead of equation 1.79, considerable improvement can be achieved.

$$e_{ed} = e_d'' + (x_q'' - x_d')\left\{\lambda\, i_{q\,new} + (1-\lambda)\, i_{q\,old}\right\} \qquad 1.82$$

where λ is an acceleration factor. λ will usually be between 1 and 1.2, depending on the magnitude of the difference $(x_q'' - x_d')$. How the improvement is achieved is illustrated graphically in Fig. 1.15.

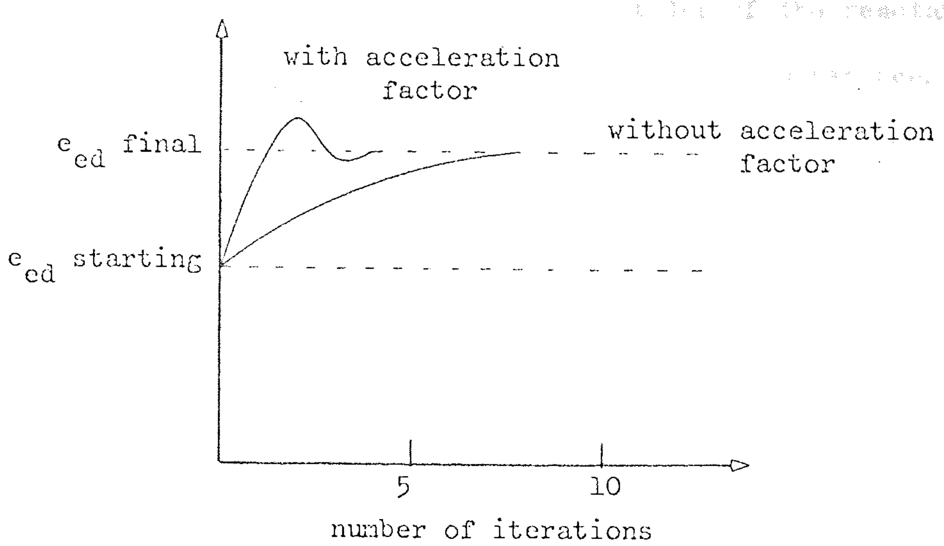


Fig. 1.15 Illustration of the improvement in the number of iterations with acceleration factor

It has been found that convergence is not achieved if in equation 1.79, x_q'' is made greater than x_d'. Had it not been so, the same iterative routine could be used for machine model 4 by replacing x_q'' by x_q. It may be seen that if x_q'' is made equal to x_q , equations 1.29 and 1.30 reduce to equations 1.33 and 1.34 for machine model 4. A voltage on the quadrature axis has to be defined for machine model 4, as follows:

$$e_{eq} = v_q + r_a\, i_q + x_q\, i_d \qquad 1.83$$

and this is related to e_q' by the equation

$$e_{eq} = e_q' + (x_q - x_d')\, i_d \qquad 1.84$$

The matrix equation corresponding to equation 1.80 is now

$$\begin{bmatrix} 0 \\ e_{eq} \end{bmatrix} = \begin{bmatrix} r_a & -x_q \\ x_q & r_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad 1.85$$

A similar process as before can be followed but this time calculating i_d during the iterations.

The equation when an acceleration factor is used would be

$$e_{eq} = e_q' + (x_q - x_d') \left\{ \lambda i_{d\ new} + (1 - \lambda)\, i_{d\ old} \right\} \qquad 1.86$$

Here again convergence could not be achieved when x_q was made less than x_d'. Thus the iterative routine for model 4 could not be used for model 3. In this iterative technique, the axis to be selected to define the voltage depends on the relative magnitudes of the reactances involved. An iterative technique very similar to the author's has been used in reference 9. However, the present one covers a wider field.

1.6.4 Description of the Multi-machine Transient Stability Program

A multi-machine transient stability program has been written incorporating all the synchronous machine models discussed earlier and an induction machine model. The induction machine model used in the program employs generalised d - q axis equations[23] and accounts for rotor transients. The equations describing the model are

$$p\, e_d' = -\left\{ e_d' - (x - x')\, i_q \right\} / T_{do}' + e_q'\, p\theta \qquad 1.87$$

$$p\, e_q' = -\left\{ e_q' + (x - x')\, i_d \right\} / T_{do}' - e_d'\, p\theta \qquad 1.88$$

$$p^2\theta = (T_e - T_m)/M \qquad 1.89$$

The electrical torque T_e is given by

$$T_e = e_d'\, i_d + e_q'\, i_q \qquad 1.90$$

The mechanical torque T_m is negative for motoring action, and $\frac{p\theta}{\omega_o}$ is the normal induction machine slip. Other terms are defined in the nomenclature.

The matrix equation connecting the machine internal and terminal voltages is given by

$$\begin{bmatrix} e_d' \\ e_q' \end{bmatrix} = \begin{bmatrix} r_a & -x' \\ x' & r_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad 1.91$$

Due to the absence of saliency the calculations are straightforward.

For synchronous machine models 2, 3 and 4, the solution of network and machine algebraic equations is obtained by the iterative technique described in the last section. In model 2, d and q axis subtransient reactances are used in the equations connecting machine internal and terminal voltages. In general any one of these can be greater or less than the other. Because of the necessity of selecting the axis to define the new voltage for iterative calculations depending on the relative magnitudes of these reactances a compromise was made. It has been found that in synchronous machines whenever x_d'' and x_q'' are appreciably different x_q'' is always greater than x_d''. Whenever x_d'' is greater than x_q'' the difference is small. Therefore the q - axis was selected to define the new voltage for iterative calculations. Whenever x_q'' is less than x_d'' they are assumed equal. The new voltage defined is then

$$e_{eq} = v_q + r_a i_q + x_q'' i_d \qquad 1.92$$

$$\text{also } e_{eq} = \left\{ -\frac{1}{\omega_o} p\, e_d'' + \left(1 + \frac{p\delta}{\omega_o}\right) e_q'' \right\} + (x_q'' - x_d'')\, i_d \qquad 1.93$$

and the matrix equation connecting the machine internal and terminal voltages is given by

$$\begin{bmatrix} \frac{1}{\omega_o}\, p\, e_q'' + (1 + \frac{p\delta}{\omega_o})\, e_d'' \\ e_{eq} \end{bmatrix} = \begin{bmatrix} r_a & -x_q'' \\ x_q'' & r_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \qquad 1.94$$

At the start of the step-by-step integration or after any change in the network matrix, the matrix $(Y^{-1} + Z)^{-1}$ is formed. Since the machine saliencies have been removed by defining new axis voltages it is not necessary to write the matrix equation connecting voltages and currents in the network reference frame in real form separating real and imaginary parts, which involves working with a matrix of size 2 n x 2 n, where n is the total number of machines. It is more convenient to work with the equation in complex form. When written in the network reference frame equation 1.77 reduces to

$$I = (Y^{-1} + Z)^{-1} E \qquad 1.95$$

Where I and E are the vectors of complex currents and voltage in the network reference frame, Y is the complex network nodal admittance matrix and Z is a complex diagonal matrix, the diagonal elements being of the form $r_a + jx$, x being the reactance depending on the machine model used. In machine model 1, calculations are performed with terminal voltages and therefore corresponding elements of the Z matrix are zero.

For machine models 2, 3, 4 and 5, and also for the induction machine model, the voltages are known at every stage of integration and currents are to be calculated, whereas the reverse is true for machine model 1. Therefore some matrix manipulation is necessary when both types of models are present. Denoting the matrix $(Y^{-1} + Z)^{-1}$ by YY, equation 1.95 is written in a slightly expanded form, separating the quantities corresponding to model 1 and the rest, as

$$\begin{bmatrix} I_1 \\ \hline I_2 \end{bmatrix} = \left[\begin{array}{c|c} YY_{11} & YY_{12} \\ \hline YY_{21} & YY_{22} \end{array}\right] \begin{bmatrix} E_1 \\ \hline E_2 \end{bmatrix} \qquad 1.96$$

Where the subscript $_1$ corresponds to machine model 1 and the subscript $_2$ to the rest. Actually E_1 should be identified as V_1 as this is the vector of the terminal voltages.

Interchanging the positions of I_1, and E_1, one obtains

$$\begin{bmatrix} E_1 \\ \hline I_2 \end{bmatrix} = \left[\begin{array}{c|c} YY_{11}^{-1} & -YY_{11}^{-1}\ YY_{12} \\ \hline YY_{21}\ YY_{11}^{-1} & YY_{22}-YY_{21}\ YY_{11}^{-1}\ YY_{12} \end{array}\right] \begin{bmatrix} I_1 \\ \hline E_2 \end{bmatrix} \qquad 1.97$$

This equation can conveniently be used to calculate the required quantities.

It may be noted that in the voltage vector E in equation 1.95, the voltage components due to the machine models 1, 2, 3 and 4 are obtained after transformation from machine to network reference frame. In the case of induction machines or synchronous machines represented by constant voltage behind transient reactance (model 5), this transformation is not required. However, for synchronous machine model 5, the components of the constant voltage along the network D, Q axes have to be found. In the same way, the current components obtained from network solution need not be transformed to machine frames for these models.

When one or more infinite busbars are present they are handled in the same way. However, the components of the voltages along the D and Q axes remain fixed.

The input data for the program consists of a set of integers showing the number and type of machines, a set of real numbers giving the integration step lengths, the interval at which the results are to be printed and the times of applying the disturbances, the machine and control system parameters, the initial load flow results and the network matrices corresponding to the various states.

The initial rotor angle δ, machine axis voltages and currents, power output etc. are calculated as follows:

Machine active and reactive powers, P and Q, terminal voltage magnitude and angle, V_t and α are supplied by the load flow program. From Fig. 1.14

$$I \text{ (machine terminal current)} = \sqrt{(P^2 + Q^2)} / V_t \qquad 1.98$$

$$\theta \text{ (power factor angle)} = \tan^{-1} Q/P \qquad 1.99$$

Φ (angle between machine terminal voltage and q-axis)

$$= \tan^{-1} \frac{x_q\, I \cos\theta - r_a\, I \sin\theta}{V_t + x_q\, I \sin\theta + r_a\, I \cos\theta} \qquad 1.100$$

$$\delta = \Phi + \alpha \qquad 1.101$$

$$v_d = V_t \sin\Phi \qquad 1.102$$

$$v_q = V_t \cos\Phi \qquad 1.103$$

$$i_d = I \sin(\Phi + \theta) \qquad 1.104$$

$$i_q = I \cos(\Phi + \theta) \qquad 1.105$$

Calculation of other quantities then follows directly.

1.6.5 A Typical Multi-machine Study

Some results of a transient stability study of a multi-machine system will now be presented to illustrate the use of the program. This represents part of a study undertaken by English Electric Power Transmission Ltd. on a system in North-West India in which the author participated and the program just described was used to obtain the main transient stability results. The main object of the study was the determination of characteristics of static excitation systems on some of the hydrogenerators of the system.

The original system supplied was too big to permit a detailed optimisation study in an economic way. It was, therefore, reduced to a five machine system, retaining full identity of the hydrogenerators under investigation and combining the rest of the machines into two groups. The reduced system is shown in Fig. 1.16. In this reduced system, a few additional nodes were also retained, in order to simulate faults etc. adequately, as required in carrying out the studies in detail. These additional nodes were, however, eliminated before preparing the final network matrices for the transient stability program. The line constants are given in Appendix A 1.2.

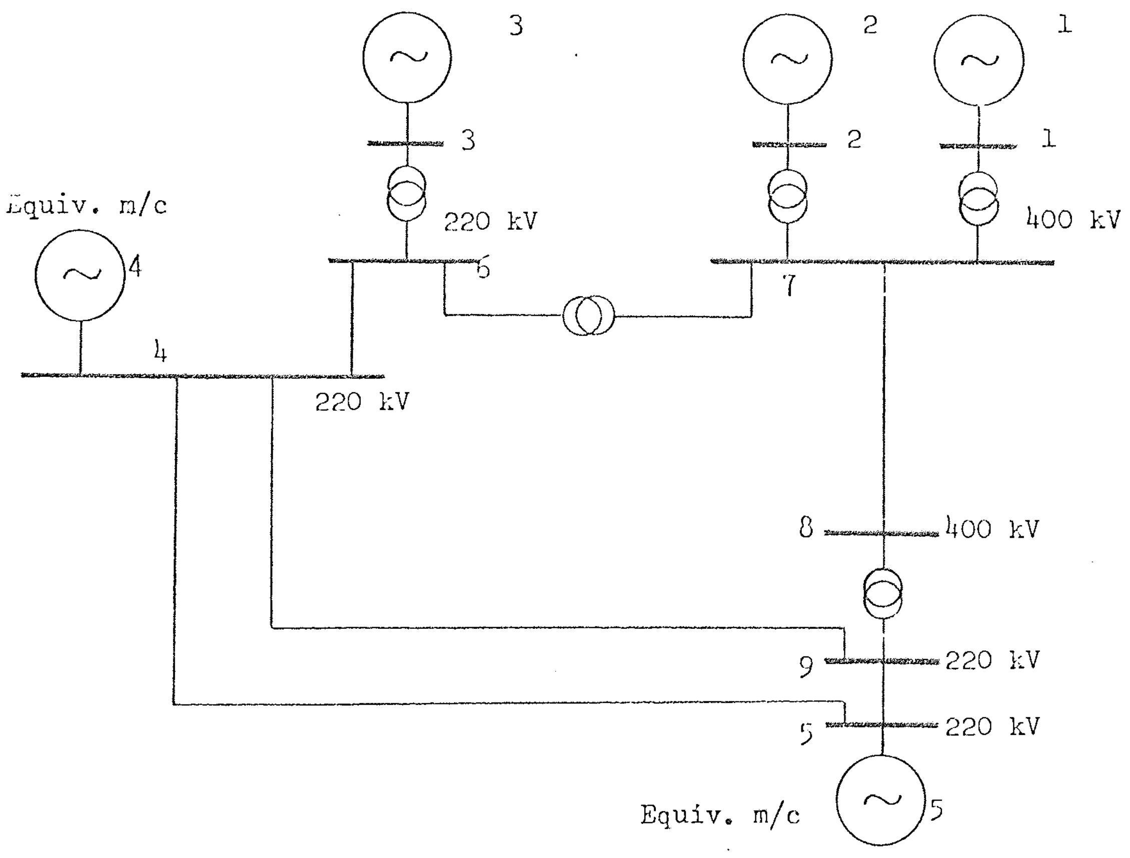


Fig. 1.16 Schematic diagram of the reduced multi-machine system studied

The four hydrogenerators - two feeding the 200 kV busbar and two the 400 kV busbar - being identical, were all represented in exactly the same way, and with identical characteristics. However, the two machines supplying the 200 kV bus were considered together as one group, while the two supplying the 400 kV bus were separately identified. Each machine was represented by synchronous machine model 3 described in section 1.2, as the available data did not permit representation in any further detail. The remaining machines, being equivalent generators of the rest of the system, were represented by constant voltage behind equivalent transient reactance (model 5).

The electrohydraulic governor and prime mover of each machine were represented as shown in Fig. 1.17. The transfer block on the extreme right simulates the behaviour of the pipe-line in a conventional way.

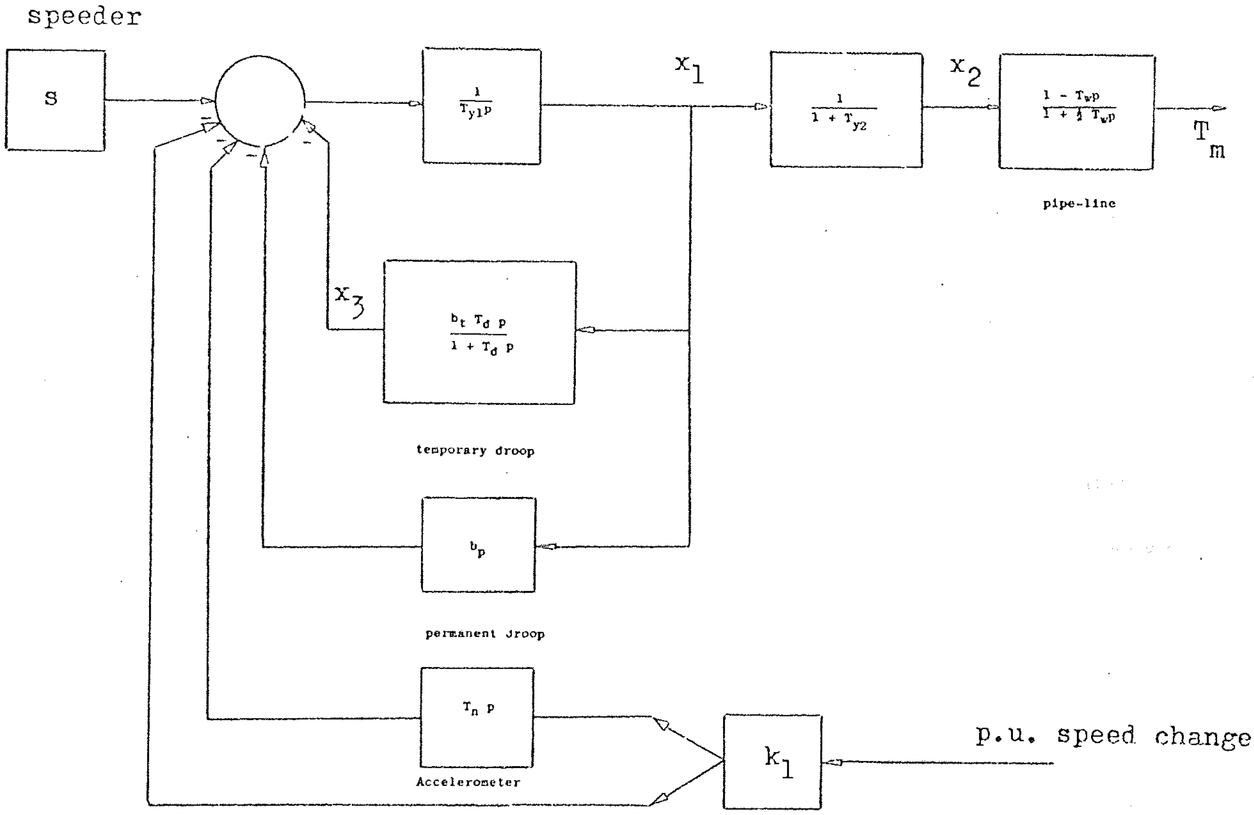


Fig. 1.17 Block diagram of the electrohydraulic governing system

The static excitation system was represented as shown in Fig. 1.18. The time constant in the main transfer function block being very small (10 ms), it was approximated to a simple gain rather than a lag function.

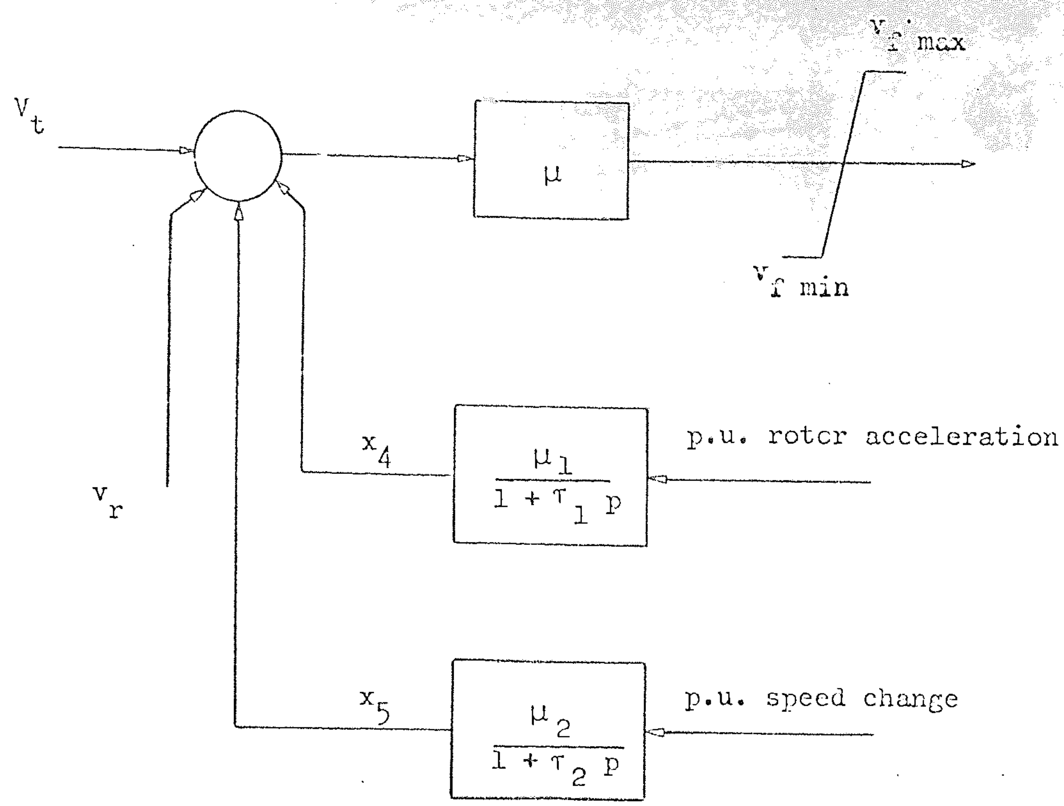


Fig. 1.18 Block diagram of the static excitation system

A number of computer runs was taken to come to a conclusion about the acceptable values of the parameters of the excitation system. However, full description of the computer results is outside the scope of this thesis. The results of a typical transient stability run are shown in Fig. 1.19. The values of the parameters for this particular run are given in Tables 1.6 - 1.9. Large rotor angle excursions are indicated with all machines swinging essentially together, but by the end of a certain length of time all angles have almost settled to new values with the same separations as in the steady state. During the swing the maximum angular machine separation is 68^o, occurring at .45 sec. after fault inception. There is, therefore, no loss of synchronism between machines and inspection of Fig. 1.20 shows that, after about 9 sec. speed has nearly returned to normal, while oscillations are at low level. The computer results also show that the machine output powers, although still oscillating, have at this time nearly returned to their original values. However, this settling

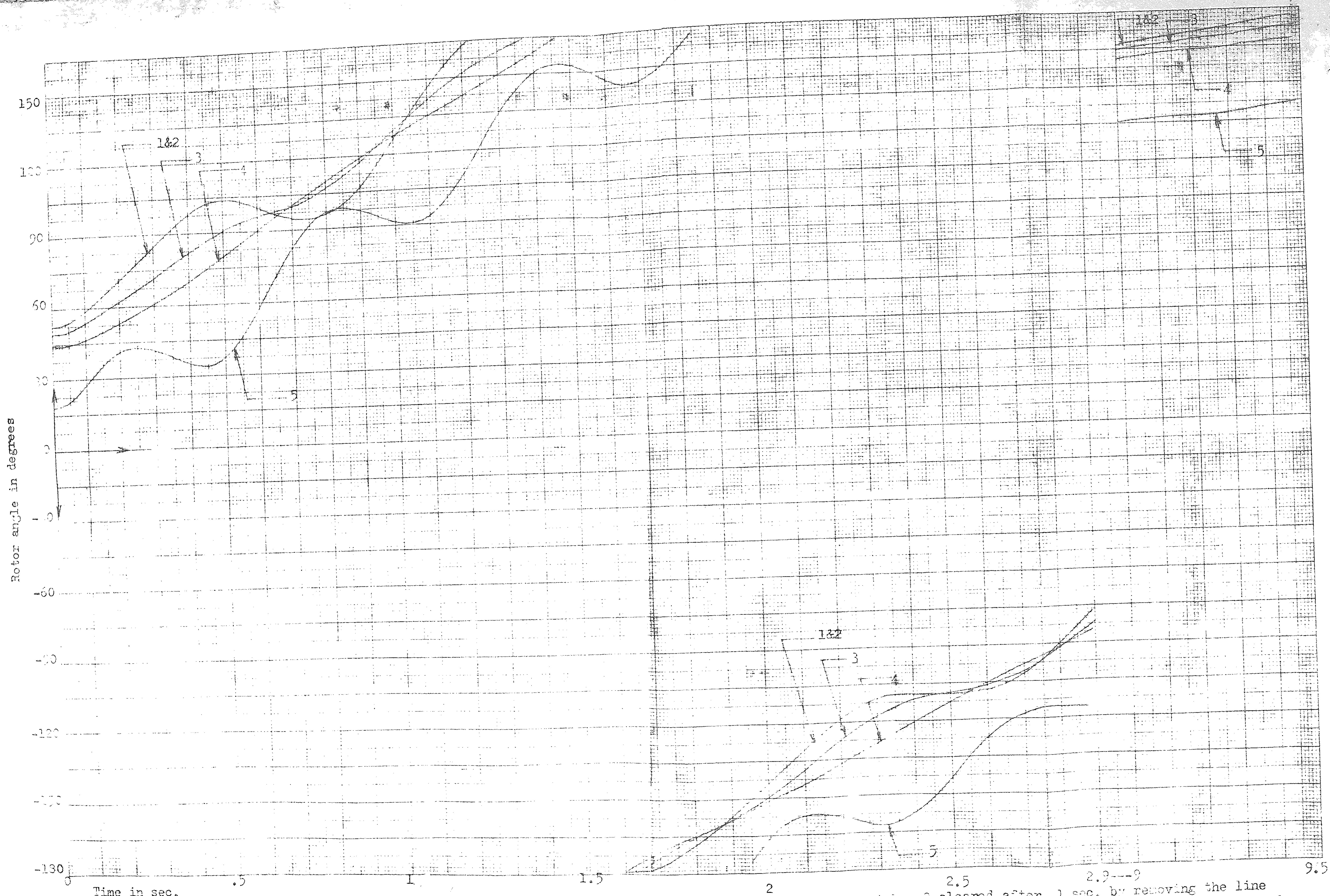


Fig. 1.19 Rotor angle excursions of the reduced 5 machine system of Fig. 1.16 following a three phase fault at bus 8, cleared after .1 sec. by removing the line between buses 7 and 8. the line is brought back after .3 sec. from clearing. Machines 1, 2, and 3, are represented by model 3; equiv. machines 4 and 5 are represented by model 5.

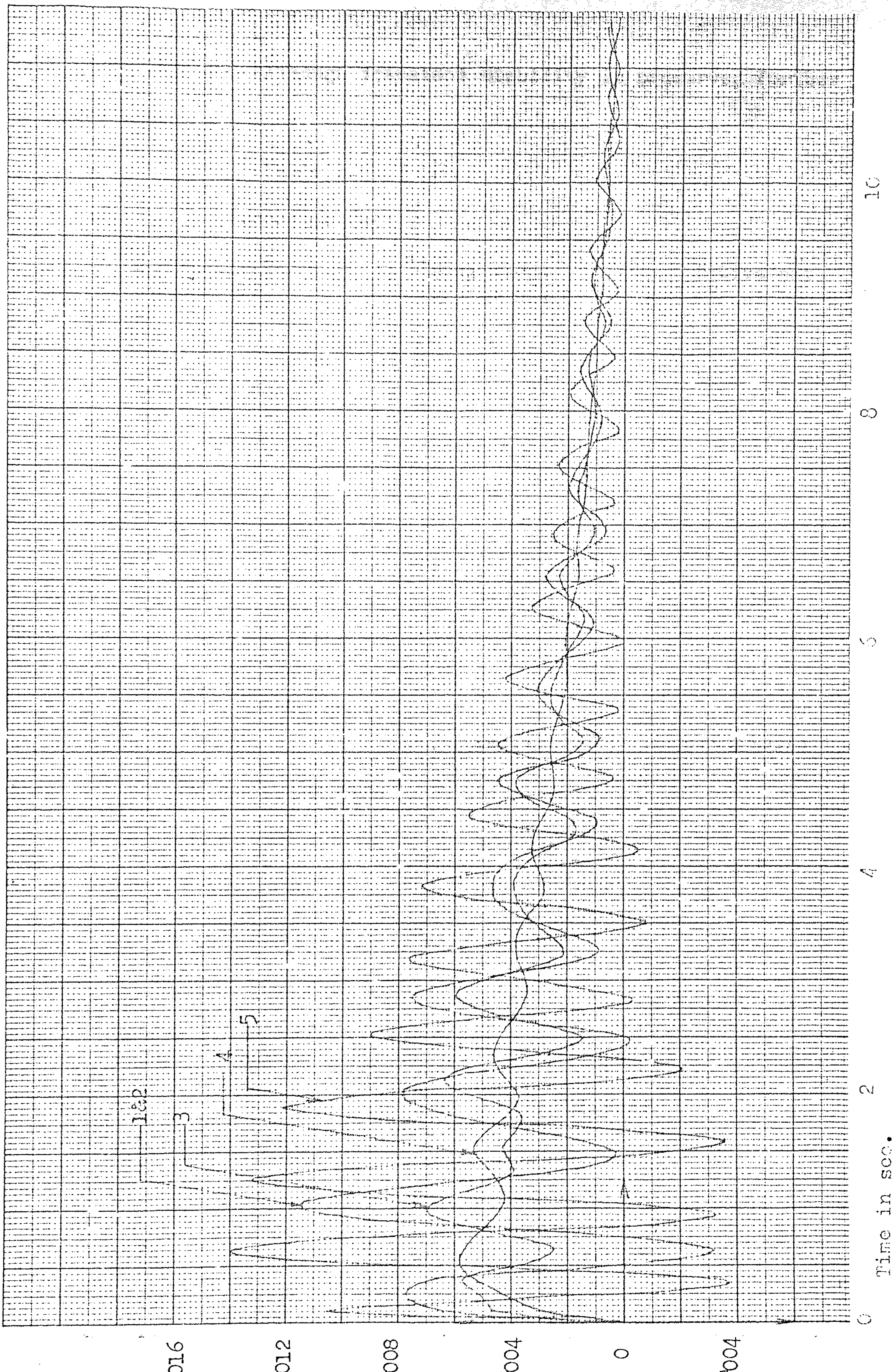


Fig. 1.20 Plots of p.u. slip versus time of the machines of Fig. 1.16 following a three phase fault at bus 8, cleared after .1 sec. by removing the line between buses 7 and 8; the line is brought back after .3 sec. from clearing. Machines 1, 2 and 3 are represented by model 3; equiv. machines 4 and 5 are represented by model 5.

time seems to be long and although transient stability is preserved further damping is desirable.

Table 1.6

Generator Data

	Hydrogenerator (each set)	Equivalent m/c at bus 4	Equivalent m/c at bus 5
x_d	0.607	–	–
x_q	0.336	–	–
x_d'	0.135	0.032125	0.112164
x_d''	0.14	–	–
x_q''/x_d''	1.2 to 1.35	–	–
K_d (damping const.)	4	10	10
T_{do}' (sec)	8.84	–	–
T_{qo}' (sec)	0.137	–	–
H	8.22	142.3	12.43
m/c rating	165 MW at .95 p.f. 11 kV 300 r.p.m.		

p.u. values are on 100 MVA base

Table 1.7

Initial Load Flow Results

M/c number	Active power	Reactive power	Terminal voltage	Angle (degrees)
1	1.5	- .1691	1.05	26.9444
2	1.5	- .1691	1.05	26.9444
3	3	.2026	1.05	25.45404
4	22.74	6.2128	1.071	15.60495
5	4.1521	3.05	1.0367	0

Table 1.8

Excitation System Data

$\mu = 400$ $\tau_1 = 1.6$ sec.

μ_1	= .14		
μ_2	= .14	τ_2	= 1.2 sec.

Field voltage ceilings - 1.895 and 2.37

Table 1.9

Prime Mover Data

k_1	= 1.5	T_n	= .6
b_p	= .04	b_t	= .4
T_{y1}	= .04 sec.	T_{y2}	= .207 sec.
T_w	= 2.07 sec.	T_d	= 10 sec.

The transient stability run corresponding to the case shown in Fig. 1.19 was repeated representing all five machines by constant voltage behind transient reactance. The results are shown in Fig. 1.21. One interesting feature may be noted. Although the initial rotor angles of the first two machines differ in the two figures the general pattern of the rotor angle variations during the transient period is very similar - at least for the first few swings. If in Figs. 1.19 and 1.21 the angular differences between the first two machines which are first represented in detail and then by simplified model are compared it can be seen that these are almost identical for the first few seconds of the transient period.

1.7 SUMMARY

This chapter has been concerned with the numerical solution of the transient stability problem. First a review and comparison of various synchronous machine models suitable for use in transient stability studies have been made.

Five models have been considered. Model 1 uses Park's basic 2-axis flux linkage and voltage equations. Models 2 - 4 are derived from model 1 after neglecting $p\Psi_d$ and $p\Psi_q$ terms and applying various simplifications. Model 5 is the classical representation of a synchronous machine by constant voltage magnitude behind transient reactance.

The results of a published system test have been used as the basis for comparison. The superiority of model 2 from the point of view of the computing time and accuracy combined has been demonstrated.

It has been shown that it is possible to simplify a given excitation control representation without affecting its overall effect on transient stability significantly.

Following a discussion of the algorithms for the use of the models in multi-machine systems a digital computer program incorporating all the synchronous machine models discussed, and an induction machine model, has been described. Using some of the synchronous machine models in multi-machine studies the algebraic solution at each step-by-step integration stage becomes time consuming due to the presence of transient or subtransient saliency. An iterative method developed by the author has been described and shown to reduce computing time significantly. As an illustration of the use of the program some results of a practical multi-machine transient stability study have been presented.

When representing a synchronous machine by model 5 the angle in the equation of motion of the rotor is not the true rotor angle. However, in a multi-machine transient stability study the important quantities are relative rotor angles and from that point of view this model has produced results very similar to those obtained by employing detailed machine representation in the multi-machine study illustrated.

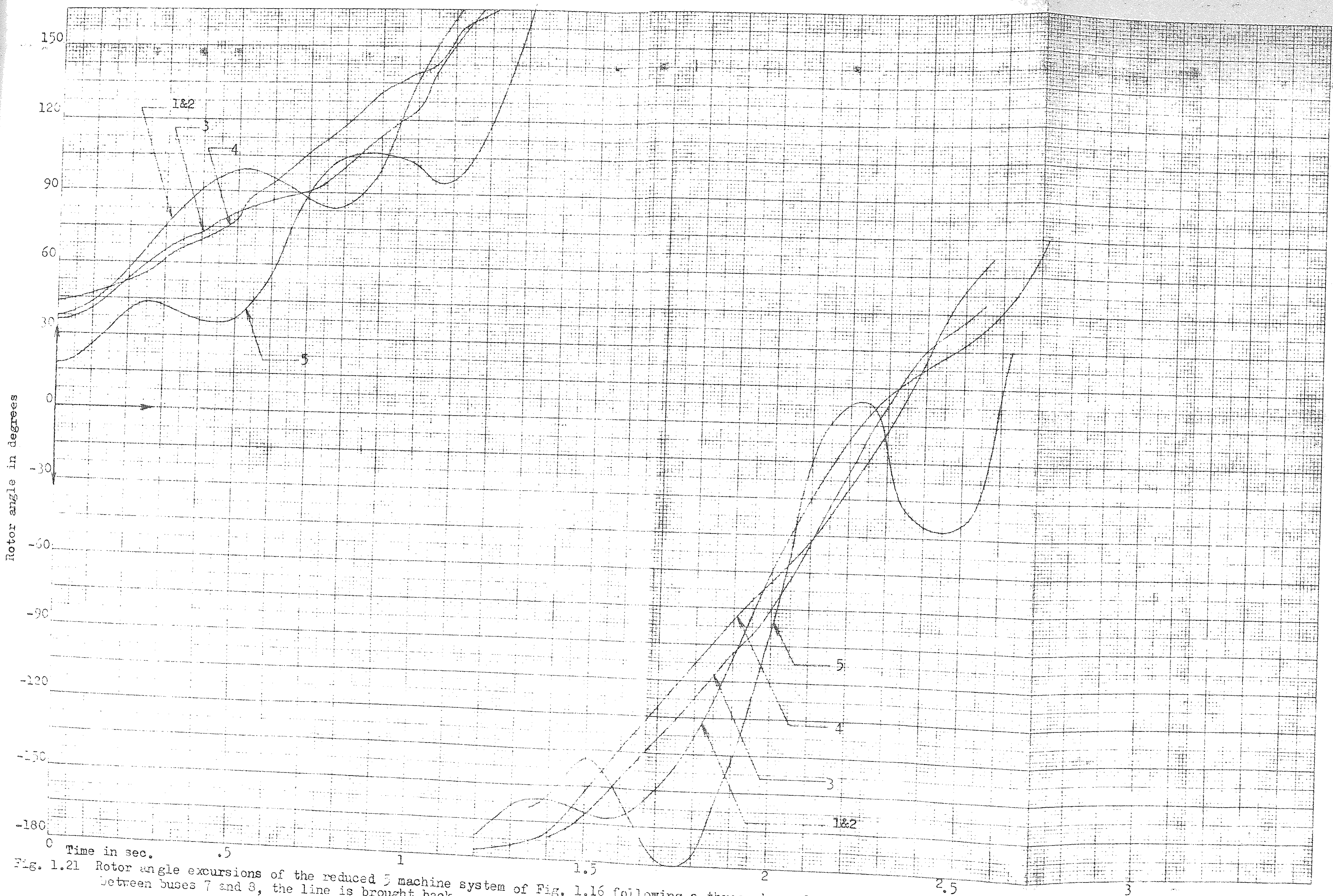


Fig. 1.21 Rotor angle excursions of the reduced 5 machine system of Fig. 1.16 following a three phase fault at bus 8, cleared after .1 sec. by removing the line between buses 7 and 8, the line is brought back after .3 sec. from clearing- all machines represented by model 5.

CHAPTER 2

INTRODUCTION TO LIAPUNOV'S DIRECT METHOD

2.1 INTRODUCTION

It has been shown in Chapter 1 that a compresensive analysis of transient stability of a power system can be undertaken when the differential and algebraic equations describing the system are solved by numerical techniques. However, the overall accuracy may be questionable having regard to the uncertainties and difficulties in obtaining correct data and also to the cumulative error inherent in any numerical method. The total computing time may also be prohibitive. An alternative approach would be to use some other approach where it is not necessary to have the actual solution of the set of differential equations. The second or direct method of Liapunov offers such an approach.

The stability of a system of linear differential equations represented in vector matrix notation by

$$\dot{x} = A\, x \qquad 2.1$$

depends on the eigenvalues of the matrix A. If all the eigenvalues have negative real parts the system is stable. The stability thus established ensures stability of the system in the entire state space where equation 2.1 is valid. A space in which x_i $(i = 1, 2..n)$ are co-ordinates is defined as the state space of the system. In a non linear system stability may be confined within a certain region around the equilibrium position, which means if the perturbations are not too large the system will return to equilibrium following such a perturbation. Through a suitably constructed Liapunov function, in addition to establishing the stability of the equilibrium state, an estimate of the size of the region of stability in a non linear system may be obtained. The trajectory of the system starting from any state within this region will tend to the stable equilibrium state as time tends to infinity.

2.2 TERMINOLOGY, DEFINITIONS AND THEOREMS OF LIAPUNOV [24 - 29]

Most dynamic systems can be represented by a vector differential equation of the general form

$$\frac{dy}{dt} = Y(y, t) \qquad 2.2$$

where y is an n - vector of the variables y_1, y_2.... describing the state of the system and Y (y, t) is the n - vector - function of these variables and t, defined in some fixed region Ω in the space of the variables.

In cases when t does not appear explicitly on the right hand side of the equation 2.2, it assumes the simpler form

$$\frac{dy}{dt} = Y(y) \quad \text{or} \quad \dot{y} = Y(y) \qquad 2.3$$

The set of non linear differential equations describing the dynamics of a power system is in general reducible to this form.

Steady state processes are described by the constant vector

$$y = y_* \qquad 2.4$$

which is the solution of the vector equation

$$Y(y) = 0 \qquad 2.5$$

Physically realizable steady states correspond uniquely to the so-called stable solutions 2.4, while physically unrealizable ones correspond to the unstable solutions.

Making the following change in variables

$$y = y_* + x \qquad 2.6$$

equation 2.3 reduces to

$$\dot{x} = X(x) \qquad 2.7$$

where $$X(x) = Y(y_* + x) \qquad 2.8$$

Equation 2.6 actually shifts the origin to a point with co-ordinates y_{*1}, y_{*2} etc. As a result the constant vector y_* of the final steady state has the corresponding null vector (zero solution)

$$x_* = 0 \qquad 2.9$$

In Liapunov's terminology, the solution 2.9 is said to be unperturbed motion, while equation 2.7 is the equation of perturbed motion.

Further, at some instant taken to be the origin of the time reference $t = 0$, let the vector x assume the initial value x_o. x_o is called the vector of initial perturbations.

Let at every point of Ω there exist continuous partial derivatives $\partial X_h / \partial x_k$. Then corresponding to each vector of initial perturbations there is a unique and continuous solution.

$$x = x(t, x_o) \qquad 2.10$$

As distinct from solution 2.9, the solution 2.10 is said to be perturbed motion.

2.2.1 Definitions

The unperturbed motion 2.9 is said to be stable in the sense of Liapunov, if for any specified positive number ϵ, no matter how small, it is possible to choose another number $\delta(\epsilon)$, such that

$$\|x_o\| \leq \delta \qquad 2.11$$

implies $\|x(t)\| < \epsilon$ 2.12

for all $t > 0$. In the contrary case the unperturbed motion is called unstable.

The unperturbed motion is said to be asymptotically stable if

a) it is stable in the sense of Liapunov and

b) $\lim_{t \to \infty} x(t) = 0$

It is this kind of stability with which we will be mostly concerned here.

Inequality 2.11 limits the totality of the initial perturbations of the system; inequality 2.12 limits the character of the course of its perturbed motion.

If $\lim_{t \to \infty} x(t) = 0$ occurs for all the x_o that belong to Ω, the unperturbed motion is asymptotically stable in the large. If, however, this requires x_o to be sufficiently small, the unperturbed motion is asymptotically stable in the small.

A function V (x) is said to be positive (negative) definite in some neighbourhood of the origin, if $V(0) = 0$ and $V(x) > 0$ (< 0) for $x \neq 0$. If $V(0) = 0$ and $V(x) \geq 0$ (≤ 0), the function is said to be positive (negative) semi-definite.

2.2.2 Stability and Instability Theorems

a) Suppose there exists a positive-definite function V (x) which is continuous together with its first partial derivatives in some neighbourhood Ω of the origin and whose total derivative $\frac{dV}{dt}$ along every trajectory of 2.7 is negative - semi-definite, then the origin is stable.

The function V (x) is called a Liapunov function.

b) If $\frac{dV}{dt}$ is negative definite in Ω, then the stability is asymptotic.

An extension of the above theorem[29] is:

c) If $\frac{dV}{dt}$ is negative - semi-definite but not identically zero along any trajectory, then the origin is asymptotically stable.

d) Let V (x) with V (0) = 0 have continuous first partial derivatives in Ω. Let $\frac{dV}{dt}$ be positive definite and let V be able to assume positive values arbitrarily near the origin. Then the origin is unstable.

The proofs of the above theorems are based on the fact that V (x) being positive definite V = C = constant represents a one - parameter family of closed surfaces surrounding the origin in the neighbourhood Ω of the origin. This is illustrated in Fig. 2.1 for a two dimensional case. For $\dot{V}$ negative definite the trajectory follows a path similar to that shown.

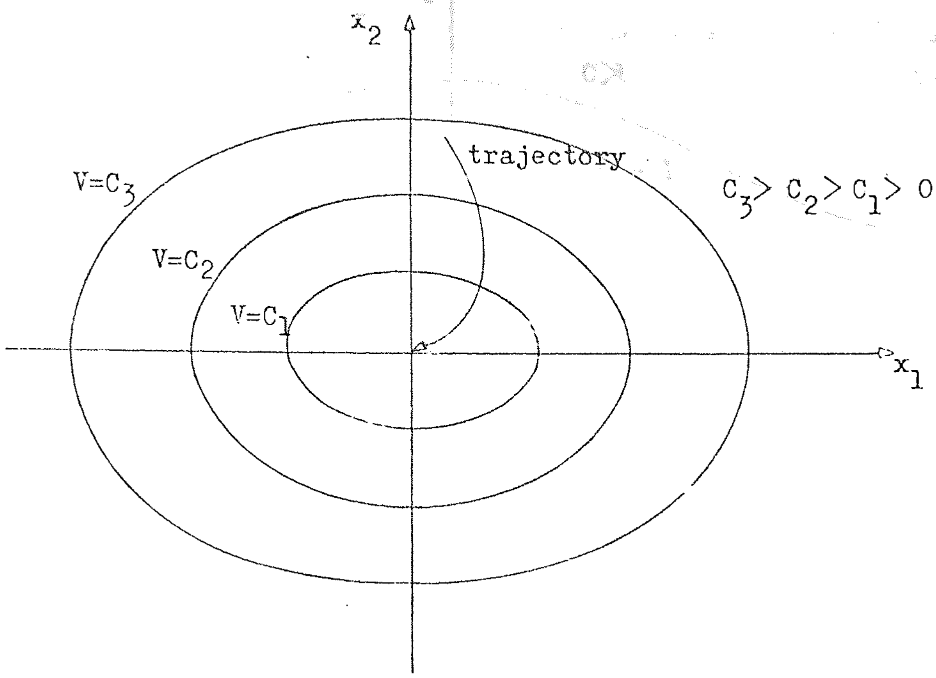


Fig. 2.1 V = constant curves

2.2.3 Illustration of Positive Definite Functions and their Closedness

Of the three functions

$V_1 = x_1^2 + x_2^2 + x_3^2$, $V_2 = (x_1 + x_2)^2 + x_3^2$ and

$V_3 = x_1^2 - x_2^2 + x_3^2$

the function V_1 is positive-definite, the function V_2 is positive - semi-definite and the function V_3 is indefinite.

For the above positive definite function V_1, V = C surfaces are closed for any C. In general, however, the surfaces V = C are closed only if C is sufficiently small. An example is provided by the function

$$V = \frac{x_1^2}{1 + x_1^2} + x_2^2$$

This defines a family of closed curves V = C only if C ≤ 1 (Fig. 2.2).

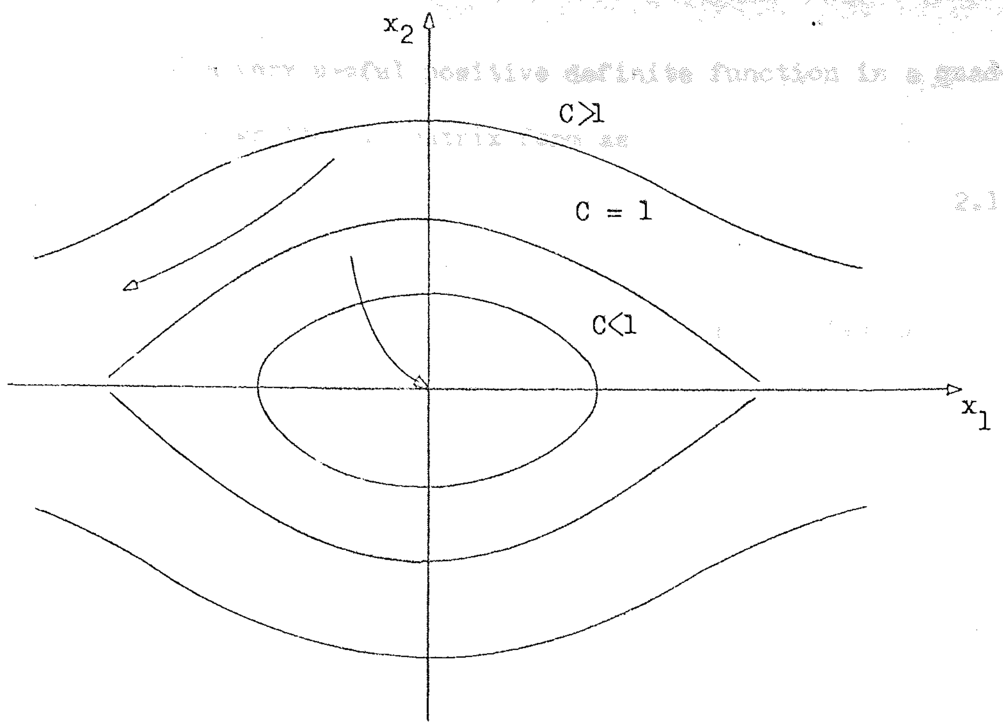


Fig. 2.2 Illustration of closedness of functions

Consequently, this can serve as a Liapunov function only for an investigation of stability in which the disturbances are limited by the condition

$$\frac{x_{1o}^{2}}{1 + x_{1o}^{2}} + x_{2o}^{2} < 1$$

Another example is given by the function

$$V = \int_{o}^{x_1} \Phi(x)\, dx + x_2^{2}$$

where $\Phi(x)$ satisfies the conditions

$$\Phi(0) = 0 , \; x_1 \Phi(x_1) > 0 \;\; \text{for } x_1 \neq 0$$

The curves $\int_o^{x_1} \Phi(x)\, dx + x_2^2 = C$ naturally must be closed if C is sufficiently small. But if $\int_o^{x_1} \Phi(x)\, dx$ tends to a limit a as $x_1 \to \infty$, then the curves V = C will be closed only for C < a. This emphasizes the need to verify whether the curves V = C are closed whenever C is not sufficiently small.

Closure of the curves is assured if, in addition to the above, V approaches infinity as $\| x \| \to \infty$. In this case V is said to be unbounded.

The simplest and a very useful positive definite function is a quadratic form which may be written in matrix form as

$$V(x) = x' P x \qquad 2.13$$

where P is a real symmetric matrix

The necessary and sufficient conditions in order that V (x) be positive definite, as given by J.J. Silvester, are that the successive principal minors of the matrix P be positive, i.e.

$$p_{11} > 0 , \quad \begin{vmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{vmatrix} > 0 , \quad \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{vmatrix} > 0 \; \ldots . \qquad 2.14$$

This test is, however, unsuitable for large matrices due to the inconvenience of evaluating determinants of large order. A much better method, which is easy to programme and which has been used in this thesis, is to use the fact that P is positive if and only if it can be expressed as the product of two matrices, LL', where L is a real lower triangular nonsingular matrix[30]. The test for nonsingularity can be made very easily as the determinant of a triangular matrix is simply the product of the diagonal elements.

2.2.4 Stability Determined by the Linear Approximation

Considering again the equation 2.7

$$\dot{x} = X(x)$$

in the majority of problems X (x) can be expanded into a convergent power series in a sufficiently small region about the origin. Then equation 2.7 can be written as

$$\dot{x} = A x + F(x) \qquad 2.15$$

where A is a constant nonsingular matrix and the vector function F (x) does not contain terms of order less than 2.

A sufficient condition for the origin of the nonlinear system 2.15 to be asymptotically stable is that the eigenvalues of the matrix A all have negative real parts. If there is an eigenvalue with positive real part the origin is unstable.

The critical cases are those for which several of the eigenvalues of the matrix A of the linear terms are zero or pure imaginary in pairs, while the remaining roots have negative real parts. In all these cases the stability cannot be determined by investigating only the linear terms. As was shown by Liapunov, in critical cases the stability is determined by the form of the nonlinearities and then it becomes necessary to consider equation 2.7 in its original form. However, for our purposes it will not be necessary to deal with these special cases.

The above theorem of Liapunov is important because it puts on a strong mathematical basis the engineer's linearization technique in studying stability under small disturbances.

2.3 EXTENT OF ASYMPTOTIC STABILITY

Asymptotic stability is a local concept, and merely to have established asymptotic stability does not necessarily mean that the system will operate satisfactorily. Some knowledge of the size of the region of asymptotic stability is always desirable.

There are several ways of determining the extent of asymptotic stability. One which will be used here in studying power system problems is as follows[29]. The Liapunov function V is formed such that $\dot{V}$ is at least negative - semi-definite in the whole state space, and V is positive definite only in a finite region around the origin. If the largest V = const. closed surface is found, the region within this surface is the region of stability. This is illustrated in Fig. 2.3 when the number of variables is two. Here the region inside V = a is the stability domain.

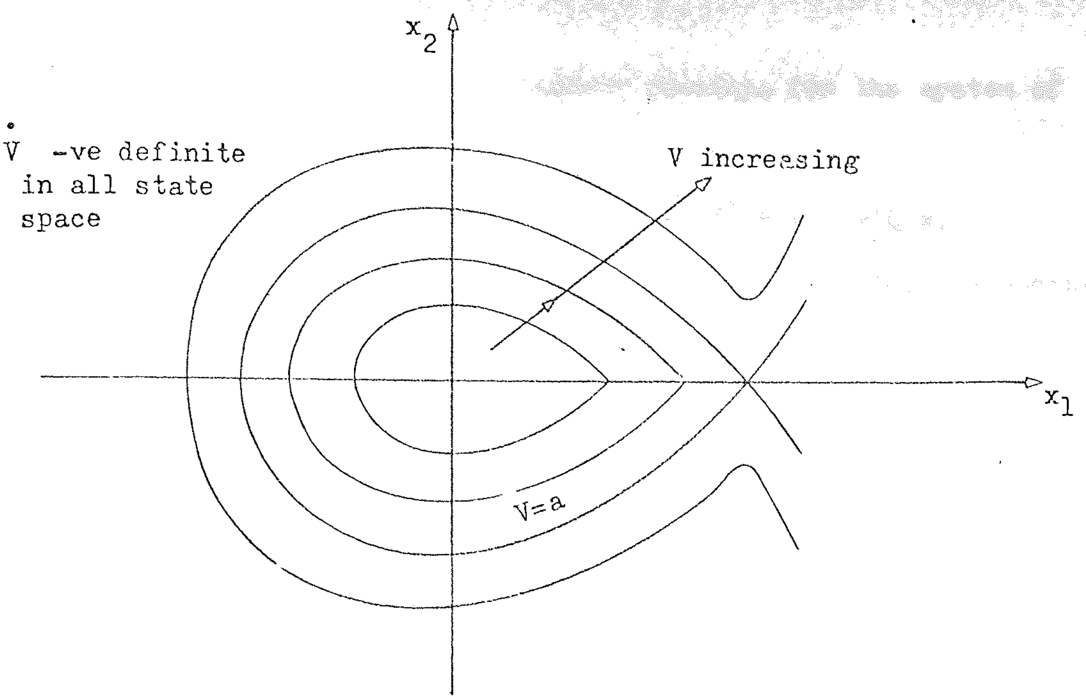


Fig. 2.3 Estimation of the region of stability

2.4 CONSTRUCTION OF LIAPUNOV FUNCTIONS

Although Liapunov's theorems give sufficient conditions for stability or asymptotic stability they do not give any indication of how a Liapunov function can be constructed in the general case. Also for a particular system there is not a unique Liapunov function. The fact that a particular Liapunov function ensures stability in a certain region around the equilibrium point, does not necessarily mean that the motions are unstable outside this region. On the other hand for a stable or asymptotically stable equilibrium state a Liapunov function with the required properties always exists.

For linear time-invariant systems Liapunov himself pointed out a method for constructing Liapunov function. This is contained in the following theorem[27, 31].

The equilibrium state x = 0 of a linear time-invariant system given by equation 2.1 is asymptotically stable if and only if given any symmetric, positive-definite matrix Q there exists a symmetric, positive-definite matrix P which is the unique solution of

$$A'P + PA = -Q \qquad 2.16$$

The scalar function $x'Px$ is a Liapunov function for the system of equation 2.1.

If $V(x) = x'Px$, then $\dot{V}(x) = x'(A'P + PA)x = -x'Qx$, where $A'P + PA = -Q$. If Q is chosen to be positive definite, the necessary and sufficient conditions for the linear system to be stable is that P is positive definite. The equation 2.16 has a unique solution for P if $\lambda_i + \lambda_j \neq 0$ for all i, j, where λ_i are the eigenvalues of the matrix A.

For nonlinear systems, however, such a straightforward method does not exist. Liapunov functions for many important cases have been formed mainly from the investigator's intuition and experience. However, a lot of work has been done in developing systematic methods of construction of Liapunov function. None of them can be said to be entirely successful. While one method may prove to be useful in a particular case it may fail in another. Also none of the methods devised so far is really effective in generating Liapunov functions for higher order systems with general nonlinearities. Only a few of them will be mentioned here. Lu'r'e devised a method of construction applicable to certain nonlinear control systems[24, 25]. The variable gradient method of Schultz and Gibson[32] provides a systematic way of constructing Liapunov functions through the introduction of a variable gradient and auxiliary equations. This method has been found useful in many investigations of stability of third or fourth order system. In Zubov's method[33] of generating Liapunov functions, which would determine the stability domain in a nonlinear system, there is no theoretical difficulty in finding a Liapunov function for an asymptotically stable system. However, in the general case a series solution has to be performed and the use of a digital computer becomes necessary because of the large number of coefficients to be determined. Even then the computational difficulties make the use of the method impracticable for systems of order higher than the third.

A practical-sized power system is described by a differential equation of order much higher than third or fourth and construction of a Liapunov function presents a real problem. However, when the system is described by a simplified mathematical model a Liapunov function can be formed by inspection. The Liapunov functions used in this thesis in connection with transient and steady state stability studies were all first formed in this way. These either represent or resemble the total transient energy of the system.

2.5 ESTIMATION OF TRANSIENT BEHAVIOUR

If the stability of a given system has been established by means of a Liapunov function, the latter can also be used to give information about transient response[27]. This is based on the observation that the Liapunov function can be regarded as a measure of distance in the state space. For a uniformly asymptotically stable system, a Liapunov function with negative definite $\dot{V}$ always exists. A parameter η may be defined as

$$\eta = \min(-\dot{V}/V) \qquad 2.17$$

in some region of the state space excluding the origin.

Equation 2.17 leads to the following inequality

$$V(x,t) \leq V(x_o,t_o) \exp\left[-\eta(t-t_o)\right] \qquad 2.18$$

where $V(x_o,t_o)$ corresponds to the initial value of V starting at $x = x_o$ at $t = t_o$.

Interpreting V as the distance from the origin, equation 2.18 gives an estimate of how fast equilibrium is approached.

$1/\eta$ can be interpreted as the largest time-constant relating to changes in the Liapunov function over a certain region in the state space and may be regarded as a figure of merit of the system. Since V may be expanded in series starting with quadratic terms in x, this time constant is about half the conventional time constant defined for the system.

The specific value of η depends on the Liapunov function chosen. Therefore it may be desirable to choose V so as to maximise η. It is usually not known in advance just how this can be done.

If $\dot{V}$ is only negative - semi-definite, one may be able to conclude asymptotic stability. But then $\eta = 0$ and equation 2.18 cannot be used to estimate the rate of approach to equilibrium. It may, however, be useful to consider the values of $(-\dot{V}/V)$ at different points of the state space. This will give some indication of how fast the system is approaching equilibrium at any particular state.

For a linear time-invariant system given by equation 2.1, if asymptotically stable, a Liapunov function and its time derivative are given by $V(x) = x'Px$ and $\dot{V}(x) = -x'Qx$ where P and Q are related to each other by equation 2.16. The figure of merit as found from equation 2.17 is then $\eta = \min (x'Q\,x/x'P\,x)$. It can be shown[27] that η is the minimum eigenvalue of QP^{-1}.

2.6 SYSTEM OPTIMISATION

The second method of Liapunov is closely related to problems in the dynamic optimisation of systems[27]. Let $\rho(x)$ be a continuous, non-negative function with $\rho(0) = 0$, which can serve as the error criterion of the system. The performance index of the system can be defined as the integrated error criterion by $V(x) = \int_0^\infty \rho(x(t))\,dt$ 2.19

This integral is a positive number. When this is infinite the upper limit of integration can be replaced by T, where T denotes an interval of time over which the system behaviour is of interest. If the system contains adjustable parameters these can be selected in such a way so as to minimize $V(x)$ with respect to those parameters.

As ρ is non-negative the integral in 2.19 can be zero only if $\rho(x(t))$ is zero for all $t \geq 0$ and if ρ is positive definite this implies $x = 0$; this shows V is positive definite. Further if V is finite in some neighbourhood of the origin $\dot{V}(x) = -\rho(x) < 0$ for $x \neq 0$. Thus the system is asymptotically stable and $V(x)$ is a Liapunov function.

The theorems and the concepts discussed in this chapter form the theoretical basis for the subsequent chapters.

CHAPTER 3.

TRANSIENT STABILITY STUDIES BY THE METHOD OF LIAPUNOV

3.1 INTRODUCTION

It has been shown in Chapter 1 that a detailed mathematical description of the dynamics of a power system containing several synchronous machines requires a large number of nonlinear differential equations. Studying stability properties of such a system by Liapunov's direct method will involve constructing a suitable Liapunov function satisfying Liapunov's theorems as stated in Chapter 2. Due to the extreme difficulties in forming Liapunov functions in the general nonlinear case the maximum possible number of simplifying assumptions has to be made in order to keep the number of differential equations and nonlinearities to a minimum. Attempts have been made to apply Liapunov's method to power system transient stability studies representing synchronous machines by Park's two-axis equations but the power systems of necessity have had to be of the simplest type - consisting of a single synchronous machine connected to an infinite busbar through an external reactance. A number of papers has appeared in the last few years[34-37]. In all these the simplest form of two-axis representation was used without representing the control equipment adequately. The stability regions obtained were of a weak nature. Considering the effort required in obtaining a Liapunov function even for this simple type of power system and the quality of results obtained from that it does not seem worthwhile to use this method in preference to a numerical method of solving differential equations and establishing stability behaviour from the solution especially when this can be done very quickly on a digital computer in the simple single machine case. However, representing synchronous machines by constant voltage magnitude behind transient reactance, the Liapunov function can be formed from inspection without undue difficulties even for a multi-machine system. In all the multi-machine transient stability studies using Liapunov's direct method reported in recent papers[38-47] this representation of synchronous machines has been used. Although the mathematical model of the power system based on this synchronous machine representation is open to question when an accurate solution is desired the method is worth pursuing further considering the savings in computer time in multi-machine studies and the new insight into the problem it provides. As pointed out in Chapter 1 this simplification in synchronous machine representation is not as drastic as it may appear, because when the question of relative motions of the machines in a system is concerned this representation often produces the right picture. Therefore in what follows the synchronous machines will be represented by constant voltage magnitude behind transient reactance. The Liapunov functions used for various cases will either represent or resemble the total transient energy of the system. The question of transient stability will be considered in this chapter. Examination of local stability of the equilibrium states using the same Liapunov functions will be discussed in the next chapter.

3.2 TRANSIENT STABILITY OF ONE AND TWO MACHINE SYSTEMS

A one or two machine system is the simplest form of power system. Although very few actual power systems, if any, are one or two machine systems an analysis of this is of great academic importance as it provides the basic understanding of the stability problem. Also, qualitatively the characteristics of a one or two machine system usually represent those of a multi-machine system. If in a stability study the system considered consists of a single machine or a group of machines connected through a transmission line to a large system, it can be approximated reasonably accurately by a single machine infinite busbar system. Also in many power systems, for certain types of disturbances, the machines divide into two groups which swing with respect to each other while the machines in the same group swing more or less together. For analytical purposes in such cases each group can be replaced by one equivalent machine.

In the analysis which follows in the rest of this chapter and in the next chapter it will be seen that the effect of transfer conductance can be taken into account rigorously in studies of one and two machine systems. However, this has to be neglected in multi-machine studies in order to make the application of the method mathematically rigorous.

3.2.1 Single Machine-Infinite Busbar System

Based on the simplifications in system representation outlined in section 3.2 the equation of motion of the synchronous machine connected to an infinite bus through a transmission line is given by

$$M \frac{d^2\delta}{dt^2} = P_m - P_e - K_d \frac{d\delta}{dt} \qquad 3.1$$

$$\text{where } P_e = E_1^2 G_{11} + E_1E_2G_{12} \cos\delta + E_1E_2B_{12} \sin\delta \qquad 3.2$$

E_1 and E_2 are the voltage behind the transient reactance of the synchronous machine and the infinite busbar voltage respectively; $G_{11} + jB_{11}$ and $G_{12} + jB_{12}$ are the self and mutual admittances with respect to machine internal bus and the infinite bus.

Equation 3.1 is equivalent to

$$\frac{d\delta}{dt} = \omega \qquad 3.3$$

$$M \frac{d\omega}{dt} = P_m - P_e - K_d\omega \qquad 3.4$$

Equilibrium or singular points are obtained by putting the right hand sides of equations 3.3 and 3.4 equal to zero. Thus one singular point exists at $\omega = 0$, $\delta = \theta - \alpha$ and a second at $\omega = 0$, $\delta = \pi - \theta - \alpha$, where $\theta = \sin^{-1} \dfrac{P_m - E_1^2 G_{11}}{E_1 E_2 Y_{12}}$ and $\alpha = \tan^{-1} \dfrac{G_{12}}{B_{12}}$, $Y_{12} = \sqrt{G_{12}^2 + B_{12}^2}$.

The equilibrium state given by $\omega = 0$, $\delta = \delta_* = \theta - \alpha$ is stable whereas the other is unstable as will be seen later.

Shifting the origin to the stable equilibrium point δ_* by change of variable $\delta = \delta_* + \delta'$ and then removing the ' equations 3.3 and 3.4 can be written as

$$\frac{d\delta}{dt} = \omega \qquad 3.5$$

$$M \frac{d\omega}{dt} = P_m - E_1^2 G_{11} - E_1 E_2 G_{12} \cos(\delta_* + \delta) - E_1 E_2 B_{12}$$
$$\sin(\delta_* + \delta) - K_d \omega \qquad 3.6$$

The equilibrium points after the change of origin will be given by $\omega = 0$, $\delta = 0$ and $\omega = 0$, $\delta = \pi - 2\delta_* - ?\alpha$

A Liapunov function can be chosen as

$$V(\omega,\delta) = \frac{1}{2} M\omega^2 + \int_0^{\delta} \Big\{ - P_m + E_1^2 G_{11} + E_1 E_2 G_{12} \cos(\delta_* + \delta')$$
$$+ E_1 E_2 B_{12} \sin(\delta_* + \delta') \Big\} d\delta' \qquad 3.7$$

Stationary points are given by $\frac{\partial V}{\partial \omega} = 0$ and $\frac{\partial V}{\partial \delta} = 0$

or $\omega = 0$ and

$$- P_m + E_1^2 G_{11} + E_1 E_2 G_{12} \cos(\delta_* + \delta) + E_1 E_2 B_{12} \sin(\delta_* + \delta) = 0$$

Thus these are the same as the equilibrium states of the system given by equations 3.5 and 3.6.

The matrix H of second derivatives of V is given by

$$H = \begin{bmatrix} M & 0 \\ 0 & E_1 E_2 Y_{12} \cos(\delta_* + \delta + \alpha) \end{bmatrix} \qquad 3.8$$

Provided $\cos(\delta_* + \alpha)$ is positive, H is positive definite at the equilibrium state $\omega = 0$, $\delta = 0$ and indefinite at the equilibrium state $\omega = 0$, $\delta = \pi - 2\delta_* - 2\alpha$. Thus the stationary point $\omega = 0$, $\delta = 0$ is a point of local minimum and the other is a saddle point.

At $\omega = 0$, $\delta = 0$, $V(\omega,\delta) = 0$ and thus V is positive definite in the neighbourhood of the origin.

The total time derivative of V is given by

$$\dot{V} = \frac{\partial V}{\partial \omega} \dot{\omega} + \frac{\partial V}{\partial \delta} \dot{\delta}$$

Substituting the expressions for $\dot{\omega}$ and $\dot{\delta}$ from equations 3.6 and 3.5,

$$\dot{V} = - K_d \omega^2 \qquad 3.9$$

By Liapunov's theorems stated in section 2.2.2 of chapter 2, the equilibrium state $\omega = 0$, $\delta = 0$ is asymptotically stable whereas the equilibrium state $\omega = 0$, $\delta = \pi - 2\delta_* - 2\alpha$ is unstable.

The curves given by $V(\omega, \delta) = C$ (constant) will obviously be closed for C sufficiently small. In order to establish the region of stability around the stable equilibrium point one has to find the largest $V = C$ closed curve.

Calling the part of the V function given by equation 3.7 which involves the integral, $F(\delta)$ we find that $F(\delta)$ has a minimum at $\delta = 0$ and a maximum at $\delta = \delta_u = \pi - 2\delta_* - 2\alpha$ corresponding to the stable and unstable equilibrium states respectively. The minimum value is 0 and the maximum value is $(-P_m + E_1^2 G_{11})\,\delta_u + 2 E_1 E_2 Y_{12} \cos(\delta_* + \alpha)$. Actually $F(\delta)$ represents the area shown shaded in Fig. 3.1. It may be seen that it increases with δ until δ becomes equal to δ_u and then goes on decreasing eventually becoming negative. Referring to the maximum value of $F(\delta)$

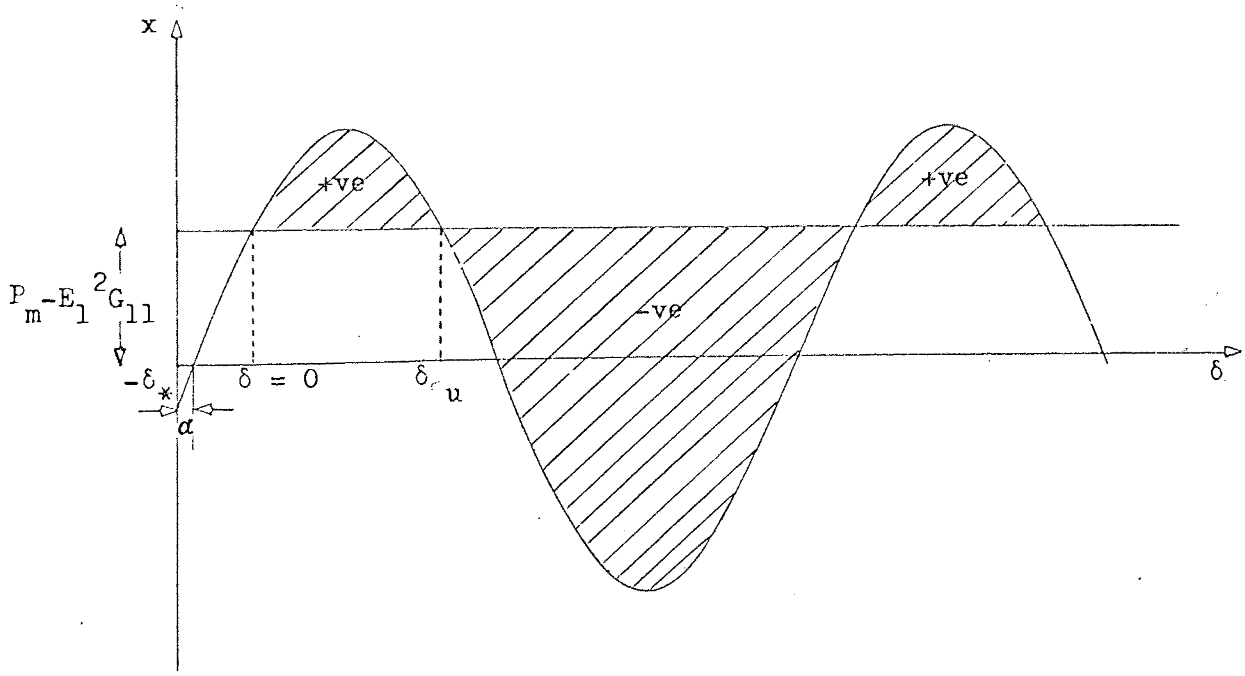


Fig. 3.1 Plot of $x = E_1 E_2 Y_{12} \sin(\delta_* + \delta + \alpha)$ versus δ

as ℓ, the curves $V = C$ will be closed for $C < \ell$ and open for $C > \ell$. This can be seen by expressing ω in terms of the other variables, thus

$$\omega = \pm\sqrt{\frac{2}{M}\left\{V - F(\delta)\right\}} \qquad 3.10$$

For $V < \ell$ two distinct values of ω will be obtained as δ is increased until $F(\delta)$ becomes equal to V. Beyond this the values of ω will no longer be real. For $V > \ell$ two distinct values of ω will be obtained for all positive δ's. V = constant curves are shown in Fig. 3.2.

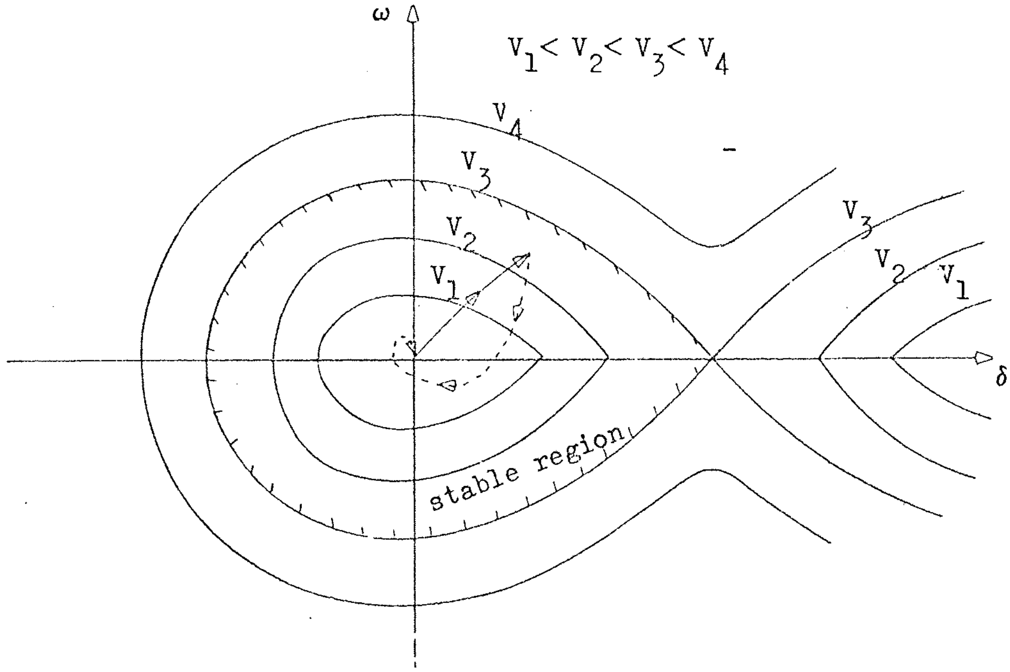


Fig. 3.2 V = constant curves

If the system is operating initially at $\delta = 0$ and a fault occurs the trajectory will follow a path similar to that shown by the solid line curve with arrows. If the fault is cleared in time before the trajectory leaves the stable region the system will remain stable; the trajectory will take a path similar to that shown by the dotted curve.

Actually for this particular type of disturbance it is not necessary to plot the curves. If substitution of the values of ω and δ at the time of the clearance of fault in the expression for V results in a value of V less than ℓ the system will remain stable. An unstable condition is indicated if $\delta \geq$, δ_u or $V > \ell$. Thus ℓ determines the extent of asymptotic stability in this case.

3.2.2 Relationship with Equal Area Criterion

It will now be shown that the choice of the above energy type Liapunov function is equivalent to the well known equal area criterion[12,13] applicable to simple one machine systems. The power angle characteristics for the system considered is shown in Fig. 3.3 for the case when $E_1 = E_2$. Curves OBH and OFH correspond to the prefault and fault conditions respectively.

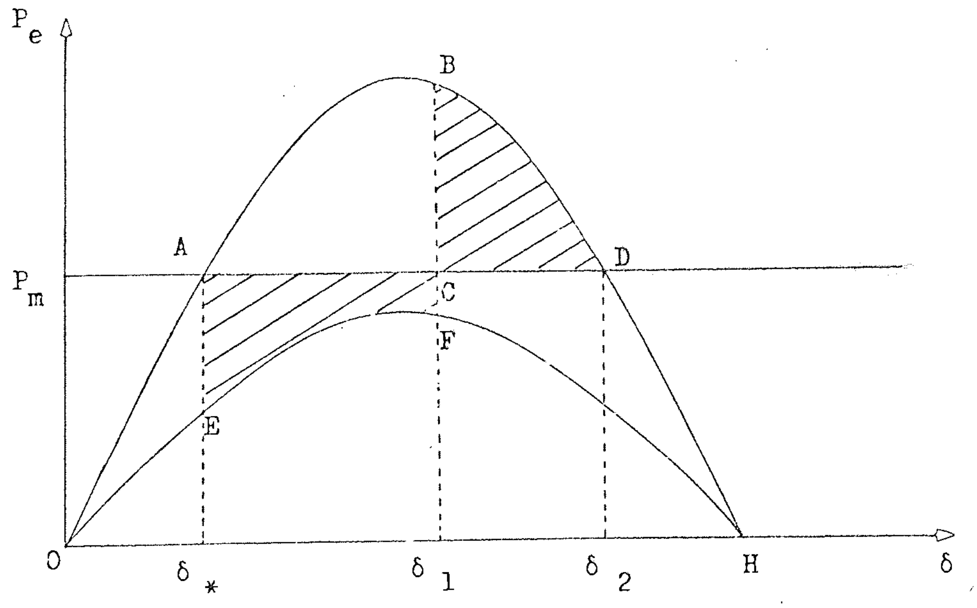


Fig. 3.3 Power-angle characteristics of the one-machine system, $E_1 = E_2$

The Liapunov function chosen is

$$V(\omega, \delta) = \frac{1}{2} M\omega^2 + \int_0^{\delta} (-P_m + P_e)\, d\delta$$

$$\ell = \int_0^{\delta_2 - \delta_*} (-P_m + P_e)\, d\delta = \text{area ABD}$$

When the fault occurs the power output falls and the machine accelerates. Suppose the fault is removed when the machine angle reaches δ_1. During this period ω has increased from zero to say ω_1. Thus at the time of fault clearance

$$V = \frac{1}{2} M\omega_1^{\ 2} + \int_0^{\delta_1 - \delta_*} (-P_m + P_e)\, d\delta$$

$$= \frac{1}{2} M\omega_1^{\ 2} + \text{area ABC}$$

Now combining the equations 3.3 and 3.4 into one during the faulted period

$$\frac{d\omega}{d\delta} = \frac{P_m - P_{el}}{M\omega} \qquad 3.11$$

Here the damping coefficient K_d is taken as zero. This will imply the system is stable but not asymptotically stable.

From equation 3.11 after integrating we obtain

$$\frac{1}{2} M\omega_1^{\ 2} = \int_0^{\delta_1 - \delta_*} (P_m - P_{el})\, d\delta$$

$$= \text{area ACFE}$$

$\therefore$ V = area ACFE + area ABC

As long as $V < \ell$ the system is stable. In the limiting case $V = \ell$ or area ACFE + area ABC = area ABD

or area ACFE = area BDC

This is the equal area criterion.

3.2.3 Liapunov Function Incorporating Damping Coefficient

Some improvement in the stability region may be obtained by including the damping term in the Liapunov function. A modified form of Liapunov function as suggested in references 45 and 49 may be used. Consider a Liapunov function $V(\omega, \delta) = \frac{1}{2} M\omega^2 + \frac{1}{2}\frac{K_d}{\beta}\delta^2 + \frac{M}{\beta}\delta\omega + F(\delta)$ 3.12

where $F(\delta) = \int_0^{\delta} (-P_m + P_e)\, d\delta$ and β is some arbitrary positive constant with $K_d\beta \geq M$.

Obviously this is positive definite in some neighbourhood of the origin.

$$\dot{V} = -K_d\,\omega^2 - \frac{1}{\beta}\delta\,(-P_m + P_e)$$

is negative definite as long as δ $(- P_m + P_e)$ is positive.

If V_m is the minimum value of V along the straight line $\delta = \delta_u$ in the $\delta - \omega$ plane, it can be seen that the curves $V = C$ are closed for $C < V_m$. V_m is given by

$$V_m = \frac{K_d \beta - M}{2 \beta^2} \delta_u^2 + F(\delta_u) \qquad 3.13$$

Substituting this into equation 3.12, the boundary of a region of asymptotic stability is obtained as

$$M \left(\omega + \frac{\delta}{\beta}\right)^2 + \frac{K_d\beta - M}{\beta^2} (\delta^2 - \delta_u^2) + 2 F(\delta) - 2 F(\delta_u) = 0 \qquad 3.14$$

This is shown by curve a in Fig. 3.4. The union of the regions for various β's satisfying $K_d\beta \geq M$ can give an improved estimate of the stability region.

It is possible to obtain a still larger region of stability using the Liapunov function[45] given by equation 3.12. The region enclosed by the open contour $V(\omega, \delta) = V(0, \delta_u)$ and the straight lines $\delta = \delta_u$ is a region of asymptotic stability. This is shown by the contour ABC in Fig. 3.4. Any trajectory starting inside this region does not intersect ABC since $V(\omega,\delta) < V(0, \delta_u)$ and $\dot{V}(\omega,\delta)$ is negative definite inside the region. The trajectory cannot intersect AC either, since $\frac{d\delta}{dt} = \omega < 0$ on AC.

In Fig. 3.4 curve b is the one corresponding to the Liapunov function discussed in section 3.2.1. The general improvement in the stability region can be seen. However the improvement in the first quadrant is the most important in usual transient stability problems, and for normal values of K_d the improvement is very little.

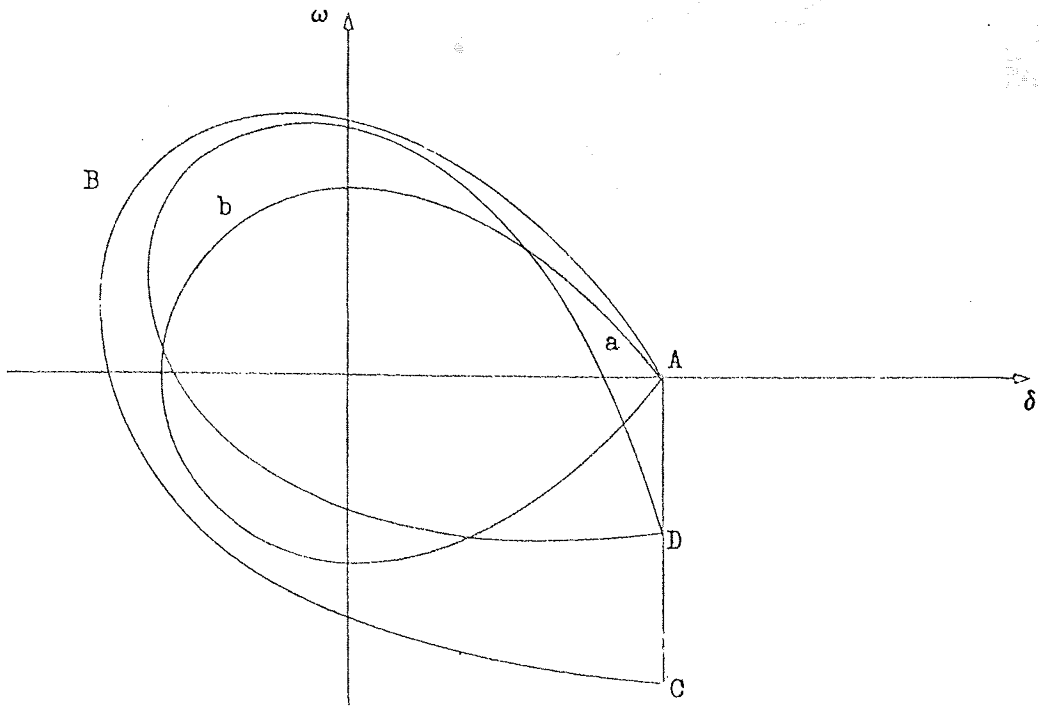


Fig. 3.4 Regions of asymptotic stability for various Liapunov functions

With the above modified Liapunov function a generalised equal area criterion may be developed[51] which can be applied to the case of a sustained fault or permanent switching out of a line in the transmission system. This is illustrated in Fig. 3.5. OAC is the power angle curve for the prefault condition; OBC is that following a fault or switching out of a line; E is the original operating position at an angle δ_1. If the system remains stable it will settle down at K at an angle δ_*. It has been shown above that the system is stable for $V(\omega,\delta) \leq V(0, \delta_u)$ where $\delta_u = \delta_2 - \delta_*$.

$$\text{Now} \quad V(0, \delta_u) = \frac{1}{2}\frac{K_d}{\beta}\delta_u^2 + F(\delta_u)$$

$$\text{Initially } \omega = 0, \; \delta = \delta_1 - \delta_*$$

$$\therefore \quad V(\omega, \delta) = \frac{1}{2}\frac{K_d}{\beta}\delta^2 + F(\delta)$$

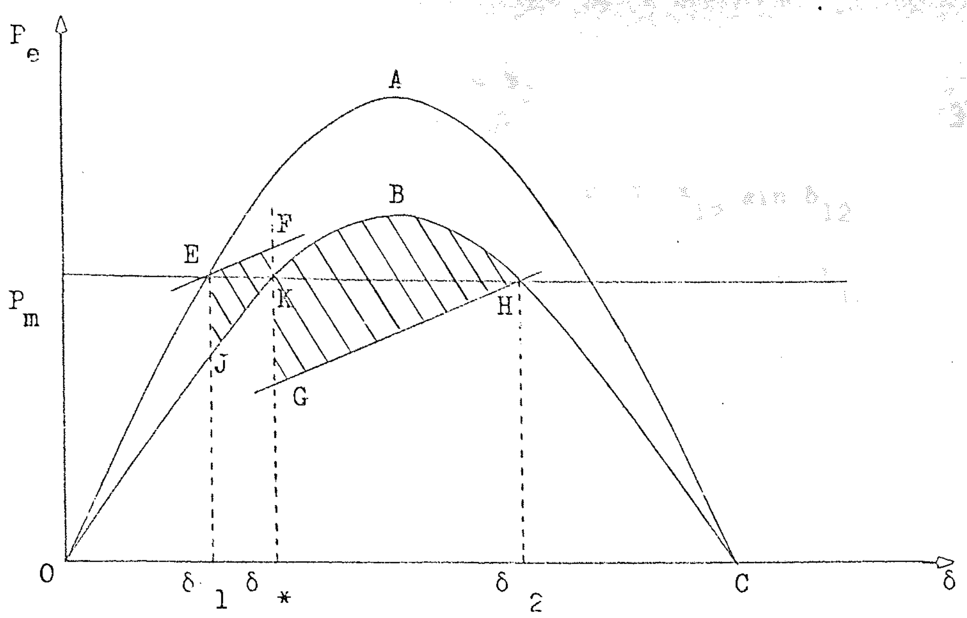


Fig. 3.5 Power-angle characteristic of the one-machine system, $E_1 = E_2$

$$\therefore \text{ For stability } \frac{1}{2}\ \frac{K_d}{\beta}\ \delta^2 + F\ (\delta) \leq \frac{1}{2}\ \frac{K_d}{\beta}\ \delta_u^{\ 2} + F\ (\delta_u) \qquad 3.15$$

Now if two parallel lines EF and GH are drawn at an angle $\theta = \tan^{-1} \frac{K_d}{\beta}$ to the horizontal axis as shown in Fig. 3.5 area EFK will be equal to $\frac{1}{2}\ \frac{K_d}{\beta}\ \delta^2$ and area GKH will be equal to $\frac{1}{2}\ \frac{K_d}{\beta}\ \delta_u^{\ 2}$. Area EKJ and area KBH are equal to $F\ (\delta)$ and $F\ (\delta_u)$ respectively. Therefore for stability area EFKJ $\leq$ area KBHG.

For the more usual case of a fault and its subsequent clearing such a general criterion cannot be developed.

3.2.4 Two Machine System

A two machine system may be replaced by an equivalent system of one machine and an infinite busbar. The equations of motion of the two machine rotors are given by

$$M_1\ \frac{d^2\ \delta_1}{dt^2} = P_{m1} - P_{e1} - K_{d1}\ \frac{d\delta_1}{dt} \qquad 3.16$$

$$M_2 \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} - K_{d2} \frac{d\,\delta_2}{dt} \qquad 3.17$$

where $P_{e1} = E_1^2 G_{11} + E_1 E_2 G_{12} \cos \delta_{12} + E_1 E_2 B_{12} \sin \delta_{12}$

and $P_{e2} = E_2^2 G_{22} + E_1 E_2 G_{12} \cos \delta_{12} - E_1 E_2 B_{12} \sin \delta_{12}$

Combining equations 3.16 and 3.17, assuming $\frac{K_{d1}}{M_1} = \frac{K_{d2}}{M_2}$,

$$\frac{M_1 M_2}{M_1+M_2} \frac{d^2 \delta_{12}}{dt^2} = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2} - \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 + M_2} - \frac{K_{d1} M_2}{M_1 + M_2} \frac{d\delta_{12}}{dt} \qquad 3.18$$

Substituting expressions for P_{e1} and P_{e2} in equation 3.18 we get

$$M \frac{d^2 \delta_{12}}{dt^2} = P_1 - P_2 - A \cos (\delta_{12*} + \delta_{12}) - B \sin (\delta_{12*} + \delta_{12}) - K \frac{d\,\delta_{12}}{dt} \qquad 3.19$$

where $M = \frac{M_1 M_2}{M_1 + M_2}$

$$P_1 = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2}$$

$$P_2 = \frac{M_2 E_1^2 G_{11} - M_1 E_2^2 G_{22}}{M_1 + M_2}$$

$$A = \frac{M_2 - M_1}{M_1 + M_2} E_1 E_2 G_{12} , \qquad B = E_1 E_2 B_{12}$$

$$K = K_{d1} \frac{M_2}{M_1 + M_2}$$

and the origin is shifted to the equilibrium point δ_{12*}

As before a Liapunov function can be chosen as

$$V = \frac{1}{2} M \omega_{12}^2 + \int_0^{\delta_{12}} \left\{ - P_1 + P_2 + A \cos (\delta_{12*} + \delta_{12}') + B \sin (\delta_{12*} + \delta_{12}') \right\} d\,\delta_{12}' = \frac{1}{2} M \omega_{12}^2 + F(\delta_{12}) \qquad 3.20$$

The total time derivative of V can be calculated as

$$\dot{V} = \frac{\partial V}{\partial \omega_{12}} \dot{\omega}_{12} + \frac{\partial V}{\partial \delta_{12}} \dot{\delta}_{12} = - K \omega_{12}^2 \qquad 3.21$$

The positive definiteness of V can be easily checked as before and therefore the system is asymptotically stable. In the neighbourhood of the origin $V = C$ curves will be closed for $C \leq F(\delta_{12u})$ where δ_{12u} is the solution for the unstable equilibrium state.

3.3 MULTI-MACHINE SYSTEM

Following the assumptions made in dealing with one and two machine systems the equation of motion of the rotor of any individual machine in a multi-machine system in the absence of an infinite busbar can be written as

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{mi} - P_{ei} - K_{di} \frac{d\delta_i}{dt}, \quad i = 1, 2 \ldots n \qquad 3.22$$

$$\text{where} \quad P_{ei} = \sum_{j=1}^{n} E_i E_j \left\{ G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right\} \qquad 3.23$$

E_1, E_2 etc. are the voltages behind transient reactances of the machines, δ_1, δ_2 etc. are the individual machine angles and $\delta_{12} = \delta_1 - \delta_2$ etc.

However as explained later it will not be possible to include the effect of mutual conductances since with these present, the energy type Liapunov functions as used here will not be positive definite around the equilibrium state as required by Liapunov's theorems. Neither will the total time derivative of the function be negative definite or semidefinite.

Neglecting mutual conductances the expressions for P_{ei} become

$$P_{ei} = E_i^2 G_{ii} + \sum_{j=1}^{n} E_i E_j B_{ij} \sin \delta_{ij} \qquad 3.24$$

The initial steady state values of the P_m's, E's, δ's are known from preliminary load flow calculations. According to the assumptions, the P_m's and E's are maintained constant throughout. After a system fault and its subsequent clearing the system passes through a transient state and if stability is maintained it settles down to a new equilibrium state. We intend to find out the stability region around this equilibrium state. If the final network condition is different from the original one the new equilibrium state will be different from the initial steady state. From equation 3.22, the equilibrium states are given by the solutions

$$P_{mi} - P_{ei} = 0 \;,\; i = 1, 2 \ldots n \qquad 3.25$$

An examination of equations 3.24 and 3.25 shows that corresponding to n equations there are only n - 1 independent variables. To get round this difficulty an extra variable is introduced. It is assumed P_m for all the machines change by an amount proportional to the respective inertias i.e.

$$\Delta P_{mi} = \frac{M_i}{\sum_{j=1}^{n} M_j} \Delta P \qquad 3.26$$

$$\text{where } \Delta P = \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi}) \qquad 3.27$$

The solutions to equation 3.25 can now be obtained. Taking the solution δ_{i*} corresponding to the stable equilibrium state as the origin, the equation of perturbed motion can be written as

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{mi} + \Delta P_{mi} - E_i^2 G_{ii} - \sum_{j=1}^{n} E_i E_j B_{ij} \sin(\delta_{ij*} + \delta_{ij}) - K_{di} \frac{d\delta_i}{dt} \qquad 3.28$$

$$i = 1, 2 \ldots n$$

The above equation is written in terms of absolute machine angles. However if stability is judged from the keeping-in-step condition of the machines in the system it is only necessary to investigate the relative motions of the machine rotors. The equations of motion of the machines relative to a reference machine preferably the one with the largest inertia can be written as

$$\frac{d^2 \delta_{in}}{dt^2} = \frac{P_{mi} - P_{ei}}{M_i} - \frac{P_{mn} - P_{en}}{M_n} - K \frac{d\delta_{in}}{dt} \qquad 3.29$$

$$i = 1, 2 \ldots n - 1$$

where n is the number of the reference machine.

Here uniform damping is assumed i.e. $\frac{K_{d1}}{M_1} = \frac{K_{d2}}{M_2} = \ldots = K$. The minimum value of $\frac{K_{di}}{M_i}$ is used for K. This is justified because higher damping is known to aid stability and hence using a low value for it would not invalidate the results. After all, the damping terms are retained in the equations merely to ensure asymptotic stability.

It may be noted that equation 3.29 allows for variations of machine input powers provided they are proportional to the machine inertias as given by equation 3.26.

The equilibrium states are given by the solutions

$$\frac{P_{mi} - P_{ei}}{M_i} - \frac{P_{mn} - P_{en}}{M_n} = 0, \quad i = 1, 2 \ \ldots \ n-1 \qquad 3.30$$

Two Liapunov functions have been used here corresponding to the system represented by equation 3.28 and equation 3.29. These are referred to by Liapunov functions A and B respectively.

3.3.1 Liapunov Function A

A Liapunov function for the system represented by equation 3.28 can be taken as

$$V(\omega,\delta) = \sum_{i=1}^{n} \tfrac{1}{2} M_i \omega_i^2 + \sum_{i=1}^{n} \int_0^{\delta_i} (-P_{mi} - \Delta P_{mi} + E_i^2 G_{ii})\, d\delta_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \int_0^{\delta_{ij}} E_i E_j B_{ij} \sin(\delta_{ij_*} + \delta_{ij})\, d\delta_{ij}$$

$$\text{or } V(\omega,\delta) = \sum_{i=1}^{n} \tfrac{1}{2} M_i \omega_i^2 + \sum_{i=1}^{n} (-P_{mi} - \Delta P_{mi} + E_i^2 G_{ii})\, \delta_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j B_{ij} \left\{ \cos \delta_{ij_*} - \cos(\delta_{ij_*} + \delta_{ij}) \right\} \qquad 3.31$$

Here $V(0, 0) = 0$

$$\text{and } \dot{V} = \sum_{i=1}^{n} \left(\frac{\partial V}{\partial \delta_i} \dot{\delta}_i + \frac{\partial V}{\partial \omega_i} \dot{\omega}_i \right)$$

$$= - \sum_{i=1}^{n} K_{di} \, \omega_i^{\,2}$$

This is negative semidefinite.

At the origin $\frac{\partial V}{\partial \omega_i} = 0$

$$\text{and } \frac{\partial V}{\partial \delta_i} = - P_{mi} - \Delta P_{mi} + E_i^{\,2} G_{ii} + \sum_{j=1}^{n} E_i E_j B_{ij} \sin \delta_{ij_*} = 0$$

for $i = 1, 2 \ldots n$

Thus V is stationary at the origin, and for V to be positive definite in the neighbourhood of the origin V (0, 0) should be a minimum i.e. the matrix H of the second derivatives of V should be positive definite. H is given by

$$H = \left[\begin{array}{c|c} M & \\ \hline & P \end{array} \right]$$

where M is a diagonal matrix given by

$$M = \text{diag} \, (M_1, M_2, \ldots)$$

The elements of P are given by $p_{ij} = - E_i E_j B_{ij} \cos \delta_{ij*}$ for $i \neq j$ and $P_{ii} = -$ (sum of the off diagonal elements)

It is clear that the requirement for H to be positive definite is that P be positive definite. However the matrix P is singular. This is because, among the variables δ_{12}, δ_{13} etc., there are only n-1 independent variables. Therefore the positive definiteness of $\overline{P}$ which is derived from P by deleting one row and column will ensure the positive definiteness of the V function in the neighbourhood of the origin in terms of the independent variables. Equation 3.31 is very similar to the expression for the energy integral used in references 52 and 53 and this form of Liapunov function has been used in reference 39. However, the positive definiteness of the function in the neighbourhood of the origin which ensures its stability is not clearly demonstrated. Also the conditions given there for stability of the equilibrium states are only necessary, not sufficient.

It may be noted that retaining the terms involving mutual conductances in the expressions for machine power outputs and adding corresponding terms in the expression for the Liapunov function, $\frac{\partial V}{\partial \delta_i}$ will no longer be equal to zero at the origin. Thus the condition for positive definiteness of V around the origin, which requires V to be zero and minimum at the origin, will not be satisfied. It will also follow that $\dot{V}$ will not be negative definite or negative semidefinite with mutual conductances included.

3.3.2 Liapunov Function B

A Liapunov function for the system represented by equation 3.29 can be taken as

$$V(\omega,\delta) = \frac{1}{2\sum_{i=1}^{n} M_i} \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} M_i M_j \omega_{ij}^2 + \sum_{i=1}^{n-1}\int_0^{\delta_{in}} (-P_{mi} - \Delta P_{mi} + E_i^2 G_{ii}) d\delta_{in} + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\int_0^{\delta_{ij}} E_i E_j B_{ij} \sin(\delta_{ij*} + \delta_{ij})\, d\delta_{ij}$$

$$\text{with } \Delta P_{mi} = \frac{M_i}{\sum_{j=1}^{n} M_j} \sum_{j=1}^{n} (E_j^2 G_{jj} - P_{mj})$$

$$\text{or } V(\omega,\delta) = \frac{1}{2\sum_{i=1}^{n} M_i} \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} M_i M_j \omega_{ij}^2 + \sum_{i=1}^{n-1} (-P_{mi} - \Delta P_{mi} + E_i^2 G_{ii})\delta_{in}$$
$$+ \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_i E_j B_{ij}\left\{\cos\delta_{ij*} - \cos(\delta_{ij*} + \delta_{ij})\right\} \qquad 3.32$$

Here V (0, 0) = 0, and

$$\dot{V} = -\frac{K}{\sum_{i=1}^{n} M_i} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} M_i M_j \omega_{ij}^2$$

which is negative semi-definite.

At the origin $\frac{\partial V}{\partial \omega_{in}} = 0$

and $\frac{\partial V}{\partial \delta_{in}} = - P_{mi} - \Delta P_{mi} + E_i^2 G_{ii} + \sum_{j=1}^{n} E_i E_j B_{ij} \sin \delta_{ij*} = 0$

Since the solution $\frac{P_{mi} - P_{ei}}{M_i} - \frac{P_{mn} - P_{en}}{M_n} = 0$ also satisfies

$$P_{mi} + \Delta P_{mi} - P_{ei} = 0, \text{ where } \Delta P_{mi} = \frac{M_i}{\sum_{j=1}^{n} M_j} \sum_{j=1}^{n} (E_j^2 G_{jj} - P_{mj})$$

Thus V is stationary at the origin, and as before for V to be positive definite in the neighbourhood of the origin the (n-1) th order matrix $\overline{P}$ must be positive definite. Equation 3.32 is similar to the expression for the energy integral given in reference 54, and this form of Liapunov function has been used in references 42 and 47. Reference 47 gives a derivation of this function applying a certain criterion due to Popov. However, as has been pointed out in reference 48, this function is invalid for the system as represented in reference 47 as for that system the function is not positive definite in the neighbourhood of the origin.

3.3.3 Liapunov Function when an Infinite Busbar is Present.

In the presence of an infinite busbar all the machine angles are measured with reference to that busbar. The equation of perturbed motion in an n-machine-infinite busbar system is given by

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{mi} - E_i^2 G_{ii} - \sum_{j=1}^{n} E_i E_j B_{ij} \sin (\delta_{ij*} + \delta_{ij}) - E_i E_\alpha B_{i\alpha} \sin (\delta_{i*} + \delta_i) - K_{di} \frac{d \delta_i}{dt} \qquad 3.33$$

for i = 1, 2 ... n,

where the subscript α refers to the quantities relating to the infinite busbar. A Liapunov function can be chosen as

$$V(\omega,\delta) = \sum_{i=1}^{n} \frac{1}{2} M_i \omega_i^{\,2} + \sum_{i=1}^{n} \int_{o}^{\delta_i} \Big\{ (-P_{mi} + E_i^{\,2} G_{ii} + E_i E_\alpha B_{i\alpha} \sin$$

$$(\delta_{i*} + \delta_i) \Big\} d\delta_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \int_{o}^{\delta_{ij}} E_i E_j B_{ij} \sin(\delta_{ij*} + \delta_{ij})\, d\,\delta_{ij} \qquad 3.34$$

(Note that equation 3.34 follows directly from equation 3.31 by letting $M_n = \alpha$)

As before $V(0, 0) = 0$ and

$$\dot{V} = - \sum_{i=1}^{n} K_{di}\, \omega_i^{\,2}$$

At the origin $\frac{\partial V}{\partial \omega_i} = 0$ and

$$\frac{\partial V}{\partial \delta_i} = - P_{mi} + E_i^{\,2} G_{ii} + E_i E_\alpha B_{i\alpha} \sin \delta_{i*} + \sum_{j=1}^{n} E_i E_j B_{ij} \sin \delta_{ij*} = 0$$

Thus V is stationary at the origin and for V to be positive definite in the neighbourhood of the origin the matrix P must be positive definite. The off-diagonal elements of P will be the same as before; however, the diagonal elements will have an extra term $E_i E_\alpha B_{i\alpha} \cos \delta_{i*}$.

3.3.4 Transient Stability Region.

Having established the positive definiteness of the V function in the neighbourhood of the origin which ensures stability of the equilibrium state we proceed to establish the size of the stability region i.e. the largest value ℓ of C for which V = C describes a closed surface surrounding the origin. While the task is simple in the two dimensional case of a single machine infinite busbar system, it is not so obvious in the multi-machine case because of the large dimensionality. Unlike the single machine case there exist many equilibrium states in a multi-machine system. At the equilibrium states $\frac{\partial V}{\partial \omega_i}$, and $\frac{\partial V}{\partial \delta_i}$, for i = 1, 2 ... n, are zero and thus V is stationary there. Since in the matrix H of the second derivative of V the submatrix M is always positive definite, for stability considerations we need to study the properties of $\overline{P}$ only. $\overline{P}$ is positive definite at the stable equilibrium state, taken as the origin, (already established.) At other stationary points of V, $\overline{P}$ will either be indefinite or negative definite. In either case the matrix H will be indefinite and thus these stationary points represent saddle points, and the equilibrium states are unstable. It is one of these saddle points which will determine the stability region. The value of V evaluated at this point will serve as ℓ. It is not, however, apparent which of the unstable equilibrium states should be used for this purpose. In actual practice ℓ can be calculated in one of the following ways.

If the system disturbance is such that it is likely to cause one particular machine to accelerate much faster than the others resulting in instability of that machine before the others have deviated appreciably from their equilibrium positions, the solution of the equilibrium state showing an unstable condition of this particular machine can be used in the expression for V to calculate ℓ.

If the situation is such that several of the machines can accelerate more or less at the same rate, then the several equilibrium states corresponding to the unstable conditions of each of these machines separately and all of them together can be found. The value of V can be calculated at each of these unstable equilibrium states and the lowest one may be used as ℓ.

Another way of estimating ℓ is discussed in a later section.

3.3.5 Solution for the Equilibrium States

The equilibrium states are given by the solution of equation 3.25 or 3.30 depending on whether the system is described by equation 3.22 or 3.29. A number of methods is available for solving this type of nonlinear equations. A method which has been found very efficient and used to solve all the equations of this type, in this and the next chapter, uses a technique for function minimisation by conjugate gradients. The technique of function minimisation is described in reference 55. A function, which is the sum of the squares of the expressions on the left-hand sides of equations 3.25 or 3.30, is formed and minimised. The function will have its minimum value at the solution and the minimum value is zero. In this method of minimisation, in addition to the function itself, the vector of its first partial derivatives with respect to the variables is also necessary. Expressions for the function f to be minimised, and its partial derivatives $\frac{\partial f}{\partial \delta_{in}}$, for i = 1, 2 ... n−1, with regard to the solution of equation 3.30 are given below.

$$f = \sum_{i=1}^{n-1} \frac{1}{2} \left\{ (P_{mi} - P_{ei}) - \frac{M_i}{M_n} (P_{mn} - P_{en}) \right\}^2 \qquad 3.35$$

$$\begin{aligned} \frac{\partial f}{\partial \delta_{in}} = & \left\{ (P_{mi} - P_{ei}) - \frac{M_i}{M_n} (P_{mn} - P_{en}) \right\} \left\{ \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \right. \\ & \left. - \frac{M_i}{M_n} E_i E_n (G_{in} \sin \delta_{in} + B_{in} \cos \delta_{in}) \right\} \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \left\{ (P_{mj} - P_{ej}) - \frac{M_j}{M_n} (P_{mn} - P_{en}) \right\} \left\{ E_i E_j (G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) \right. \\ & \left. - \frac{M_j}{M_n} E_i E_n (G_{in} \sin \delta_{in} + B_{in} \cos \delta_{in}) \right\} \end{aligned} \qquad 3.36$$

where P_{ei} is given by equation 3.23.

For the sake of generality the mutual conductances have been retained in the expressions. It may be noted that equation 3.25 will have the same solution for δ_{in}'s ; ΔP will be given by

$$\Delta P = \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi}) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 2 E_i E_j G_{ij} \cos \delta_{ij} \qquad 3.37$$

We wish to find the solutions for the stable and unstable equilibrium states of the final post-fault system. To start, an initial estimate of the variables is required. The function minimisation procedure will then enable the solution to converge to the nearest equilibrium point. Normally the stable equilibrium state solution of the post-fault system will be close to the steady state solution of the pre-fault system. Hence the starting values of the δ_{in}'s can be chosen to be the steady state values as given by the initial load-flow solution of the pre-fault system.

The initial estimate for the unstable equilibrium state will depend on which of the many unstable equilibrium states present is desired. If the unstable equilibrium state corresponding to one particular machine becoming unstable while the others are staying more or less near their steady state conditions is sought, the initial angles can be chosen such that δ_{in} for that machine is the complement of that for the stable equilibrium state while the others are the same as for the stable equilibrium state. The initial estimates for other types of unstable equilibrium states can be similarly made.

3.3.6 Procedure for Transient Stability Study

The main steps in studying transient stability, whether in investigating the maintenance of stability following a disturbance or a series of disturbances or in determining the critical clearing time after the occurence of a system fault, using Liapunov's direct method can be summarised thus:

a) Load flow for the pre-fault system, whence the machine internal voltages and angles will be determined.

b) Formation of the network admittance matrix with respect to the machine internal buses corresponding to the various faulted and the final post fault system.

c) Solution for the stable and unstable equilibrium states of the final post fault system as described in section 3.3.5.

d) Evaluation of ℓ, the constant to determine transient stability region, as outlined in section 3.3.4.

e) Numerical step-by-step integration during the disturbed state.

In order to ensure that the system trajectory remains within the stability region during the disturbed period, the value of V is checked for $V < \ell$ at each integration step during the step-by-step calculation. If V is found to be less than ℓ throughout the disturbed period up to the instant of the last switching operation, the system can be considered stable and further step-by-step calculation will not be necessary.

If one wishes to determine the critical clearing time after the occurence of a fault V can be calculated from the values of the variables found during the step-by-step integration of the faulted system and checked against ℓ. Critical clearing time will be given by the time at which V just reaches the value ℓ.

3.3.7 Description of the Computer Programs

Each of the steps from a to c are carried out using separate computer programs. Steps d and e are incorporated in one program which is the main transient stability program. However if desired all the steps could easily be incorporated in one single program. When an investigation of particular methods is the main object it is advantageous to use separate programs to handle different parts of the problem.

Initial load flow results are obtained from an already existing load flow program and machine internal voltages and angles are calculated from these. These results are checked by the program written to solve for the equilibrium states. For this purpose a pre-fault network admittance matrix is used. Network admittance matrices with respect to the machine internal buses, for pre-fault, fault and post-fault conditions are calculated using a program written in conjunction with the multi-machine transient stability program described in Chapter 1.

Originally two programs were written to solve for the equilibrium states corresponding to the solution of equation 3.25 and 3.30. Later it was realized that these two equations have the same solution for δ_{in}'s; ΔP, the extra variable in equation 3.25, is given by equation 3.37. Therefore the use of the program corresponding to the solution of equation 3.30 was taken as standard as it reduced the number of variables by one. The main body of the procedure for function minimisation was taken from reference 55 with a few minor alterations. A few of the inner iterative loops were found unnecessary and sometimes troublesome for solving equations of the type considered here and they were therefore removed thus improving on the speed of convergence. In addition to the main procedure a procedure which calculates the function value and the vector of the first partial derivatives and a procedure to deal with the output after every complete set of iterations are also part of the program. These two latter procedures are used as parameters in the main function minimisation procedure. The input data start with two constants specifying the number of runs and number of variables. This is followed by the machine quantities, the order in which they are read, for each machine in turn, being - initial estimate of the angle, mechanical input power, internal voltage and moment of inertia. Next comes the network admittance matrix. The real and imaginary parts are read separately - these being symmetrical only the lower triangular parts are entered as data. This is followed by a constant specifying the estimate of the function at the solution, read as zero, a constant specifying the maximum number of complete sets of iterations permitted before the program is thrown off the machine and a constant specifying the accuracy of the convergence. A sample of the input and output is shown in appendix A 3.1.

Corresponding to the two Liapunov functions A and B discussed in sections 3.3.1 and 3.3.2 two transient stability programs have been written. The first one refers to the system described by equation 3.28 while the other one refers to the same system described by equation 3.29. Both approaches should produce similar results. The input data to both the programs are very much the same. First the constants which specify the number of runs, size of the system etc. are read. This is followed by the machine quantities including the machine internal angles corresponding to the stable and unstable equilibrium state solutions and the initial steady state. More constants specifying the step length, total number of integration steps etc. are next read. The data concludes with the network admittance matrices. These are read in lower triangular form, the matrices of the real and imaginary parts being read separately. ℓ is calculated at the beginning of the computation. This is the value of the V function at the unstable equilibrium state specified in the data. The computation then moves into the step-by-step integration part. With the fault on, the value of V is calculated at each integration step using the current values of the machine angles, and compared with ℓ. The programs in this present form seek to determine the critical clearing time. However they can be modified, if desired, to handle other situations also, as explained in section 3.3.6. The figure of merit $(-\dot{V}/V)$ is also calculated at each integration step. This gives an indication of how fast the origin will be approached at that particular instant, if the fault is removed. The programs output, the values of the machine angles, V and $-\dot{V}/V$ at each integration step. Critical clearing time is indicated when V just reaches ℓ. Computation is continued for some time after the fault is removed. A typical computer input and output is given in appendix A 3.2.

3.3.8 Numerical Example

Results of transient stability studies on a 6 machine system will now be presented. A test system prepared by The South of Scotland Electricity Board to test their power system programs has been used here as an example. The original 150 busbar system was reduced to a 6 busbar system retaining only the busbars corresponding to the 6 machines. The first 5 machines are within the S.S.E.B. system while the 6th one represents an equivalent machine for the Central Electricity Generating Board system. The data for the 6 machines are listed in Table 3.1. The p.u. values have been calculated on a 1000 MVA base.

Table 3.1

Synchronous Machine Data

Machine Number	M	x_d'	M/c rating MVA	K_d
1	.0063	2.16	3x75	.8
2	.0186	.608	2x350	1.5
3	.00707	1.65	2x110	1
4	.00675	2.06	6x30	.75
5	.00372	2.47	2x150	.5
6	6.37	.5	Eqv. m/c for the C.E.G.B. system	1

The network matrices for pre-fault and post-fault conditions are given in Tables 3.2 - 3.4; those for fault conditions are given in appendix A 3.3. Mutual conductances are included in the network matrices given below. However, these are neglected in the actual study. This means, the off-diagonal elements of the G matrix are taken as zero and the diagonal elements are adjusted accordingly. Initial load flow results are given in Table 3.5. Solutions for stable and unstable equilibrium states are given in Tables 3.6 - 3.7.

Table 3.2

Pre-fault Network Matrices with respect to the Machine Terminals

G Matrix

.188757			
-.053155	1.973114		
.000002	-.112323	.305647	
-.059051	-.050041	-.000264	.211027

.000001	-.003018	-.017299	-.00009	.148201	
.000115	-1.574549	-.111943	-.068548	-.037182	1.869282

B Matrix

-1.288403					
1.068633	-14.58122				
.000012	1.165181	-2.203888			
.204962	.489691	.001901	-.907932		
.000004	.106677	.226923	.000771	-.693535	
.001323	11.89943	.925913	.227481	.375603	-13.4078

Table 3.3

Pre-fault Network Matrices with respect to the Machine Internal Buses

G Matrix

.016135					
.014992	.059414				
.004126	.016589	.019474			
.00153	.00768	.002149	.025334		
.002715	.011816	.006673	.001744	.022776	
.013763	.053818	.018489	.006345	.014724	.080023

B Matrix

-.31739					
.117781	-1.025135				
.032014	.167574	-.426193			
.040347	.108536	.030743	-.297566		
.015463	.080812	.042162	.015232	-.243513	
.121681	.633019	.200404	.122022	.10476	-1.088374

Table 3.4

Post-fault Network Matrices with respect to the Machine Internal Buses following Faults at the Terminal of Machine 1 or 4.

G Matrix

.016085					
.015146	.059458				
.004173	.0166	.019477			
.001494	.007464	.002085	.025514		
.00274	.011821	.006674	.001709	.022777	
.013957	.053855	.018497	.006086	.014728	.080046

B Matrix

-.314298					
.118218	-1.025082				
.0321	.167583	-.426192			
.03624	.107973	.030634	-.292335		
.015494	.080815	.042163	.015193	-.243513	
.121865	.633035	.200406	.12179	.10476	-1.088376

Post fault network matrices following a fault at the terminal of machine 3 are the same as the pre-fault matrices given in Table 3.3.

Table 3.5

Initial Load Flow Results

Machine Number	Machine Loading P	Q	Terminal Voltage	Angle degrees	M/c Internal Voltage	Angle degrees
1	.16246	-.0473	.9398	7.02	.9111	31.2122
2	.3705	-.0585	1	.905	.9904	14.052
3	-.21965	-.0499	1.0075	-7.08	.9932	-28.3144
4	.14586	-.0594	.9499	9.88	.8799	30.949
5	.05966	-.0132	1	-4.63	.9786	4.0311
6	6.63_{10}^{-5}	-3.61_{10}^{-6}	.9989	0	.9989	.0019

Table 3.6

Solution for the Stable Equilibrium States - Mutual Conductances Neglected.

Machine Number	Angles in Radians	
	Corresponding to matrices given in Table 3.3	Corresponding to matrices given in Table 3.4
1	.525292	.523371
2	.227223	.227187
3	-.533915	-.533946
4	.526257	.52889
5	.050869	.050866

Table 3.7

Solution for Unstable Equilibrium States - Mutual Conductances Neglected.

	Angles in Radians		
Machine Number	Machine 1 unstable	Machine 3 unstable	Machine 4 unstable
1	2.806505	.640734	.645873
2	.262385	.276838	.259435
3	-.621252	-2.273486	-.612156
4	.664447	.640901	2.798693
5	.052013	.046297	.05246

Critical clearing times for faults at the terminals of machines 1, 3 and 4 were evaluated by using the two transient stability programs discussed in section 3.3.7. These, as predicted by the program using the Liapunov function A, are .27, .17 and .30 sec. respectively. Those predicted by the program using the Liapunov function B are .27, .17 and .32 sec. respectively. These critical clearing times were checked by the multi-machine transient stability program described in Chapter 1. Curves showing rotor angle excursions are shown in Figs. 3.6, 3.9 and 3.12 for faults on machines 1, 3 and 4 respectively. Solid line curves are those corresponding to the clearing times for which the system is just

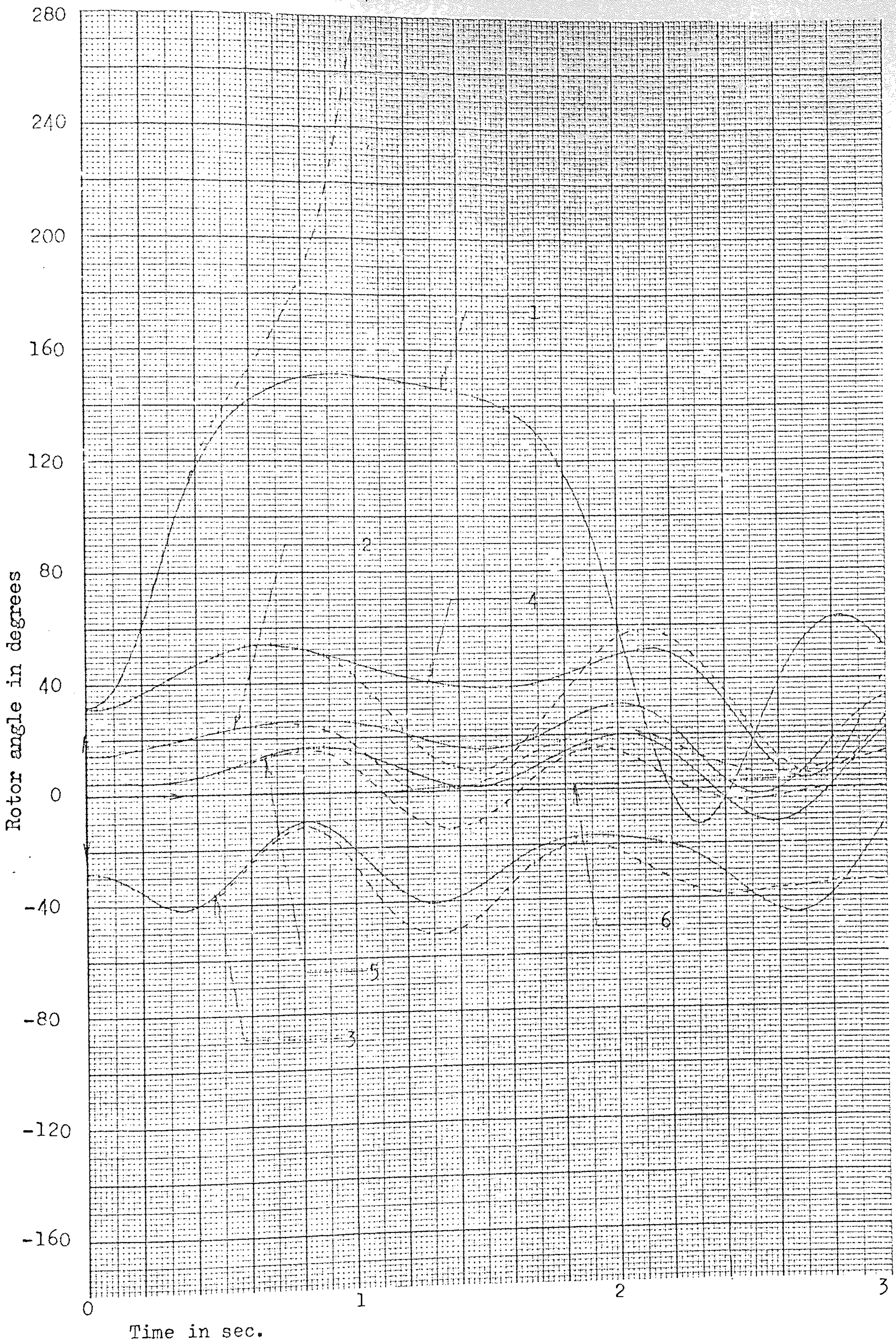


Fig. 3.6 Rotor angle excursions of the 6 machine S.S.E.B. system following a three phase fault at the terminals of machine 1

—— stable for fault duration .26 sec.

----- unstable for fault duration .27 sec.

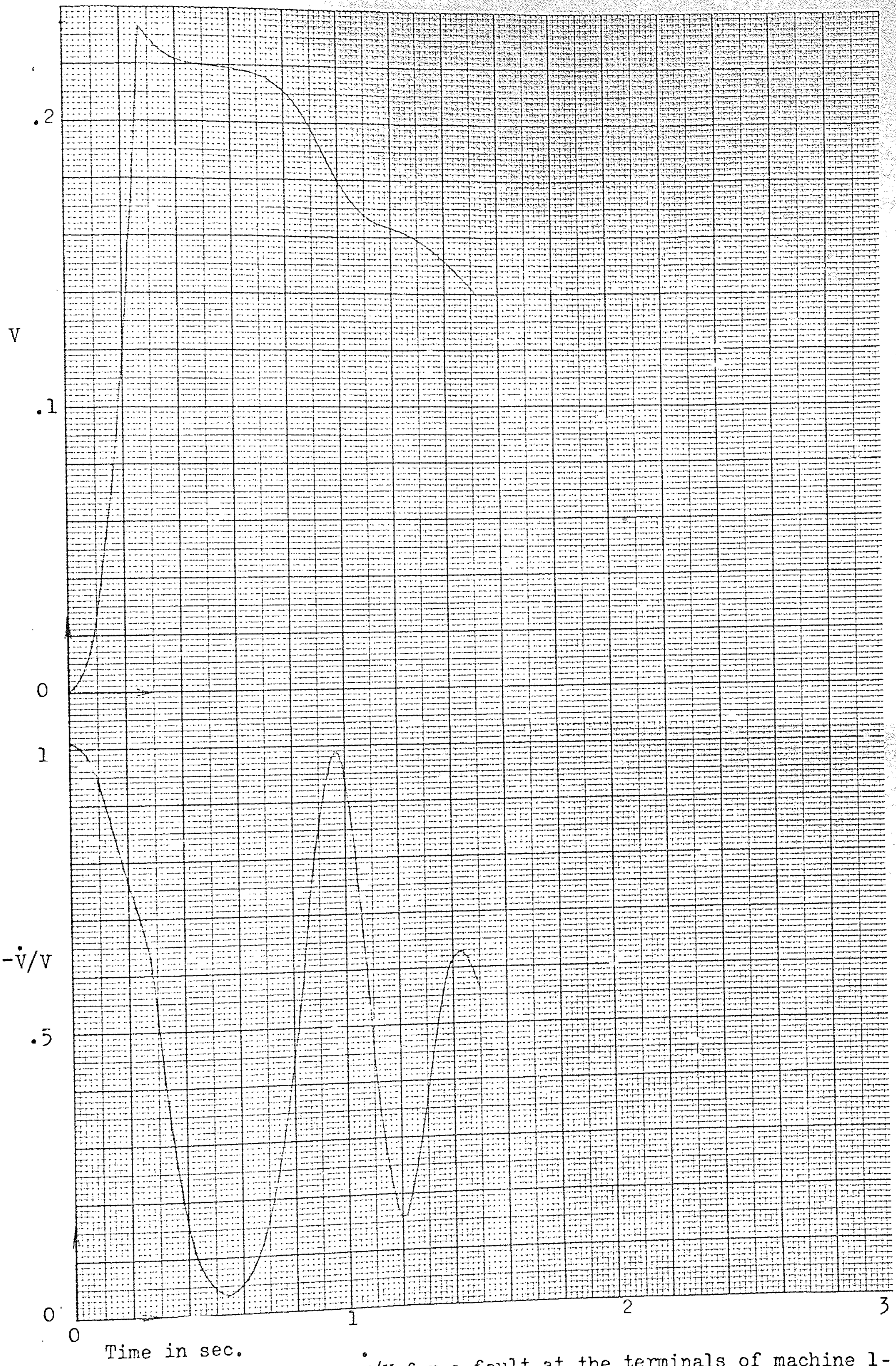


Fig. 3.7 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 1-corresponding to Liapunov function A.

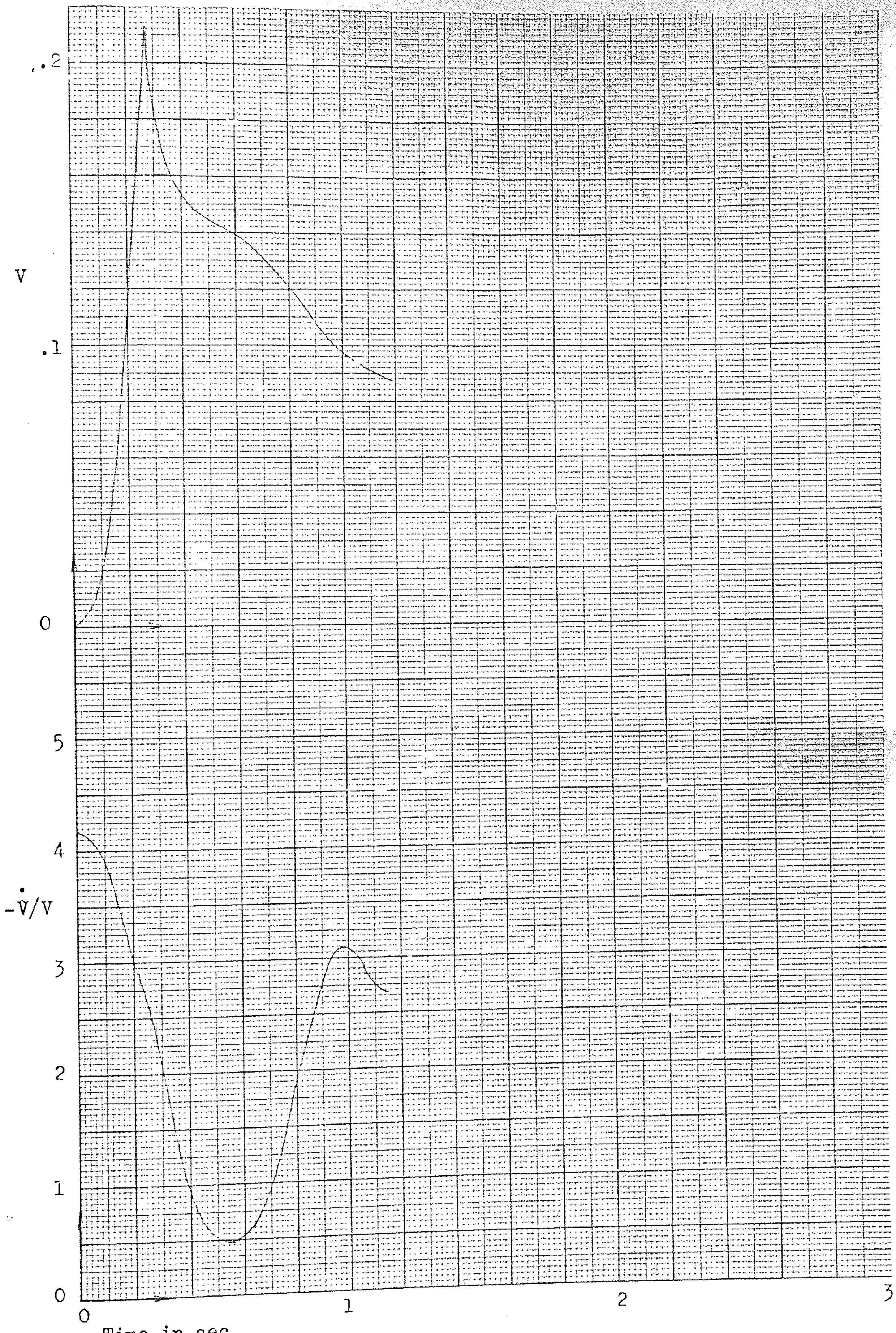


Fig. 3.8 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 1- corresponding to Liapunov function B.

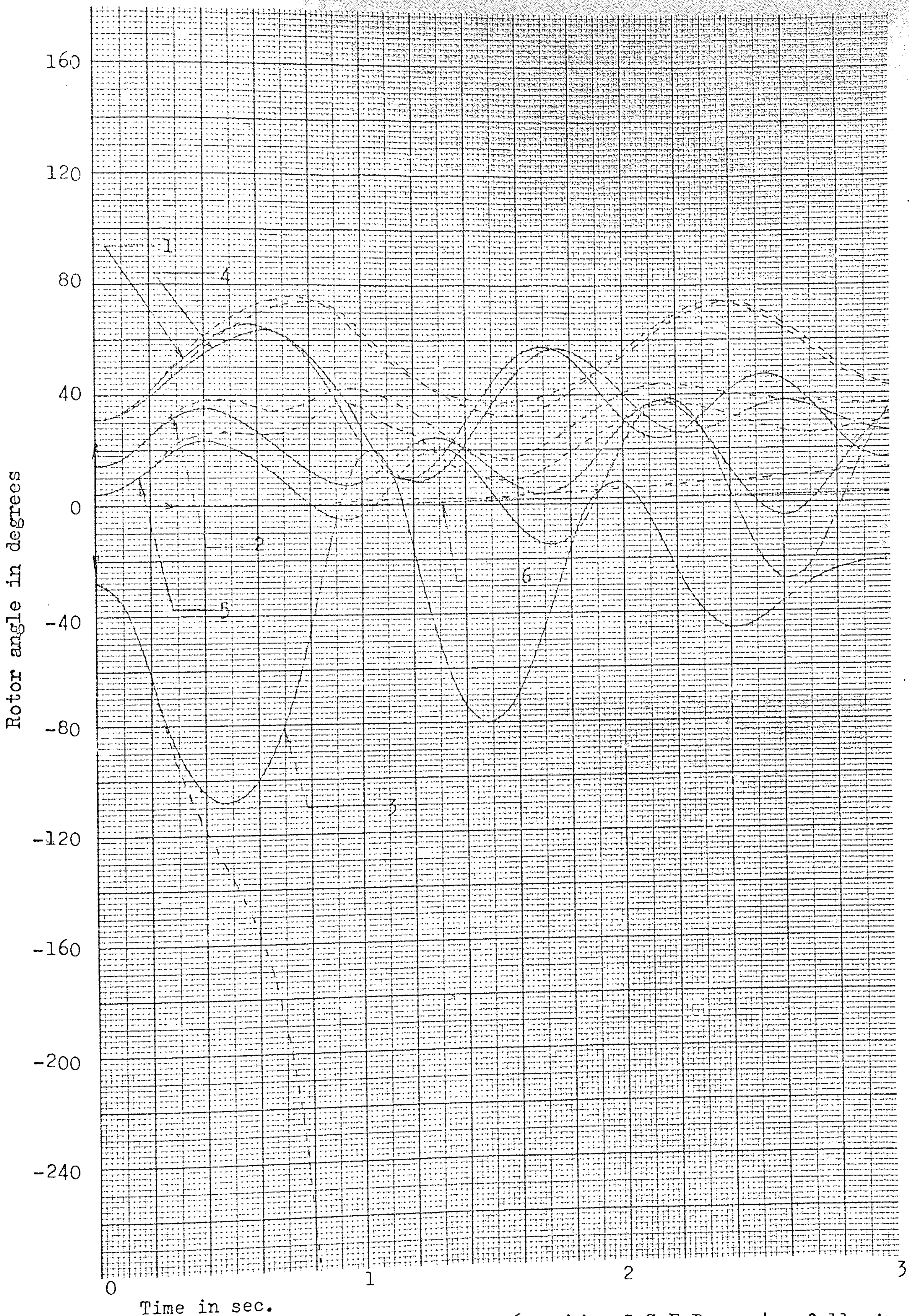


Fig. 3.9 Rotor angle excursions of the 6 machine S.S.E.B. system following a three phase fault at the terminals of machine 3

—— stable for fault duration .21 sec.

----- unstable for fault duration .22 sec.

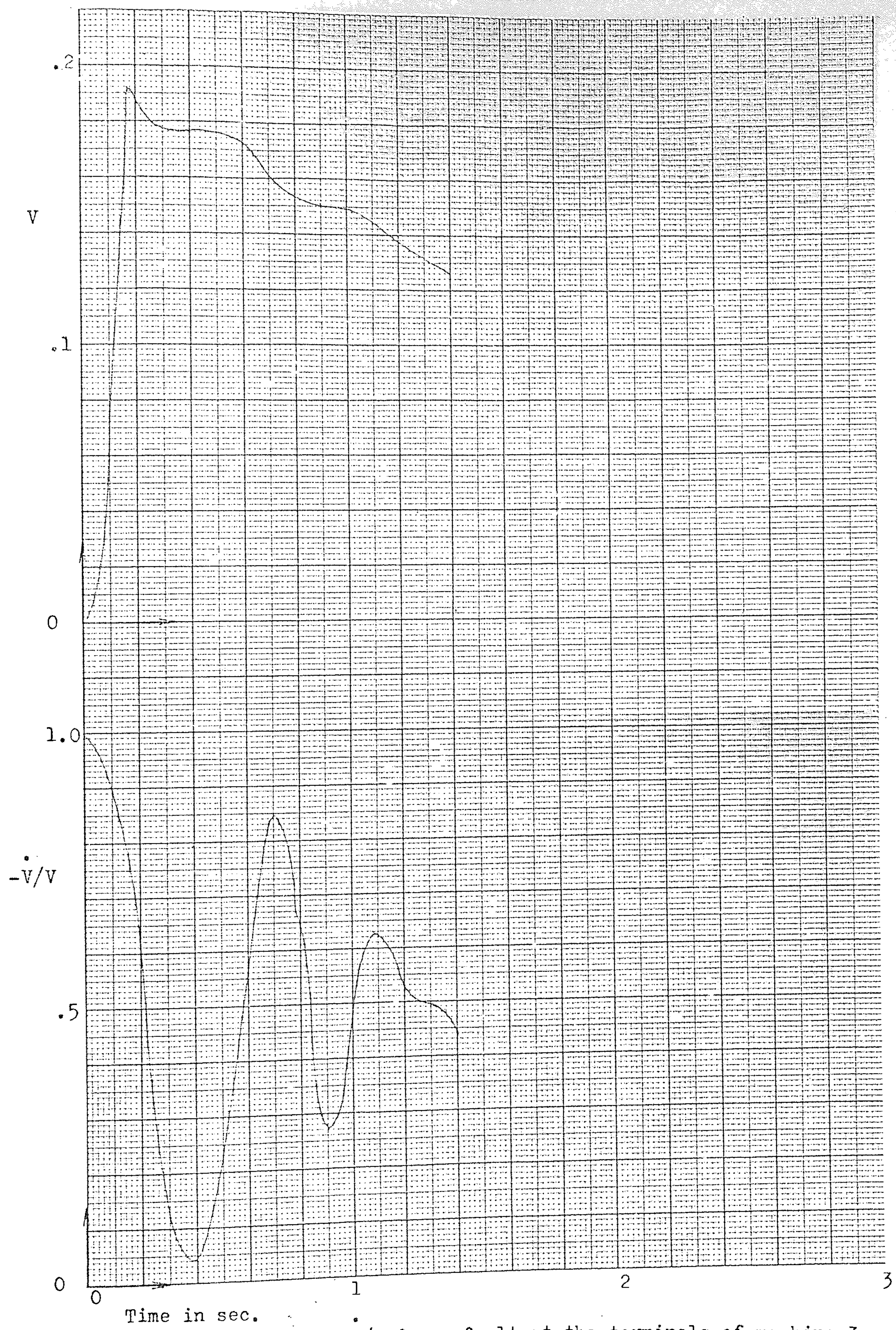


Fig. 3.10 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 3-corresponding to Liapunov function A.

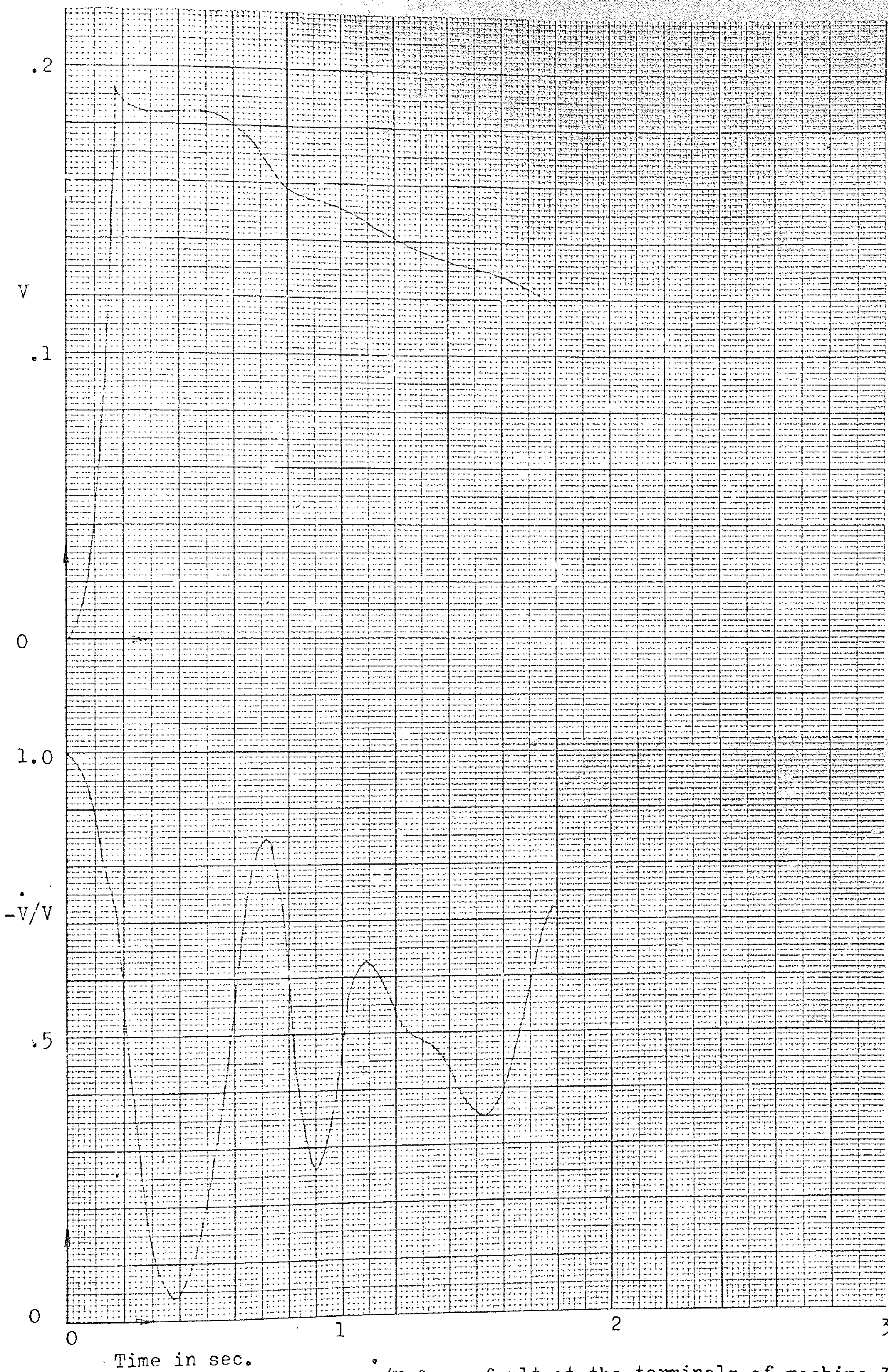


Fig. 3.11 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 3-corresponding to Liapunov function B.

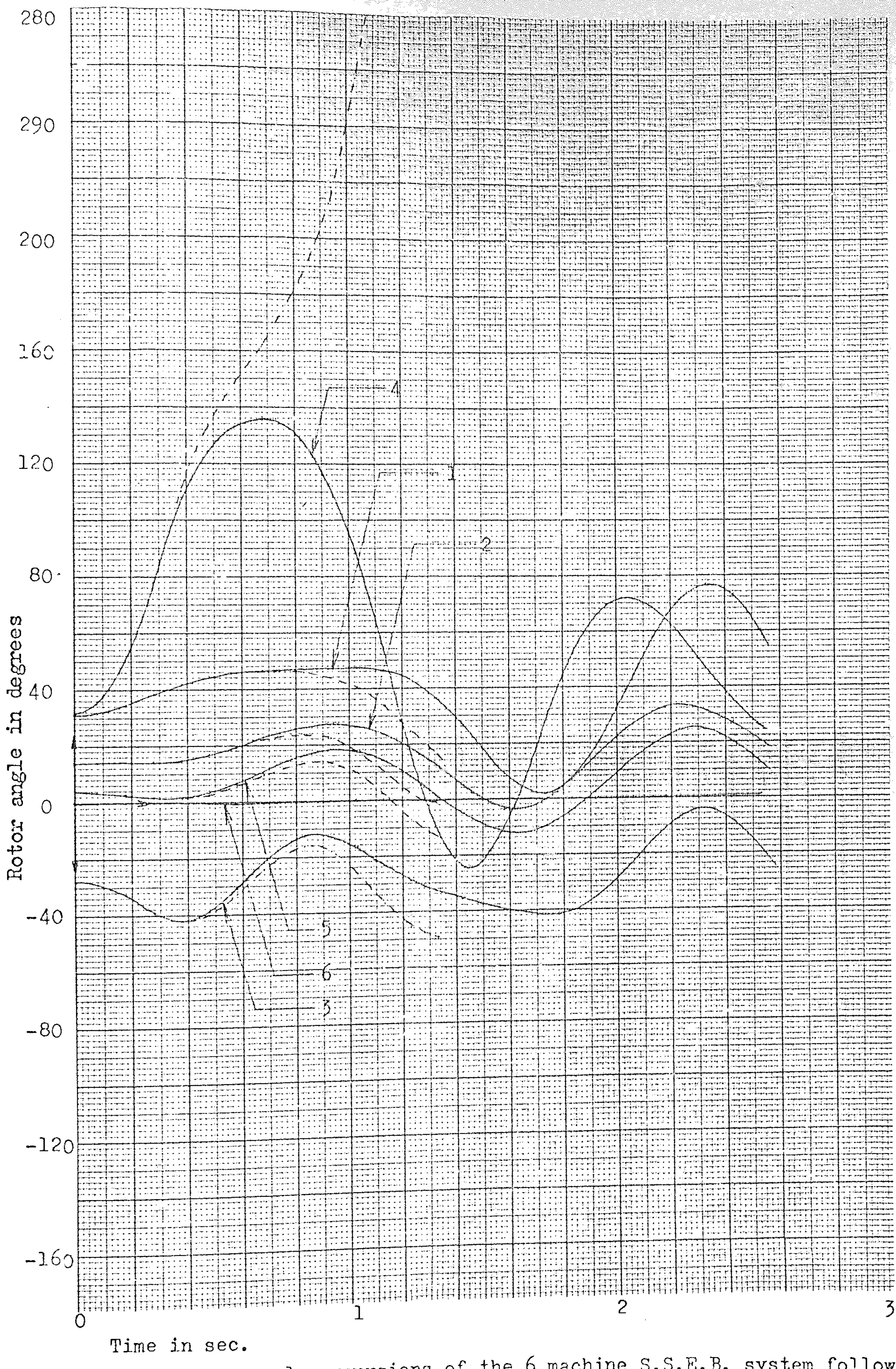


Fig. 3.12 Rotor angle excursions of the 6 machine S.S.E.B. system following a three phase fault at the terminals of machine 4

____ stable for fault duration .30 sec.

----- unstable for fault duration .31 sec.

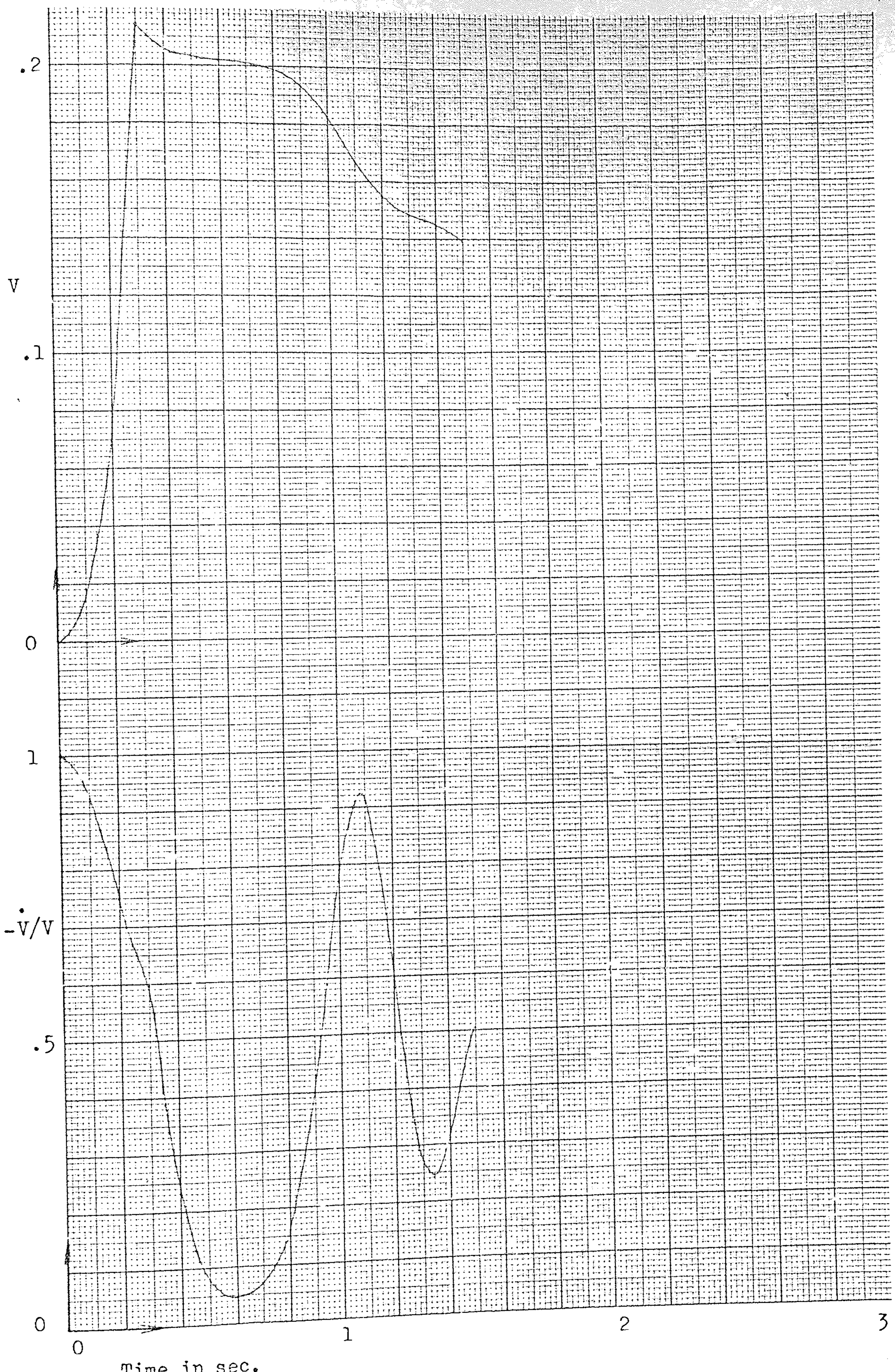


Fig. 3.13 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 4-corresponding to Liapunov function A.

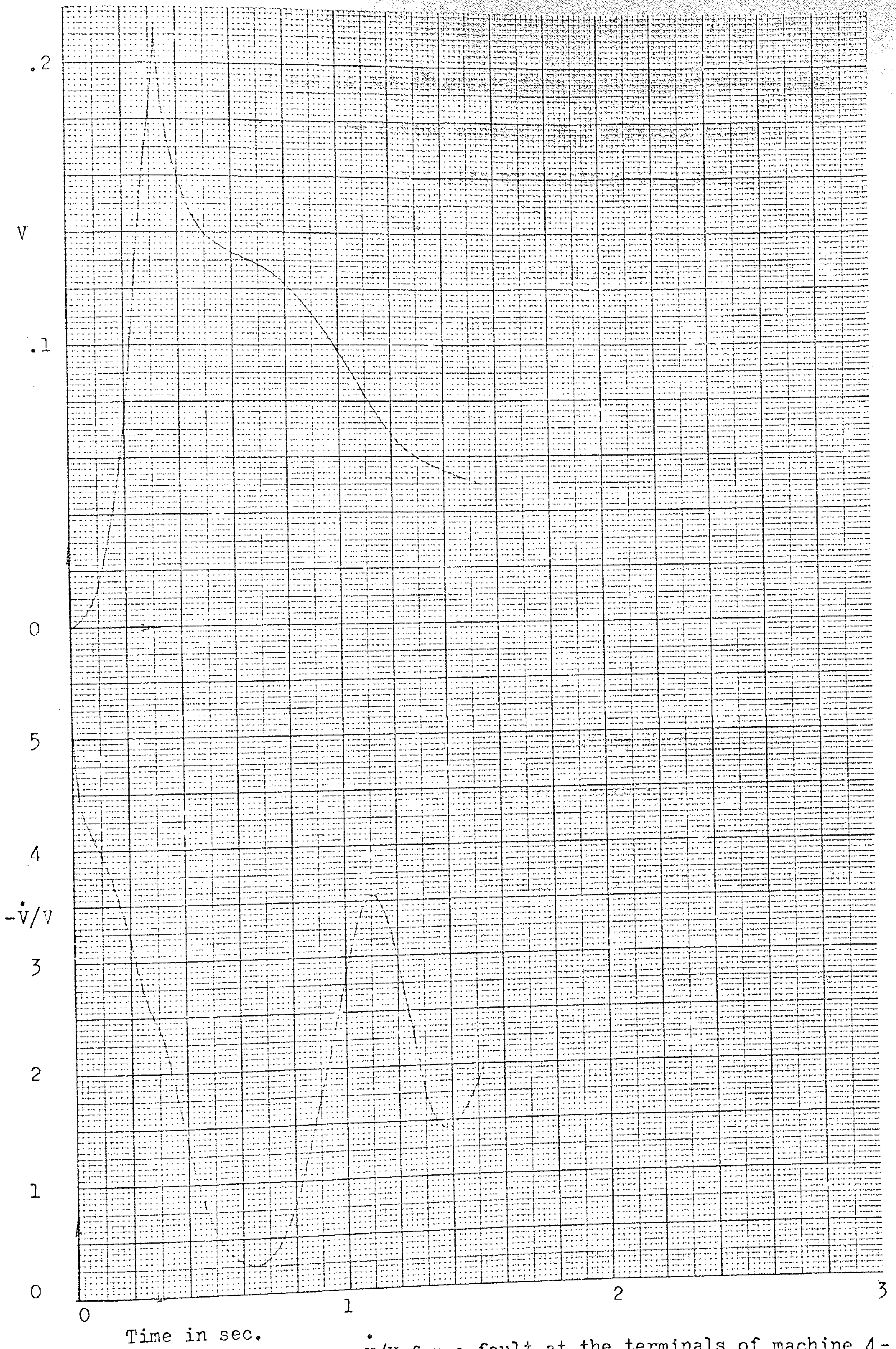


Fig. 3.14 Plots of V and $-\dot{V}/V$ for a fault at the terminals of machine 4 - corresponding to Liapunov function B.

stable. A slight increase in the clearing times will render the system unstable as illustrated by the dotted curves. The critical clearing times as shown by these curves for faults on machines 1, 3 and 4 are .26, .21 and .30 sec. respectively.

It may be seen that in all the three cases studied the results obtained by using the two alternative Liapunov functions are in close agreement with each other. Also, for faults at the terminals of machines 1 and 4, the critical clearing times as predicted by Liapunov's method agree with those obtained by using a conventional step-by-step integration procedure. The stability boundary as indicated by the value of the Liapunov function at the unstable equilibrium point closest to the final stable equilibrium point in each of these two cases is therefore exact. The slight differences in the results may be attributed to the fact that mutual conductances were neglected in both transient stability programs incorporating the Liapunov functions. For a fault on machine 3 the difference between the critical clearing times obtained by the two methods is noticeable - that obtained by Liapunov's method is considerably lower. This illustrates the fact that the stability region predicted by Liapunov's method is often conservative.

Plots of V and $-\dot{V}/V$ are shown in Figs. 3.7, 3.10 and 3.13 using the Liapunov function A for faults on machines 1, 3 and 4 respectively. The corresponding curves using the Liapunov function B are shown in Figs. 3.8, 3.11 and 3.14.

3.3.9 An Approximate Method of Estimating ℓ

Two ways of estimating ℓ have been discussed in section 3.3.4. An approximate method of estimating ℓ will now be presented, which removes the need for solving for the unstable equilibrium states. The approximation is based on the following consideration.

In the equilibrium states ω's will be zero and δ's will have values zero or non-zero depending upon whether the equilibrium state is stable or unstable. If the system is such that during a disturbance one particular machine is likely to become unstable before the others have appreciably deviated from the steady state positions it is reasonable to consider only the change in the angle of this machine assuming changes in the other machines to be negligible. Using the subscript i for this particular machine if we let the angle of this machine change and keep the angles of the others constant at their stable equilibrium state values we get expression for V, for the Liapunov function A, as

$$V_i\ (\omega_i,\ \delta_i) = \frac{1}{2} M_i \omega_i^{\ 2} + (-P_{mi} - \Delta P_{mi} + E_i^{\ 2}\ G_{ii})\ \delta_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i\ E_j\ B_{ij} \left\{ \cos\delta_{ij*} - \cos\ (\delta_{ij*} + \delta_i) \right\}$$

$$= \frac{1}{2} M_i\ \omega_i^{\ 2} + (-P_{mi} - \Delta P_{mi} + E_i^{\ 2}\ G_{ii})\ (\delta_i - \sin\delta_i) + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i\ E_j\ B_{ij} \cos\delta_{ij*}\ (1 - \cos\delta_i) \qquad 3.38$$

Thus the V function in a multi-machine system is treated as a function of two variables ω_i, δ_i as in the single machine system and from arguments similar to those made in discussing the single machine system in section 3.2.1, ℓ will be given by the maximum value of $V_i\ (\delta_i)$. The maximum value of $V_i\ (\delta_i)$ occurs at a value of δ_i given by

$$\delta_{i\ max} = 2\ \tan^{-1} \frac{\sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j B_{ij} \cos\delta_{ij*}}{P_i + \Delta P_{mi} - E_i^{\ 2}\ G_{ii}} \qquad 3.39$$

If it is not obvious that the system machines are going to behave in the way as assumed above, a number of maxima of $V_i\ (\delta_i)$ for i=1, 2...etc.

may be calculated and the minimum value of these maxima can be taken as ℓ. Now of course only the machines which are most likely to become unstable due to the particular disturbance need be considered.

If any two or more of the machines are likely to swing and become unstable together, while the others remain more or less stationary, ℓ can be approximated as follows. Supposing only the angles of the ith and the jth machines change and they change equally

$$V_{i,j}\,(\omega_{i,j},\,\delta_{i,j}) = \frac{1}{2}\,(M_i + M_j)\,\omega_{i,j}^{\,2} + \Big\{(-P_{mi} - \Delta P_{mi} + E_i^{\,2}\,G_{ii}) + (-P_{mj} - \Delta P_{mj} + E_j^{\,2}\,G_{jj})\Big\}\,\delta_{i,j}$$

$$+ \sum_{\substack{k=1\\k\neq i,j}}^{n} E_i\,E_k\,B_{ik}\Big\{\cos\delta_{ik*} - \cos(\delta_{ik*} + \delta_{i,j})\Big\} + \sum_{\substack{k=1\\k\neq i,j}}^{n} E_j\,E_k\,B_{jk}\Big\{\cos\delta_{jk*} - \cos(\delta_{jk*} + \delta_{i,j})\Big\} \qquad 3.40$$

As before ℓ is the maximum value of $V_{i,j}\,(\delta_{i,j})$, which occurs when

$$\delta_{i,j\,max} = 2\tan^{-1}\frac{\sum\limits_{\substack{k=1\\k\neq i,j}}^{n} E_i\,E_k\,B_{ik}\cos\delta_{ik*} + \sum\limits_{\substack{k=1\\k\neq i,j}}^{n} E_j\,E_k\,B_{jk}\cos\delta_{jk*}}{(P_{mi} + \Delta P_{mi} - E_i^{\,2}\,G_{ii}) + (P_{mj} + \Delta P_{mj} - E_j^{\,2}\,G_{jj})} \qquad 3.41$$

Similar arguments are also applicable while using the Liapunov function B and expressions for ℓ will be exactly the same.

In Table 3.8 the values of ℓ calculated by the approximate method are compared with those of ℓ calculated in the transient stability programs for the three cases studied.

Table 3.8

Comparison Between the Values of ℓ Calculated by Exact and Approximate Method.

Case Number	Exact Value of ℓ	ℓ calculated by the approximate method.
1	.237	.256
2	.193	.204
3	.22	.238

It can be seen that the values of ℓ calculated by the approximate method are slightly higher than the actual values and therefore the results obtained by using the approximate method will be somewhat optimistic.

3.4 SUMMARY

In this chapter the application of Liapunov's direct method to power system transient stability studies has been demonstrated. Through the use of energy type Liapunov functions transient stability regions of a single machine, a two machine and a multi-machine system have been investigated. Establishing the stability region in the general case often presents a difficulty. This has been shown to be a simple matter in the single machine case and using an energy type Liapunov function is equivalent to the well-known equal area criterion. Although in a multi-dimensional system it is extremely difficult to decide upon the stability region on a rigorous mathematical basis, the criterion used here has produced satisfactory results. In the multi-machine case two alternative Liapunov functions, depending on the way the system is described mathematically, have been considered. Similar functions have been used in connection with transient stability studies by other workers in this field. However, as has been shown, those were not entirely satisfactory from a rigorous mathematical point of view. Some modifications in the Liapunov functions and in the system representation have removed the difficulties.

A study of the critical clearing times for faults at various points of a six machine system has been included as an example.

The approximate method of estimating the stability region without actually solving for the unstable equilibrium state has been shown to produce acceptable results.

CHAPTER 4.

LIAPUNOV FUNCTIONS AND STEADY STATE STABILITY

4.1 INTRODUCTION

In Chapter 3 it has been shown how energy type Liapunov functions can be used to study transient stability problems in a multi-machine power system when the synchronous machines in the system are represented by constant voltage behind transient reactance. In this chapter the use of the same Liapunov functions in studying the steady state stability, which is the study of the stability of the equilibrium states following a small perturbation, for systems represented similarly, will be discussed. The stability of the equilibrium states of power systems represented in any arbitrary manner is considered in the next two chapters through the application of Liapunov's theorem on stability determined by the linear approximation.

4.2 CRITERION FOR STEADY STATE STABILITY

In Chapter 3 a simple criterion for stability of the equilibrium states has been obtained from either of the Liapunov functions A or B by considering the local minima of the functions. This is the positive definiteness of the matrix $\overline{F}$. The same criterion can be derived in another way[50] which will bring out more clearly a further simplification.

4.2.1 Single Machine Infinite Bus System.

Consider the Liapunov function given by equation 3.7 in Chapter 3 for a one machine infinite bus system. This can be written, after carrying out the integration and expanding the sine and cosine terms in power series, as

$$V = \frac{1}{2} M\omega^2 - (P_m - E_1^2 G_{11})\delta + E_1 E_2 G_{12}\left\{(\delta - \frac{\delta^3}{3!} + \ldots)\cos\delta_* + (1 - \frac{\delta^2}{2!} + \ldots)\sin\delta_* - \sin\delta_*\right\} + E_1 E_2 B_{12}\left\{\cos\delta_* - (1 - \frac{\delta^2}{2!} + \ldots)\cos\delta_* + (\delta - \frac{\delta^3}{3!} + \ldots)\sin\delta_*\right\}$$

$$= \frac{1}{2} M \omega^2 - E_1 E_2 G_{12} \sin\delta_* \frac{\delta^2}{2!} + E_1 E_2 B_{12} \cos\delta_* \frac{\delta^2}{2!} + \text{terms of higher degree.} \quad 4.1$$

Thus near the origin V is positive definite and the equilibrium is stable, if $- E_1 E_2 G_{12} \sin\delta_* + E_1 E_2 B_{12} \cos\delta_*$ is positive. Therefore, stability exists for $-\frac{\pi}{2} - \alpha < \delta_* < \frac{\pi}{2} - \alpha$

where $\alpha = \tan^{-1}(G_{12}/B_{12})$

Under normal circumstances α is negative and thus the stability limit extends beyond 90°.

4.2.2 Two Machine System

For a two machine system the Liapunov function is given by equation 3.20 of Chapter 3. As before, this can be written in an expanded form as

$$V = \frac{1}{2} M\omega_{12}^2 - A\sin\delta_{12*}\frac{\delta_{12}^2}{2!} + B\cos\delta_{12*}\frac{\delta_{12}^2}{2!} + \text{terms of higher degrees} \quad 4.2$$

where A and B are as given in Chapter 3.

Near the origin V is positive definite and the equilibrium is stable

if $\quad - A\sin\delta_{12*} + B\cos\delta_{12*} > 0$

or $\quad - \frac{M_2 - M_1}{M_1 + M_2} E_1 E_2 G_{12}\sin\delta_{12*} + E_1 E_2 B_{12}\cos\delta_{12*} > 0$

For $M_1 = M_2$ stability exists for $-\frac{\pi}{2} < \delta_{12*} < \frac{\pi}{2}$

For $M_2 >> M_1$ stability exists for $-\frac{\pi}{2} - \alpha < \delta_{12*} < \frac{\pi}{2} - \alpha$ as in the single machine case, where α is given by $\alpha = \tan^{-1}(G_{12}/B_{12})$.

For $M_1 >> M_2$ stability exists for $-\frac{\pi}{2} + \alpha < \delta_{12*} < \frac{\pi}{2} + \alpha$

The above results agree with the well established results[12] for the steady state stability limits of a single machine or two machine system.

4.2.3 Multi-Machine System

For a multi-machine system the Liapunov function given by equation 3.31 can be written in an expanded form as

$$V = \frac{1}{2}\sum_{i=1}^{n} M_i \omega_i^2 + \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_i E_j B_{ij} \cos \delta_{ij*} \, \delta_{ij}^2 + \text{terms of higher degree} \qquad 4.3$$

It may be seen that near the origin V is positive definite and hence the equilibrium is stable if the $E_i E_j B_{ij} \cos \delta_{ij*}$'s, for i = 1,2, ... n-1, j = i+1, i+2, ... n, are positive. These conditions are very easy to apply in practice. Once the steady state operating point is determined, its stability is confirmed if all the angular separations between machine pairs are less than 90°.

The above conditions are however sufficient and they may not all be necessary. The necessary and sufficient condition for the expression given by equation 4.3 to be positive definite in the neighbourhood of the origin, in terms of the independent variables δ_{in}, $i = 1, 2 \ldots n - 1$, thus ensuring stability of the equilibrium state, can be obtained from a a consideration of the positive definiteness of a quadratic form, and is the same as that derived in Chapter 3 from a consideration of the local minimum of the V function, viz. the matrix $\overline{P}$ is positive definite. It may also be seen that the necessary conditions for the matrix $\overline{P}$ and thus the quadratic part in equation 4.3 to be positive definite is given by

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} E_i \, E_j \, B_{ij} \cos \delta_{ij*} > 0, \text{ for } i = 1, 2 \ldots n \qquad 4.4$$

Exactly the same set of stability conditions will be obtained starting with the Liapunov function given by equation 3.32.

For a system with an infinite busbar present, the Liapunov function given by equation 3.34 can be written in an expanded form as

$$V = \frac{1}{2} \sum_{i=1}^{n} M_i \, \omega_i^2 + \frac{1}{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i \, E_j \, B_{ij} \cos \delta_{ij*} \, \delta_{ij}^2 +$$

$$\frac{1}{2} \sum_{i=1}^{n} E_i \, E_\alpha \, B_{i\alpha} \cos \delta_{i*} \, \delta_i^2 + \text{terms of higher degrees} \qquad 4.5$$

Therefore a set of sufficient conditions for stability of the equilibrium state is given by - the angular separations between machine pairs as well as the individual machine angles are less than 90°. The necessary and sufficient condition for the expression given by equation 4.5 to be positive definite in the neighbourhood of the origin can be obtained from a consideration of the positive definiteness of the quadratic portion, and is the same as obtained in section 3.3.3 of Chapter 3. A set of necessary conditions is given by

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} E_i \, E_j \, B_{ij} \cos \delta_{ij*} + E_i \, E_\alpha \, B_{i\alpha} \cos \delta_{i*} > 0, \text{ for } i = 1, 2, \ldots n \qquad 4.6$$

It may be noted that the above stability conditions are independent of the machine moment of inertias.

4.3 A STEADY STATE STABILITY STUDY

The criterion for the steady state stability developed in the preceding section will now be used to determine the steady state stability limit in a power system. In studying transient stability problems using simplified machine representation, the voltage behind transient reactance representation has been justified. In steady state stability studies it is more customary to use synchronous reactances in place of transient ones[12,13]. However, this representation cannot possibly take any account of the effect of automatic excitation control. In fact in the presence of automatic excitation control systems the problem of steady state stability is actually one of dynamic stability and the same machine representation as in transient stability studies should be more logical. However, to allow for the fact that the simple machine representation by constant voltage behind transient reactance may not be valid under all possible operating conditions, some other fictitious reactance in place of the transient reactance may be used. How this reactance may be selected is not clearly known. However, since this particular aspect is outside the scope of this thesis it is assumed here that the transient reactance is adequate for the present purpose of illustration.

In determining the steady state stability limit of a multi-machine system for practical purposes it is convenient to make a few further assumptions. According to these all loads except synchronous motors are represented by constant impedances and all nodes except those where synchronous machines are connected are eliminated. It is also necessary to define some criteria for steady state stability limit. In keeping with the generally accepted criterion of steady state stability limit[12], the criteria used here are summarised thus.

The initial steady state operating condition as found from a load flow solution is first tested for stability. The loading of the most heavily loaded machine is gradually increased and another machine, preferably the one with a large capacity or an infinite busbar, if present, is allowed to act as the balancing station while keeping the power outputs of the remaining machines unchanged. The terminal voltages of the machines are assumed constant during the load redistribution or they may be assumed to vary from one loading condition to another according to a pre-specified manner. At each new loading the steady state stability is tested until an unstable state is indicated.

The change in loading can be effected in two ways. First, the active power output may be increased gradually, and if P_o and P_{lim} are the initial and limiting powers, the steady state stability margin can be taken as $\frac{P_{lim} - P_o}{P_o} \times 100$ per cent. Alternatively, the leading reactive power of the machine can be gradually increased with the active power remaining at the initial value and the stability margin can similarly be taken as $\frac{Q_{lim} - Q_o}{Q_o} \times 100$ per cent, where Q_o and Q_{lim} are the initial and limiting values of the reactive powers respectively. It may be noted that varying the reactive power in a specified manner keeping the active power constant will necessitate a change in the terminal voltage. If desired, the stability margin can be evaluated for all the machines in turn and the lowest of these can be taken as the safe stability margin for the system.

In using either of the above criteria it is necessary to solve for the steady state operating condition from the set of power flow equations for each step change in the loading.

The expression for the active power output of each machine in an n machine system is given by

$$P_{ei} = \sum_{j=1}^{n} V_i V_j \left\{ G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right\} , \quad i = 1,2, \dots n \qquad 4.7$$

where $V_i \angle \theta_i$'s are the terminal voltages and $G_{ii} + j B_{ii}$ and $G_{ij} + j B_{ij}$ are the self and mutual admittances with respect to the machine terminals. Numbering the loaded and the balancing machines by 1 and n respectively and expressing the relative angles θ_{ij} 's as $\theta_{in} - \theta_{jn}$, it may be seen that there are only n - 1 independent variables. In applying the first criterion of stability limit where the active power limit is sought, the first n - 1 equations are solved for the θ_{in} 's, i = 1, 2, ... n - 1. The active power of the balancing machine can then be calculated by substituting the values of θ_{ij} 's in the nth equation. The reactive powers can be calculated from the expression

$$Q_i = \sum_{j=1}^{n} V_i V_j \left\{ G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right\}, \quad i = 1, 2, \ldots n \qquad 4.8$$

When an infinite busbar is present, it will act as the balancing station and measuring the angles with respect to this busbar there will be n independent variables and n equations to solve for those.

In applying the second criterion where the reactive power limit is sought with the active power outputs of the machines constant except for the balancing machine, one extra equation viz. the expression for the reactive power output of machine 1 and a corresponding extra variable, its terminal voltage, are introduced. The equations to be solved for n variables in this case are

$$Q_1 = \sum_{j=1}^{n} V_1 V_j \left\{ G_{1j} \sin \theta_{1j} - B_{1j} \cos \theta_{1j} \right\} \qquad 4.9$$

$$\text{and } P_{ei} = \sum_{j=1}^{n} V_i V_j \left\{ G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right\}, \quad i = 1, 2, \ldots n - 1$$

and the variables are V_1 and θ_{in}, i = 1, 2, ... n - 1

The solution of the equations 4.7 or 4.9 can be obtained by the function minimisation technique discussed in Chapter 3. The expressions for the function to be minimised and its first partial derivatives in the case of equation 4.7 are given by

$$f = \frac{1}{2} \sum_{i=1}^{n-1} \left\{ P_{ei} - \sum_{j=1}^{n} V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \right\}^2 \qquad 4.10$$

$$\frac{\partial f}{\partial \theta_{in}} = \left\{ P_{ei} - \sum_{j=1}^{n} V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \right\} \times$$

$$\left\{ \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \right\}$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \left\{ P_{ej} - \sum_{k=1}^{n} V_j V_k (G_{jk} \cos \theta_{jk} + B_{jk} \sin \theta_{jk}) \right\} \times$$

$$\left\{ V_i V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) \right\} \qquad 4.11$$

$i = 1, 2, \ldots n - 1$

Those for equation 4.9 can similarly be obtained.

4.3.1 Discussion of the Computer Programs

Two computer programs have been developed corresponding to the two criteria of stability limit discussed in the preceding section. The algorithms for both the programs are very similar. The input data starts with a few constants specifying the number of runs, number of step increments in power, size of the system etc. This is followed by the initial load flow results and the network admittance matrices both with respect to the machine terminals and with respect to the machine internal buses. This is followed by a few more constants specifying the number of iterations, accuracy desired etc. in connection with the solution of the power flow equations. The data concludes with a set of power levels at which the stability is to be tested until instability is detected.

The calculation starts with the solution of the power flow equations by function minimisation technique. At the end of the solution the results are printed out. The active power of the balancing machine and the reactive powers of all the machines are then calculated. This is followed by the calculation of the machine internal voltages and angles. To test for the steady state stability first the sufficient conditions as developed in section 4.2 are tried. If these are not satisfied the necessary conditions are tried. If a conclusion about stability can be drawn from these, the matrix $\bar{P}$ is not required to be formed. If only the necessary conditions are satisfied the matrix $\bar{P}$ is formed and its positive definiteness is tested. A procedure to do this by a method described in Chapter 2 is included in the programs.

If the system is proved to be stable the next power level is selected and the whole calculation is repeated. This continues until instability is indicated.

Typical inputs and outputs for the two programs are given in appendix A 4.1.

While evaluating the stability limit by stepping up the active power of the ith machine it may sometimes be necessary to alter the step change to realize uniform change in δ_{in} near the stability limit. This can be avoided by stepping up the angle θ_{in} instead of the active power i.e. by treating the active power of the ith machine as one of the variables instead of θ_{in}. A second version of the program was therefore prepared using a step change in θ_{in}. However, no significant difference was noticed between the workings of the two versions for the example presented here.

4.3.2 Illustrative Example

The same 6 machine South of Scotland Electricity Board test system as used in Chapter 3 in connection with transient stability studies has been used here to study the steady state stability margin employing the method developed in the earlier sections. The network admittance matrices and the initial operating conditions relevant to the present steady state stability study are given in tables 3.2, 3.3 and 3.5 of Chapter 3. As the purpose of this example is to illustrate the validity of the method, the stability margin as obtained by this method is compared with that obtained by other methods. In this example the loading of machine 1 is gradually increased until the stability limit is reached and the stability margin is then calculated. However if detailed analysis is required the margin for each machine in turn may be calculated.

The stability margin was first determined by increasing the active power loading of machine 1. The conditions specified were constant active power outputs of the rest of the machines except the last one which acts as the balancing machine and constant terminal voltages of all the machines. Therefore for the power flow solutions the network admittance matrix with respect to the machine terminals was used whereas to apply the stability criterion the matrix with respect to the machine internal bus was used.

Next the stability margin was determined by increasing the reactive loading. The conditions specified were constant active power outputs of all the machines except the last one and constant terminal voltages of all the machines except the first one. The network admittance matrices for the power flow solutions and stability criterion were the same as before.

The limiting values of the active and reactive powers at machine 1 which will maintain stability according to the stability criterion given in section 4.2 were found to be .66 p.u. and -.225 (leading) p.u. respectively.

4.4 COMPARISON WITH OTHER METHODS

The results obtained in the preceding section will now be compared with those obtained by using Crary's criteria a and c[12] and a state space approach. Before stating the results algorithms to apply criteria a and c efficiently will be presented. Detailed discussion of the state space approach is given in later chapters.

4.4.1 Criterion a

This criterion is derived from a consideration of the equations of motion of the machine rotors for small oscillations. The assumptions necessary are the same as made in developing the criterion in section 4.2 through the use of Liapunov functions except that here the damping torque is also neglected.

The equation of motion of any of the machine rotors is given by

$$M_i \frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{n} E_i E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \qquad 4.12$$

$$i = 1, 2, \ldots n$$

The equation for small oscillations around the equilibrium state is given by

$$M_i \frac{d^2\Delta\delta_i}{dt^2} = \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j (G_{ij} \sin \delta_{ij*} - B_{ij} \cos \delta_{ij*}) \Delta \delta_{ij} \qquad 4.13$$

where $\Delta \delta_{ij} = \Delta \delta_i - \Delta \delta_j$

Equation 4.13 is a set of second order linear differential equations with first order derivative terms absent. The criterion for stability is that the eigenvalues of the matrix A are all real and negative, where A is given by

$A = MP$, where

$M = \mathrm{diag}(1/M_1, 1/M_2, \ldots 1/M_n)$ and

$$P = \begin{bmatrix} \sum_{j=2}^{n} E_1E_j (G_{1j}\sin\delta_{1j}-B_{1j}\cos\delta_{1j}) & E_1E_2(-G_{12}\sin\delta_{12}+B_{12}\cos\delta_{12}) & E_1E_3(-G_{13}\sin\delta_{13}+B_{13}\cos\delta_{13}) \cdots \\ E_1E_2(G_{12}\sin\delta_{12}+B_{12}\cos\delta_{12}) & \sum_{\substack{j=1\\ j\neq 2}}^{n} E_2E_j(G_{2j}\sin\delta_{2j}-B_{2j}\cos\delta_{2j}) & E_2E_3(-G_{23}\sin\delta_{23}+B_{23}\cos\delta_{23}) \cdots \\ \cdots\cdots & \cdots\cdots & \\ E_1E_n(G_{1n}\sin\delta_{1n}+B_{1n}\cos\delta_{1n}) & E_2E_n(G_{2n}\sin\delta_{2n}+B_{2n}\cos\delta_{2n}) \cdots & \sum_{j=1}^{n-1} E_nE_j(G_{nj}\sin\delta_{nj}-B_{nj}\cos\delta_{nj}) \end{bmatrix}$$

If any one of the eigenvalues is real and positive the system is unstable.

4.4.2 Criterion c

In this conservative criterion the synchronising power coefficients $(\frac{d P_e}{d\delta})$ of all the machines in a system are checked to be positive. It is assumed that the increment in the power output of one machine is absorbed by one other machine with all the other machines operating at constant powers. If this is done in turn with all the machines of the system and in all the cases the synchronising power coefficient turns out to be positive the system is stable. Usually the two machines having the largest angular displacement are taken as the loaded and balancing machines since these two are the least stable. All the other machines are treated as constant power machines. Numbering the loaded and balancing machines by 1 and n, $d P_{e1}/d\delta_{1n}$ and $d P_{en}/d\delta_{n1}$ should both be positive for stability. These can be calculated as follows.

The power output of any of the machines in a system, according to the assumptions made in the previous sections, is a function of the relative rotor angles only and is given by

$$P_{ei} = \sum_{j=1}^{n} E_i E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \qquad 4.14$$

$$i = 1, 2, \dots n$$

$$\text{with } d P_2 = d P_3 = \dots = d P_{n-1} = 0$$

$$\frac{d P_1}{d \delta_{1n}} = \frac{\partial P_1}{\partial \delta_{1n}} + \frac{\partial P_1}{\partial \delta_{2n}} \frac{d \delta_{2n}}{d \delta_{1n}} + \frac{\partial P_1}{\partial \delta_{3n}} \frac{d \delta_{3n}}{d \delta_{1n}} \dots + \frac{\partial P_1}{\partial \delta_{n-1,n}} \frac{d \delta_{n-1,n}}{d \delta_{1n}}$$

$$0 = \frac{\partial P_2}{\partial \delta_{1n}} + \frac{\partial P_2}{\partial \delta_{2n}} \frac{d \delta_{2n}}{d \delta_{1n}} + \dots + \frac{\partial P_2}{\partial \delta_{n-1,n}} \frac{d \delta_{n-1,n}}{d \delta_{1n}}$$

$$\cdots\cdots$$

$$\cdots\cdots$$

$$\frac{d P_n}{d \delta_{1n}} = \frac{\partial P_n}{\partial \delta_{1n}} + \frac{\partial P_n}{\partial \delta_{1n}} \frac{d \delta_{2n}}{d \delta_{1n}} + \dots + \frac{\partial P_n}{\partial \delta_{n-1,n}} \frac{d \delta_{n-1,n}}{d \delta_{1n}}$$

Or in matrix form

$$\begin{bmatrix} 1 & -\frac{\partial P_1}{\partial \delta_{2n}} & -\frac{\partial P_1}{\partial \delta_{3n}} & \cdots & -\frac{\partial P_1}{\partial \delta_{n-1,n}} & 0 \\ 0 & -\frac{\partial P_2}{\partial \delta_{2n}} & -\frac{\partial P_2}{\partial \delta_{3n}} & \cdots & -\frac{\partial P_2}{\partial \delta_{n-1,n}} & 0 \\ \cdot & \cdot & \cdot & & \cdot\;\cdot\;\cdot & \\ \cdot & \cdot & \cdot & & \cdot\;\cdot\;\cdot & \\ 0 & -\frac{\partial P_n}{\partial \delta_{2n}} & -\frac{\partial P_n}{\partial \delta_{3n}} & \cdots & -\frac{\partial P_n}{\partial \delta_{n-1,n}} & 1 \end{bmatrix} \begin{bmatrix} \frac{d\,P_1}{d\,\delta_{1n}} \\ \frac{d\,\delta_{2n}}{d\,\delta_{1n}} \\ \vdots \\ \frac{d\,\delta_{n-1,n}}{d\,\delta_{1n}} \\ \frac{d\,P_n}{d\,\delta_{1n}} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_{1n}} \\ \frac{\partial P_2}{\partial \delta_{1n}} \\ \vdots \\ \vdots \\ \frac{\partial P_n}{\partial \delta_{1n}} \end{bmatrix} \qquad 4.15$$

From this $d\,P_1/d\,\delta_{1n}$ and $d\,P_n/d\,\delta_{n1}$ can be calculated

$\partial P_1/\partial\delta_{1n}$, $\partial P_1/\partial\delta_{2n}$ etc. are given by

$$\frac{\partial P_i}{\partial \delta_{in}} = \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i\,E_j\,(-G_{ij}\sin\delta_{ij} + B_{ij}\cos\delta_{ij})$$

$$i = 1, 2, \ldots n-1$$

$$\frac{\partial P_i}{\partial \delta_{jn}} = E_i\,E_j\,(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij})$$

$$i = 1, 2, \ldots n$$

$$j = 1, 2, \ldots n-1 \quad i \neq j$$

4.4.3 Comparison of Results

The steady state stability studies described in section 4.3 were repeated using criterion a, criterion c and a state space approach. For this particular example these three methods produced almost identical values for the stability limits. For comparison the results from all four methods are shown in Table 4.1.

Table 4.1

Comparison of the Steady State Stability Limits Obtained by Different Methods.

Method Used	Limiting value of the active power loading of machine 1 in p.u.	Limiting value of the reactive power loading of machine 1 in p.u.
Liapunov Criterion	.66	-.225

Criterion a	.61	-.22
Criterion c	.61	-.22
State space approach	.61	-.22

It can be seen that the stability limits as found by applying criteria a and c and the state space approach are somewhat lower than those predicted by the criterion deduced from Liapunov functions. The reason can be attributed to the fact that in applying this criterion mutual conductances were neglected.

4.5 SUMMARY

A quick method of testing for the steady state stability in a multi-machine power system using a criterion derived from Liapunov functions has been presented. Most of the assumptions made here have been used in applying the classical criteria of steady state stability in a multi-machine system[12]. Two of these criteria have been reviewed in section 4.4. Efficient algorithms to apply these criteria in multi-machine systems have been developed there. The steady state stability limits of the same six machine system as used in Chapter 3 in connection with transient stability have been studied using the Liapunov criterion. The results have been compared with those obtained by employing the two classical criteria reviewed and a state space approach.

CHAPTER 5

STATE SPACE APPROACH TO POWER SYSTEM DYNAMIC STABILITY STUDY - APPLICATION TO SINGLE-MACHINE SYSTEM

5.1 THE DYNAMIC STABILITY PROBLEM

As has been stated in the general introduction the term dynamic stability is used here in connection with the behaviour of the system machines under small disturbances that are constantly present in any power system in normal operating conditions. With the introduction of high performance excitation and prime mover control systems it has been necessary to investigate their effects on dynamic stability to ensure proper selection of the control parameters. In an inter-connected system due to an improper selection of the control parameters a stable equilibrium state may not exist at all; oscillations once started can build up gradually although they may not be apparent during the first few cycles. On the other hand a well designed control system can extend the dynamic stability margin considerably. Examples of stable and unstable cases are illustrated in Fig. 5.1.

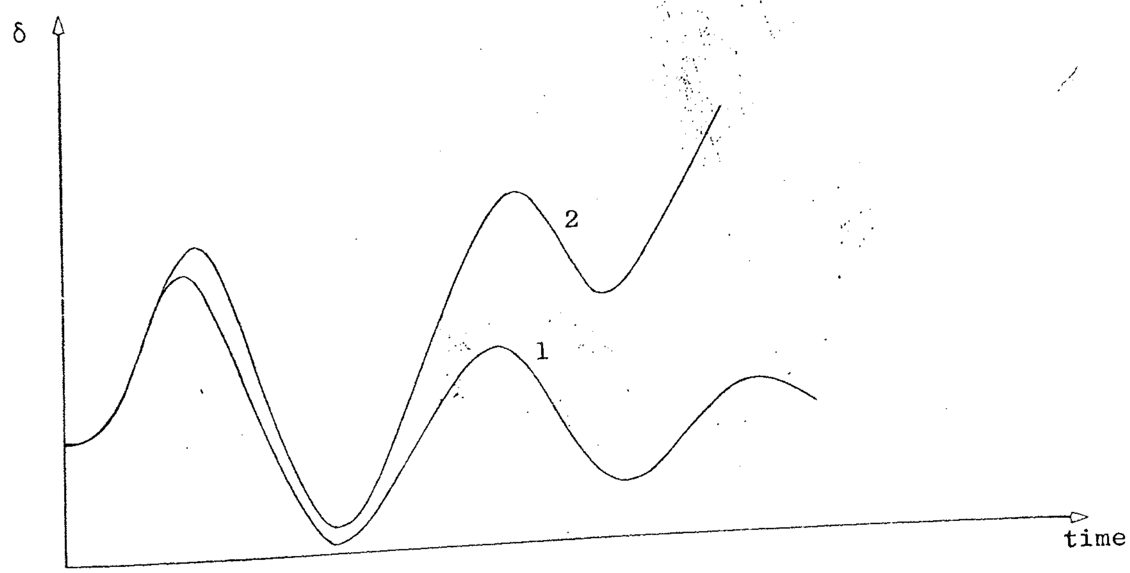


Fig. 5.1 Illustration of stable and unstable oscillations

1 - stable

2 - unstable

An efficient method of determining dynamic stability applicable to both the simple case of a power system consisting of a single synchronous machine connected to an infinite bus through an external impedance or the more general case of a multi-machine system is desirable.

In the past dynamic stability studies have usually been confined to the single machine - infinite busbar system because of the huge amount of algebra involved in applying the classical techniques for studying the stability of the equilibrium state. The equations describing the system were linearised around an operating point and a well known criterion e.g. Routh-Hurwitz or Nyquist criterion was applied[56-64]. The justification of the linearisation technique follows from Liapunov's theorem on stability determined by the linear approximation which has been stated in Chapter 2. In applying either of the above criteria the system response is written in the form

$$\frac{\Delta\delta}{\Delta T_m} = \frac{A\ (p)}{B\ (p)} \quad \text{or} \quad \frac{\Delta\delta}{\Delta v_f} = \frac{C\ (p)}{B\ (p)}$$

by eliminating other variables. The condition for stability is that none of the zeros of B (p) will have a positive real part. Whether any of the zeros has a positive real part is tested by Nyquist or Routh-Hurwitz criterion. This method of testing stability in effect formulates the problem, after linearisation, in terms of a single high-order differential equation. B (p) = 0 corresponds to the characteristic equation. For stability all the roots of this equation must have negative real parts. However, in keeping with modern control theory a state space approach in power system modelling is desirable in studying the dynamic stability since in addition to providing all the information as obtainable from other approaches this method provides certain extra advantages and can handle with sufficient ease far more complicated systems than a simple single machine-infinite busbar one. Although dynamic stability studies on a single machine system serve some useful purposes, in many cases it is necessary to represent the various machines in an interconnected power system in detail to study the effect of interaction of the various dynamic elements of the system.

After linearisation of the equations representing the power system dynamics around an operating point, which will retain the behaviour of the system under small disturbances, they can be reduced to the state space form

$$\dot{x} = A\,x$$

where x is an n-vector of the variables representing the state of the system and A is a matrix of constant coefficients, called the coefficient matrix.

For asymptotic stability the eigenvalues of the matrix A must have negative real parts. If there is an eigenvalue with positive real part the equilibrium is unstable. It may be noted that the eigenvalues of A are identical to the roots of the characteristic equation $B(p) = 0$.

Eigenvalues of A will either be real or complex. If complex they will be comprised of conjugate pairs. By studying these eigenvalues much of the information on the system can be obtained. For a stable system the larger the magnitude of the real part the faster the transient will disappear. Any increase in the magnitude of the real part means a corresponding increase in damping. The magnitude of the imaginary parts indicates the frequency of oscillation of the transients.

While the eigenvalues of the A matrix can be easily obtained by some standard techniques efficient methods must be used to form the A matrix both in the simple cases of single machine and in the more complicated cases of multi-machine systems. Several papers have appeared recently[65-72] on this subject. Although the methods presented work reasonably well using the machine models discussed in the papers they are not generally applicable when one wishes to employ wide varieties of machine and control system models. Also in the methods published so far many unnecessary complications, computations and uses of computer storage space are involved. It is possible to avoid these and form the coefficient matrix in general using the minimum amount of computation and storage space, and this basically forms the subject matter of this and the next chapter. In this chapter formation of the coefficient matrix in the single machine-infinite busbar system using various synchronous machine and control system models will be discussed. An example is selected to illustrate the use of the state space method in determining the dynamic stability limit. As a by product the effects of different synchronous machine models on dynamic stability limits will be evaluated.

5.2 FORMATION OF THE COEFFICIENT MATRIX

In this section the formation of the coefficient matrix in the simple case of a power system consisting of a single synchronous machine connected through an external impedance to an infinite busbar will be demonstrated. All the synchronous machine models discussed in Chapter 1 will be considered. The same numbering system as in Chapter 1 will be maintained. However because of the simplicity of model 5, this will be considered first before going into the more detailed ones.

5.2.1 Coefficient Matrix for Synchronous Machine Model 5

This is the simplest form of synchronous machine representation in power system stability studies. The machine is represented by a constant voltage magnitude behind transient reactance. This model is not a realistic representation in dynamic stability studies since control systems, which have a very great influence in determining dynamic stability properties, are not taken into account in their actual form. In the numerical example provided, this model will not therefore be considered. However the model serves as a good example for illustrating the state space approach in simple terms. The coefficient matrix can be formed in this case simply by inspection.

With this representation the equation of motion for a single machine connected through an external impedance to an infinite busbar is given by

$$M \frac{d^2\delta}{dt^2} = P_m - E_1^{\,2}\, G_{11} - E_1\, E_2\, G_{12} \cos\delta - E_1\, E_2\, B_{12} \sin\delta - \frac{K_d}{\omega_o} \frac{d\delta}{dt} \qquad 5.1$$

where the notations are the same as in Chapter 3.

Linearising equation 5.1 about the equilibrium point

$$M \frac{d^2\Delta\delta}{dt^2} = E_1\, E_2\, G_{12} \sin\delta_* \;\Delta\delta - E_1\, E_2\, B_{12} \cos\delta_* \;\Delta\delta - \frac{K_d}{\omega_o} \frac{d\Delta\delta}{dt} \qquad 5.2$$

Equation 5.2 can be written as two first-order equations.

Calling $\dfrac{d\Delta\delta}{dt} = \Delta\omega$,

$$\frac{d\Delta\omega}{dt} = \frac{E_1\, E_2\, G_{12}}{M} \sin\delta_* \;\Delta\delta - \frac{E_1\, E_2\, B_{12}}{M} \cos\delta_* \;\Delta\delta - \frac{K_d}{\omega_o M}\, \Delta\omega$$

These can be expressed in the matrix form as

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{-E_1\, E_2}{M} \left(B_{12} \cos\delta_* - G_{12} \sin\delta_*\right) & -\dfrac{K_d}{\omega_o M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

$$\text{or} \begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \qquad 5.3$$

where $a = \dfrac{E_1\, E_2}{M} \left(B_{12} \cos\delta_* - G_{12} \sin\delta_*\right)$

and $b = \dfrac{K_d}{\omega_o M}$

This is the state-space representation of the system given by equation 5.2. The A matrix is given by

$$A = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} \qquad 5.4$$

The eigenvalues can be readily seen to be

$$\lambda = \frac{1}{2} \left(-b \pm \sqrt{b^2 - 4a} \right)$$

Thus so long as a is positive the system is stable.

When a synchronous machine is represented more realistically by Park's 2-axis equations and the representations of the various control equipment are also considered the coefficient matrix cannot be formed by simple inspection since in the original equations describing the system there will be many other variables apart from the state variables, which are of course expressible in terms of the state variables. These extra variables have to be eliminated in order to describe the system in a state-space form. The process of elimination becomes more and more complicated as the details of system representation are increased. Using ordinary algebra the task becomes almost prohibitive after a certain stage. However through the efficient use of matrix algebra one can overcome this difficulty. This will be demonstrated in the next few sections. The coefficient matrix will be derived in turn for synchronous machine models 1, 2, 3 and 4.

5.2.2 Coefficient Matrix For Synchronous Machine Model 1

The equations for this model, which is based on Park's generalised two axis flux linkage and voltage equations, are given in section 1.2.1 of Chapter 1. After linearisation about an operating point they can be written as

$$\Delta v_f = T_{do}\, p\, \Delta\Psi_{fd} + x_{ad}\, \Delta i_{fd} \tag{5.5}$$

$$\text{where } T_{do} = \frac{x_{ad}}{x_{ffd}}\, T_{do}'$$

$$\Delta v_d = \frac{1}{\omega_o}\, p\, \Delta\Psi_d - \Delta\Psi_q - \Psi_q \frac{\Delta\omega}{\omega_o} - r_a\, \Delta i_d \tag{5.6}$$

$$0 = \frac{1}{\omega_o}\, p\, \Delta\Psi_{1d} + r_{1d}\, \Delta i_{1d} \tag{5.7}$$

$$\Delta v_q = \frac{1}{\omega_o}\, p\, \Delta\Psi_q + \Delta\Psi_d + \Psi_d \frac{\Delta\omega}{\omega_o} - r_a\, \Delta i_q \tag{5.8}$$

$$0 = \frac{1}{\omega_o}\, p\, \Delta\Psi_{1q} + r_{1q}\, \Delta i_{1q} \tag{5.9}$$

$$\Delta\Psi_{fd} = x_{ffd}\, \Delta i_{fd} - x_{ad}\, \Delta i_d + x_{ad}\, \Delta i_{1d} \tag{5.10}$$

$$\Delta\Psi_d = x_{ad}\ \Delta i_{fd} - x_d\ \Delta i_d + x_{ad}\ \Delta i_{1d} \qquad 5.11$$

$$\Delta\Psi_{1d} = x_{ad}\ \Delta i_{fd} - x_{ad}\ \Delta i_d + x_{11d}\ \Delta i_{1d} \qquad 5.12$$

$$\Delta\Psi_q = -x_q\ \Delta i_q + x_{aq}\ \Delta i_{1q} \qquad 5.13$$

$$\Delta\Psi_{1q} = -x_{aq}\ \Delta i_q + x_{11q}\ \Delta i_{1q} \qquad 5.14$$

$$\Delta T_e = \Psi_d\ \Delta i_q + \Delta\Psi_d\ i_q - \Psi_q\ \Delta i_d - \Delta\Psi_q\ i_d \qquad 5.15$$

$$p\ \Delta\delta = \Delta\omega \qquad 5.16$$

$$M\ p\Delta\omega = \Delta T_m - \Delta T_e - K_d\ \frac{\Delta\omega}{\omega_o} \qquad 5.17$$

$$V_t\ \Delta V_t = v_d\ \Delta v_d + v_q\ \Delta v_q \qquad 5.18$$

Similarly the equations for the transmission line can be linearised as

$$\Delta v_d = V_b \cos\delta\ \Delta\delta + R_t\ \Delta i_d - X_t\ \Delta i_q \qquad 5.19$$

$$\Delta v_q = -V_b \sin\delta\ \Delta\delta + R_t\ \Delta i_q + X_t\ \Delta i_d \qquad 5.20$$

If the same detailed excitation control system as discussed in section 1.4.1 of Chapter 1 is considered, the equations in linearised form are

$$p\,\Delta v_f = \frac{G_x}{K_2\tau_x}\ \Delta v_x - \frac{1}{\tau_x}\ \Delta v_f \qquad 5.21$$

$$p\,\Delta v_{m2} = -\frac{G_{m1}\ G_{vs}\ K_1}{\tau_{m1}}\ \Delta V_t + \frac{G_{m1}}{\tau_{m1}}\,\Delta v_{xs} + \frac{G_{m1}}{\tau_{m1}}\,\Delta v_{ms} - \frac{1}{\tau_{m1}}\ \Delta v_{m2} \qquad 5.22$$

$$p\,\Delta v_x = \frac{G_{m2}}{\tau_{m2}}\ \Delta v_{m2} - \frac{1}{\tau_{m2}}\ \Delta v_x \qquad 5.23$$

$$p\,\Delta v_{xs} = -\frac{G_{xs}\ G_x}{\tau_x}\ \Delta v_x + \frac{G_{xs}\ K_2}{\tau_x}\ \Delta v_f - \frac{1}{\tau_{xs}}\ \Delta v_{xs} \qquad 5.24$$

$$p\,\Delta v_{ms} = -\frac{G_{ms}\ G_{m2}}{\tau_{m2}}\,\Delta v_{m2} + \frac{G_{ms}}{\tau_{m2}}\ \Delta v_x - \frac{1}{\tau_{ms}}\ \Delta v_{ms} \qquad 5.25$$

The governor and turbine equations given in section 1.4.2 of Chapter 1 can be written in linearised form as

$$p\ \Delta\gamma_1 = \Delta\gamma_2 \qquad 5.26$$

$$p\ \Delta\gamma_2 = -\frac{1}{\tau_1\ \tau_2}\ \Delta\gamma_1 - \frac{\tau_1+\tau_2}{\tau_1\ \tau_2}\ \Delta\gamma_2 - \frac{G_1\ G_2}{\tau_1\ \tau_2}\ \Delta\omega \qquad 5.27$$

$$p\,\Delta T_m = \frac{G_3}{T_3}\,\Delta\gamma_1 - \frac{1}{\tau_3}\,\Delta T_m \qquad 5.28$$

These linearised equations can be written in matrix form as in equation 5.29.

This matrix equation can be written in short as

$$\begin{bmatrix} \dot{x}_1 \\ \hdashline 0 \end{bmatrix} = \left[\begin{array}{c:c} M_1 & M_2 \\ \hdashline M_3 & M_4 \end{array}\right] \begin{bmatrix} x_1 \\ \hdashline x_2 \end{bmatrix} \qquad 5.30$$

where x_1 is the vector of the state variables, x_2 is the vector of the variables to be eliminated and M_1, M_2, M_3, M_4 are the submatrices as indicated in equation 5.29.

By elementary matrix manipulation

$$\left[\dot{x}_1\right] = \left[M_1 - M_2\,M_4^{-1}\,M_3\right]\left[x_1\right] \qquad 5.31$$

This is a state space representation of the system and the coefficient matrix A is given by

$$A = M_1 - M_2\,M_4^{-1}\,M_3 \qquad 5.32$$

Thus the method of forming the coefficient matrix is straighforward. The steps to follow are:

a) Linearise the equations about the operating point

b) Arrange them in the form of a vector matrix equation separating the state variables and other variables and

c) form the coefficient matrix by simple matrix reduction.

The control system representations used in this model are fairly detailed. As has been shown in Chapter 1 in connection with transient stability studies, these representations can be simplified without altering the transient response of the main variables of interest. Therefore the effect of this simplification on the dynamic stability was also investigated. The equations are given in sections 1.4.1 and 1.4.2 of Chapter 1. These in linearised form are, for the control system

$$p\Delta v_f = -\frac{1}{\tau_e}\,\Delta v_f - \frac{\mu}{\tau_e}\,\Delta v_s - \frac{\mu}{\tau_e}\,\Delta V_t \qquad 5.33$$

$$
\left[\begin{array}{c}
\Delta\dot{\delta}\\ \Delta\dot{\omega}\\ \Delta\dot{\Psi}_{fd}\\ \Delta\dot{\Psi}_{1d}\\ \Delta\dot{\Psi}_{1q}\\ \Delta\dot{\Psi}_{d}\\ \Delta\dot{\Psi}_{q}\\ \Delta\dot{v}_{f}\\ \Delta\dot{v}_{m2}\\ \Delta\dot{v}_{x}\\ \Delta\dot{v}_{xs}\\ \Delta\dot{v}_{ms}\\ \Delta\dot{T}_{m}\\ \Delta\dot{Y}_{1}\\ \Delta\dot{Y}_{2}\\ \hline \\ \\ \\ \\ \\ \\ \\ \\
\end{array}\right]
=
\begin{array}{c}
\begin{array}{ccccccccccccccccccccccc}
\Delta\delta & \Delta\omega & \Delta\Psi_{fd} & \Delta\Psi_{1d} & \Delta\Psi_{1q} & \Delta\Psi_{d} & \Delta\Psi_{q} & \Delta v_{f} & \Delta v_{m2} & \Delta v_{x} & \Delta v_{xs} & \Delta v_{ms} & \Delta T_{m} & \Delta Y_{1} & \Delta Y_{2} & \Delta i_{fd} & \Delta i_{1d} & \Delta i_{1q} & \Delta i_{d} & \Delta i_{q} & \Delta v_{d} & \Delta v_{q} & \Delta V_{t}
\end{array}\\
\left[\begin{array}{ccccccccccccccc|cccccccc}
 & 1 & & & & & & & & & & & & & & & & & & & & & \\
 & \frac{-K_d}{\omega_o M} & & & & -\frac{i_q}{M} & \frac{i_d}{M} & & & & & & 1/M & & & & & & \frac{\Psi_q}{M} & \frac{-\Psi_d}{M} & & & \\
 & & & & & & & \frac{1}{T_{do}} & & & & & & & & \frac{-x_{ad}}{T_{do}} & & & & & & & \\
 & & & & & & & & & & & & & & & & -\omega_o r_{1d} & & & & & & \\
 & & & & & & & & & & & & & & & & & -\omega_o r_{1q} & & & & & \\
 & \Psi_q & & & & & \omega_o & & & & & & & & & & & & \omega_o r_a & & \omega_o & & \\
 & -\Psi_d & & & & \omega_o & & & & & & & & & & & & & & \omega_o r_a & & \omega_o & \\
 & & & & & & & -\frac{1}{\tau_x} & & \frac{G_x}{K_2\tau_x} & & & & & & & & & & & & & \\
 & & & & & & & & \frac{-1}{\tau_{m1}} & & \frac{G_{m1}}{\tau_{m1}} & \frac{G_{m1}}{\tau_{m1}} & & & & & & & & & & & -\frac{G_{m1}G_{vs}K_1}{\tau_{m1}} \\
 & & & & & & & & \frac{G_{m2}}{\tau_{m2}} & -\frac{1}{\tau_{m2}} & & & & & & & & & & & & & \\
 & & & & & & & \frac{G_{xs}K_2}{\tau_x} & & \frac{-G_{xs}G_x}{\tau_x} & \frac{-1}{\tau_{xs}} & & & & & & & & & & & & \\
 & & & & & & & & -\frac{G_{ms}G_{m2}}{\tau_{m2}} & \frac{G_{ms}}{\tau_{m2}} & & \frac{-1}{\tau_{ms}} & & & & & & & & & & & \\
 & & & & & & & & & & & & \frac{-1}{\tau_3} & \frac{G_3}{\tau_3} & & & & & & & & & \\
 & & & & & & & & & & & & & & 1 & & & & & & & & \\
 & -\frac{G_1G_2}{\tau_1\tau_2} & & & & & & & & & & & & \frac{-1}{\tau_1\tau_2} & -\frac{\tau_1+\tau_2}{\tau_1\tau_2} & & & & & & & & \\
\hline
 & & -1 & & & & & & & & & & & & & x_{ffd} & x_{ad} & & -x_{ad} & & & & \\
 & & & -1 & & & & & & & & & & & & x_{ad} & x_{11d} & & -x_{ad} & & & & \\
 & & & & -1 & & & & & & & & & & & & & x_{11q} & & -x_{aq} & & & \\
 & & & & & -1 & & & & & & & & & & x_{ad} & x_{ad} & & -x_{d} & & & & \\
 & & & & & & -1 & & & & & & & & & & & x_{aq} & & -x_{q} & & & \\
V_b\cos\delta & & & & & & & & & & & & & & & & & & R_t & -X_t & -1 & & \\
-V_b\sin\delta & & & & & & & & & & & & & & & & & & X_t & R_t & & -1 & \\
 & & & & & & & & & & & & & & & & & & & & v_d & v_q & -V_t
\end{array}\right]
\end{array}
\left[\begin{array}{c}
\Delta\delta\\ \Delta\omega\\ \Delta\Psi_{fd}\\ \Delta\Psi_{1d}\\ \Delta\Psi_{1q}\\ \Delta\Psi_{d}\\ \Delta\Psi_{q}\\ \Delta v_{f}\\ \Delta v_{m2}\\ \Delta v_{x}\\ \Delta v_{xs}\\ \Delta v_{ms}\\ \Delta T_{m}\\ \Delta Y_{1}\\ \Delta Y_{2}\\ \hline \Delta i_{fd}\\ \Delta i_{1d}\\ \Delta i_{1q}\\ \Delta i_{d}\\ \Delta i_{q}\\ \Delta v_{d}\\ \Delta v_{q}\\ \Delta V_{t}
\end{array}\right]
\tag{5.29}
$$

$$p\Delta v_s = -\frac{\mu_s}{\tau_e \tau_s}\Delta v_f - \left(\frac{\mu\mu_s}{\tau_e \tau_s} + \frac{1}{\tau_s}\right)\Delta v_s - \frac{\mu\mu_s}{\tau_e \tau_s}\Delta V_t \qquad 5.34$$

and for the prime mover control

$$p\Delta P_m = \Delta y_1 \qquad 5.35$$

$$p\Delta y_1 = -\frac{1}{\tau_g \tau_h}\Delta P_m - \frac{\tau_g + \tau_h}{\tau_g \tau_h}\Delta y_1 + \frac{K_g}{\omega_o}\Delta\omega \qquad 5.36$$

In studying the dynamic stability limit in the example presented later in this chapter it was found that there was very little difference in the results employing the two control system representations. Therefore in discussing the coefficient matrix for other synchronous machine models only the simplified control system representation will be considered as this will save space. However, as will become clear, any type of control system can be considered with any machine model without difficulty.

5.2.3 Coefficient Matrix for Synchronous Machine Model 2

This model is derived from model 1 after neglecting the $p\Psi_d$ and $p\Psi_q$ terms and the equations to be considered here are those given by equations 1.21 to 1.26 in Chapter 1.

After linearisation these can be written as

$$p\Delta e_d'' = \frac{x_q - x_q''}{T_{qo}''}\Delta i_q - \frac{1}{T_{qo}''}\Delta e_d'' \qquad 5.37$$

$$p\Delta e_q' = \frac{\Delta v_f}{T_{do}'} - \frac{x_d - x_d'}{T_{do}'}\Delta i_d - \frac{\Delta e_q'}{T_{do}'} \qquad 5.38$$

$$p\Delta e_q'' = \frac{\Delta e_q'}{T_{do}''} - \frac{x_d' - x_d''}{T_{do}''}\Delta i_d - \frac{\Delta e_q''}{T_{do}''} \qquad 5.39$$

$$\Delta e_d'' = \Delta v_d + r_a \Delta i_d - x_q'' \Delta i_q \qquad 5.40$$

$$\Delta e_q'' = \Delta v_q + r_a \Delta i_q + x_d'' \Delta i_d \qquad 5.41$$

$$\begin{aligned}\Delta T_e &= \Delta e_d'' i_d + e_d'' \Delta i_d + \Delta e_q'' i_q + e_q'' \Delta i_q \\ &\quad - (x_d'' - x_q'') i_d \Delta i_q - (x_d'' - x_q'') i_q \Delta i_d \\ &= i_d \Delta e_d'' + i_q \Delta e_q'' + (v_d + r_a i_d - x_d'' i_q)\Delta i_d + (v_q + r_a i_q + x_q'' i_d)\Delta i_q\end{aligned} \qquad 5.42$$

The equation of motion and the equations for the terminal voltage and transmission line are the same as in model 1.

$$
\left[\begin{array}{c}
\Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{e}_d'' \\ \Delta\dot{e}_q' \\ \Delta\dot{e}_q'' \\ \Delta\dot{v}_f \\ \Delta\dot{v}_s \\ \Delta\dot{P}_m \\ \Delta\dot{y}_1 \\ \hline \\ \\ \\ \\ \\
\end{array}\right]
=
\begin{array}{c}
\begin{array}{cccccccccccccc}
\Delta\delta & \Delta\omega & \Delta e_d'' & \Delta e_q' & \Delta e_q'' & \Delta v_f & \Delta v_s & \Delta P_m & \Delta y_1 & \Delta i_d & \Delta i_q & \Delta v_d & \Delta v_q & \Delta V_t
\end{array} \\
\left[\begin{array}{ccccccccc|ccccc}
 & 1 & & & & & & & & & & & & \\
 & \frac{-K_d}{\omega_o M} & \frac{-i_d}{M} & & \frac{-i_q}{M} & & & \frac{1}{M} & & -\frac{v_d+r_a i_d-x_d'' i_q}{M} & -\frac{v_q+r_a i_q+x_q'' i_d}{M} & & & \\
 & & -\frac{1}{T_{qo}''} & & & & & & & & \frac{x_q-x_q''}{T_{qo}''} & & & \\
 & & & \frac{-1}{T_{d\,o}'} & & \frac{1}{T_{d\,o}'} & & & & -\frac{x_d-x_d'}{T_{do}'} & & & & \\
 & & & \frac{1}{T_{do}''} & \frac{-1}{T_{do}''} & & & & & -\frac{x_d'-x_d''}{T_{do}''} & & & & \\
 & & & & & -\frac{1}{\tau_e} & -\frac{\mu}{\tau_e} & & & & & & & -\frac{\mu}{\tau_e} \\
 & & & & & -\frac{\mu_s}{\tau_e\tau_s} & -\left(\frac{\mu\mu_s}{\tau_e\tau_s}+\frac{1}{\tau_s}\right) & & & & & & & -\frac{\mu\mu_s}{\tau_e\tau_s} \\
 & & & & & & & & 1 & & & & & \\
 & \frac{K_g}{\omega_o} & & & & & & -\frac{1}{\tau_g\tau_h} & -\frac{\tau_g+\tau_h}{\tau_g\tau_h} & & & & & \\
\hline
 & & -1 & & & & & & & r_a & -x_q'' & 1 & & \\
 & & & & -1 & & & & & x_d'' & r_a & & 1 & \\
V_b\cos\delta & & & & & & & & & R_t & -X_t & -1 & & \\
-V_b\sin\delta & & & & & & & & & X_t & R_t & & -1 & \\
 & & & & & & & & & & & v_d & v_q & -V_t
\end{array}\right]
\end{array}
\left[\begin{array}{c}
\Delta\delta \\ \Delta\omega \\ \Delta e_d'' \\ \Delta e_q' \\ \Delta e_q'' \\ \Delta v_f \\ \Delta v_s \\ \Delta P_m \\ \Delta y_1 \\ \hline \Delta i_d \\ \Delta i_q \\ \Delta v_d \\ \Delta v_q \\ \Delta V_t
\end{array}\right]
\qquad 5.43
$$

The linearised equations can be written in a matrix form as in equation 5.43 and the coefficient matrix can be formed as before.

5.2.4 Coefficient Matrix for Synchronous Machine Model 3

This model is a simplification of model 2 by neglecting the effect of the damper winding on the direct axis. The equations are those given by equations 1.27 to 1.31 in Chapter 1. After linearisation these can be written as

$$p\Delta e_d'' = \frac{x_q - x_q''}{T_{qo}''}\Delta i_q - \frac{1}{T_{qo}''}\Delta e_d'' \qquad 5.44$$

$$p\Delta e_q' = \frac{\Delta v_f}{T_{do}'} - \frac{x_d - x_d'}{T_{do}'}\Delta i_d - \frac{\Delta e_q'}{T_{do}'} \qquad 5.45$$

$$\Delta e_d'' = \Delta v_d + r_a \Delta i_d - x_q'' \Delta i_q \qquad 5.46$$

$$\Delta e_q' = \Delta v_q + r_a \Delta i_q + x_d' \Delta i_d \qquad 5.47$$

$$\Delta T_e = \Delta e_d'' i_d + e_d''\Delta i_d + \Delta e_q' i_q + e_q'\Delta i_q - (x_d' - x_q'')i_d\Delta i_q - (x_d' - x_q'')i_q\Delta i_d$$

$$= i_d\Delta e_d'' + i_q\Delta e_q' + (v_d + r_a i_d - x_d' i_q)\Delta i_d + (v_q + r_a i_q + x_q'' i_d)\Delta i_q \qquad 5.48$$

These linearised equations along with the others can be written in a matrix form as in equation 5.49 and the coefficient matrix formed.

5.2.5 Coefficient Matrix for Synchronous Machine Model 4

This is a further simplification neglecting the effects of all damper windings. The equations are those given by equations 1.32 to 1.35 in Chapter 1. After linearisation these can be written as

$$p\Delta e_q' = \frac{\Delta v_f}{T_{do}'} - \frac{x_d - x_d'}{T_{do}'}\Delta i_d - \frac{\Delta e_q'}{T_{do}'} \qquad 5.50$$

$$0 = \Delta v_d + r_a\Delta i_d - x_q\Delta i_q \qquad 5.51$$

$$\Delta e_q' = \Delta v_q + r_a\Delta i_q + x_d'\Delta i_d \qquad 5.52$$

$$\Delta T_e = \Delta e_q' i_q + e_q'\Delta i_q - (x_d' - x_q)i_d\Delta i_q - (x_d' - x_q)i_q\Delta i_d$$

$$= i_q\Delta e_q' + (x_q - x_d')i_q\Delta i_d + (v_q + r_a i_q + x_q i_d)\Delta i_q \qquad 5.53$$

These equations along with the others can be written in a matrix form as in equation 5.54 and the coefficient matrix formed.

$$
= \begin{array}{c}
\begin{array}{ccccccccccccc}
\Delta\delta & \Delta\omega & \Delta e_d'' & \Delta e_q' & \Delta v_f & \Delta v_s & \Delta P_m & \Delta y_1 & \Delta i_d & \Delta i_q & \Delta v_d & \Delta v_q & \Delta V_t
\end{array} \\
\left[\begin{array}{cccccccc|ccccc}
 & 1 & & & & & & & & & & & \\
 & -\frac{K_d}{\omega_o M} & -\frac{i_d}{M} & -\frac{i_q}{M} & & & 1/M & & -\frac{(v_d+r_a i_d-x_d' i_q)}{M} & -\frac{(v_d+r_a i_q+x_q'' i_d)}{M} & & & \\
 & & \frac{-1}{T_{qo}''} & & & & & & & \frac{x_q-x_q''}{T_{qo}''} & & & \\
 & & & \frac{-1}{T_{do}'} & \frac{1}{T_{do}'} & & & & -\frac{x_d-x_d'}{T_{do}'} & & & & \\
 & & & & -\frac{1}{\tau_e} & -\frac{\mu}{\tau_e} & & & & & & & -\frac{\mu}{\tau_e} \\
 & & & & -\frac{\mu_s}{\tau_e\tau_s} & -\left(\frac{\mu\mu_s}{\tau_e\tau_s}+\frac{1}{\tau_s}\right) & & & & & & & -\frac{\mu\mu_s}{\tau_e\tau_s} \\
 & & & & & & & 1 & & & & & \\
 & \frac{K_g}{\omega_o} & & & & & -\frac{1}{\tau_g\tau_h} & -\frac{\tau_g+\tau_h}{\tau_g\tau_h} & & & & & \\
\hline
 & & -1 & & & & & & r_a & -x_q'' & 1 & & \\
 & & & -1 & & & & & x_d' & r_a & & 1 & \\
V_b\cos\delta & & & & & & & & R_t & -X_t & -1 & & \\
-V_b\sin\delta & & & & & & & & X_t & R_t & & -1 & \\
 & & & & & & & & & & v_d & v_q & -V_t
\end{array}\right]
\end{array}
\begin{bmatrix}
\Delta\delta \\ \Delta\omega \\ \Delta e_d'' \\ \Delta e_q' \\ \Delta v_f \\ \Delta v_s \\ \Delta P_m \\ \Delta y_1 \\ \hline \Delta i_d \\ \Delta i_q \\ \Delta v_d \\ \Delta v_q \\ \Delta V_t
\end{bmatrix}
\tag{5.49}
$$

$$
\begin{bmatrix} \dot{\Delta\delta} \\ \dot{\Delta\omega} \\ \dot{\Delta e_q'} \\ \dot{\Delta v_f} \\ \dot{\Delta v_s} \\ \dot{\Delta P_m} \\ \dot{\Delta y_1} \\ \\ \\ \\ \\ \\ \end{bmatrix}
=
\begin{array}{c}
\begin{array}{cccccccccccc}
\Delta\delta & \Delta\omega & \Delta e_q' & \Delta v_f & \Delta v_s & \Delta P_m & \Delta y_1 & \Delta i_d & \Delta i_q & \Delta v_d & \Delta v_q & \Delta V_t
\end{array} \\
\begin{bmatrix}
 & 1 & & & & & & & & & & \\
 & -\frac{K_d}{\omega_o M} & -\frac{i_q}{M} & & & \frac{1}{M} & & -\frac{(x_q-x_d')i_q}{M} & -\frac{(v_q+r_a i_q+x_q i_d)}{M} & & & \\
 & & -\frac{1}{T_{do}'} & \frac{1}{T_{do}'} & & & & -\frac{x_d-x_d'}{T_{do}'} & & & & \\
 & & & -\frac{1}{\tau_e} & -\frac{\mu}{\tau_e} & & & & & & & -\frac{\mu}{\tau_e} \\
 & & & -\frac{\mu_s}{\tau_e\tau_s} & -\left(\frac{\mu\mu_s}{\tau_e\tau_s}+\frac{1}{\tau_s}\right) & & & & & & & -\frac{\mu\mu_s}{\tau_e\tau_s} \\
 & & & & & & 1 & & & & & \\
 & \frac{K_g}{\omega_o} & & & & -\frac{1}{\tau_g\tau_h} & -\frac{\tau_g+\tau_h}{\tau_g\tau_h} & & & & & \\
 & & & & & & & r_a & -x_q & 1 & & \\
 & & -1 & & & & & x_d' & r_a & & 1 & \\
V_b \cos\delta & & & & & & & R_t & -X_t & -1 & & \\
-V_b \sin\delta & & & & & & & X_t & R_t & & -1 & \\
 & & & & & & & & & v_d & v_q & V_t
\end{bmatrix}
\end{array}
\begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta e_q' \\ \Delta v_f \\ \Delta v_s \\ \Delta P_m \\ \Delta y_1 \\ \Delta i_d \\ \Delta i_q \\ \Delta v_d \\ \Delta v_q \\ \Delta V_t \end{bmatrix}
\qquad 5.54
$$

5.3 DYNAMIC STABILITY LIMITS FOR DIFFERENT MACHINE REPRESENTATIONS

In this section the effects of various synchronous machine representations on dynamic stability limits will be investigated. The same single machine infinite bus system as used in Chapter 1 to investigate the effects of various synchronous machine representations in transient stability will be considered here. To determine the dynamic stability limit the limiting values of the reactive powers, both in the lagging and leading region, are found at any particular active power output of the machine, and a curve is drawn with the limiting values of active and reactive powers as co-ordinates. This curve will separate the stable and unstable regions. A comparative study of these stability limit curves obtained from various machine models will show their relative accuracies in dynamic stability studies.

A KDF9 ALGOL computer program was written to form the coefficient matrix for any of the machine models 1 to 4 with either of the two control system discussed or without control. The operating conditions for which the coefficient matrices are desired are specified. The program reads the complete matrix of the matrix equation for small disturbances, e.g. equation 5.29, corresponding to a particular machine model being studied and the coefficient matrix is formed by matrix reduction. For each operating condition necessary changes are made in the original matrix. The output from this program is read as data for the eigenvalue program. This program uses a subroutine written in KDF9 usercode and part of the KDF9 matrix scheme. The method employed in evaluating the eigenvalues is based on the Q-R transformation technique due to Francis, described in detail in reference 73.

As has been stated before, for stability all the eigenvalues must have negative real parts. For a particular value of the active power within the stability region the reactive power in either leading or lagging region is increased in small steps. At a certain value of the reactive ... of the eigenvalues will take positive real part indicating instability. The reactive power immediately below this can be taken approximately as the limiting value. Actually in this study the limiting values were found in a way as described in reference 65. In a single machine study such as the present one, as the operating conditions are made more and more severe it will be found that a particular eigenvalue, if real, or a pair of eigenvalues, if complex, will be most affected from the stability point of view i.e. the real part of this particular eigenvalue or pair of eigenvalues will show prominent change from the negative to the positive side, others being affected much less while a few will show negligible changes. The real part of this particular eigenvalue or pair of eigenvalues, called the critical eigenvalue, can be plotted against the reactive power values as shown in Fig. 5.2 a, for example, and the stability limit may be determined from the intersection of the curve and the reactive power axis.

5.3.1 Computed Results and Discussion.

Computer results were taken to investigate mainly the effects of various synchronous machine representation in determining the dynamic stability limit. A few other topics e.g. the effect of simplifying the control system representation and changing transmission line parameters etc. were also investigated. Detailed studies of the effects of changing control system parameters in the dynamic stability limit will not be considered here since this has already been the subject of a number of papers[56-59,61-63].

The first set of computer results was taken using the same example and set of values of the machine, control and transmission line parameters as used in Chapter 1, section 1.4, in connection with transient stability. The terminal voltage magnitude was kept at unity. Numbering the machine models in the same way as in Chapter 1, the plots of the real parts of the critical eigenvalues against reactive powers for machine models 1A, 2A and 3A are shown in Figs. 5.2 a, b and c respectively. That for

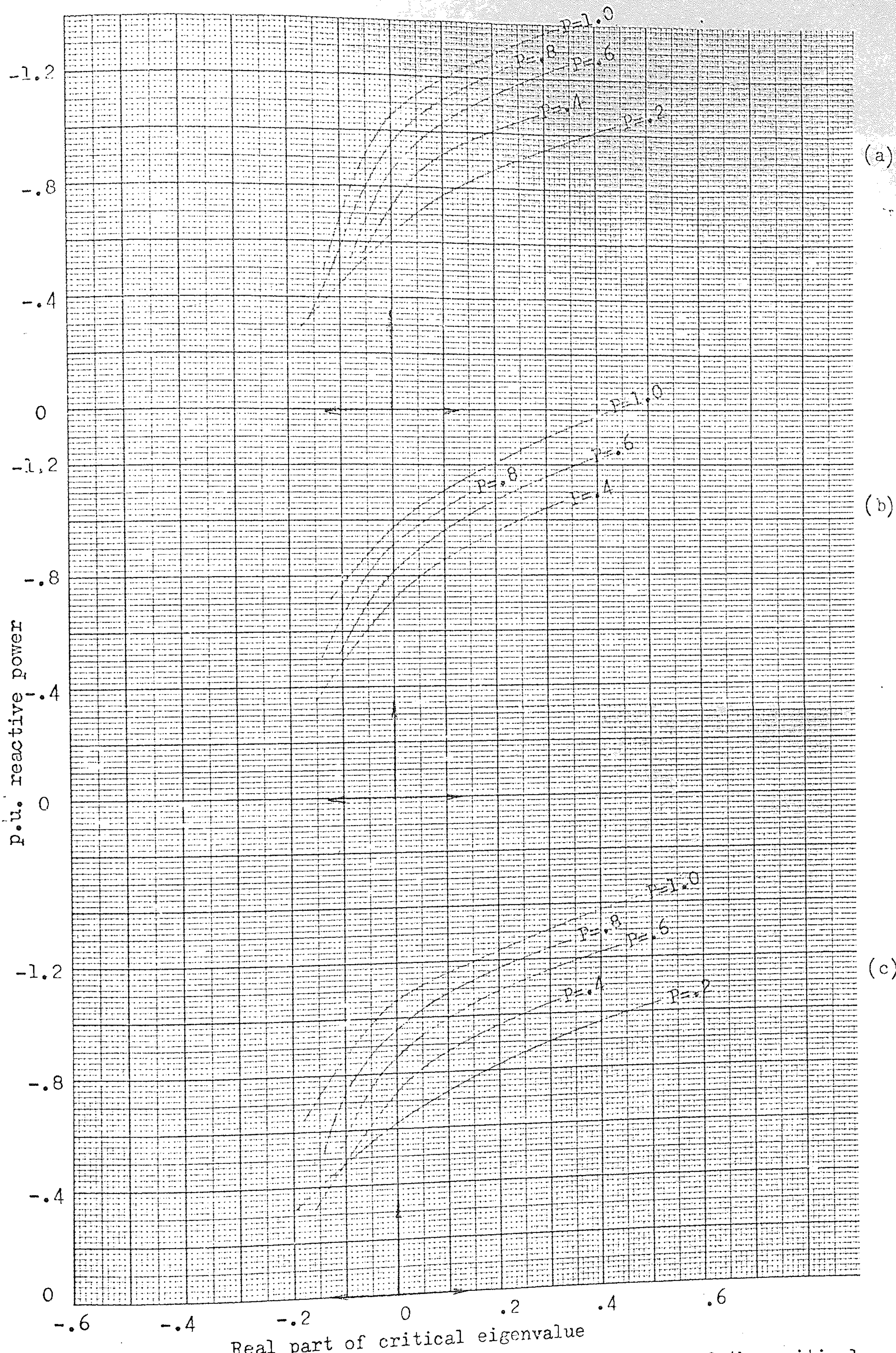
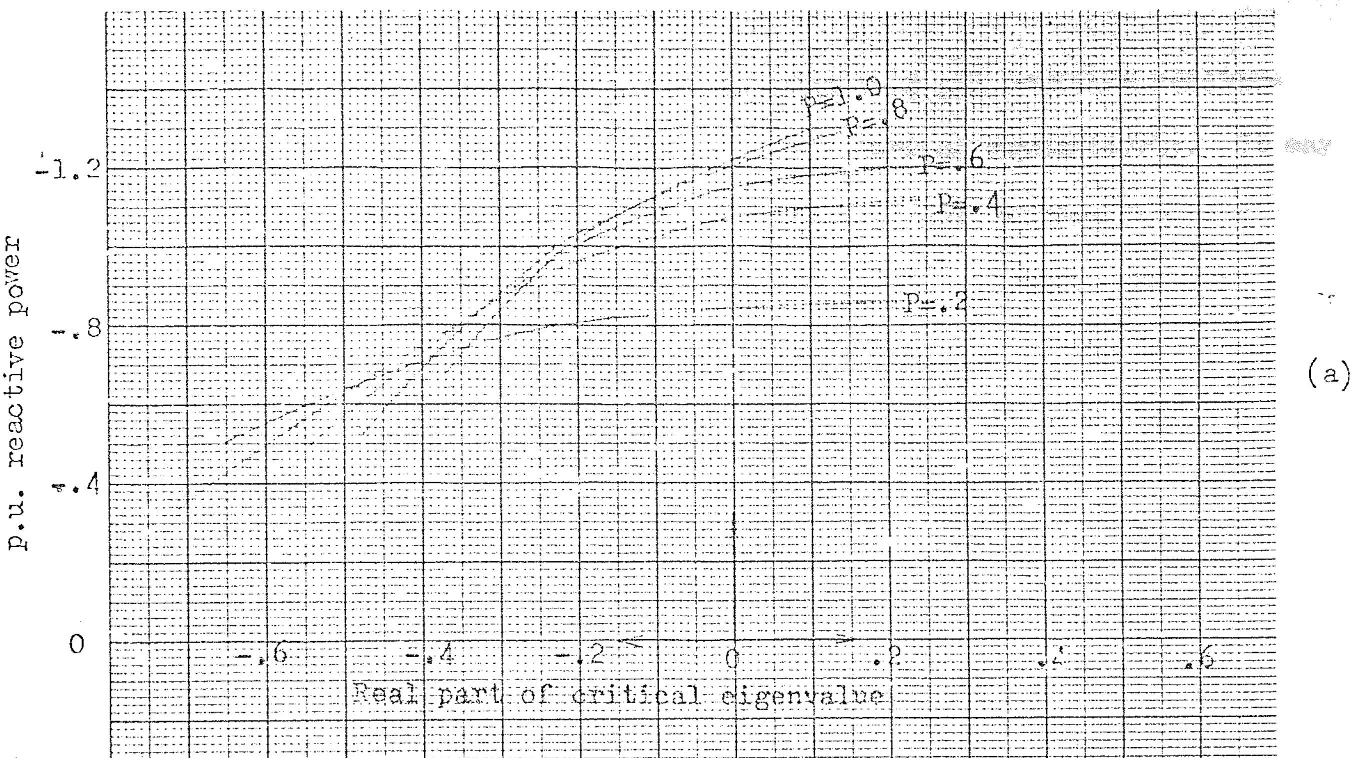


Fig. 5.2 Plots of reactive power versus the real part of the critical eigenvalue - Line impedance = .0223 + j.18
a) for machine model 1A; (b) for machine model 2A;
c) for machine model 3A



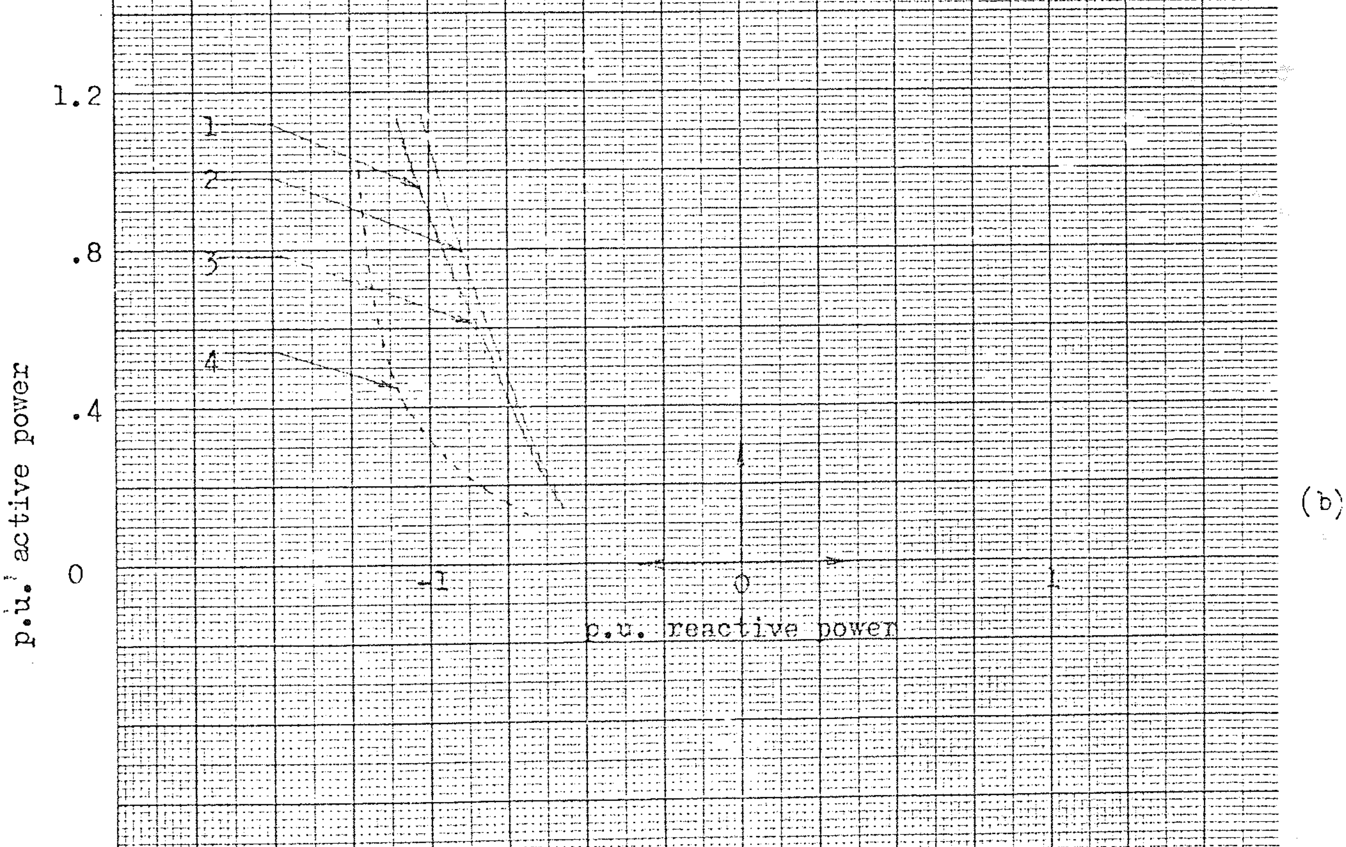


Fig. 5.3 (a) Plots of reactive power versus the real part of the critical eigenvalue - for machine model 4A - Line impedance = .0223 + j.18.

Fig. 5.3 (b) Active versus reactive power plots of dynamic stability limit curves - Line impedance = .0223 + j.18

1) for machine model 1A; (2) for machine model 2A;
3) for machine model 3A; (4) for machine model 4A.

machine model 4A is shown in Fig. 5.3a. Negative and positive reactive powers denote under-excited and over-excited regions respectively. It may be recalled that the letter A after the model number denotes the detailed control systems associated with the machine model. In these and most of the other results presented later the prime mover control has been excluded. The effect of the prime mover control in the form considered here will be shown separately.

From the plots of the critical eigenvalues the limiting values of the reactive powers can be found. With the limiting values of the active and reactive powers thus obtained stability limit curves were drawn and are shown in Fig. 5.3 b. Curves 1, 2, 3 and 4 are for machine models 1, 2, 3 and 4 respectively. Because of the low value of the transmission line impedance and the terminal conditions chosen it was not possible to extend the curves over to the over-excited region. It may be noted that while the results from machine models 1, 2 and 3 are comparable to each other those obtained from machine model 4 are optimistic.

To investigate whether the results are affected by simplifying the excitation control representation the runs were repeated using the control parameters for the simplified system which produced a transient response similar to that produced by the detailed system as discussed in Chapter 1. The limiting values of the reactive powers obtained from the plots of the critical eigenvalues were plotted against the active power values to obtain the stability limit curves. These are shown in Fig. 5.4. Curves 1, 2, 3 and 4 are for machine models 1B, 2B, 3B and 4B respectively. It may be seen that there is hardly any difference between this set of curves and the one obtained from detailed excitation system representation. Therefore in all the further studies the simplified excitation control was used.

In the next set of runs the transmission line impedance was doubled. The terminal voltage magnitude was still set at unity. This permitted the mapping of the stability limit to be extended over to the over-excited region. The stability limit curves for machine models 1, 2, 3 and 4 are

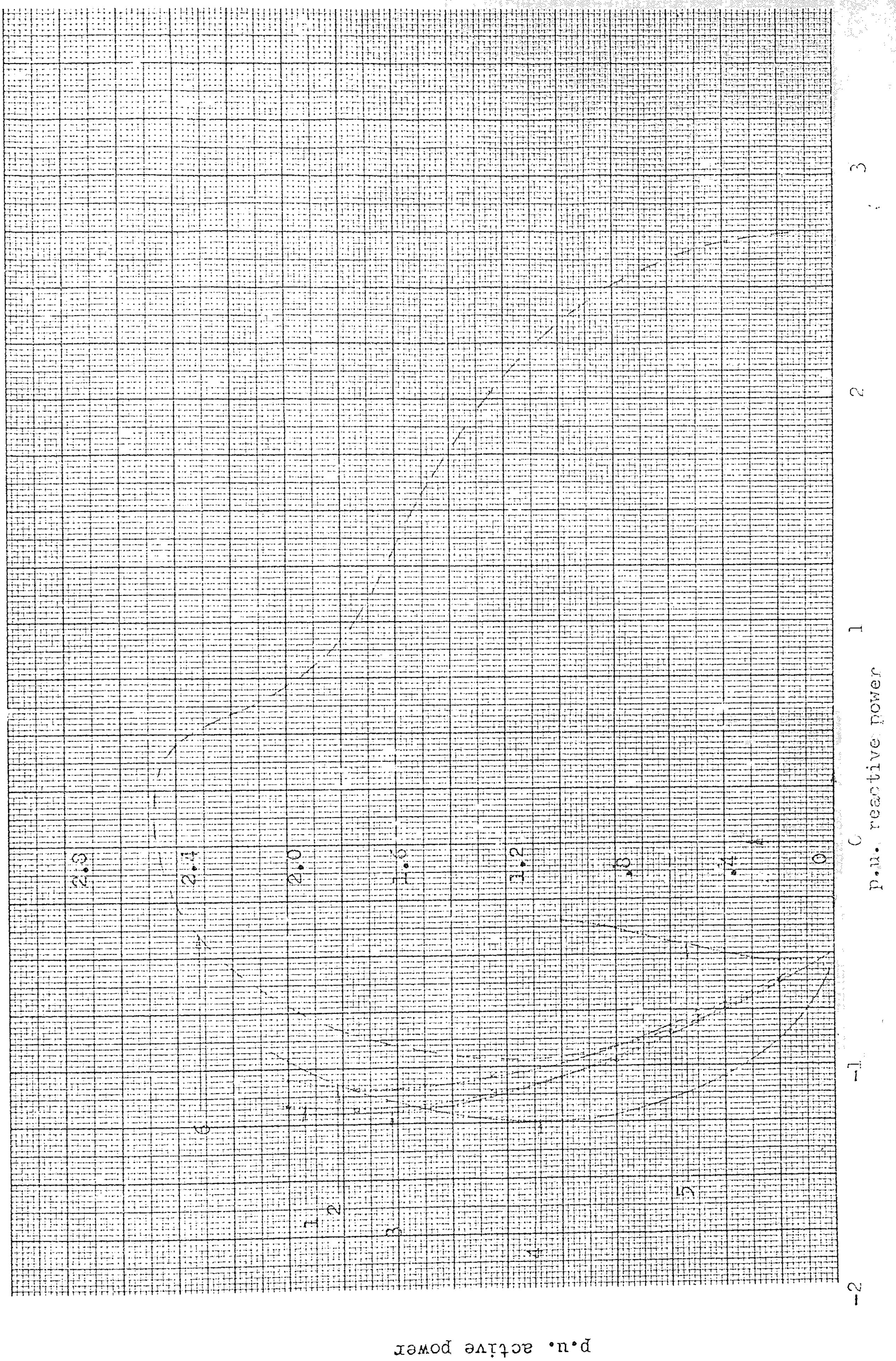


Fig. 5.4 Active versus reactive power plots of dynamic stability limit curves - Line impedance = .0223 + j.18.

1) for machine model 1B; (2) for machine model 2B;
3) for machine model 3B; (4) for machine model 4B;
5) for machine model 4 without control; (6) for machine

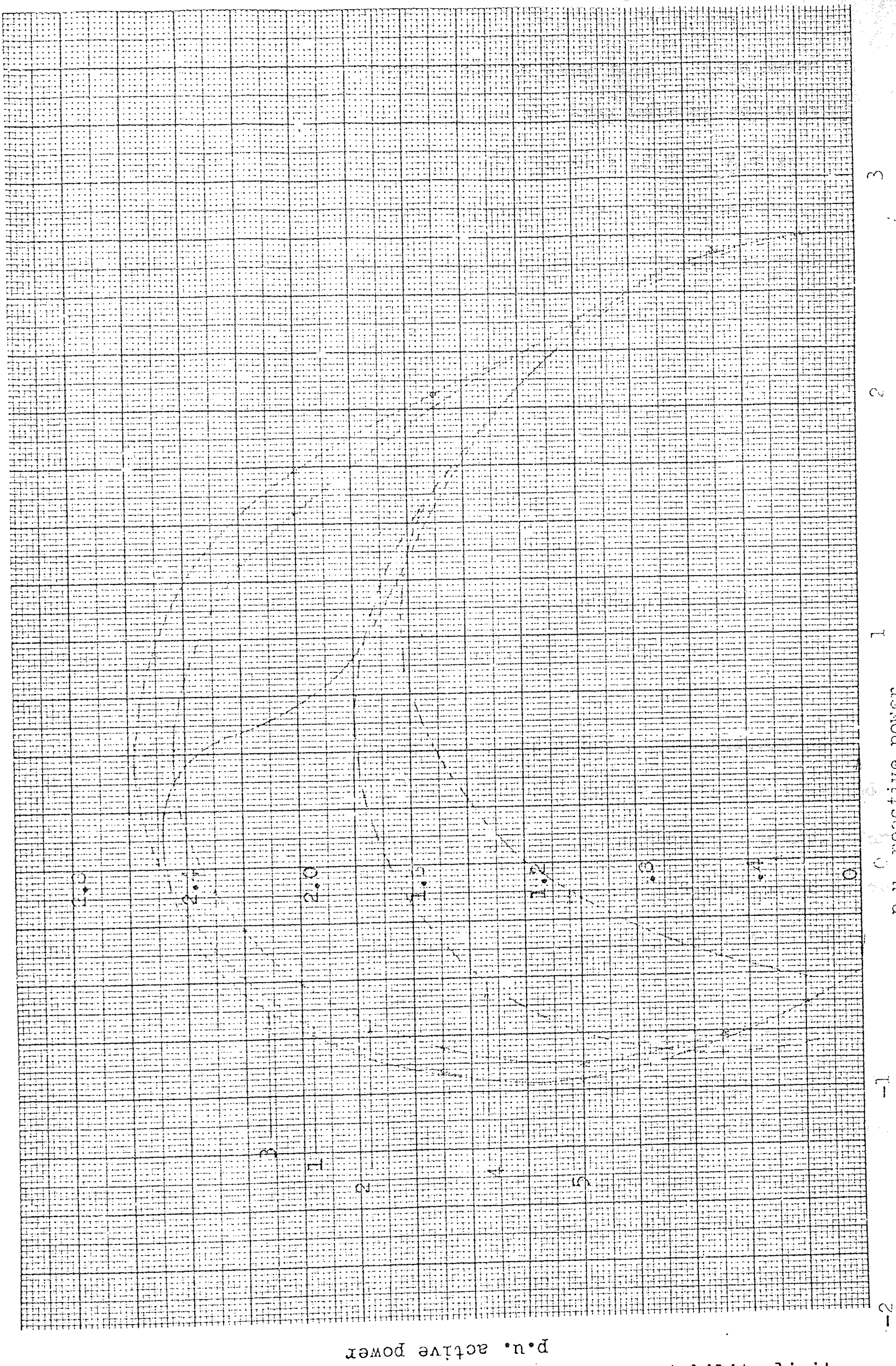


Fig. 5.5 Active versus reactive power plots of dynamic stability limit curves - Line impedance = .0446 + j.36
1) for machine model 1; (2) for machine model 2;
3) for machine model 3; (4) for machine model 4;
5) for machine model 4 without excitation control

shown as curves 1, 2, 3 and 4 respectively in Fig. 5.5. Also shown in Fig. 5.5 is the stability limit curve for the case when no excitation control was present, using machine model 4. Curve 3 of Fig. 5.5 is repeated in Fig. 5.4 for comparison purpose. From a study of Fig. 5.5 where the stability limit curves for various synchronous machine models are shown covering both the under-excited and over-excited regions, it may be seen that although there is a great difference among the stability limit curves considering the entire region, in the region of practical interest viz. the machine absorbing reactive powers at light loads, the results using machine models 1, 2 and 3 are very much the same, while the results using machine model 4 are optimistic.

The effect of the prime mover control, in the form considered here, on dynamic stability limit was then investigated. Here machine model 4 was used. Two sets of runs were taken with governor net gains of 32 and 64. The stability limit curves are shown as curve 1 and 2 respectively in Fig. 5.6. The terminal voltage magnitude and the transmission line and excitation control parameters were the same as in the previous study. For comparison curve 4 from Fig. 5.5 corresponding to the case with no prime mover control is repeated in Fig. 5.6. The negligible effect of this form of prime mover control is apparent.

The effects of reducing the terminal voltage magnitude and changing the machine inertia were also investigated. The stability limit curves are shown in Fig. 5.7. The synchronous machine model 3 was used. The transmission and control parameters were the same as in the previous study. Curve 1 is a repetition of curve 3 of Fig. 5.5. Curve 2 represents the case when the machine inertia was doubled and curve 3 represents the case when the terminal voltage magnitude was reduced by 20%.

5.4 SUMMARY

In this chapter studies of the dynamic stability of a single machine system through a state space approach have been presented. A successful use of this method requires, as a first step, the dynamic equations of the system after proper linearisation about an operating point, to be arranged in a vector matrix form in an efficient manner. Conclusions about stability can then be drawn from a study of the eigenvalues of the coefficient matrix. It has been shown how the coefficient matrix for the case of a single synchronous machine connected through an external impedance to an infinite bus and incorporating various types of control system can be formed efficiently by a matrix reduction technique. Equations have been developed employing all the synchronous machine models considered in Chapter 1.

An example has been provided in which the dynamic stability limit curves are plotted for various synchronous machine representations and a comparison has been made. Effects of certain other factors on the dynamic stability have also been discussed.

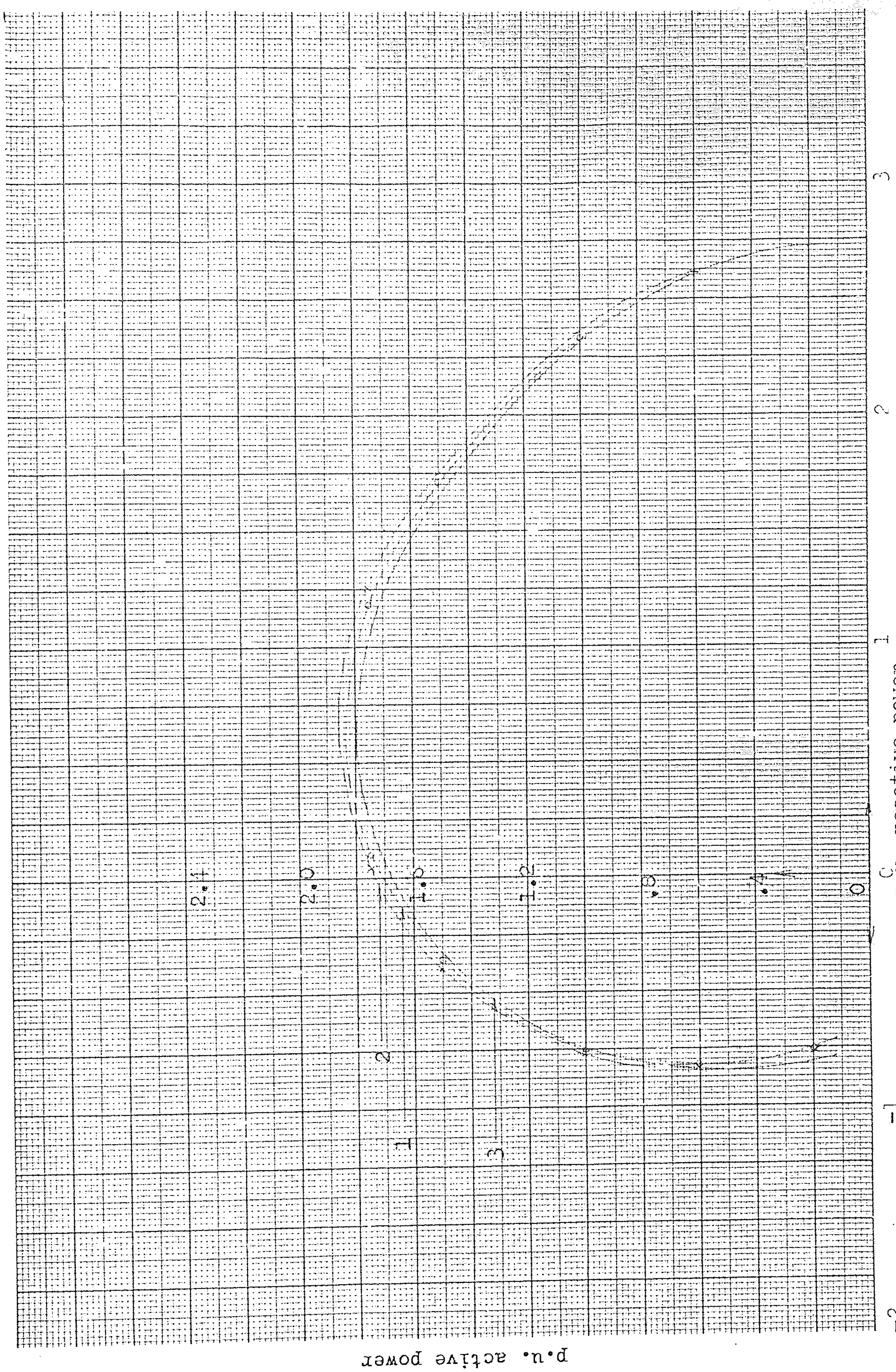


Fig. 5.6 Active versus reactive power output plots of dynamic stability limit curves with governor action present - Line impedance = .0445 +j.36
1) governor gain = 32; (2) governor gain = 64;
3) governor gain = 0.

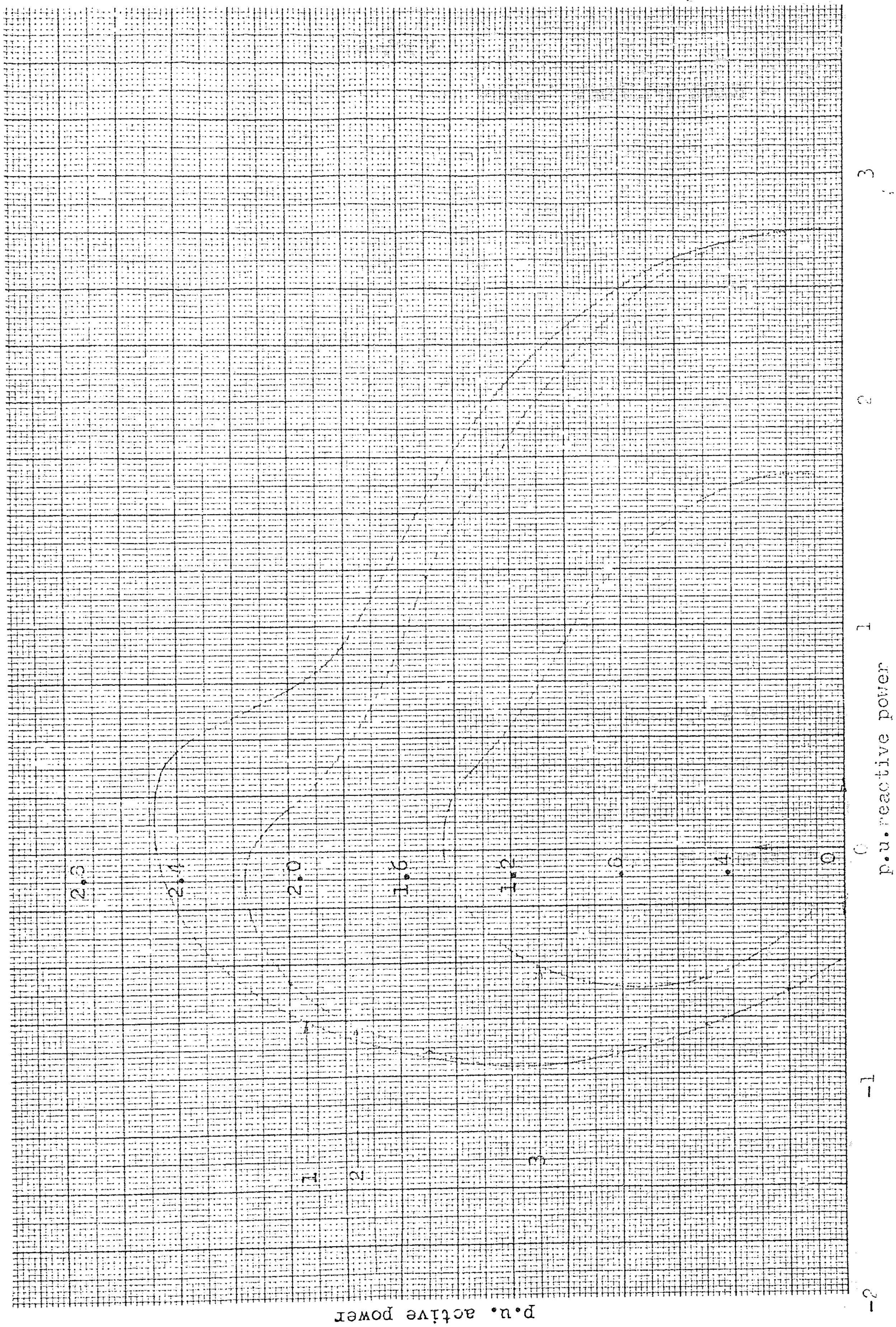


Fig. 5.7 Effects of the change of terminal voltage and machine inertia on dynamic stability limits

1) base case - line impedance = .0445 + j.36
2) machine inertia doubled
3) terminal voltage reduced by 20%

CHAPTER 6

STATE SPACE APPROACH TO POWER SYSTEM DYNAMIC STABILITY STUDY - APPLICATION TO MULTI-MACHINE SYSTEM

6.1 THE MULTI-MACHINE PROBLEM

In Chapter 5 a method has been presented to form the coefficient matrix of a single machine system, in which the coefficient matrix is obtained by simple matrix reduction. While the method is simple and straightforward and can be extended to multi-machine systems, it can be cumbersome because of the large number of equations involved to describe such systems. In this chapter a method which is not so straightforward as the above but which can handle large multi-machine systems, employing varieties of machine and control system models, much more efficiently will be described. It may be seen that in the matrix reduction method in which the non-state variables are eliminated all in one operation, in addition to the need for handling a large number of equations which obviously requires a lot of computer storage space, a large number of arithmetic operations is involved. This becomes clear when it is observed that the original matrix, e.g. the matrix in equation 5.29 of Chapter 5, from which the coefficient matrix is obtained by matrix reduction, is a sparse one. In the method to be described the system equations are divided into several groups and a number of smaller matrix equations are formed from them by eliminating the non-state variables. The final coefficient matrix is then formed by placing the elements of these smaller matrices in the appropriate positions. In this way the coefficient matrix can be formed neatly, using only the minimum amount of computation and computer storage space. This method of building up the coefficient matrix starting with smaller matrix equations is related to that described in reference 67. However it will be observed that the method described in reference 67 hardly offers any saving in the computer time and storage and any such saving is offset by the extra complications involved there.

A further advantage of following the method developed in this chapter is that the partial derivative of the coefficient matrix with respect to any particular machine or control parameter can be formed without undue difficulty. This is important, since the partial derivative is required if one wishes to study the sensitivities of the eigenvalues with respect to a system parameter.

6.2 FORMATION OF THE COEFFICIENT MATRIX

Before going into the details of the method which is really useful in forming the coefficient matrix of a multimachine system where the machines are represented by any of the models based on Park's 2-axis representation, the coefficient matrix for the simple machine representation of constant voltage behind transient reactance will be considered. As in the single machine case the coefficient matrix in this case also can be found by simple inspection. This is done, as the coefficient matrix using the simplified model will bring out more clearly a certain characteristic which applies to all coefficient matrices of a multi-machine system employing other machine models.

6.2.1 Coefficient Matrix for Simplified Machine Representation

With simplified machine representation the equation of motion for a multi-machine system is

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{n} E_i E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) - K_{di} \frac{d \delta_i}{dt} \qquad 6.1$$

$$\text{for } i = 1, 2, \ldots n$$

where the notations are the same as in Chapter 3.

After linearisation equation 6.1 can be written as

$$\frac{d^2\,\Delta\delta_i}{dt^2} = \frac{1}{M_i} \sum_{\substack{j=1\\ i\neq j}}^{n} E_i\,E_j\,(G_{ij}\sin\delta_{ij*} - B_{ij}\cos\delta_{ij*})\Delta\,\delta_{ij} - \frac{K_{di}}{M_i}\,\frac{d\Delta\,\delta_i}{dt} \qquad 6.2$$

for $i = 1, 2, \ldots n$

Calling $\frac{d\Delta\delta_1}{dt} = \Delta\omega_1$ etc. equation 6.2 can be written, for a three machine system for example, as

$$\frac{d\Delta\delta_1}{dt} = \Delta\omega_1 \qquad 6.3$$

$$\frac{d\Delta\omega_1}{dt} = -\,a\,\Delta\delta_{12} - b\,\Delta\delta_{13} - D_1\,\Delta\omega_1 \qquad 6.4$$

$$\frac{d\Delta\delta_2}{dt} = \Delta\omega_2 \qquad 6.5$$

$$\frac{d\Delta\omega_2}{dt} = a'\,\Delta\delta_{12} - c\Delta\,\delta_{23} - D_2\,\Delta\omega_2 \qquad 6.6$$

$$\frac{d\Delta\delta_3}{dt} = \Delta\omega_3 \qquad 6.7$$

$$\frac{d\,\Delta\omega_3}{dt} = b'\,\Delta\delta_{13} + c'\Delta\,\delta_{23} - D_3\,\Delta\omega_3 \qquad 6.8$$

where $a = -\frac{1}{M_1}\,(E_1\,E_2\,G_{12}\sin\delta_{12*} - E_1\,E_2\,B_{12}\cos\delta_{12*})$,

$a' = \frac{1}{M_2}\,(E_1\,E_2\,G_{12}\sin\delta_{12*} + E_1\,E_2\,B_{12}\cos\delta_{12*})$ etc.,

$\Delta\,\delta_{12} = \Delta\delta_1 - \Delta\,\delta_2$ etc. and $D_1 = \frac{K_{d1}}{M_1}$ etc.

Equations 6.3 to 6.8 can be expressed in a state space form as

$$\begin{bmatrix}\dot{\Delta\delta}_1\\ \dot{\Delta\omega}_1\\ \dot{\Delta\delta}_2\\ \dot{\Delta\omega}_2\\ \dot{\Delta\delta}_3\\ \dot{\Delta\omega}_3\end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} \Delta\delta_1 & \Delta\omega_1 & \Delta\delta_2 & \Delta\omega_2 & \Delta\delta_3 & \Delta\omega_3 \end{array} \\ \begin{bmatrix} & 1 & & & & \\ -(a+b) & -D_1 & a & & b & \\ & & & 1 & & \\ a' & & -(a'+c) & -D_2 & c & \\ & & & & & 1 \\ b' & & c' & & -(b'+c') & -D_3 \end{bmatrix} \end{array} \begin{bmatrix}\Delta\delta_1\\ \Delta\omega_1\\ \Delta\delta_2\\ \Delta\omega_2\\ \Delta\delta_3\\ \Delta\omega_3\end{bmatrix} \qquad 6.9$$

It may be seen that the columns corresponding to the $\Delta\delta$'s add up to zero. This is because the system described by equation 6.1 is actually (2n − 1) th order system instead of 2n. As a result the coefficient matrix in equation 6.9 will have one zero eigenvalue. This can be avoided, if desired, by expressing the angles of the machines with reference to a particular machine taken as a reference thus reducing the number of variables by one. For example, taking the 3rd machine as reference the variables now become $\Delta\delta_{13}$, $\Delta\delta_{23}$, $\Delta\omega_1$, $\Delta\omega_2$ and $\Delta\omega_3$, and $\Delta\dot{\delta}_{13} = \Delta\dot{\delta}_1 - \Delta\dot{\delta}_3 = \Delta\omega_1 - \Delta\omega_3$ etc.

$$\Delta\dot{\omega}_1 = -(a+b)\ \Delta\delta_{13} + a\ \Delta\delta_{23} - D_1\ \Delta\omega_1 \text{ etc.}$$

Therefore, the equation in a state space form, becomes

$$\begin{bmatrix} \Delta\dot{\delta}_{13} \\ \Delta\dot{\omega}_1 \\ \Delta\dot{\delta}_{23} \\ \Delta\dot{\omega}_2 \\ \Delta\dot{\omega}_3 \end{bmatrix} = \begin{array}{c} \begin{array}{ccccc} \Delta\delta_{13} & \Delta\omega_1 & \Delta\delta_{23} & \Delta\omega_2 & \Delta\omega_3 \end{array} \\ \begin{bmatrix} & 1 & & & -1 \\ -(a+b) & -D_1 & a & & \\ & & & 1 & -1 \\ a' & & -(a'+c) & -D_2 & \\ b' & & c' & & -D_3 \end{bmatrix} \end{array} \begin{bmatrix} \Delta\delta_{13} \\ \Delta\omega_1 \\ \Delta\delta_{23} \\ \Delta\omega_2 \\ \Delta\omega_3 \end{bmatrix} \qquad 6.10$$

The changes to be made in the coefficient matrix in equation 6.9 to arrive at the coefficient matrix in equation 6.10 are thus simple.

Apart from the zero eigenvalue the 2n − 1 eigenvalues of the two coefficient matrices will be the same. This characteristic of the coefficient matrix of a multi-machine system having a zero eigenvalue when absolute values of the machine angles are considered is also present when any other machine representation is employed and this can be avoided by reducing the order of the system by one in a similar way.

When an infinite busbar is present and the machine angles are measured with reference to that busbar the system will be of 2n th order.

and the coefficient matrix of equation 6.9 will be non singular.

6.2.2 Coefficient Matrix for Machine Model 1

The formation of the coefficient matrix for a multi-machine system using 2 axis machine representation will first be shown for machine model 1 where the various flux linkages are among the state variables. The manipulation of the machine equations in the intermediate stages before the final coefficient matrix is formed is simpler for this representation than for those in models 2, 3 or 4. The procedure for these models is shown in section 6.2.3; however since the procedure is very much the same for models 2, 3 and 4, the coefficient matrix will be formed for machine model 3 only, since this model was found most suitable for dynamic stability studies (vide Chapter 5) and also this was the model used in the example presented later in this chapter.

In a multi-machine system using 2-axis representation the relationship between the individual machine terminal voltages or currents in terms of the machine reference frame and network reference frame is given by equations of the form of equation 1.72 of Chapter 1.

For a system containing n synchronous machines the voltage relationships are

$$\begin{bmatrix} v_{D1} \\ v_{Q1} \\ v_{D2} \\ v_{Q2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \sin\delta_1 & \cos\delta_1 & & & \\ -\cos\delta_1 & \sin\delta_1 & & & \\ & & \sin\delta_2 & \cos\delta_2 & \\ & & -\cos\delta_2 & \sin\delta_2 & \\ & & & & \ddots \end{bmatrix} \begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \\ \vdots \end{bmatrix}$$

$$\text{or} \quad V = Tv \qquad 6.11$$

$$\text{and} \quad v = T^{-1} V \qquad 6.12$$

Equation 6.11 when linearised gives

$$\Delta V = T\,\Delta v + M\,\Delta\delta \qquad 6.13$$

$$\text{where } M = \begin{bmatrix} v_{d1}\cos\delta_1 - v_{q1}\sin\delta_1 & & \\ v_{d1}\sin\delta_1 + v_{q1}\cos\delta_1 & & \\ & v_{d2}\cos\delta_2 - v_{q2}\sin\delta_2 & \\ & v_{d2}\sin\delta_2 + v_{q2}\cos\delta_2 & \\ & & \ddots \end{bmatrix}$$

$$= \begin{bmatrix} -v_{Q1} & & \\ v_{D1} & & \\ & -v_{Q2} & \\ & v_{D2} & \\ & & \ddots \end{bmatrix}$$

$$\text{and } \Delta\delta = \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \vdots \end{bmatrix}$$

Similarly equation 6.12 gives

$$\Delta v = T^{-1}\ \Delta V + N\ \Delta\delta \qquad 6.14$$

$$\text{where } N = \begin{bmatrix} v_{q1} & & \\ -v_{d1} & & \\ & v_{q2} & \\ & -v_{d2} & \\ & & \ddots \end{bmatrix}$$

Similar relationships for the currents are

$$\Delta I = T\ \Delta i_s + \overline{M}\ \Delta\delta \qquad 6.15$$

$$\text{and } \Delta i_s = T^{-1}\ \Delta I + \overline{N}\ \Delta\delta \qquad 6.16$$

where

$$\Delta i_s = \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \\ \vdots \end{bmatrix}, \quad \overline{M} = \begin{bmatrix} -i_{Q1} & & & \\ i_{D1} & & & \\ & -i_{Q2} & & \\ & i_{D2} & & \\ & & & \ddots \end{bmatrix}, \text{ and } \overline{N} = \begin{bmatrix} i_{q1} & & & \\ -i_{d1} & & & \\ & i_{q2} & & \\ & -i_{d2} & & \\ & & & \ddots \end{bmatrix}.$$

The flux linkage equations in linearised form, viz. equations 5.10 to 5.14 of Chapter 5 can be written in matrix form as

$$\left[\begin{array}{c} \Delta\Psi_{fd} \\ \Delta\Psi_{1d} \\ \Delta\Psi_{1q} \\ \hline \Delta\Psi_{d} \\ \Delta\Psi_{q} \end{array}\right] = \left[\begin{array}{ccc|cc} x_{ffd} & x_{ad} & & -x_{ad} & \\ x_{ad} & x_{11d} & & -x_{ad} & \\ & & x_{11q} & & -x_{aq} \\ \hline x_{ad} & x_{ad} & & -x_{d} & \\ & & x_{aq} & & -x_{q} \end{array}\right] \left[\begin{array}{c} \Delta i_{fd} \\ \Delta i_{1d} \\ \Delta i_{1q} \\ \hline \Delta i_{d} \\ \Delta i_{q} \end{array}\right]$$

$$\text{or} \quad \left[\begin{array}{c} \Delta\Psi_r \\ \hline \Delta\Psi_s \end{array}\right] = \left[x \right] \left[\begin{array}{c} \Delta i_r \\ \hline \Delta i_s \end{array}\right] \qquad 6.17$$

Equation 6.17 can be written as

$$\left[\begin{array}{c} \Delta i_r \\ \hline \Delta i_s \end{array}\right] = \left[y \right] \left[\begin{array}{c} \Delta\Psi_r \\ \hline \Delta\Psi_s \end{array}\right] \qquad 6.18$$

where $[y]$ is the inverse of $[x]$

Equation 6.18 can be written, by appropriately partitioning the y matrix, as

$$\left[\begin{array}{c} \Delta i_r \\ \hline \Delta i_s \end{array}\right] = \left[\begin{array}{c|c} y_{11} & y_{12} \\ \hline y_{21} & y_{22} \end{array}\right] \left[\begin{array}{c} \Delta\Psi_r \\ \hline \Delta\Psi_s \end{array}\right] \qquad 6.19$$

For n machines equation 6.19 can be split up as

$$\begin{bmatrix} \Delta i_{r1} \\ \Delta i_{r2} \\ \vdots \\ \cdot \\ \\ \end{bmatrix} = \begin{bmatrix} y_{11}^{1} & y_{12}^{1} & & & \\ & & y_{11}^{2} & y_{12}^{2} & \\ & & & & \ddots \\ & & & & \\ \end{bmatrix} \begin{bmatrix} \Delta\Psi_{r1} \\ \Delta\Psi_{s1} \\ \Delta\Psi_{r2} \\ \Delta\Psi_{s2} \\ \vdots \end{bmatrix}$$

or $\Delta i_r = P \; \Delta\Psi$ 6.20

Similarly $\Delta i_s = Q \; \Delta\Psi$ 6.21

where $Q = \begin{bmatrix} y_{21}^{1} & y_{22}^{1} & & & \\ & & y_{21}^{2} & y_{22}^{2} & \\ & & & & \ddots \end{bmatrix}$

The relationship between the machine terminal currents and voltages in network reference frame is given by

$$I = YV , \qquad 6.22$$

where Y is the 2n x 2n network admittance matrix written in real form after separating the real and imaginary parts (vide Chapter 1).

Equation 6.22 can be written as

$V = ZI$, where Z is the inverse of Y.

After linearisation this gives

$$\Delta V = Z \; \Delta I \qquad 6.23$$

Substituting expressions for ΔV and ΔI from equations 6.13 and 6.15 and simplifying we get

$$\Delta v = T^{-1} Z T \; \Delta i_s + (T^{-1} Z \overline{M} + N) \; \Delta\delta \qquad 6.24$$

The differential equations, in linearised form, corresponding to the rotor flux linkages are given by equations 5.5, 5.7 and 5.9 of Chapter 5 and for n machines these can be expressed in a matrix form as

$$p \; \Delta\Psi_r = A \; \Delta i_r + \overline{\Delta v}_f$$

when $A = \text{diag}\left(-\frac{x_{ad1}}{T_{d1}}, -\omega_o r_{1d1}, -\omega_o r_{1q1}, -\frac{x_{ad2}}{T_{d2}}, -\omega_o r_{1d2}, -\omega_o r_{1q2}, \dots\right)$

and $\Delta\bar{v}_f = \begin{bmatrix} \Delta v_{f1}/T_{d1} \\ 0 \\ 0 \\ \Delta v_{f2}/T_{d2} \\ 0 \\ 0 \\ \vdots \end{bmatrix}$

Substituting the expression for Δi_r from equation 6.20, we get

$$p\ \Delta\Psi_r = \Lambda\ P\ \Delta\Psi + \overline{\Delta v}_f$$

$$\text{or } p\ \Delta\Psi_r = B\ \Delta\Psi + \overline{\Delta v}_f \qquad 6.25$$

Similarly the differential equations, in linearised form, corresponding to the stator flux linkages, viz. equations 5.6 and 5.8 of Chapter 5 can be written as

$$p\ \Delta\Psi_s = W\ \Delta v + W_r\ \Delta i_s + W_o\ \Delta\Psi \qquad 6.26$$

where $W = \text{diag}\ (\omega_o, \omega_o, \omega_o, \omega_o, \ldots)$

$W_r = \text{diag}\ (\omega_o r_{a1}, \omega_o r_{a1}, \omega_o r_{a2}, \omega_o r_{a2}, \ldots)$

and

$$W_o = \begin{bmatrix} 0 & 0 & 0 & 0 & \omega_o & & & & & \\ 0 & 0 & 0 & -\omega_o & 0 & & & & & \\ & & & & & 0 & 0 & 0 & 0 & \omega_o \\ & & & & & 0 & 0 & 0 & -\omega_o & 0 & \ddots \end{bmatrix}$$

Substituting expressions for Δv and Δi_s from equations 6.24 and 6.21 equation 6.26 reduces to

$$p\ \Delta\Psi_s = D\ \Delta\Psi + C\ \Delta\delta \qquad 6.27$$

where $D = (W\ T^{-1}\ Z\ T + W_r)\ Q + W_o$

and $C = W\ (T^{-1}\ Z\ \bar{M} + N)$

The equation of motion given in linearised form by equations 5.16 and 5.17 of Chapter 5 can be arranged in matrix form, for n machines, as

$$p\ \Delta\delta = \Delta\omega, \qquad 6.28$$

$$\text{and } p\ \Delta\omega = \Delta T_m + i\ \Delta\Psi + \Psi\ \Delta i_s + K_d\ \Delta\omega \qquad 6.29$$

where

$$\Delta T_m = \begin{bmatrix} \frac{\Delta T_{m1}}{M_1} \\ \frac{\Delta T_{m2}}{M_2} \\ \vdots \end{bmatrix}, \quad \Delta\omega = \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \\ \vdots \end{bmatrix}, \quad i = \begin{bmatrix} 0 & 0 & 0 & -\frac{i_{q1}}{M_1} & \frac{i_{d1}}{M_1} & & & & & \\ & & & & & 0 & 0 & 0 & -\frac{i_{q2}}{M_2} & \frac{i_{d2}}{M_2} \\ & & & & & & & & & & \ddots \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \frac{\Psi_{q1}}{M_1} & -\frac{\Psi_{d1}}{M_1} & & \\ & & \frac{\Psi_{q2}}{M_2} & -\frac{\Psi_{d2}}{M_2} \\ & & & & \ddots \end{bmatrix}$$

and $K_d = \operatorname{diag}\left(-\frac{K_{d1}}{M_1}, \frac{-K_{d2}}{M_2}, \ldots\right)$

Substituting the expression for Δi_s from equation 6.21, equation 6.29 becomes

$$p\,\Delta\omega = \Delta T_m + F\;\Delta\Psi + K_d\Delta\omega \qquad 6.30$$

where $F = (i + \Psi\,Q)$

Assuming, for the purpose of illustration, all the machines are fitted with similar excitation control equipment whose equations in linearised form are given by equations 5.33 and 5.34 of Chapter 5, the equation in a matrix form can be written as

$$p\;x_{ex} = K_{ex}\;x_{ex} + L\;\;\Delta v_t \qquad 6.31$$

where

$$x_{ex} = \begin{bmatrix} \Delta v_{f1} \\ \Delta v_{s1} \\ \Delta v_{f2} \\ \Delta v_{s2} \\ \vdots \\ \vdots \end{bmatrix}, \quad K_{ex} = \begin{bmatrix} \frac{-1}{\tau_{e1}} & -\frac{\mu_1}{\tau_{e1}} & & \\ -\frac{\mu_{s1}}{\tau_{e1}\tau_{s1}} & -\left(\frac{\mu_1\mu_{s1}}{\tau_{e1}\tau_{s1}} + \frac{1}{\tau_{s1}}\right) & & \\ & & -\frac{1}{\tau_{e2}} & -\frac{\mu_2}{\tau_{e2}} \\ & & -\frac{\mu_{s2}}{\tau_{e2}\tau_{s2}} & -\left(\frac{\mu_2\mu_{s2}}{\tau_{e2}\tau_{s2}} + \frac{1}{\tau_{s2}}\right) \\ & & & & \ddots \end{bmatrix}$$

$$L = \begin{bmatrix} -\frac{\mu_1}{\tau_{e1}} & & & \\ -\frac{\mu_1 \mu_{s1}}{\tau_{e1}\tau_{s1}} & & & \\ & -\frac{\mu_2}{\tau_{e2}} & & \\ & -\frac{\mu_2 \mu_{s2}}{\tau_{e2}\tau_{s2}} & & \\ & & & \ddots \end{bmatrix} \quad \text{and} \quad \Delta V_t = \begin{bmatrix} \Delta V_{t1} \\ \Delta V_{t2} \\ \vdots \end{bmatrix}$$

From equation 5.18 of Chapter 5, for n machines we get

$$\Delta V_t = V \ \Delta v \tag{6.32}$$

$$\text{where } V = \begin{bmatrix} \frac{v_{d1}}{V_{t1}} & \frac{v_{q1}}{V_{t1}} & & & \\ & & \frac{v_{d2}}{V_{t2}} & \frac{v_{q2}}{V_{t2}} & \\ & & & & \ddots \end{bmatrix}$$

From equations 6.21, 6.24, 6.31 and 6.32 we obtain

$$p \ x_{ex} = K_{ex} \ x_{ex} + G \ \Delta\Psi + H \ \Delta\delta \tag{6.33}$$

where $G = L \ V \ T^{-1} \ Z \ T \ Q$ and $H = L \ V \ (T^{-1} \ Z \ \overline{M} + N)$

The linearised prime mover control equations 5.35 and 5.36 of Chapter 5 can be written in a matrix form for n machines as

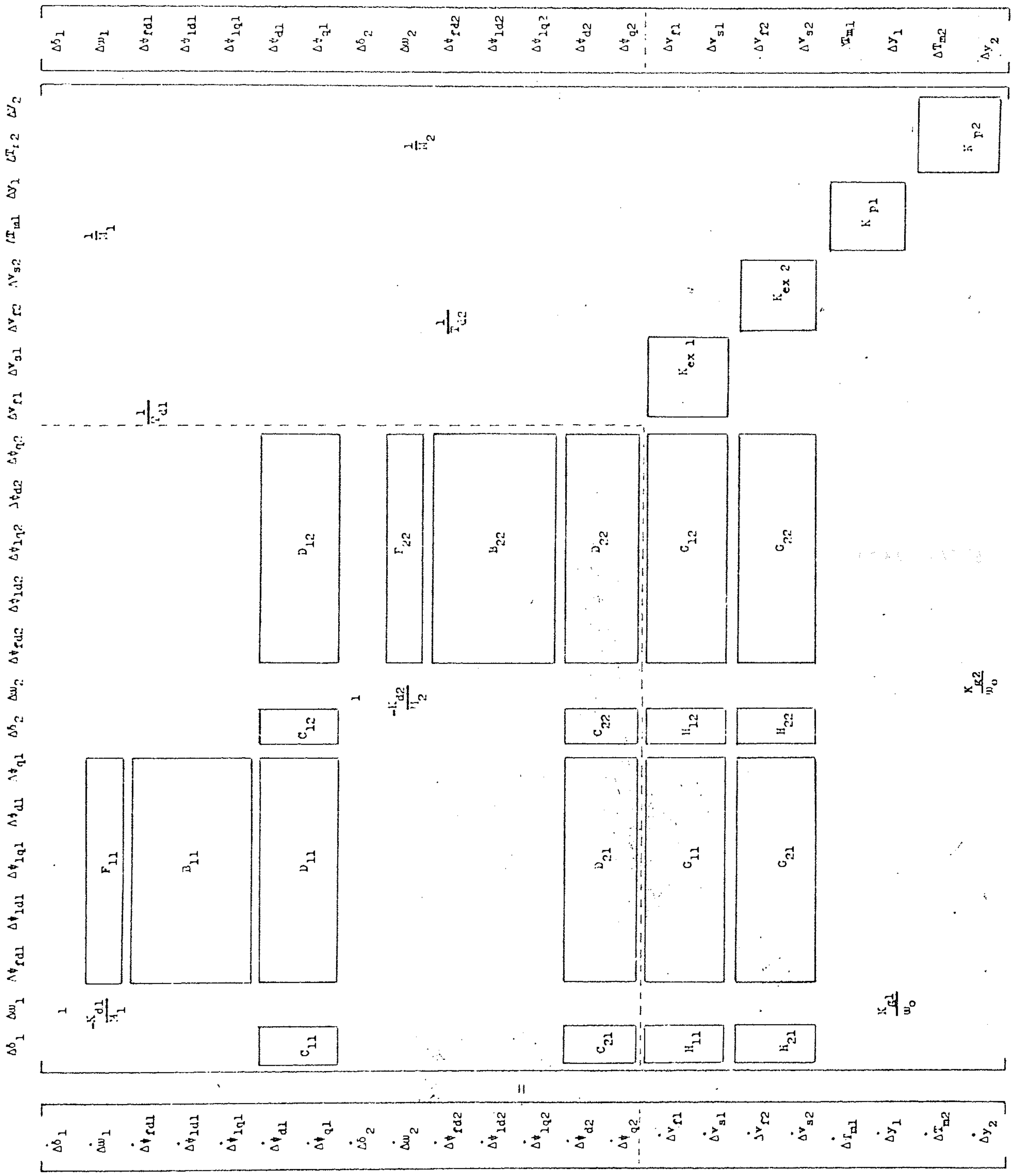
$$p \ x_p = K_p \ x_p + K_g \Delta\omega \tag{6.34}$$

$$\text{where } x_p = \begin{bmatrix} \Delta T_{m1} \\ \Delta y_1 \\ \Delta T_{m2} \\ \Delta y_2 \\ \vdots \end{bmatrix}, \quad K_p = \begin{bmatrix} & 1 & & & \\ -\frac{1}{\tau_g \tau_h} & -\frac{\tau_g+\tau_h}{\tau_g \ \tau_h} & & & \\ & & & 1 & \\ & & -\frac{1}{\tau_g\tau_h} & -\frac{\tau_g+\tau_h}{\tau_g \ \tau_h} & \\ & & & & \ddots \end{bmatrix}$$

$$\text{and} \quad K_g = \begin{bmatrix} 0 & \\ \frac{K_{g1}}{\omega_o} & \\ & 0 \\ & \frac{K_{g2}}{\omega} \end{bmatrix}$$

Equations 6.25, 6.27, 6.28, 6.30, 6.33 and 6.34 contain only the state variables and from these the coefficient matrix can be written out. For a two machine system this is illustrated by equation 6.35.

6.35



Thus the advantage of the present technique of forming the coefficient matrix for a multi-machine system with regard to the amount of computation and computer storage requirement is apparent. It may be noted that most of the matrices involved in the operation are either diagonal or block diagonal matrices and this can be taken into account during matrix multiplications.

As in the case with simple machine representation the coefficient matrix will be singular in the absence of an infinite bus and when absolute values of the machine angles are considered - the columns corresponding to the $\Delta\delta$'s will sum to zero - resulting in a zero eigenvalue. As before, this can be avoided by reducing the order of the system by one. For example, taking the n th machine as a reference, the required changes in the coefficient matrix can be achieved by deleting the row and column corresponding to $\Delta\delta_n$ and placing -1 in the intersections of the rows corresponding to $\Delta\delta_i$'s (i = 1, 2 ... n-1) and the column corresponding to $\Delta\omega_n$. Apart from eliminating the zero eigenvalue, the non-zero eigenvalues will remain unchanged regardless of which machine is taken as a reference. This can be demonstrated by applying the fundamental properties of determinants.

6.2.3 Coefficient Matrix for Machine Model 3

In machine models 2, 3 and 4 the machine internal voltages behind transient or subtransient reactances are used as variables in place of the flux linkages as in model 1. As a consequence some of the intermediate steps in the process of building up the smaller matrix equations prior to the formation of the coefficient matrix will be different. The differential and algebraic equations, in linearised form for machine model 3 are given by equations 5.44 to 5.48 in Chapter 5. For n machines the voltage equations can be written in a matrix form as

$$\Delta v + X \Delta i = \Delta e \qquad 6.36$$

where
$$\Delta v = \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \\ \vdots \end{bmatrix}, \quad \Delta e = \begin{bmatrix} \Delta e_{d1}'' \\ \Delta e_{q}'{}_{1} \\ \Delta e_{d}''{}_{2} \\ \Delta e_{q}'{}_{2} \\ \vdots \end{bmatrix}, \quad \Delta i = \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \\ \vdots \end{bmatrix},$$

and

$$X = \begin{bmatrix} r_{a1} & -x_{q}''{}_{1} & & \\ x_{d}'{}_{1} & r_{a1} & & \\ & & r_{a2} & -x_{q}''{}_{2} \\ & & x_{d}'{}_{2} & r_{a2} \\ & & & & \ddots \end{bmatrix}$$

Using equations 6.14, 6.23 and 6.15 equation 6.36 can be reduced to

$$\Delta i = A\, \Delta\delta + B\, \Delta e \qquad 6.37$$

where $B = T^{-1}\,(Z + T\,X\,T^{-1})^{-1}\,T$

and $A = \overline{N} - B\,(N + Z\,\overline{N})$

For n machines, the differential equations for the machine internal voltages can be written in a matrix form as

$$p\Delta e = P\,\Delta e + Q\,\Delta i + \Delta \overline{v}_f \qquad 6.38$$

where $P = \mathrm{diag}\left(-\frac{1}{T_{qo}''{}_{1}},\ -\frac{1}{T_{do}'{}_{1}},\ -\frac{1}{T_{qo}''{}_{2}},\ -\frac{1}{T_{do}'{}_{2}}\ \ldots\right)$,

$$Q = \begin{bmatrix} & (x_{q1}-x_{q1}'')/T_{qo}''{}_{1} & & \\ -(x_{d1}-x_{d1}')/T_{d}'{}_{o1} & & & \\ & & & (x_{q2}-x_{q}''{}_{2})/T_{qo}''{}_{2} \\ & & -(x_{d2}-x_{d}'{}_{2})/T_{do}'{}_{2} & \\ & & & & \ddots \end{bmatrix}, \quad \text{and } \Delta\overline{v}_f = \begin{bmatrix} 0 \\ \frac{\Delta v_{f1}}{T_{do}'{}_{1}} \\ 0 \\ \frac{\Delta v_{f2}}{T_{do}'{}_{2}} \\ \vdots \end{bmatrix}$$

Substituting the expression for Δi from equation 6.37 and rearranging equation 6.38 reduces to

$$p\Delta e = C\,\Delta\delta + D\,\Delta e + \overline{\Delta v_f} \qquad 6.39$$

where $C = Q\,A$ and $D = (P + Q\,B)$

The equation of motion for n machines can be written in matrix form as

$$p\,\Delta\delta = \Delta\omega \,, \qquad 6.40$$

and $$p\,\Delta\omega = \Delta T_m + i\,\Delta e + E\,\Delta i + K_d\,\Delta\omega \qquad 6.41$$

where $i = \begin{bmatrix} -\frac{i_{d1}}{M_1} & -\frac{i_{q1}}{M_1} & & & \\ & & -\frac{i_{d2}}{M_2} & -\frac{i_{q2}}{M_2} & \\ & & & & \ddots \end{bmatrix}$

and $E =$

$$\begin{bmatrix} -(v_{d1} + r_{a1}i_{d1} - x_d{'}_1 i_{q1})/M_1 & -(v_{q1} + r_{a1}i_{q1} + x_q{''}_1 i_{d1})/M_1 & & & \\ & & -(v_{d2} + r_{a2}i_{d2} - x_d{'}_2 i_{q2})/M_2 & -(v_{q2} + r_{a2}i_{q2} + x_q{''}_2 i_{d2})/M_2 & \\ & & & & \ddots \end{bmatrix}$$

Substituting the expression for Δi from equation 6.37, equation 6.41 reduces to

$$p\Delta\omega = \Delta T_m + J\,\Delta\delta + F\,\Delta e + K_d\,\Delta\omega \qquad 6.42$$

where $J = EA$ and $F = (i + EB)$

As regards excitation control equation, we can start from equation 6.31 and using equations 6.32, 6.36 and 6.37, equation 6.31 reduces to

$$p\,x_{ex} = K_{ex}\,x_{ex} + G\,\Delta e + H\,\Delta\delta \qquad 6.43$$

where $G = LV\,(I-XB)$

and $H = -\,LXAB$

Using similar prime mover control as with model 1, equation 6.34 remains unchanged.

Equations 6.39, 6.40, 6.42, 6.43 and 6.34 contain only the state variables pertaining to machine model 3 and from these the coefficient matrix can be written out. For a two machine system this is illustrated by equation 6.44.

6.2.4 Coefficient Matrix Using Mixed Machine Models

Quite often in a multimachine study it is desirable to employ different machine models to represent the various machines in the system. For example, if one or several portions of the system are represented by equivalent machines they may be represented by constant voltages behind transient reactances while the more detailed representations are reserved only for the machines whose dynamic characteristics are desired to be accurately evaluated. Also in a system there may be a number of large induction motors which may greatly affect the dynamic characteristics of the system and hence a detailed representation of these may be desirable.

In this section the formation of the coefficient matrix for a system where some of the synchronous machines are represented by model 3 and the rest by constant voltages behind transient reactances will be illustrated. To avoid using unnecessary space only the excitation control will be considered. It will however become clear that any combination of machine and control system representation can similarly be handled. The coefficient matrix for a system containing induction motors is given in appendix A 6.1.

The equations for the simplified model can be written in the 2 axis representation as

$$0 = v_d + r_a i_d - x_d' i_q \qquad 6.45$$

and

$$e_q' = v_q + r_a i_q + x_d' i_d \qquad 6.46$$

where e_q' is the constant voltage behind the transient reactance x_d'. This voltage being constant there will be no differential equation to represent flux changes.

$$
\begin{bmatrix} \dot{\Delta\delta}_1 \\ \dot{\Delta\omega}_1 \\ \dot{\Delta e}''_{d1} \\ \dot{\Delta e}'_{q1} \\ \dot{\Delta\delta}_2 \\ \dot{\Delta\omega}_2 \\ \dot{\Delta e}''_{d2} \\ \dot{\Delta e}'_{q2} \\ \hline \dot{\Delta v}_{f1} \\ \dot{\Delta v}_{s1} \\ \dot{\Delta v}_{f2} \\ \dot{\Delta v}_{s2} \\ \dot{\Delta T}_{m1} \\ \dot{\Delta y}_1 \\ \dot{\Delta T}_{m2} \\ \dot{\Delta y}_2 \end{bmatrix}
=
\begin{array}{c}
\begin{array}{cccccccccccccccc} \Delta\delta_1 & \Delta\omega_1 & \Delta e''_{d1} & \Delta e'_{q1} & \Delta\delta_2 & \Delta\omega_2 & \Delta e''_{d2} & \Delta e'_{q2} & \Delta v_{f1} & \Delta v_{s1} & \Delta v_{f2} & \Delta v_{s2} & \Delta T_{m1} & \Delta y_1 & \Delta T_{m2} & \Delta y_2 \end{array} \\
\left[\begin{array}{cccccccc|cccccccc}
 & 1 & & & & & & & & & & & & & & \\
j_{11} & -\frac{K_{d1}}{M_1} & \boxed{F_{11}} & & j_{12} & & \boxed{F_{12}} & & & & & & \frac{1}{M_1} & & & \\
\boxed{C_{11}} & & \boxed{D_{11}} & & \boxed{C_{12}} & & \boxed{D_{12}} & & & & & & & & & \\
 & & & & & & & & \frac{1}{T'_{do1}} & & & & & & & \\
 & & & & & 1 & & & & & & & & & & \\
j_{21} & & \boxed{F_{21}} & & j_{22} & -\frac{K_{d2}}{M_2} & \boxed{F_{22}} & & & & & & & & \frac{1}{M_2} & \\
\boxed{C_{21}} & & \boxed{D_{21}} & & \boxed{C_{22}} & & \boxed{D_{22}} & & & & & & & & & \\
 & & & & & & & & & & \frac{1}{T'_{do2}} & & & & & \\
\hline
\boxed{H_{11}} & & \boxed{G_{11}} & & \boxed{H_{12}} & & \boxed{G_{12}} & & \boxed{K_{ex1}} & & & & & & & \\
 & & & & & & & & & & & & & & & \\
\boxed{H_{21}} & & \boxed{G_{21}} & & \boxed{H_{22}} & & \boxed{G_{22}} & & & & \boxed{K_{ex2}} & & & & & \\
 & & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & \boxed{K_{p1}} & & & \\
 & \frac{K_{g1}}{\omega_o} & & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & & \boxed{K_{p2}} & \\
 & & & & & \frac{K_{g2}}{\omega_o} & & & & & & & & & &
\end{array}\right]
\end{array}
\begin{bmatrix} \Delta\delta_1 \\ \Delta\omega_1 \\ \Delta e''_{d1} \\ \Delta e'_{q1} \\ \Delta\delta_2 \\ \Delta\omega_2 \\ \Delta e''_{d2} \\ \Delta e'_{q2} \\ \hline \Delta v_{f1} \\ \Delta v_{s1} \\ \Delta v_{f2} \\ \Delta v_{s2} \\ \Delta T_{m1} \\ \Delta y_1 \\ \Delta T_{m2} \\ \Delta y_2 \end{bmatrix}
\qquad 6.44
$$

Written in matrix form the voltage equation, after linearisation, becomes for a number of machines

$$\Delta v + X \Delta i = \Delta e \qquad 6.47$$

where the matrix X is the same as in equation 6.36 for model 3 with x_q'' changed to x_d'. However Δe in this case will be a null vector.

For the complete system equations 6.36 and 6.47 are written as one matrix equation before forming equation 6.37.

The expression for output torque will be given by

$$T_e = e_q' \, i_q \qquad 6.48$$

After linearisation equation 6.48 becomes $\Delta T_e = e_q' \, \Delta i_q$ 6.49

Therefore the equation of motion for the machines employing simplified model in matrix form, after linearisation, becomes

$$p \, \Delta\delta = \Delta\omega \qquad 6.50$$

$$p \, \Delta\omega = E \, \Delta i + K_d \Delta\omega \qquad 6.51$$

where the matrix E is given by

$$E = \begin{bmatrix} 0 & -\frac{e_{q}'1}{M_1} & & & \\ & & 0 & -\frac{e_{q}'2}{M_2} & \\ & & & & \ddots \end{bmatrix}$$

For the complete system equations 6.41 and 6.51 are written as one matrix equation of the form of equation 6.41, the vector ΔT_m and the matrix i being modified by introducing the appropriate number of zero elements, while the new E matrix is given by

$$E = \left[\begin{array}{c|c} E \text{ for model 3} & \\ \hline & E \text{ for simplified model} \end{array}\right]$$

As before by eliminating Δ i the equation reduces to that given by equation 6.42.

The coefficient matrix for a 3 machine system with 2 machines represented by model 3 and the third by the simplified model is illustrated in equation 6.52.

$$
\begin{bmatrix} \dot{\Delta\delta}_1 \\ \dot{\Delta\omega}_1 \\ \dot{\Delta e}_{d}''{}_1 \\ \dot{\Delta e}_{q}'{}_1 \\ \dot{\Delta\delta}_2 \\ \dot{\Delta\omega}_2 \\ \dot{\Delta e}_{d}''{}_2 \\ \dot{\Delta e}_{q}'{}_2 \\ \dot{\Delta\delta}_3 \\ \dot{\Delta\omega}_3 \\ \dot{\Delta v}_{f1} \\ \dot{\Delta v}_{s1} \\ \dot{\Delta v}_{f2} \\ \dot{\Delta v}_{s2} \end{bmatrix}
=
\begin{array}{c}
\begin{array}{cccccccccccccc}
\Delta\delta_1 & \Delta\omega_1 & \Delta e_d''{}_1 & \Delta e_q'{}_1 & \Delta\delta_2 & \Delta\omega_2 & \Delta e_d''{}_2 & \Delta e_q'{}_2 & \Delta\delta_3 & \Delta\omega_3 & \Delta v_{f1} & \Delta v_{s1} & \Delta v_{f2} & \Delta v_{s2}
\end{array} \\
\begin{bmatrix}
 & 1 & & & & & & & & & & & & \\
j_{11} & -\frac{K_{d1}}{M_1} & \boxed{F_{11}} & & j_{12} & & \boxed{F_{12}} & & j_{13} & & & & & \\
\boxed{C_{11}} & & \boxed{D_{11}} & & \boxed{C_{12}} & & \boxed{D_{12}} & & \boxed{C_{13}} & & & & & \\
 & & & & & & & & & & \frac{1}{T_{do}'{}_1} & & & \\
 & & & & & 1 & & & & & & & & \\
j_{21} & & \boxed{F_{21}} & & j_{22} & -\frac{K_{d2}}{M_2} & \boxed{F_{22}} & & j_{23} & & & & & \\
\boxed{C_{21}} & & \boxed{D_{21}} & & \boxed{C_{22}} & & \boxed{D_{22}} & & \boxed{C_{23}} & & & & & \\
 & & & & & & & & & & & & \frac{1}{T_{do}'{}_2} & \\
 & & & & & & & & & 1 & & & & \\
j_{31} & & \boxed{F_{31}} & & j_{32} & & \boxed{F_{32}} & & j_{33} & -\frac{K_{d3}}{M_3} & & & & \\
\boxed{H_{11}} & & \boxed{G_{11}} & & \boxed{H_{12}} & & \boxed{G_{12}} & & \boxed{H_{13}} & & \boxed{K_{ex1}} & & & \\
 & & & & & & & & & & & & & \\
\boxed{H_{21}} & & \boxed{G_{21}} & & \boxed{H_{22}} & & \boxed{G_{22}} & & \boxed{H_{23}} & & & & \boxed{K_{ex2}} & \\
 & & & & & & & & & & & & &
\end{bmatrix}
\end{array}
\begin{bmatrix} \Delta\delta_1 \\ \Delta\omega_1 \\ \Delta e_d''{}_1 \\ \Delta e_q'{}_1 \\ \Delta\delta_2 \\ \Delta\omega_2 \\ \Delta e_d''{}_2 \\ \Delta e_q'{}_2 \\ \Delta\delta_3 \\ \Delta\omega_3 \\ \Delta v_{f1} \\ \Delta v_{s1} \\ \Delta v_{f2} \\ \Delta v_{s2} \end{bmatrix}
\qquad 6.52
$$

6.3 DERIVATIVE OF THE COEFFICIENT MATRIX

An important advantage in using the present technique is that in the final coefficient matrix the elements involving any particular parameter of the system can be easily located. This enables the partial derivative of the coefficient matrix with respect to that parameter to be formed without undue difficulty. The partial derivative of the coefficient matrix is required in calculating the sensitivity of a particular eigenvalue with respect to a system parameter. The sensitivity $(\frac{\partial \lambda_i}{\partial \mu})$ of the eigenvalue λ_i with respective a system parameter μ can be shown to be given by

$$\frac{\partial \lambda_i}{\partial \mu} = \frac{Y' \frac{\partial A}{\partial \mu} X}{Y'X}$$

where X and Y are the eigenvectors of the matrices A and A' respectively.

For example, if the sensitivity of the eigenvalue λ_i with respect to a parameter of the excitation control system, say μ_1, is desired, it is easily seen, referring to section 6.2.2 corresponding to machine model 1, that only the matrices G, H and K_{ex} contain this parameter. The derivatives with respect to μ_1 of all the other submatrices forming the final A matrix are zero. The derivative of K_{ex} can be easily formed by inspection. The derivatives of G and H can be seen to be

$$\frac{\partial G}{\partial \mu_1} = \frac{\partial L}{\partial \mu_1} VT^{-1} Z T Q$$

and

$$\frac{\partial H}{\partial \mu_1} = \frac{\partial L}{\partial \mu_1} V (T^{-1} Z \bar{M} + N)$$

since of the matrices of which G and H are formed only L contains μ.

6.4 COMPUTER PROGRAMS

A number of computer programs has been written to form the coefficient matrix of a multi-machine system corresponding to a number of individual machine representations and a number of different combinations of those. It is possible to write a single program incorporating all the synchronous machine models. However, although there is no specific programing difficulty the length of the program and programing time increases considerably due to the inclusion of many models. It was thought advisable to write separate programs to cater for individual case studies. Two programs were written to form the coefficient matrix of a multi-machine system where all the synchronous machines and their control systems were represented similarly - one using machine model 1 and the other using machine model 4. The non-synchronous loads were represented by shunt admittances. Two other programs written allow for certain combinations of machine representations. One of them incorporates synchronous machine models 4 and 5 (the simplified constant voltage magnitude behind transient reactance representation) and has provision for representing induction motor loads in detail. In all the three programs mentioned so far either of the two control systems considered in the single machine study can be incorporated.

A fourth program was written to study the dynamic stability of a practical system with special features in the control equipment - the same system described in section 1.6.5 of Chapter 1 in connection with multi-machine transient stability. As in the transient stability study three of the synchronous machines in the five machine system were represented by model 3 and the two others, which acted as equivalent machines to represent the rest of the system, by model 5. The special excitation and prime mover control system accompanying the three machines represented in detail are shown in Figs. 1.17 and 1.18 of Chapter 1. A part of the study results will be presented in section 6.5 as an illustration of the application of the present techniques.

All these programs have very similar data requirements. First the machine parameters are read. These are followed by the network admittance matrix - the real and imaginary parts being read separately. Next the load flow results (viz. the active and reactive power outputs at the machine terminals and the terminal voltage magnitudes and angles, for the particular operating condition) and the set of control data are read. The sequence in which these two sets of data are read depends on whether the coefficient matrices for different operating conditions or for different sets of control parameters are desired. For example, if the coefficient matrices for various sets of control parameters at a certain operating point are required the load flow results are read first. This arrangement enables the formation of the subsequent coefficient matrices corresponding to any change in the control parameter or operating condition to be achieved by making the minimum amount of alteration in the coefficient matrix first formed thus saving unnecessary computations. Of course any other input arrangement can be easily made.

As the machine and network equations are linearised about an operating point the steady state values of all the machine and network variables must either be read or calculated before starting the computation of the submatrices which form the coefficient matrix. After the coefficient matrix for a particular operating condition and a set of control parameters have been formed the program outputs the results and continues repeating the operation for another operating condition and/or set of control parameters until the end of data is reached.

6.5 A DYNAMIC STABILITY STUDY

As already mentioned a dynamic stability study was carried out on the same practical system whose transient stability was investigated and described in section 1.6.5 of Chapter 1. A full description of the system together with its various control equipment is given there. The purpose of the dynamic stability study was to assess whether the performance of the system under small disturbances was satisfactory with the control parameters set at values which produced satisfactory system performance under transient stability conditions. It was also intended to select control parameters for an improved dynamic performance and compare these with those which gave a satisfactory transient performance and choose the optimum control settings for the system. Conclusions about dynamic performance were made from an investigation of the eigenvalues of the coefficient matrices. This has proved to be a valuable tool for assessing the dynamic performance of a system although not the best one. An evaluation of the performance index as discussed in the next chapter serves as a better guide. However for a system such as the present one this method is an expensive and therefore often unrealistic one.

In this example the excitation and prime mover control systems are somewhat different from those considered in the discussion of the formation of the coefficient matrix in the previous sections in that, in both the excitation and prime mover control, feedback of acceleration (which is not a state variable) has been used. Also in the static excitation system representation the main forward block has been approximated by a simple gain, the time lag being very small. Although these features do not present any special difficulty it is of interest to indicate the method of dealing with them.

From Figure 1.17 of Chapter 1 the equations for the prime mover control system can be written in matrix form after linearisation as

$$p\begin{bmatrix}\Delta T_m\\ \Delta x_1\\ \Delta x_2\\ \Delta x_3\end{bmatrix}=\begin{bmatrix}-\frac{2}{T_w} & -\frac{2}{T_{y2}} & (\frac{2}{T_w}+\frac{2}{T_{y2}}) & \\ & -\frac{b_p}{T_{y1}} & & -\frac{1}{T_{y1}}\\ & \frac{1}{T_{y2}} & \frac{-1}{T_{y2}} & \\ & -\frac{b_t b_p}{T_{y1}} & & -(\frac{b_t}{T_{y1}}+\frac{1}{T_d})\end{bmatrix}\begin{bmatrix}\Delta T_m\\ \Delta x_1\\ \Delta x_2\\ \Delta x_3\end{bmatrix}$$

$$+\begin{bmatrix}0\\ \frac{-K_1}{\omega_o T_{y1}}\\ 0\\ \frac{-b_t K_1}{\omega_o T_{y1}}\end{bmatrix}\begin{bmatrix}\Delta\omega\end{bmatrix}+\begin{bmatrix}0\\ \frac{-T_n}{\omega_o T_{y1}}\\ 0\\ -\frac{b_t T_n}{\omega_o T_{y1}}\end{bmatrix}\begin{bmatrix}p\Delta\omega\end{bmatrix} \qquad 6.53$$

Similarly the excitation control equations can be written as

$$p\begin{bmatrix}\Delta x_4\\ \Delta x_5\end{bmatrix} = \begin{bmatrix}-\frac{1}{\tau_1} & \\ & -\frac{1}{\tau_2}\end{bmatrix}\begin{bmatrix}\Delta x_4\\ \Delta x_5\end{bmatrix} + \begin{bmatrix}0\\ \frac{\mu_2}{\tau_2}\end{bmatrix}\begin{bmatrix}\Delta\omega\end{bmatrix} + \begin{bmatrix}\frac{\mu_1}{\tau_1}\\ 0\end{bmatrix}\begin{bmatrix}p\Delta\omega\end{bmatrix} \quad 6.54$$

Substituting the expression for $p\Delta\omega$ from equation 6.41 and using equation 6.37 the right hand side of equations 6.53 and 6.54 can be expressed in terms of state variables.

Because of the representation of the main forward block of the excitation system by a simple gain Δv_f is no longer a state variable and it is necessary to alter equation 6.39. From Figure 1.18 of Chapter 1 $\Delta v_f = \mu(-\Delta V_t + \Delta x_4 + \Delta x_5)$ and expressing ΔV_t in terms of state variables using equations 6.32, 6.36 and 6.37, equation 6.39 can be rearranged.

The steady state operating conditions are given in section 1.6.5 of Chapter 1. In Table 6.1 four sets of eigenvalues of the four coefficient matrices corresponding to four different combinations of the excitation control parameters are shown. Any change in the system dynamic performance due to a change in the system parameters can only be assessed by a proper interpretation of the eigenvalues. In this method of studying the dynamic stability the whole multi-machine system is represented by one linear vector matrix equation and hence the eigenvalues contain information on the dynamic response of the whole set of system variables rather than a few in which we are usually interested. Any change in the control parameter of a particular machine may have effects on other machines which may be quite remote and this will be reflected in the eigenvalues.

The solution of the vector differential equation, $\dot{x} = Ax$, may be written as $x = \sum_{i=1}^{n} k_i \varepsilon^{\lambda_i t} X^i$, where the λ_i are the eigenvalues and X^i are the associated eigenvectors and k_i are arbitrary constants. It is assumed for simplicity that all the λ_i are distinct. The vector of the arbitrary constants can be found from $k = X^{-1} x_o$, where x_o is the vector of the initial values of x and X is a matrix whose columns are the vectors X^i. From the solution it is clear that for a stable system a real eigenvalue will give rise to an exponentially decaying non-oscillatory term, the decay time constant being the reciprocal of the eigenvalues, whereas a conjugate pair of complex eigenvalues will give rise to an oscillatory term having an angular frequency equal to the imaginary part; the oscillations decay exponentially with a time constant which is the reciprocal of the real part. Thus in a linear dynamic system the eigenvalues correspond to the natural modes of response. An eigenvalue with a small negative real part will indicate the presence of a transient term which is slow to die out. However the relative magnitude of that term in a particular variable will depend on the particular element of the associated eigenvector.

As in the single machine case some of the eigenvalues will be seen to be more sensitive to variations of control parameters and operating conditions than others. Since in the output the eigenvalues are printed in descending order of magnitude it is difficult to follow the locus of any particular eigenvalue with these changes. However the sensitivity of a particular eigenvalue with respect to a parameter may be calculated if desired as indicated in section 6.3.

Of the four sets of eigenvalues listed in Table 6.1, (a) corresponds to the case without any feedback of velocity or acceleration in the static excitation control system. (b), (c) and (d) correspond to the cases when velocity, acceleration and both velocity and acceleration feedbacks were employed. These four sets were selected to illustrate the effects of typical feedbacks on the dynamic response.

An examination of the eigenvalues shows that in the top half of the tables there are some eigenvalues with large imaginary parts which will give rise to highly oscillatory terms. However these eigenvalues also possess large real parts which will damp out the oscillations quickly. Thus the high frequency oscillations which may start following a small disturbance are not of any real concern since they will be damped out rapidly. Near the bottom of the table the eigenvalues are real with very small magnitudes. These will give rise to transient terms decaying very slowly. However these are most likely to be the results of the presence of the slow acting hydrogovernors and the magnitude of the corresponding terms in the variables of importance, e.g. the relative rotor angle and speed, will consequently be small. As expected these eigenvalues showed little change with changes in the control system parameters. That the terms corresponding to these small magnitude eigenvalues are most prominent in the machine input power transients can be seen referring to the corresponding eigenvectors given in Table 6.2.

The eigenvalues marked by an asterisk in Table 6.1 are thought to have the most influence in the dynamic response of the system. The imaginary parts are quite large while the real parts are not large enough to provide fast damping. Also some of them have imaginary parts of magnitudes which correspond to the frequency of the rotor oscillations observed in the transient stability study. Further information which clarifies the situation is obtained from an examination of the eigenvectors corresponding to these eigenvalues. These eigenvectors are shown in Table 6.2 for two sets of excitation feedbacks. For convenience the vector of the system variables is also shown in a separate column. In this five machine system the fifth machine was taken as a reference and therefore the rotor angle transients of the rest of the machines are referred to that machine. It can be seen that in these eigenvectors the relative magnitudes of the elements corresponding to the machine speeds are much larger than those corresponding to the rest of the variables. This means the terms corresponding to these eigenvalues are most prominent in the machine speed transients. The machine speeds are therefore likely to continue oscillating without much damping following a small disturbance. The relative rotor angle transients on the other hand will have oscillations of much reduced amplitude. A very similar behaviour was noticed in the transient stability study.

Of the four different arrangements of the excitation control feedbacks considered here the best arrangement can therefore be said to be that which resulted in the maximum magnitude in the real parts of these critical eigenvalues. As mentioned before in this study the machine and other control parameters were the same as in the transient stability study presented in section 1.5.1. of Chapter 1. The gains of the particular feedbacks under consideration are shown in Table 6.1. Identical values of the control parameters were used on all these machines. From an examination of the critical eigenvalues it may be seen that there is some improvement in the dynamic performance of the system employing acceleration feedback over that employing the other feedbacks or no feedback at all. Actually the introduction of the velocity signal seems to have a detrimental effect from the dynamic stability point of view. Although with an acceleration feedback signal an improvement in the dynamic response was obtainable no such improvement was apparent in the transient stability study. Unfortunately the author had to discontinue the study at this point; but it was later found by his ex-colleagues that using a different time constant (much lower than that shown in Table 1.8 of Chapter 1) in the acceleration feedback loop resulted in a considerable improvement in the transient stability. The corresponding improvement in the dynamic response was not so good. However, from a consideration of both transient and dynamic stability the latter time constant was thought to be more satisfactory.

6.6 SUMMARY

In this chapter dynamic stability studies by the state space approach have been extended to multi-machine systems. Due to the large number of equations involved the matrix elimination technique becomes complicated and uneconomical in multi-machine problems. A different approach has been adopted, in which the system equations are divided into several groups and a number of smaller matrix equations involving state variables only is formed by eliminating the non-state variables. The final state space equation is formed from these smaller matrix equations. Algorithms for forming the state space equations for multi-machine systems employing a number of different machine representations have been discussed.

It has been shown how the partial derivative of the coefficient matrix with respect to a system parameter can be formed easily when the coefficient matrix is formed by the method presented.

The same five machine systems whose transient stability was studied in Chapter 1 has been used here to illustrate the dynamic stability study of a multi-machine system. To draw conclusions concerning the dynamic stability or the dynamic response of a system the system eigenvalues have to be examined and properly interpreted. A discussion about the correct interpretation of the eigenvalues has been included.

Table 6.1

System eigenvalues corresponding to the various combinations of velocity and acceleration feedbacks (see Fig. 1.18, chapter 1).

(a) $\mu_1 = 0$, $\mu_2 = 0$			(b) $\mu_1 = 0$, $\mu_2 = .07$		
	$-1.3017\ 10^{1}$	$1.2867\ 10^{0}$		$-2.8621\ 10^{1}$	0
	$-1.3017\ 10^{1}$	$-1.2867\ 10^{0}$		$-2.1389\ 10^{1}$	0
	$-1.2868\ 10^{1}$	$1.2702\ 10^{0}$		$-2.0591\ 10^{1}$	0
	$-1.2868\ 10^{1}$	$-1.2702\ .0^{0}$		$-1.2144\ 10^{1}$	$1.7605\ 10^{0}$
	$-1.2305\ 10^{1}$	0		$-1.2144\ 10^{1}$	$-1.7605\ 10^{0}$
	$-1.0977\ 10^{1}$	0		$-1.2030\ 10^{1}$	$1.8135\ 10^{0}$
	$-7.3647\ 10^{0}$	0		$-1.2030\ 10^{1}$	$-1.8135\ 10^{0}$
	$-5.1513\ 10^{0}$	0		$-1.1520\ 10^{1}$	$9.5167\ 10^{-1}$
	$-4.8936\ 10^{0}$	0		$-1.1520\ 10^{1}$	$-9.5167\ 10^{-1}$
	$-4.1470\ 10^{0}$	0		$-4.3724\ 10^{0}$	0
	$-4.0641\ 10^{0}$	0		$-4.3645\ 10^{0}$	0
	$-4.0361\ 10^{0}$	0		$-4.2300\ 10^{0}$	0
	$-9.6043\ 10^{-1}$	0		$-1.0731\ 10^{0}$	0
	$-9.6029\ 10^{-1}$	0		$-1.0532\ 10^{0}$	0
	$-9.0133\ 10^{-1}$	0		$-9.7795\ 10^{-1}$	0
*	$-7.1541\ 10^{-1}$	$8.0822\ 10^{0}$		$-9.5304\ 10^{-1}$	0
*	$-7.1541\ 10^{-1}$	$-8.0822\ 10^{0}$		$-9.5112\ 10^{-1}$	0
*	$-6.5142\ 10^{-1}$	$8.0324\ 10^{0}$	*	$-6.8825\ 10^{-1}$	$8.7691\ 10^{0}$
*	$-6.5142\ 10^{-1}$	$-8.0324\ 10^{0}$	*	$-6.8825\ 10^{-1}$	$-8.7691\ 10^{0}$
	$-6.2500\ 10^{-1}$	0	*	$-6.3631\ 10^{-1}$	$8.6251\ 10^{0}$
	$-6.2500\ 10^{-1}$	0	*	$-6.3631\ 10^{-1}$	$-8.6251\ 10^{0}$
	$-6.2500\ 10^{-1}$	0		$-6.2500\ 10^{-1}$	0
	$-6.2500\ 10^{-1}$	0		$-6.2500\ 10^{-1}$	0
	$-6.2500\ 10^{-1}$	0		$-6.2500\ 10^{-1}$	0
	$-6.2500\ 10^{-1}$	0		$-5.6301\ 10^{-1}$	$9.7125\ 10^{-1}$
*	$-5.0924\ 10^{-1}$	$9.9866\ 10^{0}$		$-5.6301\ 10^{-1}$	$-9.7125\ 10^{-1}$
*	$-5.0924\ 10^{-1}$	$-9.9866\ 10^{0}$	*	$-5.3208\ 10^{-1}$	$1.0054\ 10^{-1}$
*	$-2.3168\ 10^{-1}$	$-6.9053\ 10^{0}$	*	$-5.3208\ 10^{-1}$	$-1.0054\ 10^{-1}$
*	$-2.3168\ 10^{-1}$	$6.9053\ 10^{0}$	*	$-2.7218\ 10^{-1}$	$-7.2411\ 10^{0}$
	$-1.0899\ 10^{-1}$	0	*	$-2.7218\ 10^{-1}$	$7.2411\ 10^{0}$
	$-4.0326\ 10^{-2}$	0		$-1.1517\ 10^{-2}$	0
	$-9.0134\ 10^{-3}$	0		$-9.0135\ 10^{-3}$	0
	$-9.0129\ 10^{-3}$	0		$-9.0129\ 10^{-3}$	0

(c) $\mu_1 = .07$, $\mu_2 = 0$		(d) $\mu_1 = .07$, $\mu_2 = .07$	
$-2.4801\ 10^{1}$	0	$-2.4967\ 10^{1}$	0
$-1.8347\ 10^{1}$	0	$-1.8475\ 10^{1}$	0
$-1.8167\ 10^{1}$	0	$-1.8256\ 10^{1}$	0
$-1.1833\ 10^{1}$	$1.1548\ 10^{0}$	$-1.1795\ 10^{1}$	$1.1421\ 10^{0}$
$-1.1833\ 10^{1}$	$-1.1548\ 10^{0}$	$-1.1795\ 10^{1}$	$-1.1421\ 10^{0}$
$-1.0849\ 10^{1}$	$2.6787\ 10^{0}$	$-1.1009\ 10^{1}$	$2.6022\ 10^{0}$
$-1.0849\ 10^{1}$	$-2.6787\ 10^{0}$	$-1.1009\ 10^{1}$	$-2.6022\ 10^{0}$
$-1.0084\ 10^{1}$	$2.4398\ 10^{0}$	$-1.0320\ 10^{1}$	$2.4398\ 10^{0}$
$-1.0084\ 10^{1}$	$-2.4398\ 10^{0}$	$-1.0320\ 10^{1}$	$-2.4398\ 10^{0}$
$-3.9809\ 10^{0}$	$9.5414\ 10^{0}$	$-4.0500\ 10^{0}$	0
$-3.9809\ 10^{0}$	$-9.5414\ 10^{0}$	$-3.9917\ 10^{0}$	0
$-3.9515\ 10^{0}$	0	$-3.8529\ 10^{0}$	0
$-3.8562\ 10^{0}$	0	$-3.6644\ 10^{0}$	$9.7754\ 10^{0}$
$-3.6965\ 10^{0}$	0	$-3.6644\ 10^{0}$	$-9.7754\ 10^{0}$
$-3.5005\ 10^{0}$	$8.9713\ 10^{0}$	$-3.2625\ 10^{0}$	$9.2386\ 10^{0}$
$-3.5005\ 10^{0}$	$-8.9713\ 10^{0}$	$-3.2625\ 10^{0}$	$-9.2386\ 10^{0}$
$-1.5568\ 10^{0}$	$7.0045\ 10^{0}$	$-1.5202\ 10^{0}$	$7.1968\ 10^{0}$
$-1.5568\ 10^{0}$	$-7.0045\ 10^{0}$	$-1.5202\ 10^{0}$	$-7.1968\ 10^{0}$
$-1.4037\ 10^{0}$	0	$-9.6062\ 10^{-1}$	0
$-1.0224\ 10^{0}$	0	$-9.6044\ 10^{-1}$	0
$-9.5854\ 10^{-1}$	0	$-9.1579\ 10^{-1}$	0
$-9.5846\ 10^{-1}$	0	$-6.8586\ 10^{-1}$	$5.6872\ 10^{-1}$
$-6.6852\ 10^{-1}$	0	$-6.8586\ 10^{-1}$	$-5.6872\ 10^{-1}$
$-6.6258\ 10^{-1}$	0	* $-6.6140\ 10^{-1}$	$1.0004\ 10^{1}$
* $-6.5228\ 10^{-1}$	$9.9935\ 10^{0}$	* $-6.6140\ 10^{-1}$	$-1.0004\ 10^{1}$
* $-6.5228\ 10^{-1}$	$-9.9935\ 10^{0}$	$-6.2500\ 10^{-1}$	0
$-6.2500\ 10^{-1}$	0	$-6.2500\ 10^{-1}$	0
$-6.2500\ 10^{-1}$	0	$-6.2500\ 10^{-1}$	0
$-6.2500\ 10^{-1}$	0	$-6.0471\ 10^{-1}$	0
$-3.6572\ 10^{-2}$	$-2.4840\ 10^{-2}$	$-6.0191\ 10^{-1}$	0
$-3.6572\ 10^{-2}$	$2.4840\ 10^{-2}$	$-1.1522\ 10^{-2}$	0
$-9.0135\ 10^{-3}$	0	$-9.0135\ 10^{-3}$	0
$-9.0129\ 10^{-3}$	0	$-9.0129\ 10^{-3}$	0

Table 6.2

(a) Vector of the State Variables	(b) Eigenvector corresponding to the eigenvalue $-5.0924_{10}^{-1} \pm 9.9866_{10}^{0}$ of table 6.1(a)	(c) Eigenvector corresponding to the eigenvalue $-2.3168_{10}^{-1} \pm 6.9053_{10}^{0}$ of table 6.1(a)
$\Delta\delta_{15}$	$9.1852_{10}^{-3} \pm 1.1479_{10}^{-1}$	$-8.9975_{10}^{-3} \pm 1.1417_{10}^{-1}$
$\Delta\omega_1$	$-1.5099_{10}^{-1} \pm 3.3276_{10}^{-2}$	$1.0000_{10}^{0} \pm 1.2829_{10}^{-10}$
$\Delta e_{d'1}$	$5.6569_{10}^{-3} \pm 4.4686_{10}^{-3}$	$-2.3481_{10}^{-2} \pm 3.8260_{10}^{-2}$
$\Delta e_{q'1}$	$3.7599_{10}^{-3} \pm 3.7211_{10}^{-3}$	$1.5670_{10}^{-3} \pm 7.9906_{10}^{-3}$
$\Delta\delta_{25}$	$9.1852_{10}^{-3} \pm 1.1479_{10}^{-1}$	$-8.9975_{10}^{-3} \pm 1.1417_{10}^{-1}$
$\Delta\omega_2$	$-1.5099_{10}^{-1} \pm 3.3276_{10}^{-2}$	$1.0000_{10}^{0} \pm 4.5306_{10}^{-16}$
$\Delta e_{d'2}$	$5.6569_{10}^{-3} \pm 4.4686_{10}^{-3}$	$-2.3481_{10}^{-2} \pm 3.8260_{10}^{-2}$
$\Delta e_{q'2}$	$3.7599_{10}^{-3} \pm 3.7211_{10}^{-3}$	$1.5670_{10}^{-3} \pm 7.9906_{10}^{-3}$
$\Delta\delta_{35}$	$8.5072_{10}^{-3} \pm 1.1170_{10}^{-1}$	$-1.2287_{10}^{-2} \pm 2.7944_{10}^{-2}$
$\Delta\omega_3$	$-1.1979_{10}^{-1} \pm 2.8078_{10}^{-2}$	$4.0538_{10}^{-1} \pm 4.2689_{10}^{-2}$
$\Delta e_{d'3}$	$4.4496_{10}^{-3} \pm 3.6113_{10}^{-3}$	$-1.2384_{10}^{-2} \pm 1.5095_{10}^{-2}$
$\Delta e_{q'3}$	$3.3702_{10}^{-3} \pm 2.7625_{10}^{-3}$	$3.0438_{10}^{-3} \pm 9.1869_{10}^{-4}$
$\Delta\delta_{45}$	$5.7130_{10}^{-3} \pm 1.0172_{10}^{-1}$	$-5.5324_{10}^{-3} \mp 5.0087_{10}^{-2}$
$\Delta\omega_4$	$-1.8704_{10}^{-2} \pm 5.2562_{10}^{-3}$	$-1.3501_{10}^{-1} \pm 1.4126_{10}^{-2}$
$\Delta\omega_5$	$1.0000_{10}^{0} \mp 9.1956_{10}^{-14}$	$2.0957_{10}^{-1} \mp 3.5681_{10}^{-2}$
ΔT_{m1}	$-5.8307_{10}^{-3} \pm 3.5323_{10}^{-3}$	$4.5677_{10}^{-2} \mp 5.5869_{10}^{-4}$
Δx_{11}	$6.8087_{10}^{-3} \pm 3.4838_{10}^{-3}$	$-2.7719_{10}^{-2} \pm 2.7772_{10}^{-2}$
Δx_{12}	$2.6199_{10}^{-3} \mp 2.1599_{10}^{-3}$	$-2.2406_{10}^{-2} \mp 4.4692_{10}^{-3}$
Δx_{31}	$2.7107_{10}^{-3} \pm 1.4213_{10}^{-3}$	$-1.0930_{10}^{-2} \pm 1.1272_{10}^{-2}$
ΔT_{m2}	$-5.8307_{10}^{-3} \pm 3.5323_{10}^{-3}$	$4.5677_{10}^{-2} \mp 5.5869_{10}^{-4}$
Δx_{12}	$6.8087_{10}^{-3} \pm 3.4838_{10}^{-3}$	$-2.7719_{10}^{-2} \pm 2.7772_{10}^{-2}$
Δx_{22}	$2.6199_{10}^{-3} \mp 2.1599_{10}^{-3}$	$-2.2406_{10}^{-2} \mp 4.4692_{10}^{-3}$
Δx_{32}	$2.7107_{10}^{-3} \pm 1.4213_{10}^{-3}$	$-1.0930_{10}^{-2} \pm 1.1272_{10}^{-2}$
ΔT_{m3}	$-9.2035_{10}^{-3} \pm 5.7447_{10}^{-3}$	$3.7081_{10}^{-2} \pm 3.4469_{10}^{-3}$
Δx_{13}	$1.0909_{10}^{-2} \pm 5.3993_{10}^{-3}$	$-2.4845_{10}^{-2} \pm 2.0150_{10}^{-2}$
Δx_{23}	$4.1233_{10}^{-3} \mp 3.4926_{10}^{-3}$	$-1.7784_{10}^{-2} \mp 5.5364_{10}^{-3}$
Δx_{33}	$4.3437_{10}^{-3} \pm 2.2042_{10}^{-3}$	$-9.8240_{10}^{-3} \pm 8.2060_{10}^{-3}$
Δx_{41}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$
Δx_{51}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$
Δx_{42}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$
Δx_{52}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$
Δx_{43}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$
Δx_{53}	$-5.8519_{10}^{-21} \pm 9.9528_{10}^{-21}$	$-1.9290_{10}^{-23} \mp 1.3601_{10}^{-22}$

(d)

Eigenvector corresponding to the eigenvalue $-7.1541_{10}-1 \pm 8.0822_{10}0$ of table 6.1(a)

$-1.0867_{10}-2 \mp 1.2277_{10}-1$
$1.0000_{10}0 \mp 2.6852_{10}-14$
$-2.9568_{10}-2 \mp 4.3735_{10}-2$
$2.6231_{10}-4 \mp 7.6616_{10}-3$
$1.0867_{10}-2 \pm 1.2277_{10}-1$
$-1.0000_{10}0 \mp 1.4663_{10}-10$
$2.9568_{10}-2 \pm 4.3735_{10}-2$
$-2.6231_{10}-4 \pm 7.6616_{10}-3$
$1.6787_{10}-11 \pm 3.0594_{10}-11$
$-2.6685_{10}-10 \pm 1.0890_{10}-10$
$1.2693_{10}-11 \pm 8.3411_{10}-12$
$4.1028_{10}-13 \pm 1.8737_{10}-12$
$4.2960_{10}-13 \mp 2.0376_{10}-12$
$1.0682_{10}-11 \mp 2.2422_{10}-12$
$-1.2157_{10}-11 \mp 4.8040_{10}-12$
$4.7187_{10}-2 \mp 6.6930_{10}-3$
$-3.1455_{10}-2 \mp 3.1159_{10}-2$
$-2.2392_{10}-2 \pm 7.3991_{10}-3$
$-1.2440_{10}-2 \pm 1.2630_{10}-2$
$-4.7187_{10}-2 \pm 6.6930_{10}-3$
$3.1455_{10}-2 \pm 3.1159_{10}-2$
$2.2392_{10}-2 \mp 7.3991_{10}-3$
$1.2440_{10}-2 \pm 1.2630_{10}-2$
$-2.4324_{10}-11 \pm 1.4835_{10}-11$
$2.3612_{10}-11 \pm 9.8268_{10}-12$
$1.0256_{10}-11 \mp 9.0421_{10}-12$
$9.4720_{10}-12 \pm 4.0119_{10}-12$
$1.4398_{10}-21 \pm 2.4960_{10}-21$
$1.4398_{10}-21 \pm 2.4960_{10}-21$
$1.4398_{10}-21 \pm 2.4960_{10}-21$
$1.4398_{10}-21 \pm 2.4960_{10}-21$
$1.4398_{10}-21 \pm 2.4960_{10}-21$
$1.4398_{10}-21 \pm 2.4690_{10}-21$

(e)

Eigenvector corresponding to the eigenvalue $-6.5142_{10}-1 \pm 8.0324_{10}0$ of table 6.1(a)

$6.2942_{10}-4 \pm 6.0174_{10}-2$
$-4.6454_{10}-1 \mp 6.8377_{10}-3$
$1.2703_{10}-2 \pm 1.9837_{10}-2$
$8.0155_{10}-4 \pm 4.8367_{10}-3$
$6.2942_{10}-4 \pm 6.0174_{10}-2$
$-4.6454_{10}-1 \mp 6.8377_{10}-3$
$1.2703_{10}-2 \pm 1.9837_{10}-2$
$8.0155_{10}-4 \pm 4.8367_{10}-3$
$-1.3215_{10}-2 \mp 1.2103_{10}-1$
$1.0000_{10}0 \pm 2.4800_{10}-13$
$-2.9212_{10}-2 \mp 4.3261_{10}-2$
$1.0594_{10}-3 \mp 6.1860_{10}-3$
$-3.5989_{10}-3 \pm 9.0634_{10}-3$
$-5.1248_{10}-2 \mp 7.5069_{10}-3$
$1.9209_{10}-2 \pm 2.7305_{10}-2$
$-2.1850_{10}-2 \pm 2.6086_{10}-3$
$1.4344_{10}-2 \pm 1.4555_{10}-2$
$1.0421_{10}-2 \mp 3.2043_{10}-3$
$5.6702_{10}-3 \pm 5.8980_{10}-3$
$-2.1850_{10}-2 \pm 2.6086_{10}-3$
$1.4344_{10}-2 \pm 1.4555_{10}-2$
$1.0421_{10}-2 \mp 3.2043_{10}-3$
$5.6702_{10}-3 \pm 5.8980_{10}-3$
$9.3883_{10}-2 \mp 1.2613_{10}-2$
$-6.2663_{10}-2 \mp 6.1741_{10}-2$
$-4.4653_{10}-2 \pm 1.4453_{10}-2$
$-2.4781_{10}-2 \mp 2.5028_{10}-2$
$6.3365_{10}-22 \mp 2.3210_{10}-22$
$6.3365_{10}-22 \mp 2.3210_{10}-22$
$6.3365_{10}-22 \mp 2.3210_{10}-22$
$6.3365_{10}-22 \mp 2.3210_{10}-22$
$6.3365_{10}-22 \mp 2.3210_{10}-22$
$6.3365_{10}-22 \mp 2.3210_{10}-22$

(f)

Eigenvectors corresponding to the eigenvalue $-6.5228_{10}-1 \pm 9.9535_{10}0$ of table 6.1(c)

$1.5287_{10}-4 \pm 1.1935_{10}-1$

$-1.9278_{10}-1 \mp 7.6319_{10}-2$

$4.3572_{10}-3 \pm 7.6241_{10}-3$

$-1.3124_{10}-2 \pm 7.3971_{10}-3$

$1.5287_{10}-4 \pm 1.1935_{10}-1$

$-1.9278_{10}-1 \mp 7.6319_{10}-2$

$4.3572_{10}-3 \pm 7.6241_{10}-3$

$-1.3124_{10}-2 \pm 7.3971_{10}-3$

$6.1466_{10}-5 \pm 1.1376_{10}-1$

$-1.3694_{10}-1 \mp 7.3592_{10}-2$

$2.1585_{10}-3 \pm 6.3639_{10}-3$

$-9.3064_{10}-3 \pm 4.5497_{10}-3$

$7.6676_{10}-3 + 1.0143_{10}-1$

$-1.8666_{10}-2 \pm 1.0464_{10}-2$

$1.0000_{10}0 \pm 9.2498_{10}-13$

$-9.2476_{10}-3 \mp 4.1414_{10}-4$

$4.9177_{10}-3 \pm 9.0687_{10}-3$

$4.5775_{10}-3 \mp 4.6322_{10}-4$

$1.9320_{10}-3 \pm 3.6491_{10}-3$

$-9.2476_{10}-3 \mp 4.0414_{10}-4$

$4.9177_{10}-3 \pm 9.0687_{10}-3$

$4.5775_{10}-3 \mp 4.6322_{10}-4$

$1.9320_{10}-3 \pm 3.6491_{10}-3$

$-1.3704_{10}-2 \mp 2.2092_{10}-3$

$5.7488_{10}-3 \pm 1.4363_{10}-2$

$6.8988_{10}-3 \pm 1.0562_{10}-4$

$2.2435_{10}-3 \pm 5.7711_{10}-3$

$-8.2269_{10}-3 \mp 3.8670_{10}-3$

$4.4940_{10}-22 \pm 6.8243_{10}-22$

$-8.2269_{10}-3 \mp 3.8670_{10}-3$

$4.4940_{10}-22 \pm 6.8243_{10}-22$

$-5.7910_{10}-3 \mp 3.5949_{10}-3$

$4.4940_{10}-22 \pm 6.8243_{10}-22$

(g)

Eigenvectors corresponding to the eigenvalue $-6.6140_{10}-1 \pm 1.0004_{10}1$ of table 6.1(d)

$-9.9830_{10}-5 \pm 1.2039_{10}-1$

$-2.0431_{10}-1 \mp 8.0625_{10}-2$

$4.5231_{10}-3 \pm 7.9535_{10}-3$

$-1.4046_{10}-2 \pm 8.9010_{10}-3$

$-9.9830_{10}-5 \pm 1.2039_{10}-1$

$-2.0430_{10}-1 \mp 8.0625_{10}-2$

$4.5231_{10}-3 \pm 7.9535_{10}-3$

$-1.4046_{10}-2 \pm 8.9010_{10}-3$

$-3.8888_{10}-4 \pm 1.1429_{10}-1$

$-1.4305_{10}-1 \mp 7.9480_{10}-2$

$2.1402_{10}-3 \pm 6.6491_{10}-3$

$-1.0105_{10}-2 \pm 5.3556_{10}-3$

$7.7723_{10}-3 \pm 1.0128_{10}-1$

$-1.8336_{10}-2 \pm 1.0766_{10}-2$

$1.0000_{10}0 \pm 5.0821_{10}-14$

$-9.8014_{10}-3 \mp 4.0561_{10}-4$

$5.2237_{10}-3 \pm 9.6097_{10}-3$

$4.8495_{10}-3 \mp 5.0132_{10}-4$

$2.0524_{10}-3 \pm 3.8669_{10}-3$

$-9.9014_{10}-3 \mp 4.0561_{10}-4$

$5.2237_{10}-3 \pm 9.6097_{10}-3$

$4.8495_{10}-3 \mp 5.0132_{10}-4$

$2.0524_{10}-3 \pm 3.8669_{10}-3$

$-1.4401_{10}-2 \mp 2.5111_{10}-3$

$5.8426_{10}-3 \pm 1.5216_{10}-2$

$7.2621_{10}-3 \pm 2.0535_{10}-4$

$2.2777_{10}-3 \pm 6.1129_{10}-3$

$-8.7199_{10}-3 \mp 4.0866_{10}-3$

$-3.4935_{10}-4 \pm 8.9476_{10}-4$

$-8.7199_{10}-3 \mp 4.0866_{10}-3$

$-3.4935_{10}-4 \pm 8.9476_{10}-4$

$-6.0425_{10}-3 \mp 3.8690_{10}-3$

$-3.4531_{10}-4 \pm 6.2685_{10}-4$

(h)

Eigenvector corresponding to the eigenvalue $-9.0129_{10}-3$ of table 6.1(a)

$3.4958_{10}-1$
$-3.1507_{10}-3$
$1.6330_{10}-1$
$-2.6829_{10}-3$
$-3.4957_{10}-1$
$3.1507_{10}-3$
$-1.6330_{10}-1$
$2.6830_{10}-3$
$-2.4172_{10}-6$
$3.0232_{10}-8$
$-1.1118_{10}-6$
$5.5934_{10}-9$
$-1.6497_{10}-7$
$9.9507_{10}-9$
$8.4286_{10}-9$
$1.0000_{10}0$
$9.7071_{10}-1$
$9.7253_{10}-1$
$-3.8464_{10}-2$
$-9.9999_{10}-1$
$-9.7070_{10}-1$
$-9.7251_{10}-1$
$3.8463_{10}-2$
$-1.4002_{10}-5$
$-1.3592_{10}-5$
$-1.3618_{10}-5$
$5.3851_{10}-7$
$-5.0310_{10}-23$
$-5.0310_{10}-23$
$-5.0310_{10}-23$
$-5.0310_{10}-23$
$-5.0310_{10}-23$
$-5.0310_{10}-23$

(i)

Eigenvector corresponding to the eigenvalue $-9.0129_{10}-3$ of table 6.1(d)

$3.4390_{10}-1$
$-3.0995_{10}-3$
$1.6065_{10}-1$
$3.9515_{10}-3$
$-3.4390_{10}-1$
$3.0995_{10}-3$
$-1.6064_{10}-1$
$-3.9514_{10}-3$
$-1.6336_{10}-6$
$3.3132_{10}-8$
$-7.5133_{10}-7$
$-2.2851_{10}-8$
$-1.1168_{10}-7$
$1.9345_{10}-8$
$1.8412_{10}-8$
$1.0000_{10}0$
$9.7071_{10}-1$
$9.7253_{10}-1$
$-3.8464_{10}-2$
$-9.9999_{10}-1$
$-9.7070_{10}-1$
$-9.7252_{10}-1$
$-3.8463_{10}-2$
$-9.5871_{10}-6$
$-9.3064_{10}-6$
$-9.3238_{10}-6$
$3.6858_{10}-7$
$1.9181_{10}-6$
$-2.2014_{10}-4$
$-1.9181_{10}-6$
$2.2014_{10}-4$
$-1.0824_{10}-11$
$2.3386_{10}-9$

CHAPTER 7

POWER SYSTEM OPTIMISATION

7.1 APPLICATION OF OPTIMAL CONTROL THEORY TO POWER SYSTEMS

One of the major advantages of describing the small signal dynamics of a power system in terms of a set of first order differential equations instead of a single higher order differential equation is that many of the established results of modern control theory can be applied. A power system is in effect a closed loop control system with multiple inputs and outputs and an application of modern control theory follows from a state space representation of the system. Of particular importance to power system designers is the evaluation of a performance index based either on the concepts of optimal control theory or on a Liapunov function. In applying a well known result of optimal control the linearised equation of perturbed motion of the power system is written in the state space form

$$\dot{x} = Ax + Bu \qquad 7.1$$

where x is the vector of the state variables as before, u is the control vector and A and B are matrices.

A quadratic performance index of the form

$$J = \int_0^T (x'Qx + u'Ru)\, dt \qquad 7.2$$

is then chosen and minimised. The elements of the positive definite matrices Q and R are selected after appropriate weighting which is usually guided by experience. The object of minimising J is to minimise the sum of the error response and the control effort. It may be noted that taking Q as a unit matrix, $\int_0^T x'Qx\, dt$ constitutes the familiar integral squared error criterion.

In studying the dynamic stability through a state space approach as reported in Chapters 5 and 6, the second term of the right hand side of equation 7.1 was absent. This was due to the fact that the control vector was either constant or expressed beforehand in terms of the state variables in which case the two terms on the right hand side of equation 7.1 were combined together and the stability of the system $\dot{x} = Ax$ was investigated. In the present case the object is to find the optimal control u for a stable system which will minimise the expression given by equation 7.2.

The same methods as followed in deriving state space equations of the form $\dot{x} = Ax$ can be used in deriving equation 7.1 from a set of differential and algebraic equations. For example, the original set of linearised differential and algebraic equations can be written in a vector matrix form as

$$\begin{bmatrix} \dot{x} \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} M_1 & M_2 \\ \hline M_3 & M_4 \end{array}\right] \begin{bmatrix} x \\ \hline x_1 \end{bmatrix} + \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \qquad 7.3$$

where x = the vector of the state variables,

x_1 = the vector of the algebraic variables,

u = the control vector,

and the matrices have been partitioned appropriately.

By elementary matrix reduction similar to that applied in Chapter 5, equation 7.3 reduces to the form of equation 7.1, where

$$A = M_1 - M_2 M_4^{-1} M_3,$$

$$\text{and} \quad B = B_1 - M_2 M_4^{-1} B_2$$

It is shown in any standard text book on modern control theory (e.g. ref. 74) that the optimal control u is given by

$$u = -R^{-1} B' K x \qquad 7.4$$

where K is a positive definite symmetric matrix satisfying the equation

$$-A'K - KA + KBR^{-1} B'K - Q = 0 \qquad 7.5$$

Therefore, the system given by equation 7.1 reduces, with the optimal control, to

$$\dot{x} = A_1 x$$

$$\text{where } A_1 = A - BR^{-1} B'K$$

Although the method is attractive it has to be limited to low-order systems because of the necessity of solving equation 7.5 to obtain the matrix K. The solution becomes extremely time consuming when the order of the system exceeds about ten.

Also it can be seen from equation 7.4 that the optimal control is given by a linear combination of all the state variables not all of which are directly measurable and available and thus the real advantage of a rigorous theory may not be realized. This is so in a multi-machine system where the optimal control law will suggest feedbacks of variables of one machine into another. These, along with a less important fact that the optimal control law depends on the choice of the matrices Q and R in the expression for the performance index, do not render the optimal control theory particularly useful in the design of the controls in a multi-machine system in which power system engineers are more interested. As it stands this can be helpful only in the design of controls in the relatively simple single machine infinite bus systems. Some work on this aspect have been reported in references 75 and 76.

7.2 PERFORMANCE INDEX THROUGH THE USE OF LIAPUNOV FUNCTIONS

A useful alternative, which is more suitable to multi-machine problems, will be, not to establish the optimal control law in the way as described above, but to preselect the controls as combinations of some of the measurable and available state variables which are likely to produce improved dynamic response; then obtain an estimate of $\int_0^\infty x'Qx \, dt$ through the use of a Liapunov function and adjust the control parameters so as to reduce the numerical value of the integral. Thus once again we start with equation $\dot{x} = Ax$

To calculate the performance index given by

$$J = \int_0^\infty x'Qx \, dt \tag{7.6}$$

where Q is a positive definite or semidefinite symmetric matirx, we make use of Liapunov's theorem on linear time-invariant systems stated in Chapter 2.

Choosing a Liapunov function and its time derivative as

$$V(x) = x'Px \quad , \qquad 7.7$$

$$\dot{V}(x) = -x'Qx \quad ; \qquad 7.8$$

where the positive definite symmetric matrix P is determined from the equation

$$A'P + PA = -Q \quad , \qquad 7.9$$

we have $\int_0^{\infty} x'Q\,x\,dt = -\int_0^{\infty} dV(x)$

For an asymptotically stable system $x(\infty) = 0$ and noting that $V(0) = 0$ we have

$$\int_0^{\infty} x'Q\,x\,dt = V(x_0) = x_0'Px_0 \qquad 7.10$$

Thus the performance index is equal to the value of the Liapunov function $x'Px$ at $t = 0$.

7.3 SOLUTION OF THE EQUATION $A'P + PA = -Q$

The equation $A'P + PA = -Q$ has a unique solution for P if $\lambda_i + \lambda_j \neq 0$ for all i,j, where λ_i are the eigenvalues of the coefficient matrix A. This condition will obviously be satisfied for an asymptotically stable system. It is assumed that before trying to optimise a system performance its stability has been properly checked. Two methods of solving the above equation will be considered here; one is the direct solution and the other a solution by a matrix transformation.

7.3.1 Direct Solution

To solve for the matrix P we need to find the elements on and above (or below) the leading diagonal of the P matrix, which are given by the solution of the $n(n+1)/_2$ equations arising out of the matrix equation 7.9. These $n(n+1)/_2$ linear equations can be written in matrix form as

$$Bp = q \quad , \qquad 7.11$$

where the vectors p and q consist of the elements on and above the leading diagonals of the symmetric matrices P and Q as shown below

$$p = \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ \vdots \\ p_{22} \\ p_{23} \\ \vdots \\ p_{nn} \end{bmatrix} , \quad q = \begin{bmatrix} q_{11} \\ q_{12} \\ q_{13} \\ \vdots \\ q_{22} \\ q_{23} \\ \vdots \\ q_{nn} \end{bmatrix}$$

and the matrix B is formed from the elements of the matrix A. The matrix B is given by equation 7.12.

A digital computer program has been written to solve for the matrix P from equation 7.9, check for its positive definiteness and compute $x_0'Px_0$ to give the performance index. The matrices A and Q and the vector of initial perturbations, x_0, are read as input data. The matrix Q is taken as a diagonal matrix and therefore only the diagonal elements are read. The matrix B is formed inside the program from the elements of A. The logic used to form B follows from an inspection of the way in which the elements of A are distributed in B. Once the matrix B and the vector q are formed, the vector p can be easily computed and the matrix P can be formed from it.

Although the method is straightforward the computation time goes up considerably with the size of the system. This is due to the fact that for an nth order system one has to deal with $n(n + 1)/_2$ simultaneous equations to solve for P in the above way and hence the computation time is roughly proportional to n^6. For multi-machine power systems the method, therefore, cannot be used economically. For example an evaluation of the performance index of the 33rd order system illustrated in Chapter 6 would necessitate a solution of 561 simultaneous equations. However, the method is

$$B = \begin{array}{c}
\begin{array}{ccccccccccccccc}
p_{11} & p_{12} & p_{13} & p_{14} & \cdots & p_{22} & p_{23} & p_{24} & \cdots & p_{33} & p_{34} & \cdots & p_{44} & \cdots & p_{nn}
\end{array} \\
\begin{bmatrix}
a_{11} & a_{21} & a_{31} & a_{41} & \cdots & & & & & & & & & & \\
a_{12} & a_{22}+a_{11} & a_{32} & a_{42} & \cdots & a_{21} & a_{31} & a_{41} & \cdots & & & & & & \\
a_{13} & a_{23} & a_{33}+a_{11} & a_{43} & \cdots & & a_{21} & & & a_{31} & a_{41} & \cdots & & & \\
a_{14} & a_{24} & a_{34} & a_{44}+a_{11} & \cdots & & & a_{21} & & & a_{31} & & a_{41} & \cdots & \\
\vdots & \vdots & \vdots & \vdots & & & & & \vdots & & & \vdots & & \vdots & \vdots \\
 & a_{12} & & & & a_{22} & a_{32} & a_{42} & \cdots & & & & & & \\
 & a_{13} & a_{12} & & & a_{23} & a_{33}+a_{22} & a_{43} & \cdots & a_{32} & a_{42} & \cdots & & & \\
 & a_{14} & & a_{12} & & a_{24} & a_{34} & a_{44}+a_{22} & \cdots & & a_{32} & & a_{42} & \cdots & \\
 & \vdots & & & \vdots & \vdots & \vdots & \vdots & & & & \vdots & & \vdots & \vdots \\
 & & a_{13} & & & & a_{23} & & & a_{33} & a_{43} & \cdots & & & \\
 & & a_{14} & a_{13} & & & a_{24} & a_{23} & & a_{34} & a_{44}+a_{33} & \cdots & a_{43} & \cdots & \\
 & & \vdots & & \vdots & & \vdots & & \vdots & \vdots & \vdots & & & \vdots & \vdots \\
 & & & a_{14} & & & & a_{24} & & & a_{34} & & a_{44} & \cdots & \\
 & & & \vdots & & & & \vdots & & & \vdots & & \vdots & & \vdots \\
 & & & & a_{1n} & & & & a_{2n} & & & a_{3n} & & a_{4n} \cdots & a_{nn}
\end{bmatrix}
\end{array}$$

7.12

well suited to smaller sized systems e.g. a single machine infinite bus system. Some study results using the above method have been reported in reference 66. There, the performance index for various combinations of the parameters is calculated. The results are plotted to show the variations of the performance index with change in certain parameter values so that a selection of the parameter values for acceptable small signal performance can be made. However, if a minimum value of the performance index exists within an acceptable range of the parameter values, it can be found by minimising the performance index using a suitable function minimization technique.

This approach was, however, not pursued any further since the main object of the present work was to develop a realistic method for assessing the small signal performance of multi-machine systems. It was noted that if the matrix A is diagonalised by applying a certain matrix transformation the amount of computation in solving for P could be greatly reduced. This is discussed in the next section.

7.3.2 Solution by Matrix Transformation and an Example of its Use.

Using the relation $T^{-1} AT = D$, 7.13

where D is a diagonal matrix composed of the eigenvalues of A and T is a square matrix whose columns are the corresponding eigenvectors, we have

$$A = TDT^{-1} \text{ and } A' = (T^{-1})' DT'$$

It is assumed that the eigenvalues of A are all distinct. Using these relations equation 7.9 reduces to

$$(T^{-1})' DT' P + PTDT^{-1} = -Q \qquad 7.14$$

Premultiplying and postmultiplying both sides by T' and T respectively equation 7.14 reduces to

$$DV + VD = -S \qquad 7.15$$

where $V = T'PT$ and $\dot{S} = T'QT$

Equation 7.15 in expanded form can be written as

$$\begin{bmatrix} 2\lambda_1 v_{11} & (\lambda_1+\lambda_2) v_{12} & (\lambda_1+\lambda_3) v_{13} & \cdots \\ (\lambda_1+\lambda_2) v_{12} & 2\lambda_2 v_{22} & (\lambda_2+\lambda_3) v_{23} & \cdots \\ (\lambda_1+\lambda_3) v_{13} & (\lambda_2+\lambda_3) v_{23} & 2\lambda_3 v_{33} & \cdots \end{bmatrix} = -\begin{bmatrix} s_{11} & s_{12} & s_{13} & \cdots \\ s_{12} & s_{22} & s_{23} & \cdots \\ s_{13} & s_{23} & s_{33} & \cdots \end{bmatrix} \quad 7.16$$

Therefore, the elements of V can be easily calculated by equating the corresponding elements of the two matrices of equation 7.16.

Once V is found P is calculated from the relation $P = (T^{-1})'VT^{-1}$. This has proved to be a much better way of solving for P when P is large.

A digital computer program has been written to solve for P and then calculate the performance index using the above method. Although following this technique the improvements in both the computing time and computer storage requirement over the direct method of solution were considerable, the computing time was still high enough to warrant the number of runs to be kept to a minimum.

Therefore instead of using the method in actually optimising the dynamic response of the system illustrated in Chapter 6, which would have involved a large number of expensive computer runs, it was used to supplement the conclusions made from the eigenvalue analysis. In selecting the diagonal matrix Q, the elements weighting against deviations in the machine angles and speeds were taken as 10 while the rest were taken as unity. Although this selection of the Q matrix was arbitrary it was intended to give more emphasis to the machine angle and speed transients, in which we are mostly interested. In the vector of initial perturbations, x_o, the elements corresponding to the machine angles and speeds were taken as unity and the rest were taken as zero.

It has been shown in Chapter 6 that of the several excitation arrangements considered, the one employing an acceleration feedback gave the best result. The results presented in this section show how the performance index changes with changes in the gains in the main forward block and the acceleration feedback. Due to the long computing time involved in each run only four runs were taken. The results are shown below.

Table 7.1

Performance Index for Various Sets of Control Parameters.

CASE NO.	CONTROL PARAMETERS	PI
1	$\mu = 400,\ \mu_1 = .15, \mu_2 = 0$	1195.5
2	$\mu = 400,\ \mu_1 = .1, \mu_2 = 0$	1148.7
3	$\mu = 400,\ \mu_1 = .07, \mu_2 = 0$	1120.2
4	$\mu = 200,\ \mu_1 = .15, \mu_2 = 0$	1102.9

Thus with the present choice of the Q matrix, case 4 indicates the best small signal performance of the four cases considered.

7.4 SUMMARY

In this chapter a discussion of the optimisation of a power system small signal performance through the use of Liapunov functions has been presented. The possibility of applying a well established result of optimal control theory has been considered first and the difficulties of applying the result in multi-machine power systems have been pointed out. An alternative approach of estimating the performance index in the form of the sum of the integral error squared, which involves the solution of Liapunov's matrix equation, and attempting to reduce its numerical value by adjusting the system parameters has been considered to be the more practical one.

The solution of Liapunov's matrix equation is time consuming when large matrices are involved, if a direct solution is attempted. The computing time and the computer storage requirement can however be reduced considerably using a matrix transformation technique.

CHAPTER 8

CONCLUSIONS

The aim of this thesis has been to give a comprehensive account of the mathematical methods most suited to the transient and dynamic stability studies of practical power systems. Three aspects of the topic have been considered. These are a) numerical solution of the transient stability problem, b) transient and steady state stability studies by Liapunov's direct method through the use of suitably constructed Liapunov functions, and c) studies of dynamic stability and optimisation problems through state space approaches.

In considering the numerical solution of the transient stability problem, which has been the subject matter of Chapter 1, special attention has been given to the representation of synchronous machines in the system. Five synchronous machine models ranging from a very comprehensive representation to the most simple one have been considered. Model 1 uses Park's basic 2-axis flux linkage and voltage equations. An advantage of using this model is that it takes into account various damping effects due to the presence of rotor iron circuits. Another advantage lies in the ease with which the network solution can be obtained during multi-machine calculations. However, the very small integration step length required prohibits the use of this model in all but specialised studies.

Model 2 derived after neglecting the $p\Psi_d$ and $p\Psi_q$ terms and thereby permitting a much bigger integration step length has been shown to be capable of producing results of accuracy comparable with that from model 1. If the direct and quadrature axis subtransient reactances are equal, which is nearly so in many synchronous machines, the network calculation has been shown to be as straightforward as with model 1. Model 2 is therefore recommended for general use when sufficient machine data is available. However availability of sufficient machine data is an acute problem with many existing synchronous machines. This led to the consideration of models 3 and 4 which are derived after further simplifications. Models 3 and 4 permit of a larger integration step length than is possible with model 2, although accuracy of the predicted results is somewhat poor. However, due to the introduction of saliency in the machine algebraic equations the network calculation becomes much more involved and although using a larger integration step, the overall computation time is increased. An iterative process has been described which reduces this considerably. A similar iterative process has been described in reference 9, but the method developed in this thesis covers a wider field. The need for selecting the auxiliary voltage on the appropriate axis depending on the relative magnitudes of the reactances involved has been pointed out.

It has been shown previously[21] that saturation when considered in a manner described in section 1.3 has no appreciable effect in influencing transient stability. In the method described in section 1.3 it has been assumed that the leakage reactances are unaffected by saturation. It is suggested therefore that a future work should consider saturation in the leakage reactances by some means.

The excitation control representation used can be considered sufficiently accurate for predicting the transient performance of the synchronous machine used here as an example. It has been shown that a given control representation can be simplified without affecting its overall effect significantly.

The prime mover control representation as used here with the various machine models is considered accurate enough only for the brief period immediately after a disturbance. The disagreement between the calculated and experimentally obtained curves for rotor angle excursions after the first swing is thought largely due to the inadequate governor and turbine representation. This disagreement is equally prominent with all the machine models. The use of a single time constant to represent valve opening and closing is thought largely responsible for this. However this form of prime mover control representation has the virtue of simplicity and can be considered adequate in cases where pole slipping does not occur and therefore the speed variation is not large. In cases where pole slipping occurs and the performance of the prime mover control has a major influence on the rotor angle transients an elaborate representation which simulates the prime mover control more accurately would be required.

The damping due to the flow of eddy currents in the rotor iron has not been accounted for satisfactorily in machine models 3, 4 and 5 as is evident from the rotor angle transients computed using these models. From the close agreement between the test and calculated results during the first swing and the fair agreement between them during the subsequent swings using models 1 and 2, it appears that the representation of damping by one equivalent short circuited winding on each of the d and q axes is adequate although not entirely satisfactory in studies of the nature reported in this thesis. A rigorous consideration of the rotor iron eddy current damping would necessitate the use of model 1, which is the full set of Park's two axis equations, with extra refinement in selecting the damper winding impedances. This may be achieved by introducing variable impedances for the damper windings, the impedance at any stage depending on the state of the rotor iron. This would assume importance in cases involving pole slipping. The development of an efficient method of evaluating the appropriate impedance at each integration step is therefore desirable. Some work has been reported in reference 11. Although good agreement between computed and test results has been obtained there the various assumptions made leave doubt about the validity of the method for general use. It may be noted that the benefit of any improvement in model 1 may have to be restricted mainly to studies of single machine systems since, as already pointed out, the use of a small integration step renders the use of this model uneconomical for multi-machine systems.

Some remarks about using model 5 in stability studies will be in order. Due to the special representation the calculated rotor angle in the initial steady state condition will be much lower than its true value. Therefore during transients this model will not produce the true picture of the rotor angle variations of individual machines. In multi-machine studies, however, the important quantities are the relative rotor angles. From that point of view this model can produce pictures very similar to those obtained by employing detailed machine models. That this simple representation can in general produce results of acceptable accuracy has been verified in the past[12,13]. Similar conclusions have been reached from the multi-machine transient stability results presented in section 1.6.5. In the single machine study presented in the earlier part of Chapter 1 no meaningful comparison could be made between the calculated result using this model and the test result. However critical clearing time as obtained using this model was compared with that obtained using model 2. Both were of the order of .4 second. Thus at least for this particular operating condition these two machine models were equivalent as far as critical clearing time was concerned. The use of this model is therefore recommended where extreme accuracy is not required and only information about overall transient behaviour is desired. Also in multi-machine studies the machines away from the centre of disturbance, quite often grouped together as equivalent machines, can be adequately represented by this model.

A multi-machine transient stability calculation procedure incorporating all the machine models has been described. A computer program of this kind is useful in system studies where it is desired to represent the synchronous machines as accurately as possible but available data do not permit all of them to be represented in the same degree of detail.

In discussing the synchronous machine equations and the transient stability problem, unbalanced operation has not been considered. However, in a multimachine system unbalanced fault conditions can be studied satisfactorily by preparing the positive, negative and zero sequence networks of the system and connecting them at the point of the fault in a way depending on the type of the fault. Thus the procedure basically reduces to inserting a shunt impedance in the network at the point of the fault[12,13], the shunt impedance being calculated from the negative and zero sequence networks. The braking effect due to the negative sequence currents flowing in the generators can be taken into account as described in reference 12.

In chapter 3 an application of Liapunov's direct method to power system transient stability studies has been demonstrated. A power system with its component machines and their control equipment represented fully is a nonlinear system with multiple nonlinearities. The direct method of Liapunov is based on the formation of a positive definite function with certain properties which guarantee stability. The function is not easy to determine in the general nonlinear case. Although some methods are available to assist in a systematic construction of Liapunov functions their use is usually restricted to systems representable by second or third order differential equations. Therefore to take into account the effects of flux variation, excitation and prime mover control, it is necessary to apply too many simplifications and approximations. Also with the inclusion of these effects the power system has to be limited to a single machine-infinite bus system. Because of this the results will be of little practical value. Unless some general methods are developed for generating Liapunov functions, irrespective of the type of nonlinearities (which is very unlikely in the near future), Liapunov's method will have limited practical importance in power system transient stability studies. Only when the system synchronous machines are representable by constant voltage behind transient reactance and other asynchronous machines and nonlinear loads etc. are replaceable by constant impedances can this method be profitably used. Although this simplification may seem to be a very drastic one, in many cases it produces acceptable results as has been verified in the past and confirmed again in a study reported in chapter 1. As has been shown in chapter 3, with this system representation Liapunov functions can be formed by inspection even for multi-machine systems. However, in a multi-machine study in order to establish the positive and negative definiteness of the Liapunov function and its time derivative respectively in a region around the equilibrium point, the network mutual conductances have to be neglected. Fortunately in most practical cases this is justified.

Two alternative Liapunov functions have been considered depending on the mathematical description of the multi-machine system. Similar Liapunov functions have been used previously in connection with transient stability studies by other workers in this field. However, as has been shown, those were not entirely satisfactory from a rigorous mathematical point of view. The difficulties have been removed by modifying the Liapunov functions and the system representation. In the study of the critical clearing time for faults at various points of a six machine system, included as an example, both Liapunov functions have been shown to produce similar results. It may be noted that the Liapunov functions for all the cases considered here are closely related to the system transient energy.

A further difficulty in applying Liapunov's method is due to the fact that in power system transient stability studies we are confronted with the problem of establishing the stability region as opposed to the global stability. As has been demonstrated, in the single machine case it is a simple matter to establish the stability region and using an energy type Liapunov function is equivalent to the well-known equal area criterion.

However, in a multi-dimensional system it is extremely difficult to decide upon the stability region on a rigorous mathematical basis. Although the criterion used in the thesis to establish the transient stability region has produced correct results, it may leave doubt in some cases.

In spite of the difficulties Liapunov's direct method is attractive since using this method it is possible to get an estimate of the stability region, although conservative in most cases, without going through the extensive step-by-step solution of the system differential equations. Once the stability region is established it is a simple task to calculate the critical clearing time. Also following a disturbance or a series of disturbances the stability can be predicted from a knowledge of the values of the system variables at the end of the disturbance. Since at the moment it is possible to construct suitable Liapunov functions when simplified system representation is employed, Liapunov's method should find immediate application whenever this simplification is permissible. The transient stability programs using the two Liapunov functions considered can be successfully used (a) to obtain quick and approximate results during initial system planning, and (b) to supplement general transient stability programs based on conventional numerical methods.

The approximate method of estimating the stability region without actually solving for the unstable equilibrium state has been shown to produce acceptable results.

In chapter 4 a simple criterion for the steady state stability of a multi-machine system has been developed from the energy type Liapunov functions used in chapter 3 for the transient stability study. Using this criterion the results obtained for single machine and for two machine systems have been shown to be equivalent to those given in reference 12. As an example of the use of the criterion in multi-machine systems the steady state stability limits of the same six machine system, whose transient stability was studied in chapter 3, have been studied using the criterion. Most of the assumptions made here have been used in applying the classical criteria of steady state stability of a multi-machine system[12]. Two of these criteria have been reviewed and efficient algorithms to apply these in multi-machine systems have been developed. The steady state stability study of the six machine system has been repeated using the various stability criteria. The stability limits obtained through the two classical criteria are identical and agree with those obtained through a rigorous state space analysis. However, there is a difference between these and the results obtained from the Liapunov criterion. In deriving this criterion the mutual conductances were neglected. The discrepancies in the results must have been due to the presence of finite mutual conductances since with these negligibly small the criterion is otherwise mathematically rigorous. Unfortunately the stability limits obtained by applying this criterion are somewhat optimistic and therefore cannot be wholly relied upon. However, the effort and time needed to apply this criterion are negligible compared to those needed for other criteria. This criterion can therefore be used to obtain an initial estimate of stability limits with a subsequent checking by some other criterion, preferably by a state space approach.

In chapters 5, 6 and 7 dynamic stability and small signal dynamic response have been studied through a state space representation of the system. The methods are based on Liapunov's theorems on stability through the linear approximation valid for small perturbations. The system equations are linearised about an operating point and expressed in a state space form. Conclusions about stability are drawn from a study of the eigenvalues of the coefficient matrix. Although the state space approach is a powerful method in dynamic stability studies, in order to render the method attractive efficient means must be available to express the system equations in a state space form. In chapter 5 it has been shown how the coefficient matrix for the case of a single synchronous machine connected through an external impedance to an infinite bus and incorporating various types of control systems can be formed efficiently by a matrix reduction technique. Equations have been developed employing all the synchronous machine models considered in chapter 1. The same technique can be extended to multi-machine systems. However, because of the large number of equations involved in a multi-machine system the method becomes cumbersome. A different approach is therefore desirable and this has been discussed in chapter 6.

Although a separate computer program was used to find the eigenvalues of the coefficient matrix and the stability limit was determined from a plot of the critical eigenvalue against reactive power, an algorithm can be prepared to find the stability limit automatically. This will necessitate the working of the coefficient matrix program and the eigenvalue program in conjunction with one another.

Effects of synchronous machine representation in dynamic stability studies have been investigated. Although the dynamic stability limit curves obtained by using different machine models differ appreciably in certain parts of the under excited and over excited regions, in the region of practical interest results from machine models 2 and 3 agree closely with that from the most detailed model 1. Since model 3 uses the least number of equations this is to be recommended in general studies. However, the choice of a machine model might as easily depend on certain other factors one of which is the availability of sufficient machine data.

Along with the effects of machine representation the effects of changing the transmission line parameters, the machine inertia and the terminal voltage on dynamic stability have been considered. The results agree with those obtained by other workers[64,65].

In chapter 6 dynamic stability studies by the state space approach have been extended to multi-machine systems. A method has been presented to form the coefficient matrix through an efficient use of matrix algebra.

It has been shown that with a proper grouping of the machine and system equations and working through a number of smaller matrix equations the amount of computation and computer storage requirement can be kept to a minimum. This is particularly important in multi-machine dynamic stability studies employing varieties of machine and control system models. A method of forming the coefficient matrix of a multi-machine system starting with smaller matrix equations has been described in reference 67. However, the small reduction in the amount of computation and computer storage requirement in the method described in reference 67 hardly justifies the extra complications involved there.

In studying the sensitivity of an eigenvalue with respect to a particular system parameter the partial derivative of the coefficient matrix with respect to that parameter is needed. As has been shown this becomes very simple to obtain when the coefficient matrix is formed by the method presented. Also any modification in the coefficient matrix due to a change in the system parameters or operating condition can in most cases be easily achieved.

The advantage of using a state space method in dynamic stability studies is that it is independent of the type of disturbance so long as the latter is small enough to render the linearisation of the system equations valid. Selection of a particular set of state variables is immaterial since the eigenvalues of the system will remain unaltered by a different choice of the state variables although the coefficient matrix will be different.

In the absence of an infinite busbar the coefficient matrix of a multi-machine power system is singular when the equations are formulated in terms of the absolute values of the rotor angles. The coefficient matrix will consequently have a zero eigenvalue. It has been shown that this can be avoided by taking one machine as a reference and expressing the rotor angles of the other machines relative to that reference. In doing so the order of the coefficient matrix is reduced by one. Apart from eliminating the zero eigenvalue the remaining eigenvalues remain unchanged regardless of which machine is taken as a reference.

It has been stated that information about stability or instability can be obtained from an examination of the signs of the real parts of the eigenvalues. For stability they must all be negative. To obtain information on the dynamic response the eigenvalues have to be properly interpreted. In a linear dynamic system the eigenvalues correspond to the natural modes of response. The imaginary parts give the angular frequency of response while the real parts give the reciprocal of the decay time. Whether a particular mode has much influence on a certain variable can be established from an examination of the associated eigenvector. The presence of an eigenvalue with a small negative real part does not necessarily mean that the system is marginally stable. It merely means that there is a transient in the system which is slow to die out. That transient may be quite small in the variables of most interest. In the example of the dynamic stability study of a multi-machine system presented in section 6.4, three of the system eigenvalues were real and negative with very small magnitude and remained virtually unchanged with changes in the values of the control parameters. An examination of the corresponding eigenvectors revealed that the transients associated with these eigenvalues were small in the variables $\Delta\delta$, $\Delta\omega$ etc.

Another important advantage of the state-space approach lies in the fact that the coefficient matrix represents the whole system rather than a particular machine or a part of the system. If the system is known to develop objectionable oscillations of certain frequencies following a small disturbance, the corresponding eigenvalues can be located in the list of the system eigenvalues and the possibility of a quick damping of the oscillations by adjustments of the control parameters can be investigated.

In addition to an easier determination of dynamic stability or instability from an inspection of the system eigenvalues, the state space modelling of power systems enables the results from modern control theory to be applied. This has been discussed in chapter 7. The application of optimal control theory in multi-machine systems however, gives rise to several difficulties. The most important is the computational difficulty in establishing the control law. Also once the control law is obtained it is not very easy to realize that law in the actual system. An alternative approach in optimising the system performance under small perturbations is to use a slightly different form of performance index which does not involve the control parameters and obtain a numerical estimate of that index through the use of Liapunov's stability theorem for linear time invarient systems. The system parameter values can then be selected from various combinations to give a better system performance as indicated by the numerical value of the performance index. Alternatively, the performance index can be minimised by a suitable function minimisation technique.

The Liapunov approach requires the solution of a matrix equation, which is time consuming when large matrices are involved, if direct solution is attempted. The computing time and the computer storage requirement can, however, be reduced considerably using a matrix transformation technique.

It may be noted that the evaluation of the performance index gives a numerical estimate of the system dynamic response following a small perturbation. The estimate is in the form of the sum of the integral error squared for all the variables with appropriate weighting for each and as such it is a better and much faster method than plotting the responses of all the variables using a step-by-step integration method. For the same reason an estimate of the performance index provides a better knowledge of the dynamic response than is obtainable from an analysis of the system eigenvalues and eigenvectors.

APPENDICES

A 1.1 DERIVATION OF SYNCHRONOUS MACHINE MODEL 2

From equations 1.1 to 1.10 the following expressions can be obtained[4].

$$\psi_d = \frac{1 + T_{kd}p}{1 + (T_1 + T_{do}')p + T_{do}'\,T_{do}''\,p^2}\,v_f - \frac{1 + (T_2 + T_d')\,p + T_d'\,T_d''\,p^2}{1 + (T_1 + T_{do}')\,p + T_{do}'T_{do}''\,p^2}\,x_d i_d \qquad \text{A.1}$$

$$\psi_q = -\frac{1 + T_q''\,p}{1 + T_{qo}''p}\,x_q\,i_q \qquad \text{A.2}$$

The time constants in these expressions are given by:

$$T_{do}' = \frac{1}{\omega r_f}(x_f + x_{ad}) \qquad \text{A.3}$$

$$T_{do}'' = \frac{1}{\omega r_{1d}}\left(x_{1d} + \frac{x_{ad}\,x_f}{x_{ad} + x_f}\right) \qquad \text{A.4}$$

$$T_d' = \frac{1}{\omega r_f}\left(x_f + \frac{x_{ad}\,x_a}{x_{ad} + x_a}\right) = T_{do}'\,\frac{x_d'}{x_d} \qquad \text{A.5}$$

$$T_d'' = \frac{1}{\omega r_{1d}}\left(x_{1d} + \frac{x_{ad}\,x_a\,x_f}{x_{ad}\,x_a + x_{ad}\,x_f + x_a\,x_f}\right) = T_{do}''\,\frac{x_d''}{x_d'} \qquad \text{A.6}$$

$$T_1 = \frac{1}{\omega r_{1d}}(x_{1d} + x_{ad}) \qquad \text{A.7}$$

$$T_2 = \frac{1}{\omega r_{1d}}\left(x_{1d} + \frac{x_{ad}\,x_a}{x_{ad} + x_a}\right) \qquad \text{A.8}$$

$$T_{kd} = \frac{x_{1d}}{\omega r_{1d}} \qquad \text{A.9}$$

$$T_{qo}'' = \frac{1}{\omega r_{1q}}(x_{1q} + x_{aq}) \qquad \text{A.10}$$

$$T_q'' = \frac{1}{\omega r_{1q}}\left(x_{1q} + \frac{x_{aq}\,x_a}{x_{aq} + x_a}\right) = T_{qo}''\,\frac{x_q''}{x_q} \qquad \text{A.11}$$

Using the relation $\frac{T_q''}{T_{qo}''}\,x_q = x_q''$ equation A.2 can be re-arranged as:

$$T_{qo}''\,p\,(\psi_q + x_q''\,i_q) = -(\psi_q + x_q\,i_q)$$

setting $\psi_q + x_q''\,i_q = -e_d''$ A.12

it follows that

$$pe_d'' = \left\{(x_q - x_q'')i_q - e_d''\right\}/T_{qo}'' \qquad 1.14$$

Similarly using the relation $\frac{T_d' \, T_d''}{T_{do}' T_{do}''} x_d = x_d''$, equation A.1 can be written as

$$T_{do}' \, T_{do}'' \, p^2 \, (\psi_d + x_d'' \, i_d) + p \left\{ (T_1 + T_{do}')\psi_d + (T_2 + T_d') \, x_d i_d - T_{kd} v_f \right\}$$

$$= v_f - (\psi_d + x_d \, i_d)$$

Setting $\psi_d + x_d'' \, i_d = e_q''$ A.13

$$T_{do}' T_{do}'' \, p^2 \, e_q'' + p \left\{ (T_1 + T_{do}') \, (e_q'' - x_d'' i_d) + (T_2 + T_d') \, x_d \, i_d - T_{kd} v_f \right\}$$

$$= v_f - (x_d - x_d'') \, i_d - e_q''$$

Using, as an auxiliary variable,

$$e_q' = T_{do}'' \, p e_q'' + \frac{1}{T_{do}'} \left\{ (T_1 + T_{do}') \, (e_q'' - x_d'' \, i_d) + (T_2 + T_d') \, x_d \, i_d - T_{kd} \, v_f \right\}$$

it follows that

$$T_{do}' \, p e_q' = v_f - (x_d - x_d'') \, i_d - e_q''$$

or

$$p e_q' = \left\{ v_f - (x_d - x_d'') \, i_d - e_q'' \right\} / T_{do}' \qquad 1.15$$

and, since $x_d' = \frac{T_d'}{T_{do}'} x_d$,

$$T_{do}'' \, p e_q'' = e_q' - (x_d' - x_d'') \, i_d - e_q'' - \left\{ T_1 \, (e_q'' - x_d'' \, i_d) + T_2 \, x_d \, i_d - T_{kd} \, v_f \right\} / T_{do}'$$

or

$$p e_q'' = \left[e_q' - (x_d' - x_d'') \, i_d - e_q'' - \left\{ T_1 \, (e_q'' - x_d'' \, i_d) + T_2 \, x_d \, i_d - T_{kd} \, v_f \right\} / T_{do}' \right] / T_{do}'' \qquad 1.16$$

Substituting equations A.12 - A.13 in equations 1.7 and 1.9

$$v_d = \frac{1}{\omega_o} p \, (e_q'' - x_d'' \, i_d) + \frac{\omega}{\omega_o} (e_d'' + x_q'' \, i_q) - r_a i_d \qquad 1.17$$

$$v_q = \frac{1}{\omega_o} p \, (- e_d'' - x_q'' \, i_q) + \frac{\omega}{\omega_o} (e_q'' - x_d'' \, i_d) - r_a i_q \qquad 1.18$$

A 1.2

Table A.1

Transmission line constants for the reduced five machine system shown schematically in Fig. 1.16 of Chapter 1. All impedances and susceptances are in p.u. on 100 MVA base; all loads are represented by equivalent shunt impedances; bus numbered 0 represents ground.

Line		p.u. Impedances		p.u. Susceptances
From Bus	To Bus	R	X	B
1	7		.11	
2	7		.11	
3	6		.0415	
6	7		.05	
4	6	.0042	.0263	
4	5	-.0219	.2962	
4	9	.0123	.0654	
4	0	.05082	.01325	
5	9	.00372	.01663	
5	0	.09419	.03168	
8	9		.0286	
9	0	.89882	.18824	
7	8	.00575	.0577	1.538

A 3.1 SAMPLE INPUT OF THE PROGRAM FOR THE SOLUTION OF STABLE AND UNSTABLE EQUILIBRIUM STATES

```
JOB/2042-J88/6139/894/FUM→
12(NO OF RUNS);
RUN ONE
↑[2c3s]SOLUTION*FOR*STABLE*EQLM.*STATE[c3s]
USING*PREFAULT*NETWORK*MATRIX-TRANSFER*CONDUCTANCES*
NEGLECTED[c]↑
5(NO  OF NODES LESS ONE);
<INITIAL GUESS OF M/C ANGLE;M/C OUTPUT POWER;
INTERNAL VOLTAGE;MOMENT OF INERTIA>
.545;.16245;.911087;.0063;(M/C ONE)
.245;.3705;.990375;.0186;(M/C TWO)
-.494;-.21965;.993207;.00707;(M/C THREE)
.54;.14586;.879968;.00675;(M/C FOUR)
.0702;.05966;.978635;.00372;(M/C FIVE)
0;.0000663;.998898;6.37;(M/C SIX)
 NETWORK ADMITTANCE MATRIX
GMATRIX W R T  M/C INTERNAL BUS
.053262;
0;.164309;
0;0;.0675;
0;0;0;.044782;
0;0;0;0;.060448;
0;0;0;0;0;.187162;
BMATRIX
-.31739;
.117781;-1.025135;
.032014;.167574;-.426193;
.040347;.108536;.030743;-.297566;
.015463;.080812;.042162;.015232;-.243513;
.121681;.633019;.200404;.122022;.10476;-1.088374;
0(ESTIMATE OF THE FUNCTION AFTER MINIMISATION);
8(MAXIMUM NO  OF ITERATIONS);
10-15(ACCURACY SPECIFIED);
RUN TWO
↑[2c3s]SOLUTION*FOR*STABLE*EQLM.*STATE[c3s]
USING*POSTFAULT*NETWORK*MATRIX-TRANSFER*
CONDUCTANCES*NEGLECTED[c]↑
```

SAMPLE OUTPUT

SOLUTION FOR STABLE EQLM. STATE
USING PREFAULT NETWORK MATRIX-TRANSFER CONDUCTANCES NEGLECTED

NO. OF ITERATIONS	ANGLES IN RADIANS	GRADIENT VECTOR	FUNCTION VALUE
0	5.450000$_{10}$-1;	-5.134431$_{10}$-4;	
	2.450000$_{10}$-1;	6.601051$_{10}$-3;	
	-4.940000$_{10}$-1;	2.858975$_{10}$-3;	
	5.400000$_{10}$-1;	-9.005314$_{10}$-4;	
	7.020000$_{10}$-2;	-6.937429$_{10}$-4;	9.733918$_{10}$-5;
6	5.252904$_{10}$-1;	-3.069222$_{10}$-6;	
	2.272482$_{10}$-1;	2.716055$_{10}$-5;	
	-5.339022$_{10}$-1;	-1.835827$_{10}$-6;	
	5.262481$_{10}$-1;	-3.251116$_{10}$-6;	
	5.084314$_{10}$-2;	-3.987298$_{10}$-6;	3.927121$_{10}$-10;

CONVERGED SUCCESSFULLY

ANGLES IN RADIANS	ANGLES IN DEGREES	FUNCTION VALUE
5.252917$_{10}$-1;	3.009702$_{10}$1;	
2.272234$_{10}$-1;	1.301895$_{10}$1;	
-5.339150$_{10}$-1;	-3.059110$_{10}$1;	
5.262574$_{10}$-1;	3.015235$_{10}$1;	
5.086869$_{10}$-2;	2.914564$_{10}$0;	3.894629$_{10}$-22;

POWER MISMATCH DP= 3.239176$_{10}$-2;

SOLUTION FOR STABLE EQLM. STATE
USING POSTFAULT NETWORK MATRIX-TRANSFER CONDUCTANCES NEGLECTED

NO. OF ITERATIONS	ANGLES IN RADIANS	GRADIENT VECTOR	FUNCTION VALUE

A 3.2 SAMPLE INPUT FOR THE LIAPUNOV TRANSIENT STABILITY PROGRAM

```
JOB/2042-J88/6139/894/UPS→
↑[2c3s]TRANSIENT*STABILITY*RUN[c3s]
FAULT*APPLIED*ON*BUS*3[c3s]
POSTFAULT*NETWORK*MATRIX*IS*SAME*
AS*PREFAULT[c3s]TRANSFER*CONDUCTANCES*NEGLECTED*AFTER*FINAL*REDUCTION[c]
6(NO . OF M/CS);
<M/C INERTIA;M/C OUTPUT POWER;INTERNAL
VOLTAGE;STABLE EQULM ANGLE RAD;UNSTABLE
EQULM ANGLE RAD>
.0063;.16245;.911087;.525292;.640734;(M/C ONE)
.0186;.3705;.990375;.227223;.276838;(M/C TWO)
.00707;-.21065;.993207;-.533915;-2.273486;(M/C THREE)
.00675;.14536;.979958;.526257;.3409;(M/C FOUR)
.00372;.05966;.978635;.050869;.046297;(M/C FIVE)
6.37;.0000663;.998898;0;0;(M/C SIX)
0.5(DAMPING COEFF USED IN THE EQUATION OF MOTION);
314.159(ANGULAR VELOCITY RADIANS);
0.5(DAMPING COEFF USED IN CALCULATING
THE DERIVATIVE OF V);
<INITIAL VALUES OF M/C ANGLES RADIANS>
.525292;.227223;-.533915;.526257;.050869;0;
.01(INTEGRATION STEP LENGTH);
180(NO OF STEPS);
NETWORK ADMITTANCE MATRICES UNDER VARIOUS
CONDITIONS   W R T  M/C INTERNAL BUS
POSTFAULT
DIAGONAL ELEMENTS OF G MATRIX
.053262;.164309;.0675;.044782;.060448;.187162;
B MATRIX
-.31739;
.117781;-1.025135;
.032014;.167574;-.426193;
.040347;.108536;.030743;-.297566;
.015463;.080812;.042162;.015232;-.243513;
.121681;.633019;.200404;.122022;.10476;-1.088374;
FAULT
DIAG  ELEMENTS OF G MATRIX
.03885;.10707;0;.03752;.03699;.1236;
B MATRIX
-.323086;
.08795;-1.181242;
6.55_10-8;3.41_10-7;-.60606;
.03487;.079904;5.25_10-8;-.302813;
.007966;.041513;8.65_10-8;.008012;-.253372;
.086;.44634;4.08_10-7;.08779;.057747;-1.311602;
POSTFAULT
DIAG  ELEMENTS OF G MATRIX
.053262;.164309;.0675;.044782;.060448;.187162;
B MATRIX
-.31739;
.117781;-1.025135;
.032014;.167574;-.426193;
.040347;.108536;.030743;-.297566;
.015463;.080812;.042162;.015232;-.243513;
.121681;.633019;.200404;.122022;.10476;-1.088374;
→
```

SAMPLE OUTPUT

TRANSIENT STABILITY RUN
FAULT APPLIED ON BUS 3
POSTFAULT NETWORK MATRIX IS SAME AS PREFAULT
TRANSFER CONDUCTANCES NEGLECTED AFTER FINAL REDUCTION

UV= 0.19328484

TIME SEC.		ROTOR ANGLE	P.U. SLIP	POWER OUTPUT	POW.OUT.OF MACHINE 6	V	FIGURE OF MERIT
0.000	MACHINE 1	30.0970	0.00000000	0.00000000			
	MACHINE 2	13.0189	0.00000000	0.00000000			
	MACHINE 3	-30.5911	0.00000000	0.00000000			
	MACHINE 4	30.1523	0.00000000	0.00000000			
	MACHINE 5	2.9146	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.010	MACHINE 1	30.1263	0.00032438	0.09805558			
	MACHINE 2	13.0501	0.00034572	0.16797711			
	MACHINE 3	-30.6800	-0.00098674	-0.00000058			
	MACHINE 4	30.1753	0.00025533	0.09152790			
	MACHINE 5	2.9413	0.00029689	0.02485867	-0.05734385	0.00052058	0.99862716
0.020	MACHINE 1	30.2137	0.00064678	0.09816116			
	MACHINE 2	13.1432	0.00068886	0.16872115			
	MACHINE 3	-30.9460	-0.00196856	-0.00000059			
	MACHINE 4	30.2442	0.00050926	0.09156951			
	MACHINE 5	3.0213	0.00059190	0.02492870	-0.05830510	0.00207914	0.99452701
0.030	MACHINE 1	30.3590	0.00096685	0.09833653			
	MACHINE 2	13.2979	0.00102860	0.16995524			
	MACHINE 3	-31.3884	-0.00294548	-0.00000059			
	MACHINE 4	30.3586	0.00076165	0.09163878			
	MACHINE 5	3.1543	0.00088463	0.02504509	-0.05990023	0.00468155	0.98777120

A 3.3

Table A.2

Reduced network matrices with respect to the machine internal buses for fault at the terminal of machine 1.

G matrix

0					
0	.043715				
0	.012824	.018440			
0	.005910	.001654	.025719		
0	.009413	.006014	.001306	.022381	
0	.041064	.014979	.004991	.012426	.068289

B matrix

-.462963					
0	-1.120405				
0	.141679	-.433232			
0	.075853	.021859	-.308690		
0	.068314	.038766	.010927	-.245149	
0	.534572	.173645	.088292	.091837	-1.190084

Table A.3

Reduced network matrices with respect to the machine internal buses for fault at the terminal of machine 3.

G matrix

.015284					
.011425	.045405				
0	0	0			
.001035	.005942	0	.025168		
.001372	.005966	0	.000881	.020715	
.009738	.038322	0	.004497	.008046	.062993

B matrix

-.323086					
.087950	-1.181242				
0	0	-.606060			
.034870	.079904	0	-.302813		
.007966	.041513	0	.008012	-.253372	
.086000	.446340	0	.087790	.057747	-1.311602

Table A.4

Reduced network matrices with respect to the machine internal buses for fault at the terminal of machine 4.

G matrix

.016636					
.015583	.058962				
.004299	.016476	.019447			
0	0	0	0		
.002657	.011371	.006549	0	.022660	
.014912	.054621	.018733	0	.014409	.082395

B matrix

-.325974					
.094615	-1.087585				
.025453	.149885	-.431203			
0	0	0	-.485437		
.012198	.072023	.039673	0	-.244748	
.095683	.562892	.180541	0	.094883	-1.167094

A 4.1 SAMPLE INPUT FOR LIAPUNOV STEADY STATE STABILITY PROGRAM

```
JOB/2042-J88/6139/894/SSA→
1(NO OF RUNS);
↑[2c3s]STEADY*STATE*STABILITY*STUDY*OF*A
[c3s]6*MACHINE*SYSTEM[c3s]OUTPUT*
POWER*OF*MACHINE*1*IS*INCREASED
*UNTIL*THE[c3s]SYSTEM*BECOMES*
UNSTABLE[c3s]TRANSFER*CONDUCTANCES*
ARE*NEGLECTED[c]↑
20(MAXIMUM NO. OF POWER LEVELS OF M/C ONE FOR
WHICH STABILITY IS TO BE VERIFIED);
5(NO OF M/CS LESS ONE);
<ACTIVE POWER;REACTIVE POWER;TERMINAL
VOLTAGE;ANGLE IN DEGREES;TRANSIENT
REACTANCE>
.29;-.03957;.9398;31.2;2.16;(M/C ONE)
.3705;-.04474;1.0;2.72;.608;(M/C TWO)
-.21965;-.04965;1.0075;-6.08;1.65;(M/C THREE)
.14586;-.05476;.9499;15.7;2.06;(M/C FOUR)
.05966;-.01326;1.0;-4.04;2.47;(M/C FIVE)
-.12408;.02035;.9989;-.905;.5;(M/C SIX)
NETWORK ADMITTANCE MATRIX W R T
M/C TERMINALS
G MATRIX
[MATRIXGG];1;2;6;6;
.076669;5Z;
0;.180028;4Z;
2Z;.06382;3Z;
3Z;.033033;2Z;
4Z;.090613;0;
5Z;.077175;→
B MATRIX
[MATRIXBB];1;2;6;6;
-1.208403;1.068633;.000012;.204962;.000004;.001323;
1.068633;-14.58122;1.165181;.489691;.106677;11.89943;
.000012;1.165181;-2.203888;.001901;.226923;.925913;
.204962;.489691;.001901;-.907932;.000771;.227481;
.000004;.106677;.226923;.000771;-.693535;.375603;
.001323;11.89943;.925913;.227481;.375603;-13.4078;→
B MATRIX W R T. M/C INTERNAL BUS
[MATRIXBI];1;2;6;6;
-.317390;.117781;.032014;.040347;.015463;.121681;
.117781;-1.025135;.167574;.108536;.080812;.633019;
.032014;.167574;-.426193;.030743;.042162;.200404;
.040347;.108536;.030743;-.297566;.015232;.122022;
.015463;.080812;.042162;.015232;-.243513;.104760;
.121681;.633019;.200404;.122022;.104760;-1.088374;→
0(ESTIMATE OF THE FUNCTION AFTER MINIMISATION);
8(MAXIMUM NO. OF ITERATIONS IN ANY SOLUTION);
10-15(ACCURACY SPECIFIED);
POWER LEVELS OF M/C ONE
.16245;.2;.3;.4;.45;.5;
.55;.57;.585;.6;.615;.63;.645;
.66;.675;.69;.705;.72;.75;.8;→
```

SAMPLE OUTPUT

STEADY STATE STABILITY STUDY OF A
6 MACHINE SYSTEM
OUTPUT POWER OF MACHINE 1 IS INCREASED UNTIL THE
SYSTEM BECOMES UNSTABLE
TRANSFER CONDUCTANCES ARE NEGLECTED

OUTPUT POWER OF MACHINE 1= 0.162450

RESULTS OF POWER FLOW SOLUTIONS
USING FUNCTION MINIMISATION TECHNIQUE

NO.OF ITERATIONS	ANGLES IN RADIANS	GRADIENT VECTOR	FUNCTION VALUE
0	5.603375$_{10}$ -1;	3.095348$_{10}$ -1;	
	6.326813$_{10}$ -2;	2.058049$_{10}$ 0;	
	-9.032071$_{10}$ -2;	-1.705880$_{10}$ -1;	
	2.898117$_{10}$ -1;	-1.516852$_{10}$ -1;	
	-5.471603$_{10}$ -2;	-1.663489$_{10}$ -2;	1.085731$_{10}$ -1;
6	1.239358$_{10}$ -1;	1.535743$_{10}$ -2;	
	1.504604$_{10}$ -2;	-1.411758$_{10}$ -1;	
	-1.359632$_{10}$ -1;	-7.202287$_{10}$ -2;	
	1.791522$_{10}$ -1;	1.255530$_{10}$ -2;	
	-6.367295$_{10}$ -2;	2.001564$_{10}$ -2;	8.670156$_{10}$ -4;
12	1.192195$_{10}$ -1;	-1.354600$_{10}$ -4;	
	1.619314$_{10}$ -2;	1.511058$_{10}$ -4;	
	-1.216198$_{10}$ -1;	-4.027800$_{10}$ -5;	
	1.676873$_{10}$ -1;	-1.960156$_{10}$ -4;	
	-7.969103$_{10}$ -2;	1.948563$_{10}$ -4;	8.637294$_{10}$ -8;
18	1.194300$_{10}$ -1;	-1.290007$_{10}$ -8;	
	1.621417$_{10}$ -2;	2.382664$_{10}$ -7;	
	-1.216506$_{10}$ -1;	-8.211859$_{10}$ -8;	
	1.680532$_{10}$ -1;	-6.328272$_{10}$ -9;	
	-8.008581$_{10}$ -2;	7.556828$_{10}$ -9;	4.988167$_{10}$-16;

CONVERGED SUCCESSFULLY

ANGLES IN RADIANS	ANGLES IN DEGREES	FUNCTION VALUE
1.194300$_{10}$ -1;	6.842843$_{10}$ 0;	
1.621417$_{10}$ -2;	9.290044$_{10}$ -1;	
-1.216506$_{10}$ -1;	-6.970072$_{10}$ 0;	
1.680532$_{10}$ -1;	9.628748$_{10}$ 0;	
-8.008581$_{10}$ -2;	-4.588583$_{10}$ 0;	1.333678$_{10}$-19;

FINAL RESULTS

	ACTIVE POWER	REACTIVE POWER	INTERNAL VOLTAGE-PU	INTERNAL ANGLE-DEG
MACHINE 1	0.16245000	-0.04501234	0.91590276	29.995232
MACHINE 2	0.37050000	-0.03128431	1.00651078	12.956742
MACHINE 3	-0.21965000	-0.08083772	0.94615989	-30.220784
MACHINE 4	0.14586000	-0.03858528	0.92217053	28.784494
MACHINE 5	0.05966000	-0.01577612	0.97226510	3.223987
MACHINE 6	-0.00887089	-0.01942998	0.98918427	-1.162194

THE SYSTEM IS STABLE

A.6.1 FORMATION OF THE COEFFICIENT MATRIX FOR SYSTEMS CONTAINING INDUCTION MOTOR LOADS

In this illustration the synchronous machines in the system are assumed to be represented by either the simplified model or model 3. The induction machines are assumed to be represented by equations 1.87 to 1.91 of chapter 1.

The equations for the induction machines can be linearised and written in matrix form for n machines as

$$p\,\Delta e = P\Delta e + Q\,\Delta i + R\,\Delta s \qquad \text{A.14}$$

$$\text{where } P = \begin{bmatrix} -\frac{1}{T'_{do1}} & s_1\omega_o & & & \\ -s_1\omega_o & -\frac{1}{T'_{do1}} & & & \\ & & -\frac{1}{T'_{do2}} & s_2\omega_o & \\ & & -s_2\omega_o & -\frac{1}{T'_{do2}} & \\ & & & & \ddots \end{bmatrix},$$

$$Q = \begin{bmatrix} & \frac{x_1-x_1'}{T'_{do1}} & & & \\ -\frac{x_1-x_1'}{T'_{do1}} & & & & \\ & & & \frac{x_2-x_2'}{T'_{do2}} & \\ & & -\frac{x_2-x_2'}{T'_{do2}} & & \\ & & & & \ddots \end{bmatrix},$$

$$R = \begin{bmatrix} e'_{q1}\omega_o & & \\ -e'_{d1}\omega_o & & \\ & e'_{q2}\omega_o & \\ & -e'_{d2}\omega_o & \\ & & \ddots \end{bmatrix}, \text{ and } \Delta s = \begin{bmatrix} \Delta s_1 \\ \Delta s_2 \\ \vdots \\ \\ \end{bmatrix},$$

$$p\,\Delta s = E\,\Delta i + i\,\Delta e \,, \qquad \text{A.15}$$

$$\text{where } E = \begin{bmatrix} \frac{e_d{'}_1}{\omega_o M_1} & \frac{e_q{'}_1}{\omega_o M_1} & & & \\ & & \frac{e_d{'}_2}{\omega_o M_2} & \frac{e_q{'}_2}{\omega_o M_2} & \\ & & & & \ddots \end{bmatrix}$$

$$\text{and } i = \begin{bmatrix} \frac{i_{d1}}{\omega_o M_1} & \frac{i_{q1}}{\omega_o M_1} & & & \\ & & \frac{i_{d2}}{\omega_o M_2} & \frac{i_{q2}}{\omega_o M_2} & \\ & & & & \ddots \end{bmatrix}$$

$$\Delta v + X\,\Delta i = \Delta e \,, \qquad \text{A.16}$$

$$\text{where } X = \begin{bmatrix} r_{a1} & -x_1{'} & & & \\ x_1{'} & r_{a1} & & & \\ & & r_{a2} & -x_2{'} & \\ & & x_2{'} & r_{a2} & \\ & & & & \ddots \end{bmatrix}$$

The vectors Δe, Δi etc. are as defined in chapter 6 in connection with the coefficient matrix for synchronous machine model 3.

In the induction machine equations the axis voltages and currents are referred to a set of synchronously rotating axes which can be assumed to coincide with the network reference axes. Therefore in the equations, similar to equations 6.13 and 6.15, for the induction machines the matrix T will be a unit matrix and the vector $\Delta\delta$ will be a null vector.

For the complete system equations 6.36, 6.47 and A.16 are written as one matrix equation before forming equation 6.37.

The differential equations for the internal voltages of all the synchronous and induction machines can be collected and written in matrix form as

$$p\,\Delta e = P\,\Delta e + Q\,\Delta i + R\,\Delta s + \Delta \bar{v}_f \qquad \text{A.17}$$

The new P and Q matrices for the mixed machine models will be given by

$$P = \left[\begin{array}{c:c} P_{\text{for synch. m/c model 3}} & \\ \hdashline & P_{\text{for induction m/cs.}} \end{array}\right];$$

$$Q = \left[\begin{array}{c:c} Q_{\text{for synch. m/c model 3}} & \\ \hdashline & Q_{\text{for induction m/cs}} \end{array}\right],$$

and the matrix R is modified by introducing an appropriate number of zero elements.

Substituting the expression for Δi from equation 6.37 and rearranging, equation A.17 reduces to

$$p\,\Delta e = C\Delta\delta + D\Delta e + R\Delta s + \Delta\bar{v}_f \qquad \text{A.18}$$

where $C = QA$ and $D = (P + QB)$

The equation of motion for all the synchronous and induction machines can be written as one matrix equation of the form of equation 6.41 with appropriate modifications of the vectors ΔT_m and Δe and the matrix K_d.

$$p\Delta\omega = \Delta T_m + i\Delta e + E\Delta i + K_d\Delta\omega \qquad 6.41$$

The new matrices i and E will be given by

$$i = \left[\begin{array}{c:c} i_{\text{for synch. m/c model 3}} & \\ \hdashline \begin{array}{l}\text{zero elements corresponding}\\ \text{to the simplified synchronous}\\ \text{m/c model}\end{array} & \\ \hdashline & i_{\text{for induction m/c}} \end{array}\right]$$

$$E = \left[\begin{array}{c:c:c} E_{\text{for synch. m/c model 3}} & & \\ \hdashline & E_{\text{for simplified synch. m/c model}} & \\ \hdashline & & E_{\text{for induction m/c}} \end{array}\right]$$

By eliminating Δi as before equation 6.41 will reduce to the state space form of equation 6.42

$$p\Delta\omega = \Delta T_m + J\Delta\delta + F\Delta e + K_d\Delta\omega \qquad 6.42$$

where $J = EA$ and $F = (i + EB)$

The coefficient matrix for a four machine system with three synchronous machines - two represented by model 3 and one by the simplified model, and one induction machine is illustrated by equation A.19.

$$
\begin{bmatrix}
\dot{\Delta\delta}_1 \\ \dot{\Delta\omega}_1 \\ \dot{\Delta e}_d{}'_1 \\ \dot{\Delta e}_q{}'_1 \\ \dot{\Delta\delta}_2 \\ \dot{\Delta\omega}_2 \\ \dot{\Delta e}_d{}'_2 \\ \dot{\Delta e}_q{}'_2 \\ \hdashline \dot{\Delta\delta}_3 \\ \dot{\Delta\omega}_3 \\ \hdashline \dot{\Delta s} \\ \dot{\Delta e}_d{}'_4 \\ \dot{\Delta e}_q{}'_4 \\ \hdashline \Delta\dot{v}_{f1} \\ \Delta\dot{v}_{s1} \\ \Delta\dot{v}_{f2} \\ \Delta\dot{v}_{s2}
\end{bmatrix}
=
\begin{array}{c}
\begin{array}{cccccccccccc}
\Delta\delta_1 & \Delta\omega_1 & \Delta e_d{}'_1\ \Delta e_q{}'_1 & \Delta\delta_2 & \Delta\omega_2 & \Delta e_d{}'_2\ \Delta e_q{}'_2 & \Delta\delta_3 & \Delta\omega_3 & \Delta s & \Delta e_d{}'_4\ \Delta e_q{}'_4 & \Delta v_{f1}\ \Delta v_{s1} & \Delta v_{f2}\ \Delta v_{s2}
\end{array} \\
\begin{bmatrix}
 & 1 & & & & & & & & & & \\
j_{11} & -\frac{K_{d1}}{M_1} & \boxed{F_{11}} & j_{12} & & \boxed{F_{12}} & j_{13} & & & \boxed{F_{13}} & & \\
\boxed{C_{11}} & & \boxed{D_{11}} & \boxed{C_{12}} & & \boxed{D_{12}} & \boxed{C_{13}} & & & \boxed{D_{13}} & \frac{1}{T_{do}{}'1} & \\
 & & & & 1 & & & & & & & \\
j_{21} & & \boxed{F_{21}} & j_{22} & -\frac{K_{d2}}{M_2} & \boxed{F_{22}} & j_{23} & & & \boxed{F_{23}} & & \\
\boxed{C_{21}} & & \boxed{D_{21}} & \boxed{C_{22}} & & \boxed{D_{22}} & \boxed{C_{23}} & & & \boxed{D_{23}} & & \frac{1}{T_{do}{}'2} \\
\hdashline
 & & & & & & & 1 & & & & \\
j_{31} & & \boxed{F_{31}} & j_{32} & & \boxed{F_{32}} & j_{33} & -\frac{K_{d3}}{M_3} & & \boxed{F_{33}} & & \\
\hdashline
j_{41} & & \boxed{F_{41}} & j_{42} & & \boxed{F_{42}} & j_{43} & & & \boxed{F_{43}} & & \\
\boxed{C_{31}} & & \boxed{D_{31}} & \boxed{C_{32}} & & \boxed{D_{32}} & \boxed{C_{33}} & & r_{51} & \boxed{D_{33}} & & \\
 & & & & & & & & r_{61} & & & \\
\hdashline
\boxed{H_{11}} & & \boxed{G_{11}} & \boxed{H_{12}} & & \boxed{G_{12}} & \boxed{H_{13}} & & & \boxed{G_{13}} & \boxed{K_{ex1}} & \\
\boxed{H_{21}} & & \boxed{G_{21}} & \boxed{H_{22}} & & \boxed{G_{22}} & \boxed{H_{23}} & & & \boxed{G_{23}} & & \boxed{K_{ex2}}
\end{bmatrix}
\end{array}
\begin{bmatrix}
\Delta\delta 1 \\ \Delta\omega 1 \\ \Delta i_d{}'1 \\ \Delta e_q{}'1 \\ \Delta\delta_2 \\ \Delta\omega_2 \\ \Delta e_d{}'2 \\ \Delta e_q{}'2 \\ \hdashline \Delta\delta_3 \\ \Delta\omega_3 \\ \hdashline \Delta s \\ \Delta e_d{}'4 \\ \Delta e_q{}'4 \\ \hdashline \Delta v_{f1} \\ \Delta v_{s1} \\ \Delta v_{f2} \\ \Delta v_{s2}
\end{bmatrix}
\quad \text{A.19}
$$

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