bush, as represented by the lower orbit, whereby allowing the opposite end of the journal to lift, as seen by the upper orbit, thus demonstrating that excessive misalignment could be easily observed. If these latter conditions were allowed to continue, overheating would occur which was not only detectable by monitoring the journal running positions but also by the embedded thermocouples, further the whole of the test journal would be seen to slowly move due to the thermal growth of the bearing bush.

In order to provide confirmatory checks, several tests were repeated but with the direction of the misaligning couple reversed, repeatability of the journal trajectory was good, however, certain discrepancies were noticed when first applying the reversed couple, which was thought to be due to the oil film temperature distribution, this explanation was accepted as being plausible if not entirely correct.

Misalignment along the horizontal plane was also introduced, but only as a preliminary investigation as it was not intended to provide design charts for this misaligning condition. The object was to briefly find the magnitude of any changes made to the averaged journal trajectory. The pattern which emerged was that for all eccentricity ratios the stiffness coefficients reduced slightly and up to an eccentricity ratio of 0.5 the damping coefficients were reduced, and above 0.5 the damping tended to be increased. However, these trends should only be taken as a guide as to what should be expected for a horizontal misaligning couple, such that the design engineer is aware of the effects made to the hydrodynamics oil film under this environment.

In presenting all twenty-eight oil film coefficients the writer is conscious that it is more than plausible that several of these coefficients perhaps could have been neglected, for example, coefficients $A(xx\dot{x})$, $A(xy^2)$ and $A(yx^2)$ could possibly be discarded without effecting the overall orbital response. However, to investigate this possibility taking into account the complete range of eccentricity ratios having differing amplitudes of vibration at various forcing frequencies, would be prohibitive and not worth the amount of work involved.

The computer program solving the hydrodynamic oil film coefficients proved to be costly, for example, in order to solve the complete field equation, the number of iterations was in the order of 650 which is approximately 375,000 nodal calculations of the type given in equations (3.7) and (3.10), giving a computer running time of 12 to 15 minutes which only provides one running condition of the bearing. For this reason investigations have been limited to misalignment in the vertical plane as this is the most likely condition to be experienced in practice, however it is envisaged that other planes of misalignment will be examined at a later date.

CHAPTER THIRTEEN

CHAPTER THIRTEEN

CONCLUSIONS

13.1 Distribution of the hydrodynamic oil film pressures

The numerical analysis using the boundary condition $dP/d\theta \neq 0$ together with the selected size of grillage, produced oil film pressures conducive with experimental work published by Dubois, Mabie and Ocvirk. The oil film pressure distribution in the axial direction was shown to be severely effected for only a slight increase in misalignment, resulting in high intensity of pressure and a moving of the centre of pressure towards one end of the bearing, behaving in complete accord with the experimental test journal.

The method of calculating the pressure profile can therefore be used for estimating the load carrying capacity and oil film forces for bearings operating under both steady state and dynamic running conditions. The results also provide a firm foundation for investigating changes made about the steady equilibrium position, which are essential for computing the hydrodynamic force coefficient.

The oil film pressure distribution allowed the following observations:-

(I) Misalignment is capable of changing the equivalent length of the bearing span by offsetting the centre of pressure, thus changing the critical frequency, quite apart to changes made to the oil film force coefficients.

(II) Misalignment applied to the vertical plane generated residual couples along the horizontal plane, prior knowledge of this phenomenon allowed the test journal to be designed such that trim weights could be applied to correct any unwanted residual couples.

(III) Journals operating with an eccentricity ratio of 0.7 or less, although having a large oil film thickness compared to a bearing having an eccentricity ratio above 0.7, are indeed less capable of absorbing a misaligning couple than bearings having an eccentricity ratio above 0.7.

13.2 Journal locus aligned and misaligned conditions

The main objective of the experimental investigation described in this section was to provide an adequate representation of the journal bearing locus for various operating conditions, for subsequent analysis into the dynamic behaviour of the oil film. The equilibrium position for a steadily loaded journal bearing operating under both aligned and misaligned conditions have been analyzed, the results obtained in this study are as follows:

(I) The experimental results for a perfectly aligned journal bearing revealed that the calculated performances are in good agreement with experimental results. Thus presenting design charts capable of being used for determining the load carrying capacity, and minimum oil film thickness.

(II) The experimentally determined characteristics for a misaligned journal bearing are also shown to agree well with the computed values for a steadily loaded bearing experiencing misalignment in the vertical plane.

(III) For practical purposes the results may be summarized in the form of design charts which show the relationship between the misaligning slope, expressed as an end tilt ratio, and the Sommerfeld duty parameter at various central eccentricity ratios.

(IV) Misalignment had little effect on the oil flow and bearing averaged temperature, provided the amount of tilt was sufficiently small so as to prevent metal to metal contact.

(V) The central eccentricity ratio was reduced with the introduction of misalignment, the general pattern was a reduction of $\varepsilon = 0.1$ for full misalignment.

(VI) The attitude angle at the ends and central eccentricities are also a function of the applied misaligning couple, however, as only small changes were present, the analytical solution describing the exact behaviour had to be relied upon for providing this information.

13.3 Hagg and Sankey force coefficients

Hagg and Sankey force coefficients are of limited use and should only be used when the mass and the rotational speed of the rotor under consideration are similar. Further the amplitude of vibration should also be identical, as averaged values of the oil film stiffness are unwittingly presented due to the non linear characteristics of the oil film which occurs at surprisingly small amplitudes of vibration.

Force coefficients of the Hagg and Sankey type, should in the writers opinion only be used where such coefficients are determined experimentally from similar machines or rotors, and are the sole source of information available for computing critical speeds at the design stage.

Both the oil film stiffness and damping are shown to be effected if the bearing was made to run at an altered duty parameter. This may be achieved by making changes to either the oil viscosity, loading or rotational speed, indeed full use is made of this on actual machines which are found to operate close to a critical speed; for example by reducing the axial length of the bearing bush thus increasing the specific pressure and in turn changing the Sommerfeld number, had the effect of increasing the force coefficients without changing the remaining operational functions to any extent.

From the measured whirl trajectories, it was clear that a rotor supported on journal bearings should display two critical speeds due to the major and minor stiffness coefficients, compared to each critical speed determined on rigid supports, indeed this has been found to be true in practice but only on the more heavily loaded rotor where such phenomenon is more pronounced.

Because of the non-reciprocal nature of the oil film dynamic characteristics, the journal response for a disturbing force first rotating in the same direction as the journal rotation, and secondly in the reverse direction to the journal rotation, are shown to be quite different. The two principal differences being in the suppression in vibration amplitude for the reverse whirl conditions and also, again under reverse whirl conditions, the direction of the whirl orbit rotates in the same direction as the journal rotation, indeed over the entire working range of the bearing the whirl trajectory was never capable of being excited to rotate in the opposite direction to the journal rotation.

Hagg and Sankey force coefficients can only be considered under synchronous vibrational conditions, that is once per revolution as associated with rotor unbalance, or a fundamental critical speed, also correlation between two different machines is only possoble if either the mass is negligible, which is unlikely, or if the rotational exciting frequency is low. Failure to comply with these conditions leads to inaccuracies.

13.4 Eight force coefficients

The large discrepancies between theory and experimental results for synchronous whirl vibrations, can not be explained in terms of errors in measuring, or calculating, the Sommerfeld duty parameter or indeed any inaccuracies in measuring the amplitude of the orbital whirl, further the success of the low frequency whirl vibration produced results in good agreement between theoretical predictions and experimental observations, thus suggesting that measuring errors are minimal.

Clearly the eight calculated dynamic coefficients characterizing the dynamical behaviour of the journal bearing, excited at frequencies equal to the rotational speed of the bearing, are of uncertain accuracy when presenting realistic dynamic conditions of the oil film. As previously stated real coefficients do not necessarily approach linearity for small amplitudes of vibration, a condition which was evident when investigating synchronous whirl vibrations, even with amplitudes kept as small as 0.00005 in (0.00125 mm). Therefore previous calculations of force coefficients for a partial journal bearing of the type used throughout this thesis are in quantitative error, though they may be qualitatively correct. However, better approximations are later presented which consider non-linearity by introducing a further twenty coefficients which do not impose restrictions upon the frequency of vibration in order to obtain results conducive with physical reality.

In order to evoke the theory of small perturbations allowing the use of the eight force coefficients, the amplitudes of vibration have to be so small as to make measuring extremely difficult, if indeed such measurements could be separated from the surface irregularities of the test journal. Further any suggestion that for critical speed purposes such coefficients would suffice, may be discounted, as it would be difficult to balance a rotor to such fine limits whereby the amplitudes of vibration are sufficiently small such that non linearity may be neglected. Indeed, acceptable levels of journal vibration at 3000 R.P.M. according to British Standard for Electrical Machines, could be in the order of 0·00075 in (0.0188 mm) which for the geometry of a bearing under investigation would most certainly involve the non-linear characteristics as found experimentally.

For low frequency of vibration the velocity coefficients were found to give values greater than those determined analytically, however, the physical implication would be in producing phase errors in the angular position of the exciting force in the order of 3 to 5 degrees.

13.5 Twenty-eight coefficients - aligned conditions

The main objective of the experimental investigation described was for adequate representation of the oil film force coefficients, for subsequent analysis relating to the design of journal bearings for the purpose of assessing at the design stage the likelihood of critical speeds, and also the possibility of any other kind of instability within the running range of the machine. A method for analyzing a dynamically loaded journal bearing has been presented taking into account the second order coefficients characterizing non-linearity whereby the theoretical elliptical whirl orbit can be deduced for a given periodic disturbing force. Calculations for predicting the orbital response using only eight force coefficients are clearly in quantititative error, however better approximations are presented by making use of the non-linear terms generated by considering the second order terms of Taylors Series. It was also evident from the closeness of the predicted and measured journal trajectories that there is good agreement between the theoretical force coefficients and experimental observations, therefore although it is an impossible task to compute values of the non linear force and velocoty coefficients from experimental results, it may be deduced that values presented in chapter ten are correct.

Further observations may be listed as follows:-

(I) The assumption of Simple Harmonic Motion together with the modified method of Newton using a quadratic Taylor expansion for solving the resulting second order non-linear simultaneous equations, appeared in every way to be correct.

(II) The vibrating motion of the test journal was always in the same direction as the journal rotation, even under conditions of reverse rotation of the disturbing force, further, no half speed whirl or other instability phenomenon occurred during the tests.

(III) Journal centre trajectories using all the non linear terms approached those trajectories using the eight linear coefficients, provided the amplitude of vibration under synchronous and above synchronous vibrating speeds were kept to within 0.00005 in (0.00125 mm), having an error in amplitude of approximately 2.0 per cent, increasing to an error of 10.0 per cent for amplitudes of vibration of 0.00013 in (0.00325 mm).

Journal trajectories also approached each other when the frequency of the exciting out of balance force was low, slight errors in damping were however still present under these conditions of vibration.

(IV) The amplitude of vibration of the journal had no effect on the temperature field of the oil film wedge.

(V) The maximum vibration amplitude was kept to well within 20.0 per cent of the bearing radial clearance.

(VI) Damping was shown to be increased with a corresponding increase in the vibrational frequency, a condition conducive with normal theories of vibration.

(VII) For all cases of reverse vibration the whirl amplitude was vigorously suppressed compared with forward whirling conditions.

13.6 Twenty-eight coefficients - misaligned conditions

The extension of the twenty-eight oil film coefficients from the aligned to the misaligned condition of the journal have been shown to be capable of predicting the journal centre trajectory with good accuracy. Other results found in this study are as follows:-

(I) The design charts presented in Appendix B are capable of providing information of sufficient accuracy for determining the hydrodynamic oil film forces, and may be used with confidence when investigating rotor dynamics.

(II) The introduction of misalignment to the dynamically loaded journal bearing will have the effect of increasing the magnitude of the oil film stiffness and damping coefficients.

(III) Other operating conditions such as averaged oil film temperature, friction losses, Sommerfeld duty parameter all remained unchanged.

(IV) Throughout all the aligned and misaligned tests, the resulting journal trajectory was elliptic which closed for one complete cycle of the pericdic disturbance force, vindicating the basic hypothesis in the theoretical approach discussed in Chaptersfive and six.

(V) Excessive misalignment causes overheating of the bearing bush within a short time of applying the misaligning couple.

CHAPTER FOURTEEN

CHAPTER FOURTEEN

14.1 Recommendations for future investigations

The force coefficients have been investigated to such depths, that results presented exceed the normal capabilities usually demanded by the present generation of critical speed programs. Therefore the difficulty in suggesting recommendations for future work, is in deciding if, at this stage, improvements should be made to the theoretical methods for analyzing the dynamical behaviour of rotors, taking into account all the twenty-eight force coefficients, or whether to investigate the oil film in even greater detail, or indeed, if investigations should take the form of monitoring the various pedestal characteristics in order to evaluate the performance or change in performance, of a rotor system.

If the former is selected the analytical solution should include the rotor as a flexible system, capable of taking into account its behaviour having unbalanced assigned to various planes along the length of the rotor. If such a solution could be successfully obtained, investigations will have gone a long way in achieving "one shot" modal balancing, whereby a displacement vector obtained from the journal bearing, could determine the amount of unbalance and its location, such that it will not cause ill conditioning for the other modes of vibration.

If, however, further investigations into the behaviour of the hydrodynamic oil film are to be considered as a future research programme, the remaining outstanding items of information which may be of possible assistance to the design engineer, would be the effects of the inertia forces generated within the oil film bearings where the film is operating within the turbulant region. However, this kind of investigation could not be conducted on the existing experimental test journal, unless the speed was increased to 9.500 R.P.M. which because of the kinetic energy stored in the rotor it would be dangerous if for any reason the rotor became unstable. Further investigations not covered by the work presented in this thesis is the behaviour of the oil film when the journal is experiencing conical whirl, as would be the case for an extremely flexible rotor undergoing large amplitudes of whirl.

The dynamic characteristics of the oil film is responsible for providing information as to the behaviour of the rotor, clearly any changes experienced by the rotor caused by a change in the condition of balance, must be transmitted to the bearing supporting structure via the oil film. Therefore any monitoring system employing the pedestal vibration to determine the condition of the rotor, was dependant upon the transmissibility of the oil film, this then could form the basis for future research work whereby the transmissibility could be determined as a function of the Sommerfeld duty parameter and upon the flexibility of both the rotor and the bearing pedestal. For example, where a journal is misaligned causing an increase in the oil film force coefficients, the forces transmitted through the oil film could be increased, depending upon the pedestal stiffness. This type of information could therefore indicate foundation movement by monitoring the journal response characteristics. Further, because of the non linearity of the oil film twice the unbalance does not give twice the whirl amplitude, a condition which could be misleading if monitoring the state of rotor balance via the journal. However because the equivalent oil film force coefficients are increased due to the increased unbalance disturbing force, the transmissibility of the oil film would also be expected to change, thus the mechanical impedance of the oil film plus the bearing supporting structure may be altered as to provide displacements greater than proportional to the magnitude of unbalance.

An investigation into the above phenomenon would be of considerable value to the site engineer for predicting the change in dynamical performance of rotating equipment.

APPENDIX A

APPENDIX A

THEORETICAL ANALYSIS OF THE OIL FILM FORCES

The force exerted by the oil film onto the vibrating journal is a function of the displaced journal, ε and ϕ from the equilibrium running position, and also a function of their respective velocities. The additional oil film forces due to these perturbations may be written as Taylors series, extended to include four variables as illustrated in section 3.4.

Using equations (3.9) which describe the components of force acting along the radial and tangential directions, allows the following list of partial differential equations to be formulated:-

(I) Incremental force acting along the radial axis

$$\frac{\partial F}{\partial \varepsilon} d\varepsilon = -SF \frac{\partial f_R}{\partial \varepsilon} d\varepsilon \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \varepsilon} d\varepsilon \sin(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \phi} d\phi = -SF \frac{\partial f_R}{\partial \phi} d\phi \cos(\phi-\phi_o) + SF f_R\, d\phi \sin(\phi-\phi_o)$$

$$-SF \frac{\partial f_T}{\partial \phi} d\phi \sin(\phi-\phi_o) - SF f_T\, d\phi \cos(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} = -SF \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} \sin(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} = -SF \frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \varepsilon^2} d\varepsilon^2 = -SF \frac{\partial^2 f_R}{\partial \varepsilon^2} d\varepsilon^2 \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial \varepsilon^2} d\varepsilon^2 \sin(\phi-\phi_o)$$

$$\begin{aligned}\frac{\partial^2 F}{\partial \phi^2} d\phi^2 = & -SF \frac{\partial^2 f_R}{\partial \phi^2} d\phi^2 \cos(\phi-\phi_o) + SF \frac{\partial f_R}{\partial \phi} d\phi^2 \sin(\phi-\phi_o) \\ & + SF \frac{\partial f_R}{\partial \phi} d\phi^2 \sin(\phi-\phi_o) + SF\, f_R\, d\phi^2 \cos(\phi-\phi_o) \\ & - SF \frac{\partial^2 f_T}{\partial \phi^2} d\phi^2 \sin(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \phi} d\phi^2 \cos(\phi-\phi_o) \\ & - SF \frac{\partial f_T}{\partial \phi} d\phi^2 \cos(\phi-\phi_o) + SF\, f_T\, d\phi^2 \sin(\phi-\phi_o)\end{aligned}$$

$$\frac{\partial^2 F}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} = -SF \frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} = -SF \frac{\partial^2 f_R}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} \sin(\phi-\phi_o)$$

$$\begin{aligned}\frac{\partial^2 F}{\partial \varepsilon \partial \phi} d\varepsilon d\phi = & -SF \frac{\partial^2 f_R}{\partial \phi \partial \varepsilon} d\phi d\varepsilon \cos(\phi-\phi_o) + SF \frac{\partial f_R}{\partial \varepsilon} d\phi d\varepsilon \sin(\phi-\phi_o) \\ & - SF \frac{\partial^2 f_R}{\partial \varepsilon \partial \phi} d\varepsilon d\phi \sin(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \varepsilon} d\varepsilon d\phi \cos(\phi-\phi_o)\end{aligned}$$

$$\frac{\partial^2 F}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} = -SF \frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o) + SF \frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\phi \sin(\phi-\phi_o)$$

$$-SF \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \sin(\phi-\phi_o) - SF \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\frac{\dot{\phi}}{\omega} = -SF \frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\frac{\dot{\phi}}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\frac{\dot{\phi}}{\omega} \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} d\frac{\dot{\phi}}{\omega} d\varepsilon = -SF \frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} d\frac{\dot{\phi}}{\omega} d\varepsilon \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} d\frac{\dot{\phi}}{\omega} d\varepsilon \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} d\varepsilon d\frac{\dot{\varepsilon}}{\omega} = -SF \frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} d\varepsilon d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\varepsilon \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} d\phi d\frac{\dot{\phi}}{\omega} = -SF \frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} d\phi d\frac{\dot{\phi}}{\omega} \cos(\phi-\phi_o) + SF \frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} d\phi \sin(\phi-\phi_o)$$

$$-SF \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} d\phi d\frac{\dot{\phi}}{\omega} \sin(\phi-\phi_o) - SF \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} d\phi \cos(\phi-\phi_o)$$

For small amplitude of vibration $\sin(\phi-\phi_o) \simeq 0$ and $\cos(\phi-\phi_o) \simeq 1.0$ hence the incremental force acting along the radial axis may be written as:-

$$dF_R = -SF\{\frac{\partial f_R}{\partial \varepsilon} d\varepsilon + \frac{\partial f_R}{\partial \phi} d\phi + f_T\, d\phi + \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} + \frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} + \frac{1}{2}\frac{\partial^2 f_R}{\partial \varepsilon^2} d\varepsilon^2$$

$$+ \frac{1}{2}\frac{\partial^2 f_R}{\partial \phi^2} d\phi^2 - \frac{f_R}{2} d\phi^2 + \frac{1}{2}\frac{\partial f_T}{\partial \phi} d\phi^2 + \frac{1}{2}\frac{\partial f_T}{\partial \phi} d\phi^2 + \frac{1}{2}\frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}}{\omega}^2} d\frac{\dot{\varepsilon}^2}{\omega}$$

$$+ \frac{1}{2}\frac{\partial^2 f_R}{\partial \frac{\dot{\phi}}{\omega}^2} d\frac{\dot{\phi}^2}{\omega} + \frac{\partial^2 f_R}{\partial \phi \partial \varepsilon} d\phi d\varepsilon + \frac{\partial f_T}{\partial \varepsilon} d\phi d\varepsilon + \frac{\partial^2 f_R}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega}$$

$$+ \frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} + \frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}}{\omega} \partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} d\frac{\dot{\phi}}{\omega} + \frac{\partial^2 f_R}{\partial \varepsilon \partial \frac{\dot{\phi}}{\omega}} d\varepsilon d\frac{\dot{\phi}}{\omega} + \frac{\partial^2 f_R}{\partial \varepsilon \frac{\partial \dot{\varepsilon}}{\partial \omega}} d\varepsilon d\frac{\dot{\varepsilon}}{\omega}$$

$$+ \frac{\partial^2 f_R}{\partial \phi \partial \frac{\dot{\phi}}{\omega}} d\phi d\frac{\dot{\phi}}{\omega} + \frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}} d\phi d\frac{\dot{\phi}}{\omega} \} \qquad \text{(A-1)}$$

(II) Incremental forces acting along the tangential axis

$$\frac{\partial F}{\partial \varepsilon} d\varepsilon = -SF \frac{\partial f_R}{\partial \varepsilon} d\varepsilon \sin(\phi-\phi_o) + SF \frac{\partial f_T}{\partial \varepsilon} d\varepsilon \cos(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \phi} d\phi = -SF \frac{\partial f_R}{\partial \phi} d\phi \sin(\phi-\phi_o) - SF\, f_R\, d\phi \cos(\phi-\phi_o)$$

$$+SF \frac{\partial f_T}{\partial \phi} d\phi \cos(\phi-\phi_o) - SF\, f_T\, d\phi \sin(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} = -SF \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial F}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} = -SF \frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}} d\frac{\dot{\phi}}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \varepsilon^2} d\varepsilon^2 = -SF \frac{\partial^2 f_R}{\partial \varepsilon^2} d\phi^2 \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial \varepsilon^2} d\varepsilon^2 \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \phi^2} d\phi^2 = -SF \frac{\partial^2 f_R}{\partial \phi^2} d\phi^2 \sin(\phi-\phi_o) - SF \frac{\partial f_R}{\partial \phi} d\phi^2 \cos(\phi-\phi_o)$$

$$-SF \frac{\partial f_R}{\partial \phi} d\phi^2 \cos(\phi-\phi_o) + SF\, f_R\, d\phi^2 \sin(\phi-\phi_o)$$

$$+SF \frac{\partial^2 f_T}{\partial \phi^2} d\phi^2 \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \phi} d\phi^2 \sin(\phi-\phi_o)$$

$$-SF \frac{\partial f_T}{\partial \phi} d\phi^2 \sin(\phi-\phi_o) - SF\, f_T\, d\phi^2 \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} = -SF \frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}^2}{\omega}} d\frac{\dot{\varepsilon}^2}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} = -SF \frac{\partial^2 f_R}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}^2}{\omega}} d\frac{\dot{\phi}^2}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \varepsilon \partial \phi} d\varepsilon d\phi = -SF \frac{\partial^2 f_R}{\partial \varepsilon \partial \phi} d\varepsilon d\phi \sin(\phi-\phi_o) - SF \frac{\partial f_R}{\partial \varepsilon} d\varepsilon d\phi \cos(\phi-\phi_o)$$

$$+SF \frac{\partial^2 f_T}{\partial \varepsilon \partial \phi} d\varepsilon d\phi \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \varepsilon} d\varepsilon d\phi \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} = -SF \frac{\partial^2 f_R}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \sin(\phi-\phi_o) - SF \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o)$$

$$+SF \frac{\partial^2 f_T}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} d\phi d\frac{\dot{\varepsilon}}{\omega} \sin(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\frac{\dot\varepsilon}{\omega}\partial\frac{\dot\phi}{\omega}} d\frac{\dot\varepsilon}{\omega} d\frac{\dot\phi}{\omega} = -SF \frac{\partial^2 f_R}{\partial\frac{\dot\varepsilon}{\omega}\partial\frac{\dot\phi}{\omega}} d\frac{\dot\varepsilon}{\omega} d\frac{\dot\phi}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial\frac{\dot\varepsilon}{\omega}\partial\frac{\dot\phi}{\omega}} d\frac{\dot\varepsilon}{\omega} d\frac{\dot\phi}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\frac{\dot\phi}{\omega}\partial\varepsilon} d\frac{\dot\phi}{\omega} d\varepsilon = -SF \frac{\partial^2 f_R}{\partial\frac{\dot\phi}{\omega}\partial\varepsilon} d\frac{\dot\phi}{\omega} d\varepsilon \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial\frac{\dot\phi}{\omega}\partial\varepsilon} d\frac{\dot\phi}{\omega} d\varepsilon \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\varepsilon\partial\frac{\dot\varepsilon}{\omega}} d\varepsilon d\frac{\dot\varepsilon}{\omega} = -SF \frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot\varepsilon}{\omega}} d\varepsilon d\frac{\dot\varepsilon}{\omega} \sin(\phi-\phi_o) + SF \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot\varepsilon}{\omega}} d\varepsilon d\frac{\dot\varepsilon}{\omega} \cos(\phi-\phi_o)$$

$$\frac{\partial^2 F}{\partial\phi\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} = -SF \frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} \sin(\phi-\phi_o) - SF \frac{\partial f_R}{\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} \cos(\phi-\phi_o)$$

$$+ SF \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} \cos(\phi-\phi_o) - SF \frac{\partial f_T}{\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} \sin(\phi-\phi_o)$$

Again $\sin(\phi-\phi_o) \simeq 0$ and $\cos(\phi-\phi_o) \simeq 1.0$ hence the incremental force acting along the tangential axis may be written as:-

$$dF_T = -SF \{- \frac{\partial f_T}{\partial\varepsilon} d\varepsilon + f_R\, d\phi - \frac{\partial f_T}{\partial\phi} d\phi - \frac{\partial f_T}{\partial\frac{\dot\varepsilon}{\omega}} d\frac{\dot\varepsilon}{\omega} - \frac{\partial f_T}{\partial\frac{\dot\phi}{\omega}} d\frac{\dot\phi}{\omega}$$

$$- \frac{1}{2}\frac{\partial^2 f_T}{\partial\varepsilon^2} d\varepsilon^2 + \frac{1}{2}\frac{\partial f_R}{\partial\phi} d\phi^2 + \frac{1}{2}\frac{\partial f_R}{\partial\phi} d\phi^2 - \frac{1}{2}\frac{\partial^2 f_T}{\partial\phi^2} d\phi^2$$

$$+ \frac{f_T}{2} d\phi^2 - \frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot\varepsilon}{\omega}^2} d\frac{\dot\varepsilon}{\omega}^2 - \frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot\phi}{\omega}^2} d\frac{\dot\phi}{\omega}^2 + \frac{\partial f_R}{\partial\varepsilon} d\phi d\varepsilon$$

$$- \frac{\partial^2 f_T}{\partial\varepsilon\partial\phi} d\varepsilon d\phi - \frac{\partial f_R}{\partial\frac{\dot\varepsilon}{\omega}} d\phi\, d\frac{\dot\varepsilon}{\omega} - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot\varepsilon}{\omega}} d\phi d\frac{\dot\varepsilon}{\omega} - \frac{\partial^2 f_T}{\partial\frac{\dot\varepsilon}{\omega}\partial\frac{\dot\phi}{\omega}} d\frac{\dot\varepsilon}{\omega} d\frac{\dot\phi}{\omega}$$

$$- \frac{\partial^2 f_T}{\partial\frac{\dot\phi}{\omega}\partial\varepsilon} d\frac{\dot\phi}{\omega} d\varepsilon - \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot\varepsilon}{\omega}} d\varepsilon d\frac{\dot\varepsilon}{\omega} + \frac{\partial f_R}{\partial\frac{\dot\phi}{\omega}} + \frac{\partial f_R}{\partial\frac{\dot\phi}{\omega}} d\frac{\dot\phi}{\omega}\, d\phi - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot\phi}{\omega}} d\phi d\frac{\dot\phi}{\omega} \}$$

Equations (A-1) and (A-2) give the increase in the hydrodynamic force in the radial and tangential directions respectively, for a change in journal position and rate of change of position. For convenience it would be better to consider the magnitude of these forces in the horizontal and vertical directions i.e. x and y axis measured positive as illustrated in figure (3.3).

It can be shown that the components of the radial and tangential film forces in the horizontal and vertical axes may be written thus:-

$$F_x = F_R \sin\phi + F_T \cos\phi$$

$$F_y = F_R \cos\phi - F_T \sin\phi$$

Also the displacements of the journal in the same coordinate axes may be written in terms of the static polar coordinates thus:-

$$dx = c\varepsilon \cos\phi d\phi + cd\varepsilon \sin\phi$$

$$dy = -c\varepsilon \sin\phi d\phi + cd\varepsilon \cos\phi$$

To obtain polar incremental displacements, in terms of cartesian coordinates displacements the inverse of the above equation may be obtained by using Cramer's method thus giving:

$$d\phi = \frac{dx \cos\phi - dy \sin\phi}{c\varepsilon}$$

$$d\varepsilon = \frac{dy \cos\phi + dx \sin\phi}{c}$$

and $$d\frac{\dot{\phi}}{\omega} = \frac{d\frac{\dot{x}}{\omega} \cos\phi - d\frac{\dot{y}}{\omega} \sin\phi}{c\varepsilon}$$

$$d\frac{\dot{\varepsilon}}{\omega} = \frac{d\frac{\dot{x}}{\omega}\sin\phi + d\frac{\dot{y}}{\omega}\cos\phi}{c}$$

$$d\phi^2 = \frac{1}{(c\varepsilon)^2}[dx^2\cos^2\phi + dy^2\sin^2\phi - dxdy\sin2\phi]$$

$$d\varepsilon^2 = \frac{1}{c^2}[dx^2\sin^2\phi + dy^2\cos^2\phi + dxdy\sin2\phi]$$

$$d\frac{\dot{\phi}^2}{\omega} = \frac{1}{(c\varepsilon)^2}[d\frac{\dot{x}^2}{\omega}\cos^2\phi + d\frac{\dot{y}^2}{\omega}\sin^2\phi - d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}\sin2\phi]$$

$$d\frac{\dot{\varepsilon}^2}{\omega} = \frac{1}{c^2}[d\frac{\dot{x}^2}{\omega}\sin^2\phi + d\frac{\dot{y}^2}{\omega}\cos^2\phi - d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}\sin2\phi]$$

$$d\varepsilon d\phi = \frac{1}{c^2\varepsilon}[(dx^2-dy^2)\frac{1}{2}\sin2\phi + dxdy(\cos^2\phi - \sin^2\phi)]$$

$$d\phi d\frac{\dot{\varepsilon}}{\omega} = \frac{1}{c^2\varepsilon}[\frac{1}{2}(dxd\frac{\dot{x}}{\omega} - dyd\frac{\dot{y}}{\omega})\sin2\phi + dxd\frac{\dot{y}}{\omega}\cos^2\phi - dyd\frac{\dot{x}}{\omega}\sin^2\phi]$$

$$d\frac{\dot{\varepsilon}}{\omega}d\frac{\dot{\phi}}{\omega} = \frac{1}{c^2\varepsilon}[\frac{1}{2}(d\frac{\dot{x}^2}{\omega} - d\frac{\dot{y}^2}{\omega})\sin2\phi + d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}(\cos^2\phi - \sin^2\phi)]$$

$$d\frac{\dot{\phi}}{\omega}d\varepsilon = \frac{1}{c^2\varepsilon}[\frac{1}{2}(d\frac{\dot{x}}{\omega}dx - d\frac{\dot{y}}{\omega}dy)\sin2\phi + d\frac{\dot{x}}{\omega}dy\cos^2\phi - d\frac{\dot{y}}{\omega}dx\sin^2\phi]$$

$$d\varepsilon d\frac{\dot{\varepsilon}}{\omega} = \frac{1}{c^2}[\frac{1}{2}(dyd\frac{\dot{x}}{\omega} + dxd\frac{\dot{y}}{\omega})\sin2\phi + dyd\frac{\dot{y}}{\omega}\cos^2\phi + dxd\frac{\dot{x}}{\omega}\sin^2\phi]$$

$$d\phi d\frac{\dot{\phi}}{\omega} = \frac{1}{(c\varepsilon)^2}[dxd\frac{\dot{x}}{\omega}\cos^2\phi + dyd\frac{\dot{y}}{\omega}\sin^2\phi - \frac{1}{2}(dxd\frac{\dot{y}}{\omega} + dyd\frac{\dot{x}}{\omega})\sin2\phi]$$

Substituting the above derivatives into equations (A-1) and (A-2) gives the incremental forces acting along the horizontal and vertical axes:-

$$dF_x = -SF\{\frac{\partial f_R}{\partial \varepsilon}\frac{1}{c}(dy\cos\phi+dx\sin\phi)\sin\phi + (\frac{\partial f_R}{\partial \phi} + f_T)\frac{1}{c\varepsilon}(dx\cos\phi - dy\sin\phi)\sin\phi$$

$$+ \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}}\frac{1}{c}(d\frac{\dot{x}}{\omega}\sin\phi + d\frac{\dot{y}}{\omega}\cos\phi)\sin\phi + \frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}}\frac{1}{c\varepsilon}(d\frac{\dot{x}}{\omega}\cos\phi - d\frac{\dot{y}}{\omega}\sin\phi)\sin\phi$$

$$+ \frac{1}{2}\frac{\partial^2 f_R}{\partial \varepsilon^2}\frac{1}{c^2}(dx^2\sin^2\phi + dy^2\cos^2\phi + 2dxdy\sin\phi\cos\phi)\sin\phi$$

$$+ (\frac{1}{2}\frac{\partial^2 f_R}{\partial \phi^2} - \frac{f_R}{2} + \frac{\partial f_T}{\partial \phi})\frac{1}{(c\varepsilon)^2}(dx^2\cos^2\phi + dy^2\sin^2\phi - 2dxdy\sin\phi\cos\phi)\sin\phi$$

$$+ \frac{1}{2}\frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}}{\omega}^2}\frac{1}{c^2}(d\frac{\dot{x}}{\omega}^2\sin^2\phi + d\frac{\dot{y}}{\omega}^2\cos^2\phi + 2d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}\sin\phi\cos\phi)\sin\phi$$

$$+ \frac{1}{2}\frac{\partial^2 f_R}{\partial \frac{\dot{\phi}}{\omega}^2}\frac{1}{(c\varepsilon)^2}(d\frac{\dot{x}}{\omega}^2\cos^2\phi + d\frac{\dot{y}}{\omega}^2\sin^2\phi - 2d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}\sin\phi\cos\phi)\sin\phi$$

$$+ (\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} + \frac{\partial f_T}{\partial\varepsilon})\frac{1}{c^2\varepsilon}[(dx^2 - dy^2)\sin\phi\cos\phi + dydx(\cos^2\phi - \sin^2\phi)]\sin\phi$$

$$+ (\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}})\frac{1}{c^2\varepsilon}[(dxd\frac{\dot{x}}{\omega} - dyd\frac{\dot{y}}{\omega})\sin\phi\cos\phi - dyd\frac{\dot{x}}{\omega}\sin^2\phi + dxd\frac{\dot{y}}{\omega}\cos^2\phi]\sin\phi$$

$$+ \frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\frac{1}{c^2\varepsilon}[(d\frac{\dot{x}}{\omega}^2 - d\frac{\dot{y}}{\omega}^2)\sin\phi\cos\phi + (\cos^2\phi - \sin^2\phi)d\frac{\dot{x}}{\omega}d\frac{\dot{y}}{\omega}]\sin\phi$$

$$+ \frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\frac{1}{c^2\varepsilon}[dyd\frac{\dot{x}}{\omega}\cos^2\phi - dxd\frac{\dot{y}}{\omega}\sin^2\phi + (dxd\frac{\dot{x}}{\omega} - dyd\frac{\dot{y}}{\omega})\cos\phi\sin\phi]\sin\phi$$

$$+ \frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}}\frac{1}{c^2}[dxd\frac{\dot{x}}{\omega}\sin^2\phi + dyd\frac{\dot{y}}{\omega}\cos^2\phi + (dyd\frac{\dot{x}}{\omega} + dxd\frac{\dot{y}}{\omega})\sin\phi\cos\phi]\sin\phi$$

$$+ (\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}})\frac{1}{(c\varepsilon)^2}[dyd\frac{\dot{y}}{\omega}\sin^2\phi + dxd\frac{\dot{x}}{\omega}\cos^2\phi - (+dxd\frac{\dot{y}}{\omega} + dyd\frac{\dot{x}}{\omega})\sin\phi\cos\phi]\sin\phi$$

$$+ \frac{\partial f_T}{\partial\varepsilon}\frac{1}{c}(dy\cos\phi + dx\sin\phi)\cos\phi + (f_R + \frac{\partial f_T}{\partial\phi})\frac{1}{c\varepsilon}(dx\cos\phi - dy\sin\phi)\cos\phi$$

$$- \frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} \frac{1}{c} (d\frac{\dot{x}}{\omega} \sin\phi + d\frac{\dot{y}}{\omega} \cos\phi) \cos\phi - \frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}} \frac{1}{c\varepsilon} (d\frac{\dot{x}}{\omega} \cos\phi - d\frac{\dot{y}}{\omega} \sin\phi) \cos\phi$$

$$- \frac{1}{2} \frac{\partial^2 f_T}{\partial \varepsilon^2} \frac{1}{c^2} (dx^2 \sin^2\phi + dy^2 \cos^2\phi + 2dxdy\sin\phi\cos\phi)\cos\phi$$

$$+ (\frac{\partial f_R}{\partial \phi} - \frac{1}{2} \frac{\partial^2 f_T}{\partial \phi^2} + \frac{f_T}{2}) \frac{1}{(c\varepsilon)^2} (dx^2\cos^2\phi + dy^2\sin^2\phi - 2dxdy\sin\phi\cos\phi)\cos\phi$$

$$- \frac{1}{2} \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}}{\omega}^2} \frac{1}{c^2} (d\frac{\dot{x}}{\omega}^2 \sin^2\phi + d\frac{\dot{y}}{\omega}^2 \cos^2\phi + 2d\frac{\dot{x}}{\omega} d\frac{\dot{y}}{\omega} \sin\phi\cos\phi)\cos\phi$$

$$- \frac{1}{2} \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}}{\omega}^2} \frac{1}{(c\varepsilon)^2} (d\frac{\dot{x}}{\omega}^2 \cos^2\phi + d\frac{\dot{y}}{\omega}^2 \sin^2\phi - 2d\frac{\dot{x}}{\omega} d\frac{\dot{y}}{\omega} \sin\phi\cos\phi)\cos\phi$$

$$+ (\frac{\partial f_R}{\partial \varepsilon} - \frac{\partial^2 f_T}{\partial \varepsilon \partial \phi}) \frac{1}{c^2 \varepsilon} [(dx^2 - dy^2)\sin\phi\cos\phi + dxdy(\cos^2\phi - \sin^2\phi)]\cos\phi$$

$$+ (\frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} - \frac{\partial^2 f_T}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}}) \frac{1}{c^2 \varepsilon} [(dx\, d\frac{\dot{x}}{\omega} - dy\, d\frac{\dot{y}}{\omega})\sin\phi\cos\phi - dyd\frac{\dot{x}}{\omega}\sin^2\phi + dx\, d\frac{\dot{y}}{\omega}\cos^2\phi]\cos\phi$$

$$- \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}}{\omega} \partial \frac{\dot{\phi}}{\omega}} \frac{1}{c^2 \varepsilon} [(d\frac{\dot{x}}{\omega}^2 - d\frac{\dot{y}}{\omega}^2)\sin\phi\cos\phi + (-\sin^2\phi + \cos^2\phi) d\frac{\dot{x}}{\omega} d\frac{\dot{y}}{\omega}]\cos\phi$$

$$- \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}}{\omega} \partial \varepsilon} \frac{1}{c^2 \varepsilon} [dyd\frac{\dot{x}}{\omega} \cos^2\phi - dxd\frac{\dot{y}}{\omega} \sin^2\phi + (dxd\frac{\dot{x}}{\omega} - dyd\frac{\dot{y}}{\omega})\cos\phi\sin\phi]\cos\phi$$

$$- \frac{\partial^2 f_T}{\partial\varepsilon \partial\frac{\dot{\varepsilon}}{\omega}} \frac{1}{c^2} [dx d\frac{\dot{x}}{\omega} \sin^2\phi + dy d\frac{\dot{y}}{\omega} \cos^2\phi + (dy d\frac{\dot{x}}{\omega} + dx d\frac{\dot{y}}{\omega}) \sin\phi\cos\phi] \cos\phi$$

$$(\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}) \frac{1}{(c\varepsilon)^2} [dy d\frac{\dot{y}}{\omega} \sin^2\phi + dx d\frac{\dot{x}}{\omega} \cos^2\phi - (dx d\frac{\dot{y}}{\omega} - dy d\frac{\dot{x}}{\omega}) \sin\phi\cos\phi] \cos\phi$$

(A-3)

The above equation may be separated into components of force acting along the horizontal axis, produced by their respective displacements and velocities thus:-

Incremental force due to displacement dx

$$= - \frac{\lambda\omega}{c} [\frac{\partial f_R}{\partial\varepsilon} \sin^2\phi + (f_R - \frac{\partial f_T}{\partial\phi}) \frac{1}{\varepsilon} \cos^2\phi + (\frac{1}{\varepsilon} \frac{\partial f_R}{\partial\phi} + \frac{f_T}{\varepsilon} - \frac{\partial f_T}{\partial\varepsilon}) \sin\phi\cos\phi](dx)$$

Incremental force due to displacement dy

$$= - \frac{\lambda\omega}{c} [-(\frac{\partial f_R}{\partial\phi} + f_T) \frac{1}{\varepsilon} \sin^2\phi - \frac{\partial f_T}{\partial\varepsilon} \cos^2\phi + (\frac{1}{\varepsilon} \frac{\partial f_T}{\partial\phi} - \frac{f_R}{\varepsilon} + \frac{\partial f_R}{\partial\varepsilon}) \sin\phi\cos\phi](dy)$$

Incremental force due to velocity $d\frac{\dot{x}}{\omega}$

$$= - \frac{\lambda\omega}{c} [\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} \sin^2\phi - \frac{\partial f_T}{\varepsilon\partial\frac{\dot{\phi}}{\omega}} \cos^2\phi + (\frac{1}{\varepsilon} \frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} - \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}}) \sin\phi\cos\phi](d\frac{\dot{x}}{\omega})$$

Incremental force due to velocity $d\frac{\dot{y}}{\omega}$

$$= - \frac{\lambda\omega}{c} [- \frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} \frac{1}{\varepsilon} \sin^2\phi - \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} \cos^2\phi + (\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon} \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}}) \sin\phi\cos\phi](d\frac{\dot{y}}{\omega})$$

Incremental force due to displacement dx^2

$$= -\frac{\lambda\omega}{c^2}\left[\frac{1}{2}\frac{\partial^2 f_R}{\partial\varepsilon^2}\sin^3\phi + \left(\frac{\partial f_R}{\partial\phi} - \frac{1}{2}\frac{\partial^2 f_T}{\partial\phi^2} + \frac{f_T}{2}\right)\frac{1}{\varepsilon^2}\cos^3\phi\right.$$

$$+ \left(\frac{1}{2\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi^2} - \frac{f_R}{2\varepsilon^2} + \frac{\partial f_T}{\varepsilon^2\partial\phi} + \frac{\partial f_R}{\varepsilon\partial\varepsilon} - \frac{\partial^2 f_R}{\varepsilon\partial\varepsilon\partial\phi}\right)\cos^2\phi\,\sin\phi$$

$$\left. + \left(\frac{\partial^2 f_R}{\varepsilon\partial\phi\partial\varepsilon} + \frac{\partial f_T}{\varepsilon\partial\varepsilon} - \frac{1}{2}\frac{\partial^2 f_T}{\partial\varepsilon^2}\right)\sin^2\phi\,\cos\phi\right](dx)^2$$

Incremental force due to displacement dy^2

$$= -\frac{\lambda\omega}{c^2}\left[\left(\frac{1}{2}\frac{\partial^2 f_R}{\partial\phi^2} - \frac{f_R}{2} + \frac{\partial f_T}{\partial\phi}\right)\frac{1}{\varepsilon^2}\sin^3\phi - \frac{1}{2}\frac{\partial^2 f_T}{\partial\varepsilon^2}\cos^3\phi\right.$$

$$+\left(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\varepsilon} + \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\phi} - \frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi^2} + \frac{f_T}{2\varepsilon^2}\right)\sin^2\phi\,\cos\phi$$

$$\left. + \left(\frac{1}{2}\frac{\partial^2 f_R}{\partial\varepsilon^2} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\varepsilon\partial\phi}\right)\cos^2\phi\,\sin\phi\right](dy^2)$$

Incremental force due to displacements dxdy

$$= -\frac{\lambda\omega}{c^2}\left[\left(-\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} - \frac{\partial f_T}{\partial\varepsilon}\right)\frac{1}{\varepsilon}\sin^3\phi + \left(\frac{\partial f_R}{\partial\varepsilon} - \frac{\partial^2 f_T}{\partial\varepsilon\partial\phi}\right)\frac{1}{\varepsilon}\cos^3\phi\right.$$

$$+ \left(\frac{\partial^2 f_R}{\partial\varepsilon^2} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi^2} + \frac{f_R}{\varepsilon^2} - \frac{2}{\varepsilon^2}\frac{\partial f_T}{\partial\phi} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\varepsilon\partial\phi}\right)\sin^2\phi\cos\phi$$

$$\left. + \left(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\varepsilon} - \frac{\partial^2 f_T}{\partial\varepsilon^2} - \frac{2}{\varepsilon^2}\frac{\partial f_R}{\partial\phi} + \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi^2} - \frac{f_T}{\varepsilon^2}\right)\cos^2\phi\,\sin\phi\right]dxdy$$

Inremental force due to velocity $(d\frac{\dot{x}}{\omega})^2$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{1}{2}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}^2}{\omega}}\sin^3\phi - \frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}^2}{\omega}}\cos^3\phi$$

$$+\Big(\frac{1}{2\varepsilon^2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}^2}{\omega}} - \frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\frac{1}{\varepsilon}\Big)\cos^2\phi\,\sin\phi$$

$$+\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}^2}{\omega}}\Big)\sin^2\phi\,\cos\phi\Big]\,(d\frac{\dot{x}}{\omega})^2$$

Incremental force due to velocity $(d\frac{\dot{y}}{\omega})^2$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{1}{2\varepsilon^2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}^2}{\omega}}\sin^3\phi - \frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}^2}{\omega}}\cos^3\phi$$

$$+\Big(\frac{1}{2}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}^2}{\omega}} + \frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\frac{1}{\varepsilon}\Big)\cos^2\phi\,\sin\phi$$

$$-\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}}\Big)\sin^2\phi\,\cos\phi\Big](d\frac{\dot{y}}{\omega})^2$$

Incremental force due to velocities $d\frac{\dot{x}}{\omega}\,d\frac{\dot{y}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\sin^3\phi - \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\cos^3\phi$$

$$+\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} + \frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}^2}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}}\Big)\sin^2\phi\,\cos\phi$$

$$+\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} - \frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}^2}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}}\Big)\cos^2\phi\,\sin\phi\Big](d\frac{\dot{x}}{\omega}\,d\frac{\dot{y}}{\omega})$$

Incremental force due to displacement/velocity $dx\ d\frac{\dot{x}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}}\sin^3\phi + \frac{1}{\varepsilon^2}\Big(\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\Big)\cos^3\phi$$

$$+\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} - \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}}\Big)\sin^2\phi\cos\phi$$

$$+\Big(\frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}}$$

$$-\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\Big)\cos^2\phi\sin\phi\Big](dx\ d\frac{\dot{x}}{\omega})$$

Incremental force due to displacement/velocity $dx\ d\frac{\dot{y}}{\omega}$

$$-\frac{\lambda\omega}{c^2}\Big[-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\sin^3\phi + \frac{1}{\varepsilon}\Big(\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}}\Big)\cos^3\phi$$

$$+\Big(\frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\Big)\sin^2\phi\cos\phi$$

$$+\Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}}$$

$$+\frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\Big)\cos^2\phi\sin\phi\Big\}(dx\,d\frac{\dot{y}}{\omega})$$

Incremental force due to displacement/velocity $dy\ d\frac{\dot{y}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{1}{\varepsilon^2}\Big(\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}}\Big)\sin^3\phi - \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}}\cos^3\phi$$

$$+ \Big(\frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\Big)\cos^2\phi\sin\phi$$

$$+ \Big(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}}$$

$$- \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\Big)\sin^2\phi\cos\phi\Big]\Big(dy\ d\frac{\dot{y}}{\omega}\Big)$$

Incremental force due to displacement/velocity $dy\ d\frac{\dot{x}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[\Big(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}}\Big)\sin^3\phi - \frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\frac{1}{\varepsilon}\cos^3\phi$$

$$+ \Big(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} - \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\Big)\cos^2\phi\sin\phi$$

$$+ \Big(\frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}}$$

$$+ \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}}\Big)\sin^2\phi\cos\phi\Big]\Big(dy\ d\frac{\dot{x}}{\omega}\Big)$$

where $\lambda = \dfrac{SF}{\omega}$

$S = \dfrac{\mu N}{P} \left(\dfrac{R}{C}\right)^2$

F = Force acting on journal or reactive weight on bearing.

Also the hydrodynamic film force acting along the Y axis is given by:-

$$dFy = -\lambda\omega \Big[\frac{\partial f_R}{\partial \varepsilon} \frac{1}{c}(dy\cos\phi + dx\sin\phi)\cos\phi + \left(\frac{\partial f_R}{\partial \phi} + f_T\right) \frac{1}{c\varepsilon}(dx\cos\phi - dy\sin\phi)\cos\phi$$

$$+ \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} \frac{1}{c} \left(d\frac{\dot{x}}{\omega} \sin\phi + d\frac{\dot{y}}{\omega} \cos\phi\right)\cos\phi + \frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}} \frac{1}{c\varepsilon} \left(d\frac{\dot{x}}{\omega} \cos\phi - d\frac{\dot{y}}{\omega} \sin\phi\right)\cos\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_R}{\partial \varepsilon^2} \frac{1}{c^2} (dx^2\sin^2\phi + dy^2\cos^2\phi + 2dxdy\sin\phi\cos\phi)\cos\phi$$

$$+ \left(\frac{1}{2} \frac{\partial^2 f_R}{\partial \phi^2} - \frac{f_R}{2} + \frac{\partial f_T}{\partial \phi}\right) \frac{1}{(c\varepsilon)^2}(dx^2\cos^2\phi + dy^2\sin^2\phi - 2dxdy\sin\phi\cos\phi)\cos\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_R}{\partial \frac{\dot{\varepsilon}}{\omega}^2} \frac{1}{c^2} \left(d\frac{\dot{x}}{\omega}^2 \sin^2\phi + d\frac{\dot{y}}{\omega}^2 \cos^2\phi + 2d\frac{\dot{x}}{\omega} d\frac{\dot{y}}{\omega} \sin\phi\cos\phi\right)\cos\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_R}{\partial \frac{\dot{\phi}}{\omega}^2} \frac{1}{(c\varepsilon)^2}\left(d\frac{\dot{x}}{\omega}^2 \cos^2\phi + d\frac{\dot{y}}{\omega}^2 \sin^2\phi - 2d\frac{\dot{x}}{\omega} d\frac{\dot{y}}{\omega} \sin\phi\cos\phi\right)\cos\phi$$

$$+ \left(\frac{\partial^2 f_R}{\partial \phi \partial \varepsilon} + \frac{\partial f_T}{\partial \varepsilon}\right) \frac{1}{c^2\varepsilon} [dx^2 - dy^2)\sin\phi\cos\phi + dxdy(\cos^2\phi - \sin^2\phi)]\cos\phi$$

$$+ \left(\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}}\right) \frac{1}{c^2\varepsilon} \left[\left(dx\,d\frac{\dot{x}}{\omega} - dy\,d\frac{\dot{y}}{\omega}\right)\sin\phi\cos\phi - dy\,d\frac{\dot{x}}{\omega}\sin^2\phi + dx\,d\frac{\dot{y}}{\omega}\cos^2\phi\right]\cos\phi$$

$$+ \frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}} \frac{1}{c^2\varepsilon} \left[\left(d\frac{\dot{x}}{\omega}^2 - d\frac{\dot{y}}{\omega}^2\right)\sin\phi\cos\phi + (\cos^2\phi - \sin^2\phi)\,d\frac{\dot{x}}{\omega}\,d\frac{\dot{y}}{\omega}\right]\cos\phi$$

$$+ \frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} \frac{1}{c^2\varepsilon} \left[dy\,d\frac{\dot{x}}{\omega}\cos^2\phi - dx\,d\frac{\dot{y}}{\omega}\sin^2\phi + \left(dx\,d\frac{\dot{x}}{\omega} - dy\,d\frac{\dot{y}}{\omega}\right)\sin\phi\cos\phi\right]\cos\phi$$

$$+ \frac{\partial^2 f_R}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}} \frac{1}{c^2} \left[dx\,d\frac{\dot{x}}{\omega}\sin^2\phi + dy\,d\frac{\dot{y}}{\omega}\cos^2\phi + \left(dy\,d\frac{\dot{x}}{\omega} + dx\,d\frac{\dot{y}}{\omega}\right)\sin\phi\cos\phi\right]\cos\phi$$

$$+ \left(\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\phi}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}}\right) \frac{1}{(c\varepsilon)^2} \left[dy\,d\frac{\dot{y}}{\omega}\sin^2\phi + dx\,d\frac{\dot{x}}{\omega}\cos^2\phi - \left(dx\,d\frac{\dot{y}}{\omega} + dy\,d\frac{\dot{x}}{\omega}\right)\sin\phi\cos\phi\right]\cos\phi$$

$$+ \frac{\partial f_T}{\partial\varepsilon} \frac{1}{c} (dy\cos\phi + dx\sin\phi)\sin\phi - \left(f_R - \frac{\partial f_T}{\partial\phi}\right) \frac{1}{c\varepsilon} (dx\cos\phi - dy\sin\phi)\sin\phi$$

$$+ \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} \frac{1}{c} \left(d\frac{\dot{x}}{\omega}\sin\phi + d\frac{\dot{y}}{\omega}\cos\phi\right)\sin\phi + \frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} \frac{1}{c\varepsilon} \left(d\frac{\dot{x}}{\omega}\cos\phi - d\frac{\dot{y}}{\omega}\sin\phi\right)\sin\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_T}{\partial\varepsilon^2} \frac{1}{c^2} (dx^2\sin^2\phi + dy^2\cos^2\phi + 2dx\,dy\sin\phi\cos\phi)\sin\phi$$

$$- \left(\frac{\partial f_R}{\partial\phi} - \frac{1}{2}\frac{\partial^2 f_T}{\partial\phi^2} + \frac{f_T}{2}\right) \frac{1}{(c\varepsilon)^2} (dx^2\cos^2\phi + dy^2\sin^2\phi - 2dx\,dy\sin\phi\cos\phi)\sin\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}}{\omega}^2} \frac{1}{c^2} \left(d\frac{\dot{x}}{\omega}^2 \sin^2\phi + d\frac{\dot{y}}{\omega}^2 \cos^2\phi + 2d\frac{\dot{x}}{\omega}\, d\frac{\dot{y}}{\omega} \sin\phi\cos\phi\right)\sin\phi$$

$$+ \frac{1}{2} \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}}{\omega}^2} \frac{1}{(c\varepsilon)^2} \left(d\frac{\dot{x}}{\omega}^2 \cos^2\phi + d\frac{\dot{y}}{\omega}^2 \sin^2\phi - 2d\frac{\dot{x}}{\omega}\, d\frac{\dot{y}}{\omega} \sin\phi\cos\phi\right)\sin\phi$$

$$- \left(\frac{\partial f_R}{\partial \varepsilon} - \frac{\partial^2 f_T}{\partial \varepsilon \partial \phi}\right) \frac{1}{c^2 \varepsilon} \left[(dx^2 - dy^2)\sin\phi\cos\phi + dxdy(\cos^2\phi - \sin^2\phi)\right]\sin\phi$$

$$- \left(\frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} - \frac{\partial^2 f_T}{\partial \phi \partial \frac{\dot{\varepsilon}}{\omega}}\right) \frac{1}{c^2 \varepsilon} \left[\left(dx d\frac{\dot{x}}{\omega} - dy d\frac{\dot{y}}{\omega}\right)\sin\phi\cos\phi - dy d\frac{\dot{x}}{\omega}\sin^2\phi + dx d\frac{\dot{y}}{\omega}\cos^2\phi\right]\sin\phi$$

$$+ \frac{\partial^2 f_T}{\partial \frac{\dot{\varepsilon}}{\omega} \partial \frac{\dot{\phi}}{\omega}} \frac{1}{c^2 \varepsilon} \left[\left(d\frac{\dot{x}}{\omega}^2 - d\frac{\dot{y}}{\omega}^2\right)\sin\phi\cos\phi + (\cos^2\phi - \sin^2\phi) d\frac{\dot{x}}{\omega}\, d\frac{\dot{y}}{\omega}\right]\sin\phi$$

$$+ \frac{\partial^2 f_T}{\partial \frac{\dot{\phi}}{\omega} \partial \varepsilon} \frac{1}{c^2 \varepsilon}\left[dy d\frac{\dot{x}}{\omega}\cos^2\phi - dx d\frac{\dot{y}}{\omega}\sin^2\phi + \left(dx d\frac{\dot{x}}{\omega} - dy d\frac{\dot{y}}{\omega}\right)\cos\phi\sin\phi\right]\sin\phi$$

$$+ \frac{\partial^2 f_T}{\partial \varepsilon \partial \frac{\dot{\varepsilon}}{\omega}} \frac{1}{c^2} \left[dx d\frac{\dot{x}}{\omega}\sin^2\phi + dy d\frac{\dot{y}}{\omega}\cos^2\phi + \left(dy d\frac{\dot{x}}{\omega} + dx d\frac{\dot{y}}{\omega}\right)\sin\phi\cos\phi\right]\sin\phi$$

$$- \left(\frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}} - \frac{\partial^2 f_T}{\partial \phi \partial \frac{\dot{\phi}}{\omega}}\right) \frac{1}{(c\varepsilon)^2} \left[dy d\frac{\dot{y}}{\omega}\sin^2\phi + dx d\frac{\dot{x}}{\omega}\cos^2\phi - \left(dx d\frac{\dot{y}}{\omega} + dy\frac{\dot{x}}{\omega}\right)\sin\phi\cos\phi\right]\sin\phi$$

From the foregoing equations the following incremental forces acting along the Y axis may be obtained.

Incremental force due to displacement dx

$$= -\frac{\lambda\omega}{c}\left[\frac{\partial f_T}{\partial \varepsilon}\sin^2\phi + \left(\frac{\partial f_R}{\partial \phi} + f_T\right)\frac{1}{\varepsilon}\cos^2\phi + \left(\frac{\partial f_R}{\partial \varepsilon} - \frac{f_R}{\varepsilon} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial \phi}\right)\sin\phi\cos\phi\right](dx)$$

Incremental force due to displacement dy

$$= -\frac{\lambda\omega}{c}\left[\left(f_R - \frac{\partial f_T}{\partial \phi}\right)\frac{1}{\varepsilon}\sin^2\phi + \frac{\partial f_R}{\partial \varepsilon}\cos^2\phi + \left(\frac{\partial f_T}{\partial \varepsilon} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial \phi} - \frac{f_T}{\varepsilon}\right)\sin\phi\cos\phi\right](dy)$$

Incremental force due to velocity $d\frac{\dot{x}}{\omega}$

$$= -\frac{\lambda\omega}{c}\left[\frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}}\sin^2\phi + \frac{1}{\varepsilon}\frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}}\cos^2\phi + \left(\frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}}\right)\sin\phi\cos\phi\right]\left(d\frac{\dot{x}}{\omega}\right)$$

Incremental force due to velocity $d\frac{\dot{y}}{\omega}$

$$= -\frac{\lambda\omega}{c}\left[-\frac{1}{\varepsilon}\frac{\partial f_T}{\partial \frac{\dot{\phi}}{\omega}}\sin^2\phi + \frac{\partial f_R}{\partial \frac{\dot{\varepsilon}}{\omega}}\cos^2\phi + \left(\frac{\partial f_T}{\partial \frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial \frac{\dot{\phi}}{\omega}}\right)\sin\phi\cos\phi\right]\left(d\frac{\dot{y}}{\omega}\right)$$

Incremental force due to displacement dx^2

$$= -\frac{\lambda\omega}{c^2}\left[\frac{1}{2}\frac{\partial^2 f_T}{\partial \varepsilon^2}\sin^3\phi + \left(\frac{1}{2}\frac{\partial^2 f_R}{\partial \phi^2} - \frac{f_R}{2} + \frac{\partial f_T}{\partial \phi}\right)\frac{1}{\varepsilon^2}\cos^3\phi\right.$$
$$\left. + \left(\frac{1}{2}\frac{\partial^2 f_R}{\partial \varepsilon^2} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial \varepsilon} + \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial \varepsilon \partial \phi}\right)\sin^2\phi\cos\phi\right.$$

$$+\left(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\varepsilon} - \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\phi} + \frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi^2} - \frac{1}{2\varepsilon^2} f_T\right)\cos^2\phi\sin\phi\Big](dx^2)$$

Incremental force due to displacement dy^2

$$= -\frac{\lambda\omega}{c^2}\Big[\left(\frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi^2} - \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\phi} - \frac{1}{2\varepsilon^2} f_T\right)\sin^3\phi + \frac{1}{2}\frac{\partial^2 f_R}{\partial\varepsilon^2}\cos^3\phi$$

$$+\left(\frac{1}{2}\frac{\partial^2 f_T}{\partial\varepsilon^2} - \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\varepsilon}\right)\cos^2\phi\sin\phi$$

$$+\left(\frac{1}{2\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi^2} - \frac{f_R}{2\varepsilon^2} + \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\phi} + \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\varepsilon} - \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\partial\varepsilon}\right)\sin^2\phi\cos\phi\Big](dy^2)$$

Incremental force due to displacement $dxdy$

$$= -\frac{\lambda\omega}{c^2}\Big[\left(\frac{\partial f_R}{\partial\varepsilon} - \frac{\partial^2 f_T}{\partial\varepsilon\partial\phi}\right)\frac{1}{\varepsilon}\sin^3\phi + \left(\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} + \frac{\partial f_T}{\partial\varepsilon}\right)\frac{1}{\varepsilon}\cos^3\phi$$

$$+\left(\frac{\partial^2 f_T}{\partial\varepsilon^2} - \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\varepsilon} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\varepsilon} + \frac{2}{\varepsilon^2}\frac{\partial f_R}{\partial\phi} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi^2} + \frac{1}{\varepsilon^2} f_T\right)\sin^2\phi\cos\phi$$

$$+\left(\frac{\partial^2 f_R}{\partial\varepsilon^2} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi^2} + \frac{f_R}{\varepsilon^2} - \frac{2}{\varepsilon^2}\frac{\partial f_T}{\partial\phi} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\varepsilon} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\varepsilon\partial\phi}\right)\cos^2\phi\sin\phi\Big](dxdy)$$

Incremental force due to velocity $d\frac{\dot{x}}{\omega}^2$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}^2}\sin^3\phi + \frac{1}{2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}^2}\frac{1}{\varepsilon^2}\cos^3\phi + \left(\frac{1}{2}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}^2} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\right)\sin^2\phi\cos\phi$$

$$+\left(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}+\frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}^2}\right)\cos^2\phi\sin\phi\Big]\left(d\frac{\dot{x}}{\omega}^2\right)$$

Incremental force due to velocity $d\frac{\dot{y}}{\omega}^2$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}^2}\sin^3\phi+\frac{1}{2}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}^2}\cos^3\phi+\left(\frac{1}{2\varepsilon^2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}^2}-\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\right)\sin^2\phi\cos\phi$$

$$+\left(\frac{1}{2}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}^2}-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\right)\cos^2\phi\sin\phi\Big]\left(d\frac{\dot{y}}{\omega}^2\right)$$

Incremental force due to velocities $d\frac{\dot{x}}{\omega}\ d\frac{\dot{y}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[-\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\sin^3\phi+\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\cos^3\phi+\left(\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}^2}-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}-\frac{1}{2\varepsilon^2}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}^2}\right)\sin^2\phi\cos\phi$$

$$+\left(\frac{\partial^2 f_R}{\partial\frac{\dot{\varepsilon}}{\omega}^2}-\frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}^2}+\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\varepsilon}}{\omega}\partial\frac{\dot{\phi}}{\omega}}\right)\cos^2\phi\sin\phi\Big]d\frac{\dot{x}}{\omega}\ d\frac{\dot{y}}{\omega}$$

Incremental force due to displacement/velocity $dx\ d\frac{\dot{x}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[\frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}}\sin^3\phi+\left(\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}+\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}}\right)\frac{1}{\varepsilon^2}\cos^3\phi$$

$$+\left(\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}}+\frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}}+\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}-\frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}}+\frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\right)\cos^2\phi\sin\phi$$

$$+\left(\frac{\partial^2 f_R}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\right)\sin^2\phi\cos\phi\Big]\left(dx\ d\frac{\dot{x}}{\omega}\right)$$

<u>Incremental force due to displacement/velocity dy $d\frac{\dot{y}}{\omega}$</u>

$$= -\frac{\lambda\omega}{c^2}\Big[\left(\frac{\partial^2 f_T}{\partial\phi\,\partial\frac{\dot{\phi}}{\omega}} - \frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}}\right)\frac{1}{\varepsilon^2}\sin^3\phi + \frac{\partial^2 f_R}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}}\cos^3\phi$$

$$+\left(\frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\right)\sin^2\phi\cos\phi$$

$$+\left(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon} + \frac{\partial^2 f_T}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}}\right)\cos^2\phi\sin\phi\Big]\left(dy\ d\frac{\dot{y}}{\omega}\right)$$

<u>Incremental force due to displacement/velocity dx $d\frac{\dot{y}}{\omega}$</u>

$$= -\frac{\lambda\omega}{c^2}\Big[-\frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\sin^3\phi + \left(\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}}\right)\frac{1}{\varepsilon}\cos^3\phi$$

$$+\left(\frac{\partial^2 f_R}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\,\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\phi\,\partial\frac{\dot{\varepsilon}}{\omega}}\right)\cos^2\phi\sin\phi$$

$$+\left(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\varepsilon\,\partial\frac{\dot{\phi}}{\omega}} + \frac{\partial^2 f_T}{\partial\varepsilon\,\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\,\partial\frac{\dot{\phi}}{\omega}}\right)\sin^2\phi\cos\phi\Big]\left(dx\ d\frac{\dot{y}}{\omega}\right)$$

Incremental force due to displacement/velocity dy $d\frac{\dot{X}}{\omega}$

$$= -\frac{\lambda\omega}{c^2}\Big[\Big(\frac{\partial f_R}{\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}}\Big)\frac{1}{\varepsilon}\sin^3\phi + \frac{\partial^2 f_R}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\frac{1}{\varepsilon}\cos^3\phi$$

$$+\Big(-\frac{1}{\varepsilon}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon}\frac{\partial f_T}{\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{\partial^2 f_T}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} + \frac{1}{\varepsilon^2}\frac{\partial f_R}{\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_T}{\partial\phi\partial\frac{\dot{\phi}}{\omega}}\Big)\sin^2\phi\cos\phi$$

$$+\Big(\frac{\partial^2 f_R}{\partial\varepsilon\partial\frac{\dot{\varepsilon}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial^2 f_R}{\partial\phi\partial\frac{\dot{\phi}}{\omega}} - \frac{1}{\varepsilon^2}\frac{\partial f_T}{\partial\frac{\dot{\phi}}{\omega}} + \frac{1}{\varepsilon}\frac{\partial^2 f_T}{\partial\frac{\dot{\phi}}{\omega}\partial\varepsilon}\Big)\cos^2\phi\sin\phi\Big]\Big(dy\ d\frac{\dot{X}}{\omega}\Big)$$

The foregoing incremental forces acting along the X and Y axis characterize the non-dimensional force coefficients. Computer results for these coefficients are presented in Appendix B as design charts for varying amounts of misalignment, and are also given in figures (4.8) to (4.16) for a journal running parallel with the bearing bush.

APPENDIX B

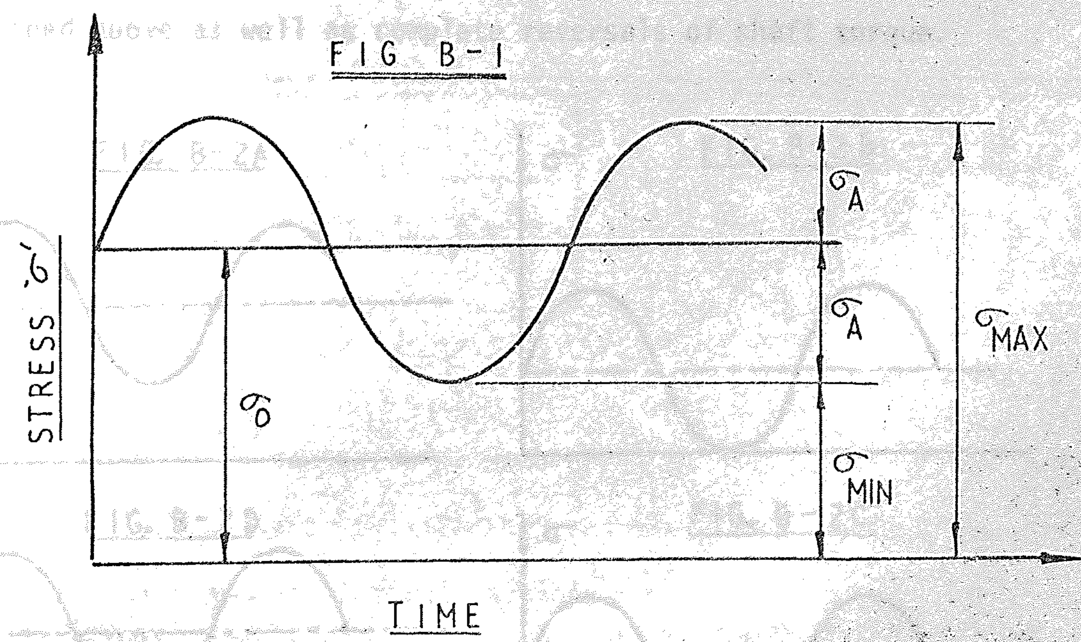
APPENDIX B

JOURNAL BEARING DESIGN AND PERFORMANCE TOGETHER WITH SAMPLE CALCULATIONS

The following information sets down a procedure for selecting the size of a journal bearing based on torque requirements etc. together with general guidance notes on acceptable oil film thickness, operating temperature, and thermal balance. Performance calculation sheets are also provided to allow the design engineer to predict both the steady state and the dynamic operating conditions.

(I) Selection of journal diameter based on torque requirements

FIG B-I



The majority of important rotor design problems involves both a fluctuating bending stress (i.e. during each revolution a point on a shaft is subjected to a compressive stress and 180 degrees later the stress changes to a tensile stress, thus a complete reversal of stress is accomplished during each shaft rotation), and secondly the shaft may also be subjected to a fluctuating torsional stress. Both these fluctuating stresses will eventually cause failure by fatiguing the steel if the stress range is great enough, and if applied a sufficient number of times. Thus when selecting the size of the journal bearing it is essential to include any cyclic variation of stress within the journal itself and accordingly choose its diameter and grade of manufacturing material.

Figure (B-1) sets out the type of stresses to be expected and figure (B-2) shows variations of (B-1) all of which may be experienced in practice when designing a rotor shaft system for a wide range of engineering applications. Figure (B-2c) represents shaft bending as described above as well as complete reversals of shaft torque.

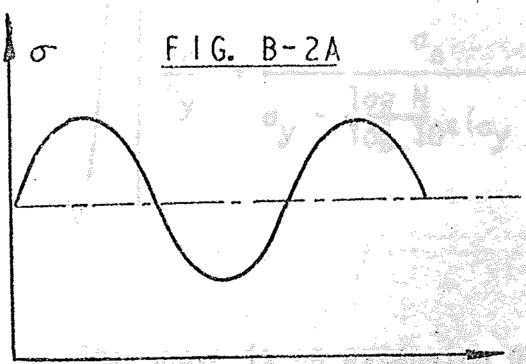


FIG. B-2A

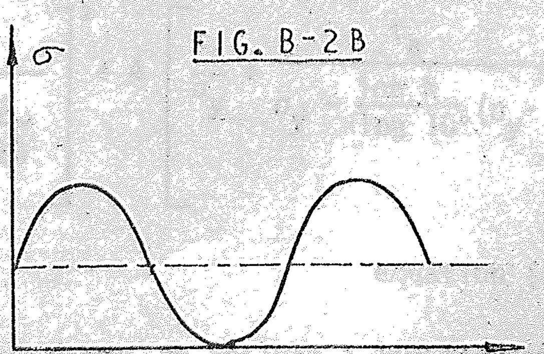


FIG. B-2B

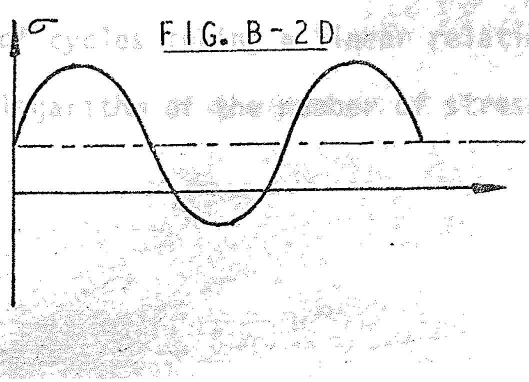


FIG. B-2D

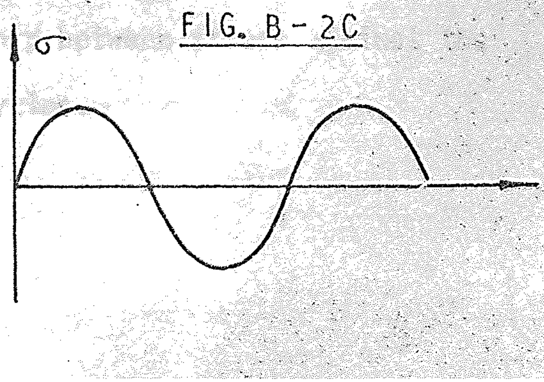


FIG. B-2C

It is usual practice, for shafts manufactured from ductile materials, to apply a stress concentration factor to the alternating component of stress, but not to the steady component.

Stress concentration factors need only be applied to shafts having an abrupt change in section, these irregularities of shape are of practical importance when designing journal bearings, because they cause variations in the stress distribution in that section of shaft continuing away from the oil throwers. Values of stress concentration factors likely to be experienced with journal bearings, are given in figures (B-3A) and (B-3B).

For a combination of steady and alternating bending stress together with a steady and alternating torsional stress, assuming the alternating components to be in phase with each other, allowed the following relationship to be derived by Soderberg(56).

FACTOR OF SAFETY BASED ON YIELD STRENGTH OF MATERIAL(μ)

$$= \frac{1.0}{\sqrt{\left[\frac{\sigma_o}{\sigma_y} + \frac{\sigma_a}{\sigma_y - \frac{\log N}{\log 10^6}\left(\sigma_y - \frac{\sigma_e}{K_{SB}}\right)}\right]^2 + 4\left[\frac{\tau_o}{\sigma_y} + \frac{\tau_a}{\sigma_y - \frac{\log N}{\log 10^6}\left(\sigma_y - \frac{\sigma_e}{K_{ST}}\right)}\right]^2}}$$

equation (B-1)

The above is an extension of Soderberg, to include a limited number of cycles taking a linear relationship between stress against the logarithm of the number of stress cycles.

where σ_o = Steady bending stress

σ_a = Alternating bending stress

σ_y = Yield strength (Tension)

σ_e = Fatigue strength or Endurance strength

τ_o = Steady torsional stress

τ_a = Alternating torsional stress

N = Number of cycles to failure

K_{SB} = Stress concentration factor (bending)

K_{ST} = Stress concentration factor (torsion)

The endurance strength, which is often referred to as the endurance limit, is dependant upon the surface finish of the material and to some extent the amount of hardness, however, for carbon steels the endurance limit is approximately 43 per cent of the ultimate tensile strength.

In the majority of cases the design of the journal does not have to take into account a limited number of stress cycles, therefore for practical purposes it would be satisfactory if the fatigue was considered for an infinite number of cycles, hence equation (B-1) may be written as follows:-

$$n = \frac{1.0}{\sqrt{\left[\frac{\sigma_o}{\sigma_y} + \frac{\sigma_a K_{SB}}{\sigma_e}\right]^2 + 4\left[\frac{\tau_o}{\sigma_y} + \frac{\tau_a K_{ST}}{\sigma_e}\right]^2}} \qquad \text{(B-2)}$$

The above equation (B-2) may be re-arranged such that for a given bending moment, together with the maximum and minimum shaft torques and a selected factor of safety, a journal diameter is computed.

$$\text{Shaft dia(m/m)} = 2.54 \times \sqrt[3]{\frac{A_n}{\sigma_y}} \qquad \text{(B-3)}$$

where $A = \sqrt{105.16M^2 + (7.41\ T_{max} - 2.86\ T_{min})^2}$

n = Factor of safety based on yield

σ_y = Yield of shaft material (Tons/in^2)

T_{max} = Maximum shaft torque (lb.ins)

T_{min} = Minimum shaft torque (lb.ins)

M = Bending Moment (lb.ins)

In deriving the above equation the following assumptions have been made:-

σ_e = 12.0 Tons/in^2

$K_{ST} = K_{SB}$ = 1.7 which is an averaged value found in practice.

Equation (B-3) will provide the size of journal, assuming of course that the rotor driving torque is transmitted through the bearing.

(II) Selection of the journal, based upon the reactive loading

The specific pressure of the journal bearing should be limited to 200 lb/in^2 (1.38 MN/m^2) for turbines and electrical rotating machines, under certain conditions this value could be increased to 250 lb/in^2 (1.73 MN/m^2) provided a good oil film is obtained under normal working conditions. Having calculated the journal size for the required torque, and provided the specific pressure is not greater than 200 lb/in^2 and not below 50 lb/in^2 (0.345 MN/m^2) for a given L/D ratio, figure (B-4) together with calculation sheet figure (B-5) will give the steady running position of the journal within the bearing bush together with the corresponding friction factor.

(III) Bearing performance calculations

The step by step calculation sheet as given in figure (B-5) should be used as described below, together with the help of the additional guidance notes. The first column (T_f) of the calculation sheet is the averaged film temperature at which the horsepower losses are to be evaluated, the second column (μ) is the oil viscosity, in centipoises, for the given temperature (T). This information may be obtained from the temperature/viscosity chart supplied by most oil manufacturers. Substituting the above information into the Sommerfeld equation in column three together with the reactive load plus the journal speed, and the geometry of the bearing, i.e. R and C, provides the Sommerfeld duty parameter, which if used with figure (B-4) allows the value of ε to be determined using the "S" curve, i.e. projection $a \rightarrow b \rightarrow c$. The value of $f(\frac{R}{C})$ in column five may be obtained from the same figure i.e. projection $c \rightarrow d$ for the eccentricity ratio (ε) previously determined in column four; also for the same value of (ε) the angle of incidence may be determined from curve (ϕ) of figure (B-4) as illustrated by the projection $c \rightarrow e$.

(IV) Minimum oil film thickness

The minimum oil film thickness which will safely separate the journal from the bearing bush, depends upon the distortion of the bush and also on the roughness of the surfaces, a further consideration is that the film thickness should be sufficiently large as to allow very small particles of foreign matter to pass through the bearing oil film, without injury to the surfaces. From the majority of bearing diameters, i.e. 4 in to 20 in dia. (100mm to 500mm) the designer should try and obtain a film thickness of 0.0015 in (0.0391 mm) or an absolute minimum of 0.00075 in (0.019 mm) for commercial babbitted bearings.

(V) Operating temperatures

Normal design operating temperatures are 60°C to 80°C; below 60°C the bearing would be under-rated, and if operating with a temperature in excess of 80°C does not allow for high ambient temperatures. The film temperature giving an economical design is 70°C and also allows a margin of safety where machines are "close coupled", possibly giving rise to load transference due to differential thermal expansion of the bearing supporting structures. The bearing white metal will allow temperatures in excess of 100°C, but at these higher temperatures oxidation of the oil will commence, and at 120°C becomes appreciable, unless synthetic oils are used.

(VI) Heat Dissipation

Method (A) Heat carried away by the oil

This method of heat extraction is by circulating oil through the bearing. The oil is supplied under pressure to the working face of the bearing. The amount of oil required to cool the bearing depends upon the inlet temperature of the oil and the expected temperature rise as it passes through the bearing. Experience shows that the temperature rise is usually in the order of half the difference between the film temperature and the oil inlet supply temperature.

$$\text{i.e. } 0.5\ (T_f - T_i)$$

T_f = Oil film temperature ^oC

T_i = Oil inlet temperature ^oC

The amount of cooling oil required to remove the heat generated within the bearing, based on a specific gravity of 0.88 and a specific heat of 0.46, is given by:-

$$G.P.M. = \frac{11.62\ (H.P.\ generated)}{(T_f - T_i)} \qquad (B-4)$$

The diameter of the oil inlet supply pipe should be chosen for the above quantity of oil, such that the flow velocity is not greater than 5 ft/sec (1.53 m/sec).

Method (B) Heat removed by natural cooling

With this method of cooling the heat generated within the oil film is transferred to the bearing pedestal by conduction through the bearing bush, and by the oil circulated by the oil rings and by side leakage. The heat then flows along different paths to the outer surface of the bearing pedestal, and then finally transferred to the surrounding air. This type of heat flow has been investigated by the writer using the electrical analogy technique, and by checking a number of machines under both test and site conditions. The rate of heat dissipated was found to be dependant on the temperature difference of the oil film and surrounding air, and the rate of movement of air in general, however, the following formula will provide a good assessment of the heat dissipated from a bearing

$$\text{H.P.dissipated} = (T_f - T_a)0.0008\,A/746 \qquad \text{(B-5)}$$

T_f = Oil film temperature ^oC

T_a = Ambient temperature of surrounding air ^oC

A = Radiation area of the pedestal (sq.in).

For small bearings all the total heat generated within the oil film may be dissipated by natural cooling. On medium size bearings the heat is removed by natural dissipation plus forced cooling by an external supply of oil, and for large bearings running at the higher speeds, all the heat generated has to be carried away by the cooling oil.

(VII) Graphical representation of the bearing thermal balance

Using the total horsepower as determined from the calculation sheet, plot the horsepower generated within the oil film wedge for various temperatures i.e. curve (1) of figure (B-6), then using equation (B-5) compute the amount of heat removed by the surrounding air, i.e. curve (2). Curve (2) may or may not cross curve (1), if it does the intersection will give the temperature at which the bearing will operate. If curve (2) does not cross curve (1), then at a temperature corresponding to 70^{0} C measure the remaining power to be removed by an external supply of cooling oil. The quantity of oil in gallons per minute may be determined by equation (B-4). The temperature of 70^{0}C is chosen as the best operating temperature, this value could be adjusted to suit any particular requirement.

(VIII) Method of determining the oil film force coefficients

If the rotor is expected to run through the critical speed, then the coefficients should be determined for two speeds below the running speed, as well as the normal speed, thus allowing a curve to be drawn giving the approximate oil film stiffness for various speeds. If, however, the critical speed is expected to be above the running speed, then the film flexibility need only be evaluated for the normal running speed of the machine. In order to demonstrate the method for evaluating the force coefficients, it would be more informative to assume the critical speed to be below the normal running speed, therefore having decided at which speeds the oil film coefficients are to be evaluated, the performance characteristics should be determined using the step by step calculation sheet given in figure (B-5), i.e. curves (4) and (5) of figure (B-6). Where these curves cross the external cooling curves (3) or (2) gives the operating temperature of the film for these reduced speeds. It should be remembered that this film temperature is for thermal equilibrium, the true film temperature will depend upon the time taken to reach the full operational speed, therefore results obtained will be on the high side, however, this method has been successfully used on heavy rotating machines.

Having obtained the expected operating temperature of the oil film, it is possible to determine the Sommerfeld number and eccentricity ratio from the calculation sheets, thus allowing the non-dimensional coefficients to be read from the charts given in this Appendix for the above determined eccentricity ratio, and also adjusted for any misalignment expected to be present in the shaft system.

(IX) Worked Example

It is desired to determine the oil film characteristics for a 33 M.W. turbo-generator having a bearing 12 in diameter by 12 in long. From experience it is known that for this size of machine the rotor passes through its first critical speed, therefore it will be necessary to compute the oil film dynamic characteristics for several speeds below the normal running speed.

The computer program available for solving critical speeds is only capable of investigating vibrations in one plane, therefore it will be necessary to contract the twenty-eight force coefficients to a single stiffness giving the weakest plane.

The starting point would be to select the speeds at which investigations are to be made, for this example 3000, 2000 and 1000 R.P.M. have been selected, where 3000 R.P.M. is the normal running speed. Using the step by step calculation sheet together with figure (B-4) will provide the total horsepower absorbed in shearing the oil for various averaged oil film temperatures, a sample calculation is given in figure (B-7) which may be plotted as shown in figure (B-8). The above procedure may be repeated for speeds 2000 R.P.M. and 1000 R.P.M., calculation sheets are not shown here, but take exactly the same form as for the speed of 3000 R.P.M.

For this size of bearing, where the horsepower generated by the oil film is as large as 70 H.P. the heat removed by the surrounding air is so small compared with this figure that it is not worth plotting, therefore all the heat generated within the bearing must be removed by an external oil supply, using equation B-4 gives

$$G.P.M. = 11.62\ (52.6)/(70-28) = 14.6$$

This quantity of oil is then supplied to the bearing irrespective of the running up speed of the rotor. The heat removed by the oil for various temperatures is shown by curve (A) and the intersection of this curve with the horsepower generated for the three selected speeds gives the expected oil film temperature, i.e.

1000 R.P.M. = 40^{o}C; 2000 R.P.M. = 57^{o}C; 3000 R.P.M. = 70^{o}C

thus for each temperature the respective Sommerfeld number and eccentricity ratio may be obtained by referring back to the calculation sheet, which for 3000 R.P.M. would be S = 0.686 and ε = 0.275.

Calculation of oil film coefficients for τ = zero

Speed = 3000 R.P.M. or 314.15 rad/sec.

Reactive force = 19000 lbs.

Sommerfeld number = 0.686

Radial clearance = 0.006 in.

Eccentricity ratio = 0.275

SW/C	=	$0.686 \times 19000/0.006$	$= 2.116 \times 10^6$
$SW\Omega/\omega C$	=	$0.686 \times 19000 \times 314.15/314.15 \times 0.006$	$= 2.116 \times 10^6$
SW/C^2	=	$0.686 \times 19000/0.006^2$	$= 3.16 \times 10^8$
$SW\Omega^2/\omega^2C^2$	=	$0.686 \times 19000 \times 314.15^2/314.15^2 \times 0.006^2$	$= 3.16 \times 10^8$
$SW\Omega/\omega C^2$	=	$0.686 \times 19000 \times 314.15/314.15 \times 0.006^2$	$= 3.16 \times 10^8$
$m\omega^2$	=	$19000 \times 314.15^2/386$	$= 4.858 \times 10^6$

Using figures (B-15) onwards allows the non-dimensional force coefficients to be obtained for the eccentricity ratio $\varepsilon = 0.275$, thus the following dimensional coefficients may be calculated.

Axx	=	1.48	$\times\ 2.166 \times 10^6$	=	3.205×10^6
Axy	=	-0.08	$\times$ "	=	-1.73×10^5
$Ax\dot{x}$	=	2.08	$\times$ "	=	4.505×10^6
$Ax\dot{y}$	=	1.6	$\times$ "	=	3.465×10^6
Axx^2	=	7.0	$\times\ 3.61 \times 10^6$	=	2.527×10^9
Axy^2	=	-2.0	$\times$ "	=	-7.22×10^8
$Axxy$	=	2.9	$\times$ "	=	1.047×10^9
$Ax\dot{x}^2$	=	0.19	$\times$ "	=	6.85×10^7
$Ax\dot{y}^2$	=	3.5	$\times$ "	=	1.264×10^9
$Ax\dot{x}\dot{y}$	=	5.7	$\times$ "	=	2.058×10^9
$Axx\dot{x}$	=	-3.3	$\times$ "	=	-1.191×10^9
$Axx\dot{y}$	=	15.3	$\times$ "	=	5.523×10^9
$Axy\dot{y}$	=	-0.3	$\times$ "	=	-1.083×10^8
$Axy\dot{x}$	=	8.6	$\times$ "	=	3.105×10^9
Ayx	=	6.8	$\times\ 2.166 \times 10^6$	=	1.473×10^7
Ayy	=	3.5	$\times$ "	=	7.581×10^6

$$A_{y\dot{x}} = 1.6 \times 2.166 \times 10^6 = 3.465 \times 10^6$$

$$A_{y\dot{y}} = 14.3 \times \text{"} = 3.079 \times 10^7$$

$$A_{yx^2} = 10.8 \times 3.61 \times 10^8 = 3.899 \times 10^9$$

$$A_{yy^2} = 3.2 \times \text{"} = 1.155 \times 10^9$$

$$A_{yxy} = 24.5 \times \text{"} = 8.845 \times 10^9$$

$$A_{y\dot{x}^2} = -0.11 \times \text{"} = -3.97 \times 10^7$$

$$A_{y\dot{y}^2} = 16.0 \times \text{"} = 5.776 \times 10^9$$

$$A_{y\dot{x}\dot{y}} = 2.5 \times \text{"} = 9.025 \times 10^8$$

$$A_{yx\dot{x}} = -31.0 \times \text{"} = -1.119 \times 10^{10}$$

$$A_{yx\dot{y}} = 31.0 \times \text{"} = 1.119 \times 10^{10}$$

$$A_{yy\dot{y}} = 62.0 \times \text{"} = 2.238 \times 10^{10}$$

$$A_{yy\dot{x}} = 33.0 \times \text{"} = 1.191 \times 10^{10}$$

The value of the exciting force, i.e. $m\omega^2 r$ is usually taken to be a maximum of 1/5 of the bearing reactive force.

The above coefficients are substituted into equations (6-4A/B/C/D) and solved according to equation (6-10B). The computer input data listing, and output results, are presented in figure (B-9) from which the displacement along the X axis may be described by the equation $x = 6.486 \sin(\omega t - 121^0)$, and along the Y axis by the equation $y = 6.421 \cos(\omega t - 81.2^0)$ which when plotted gives the whirl trajectory illustrated in figure (B-10). By noting the maximum amplitude of vibration, and the angular position of the force vector, allows the determination of stiffness coefficient along the major axis of vibration, which has the most influence in affecting the dynamic behaviour of the rotor.

Using equation (5.12) allows the Hagg and Sankey type of force coefficient for the major axis of vibration, to be evaluated as follows:

$$K_u = \frac{m\Omega^2 r}{u_0} \cos(\alpha_1 - \xi) + M\Omega^2$$

where $m\Omega^2 r = 3800$

$u_0 = 0.00075$

$\cos(\alpha_1 - \xi) = \cos(107^0)$

$M\Omega^2 = 49.2 \times 314.15^2$

giving $K_u = 3.366 \times 10^6$ lb/in.

Repeating the entire calculation for 1000 R.P.M. and 2000 R.P.M. gives:-

1000 R.P.M. = 1.89×10^6 lb/in.

2000 R.P.M. = 2.8×10^6 lb/in.

In addition to the above oil film stiffness the impedance of the bearing supporting structure has to be accounted for. Tests conducted by Parsons Peebles Limited for determining the dynamic properties of the bearing pedestals used for works testing of this 33 MW rotor, and also used in this illustrative design example, gave the following results:-

Equivalent stiffness	= 9.9×10^6 lb/in.
Equivalent damping	= 3.64×10^3 lb/in/sec.
Equivalent mass	= 40.6 lb/in/sec^2
Natural frequency	= 492 rad/sec.

Hence th supporting structure stiffness for the following speeds would be:-

1000 R.P.M. = 9.5 lb/in.

2000 R.P.M. = 8.1 lb/in.

3000 R.P.M. = 6.0 lb/in.

Therefore the equivalent stiffness of the oil film plus the bearing supporting structure is as follows:-

1000 R.P.M. = 1.58 lb/in.

2000 R.P.M. = 2.07 lb/in.

3000 R.P.M. = 2.17 lb/in.

It is now possible to present a graph of support stiffness against speed, also on the same graph the dynamic behaviour of the rotor for varying support stiffness may be presented, i.e. curves (1) and (2) of figure (B-13). The intersection of these curves with the equivalent support stiffness, curve (3), gives the expected critical frequency of the complete rotor-bearing system, which for this example is:-

N_{c1} = 1180 R.P.M. N_{c2} = 3240 R.P.M.

Actual measured critical frequencies for this rotor under test conditions gave:-

Driving end pedestal measurement

N_{c1} = 1100 R.P.M. N_{c2} = 3200 R.P.M.

Opposite driving end pedestal measurement

$$N_{c1} = 1200 \text{ R.P.M.} \qquad N_{c2} = 3060 \text{ R.P.M.}$$

If, however, the calculated critical speed for this same rotor, only considered rigid supports, the critical speeds would be:-

$$N_{c1} = 1504 \text{ R.P.M.} \qquad N_{c2} = 4228 \text{ R.P.M.}$$

The response curve recorded by Parsons Peebles for this turbo-alternator, is present in figure (B-14) and illustrates the degree of accuracy of this method together with the value of the oil film force coefficients presented in this thesis.

FIG. B—3A. STRESS CONCENTRATION FACTOR, K_{SB}, FOR THE BENDING OF A SHAFT WITH A SHOULDER FILLET.

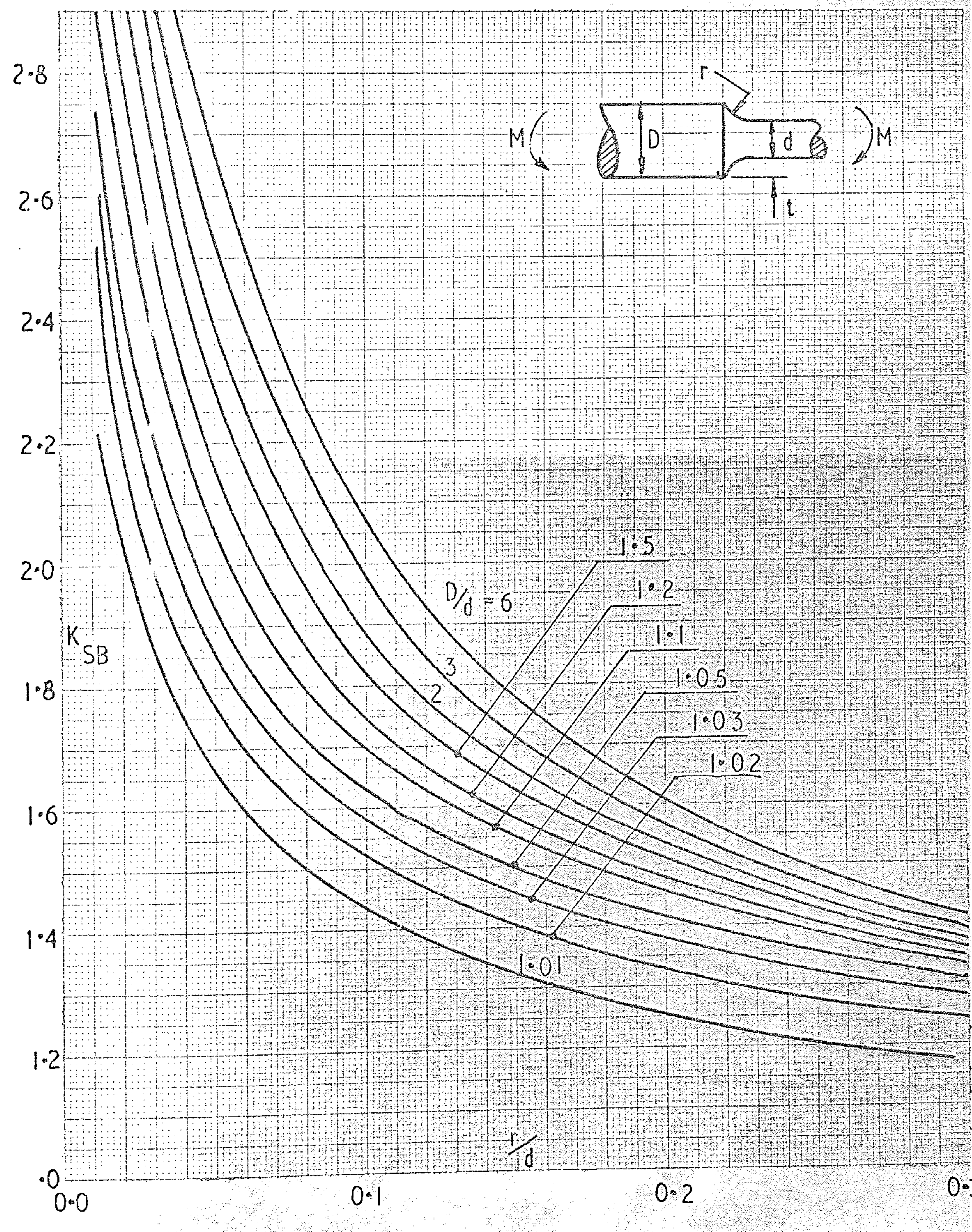
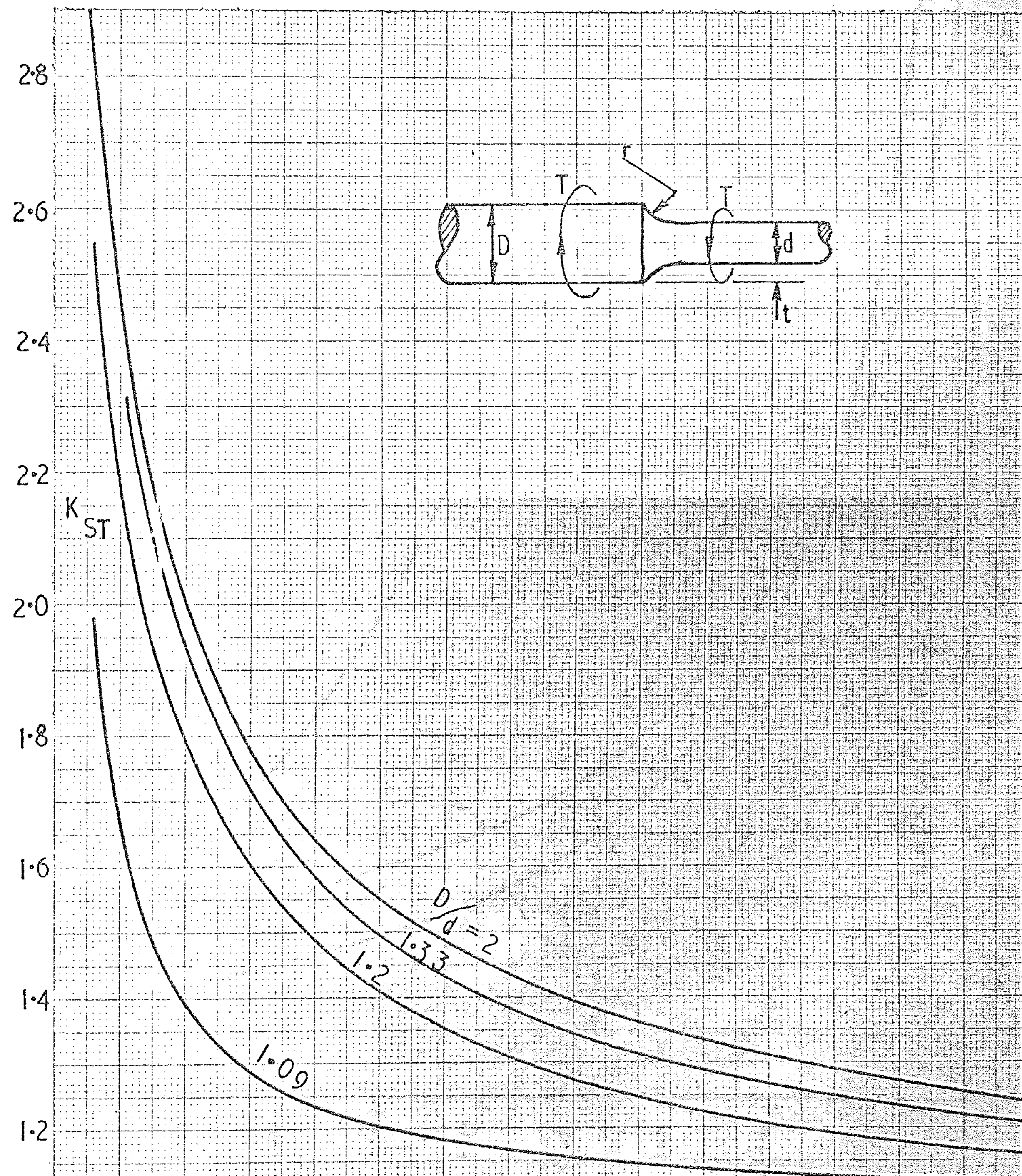


FIG. B-3B. STRESS CONCENTRATION FACTOR, K_{ST}, FOR THE TORSION OF A SHAFT WITH A SHOULDER FILLET.



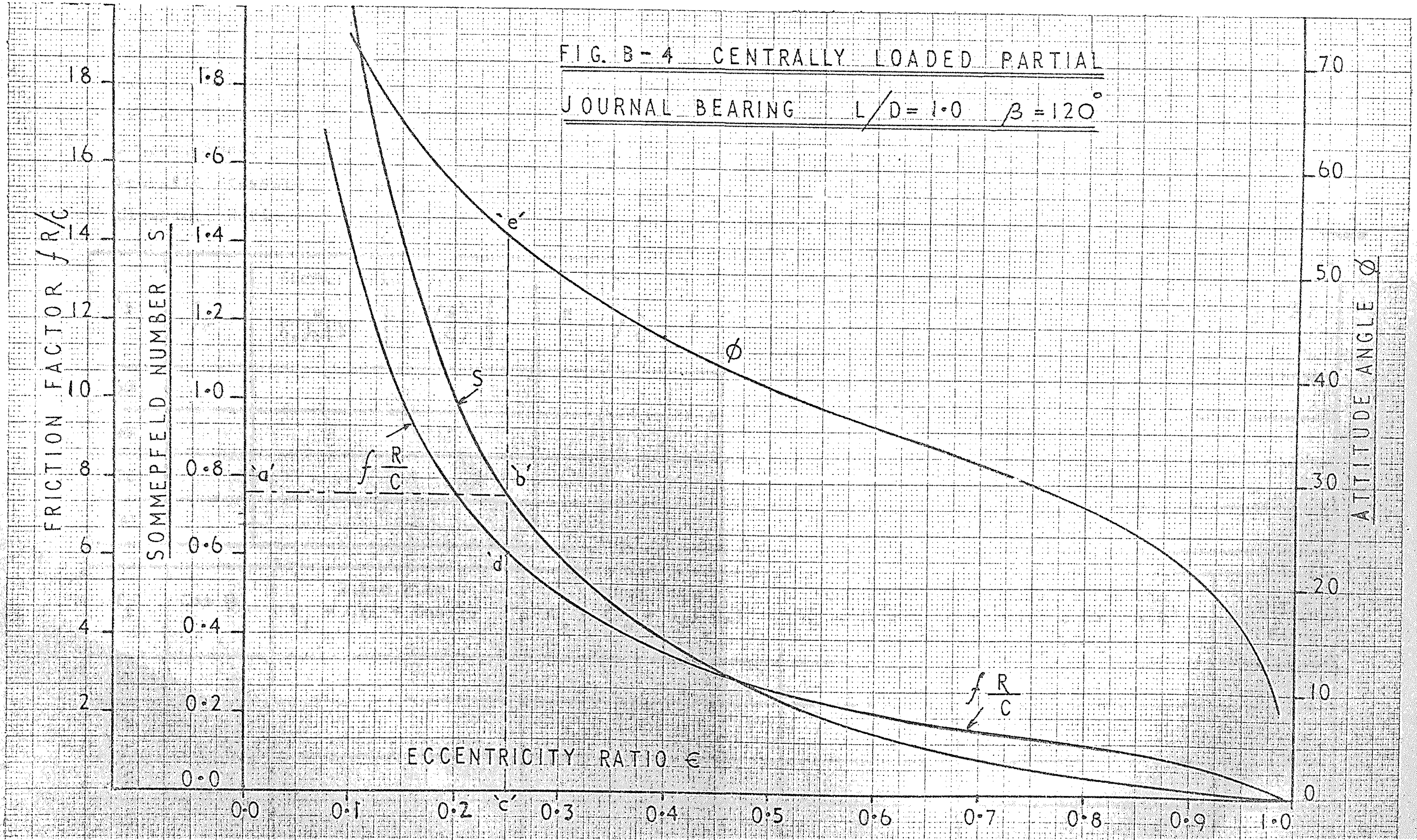


FIG. B-4 CENTRALLY LOADED PARTIAL
JOURNAL BEARING L/D = 1·0 β = 120°
FRICTION FACTOR f R/C
SOMMEPFELD NUMBER S
ATTITUDE ANGLE Ø
ECCENTRICITY RATIO ∈
18
16
14
12
10
8
6
4
2
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0·0
70
60
50
40
30
20
10
0
0·0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
a′
b′
c′
d′
e′
S
Ø
f R/C

Journal Radius	(R) =		in.	Bottom Brg. Arc	(β) =		Deg.
Journal Axial Length	(L) =		in.	Top Brg. Arc	(β_1) =		Deg.
Bearings Load	(W) =		lbs.	Ambient Temp.	(Ta) =		^oC
Radial Clearance	(C) =		in.	Oil Inlet Temp.	(Ti) =		^oC
Journal Speed	(N) =		R.P.S.	Ave. Film Temp.	(Tf) =		^oC
Specific Pressure	(P) =		lb/in^2	Oil Viscosity (μ)	at temp T_f		
Clearance Ratio	$\frac{R}{C}$ =						

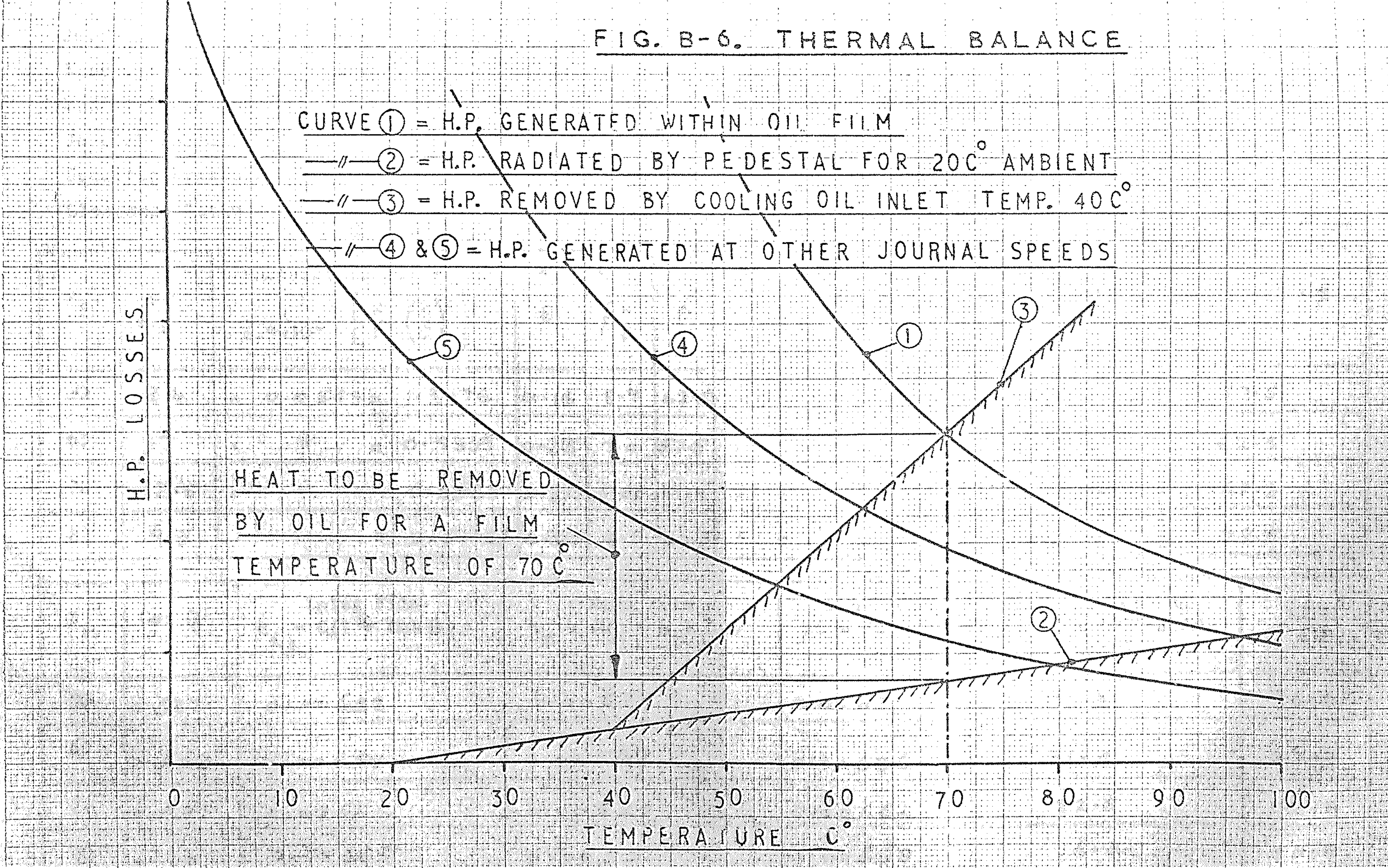
* Obtained from Design Charts

FIG B-5

T_f	$\frac{\mu}{c.p}$	Sommerfield No. = $\frac{\mu}{6.9 \times 10^6}\left[\frac{N}{P}\left(\frac{R}{C}\right)^2\right]$	* ϵ	* $f\frac{R}{C}$	* ϕ	H.P. in btm.Pad $= f\frac{R}{C} \times \frac{CPLNR}{525}$	Minimum Film $h_o = c(1-\epsilon)$	$\theta = 180-(\frac{\beta}{2}+\phi)$
50								
60								
70								
80								

T_f	Cos θ	Inlet Film $h_i = C(1 + \cos\theta)$	Mean Film $h_m = (h_o + h_i)\ 0.5$	Top Film $ht = 2C - h_m$	H.P. in Top Pad $= 1.512 \times 10^{-11} N^2 R^3 \times \mu\beta_1(L/ht)$	TOTAL H.P.
50						
60						
70						
80						

FIG. B-6. THERMAL BALANCE



Journal Radius	(R) =	6.0	in.		Bottom Brg. Arc	(β)	=	120	Deg.
Journal Axial Length	(L) =	12.0	in.		Top Brg. Arc	(β_1)	=	120	Deg.
Bearings Load	(W) =	19000	lbs.		Ambient Temp.	(Ta)	=	20	°C
Radial Clearance	(C) =	0.006	in.		Oil Inlet Temp.	(Ti)	=	28	°C
Journal Speed	(N) =	50.0	R.P.S.		Ave. Film Temp.	(Tf)	=		°C
Specific Pressure	(P) =	132	lb/in^2		Oil Viscosity (μ)	at temp T_f			
Clearance Ratio	$\frac{R}{C}$ =								

* Obtained from Design Charts

FIG. B-7

T_f	μ c.p	Sommerfield No. = $\frac{\mu}{6.9 \times 10^6}\left[\frac{N}{P}\left(\frac{R}{C}\right)^2\right]$	* ϵ	* $f\frac{R}{C}$	* ϕ	H.P. in btm. Pad $= f\frac{R}{C} \times \frac{CPLNR}{525}$	Minimum Film $h_o = c(1-\epsilon)$	$\theta = 180-(\frac{\beta}{2}+\phi)$
50	24	x 0.0549 = 1.3176	0.16	8.9	62.5	x 5.43 = 48.3	0.00504	57.5
60	17	x " = 0.9333	0.215	7.0	57.5	x " = 38.01	0.00470	62.5
70	12.5	x " = 0.6863	0.275	5.55	51.8	x " = 30.13	0.00435	68.2
80	9.3	x " = 0.51057	0.335	4.5	47.5	x " = 24.44	0.00399	72.5

T_f	Cos θ	Inlet Film $h_i = C(1 + \cos\theta)$	Mean Film $h_m = (h_o + h_i)\ 0.5$	Top Film $ht = 2C - h_m$	H.P. in Top Pad $= 1.512 \times 10^{-11} N^2 R^3 \times \mu \beta_1 (L/ht)$	TOTAL H.P.
50	0.537	0.00652	0.00579	0.00621	45.3	93.6
60	0.462	0.00658	0.00564	0.00636	31.4	69.4
70	0.371	0.0066	0.00548	0.00652	22.5	52.6
80	0.301	0.00665	0.00532	0.00668	16.4	40.8

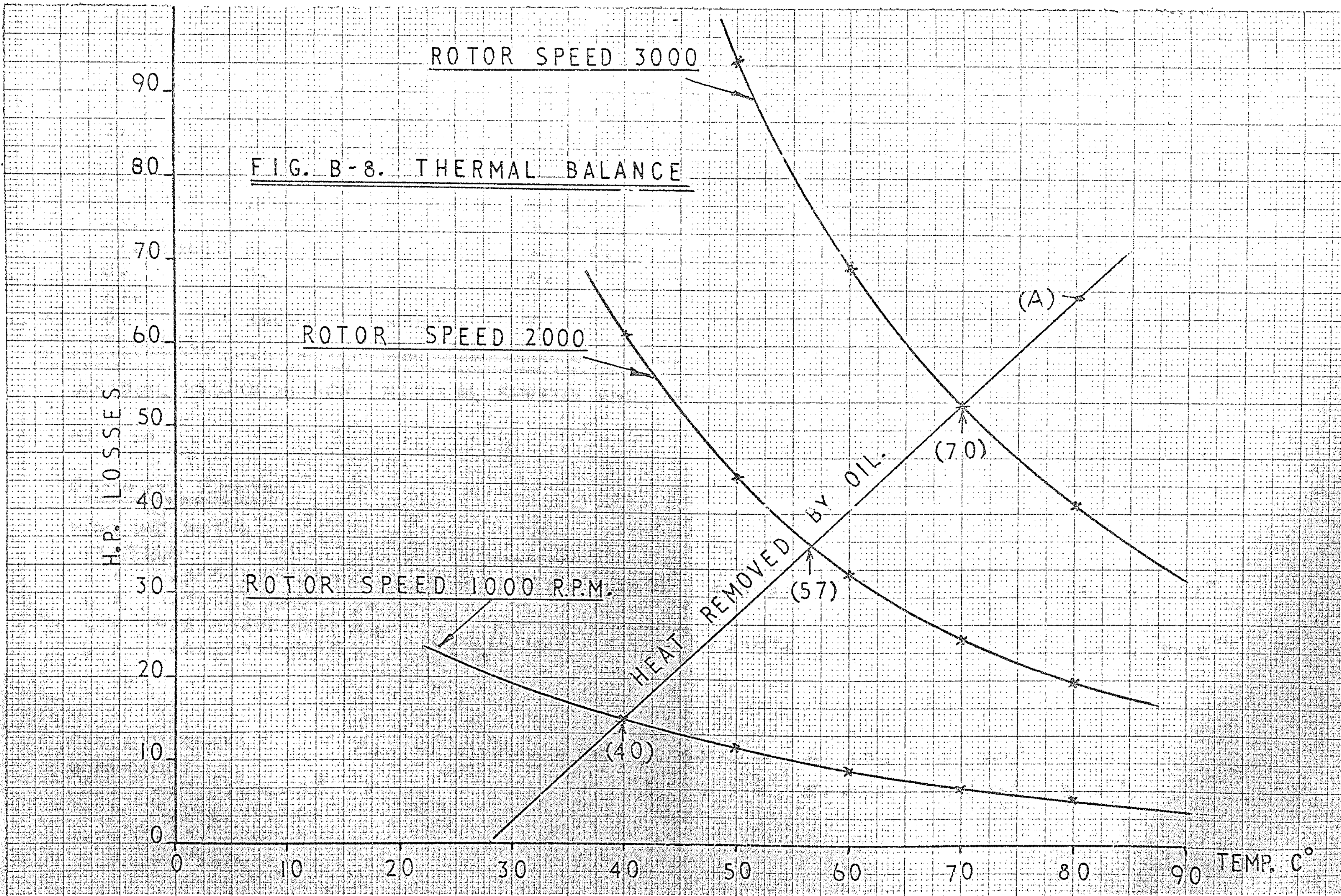


FIG. B-8. THERMAL BALANCE

INPUT DATA.

*DATA LISTING FOR JOB: FJ1412 01402

21-7-72 JOB NO 5333.

MI ISA

2.527E9	-7.224E8	6.85E7	1.264E9	-1.047E9	1.191E9	-5.523E9	3.105E9	} EQN.-1
-1.083E8	2.058E9	1.653E6	-1.73E5	4.505E6	3.465E6	0.		
3.899E9	1.155E9	-3.97E7	5.776E9	-8.845E9	1.119E10	-1.119E10	1.191E10	} EQN.-2
2.238E10	9.025E8	-1.473E7	2.723E6	3.465E6	3.079E7	3800.		
6.85E7	1.264E9	2.527E9	-7.22E8	-2.058E9	-1.191E9	3.105E9	-5.523E9	} EQN.-3
1.083E8	1.047E9	4.505E6	-3.465E6	-1.653E6	-1.73E5	3800.		
-3.97E7	5.776E9	3.899E9	1.155E9	-9.025E8	-1.119E10	1.191E10	-1.119E10	} EQN.-4.
-2.238E10	8.845E9	3.465E6	-3.079E7	1.473E7	2.723E6	0.		
0.	0.	0.	0.					
20.								
0.	3800.	-3800.	0.					

*ENDFILE

JOURNAL RESPONSE FOR CRITICAL SPEEDS AND UNBALANCE

MI ISA 21-7-72 JOB NO 5333.

OUTPUT DATA.

FORWARD WHIRL
SOLUTION

A = .555735E-03 = X_i $\therefore |X| = \sqrt{5.557^2 + 3.3463^2} = 6.486$; $\tan \frac{5.557}{-3.346} = -1.66 = 121°$

B = .781117E-04 = Y_R

C = -.334629E-03 = X_R

D = .501555E-03 = Y_i $\therefore |Y| = \sqrt{0.7811^2 + 5.015^2} = 5.075$; $\tan \frac{5.015}{0.7811} = 6.421 = 81.2°$.

REVERSE WHIRL
SOLUTION

A = -.736109E-03
B = .286279E-03
C = .391939E-03
D = -.256245E-03

Eqn of motion along X axis = $6.486 \sin(\omega t - 121°)$

" " " " Y " = $5.075 \cos(\omega t - 81.2°)$.

FIG. B-9

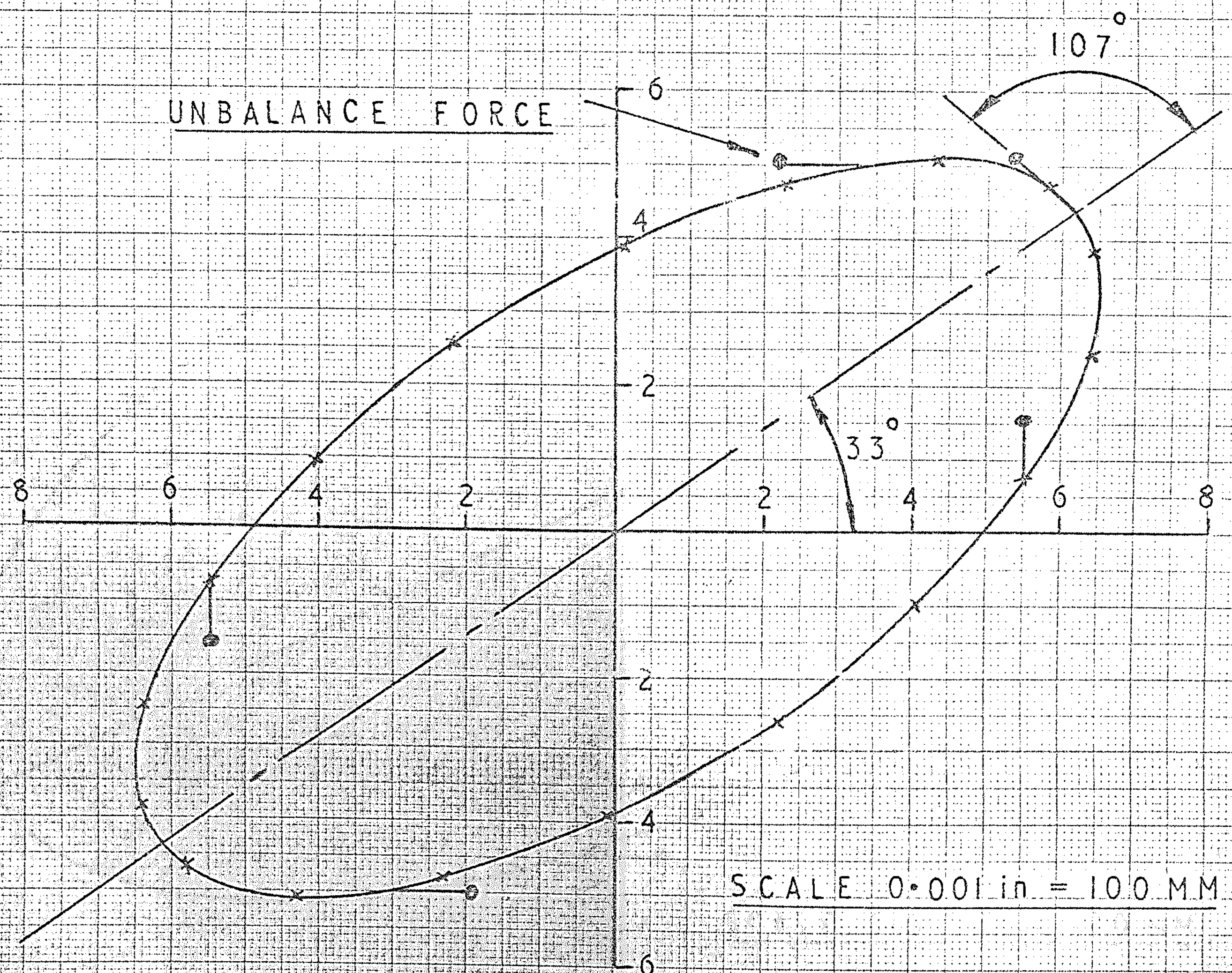


FIG. B-10.
COMPUTER RESULTS SHOWING WHIRL TRAJECTORY
JOUPNAL SPEED = 3000 R.P.M.

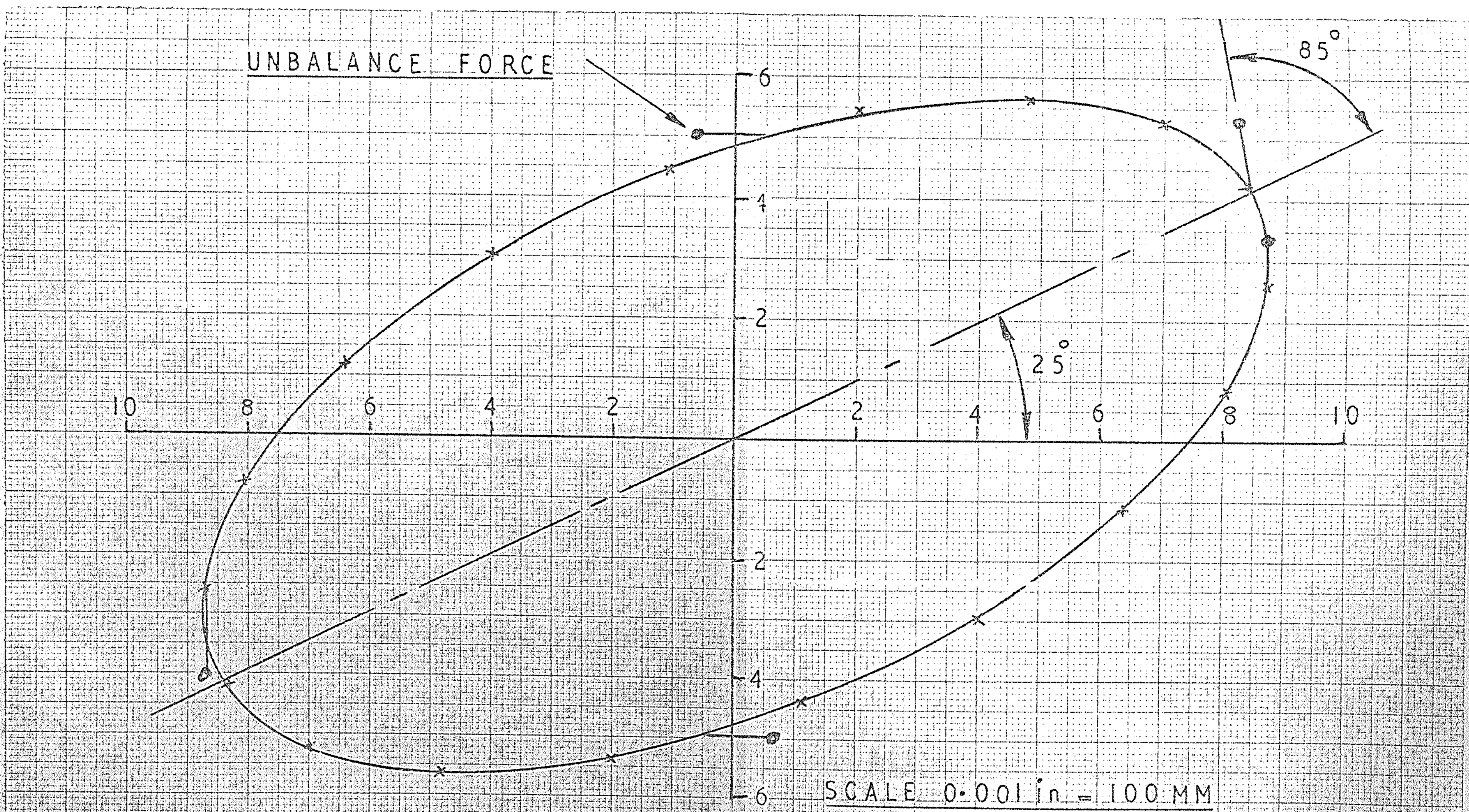


FIG. B-11.

COMPUTER RESULTS SHOWING WHIRL TRAJECTORY

JOURNAL SPEED = 2000 R.P.M.

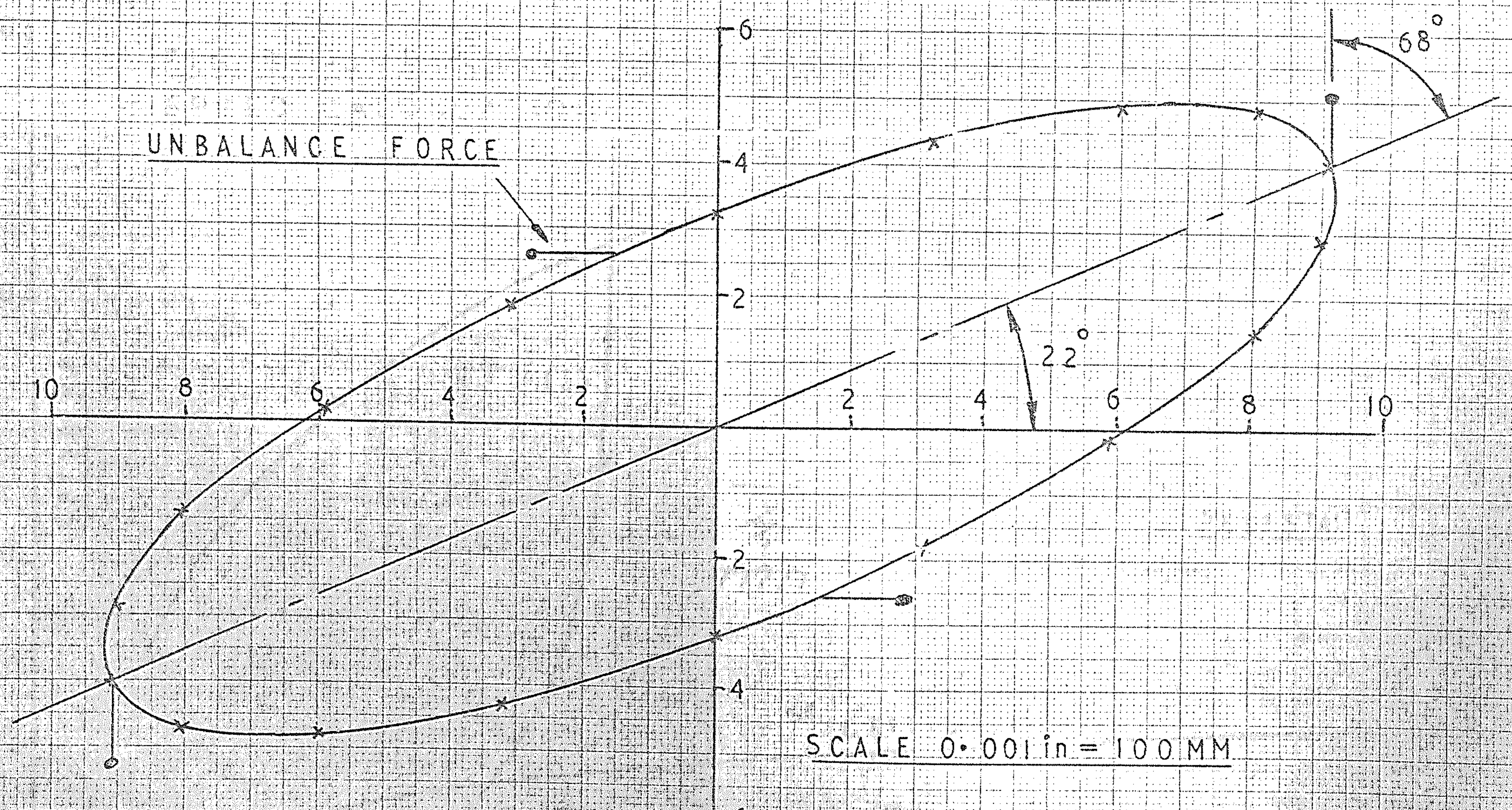
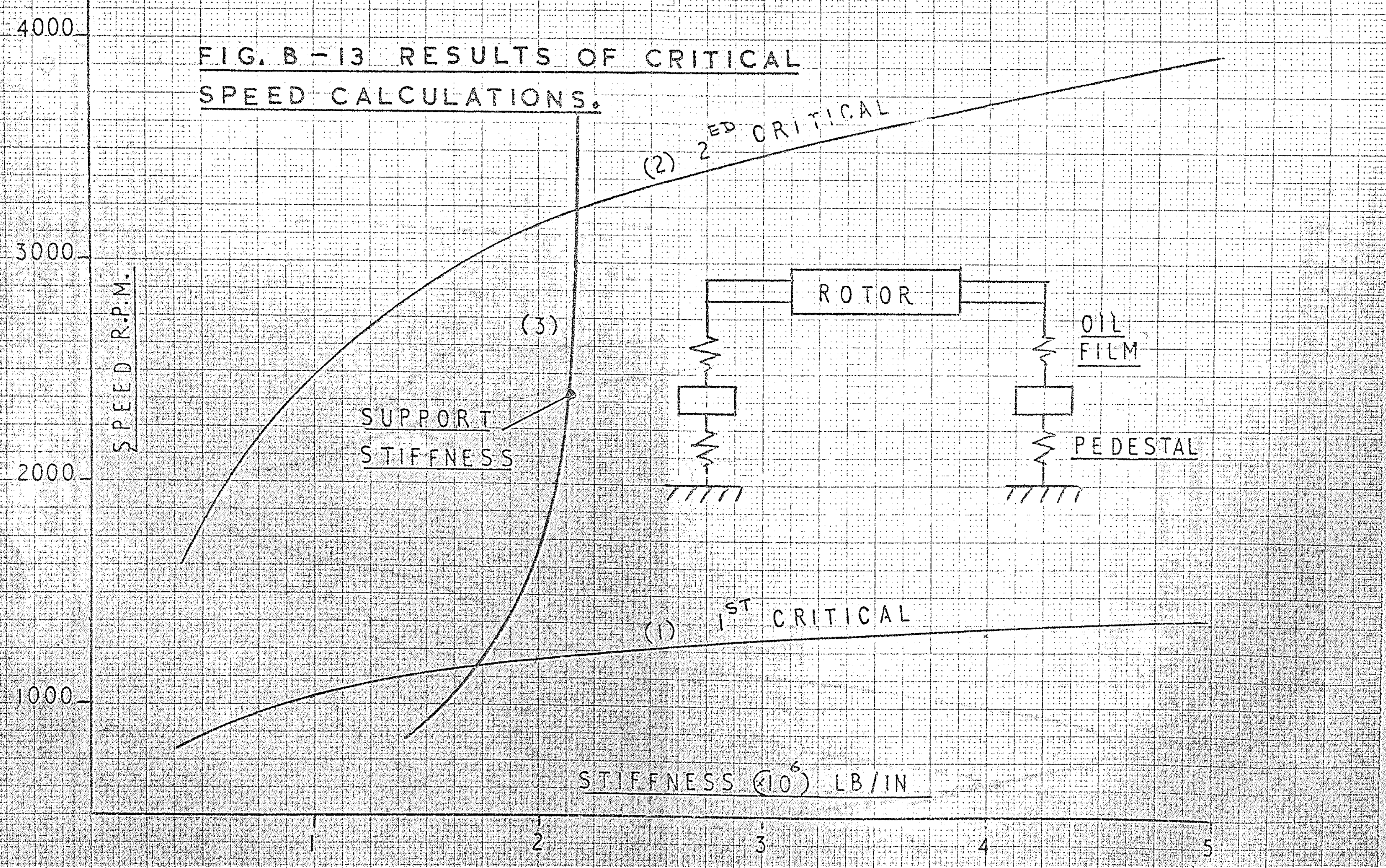


FIG. B-12.
COMPUTER RESULTS SHOWING WHIRL TRAJECTORY
JOURNAL SPEED = 1000 R.P.M.

FIG. B – 13 RESULTS OF CRITICAL SPEED CALCULATIONS.



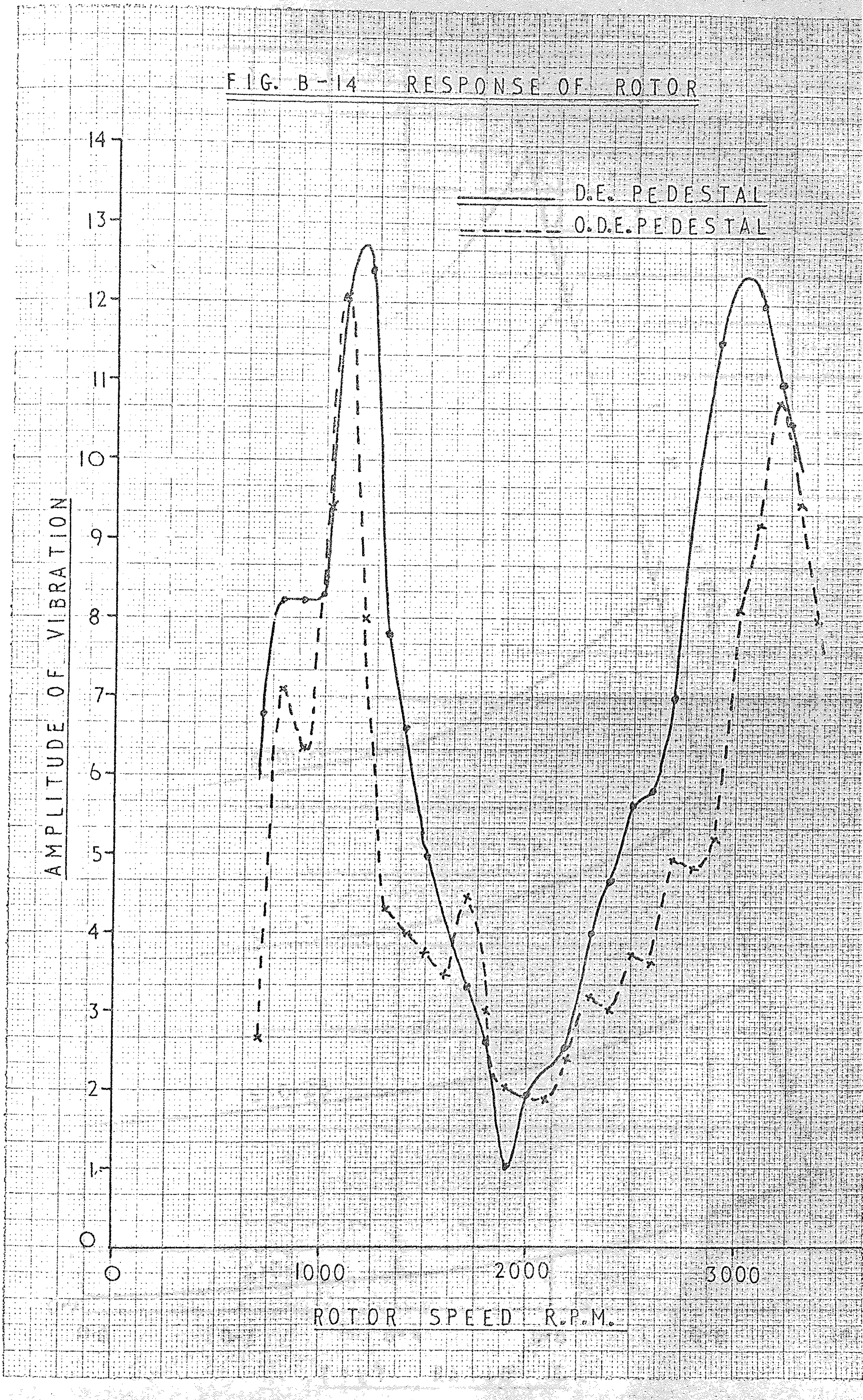
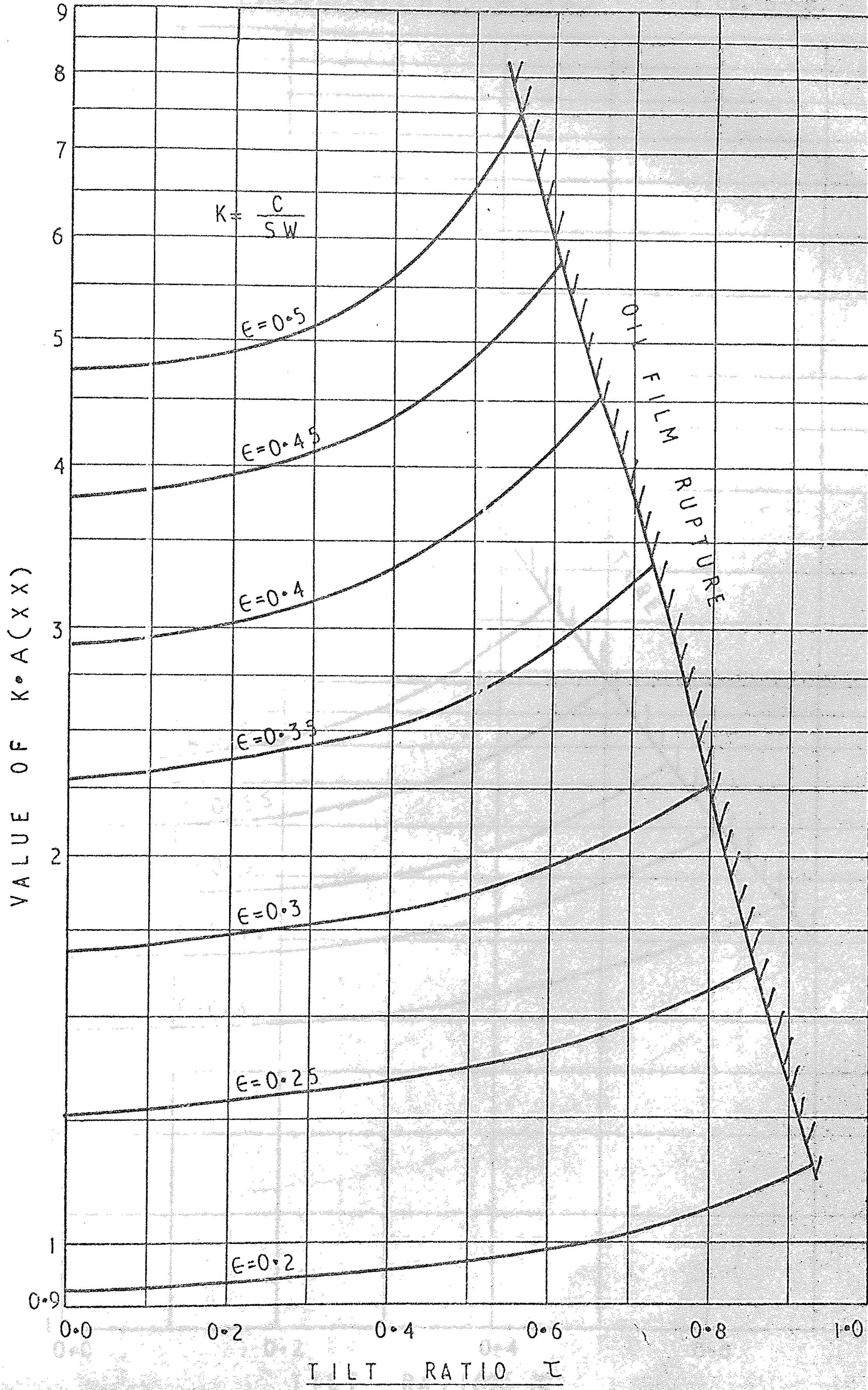
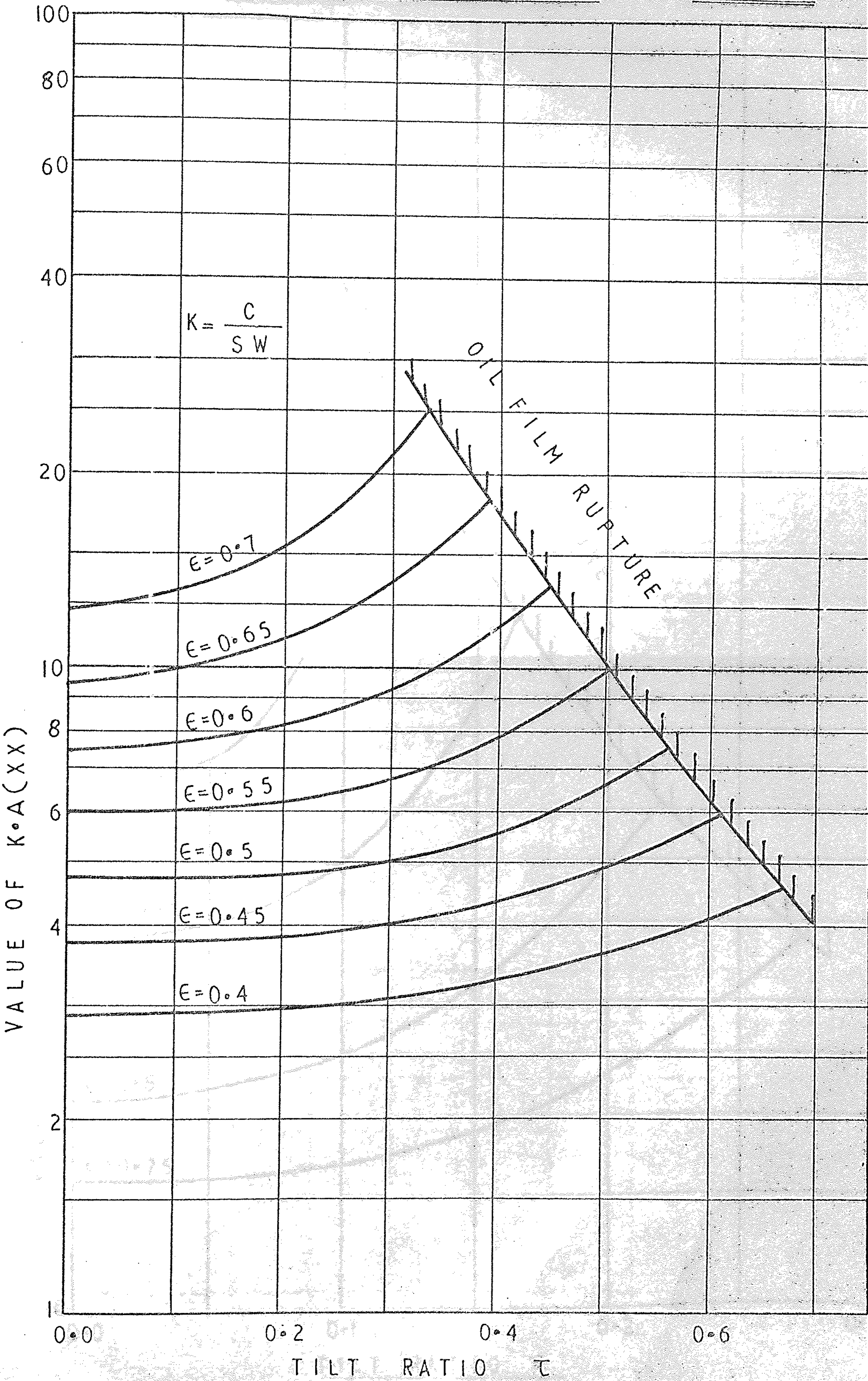


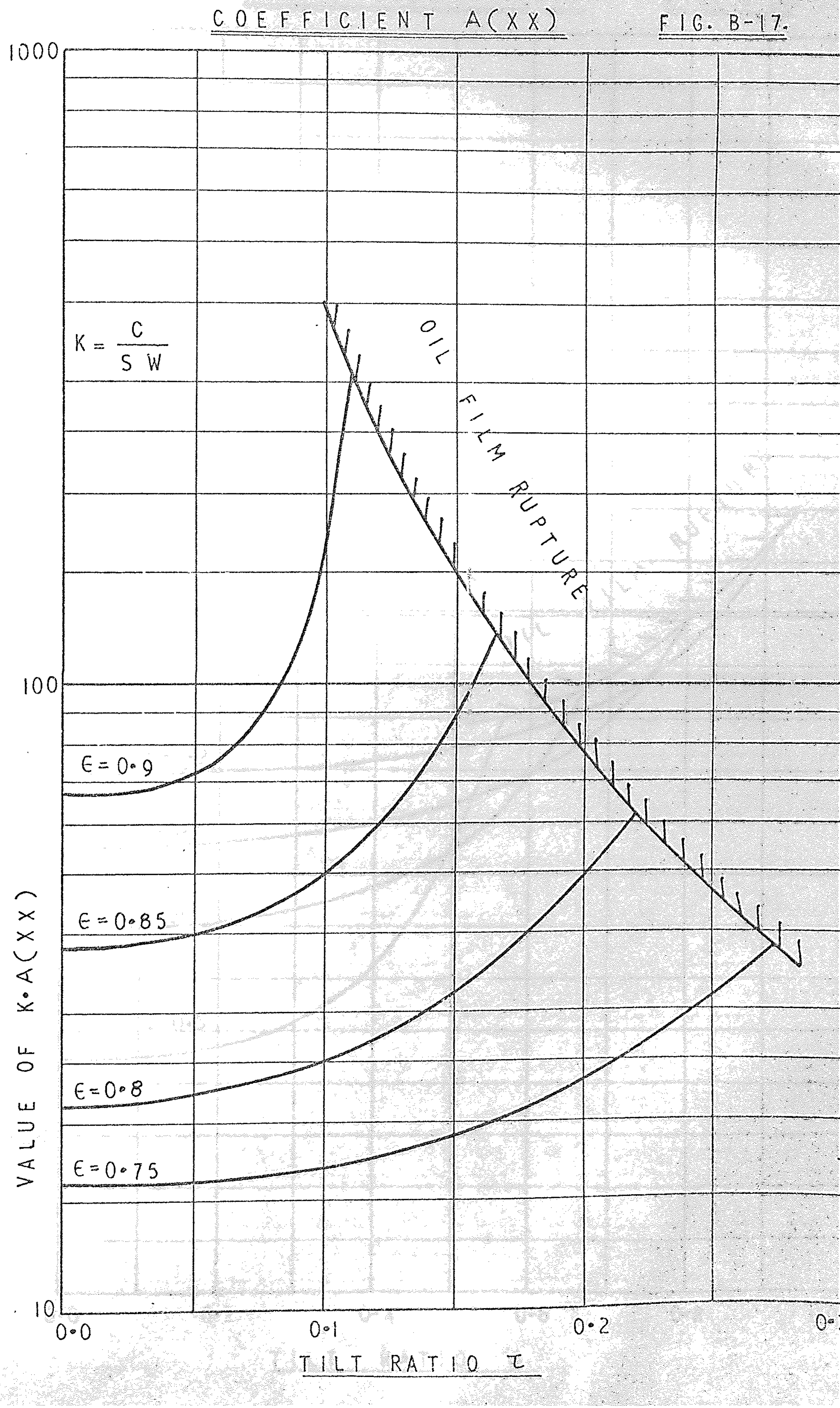
FIG. B-14 RESPONSE OF ROTOR
D.E. PEDESTAL
O.D.E. PEDESTAL
AMPLITUDE OF VIBRATION
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
0
1000
2000
3000
ROTOR SPEED R.P.M.

COEFFICIENT A(XX) FIG. B-15.

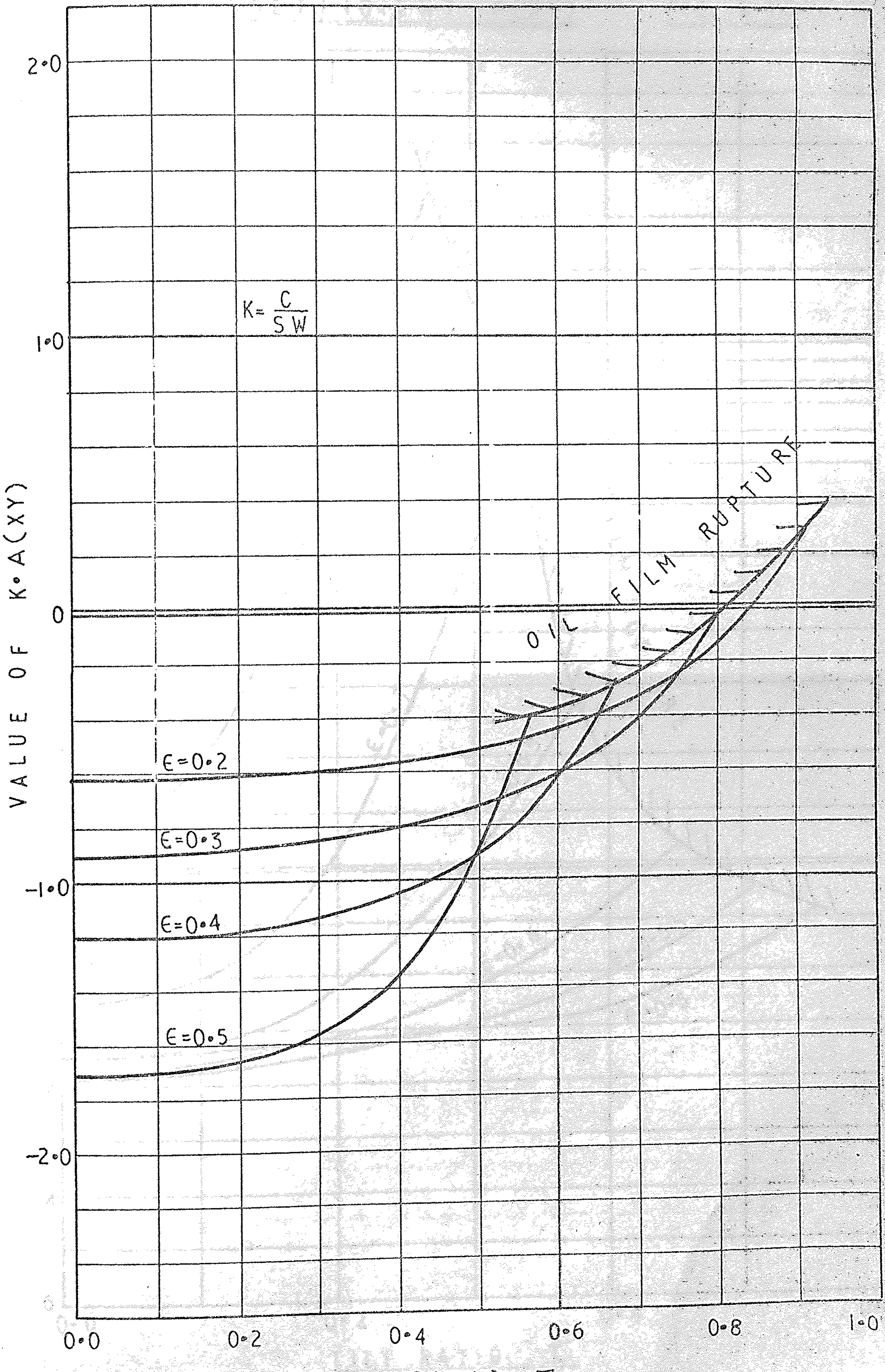


COEFFICIENT A(XX) FIG. B-16.

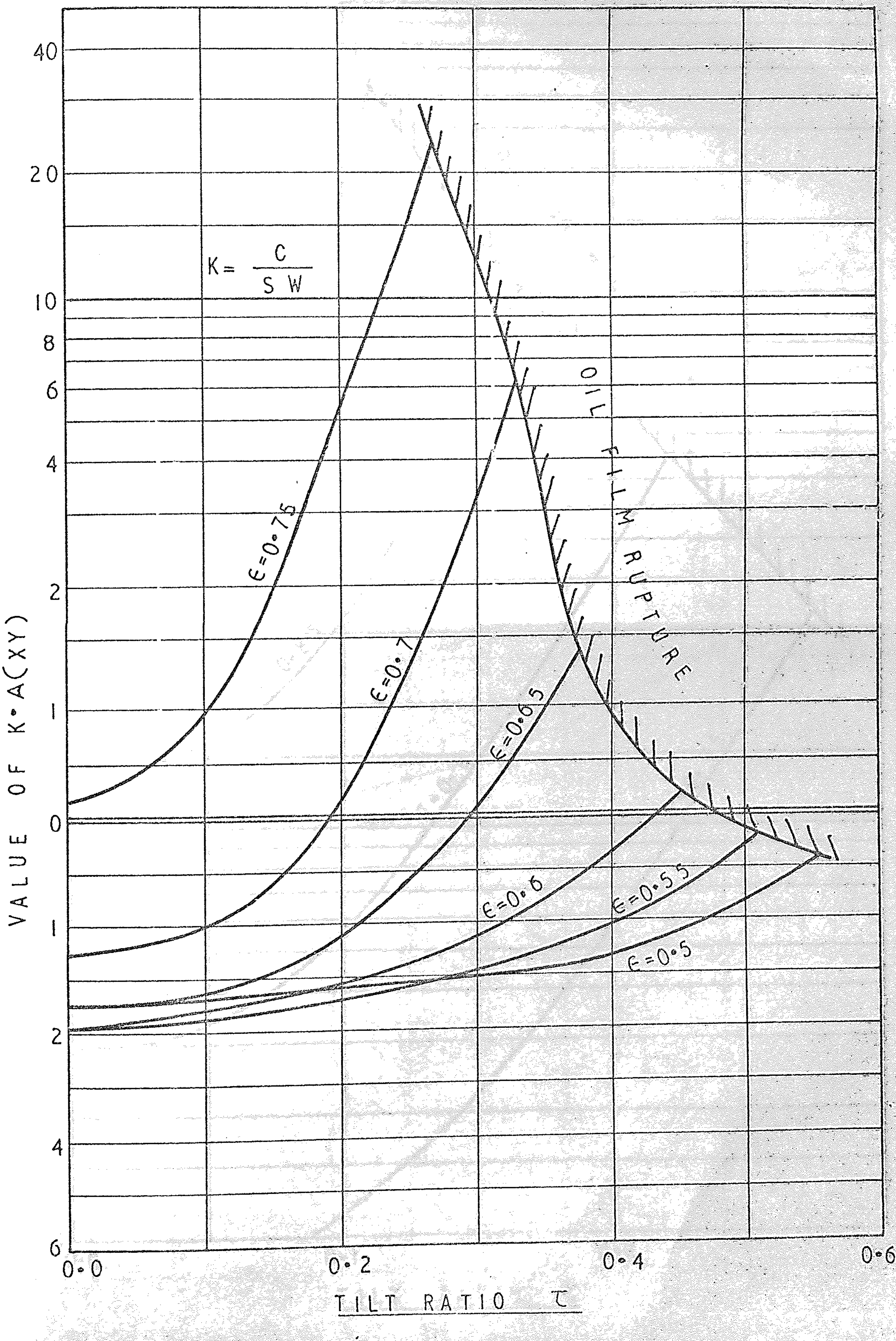



COEFFICIENT A(XX)
FIG. B-17.
1000
100
10
$K = \frac{C}{SW}$
OIL FILM RUPTURE
ϵ = 0·9
ϵ = 0·85
ϵ = 0·8
ϵ = 0·75
VALUE OF K·A(XX)
0·0
0·1
0·2
TILT RATIO τ

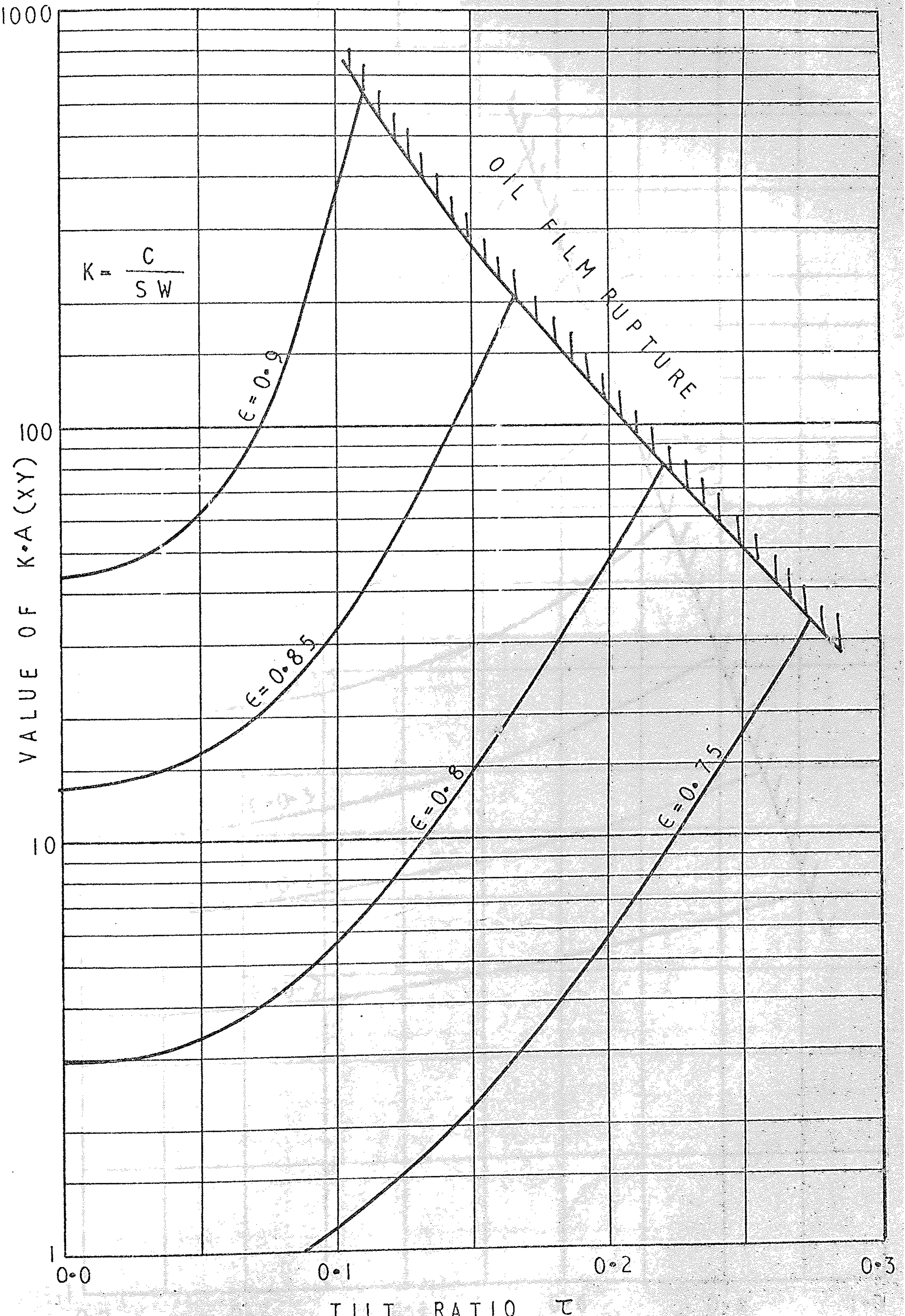
COEFFICIENT A(XY) FIG. B-18.



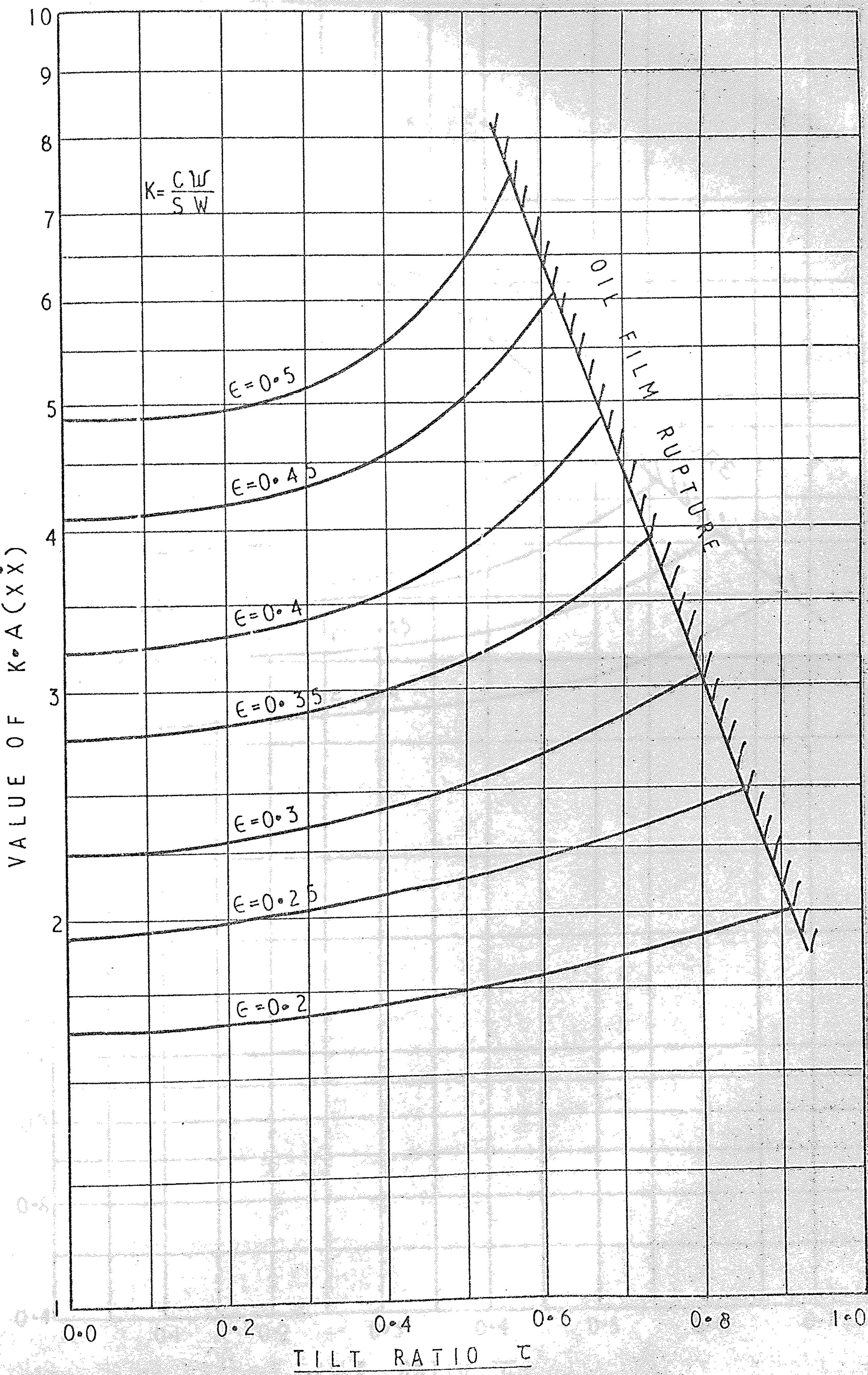
COEFFICIENT A(XY)　　FIG. B-19.



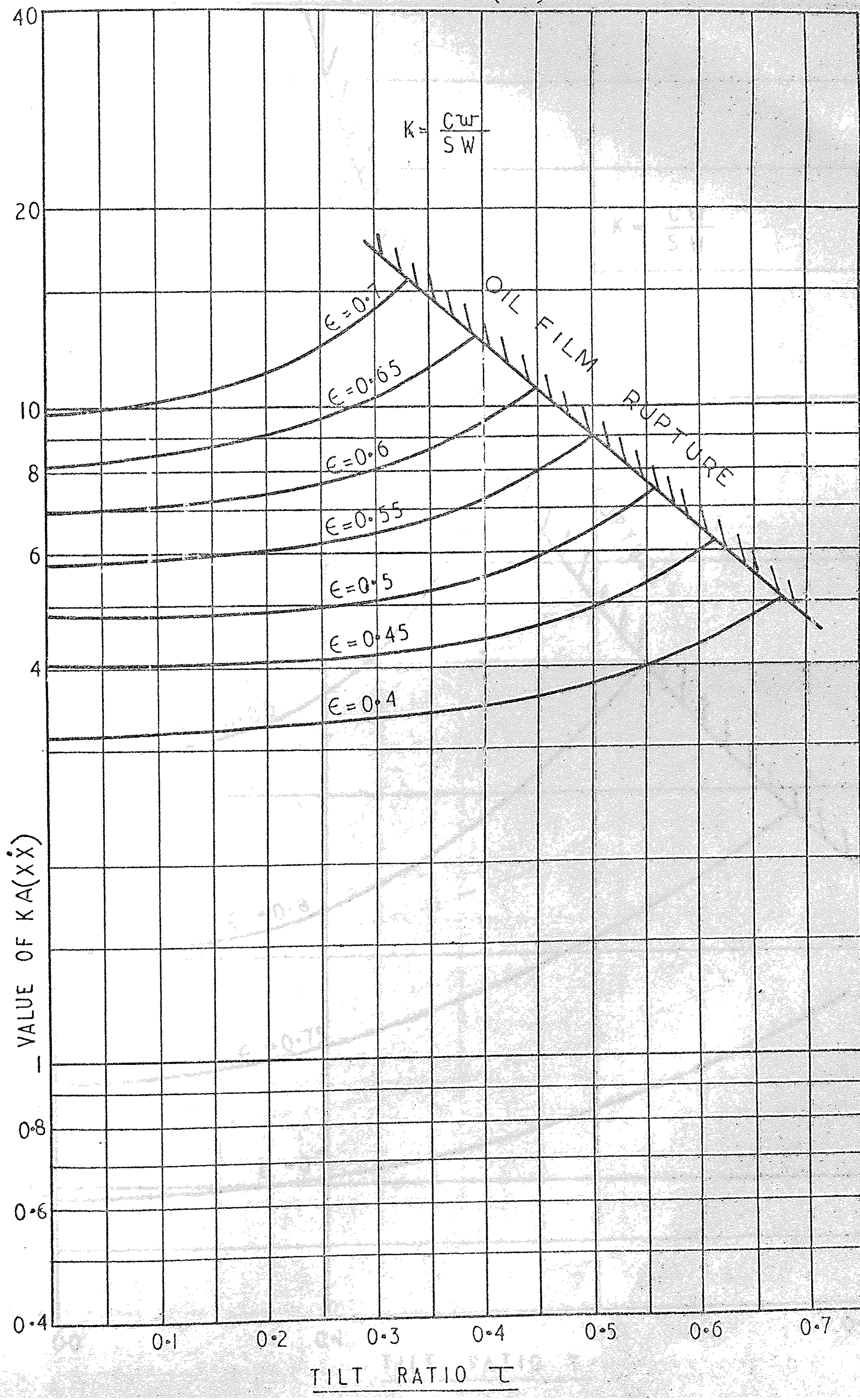
COEFFICIENT A(XY) FIG. B-20.



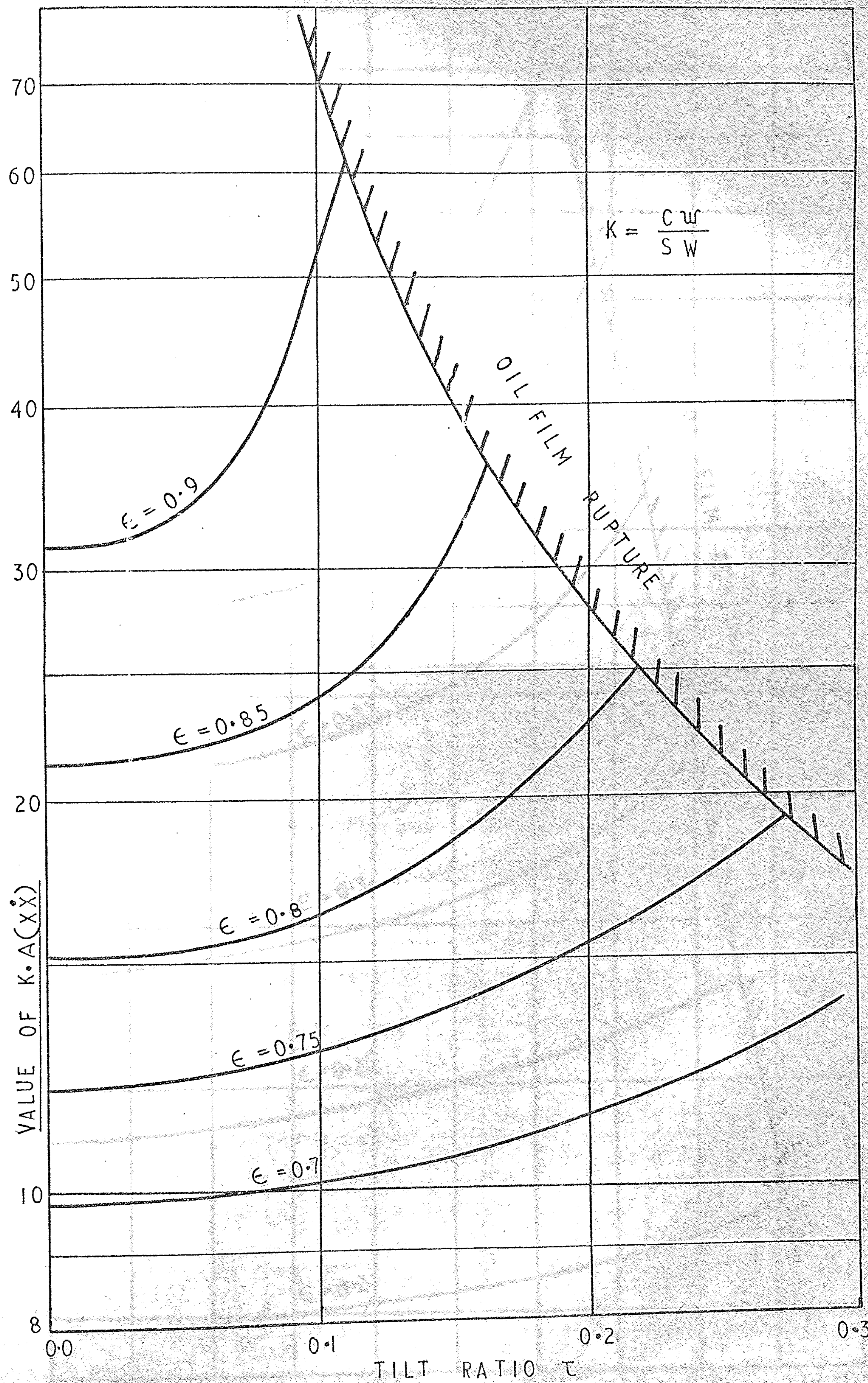
COEFFICIENT A(XẊ)



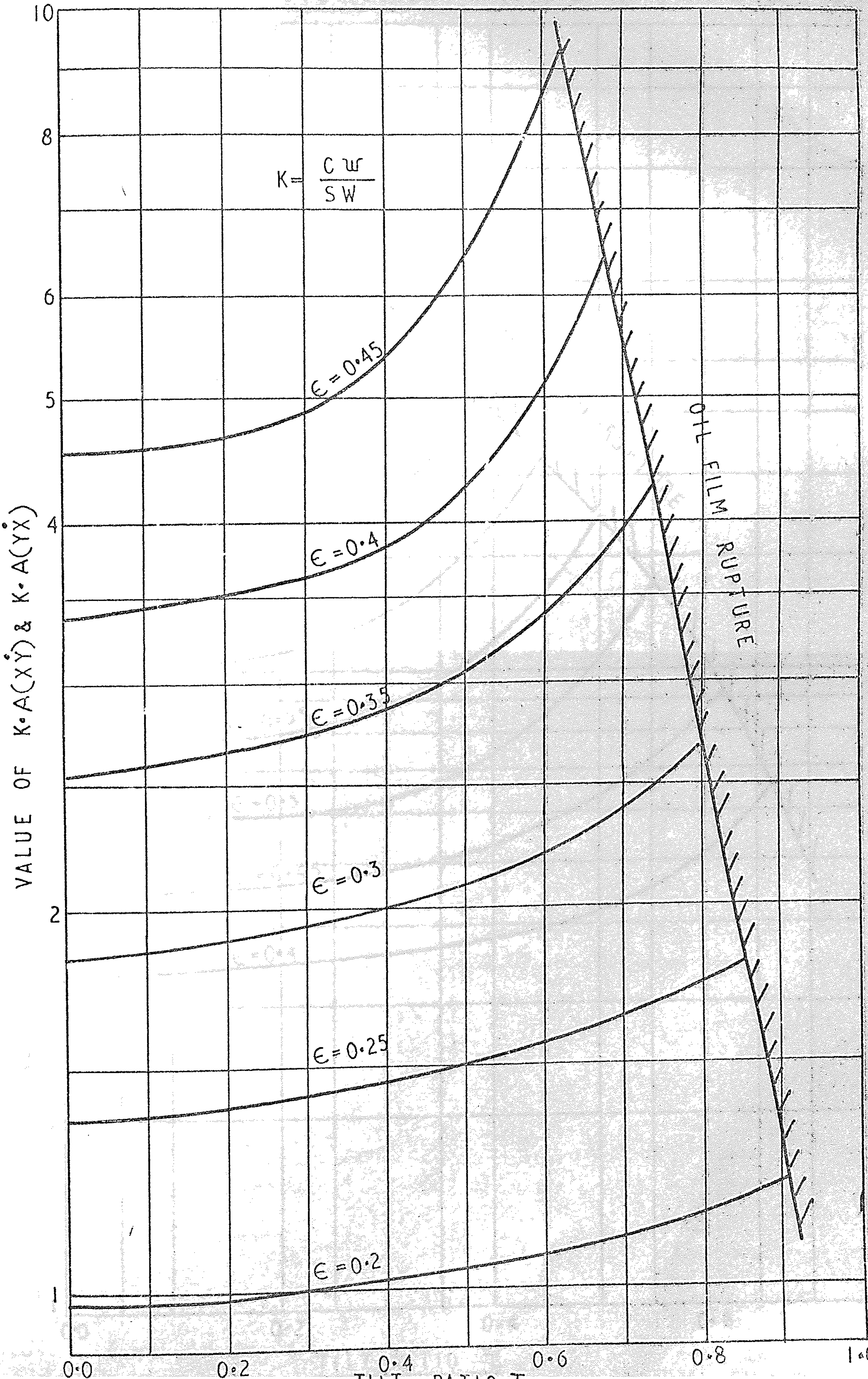
COEFFICIENT $A(x\dot{x})$ FIG. B-22.



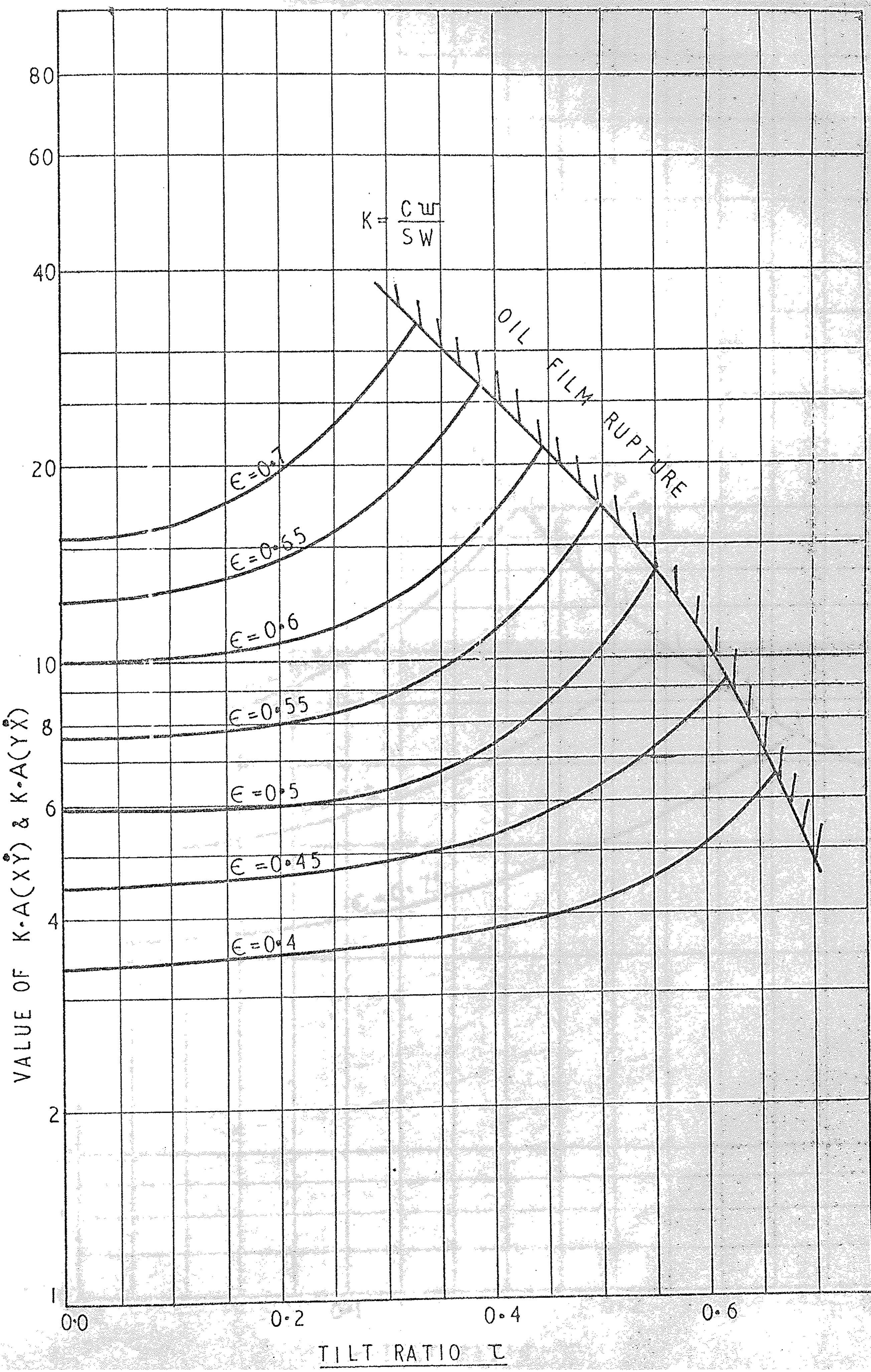
COEFFICIENT $A(X\dot{X})$ FIG. B-23.



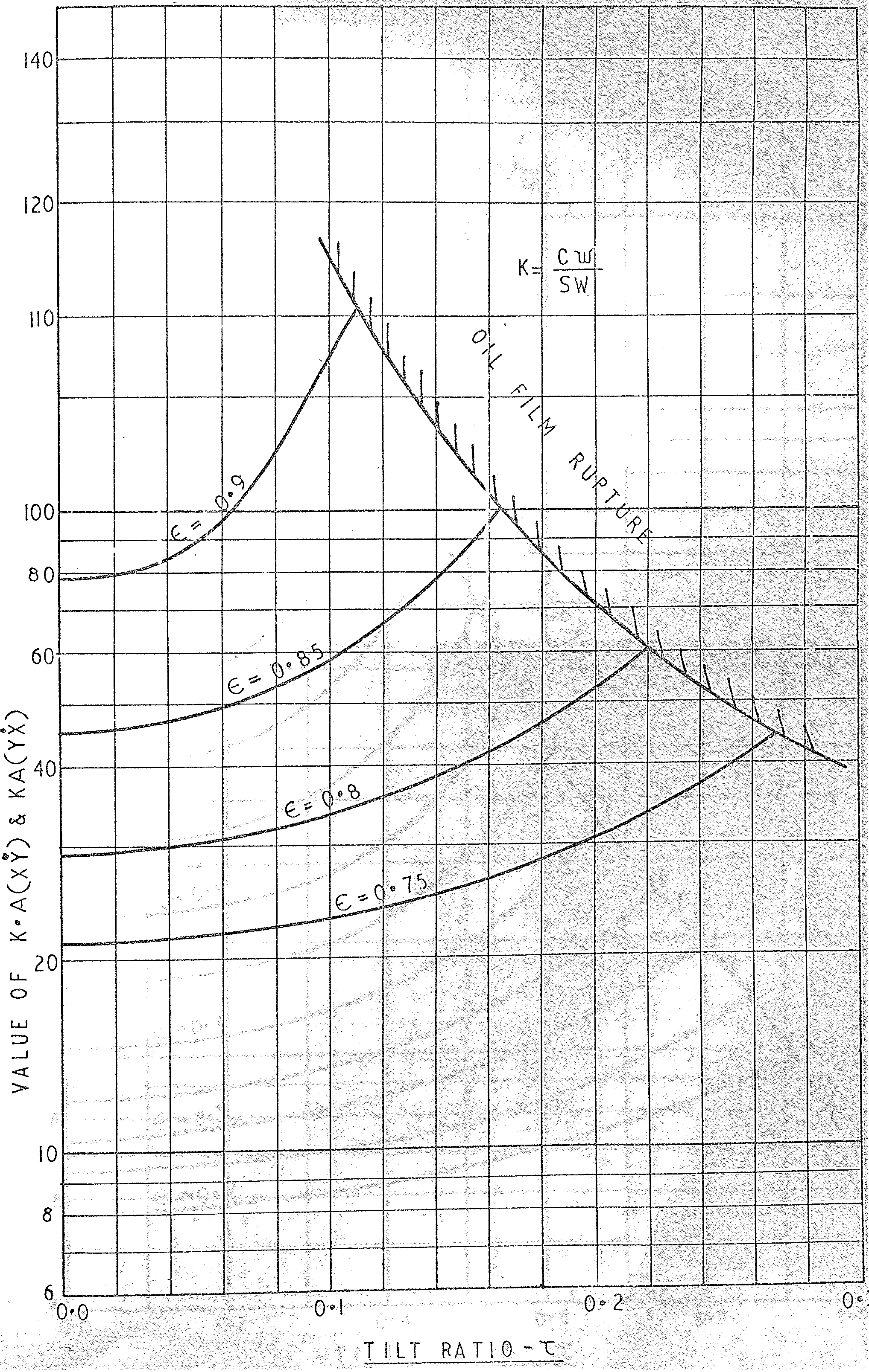
COEFFICIENT A(XẎ) & A'(YẊ)   FIG. B-24.



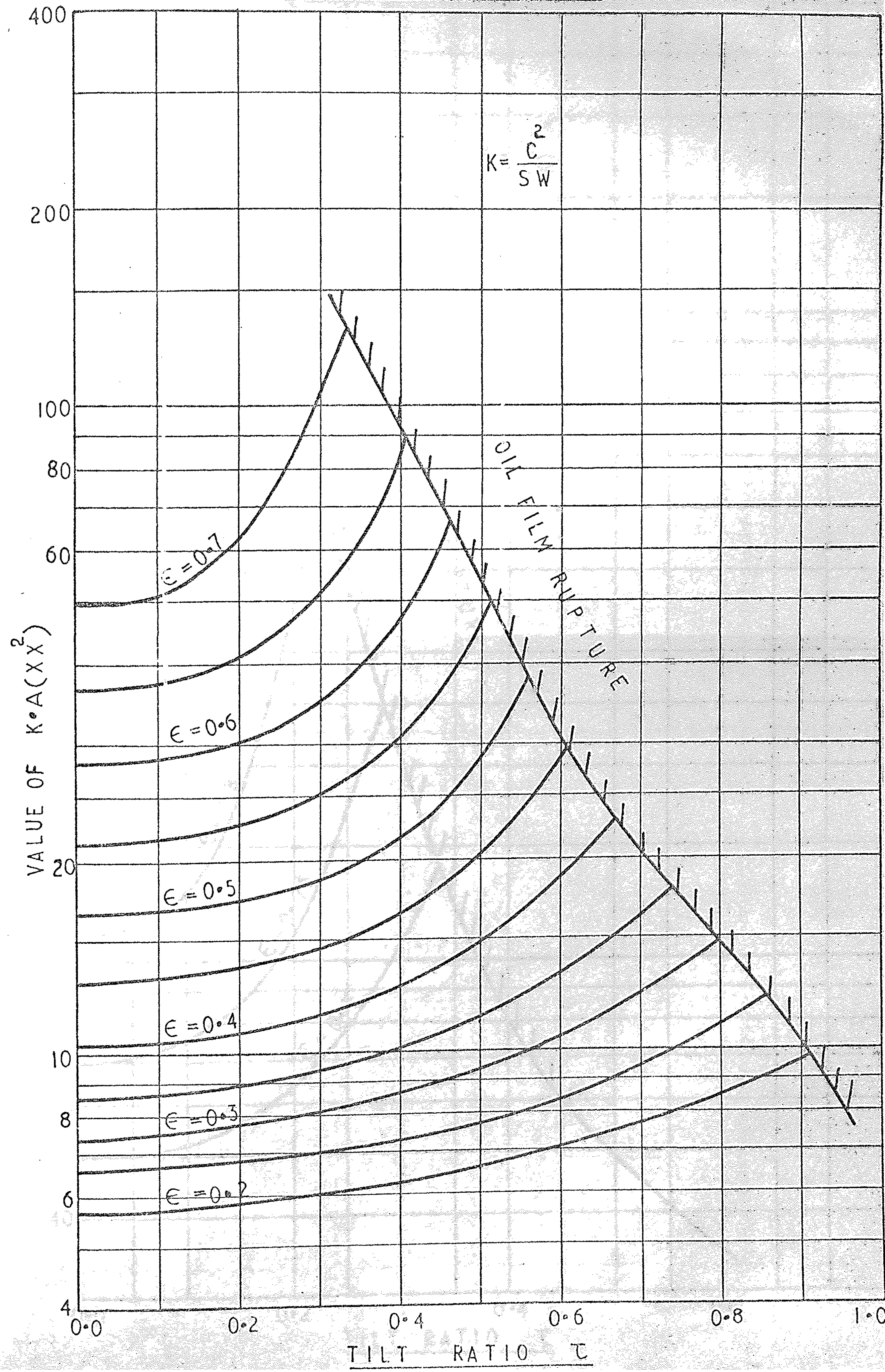
COEFFICIENT A(YẊ) & A(XẎ) FIG. B-25.



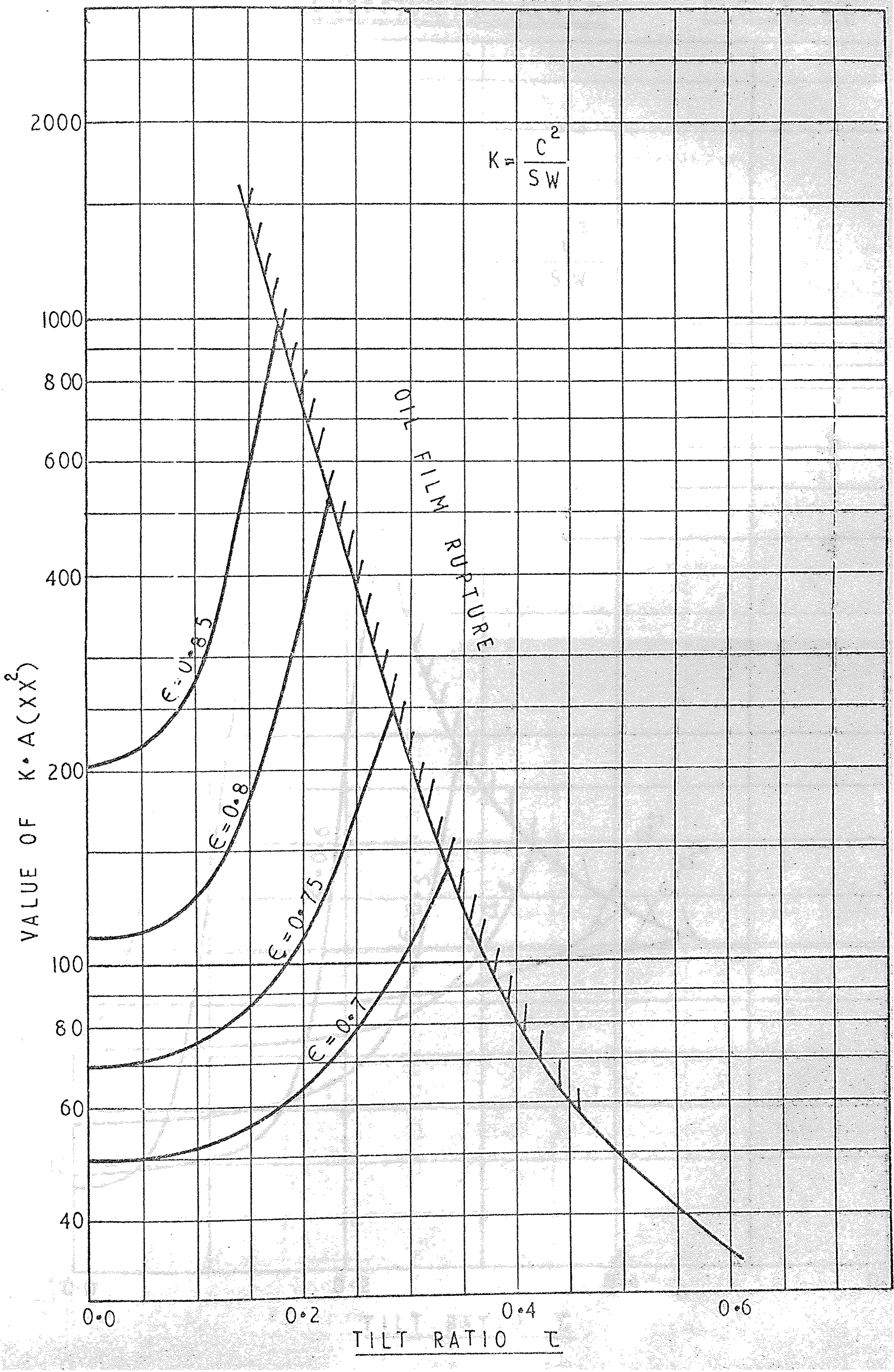
COEFFICIENT A(XẎ) & A(YẊ) FIG. B-26.



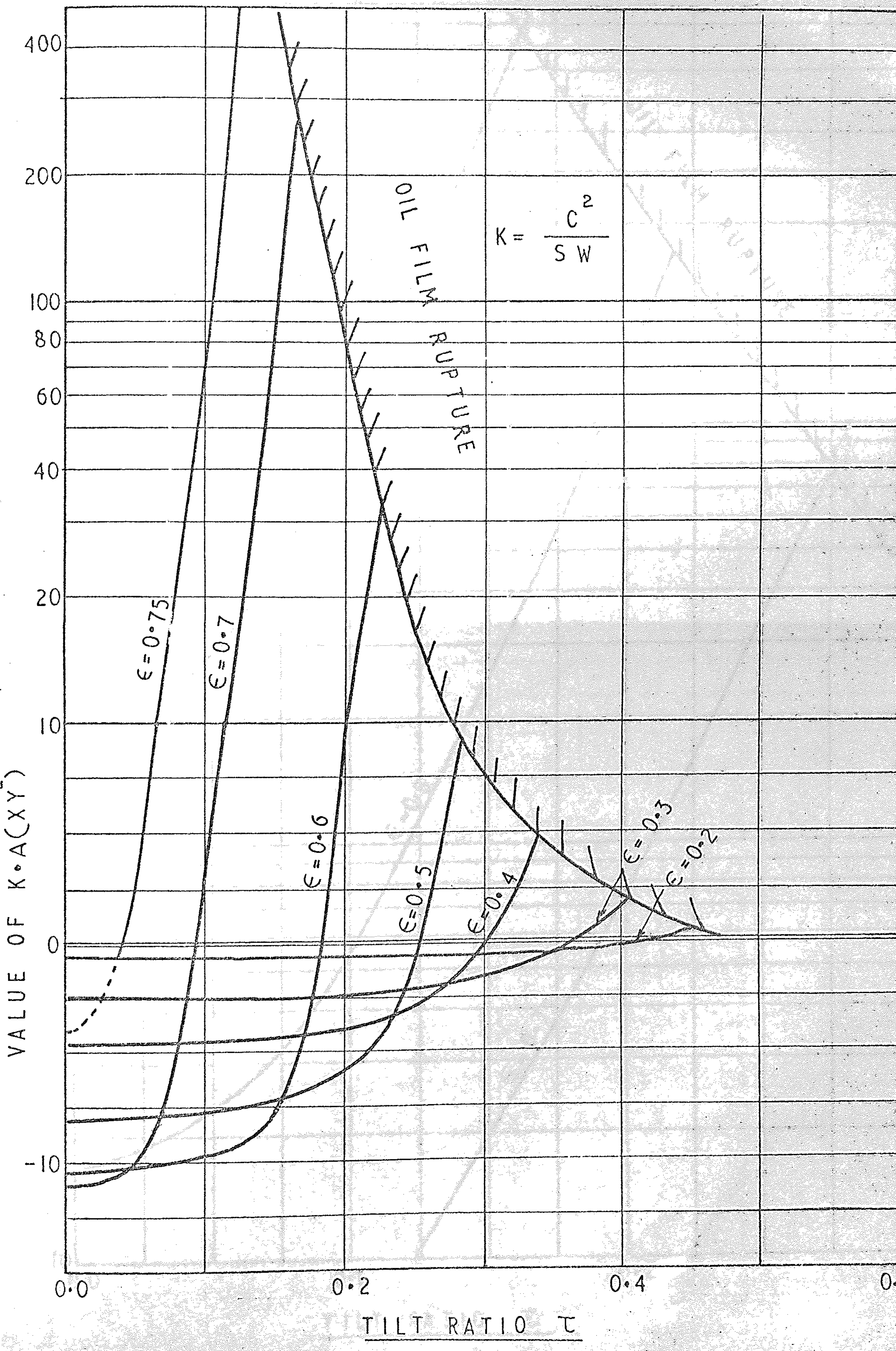
COEFFICIENT $A(XX^2)$ FIG. B-27.



COEFFICIENT $A(XX^2)$ FIG. B-28

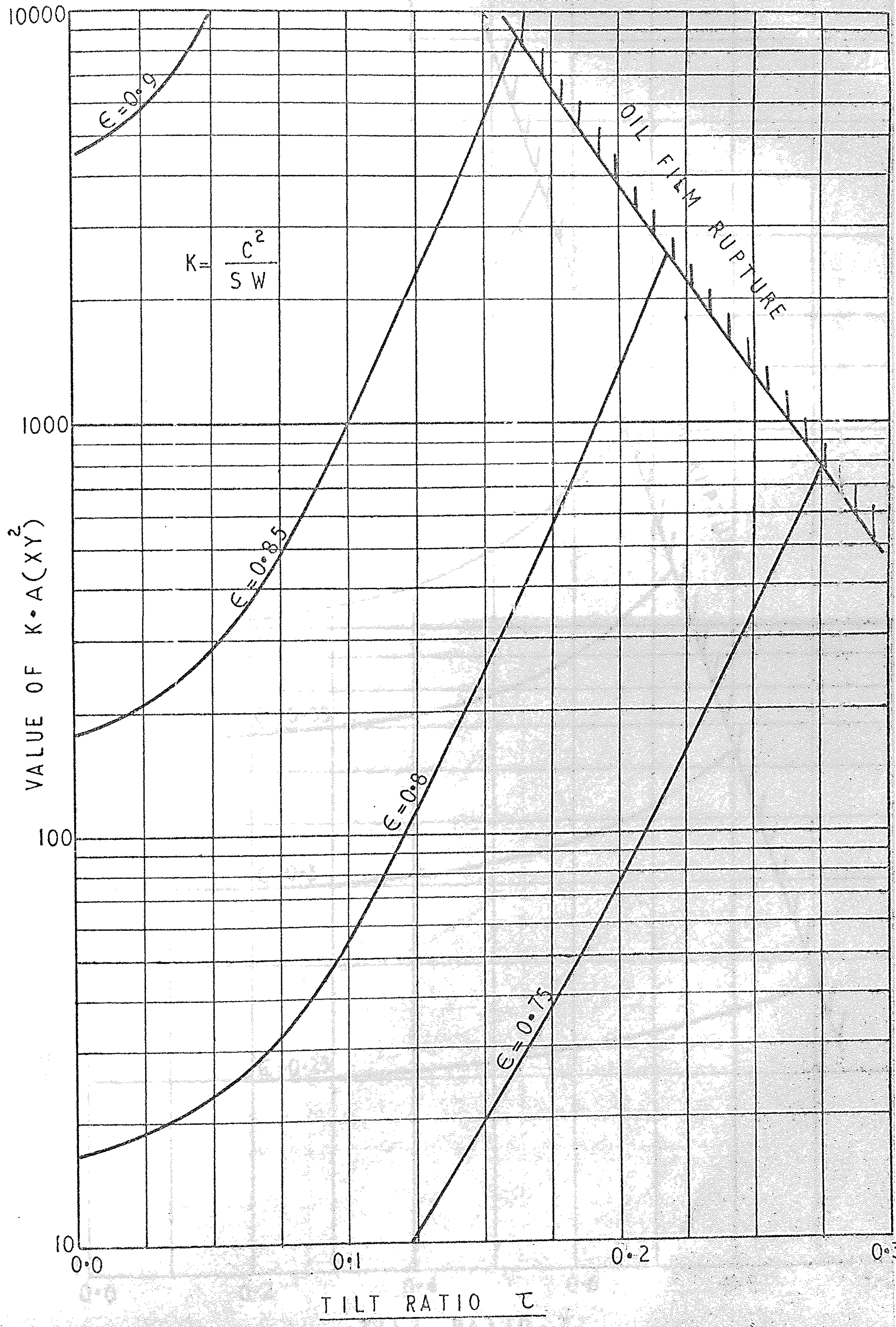


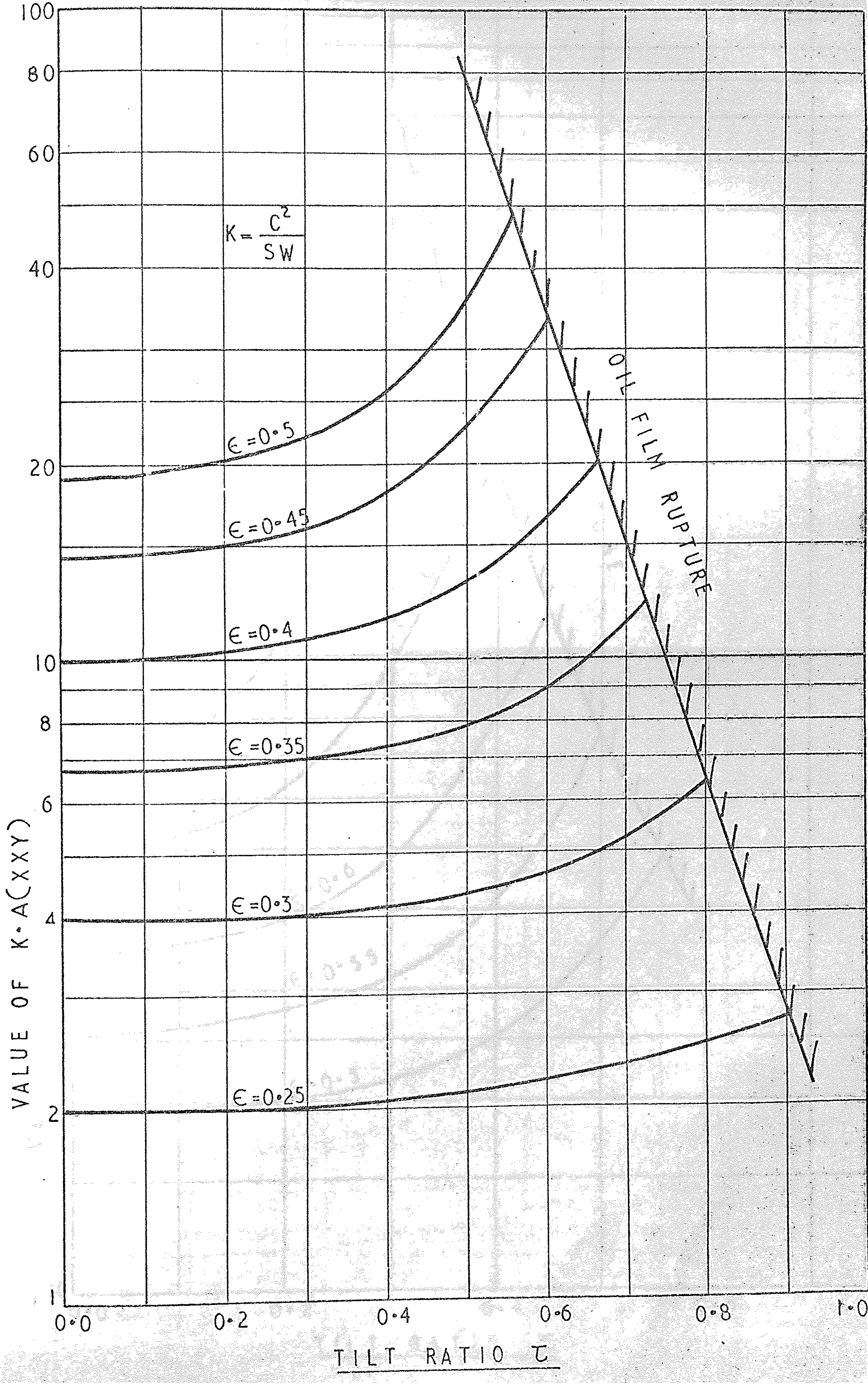
COEFFICIENT $A(XY^2)$ FIG. B-29.



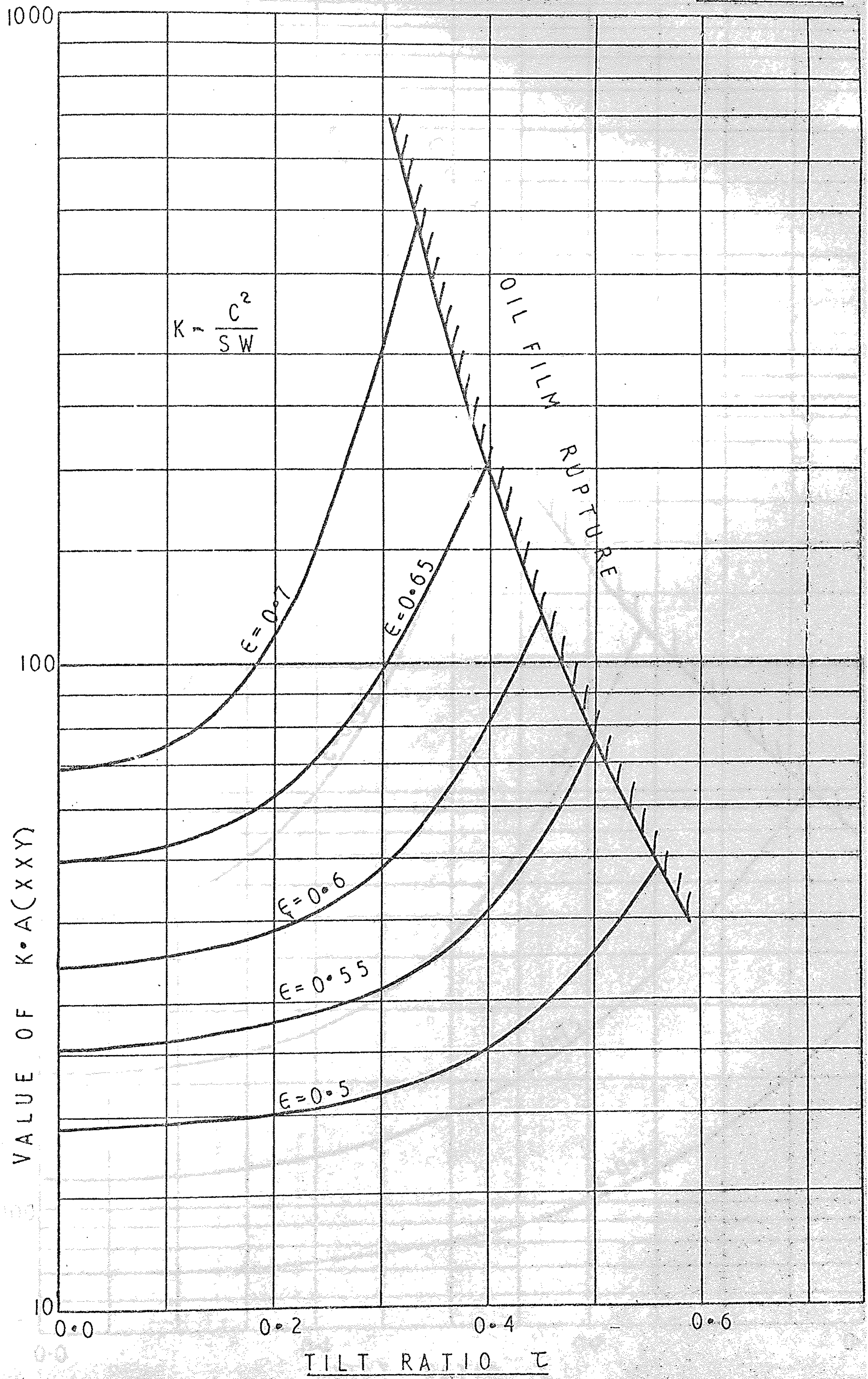
COEFFICIENT $A(XY^2)$

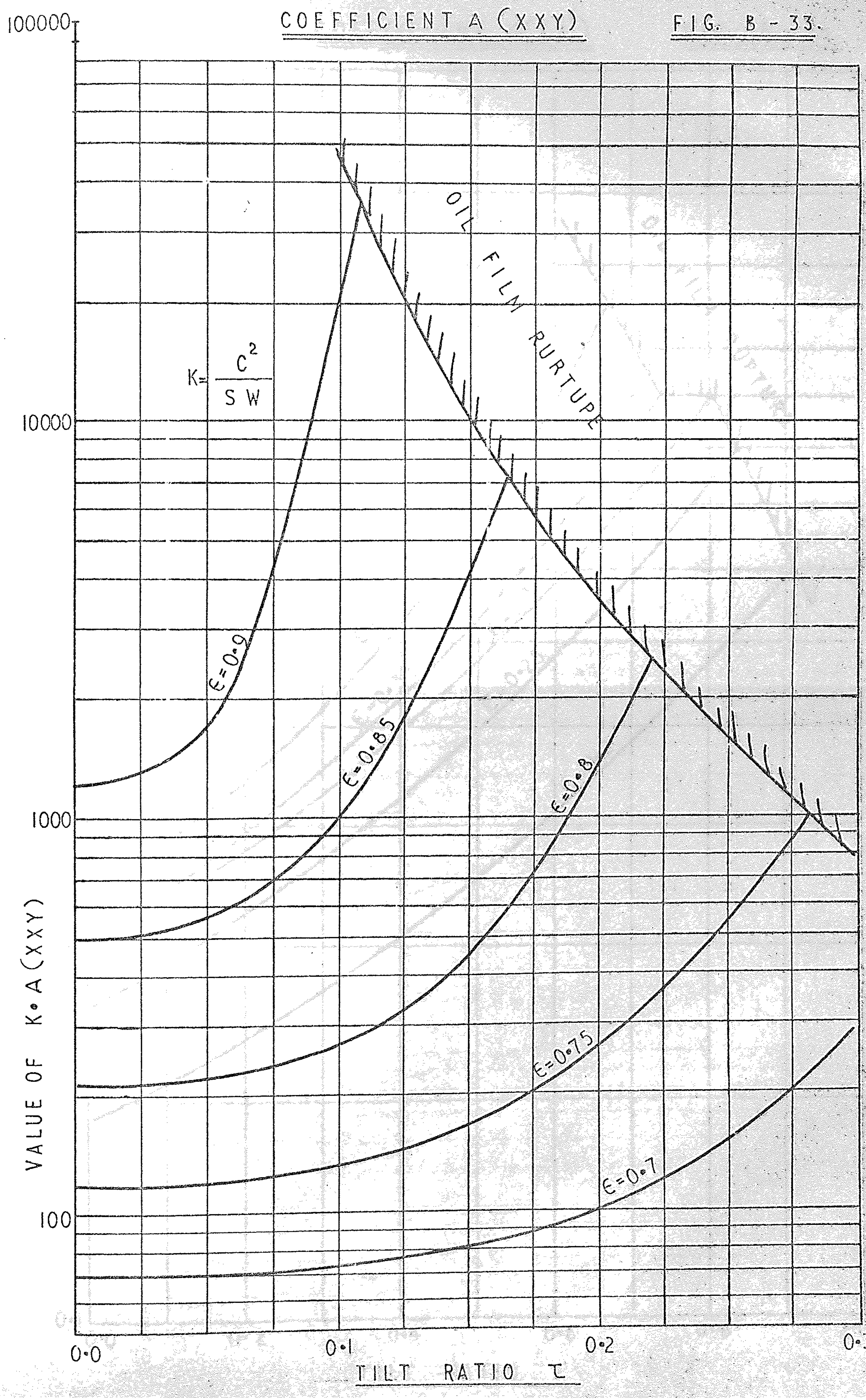
FIG. B-30.




100
80
60
40
20
10
8
6
4
2
1
$K = \frac{C^2}{SW}$
∈ = 0·5
∈ = 0·45
∈ = 0·4
∈ = 0·35
∈ = 0·3
∈ = 0·25
OIL FILM RUPTURE
VALUE OF K·A(XXY)
0·0
0·2
0·4
0·6
0·8
1·0
TILT RATIO τ

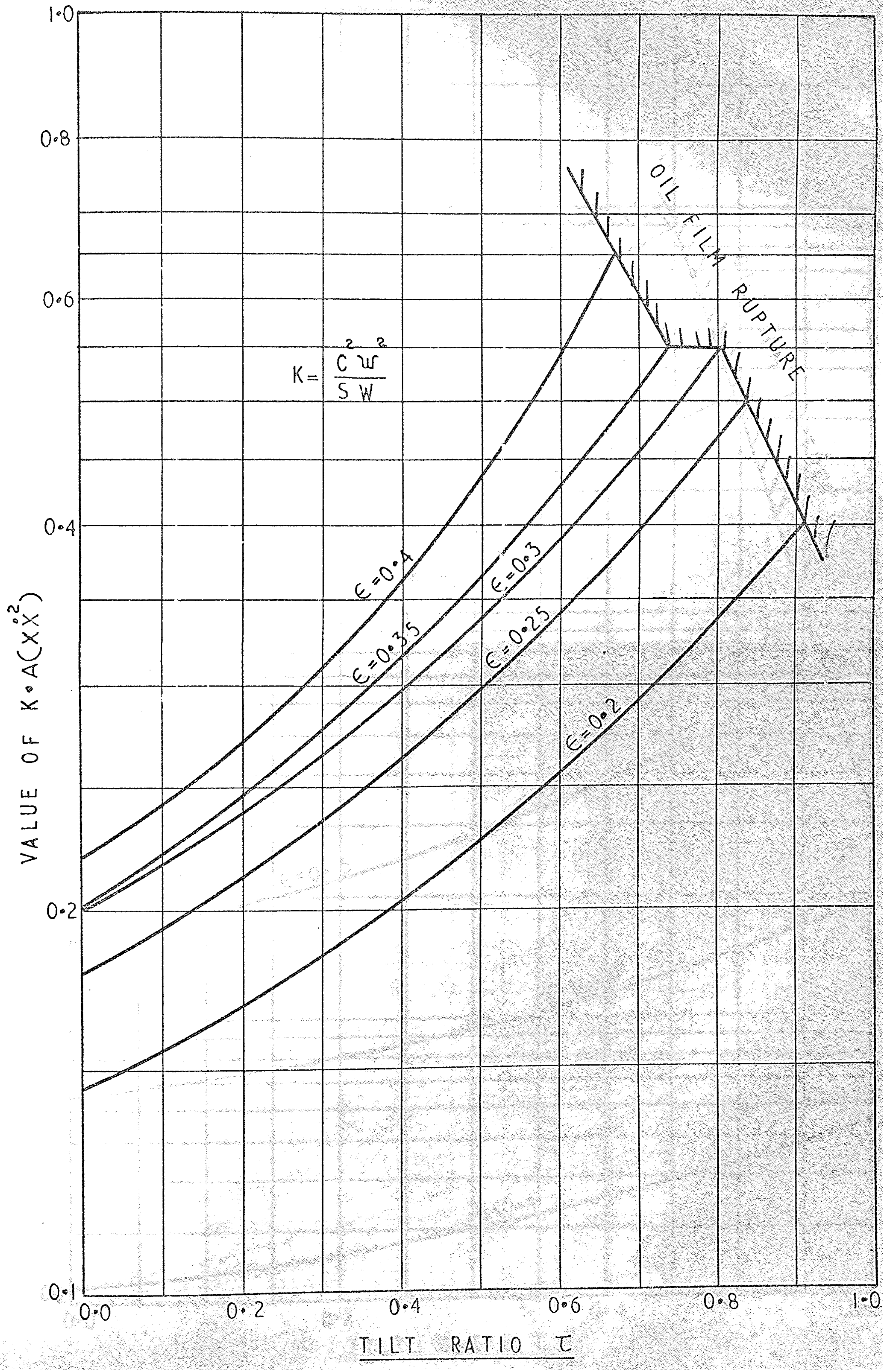
COEFFICIENT A(XXY) FIG. B-32.



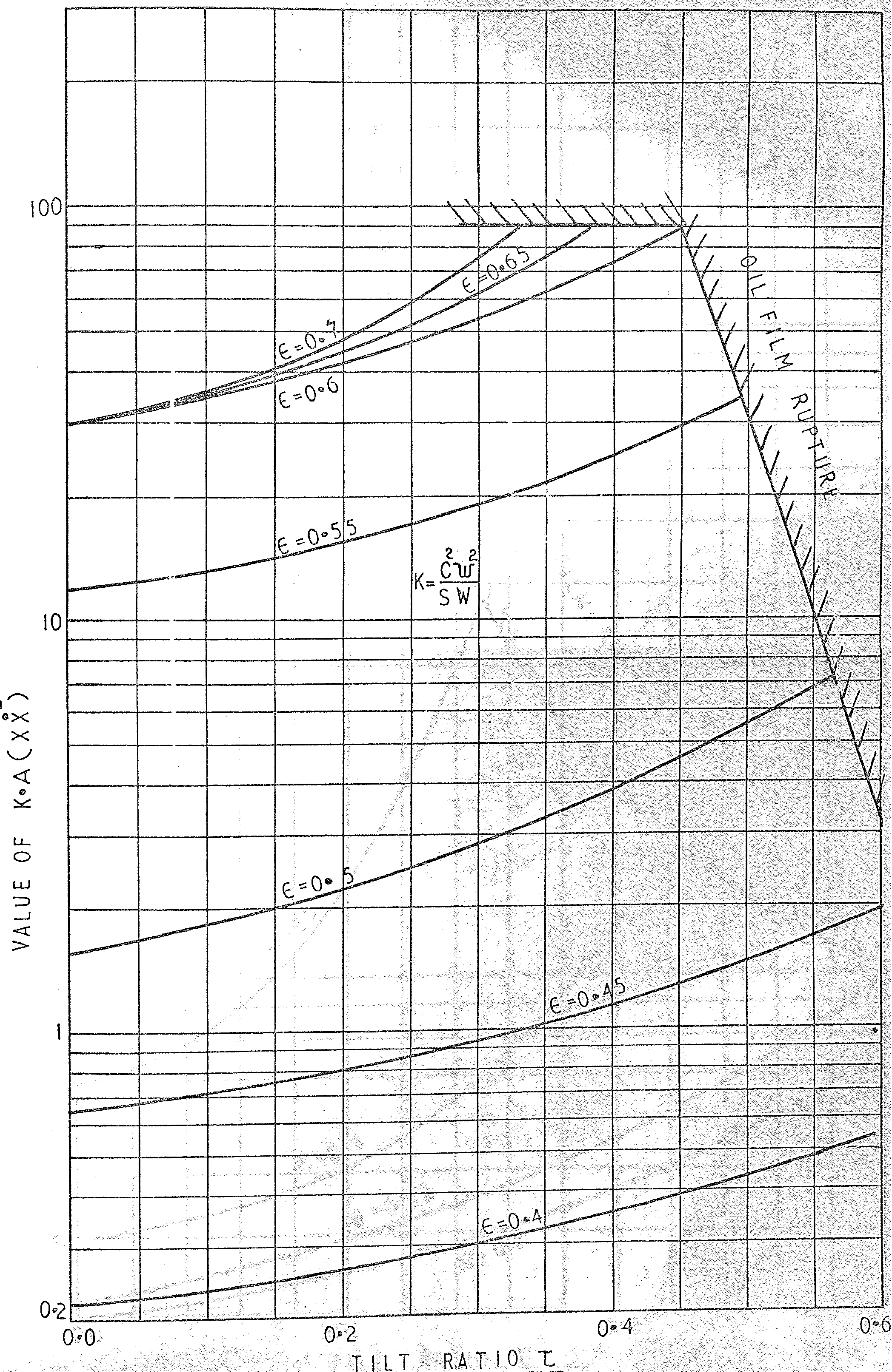


COEFFICIENT A (XXY)
FIG. B - 33.
100000
10000
1000
100
VALUE OF K • A (XXY)
$K = \frac{C^2}{S W}$
OIL FILM RURTUPE
ε = 0•9
ε = 0•85
ε = 0•8
ε = 0•75
ε = 0•7
0•0
0•1
0•2
TILT RATIO τ

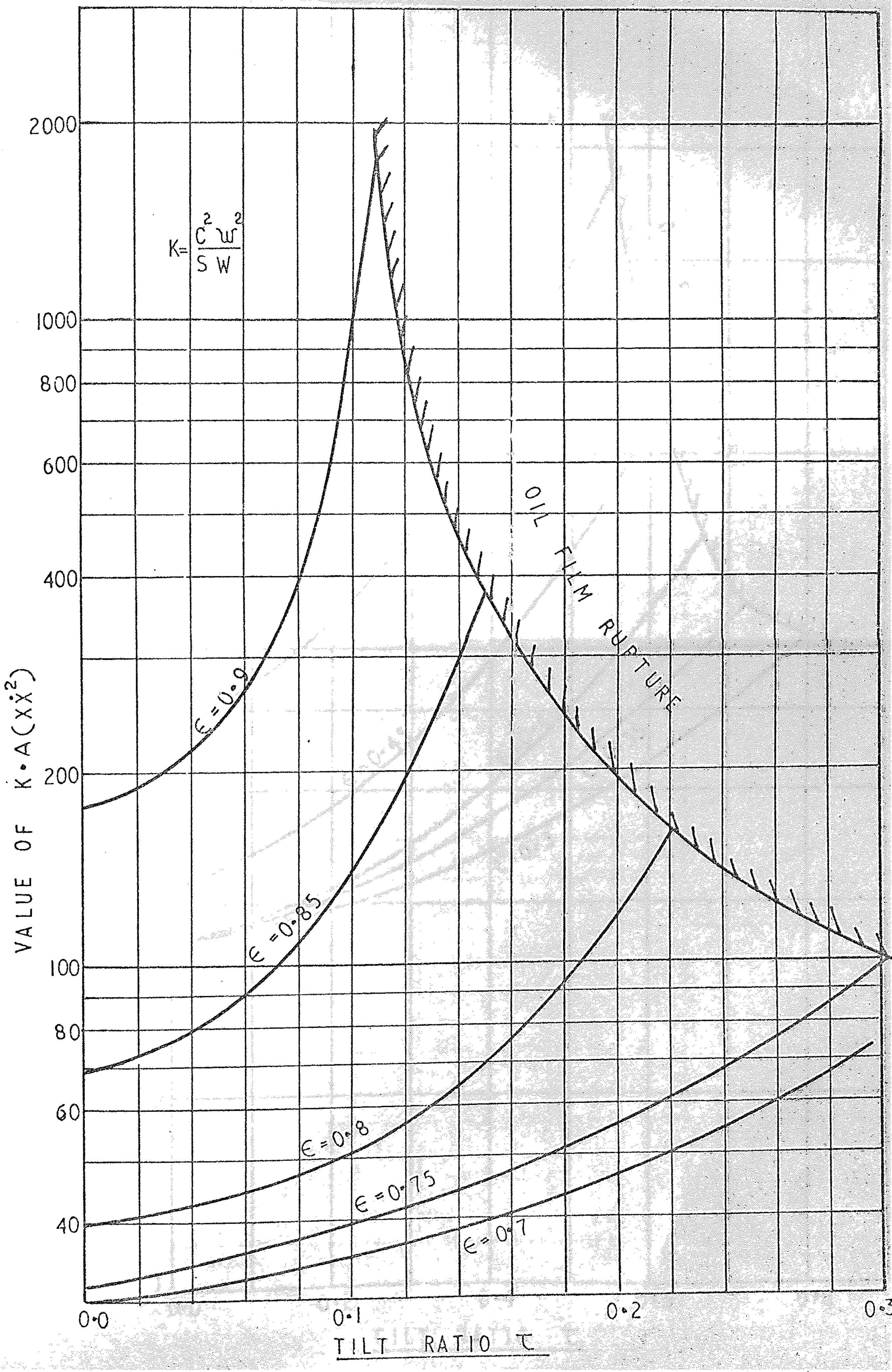
COEFFICIENT $A(X\dot{X}^2)$ FIG. B - 34.



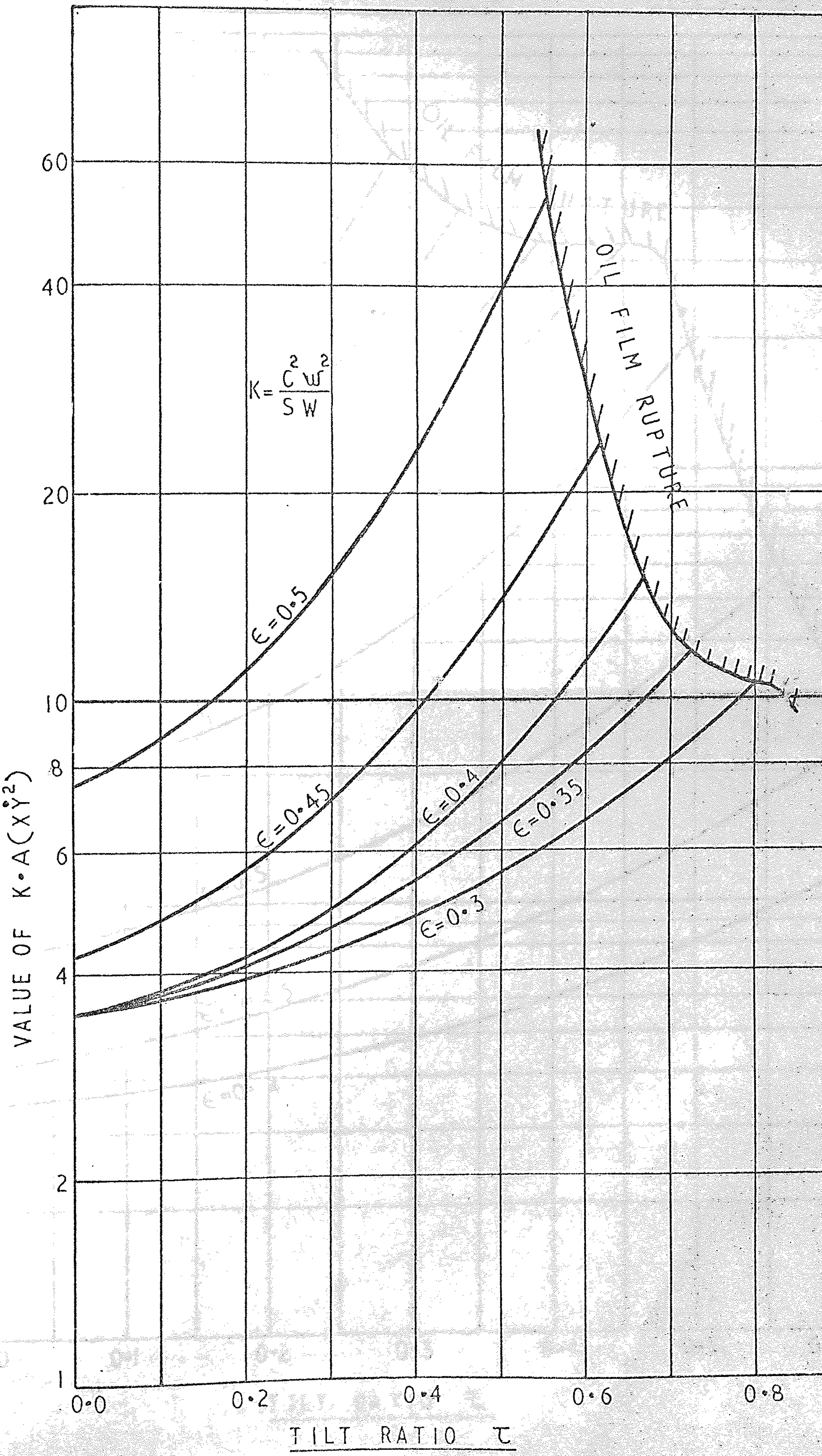
COEFFICIENT A$(X\dot{X}^2)$ FIG. B-35.



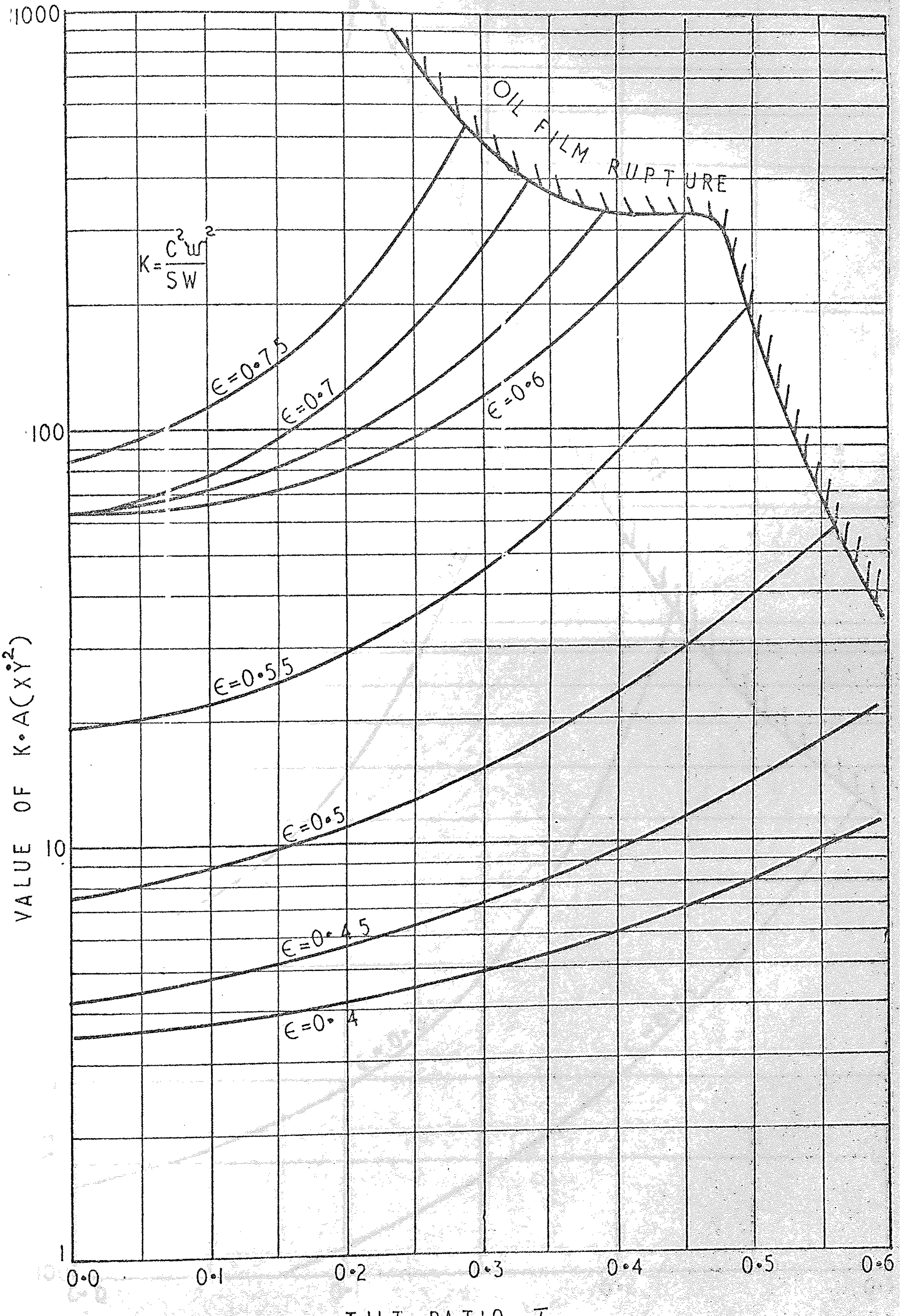
COEFFICIENT $A(X\dot{X}^2)$ FIG. B-36.



COEFFICIENT $A(X\dot{Y}^2)$ FIG. B-37.

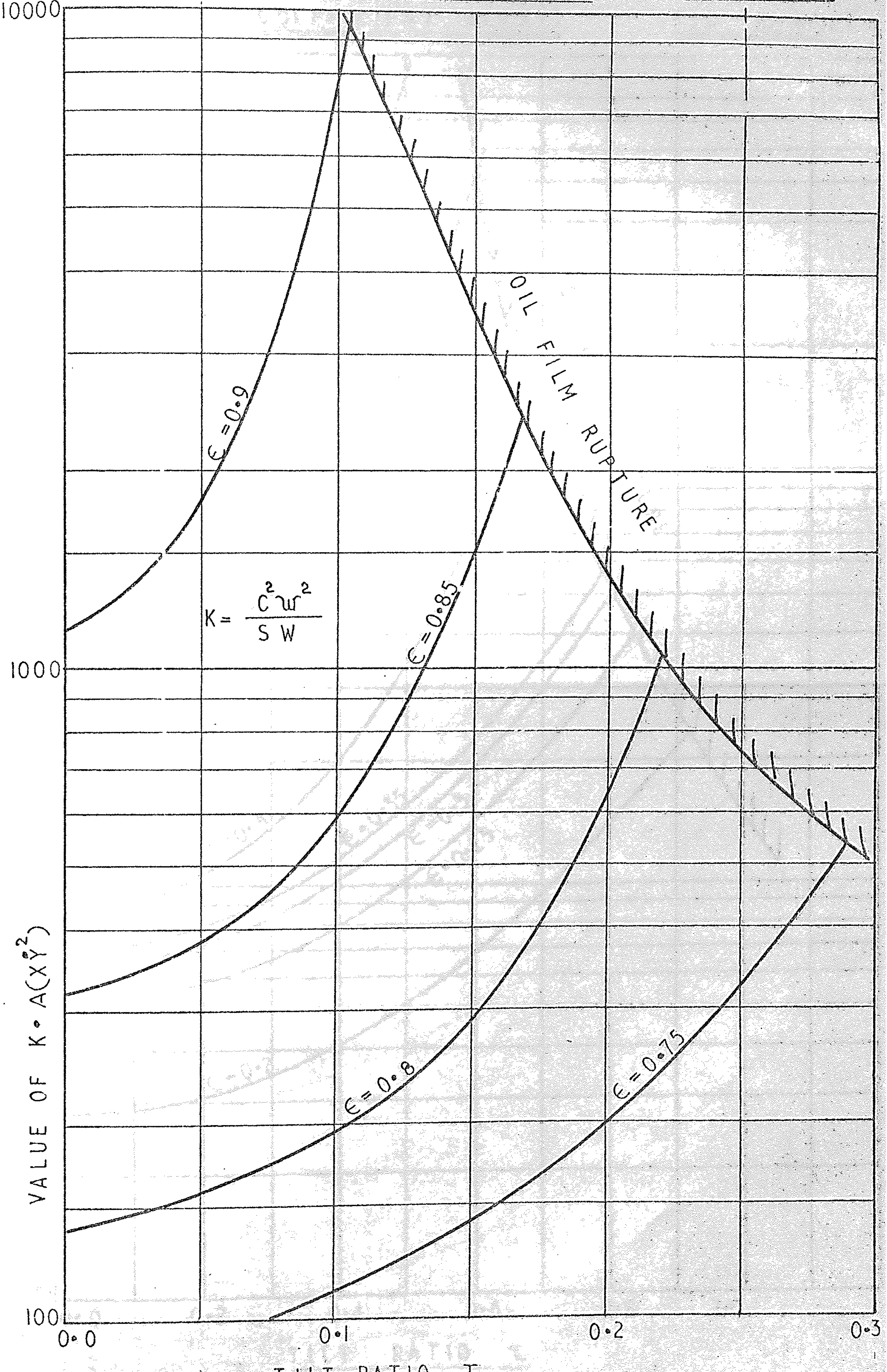


COEFFICIENT $A(X\dot{Y}^2)$ FIG. B-38.

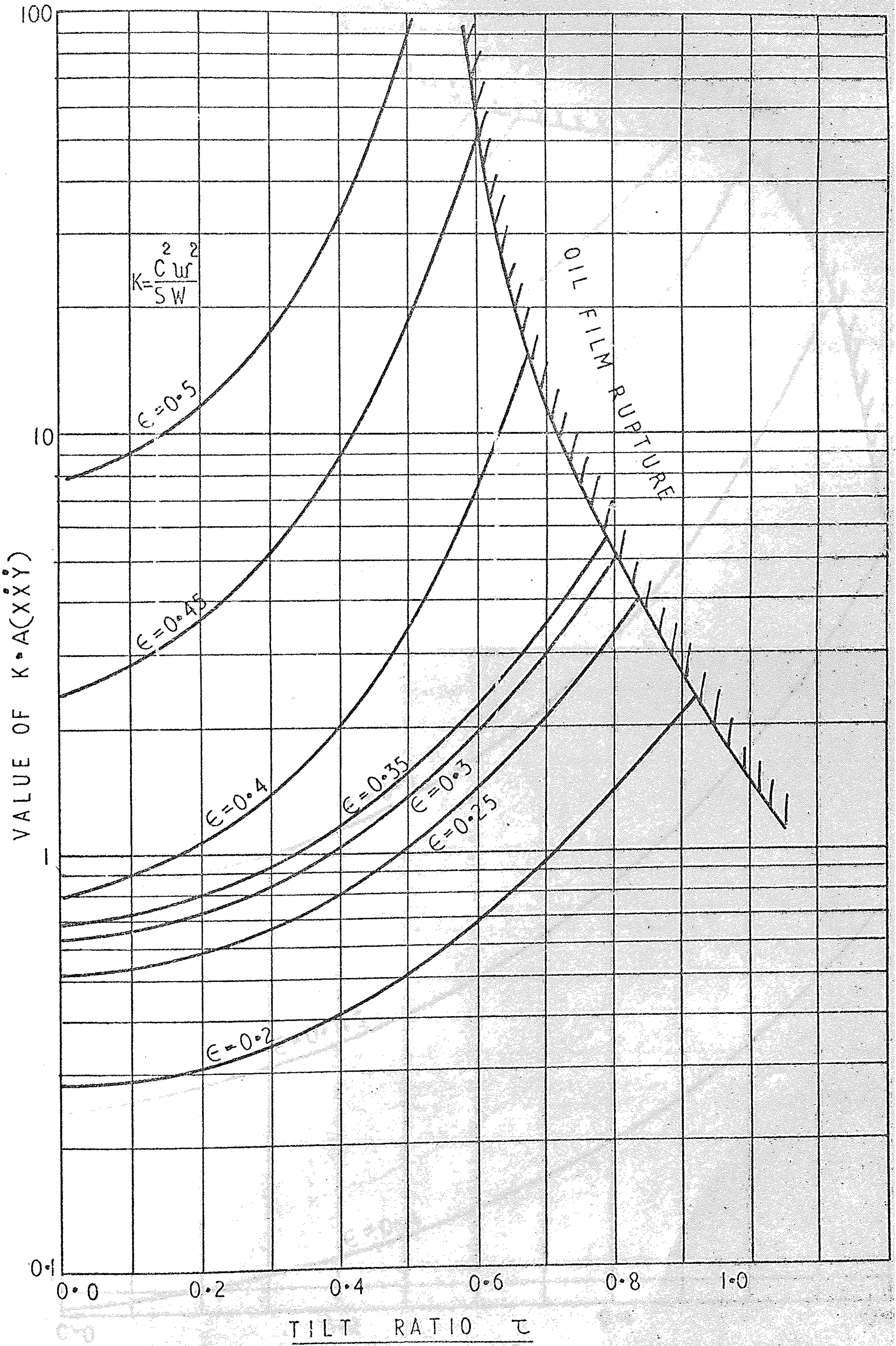


TILT RATIO τ

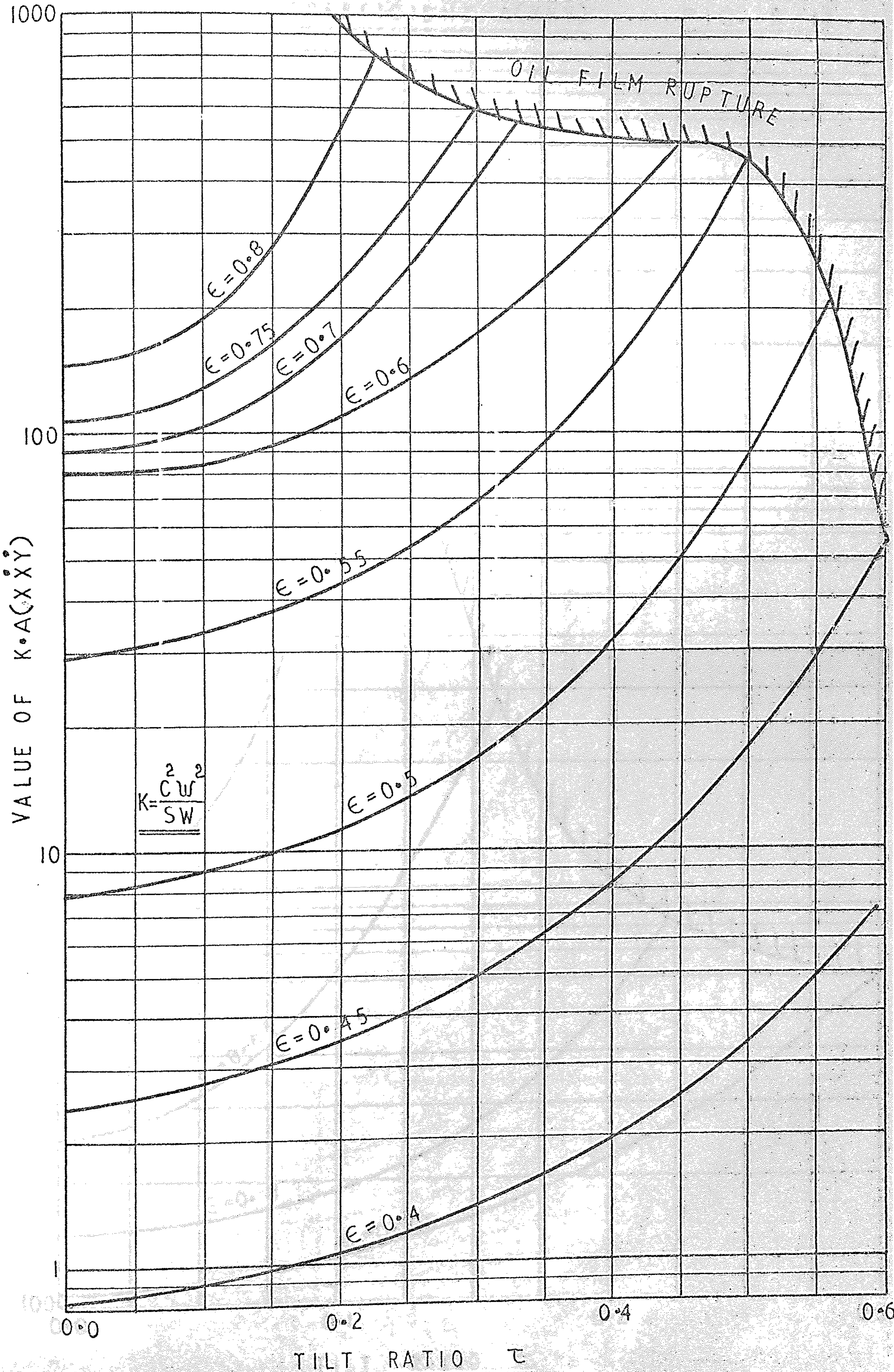
COEFFICIENT $A(X\dot{Y}^2)$ FIG. B - 39.



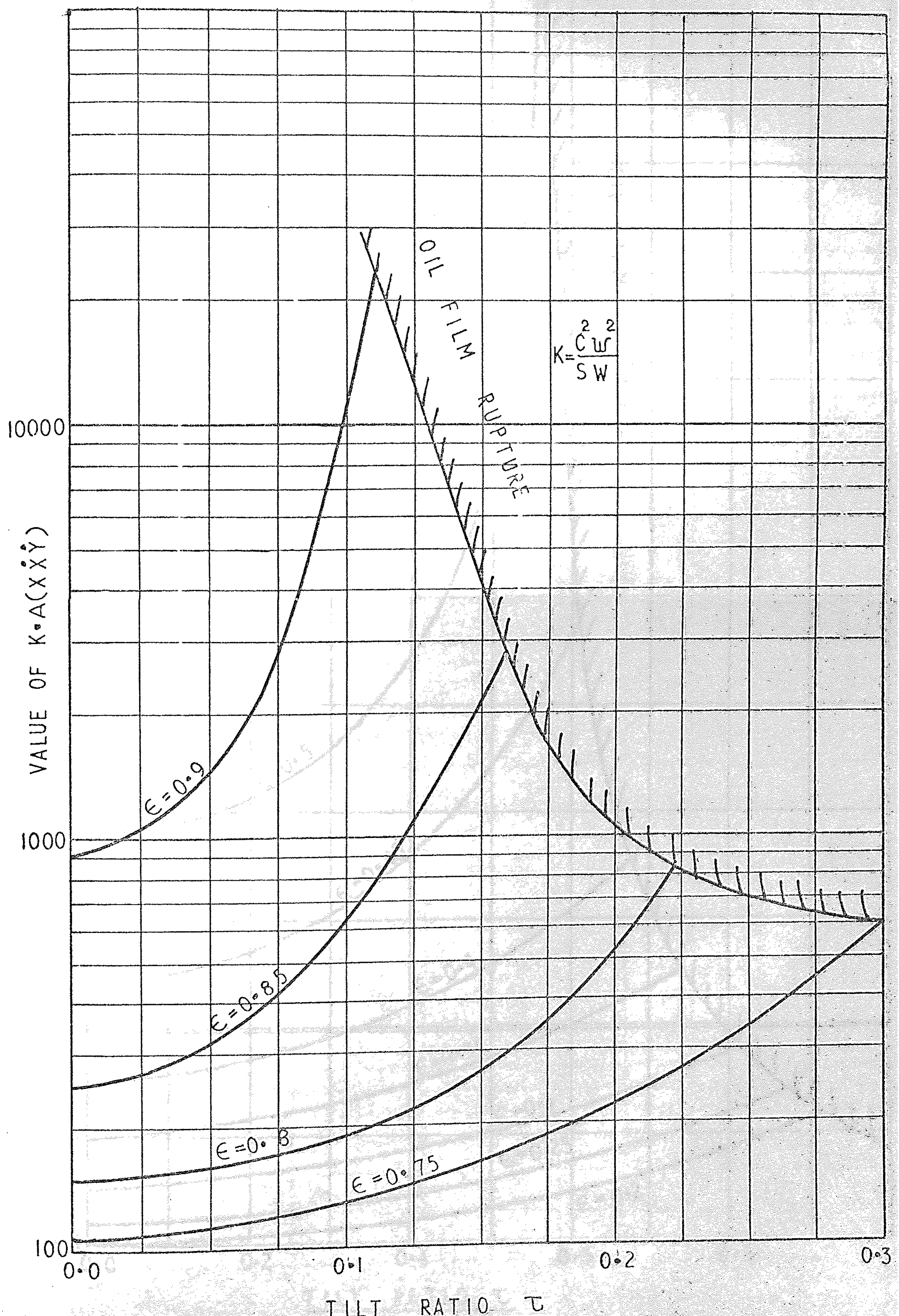
COEFFICIENT A(X Ẋ Ẏ) FIG. B-40.



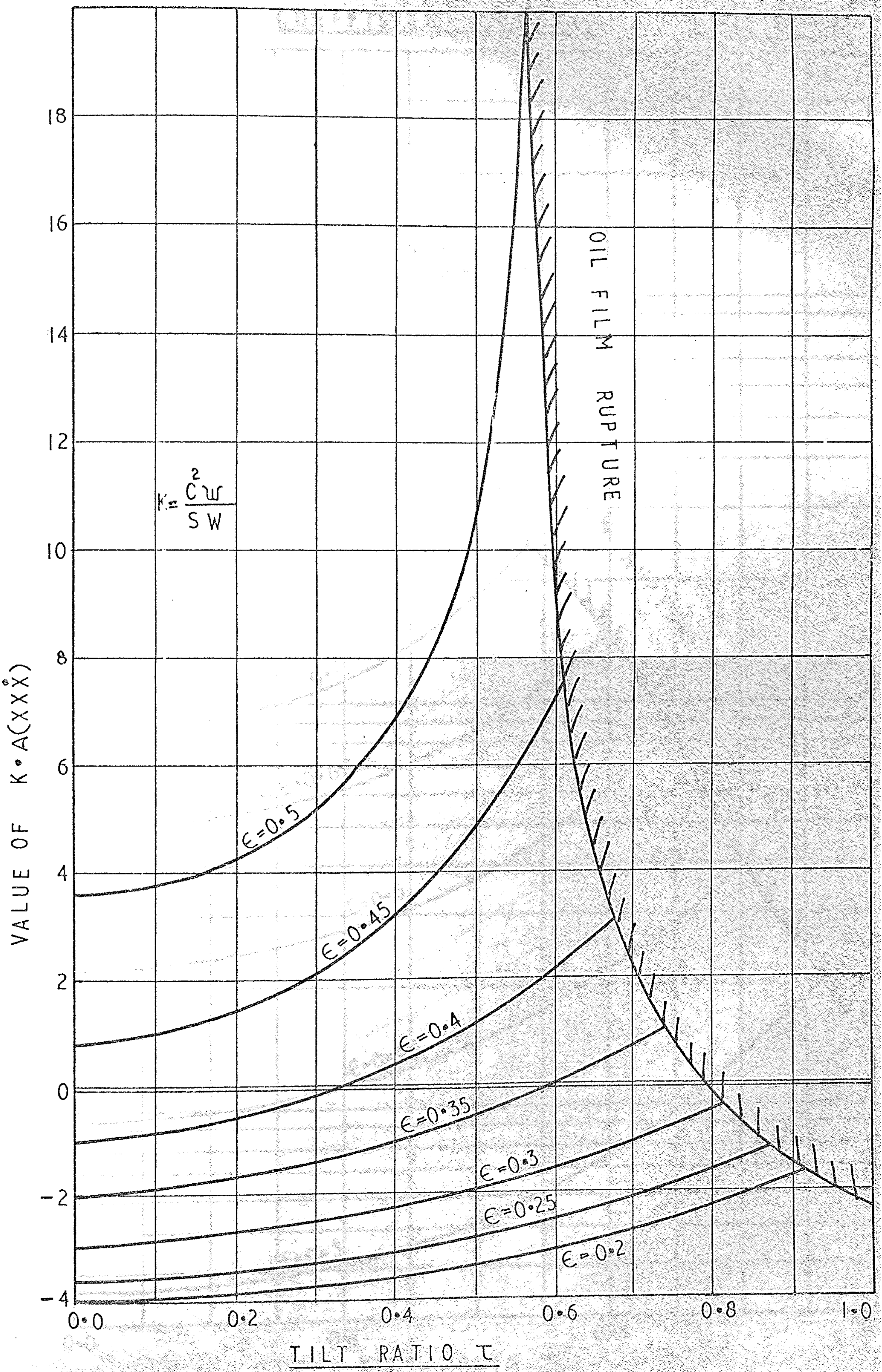
COEFFICIENT A(XẊẎ)   FIG. B-41.



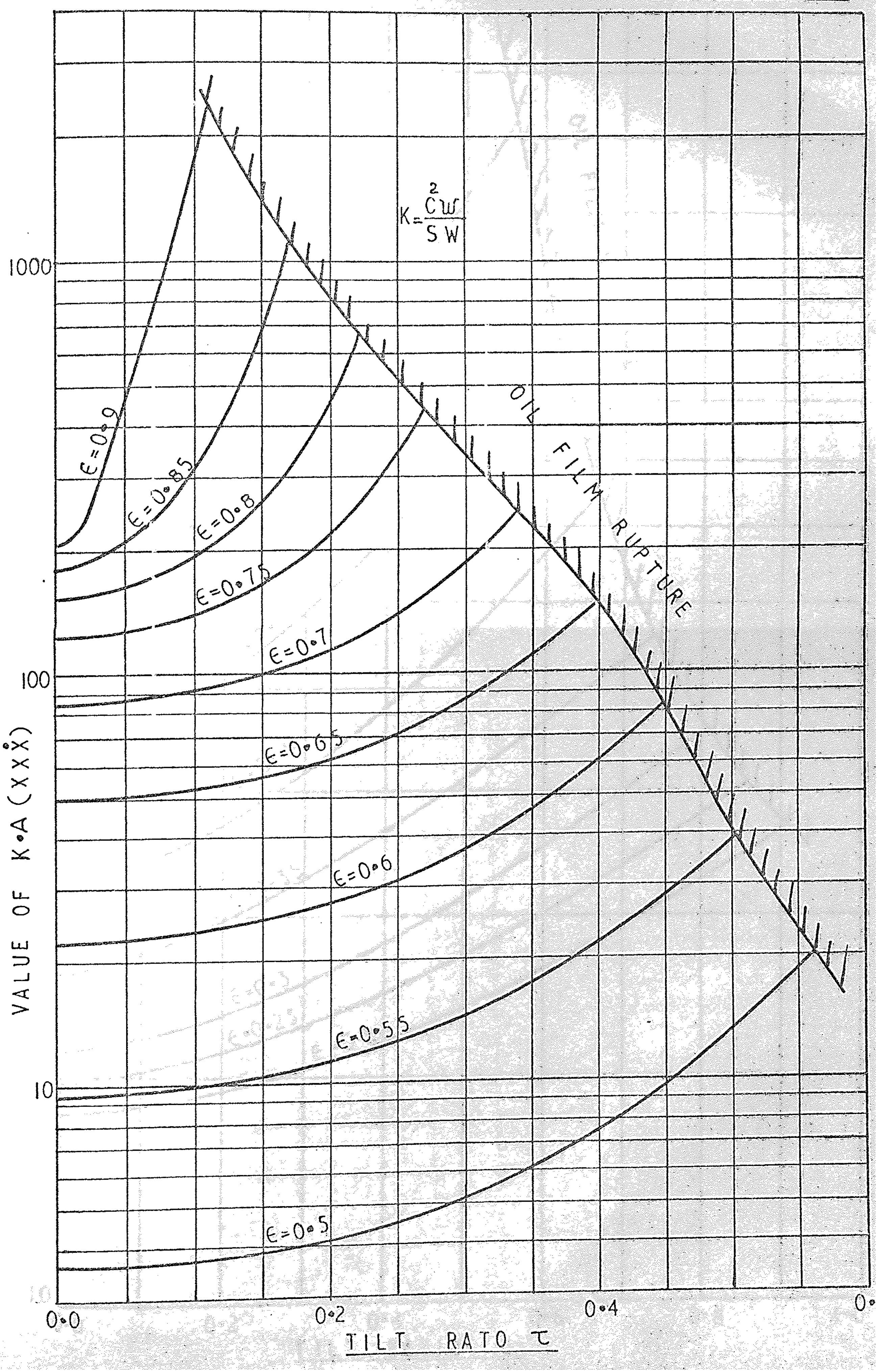
COEFFICIENT $A(X\dot{X}\dot{Y})$ FIG. B-42



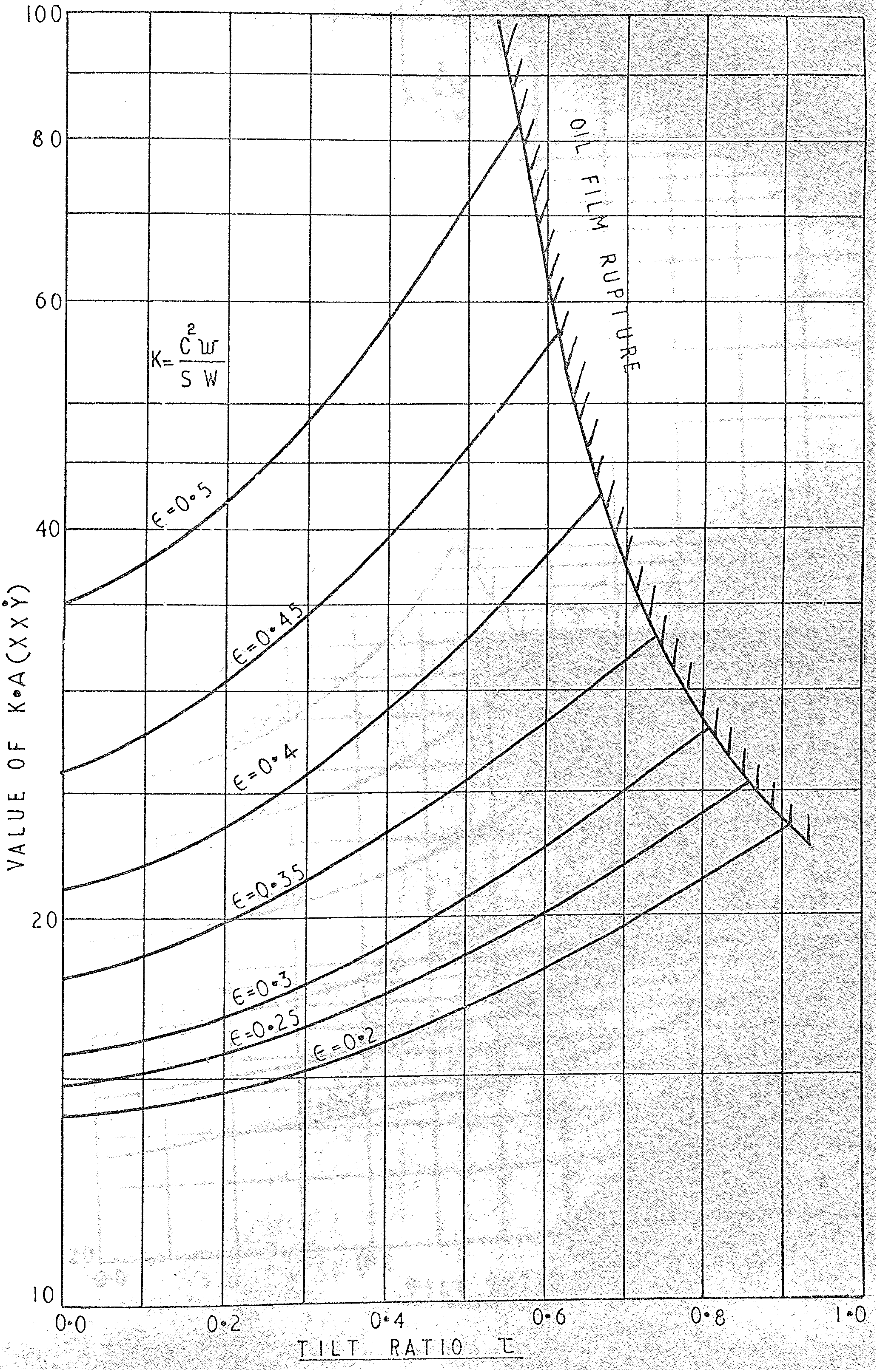
COEFFICIENT A(XXẊ) FIG. B - 43.



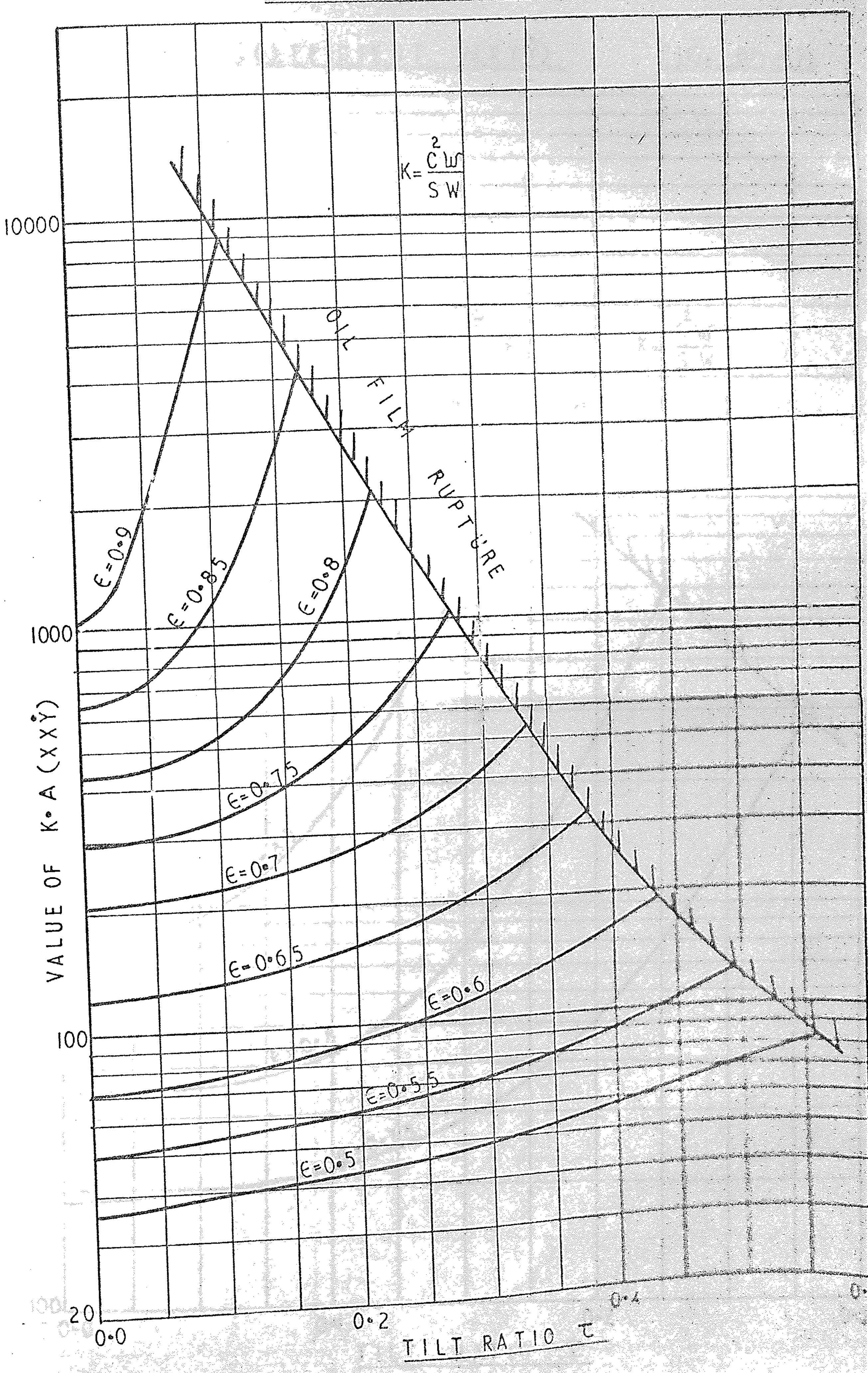
COEFFICIENT A(XXẊ) FIG. B-44.



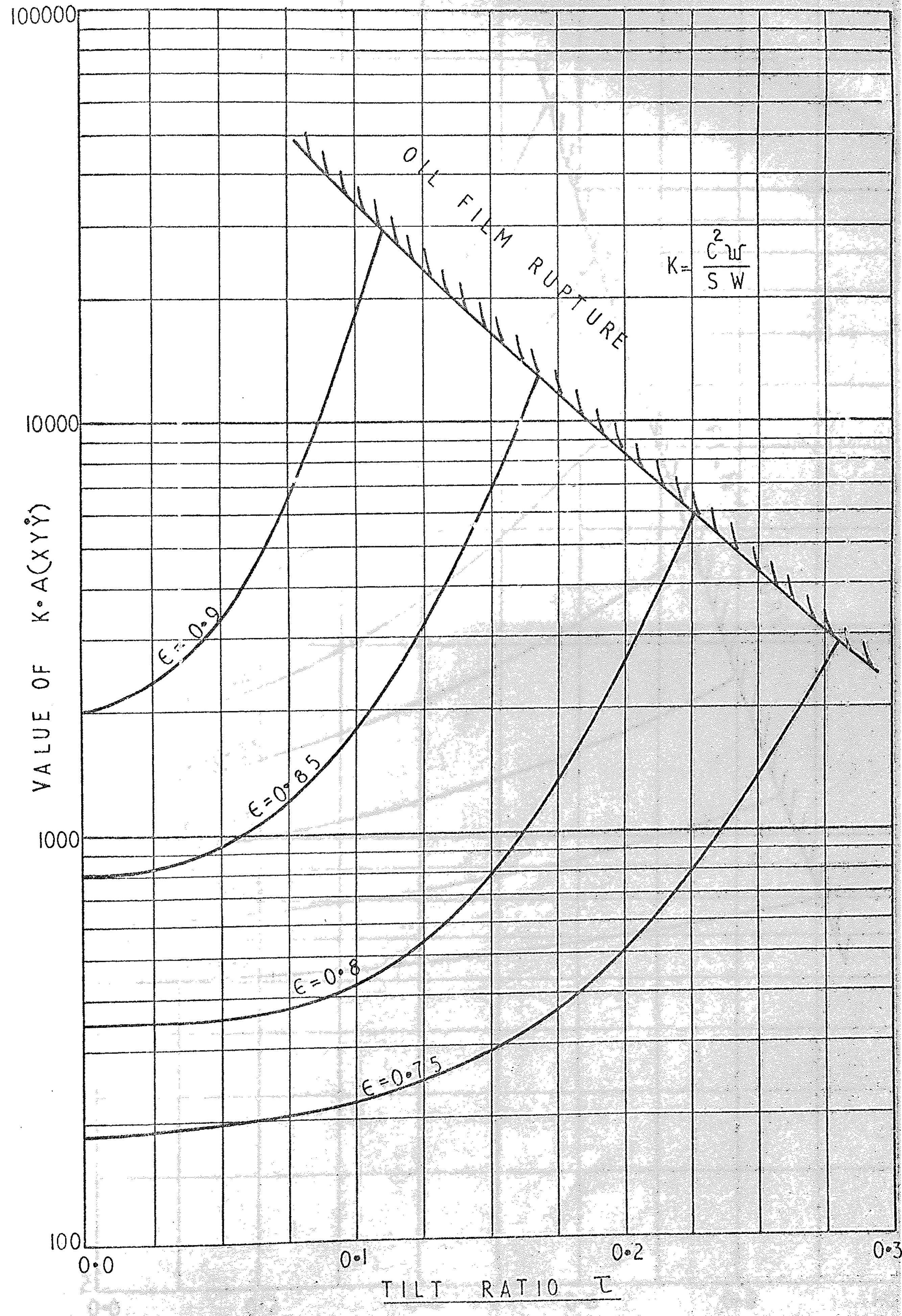
COEFFICIENT A $(XX\dot{Y})$ FIG. B-45.



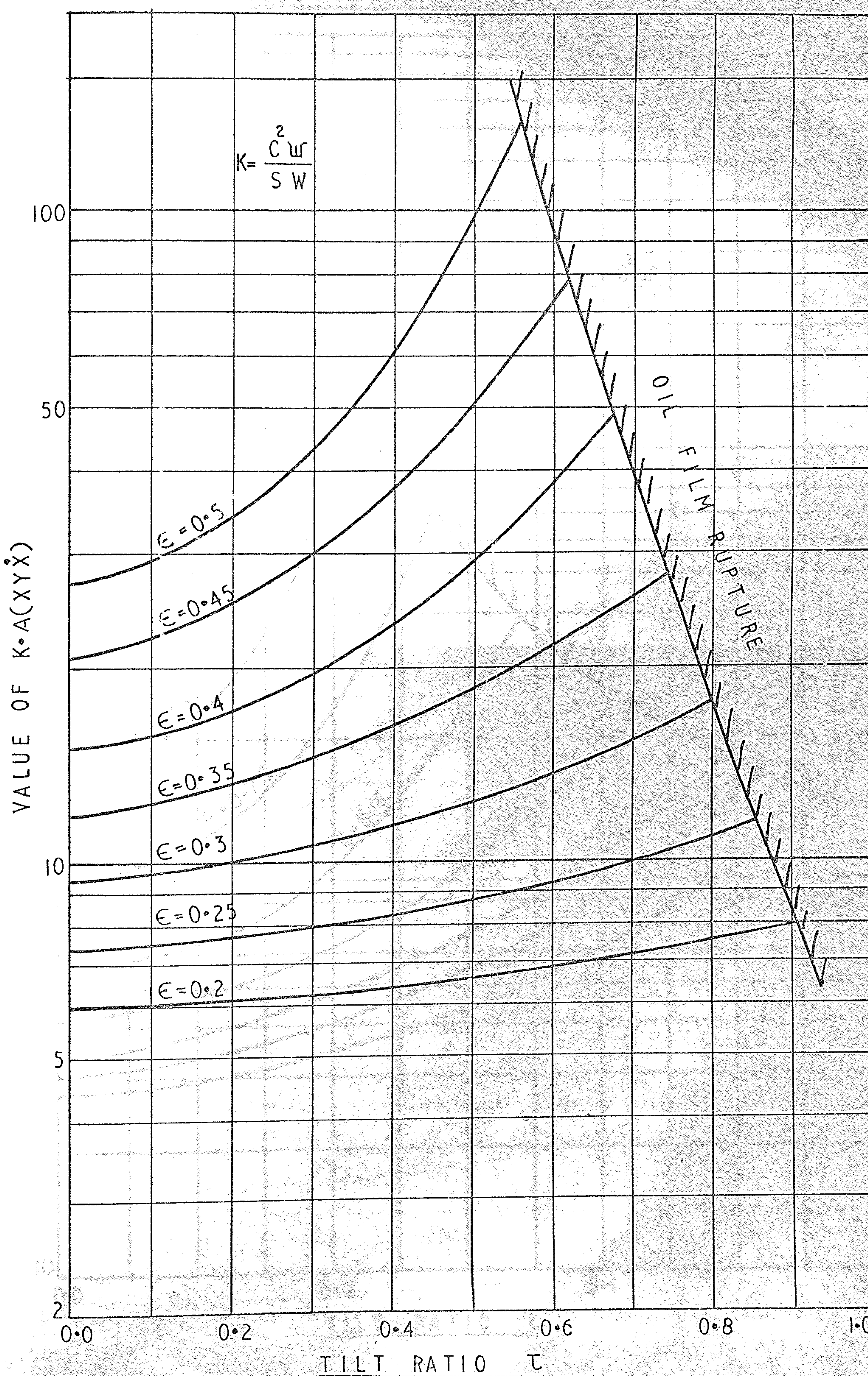
COEFFICIENT A(XXẎ) FIG. B-46.



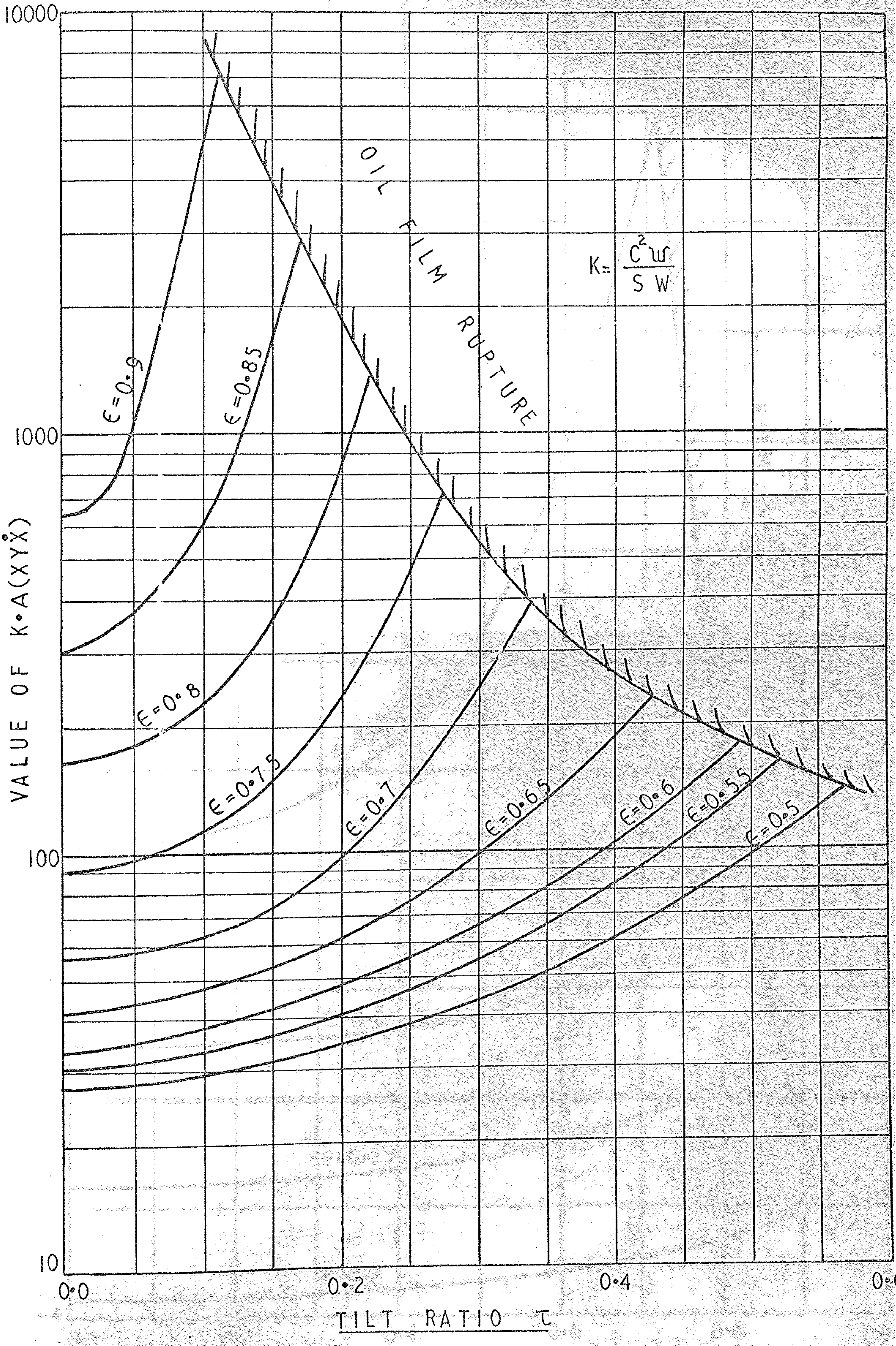
COEFFICIENT A(XYẎ) FIG. B - 47.



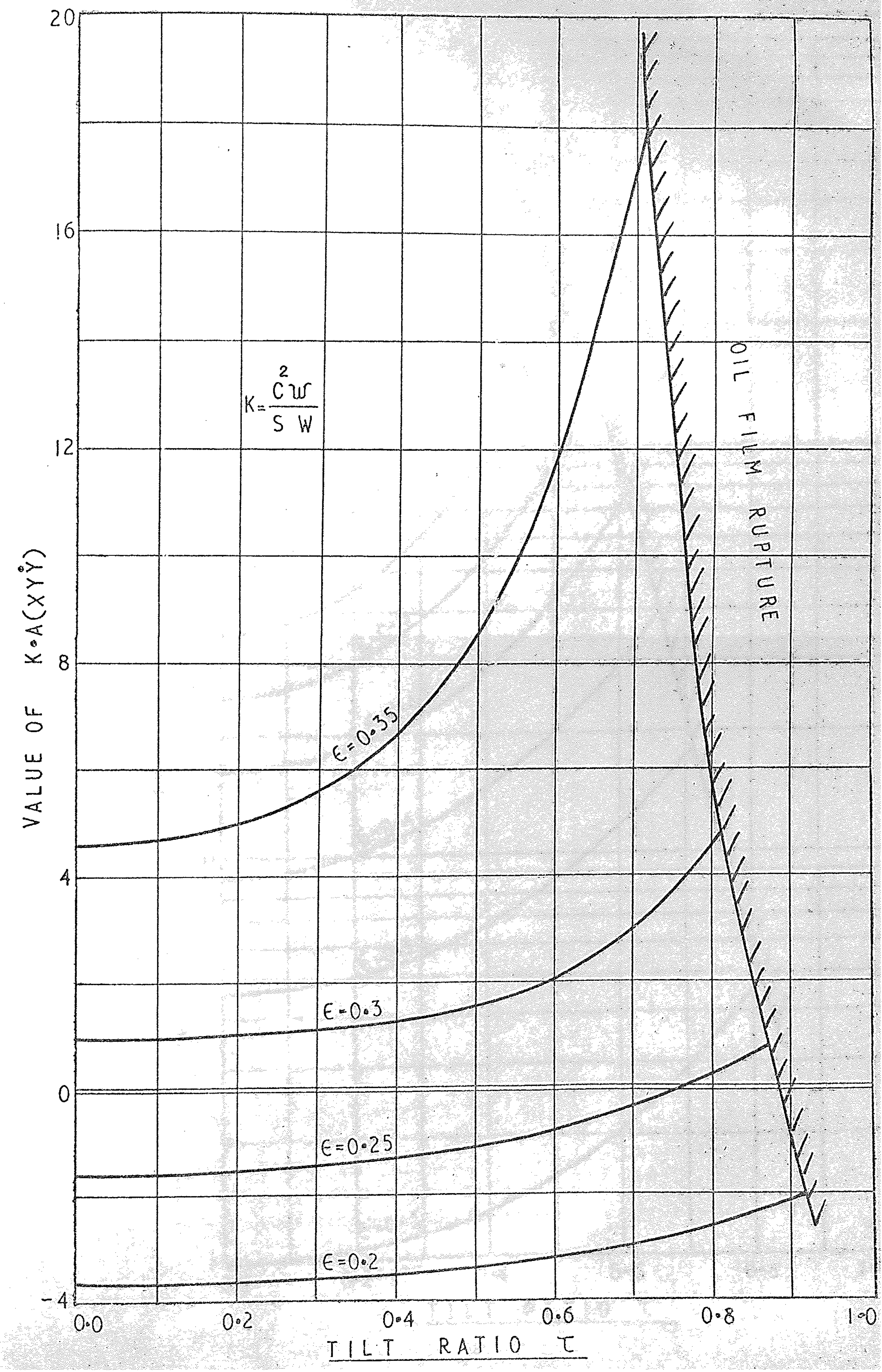
COEFFICIENT A(XYẊ) FIG. B-48.



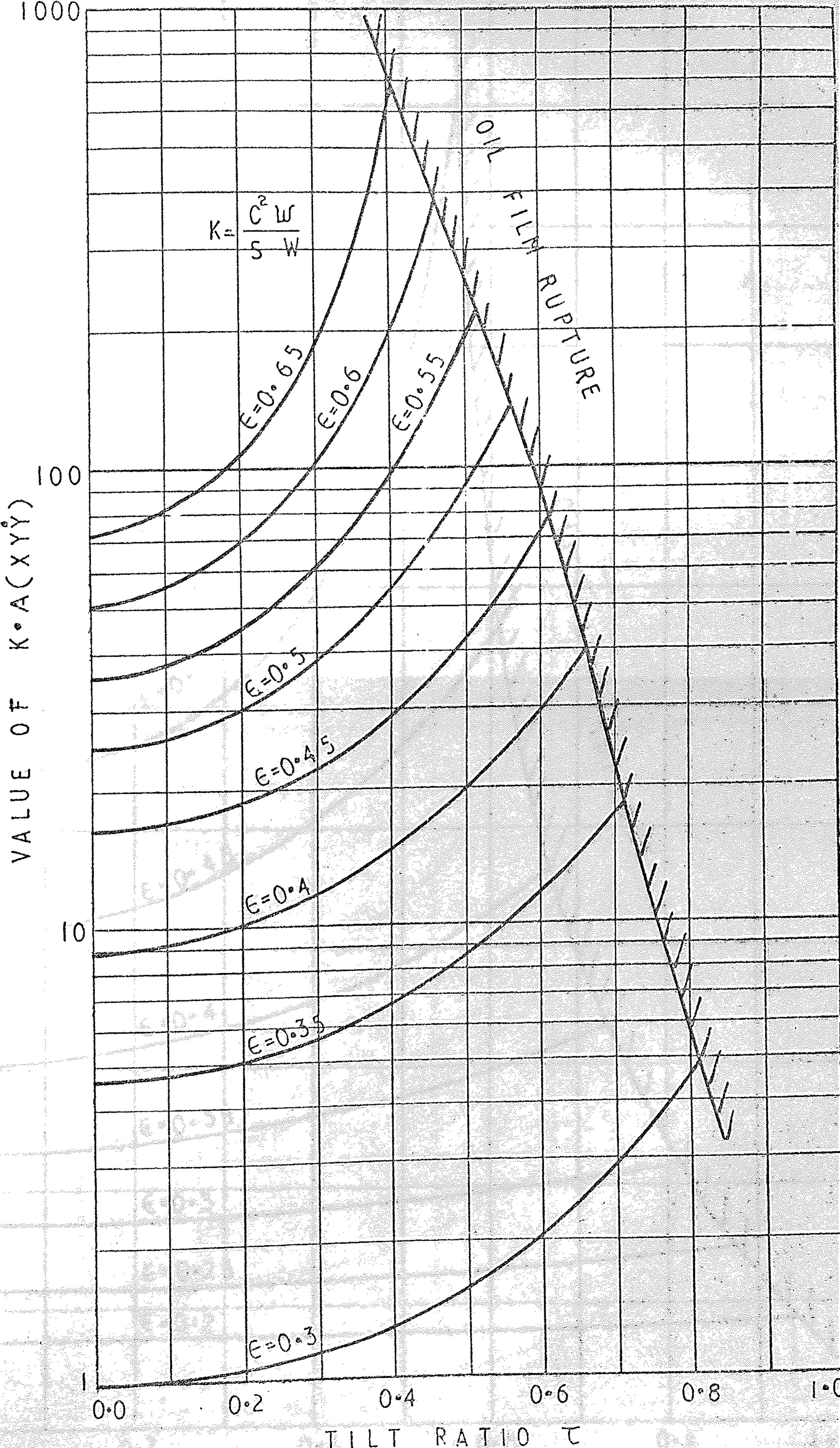
COEFFICIENT A(XYẊ) FIG. B-49.



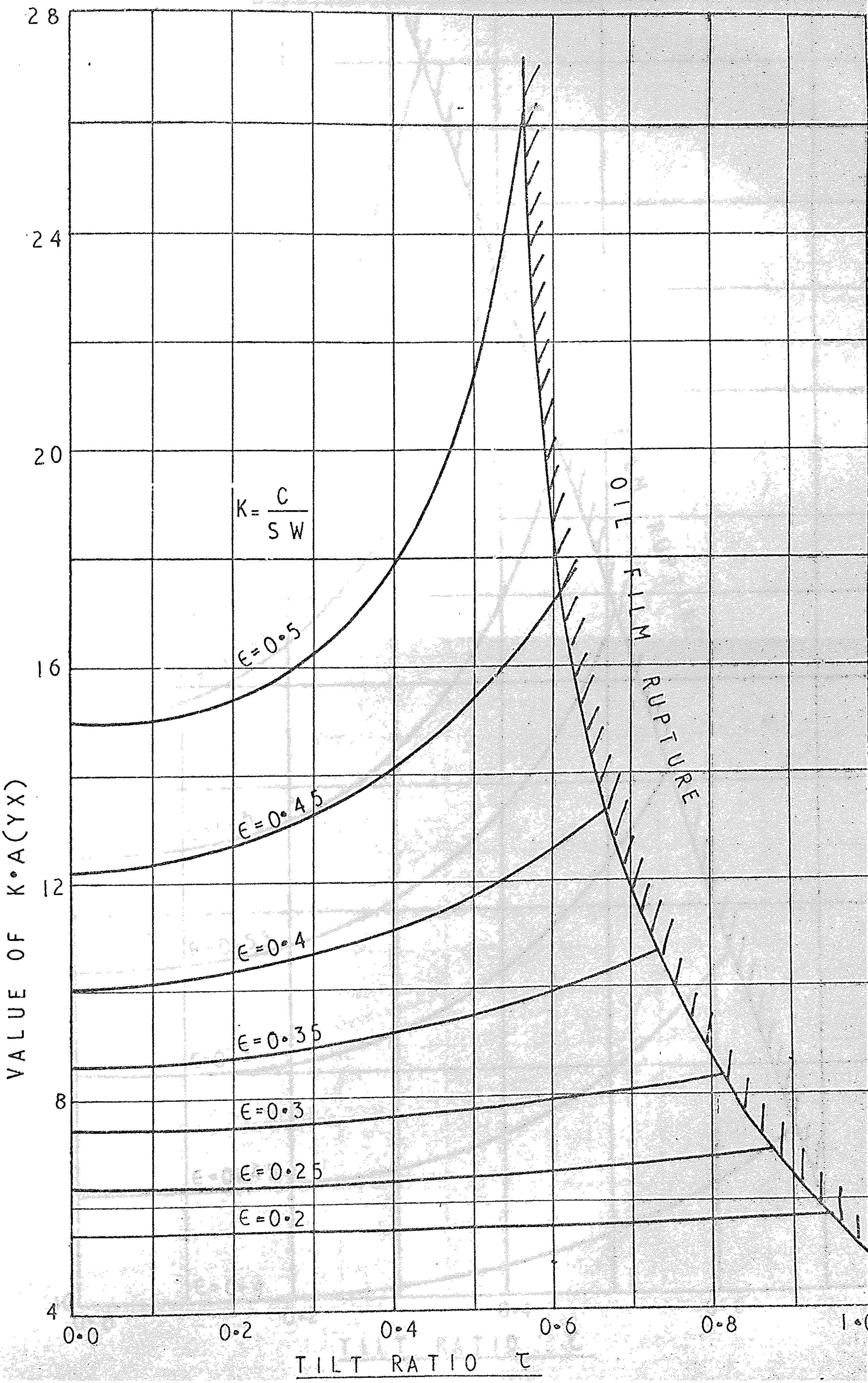
COEFFICIENT A(XYẎ) FIG. B - 50.



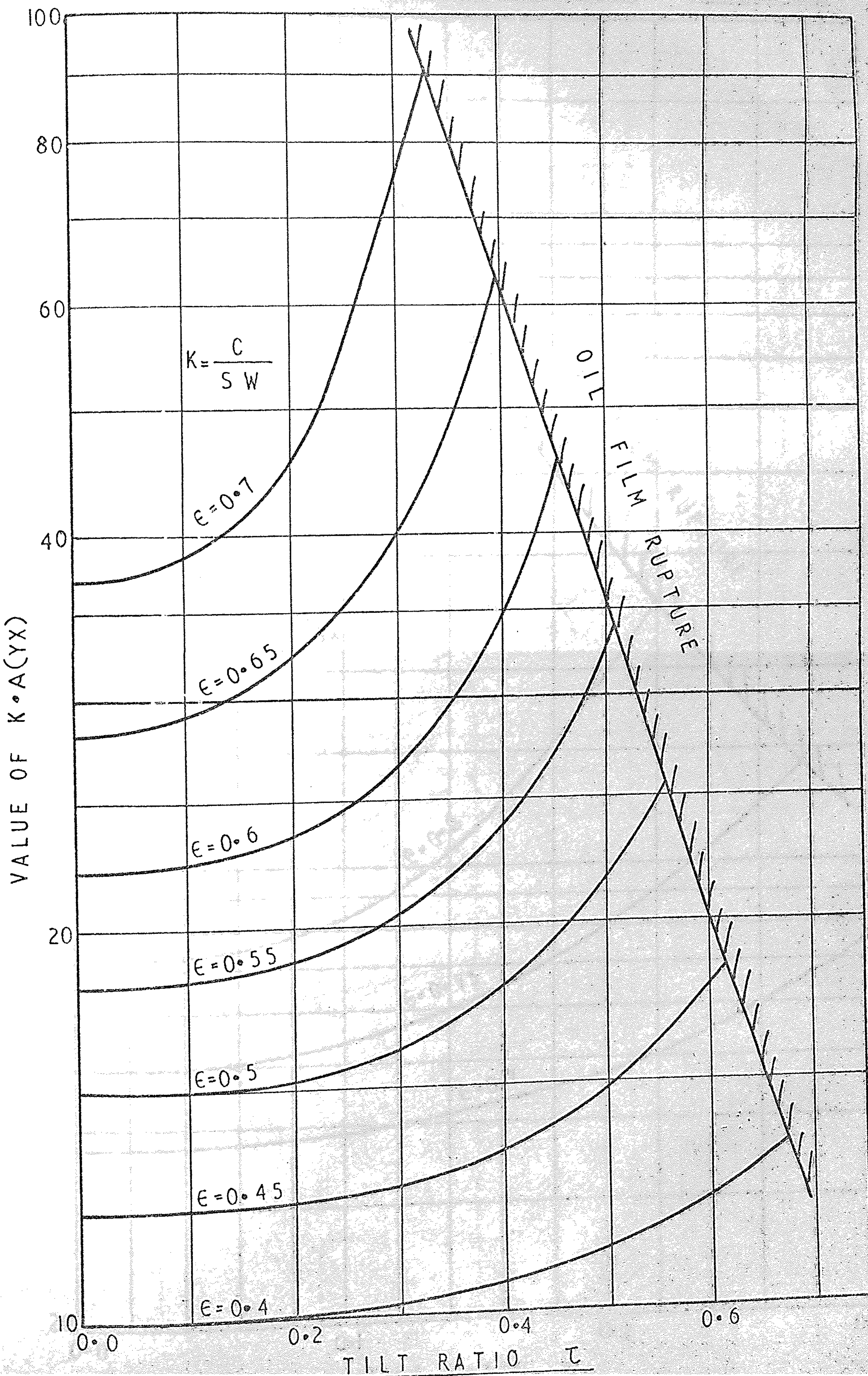
COEFFICIENT A(XYẎ) FIG. B-51.



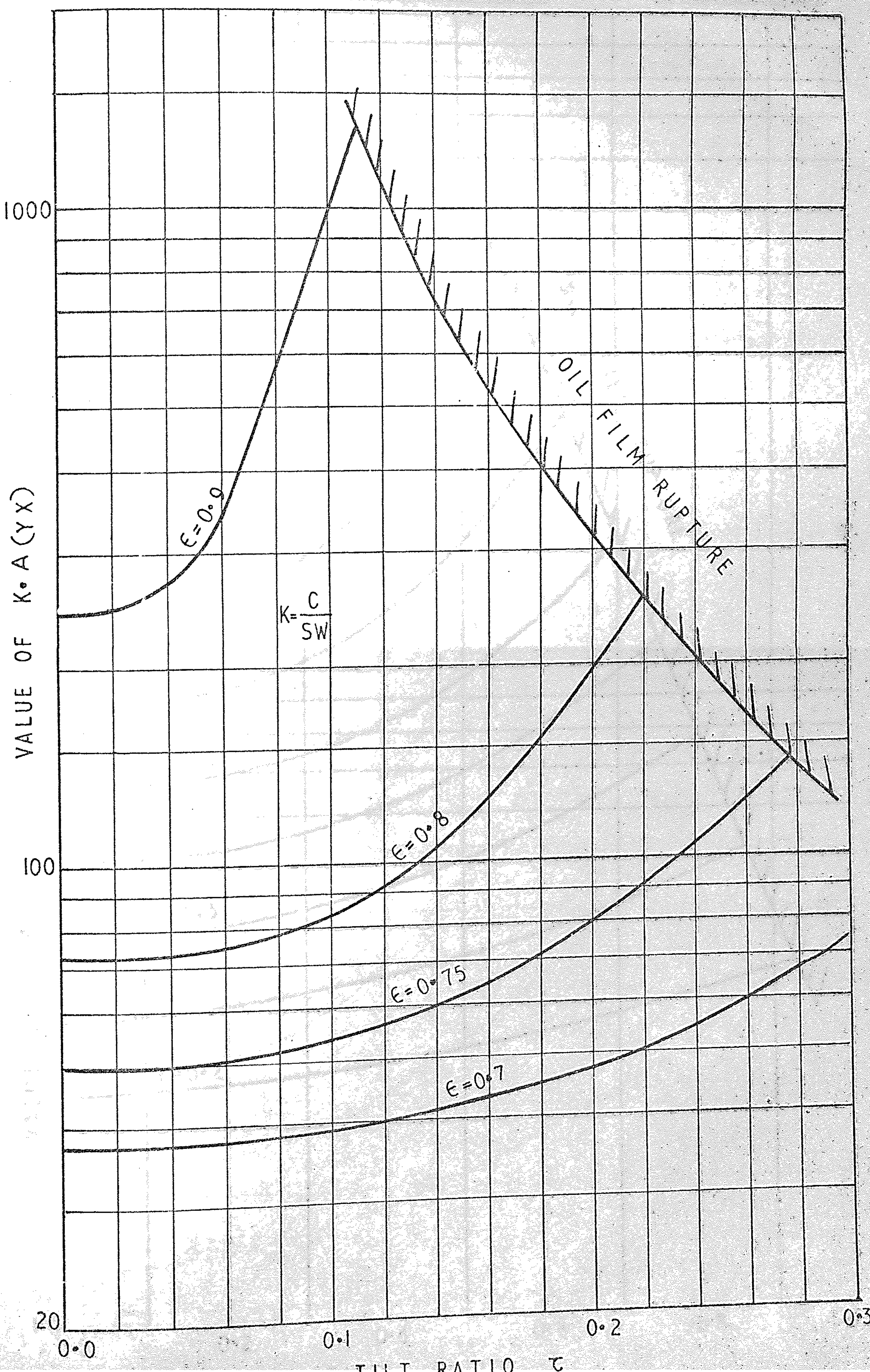
COEFFICIENT A(YX) FIG. B-52.



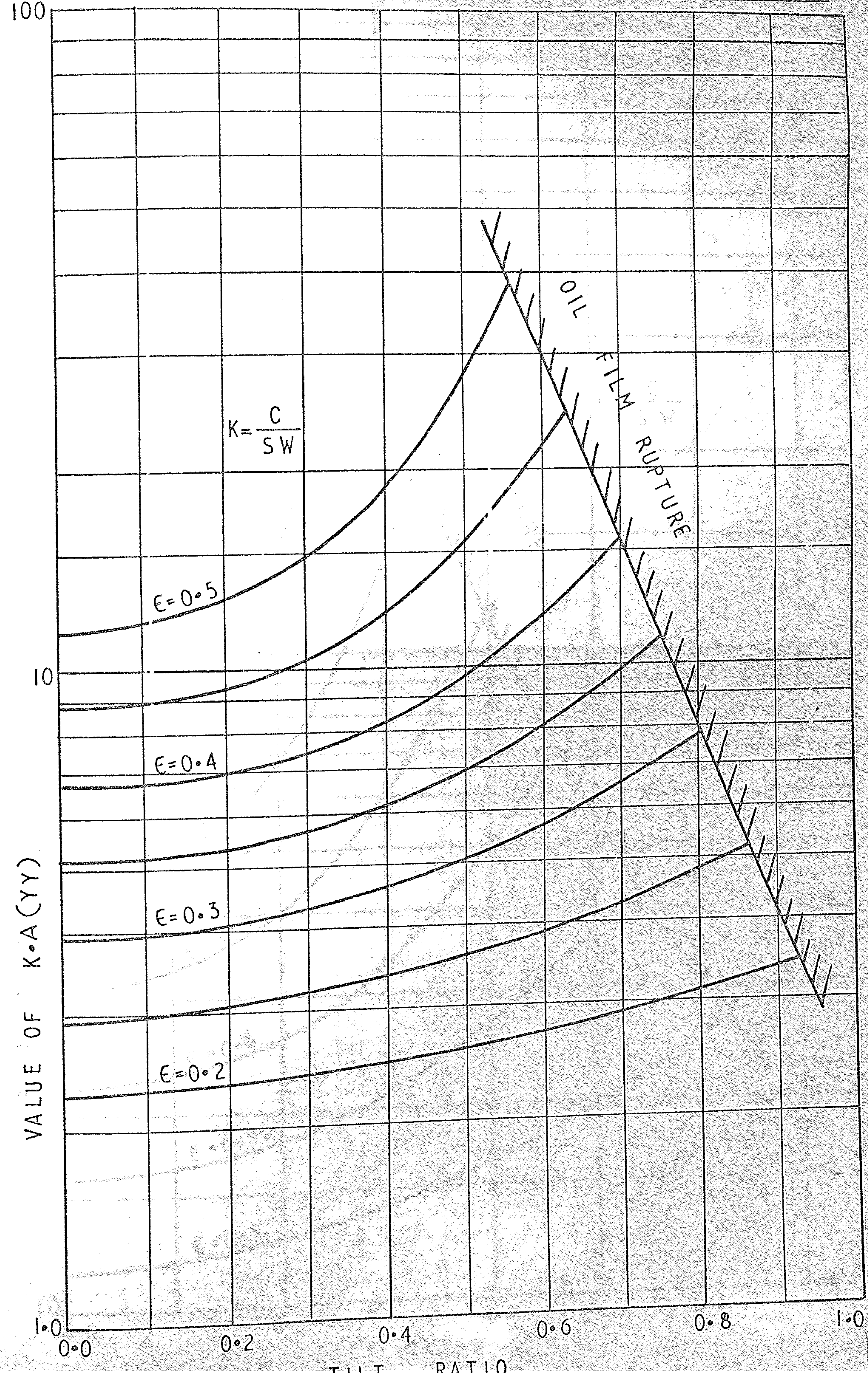
COEFFICIENT A(YX) FIG. B-53



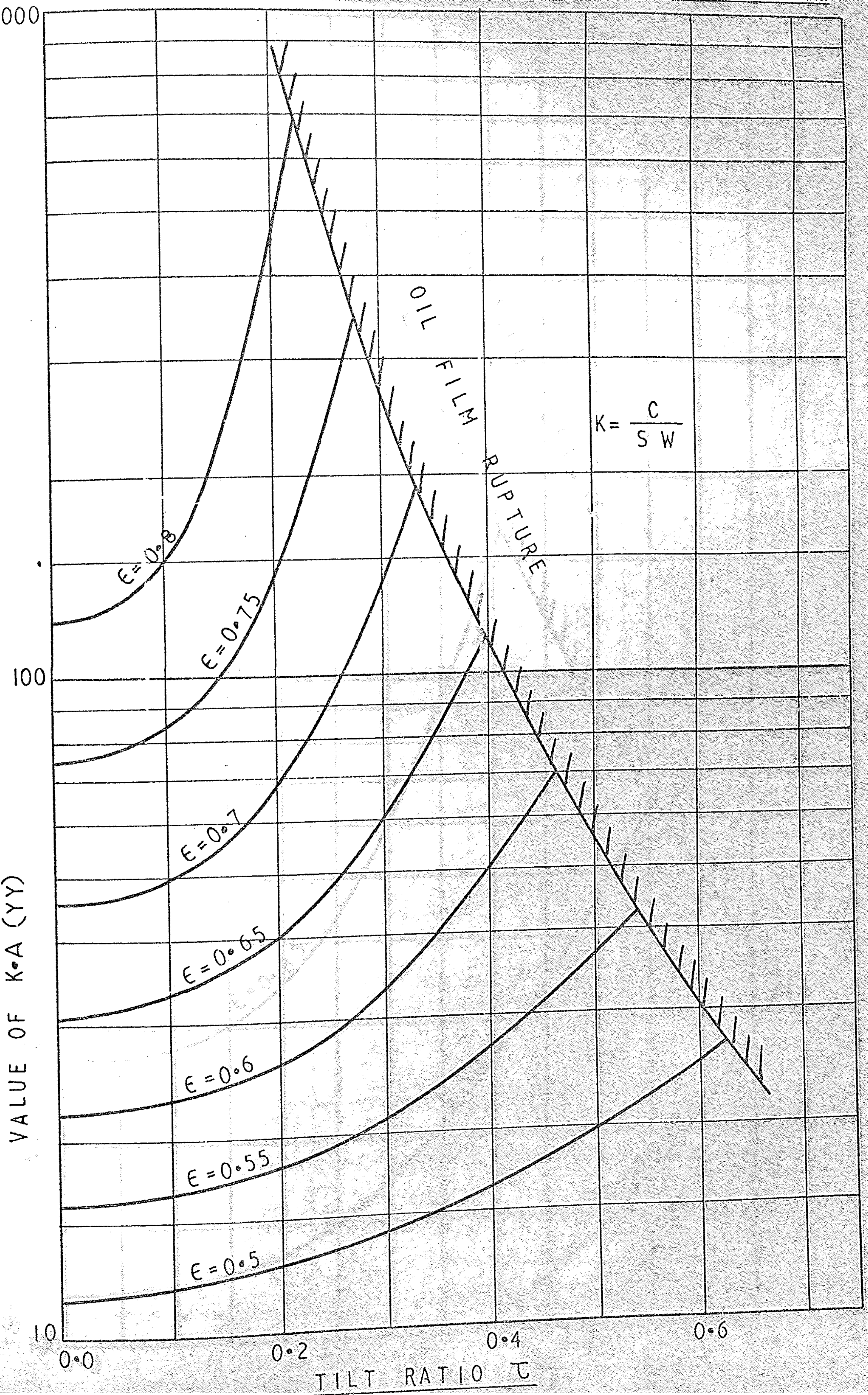
COEFFICIENT A (YX) FIG. B-54.



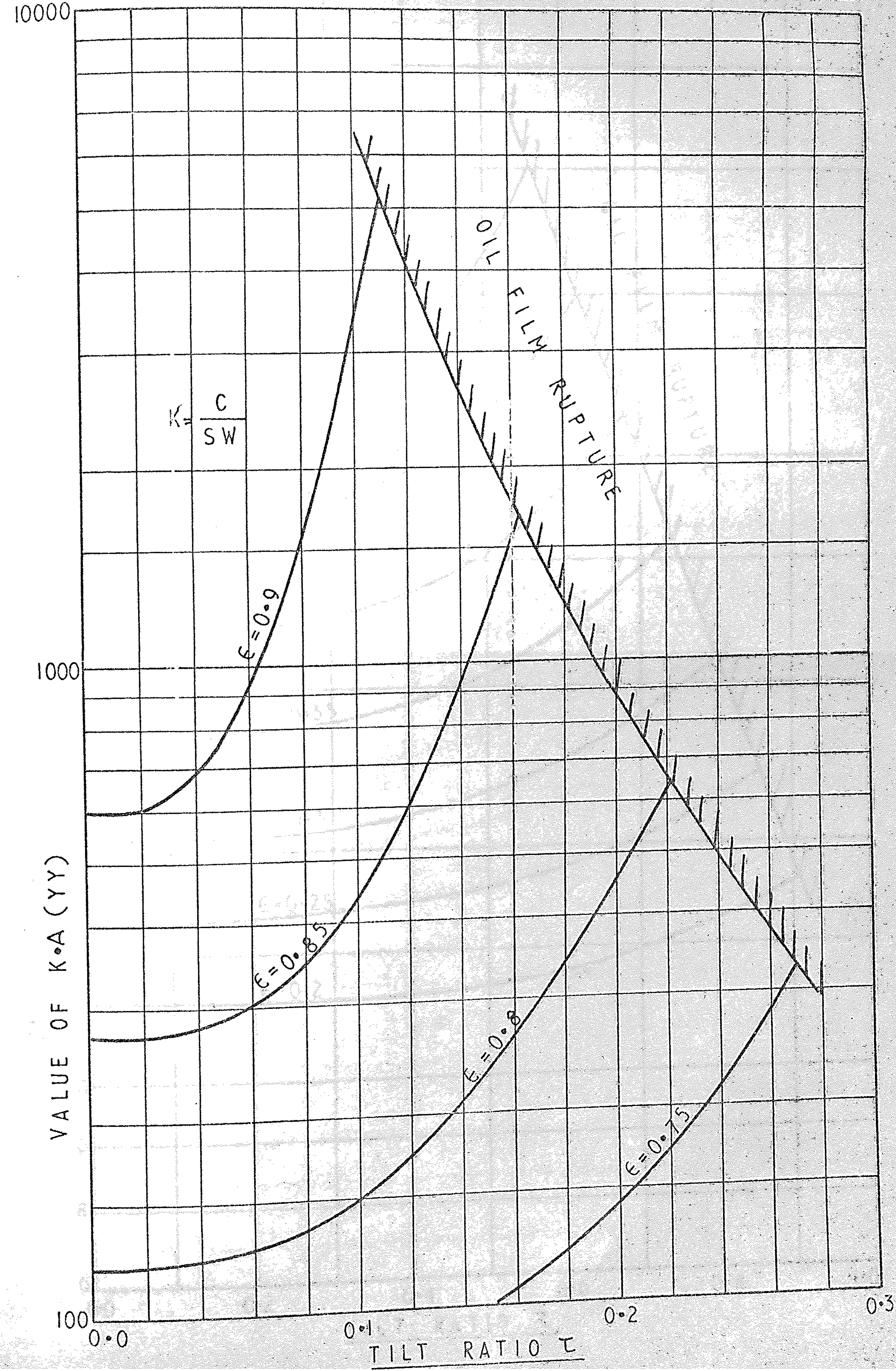
COEFFICIENT A(YY) FIG. B - 55.



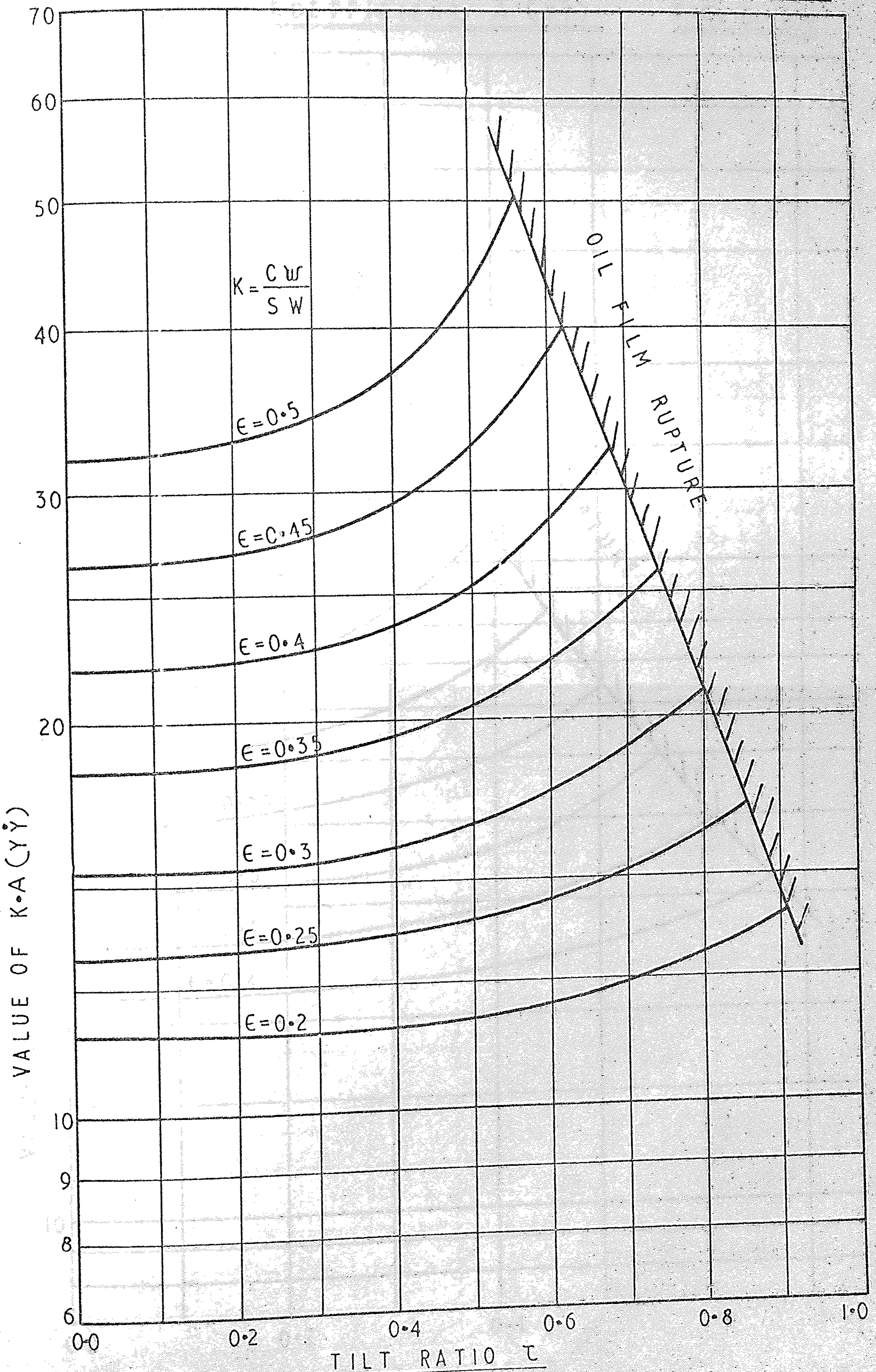
COEFFICIENT A (YY) FIG. B-56.



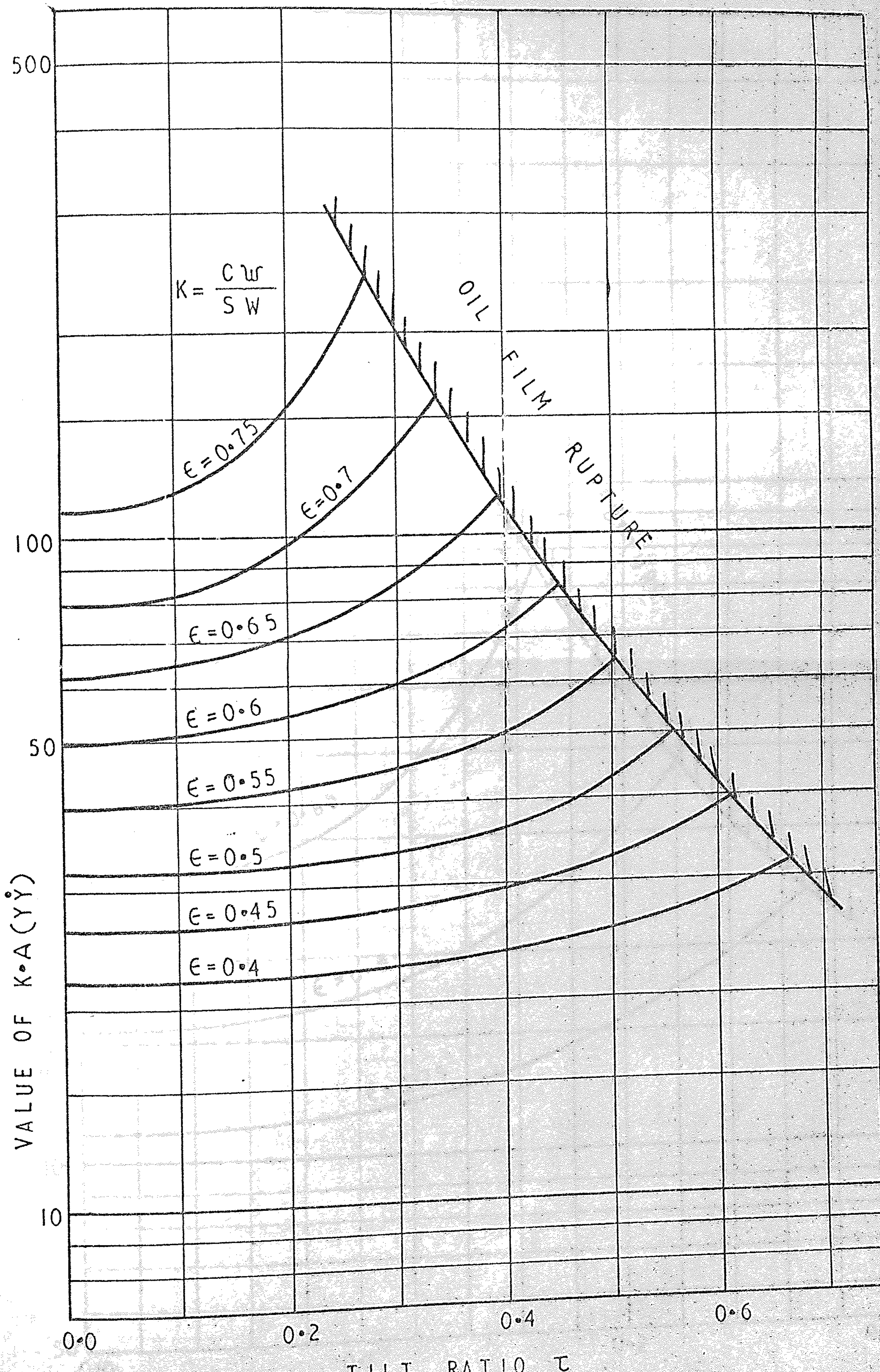
COEFFICIENT A (YY) FIG. B-57.



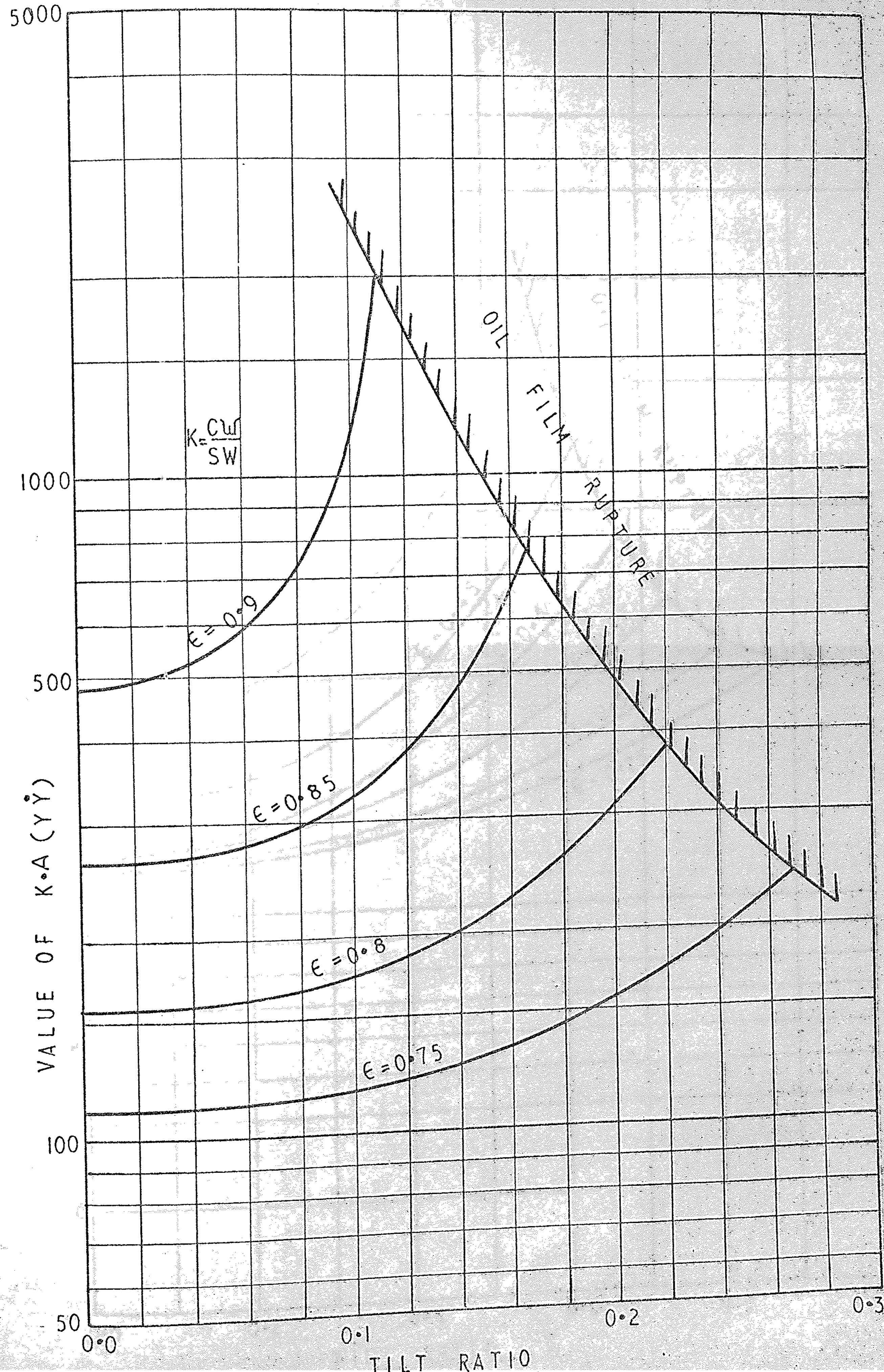
COEFFICIENT A(YẎ) FIG. B - 58.



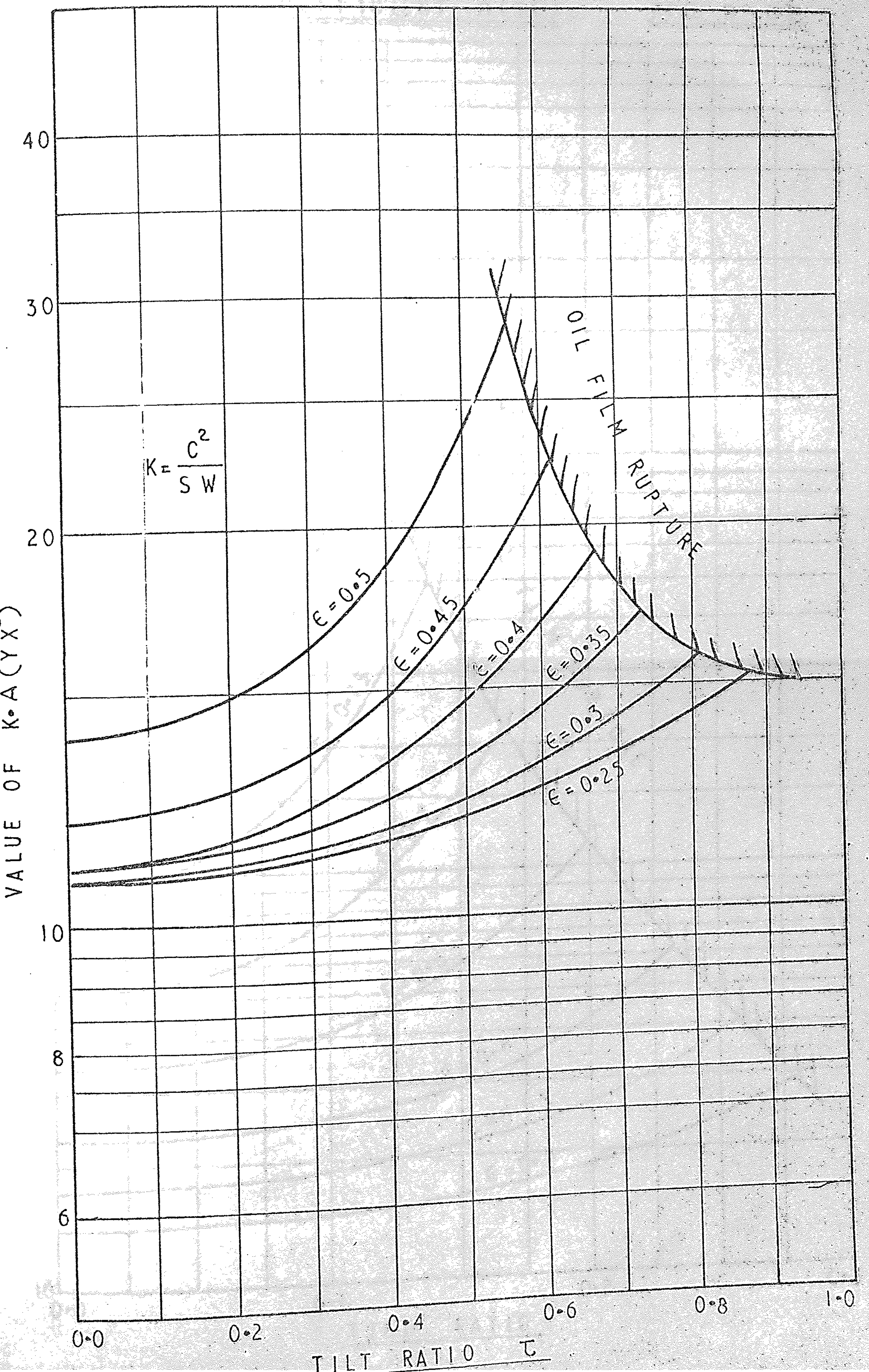
COEFFICIENT A(YẎ) FIG. B.-59.



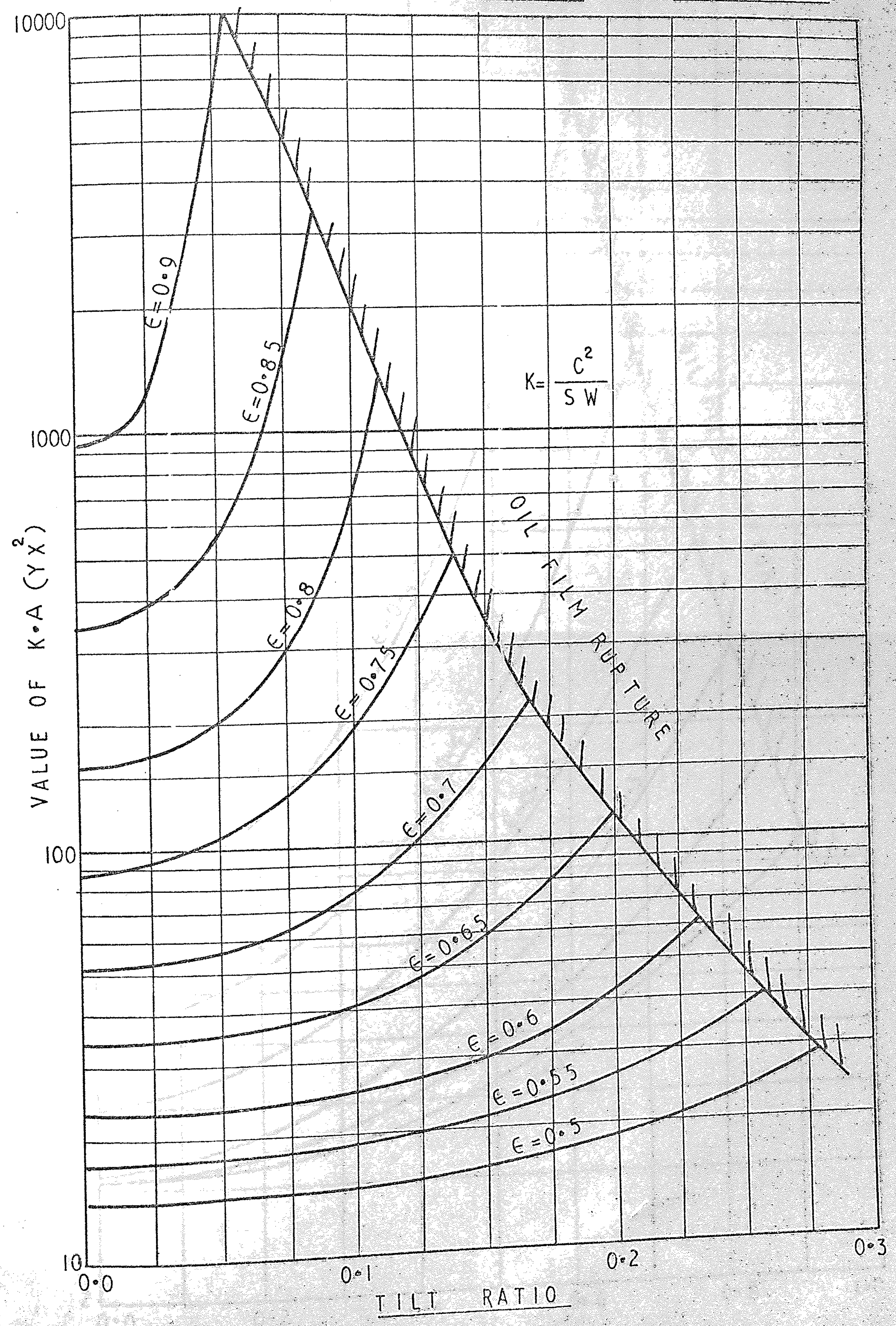
COEFFICIENT A (YẎ) FIG. B-60.



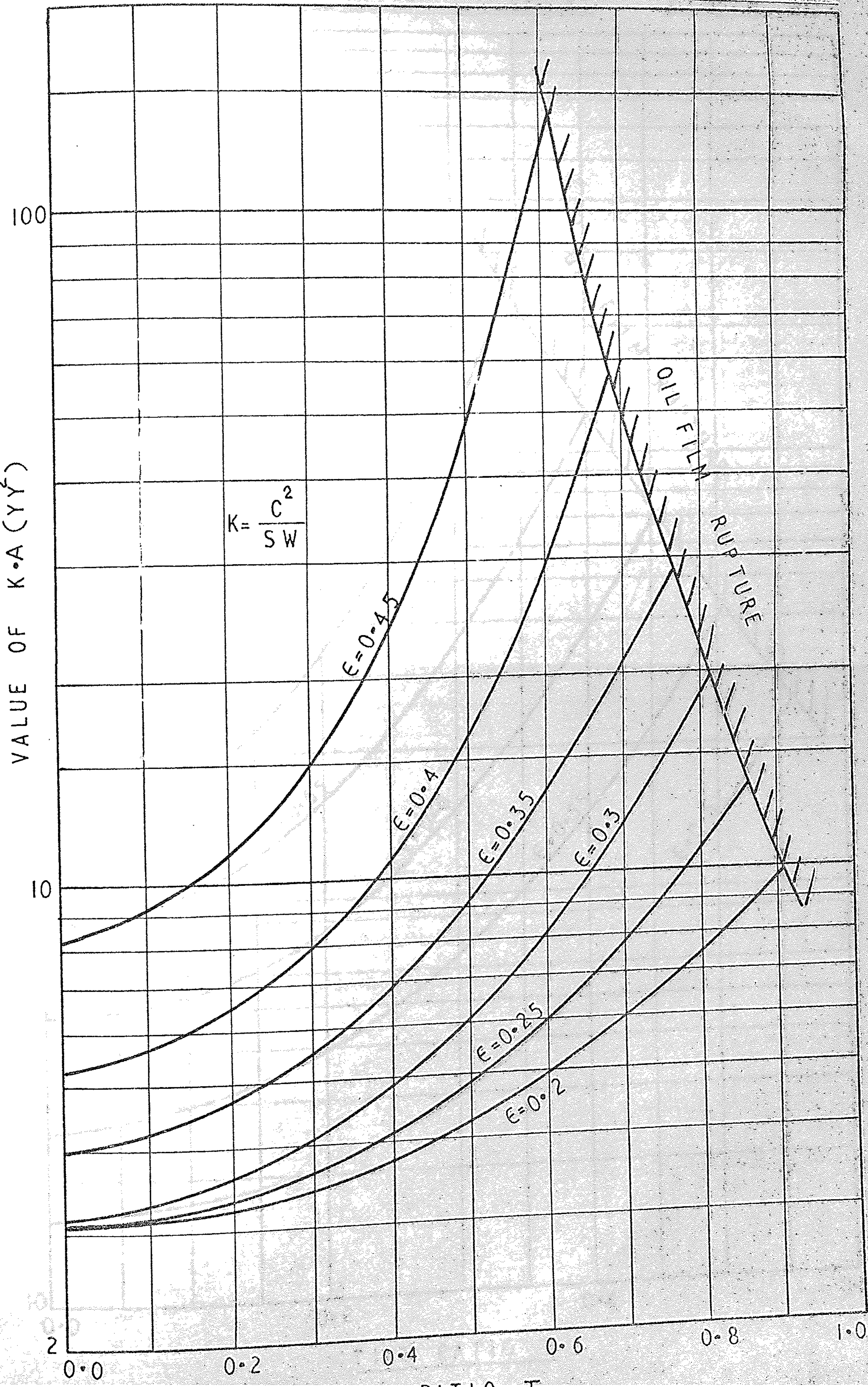
COEFFICIENT $A(YX^2)$ FIG. B-61.



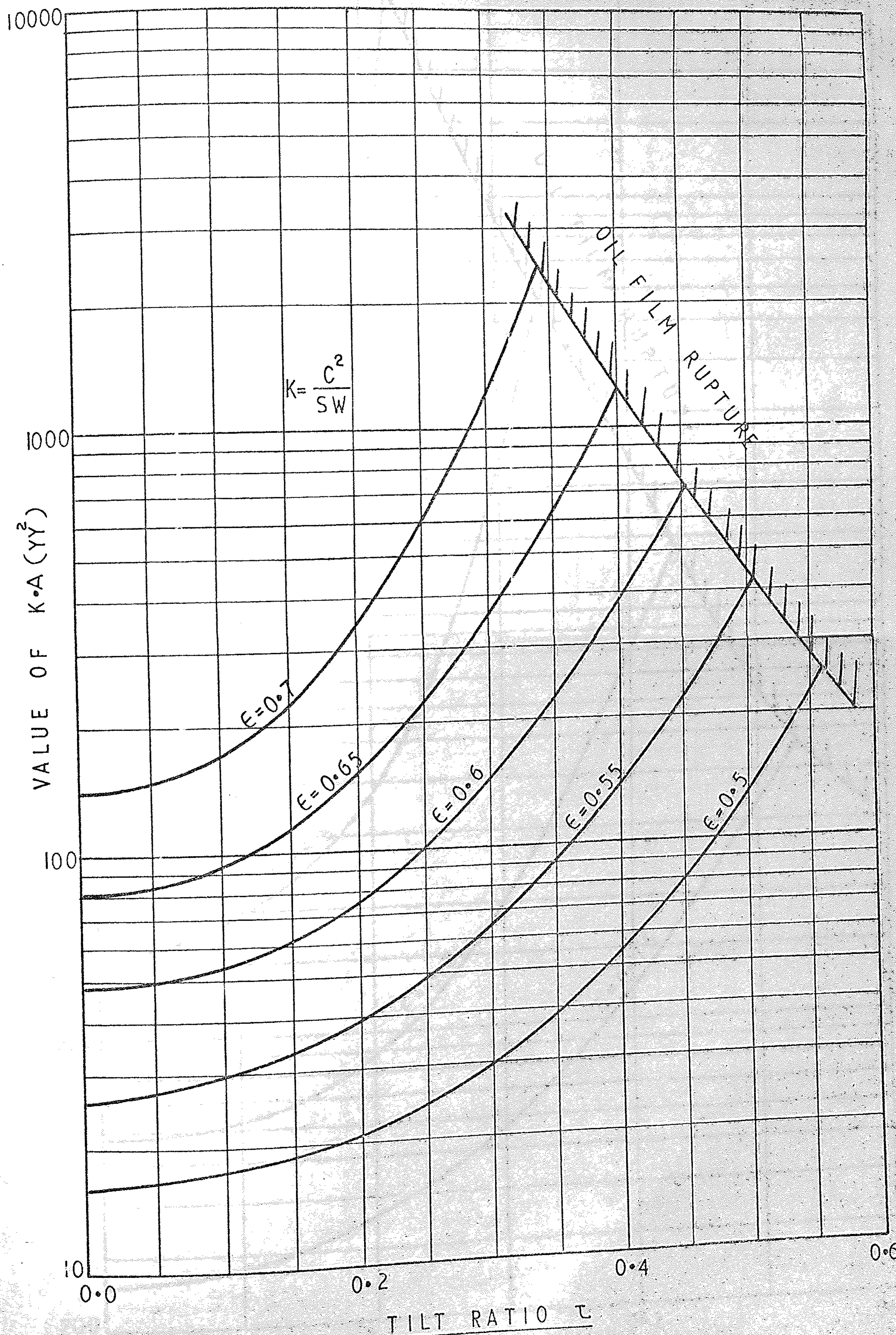
COEFFICIENT $A(YX^2)$ FIG. B-62.



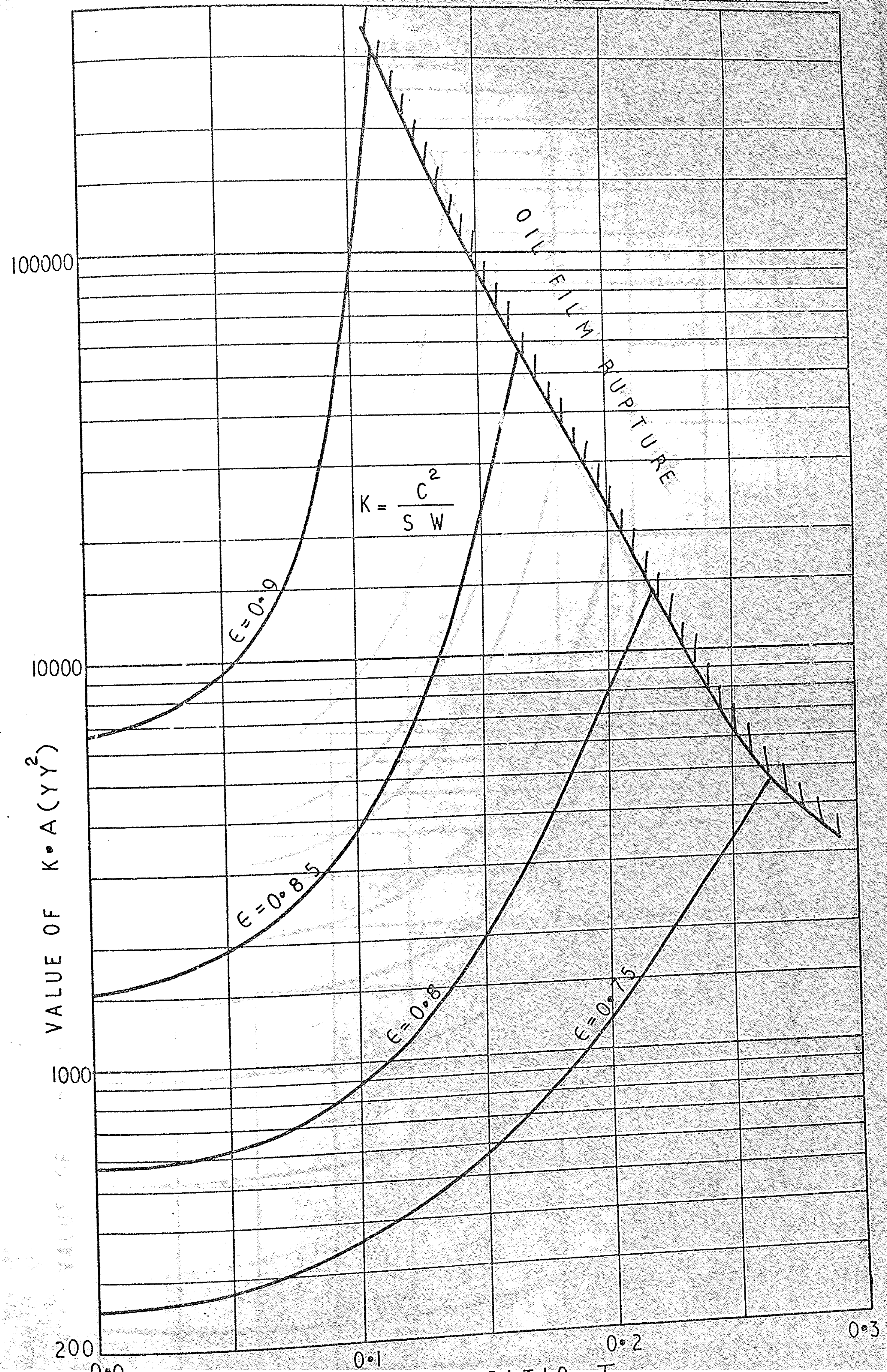
COEFFICIENT A(YY²) FIG. B-63.



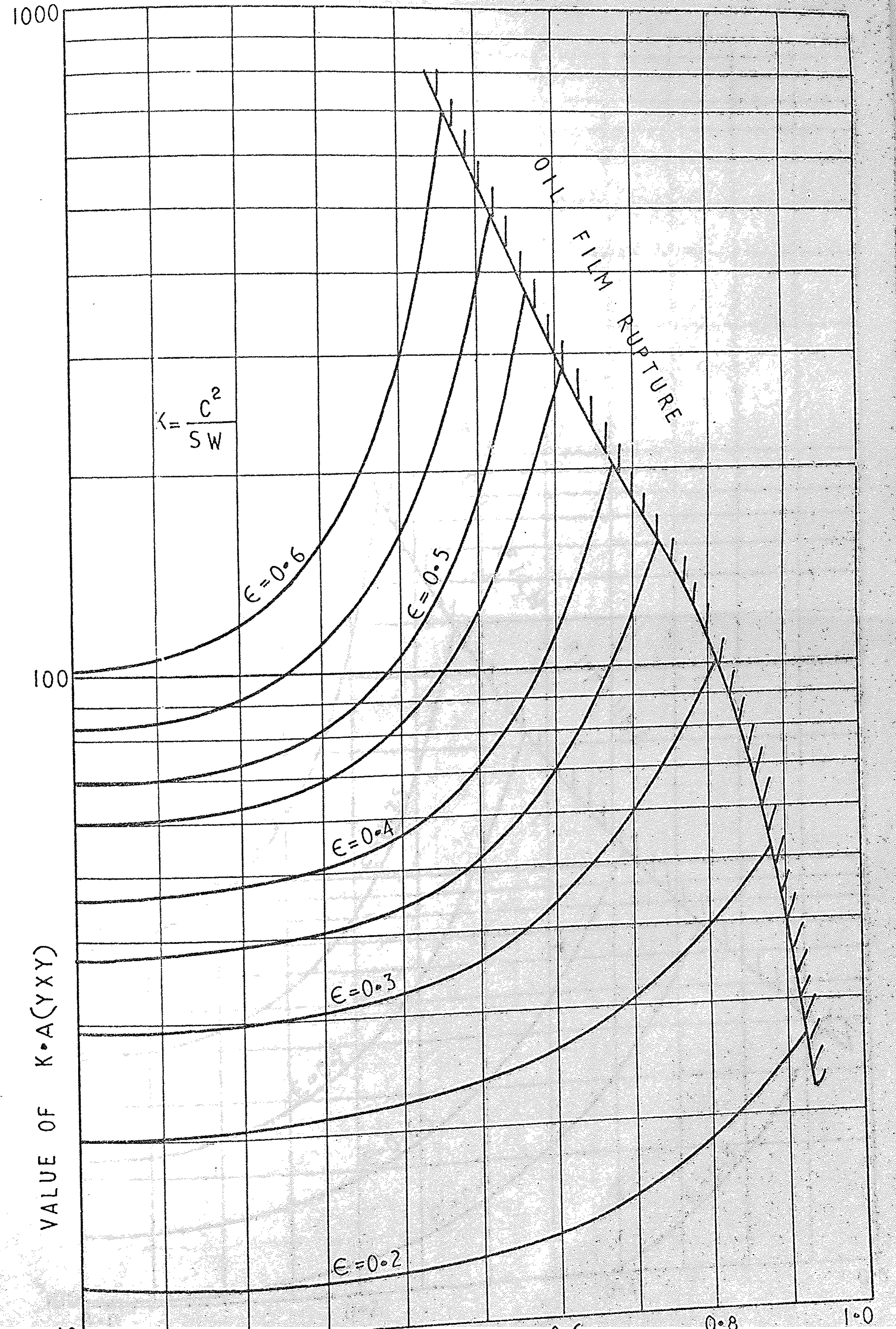
COEFFICIENT A (YY²) FIG. B-64.



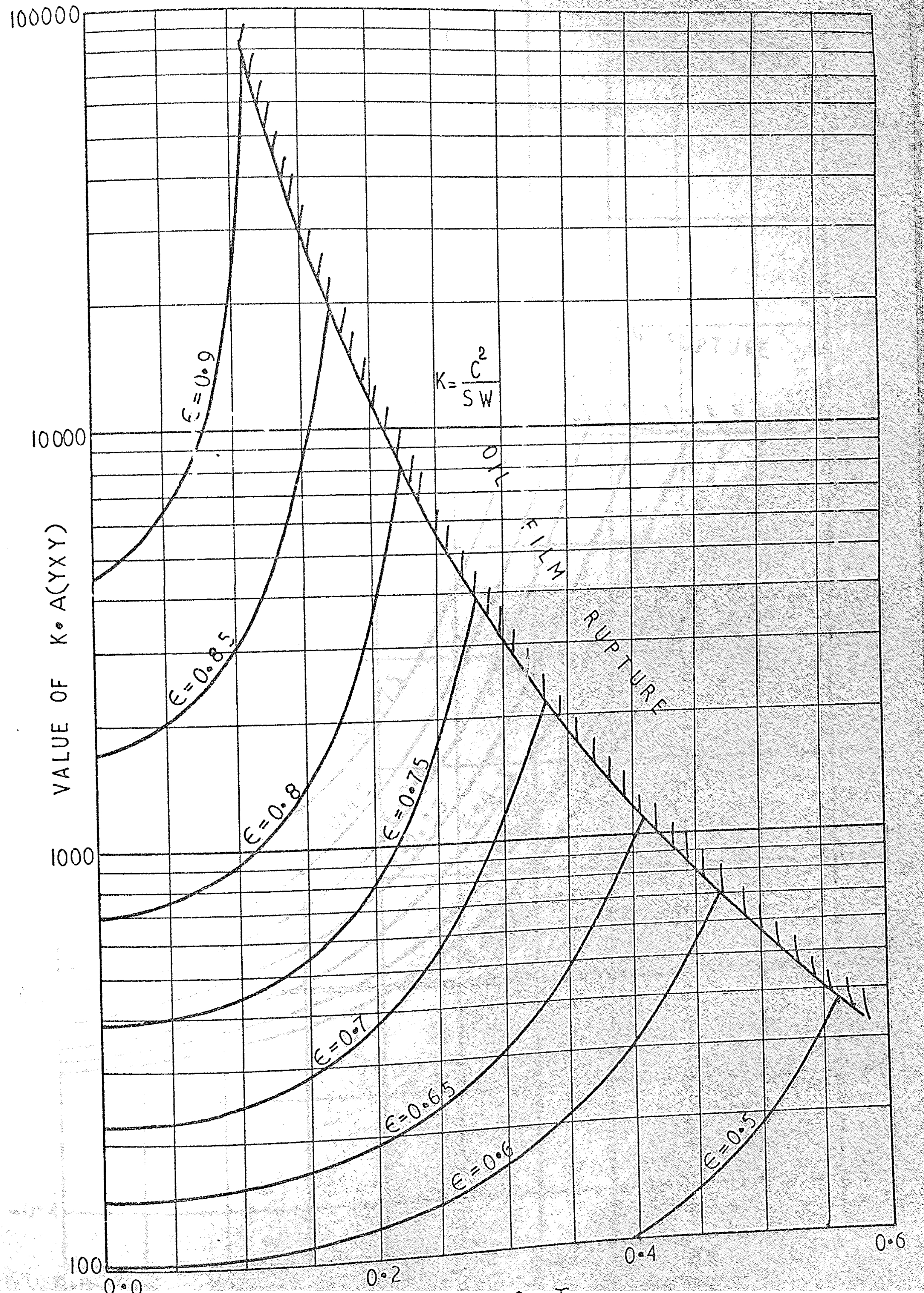
COEFFICIENT $A(YY^2)$ FIG. B-65.



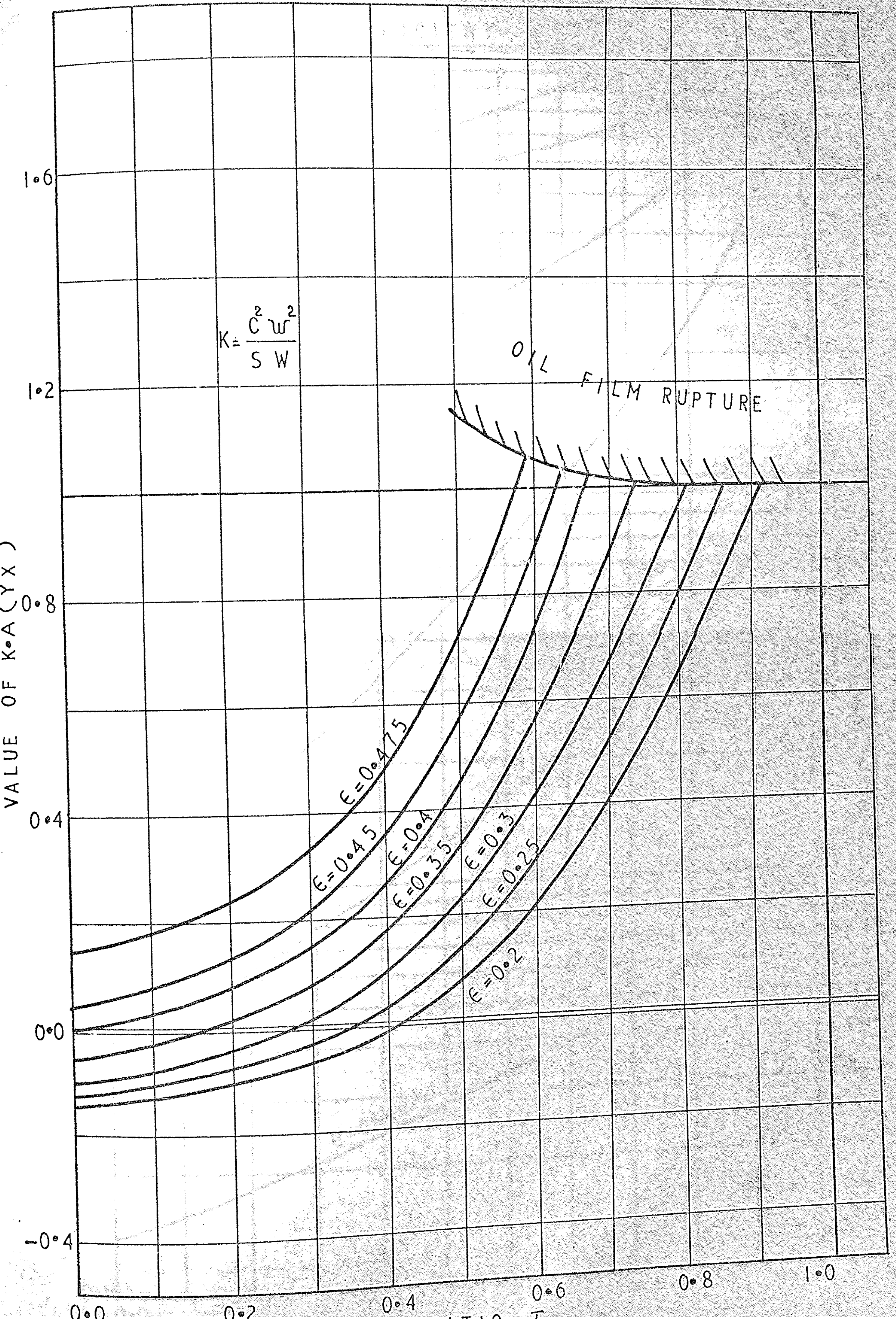
COEFFICIENT A(YXY) FIG. B-66.



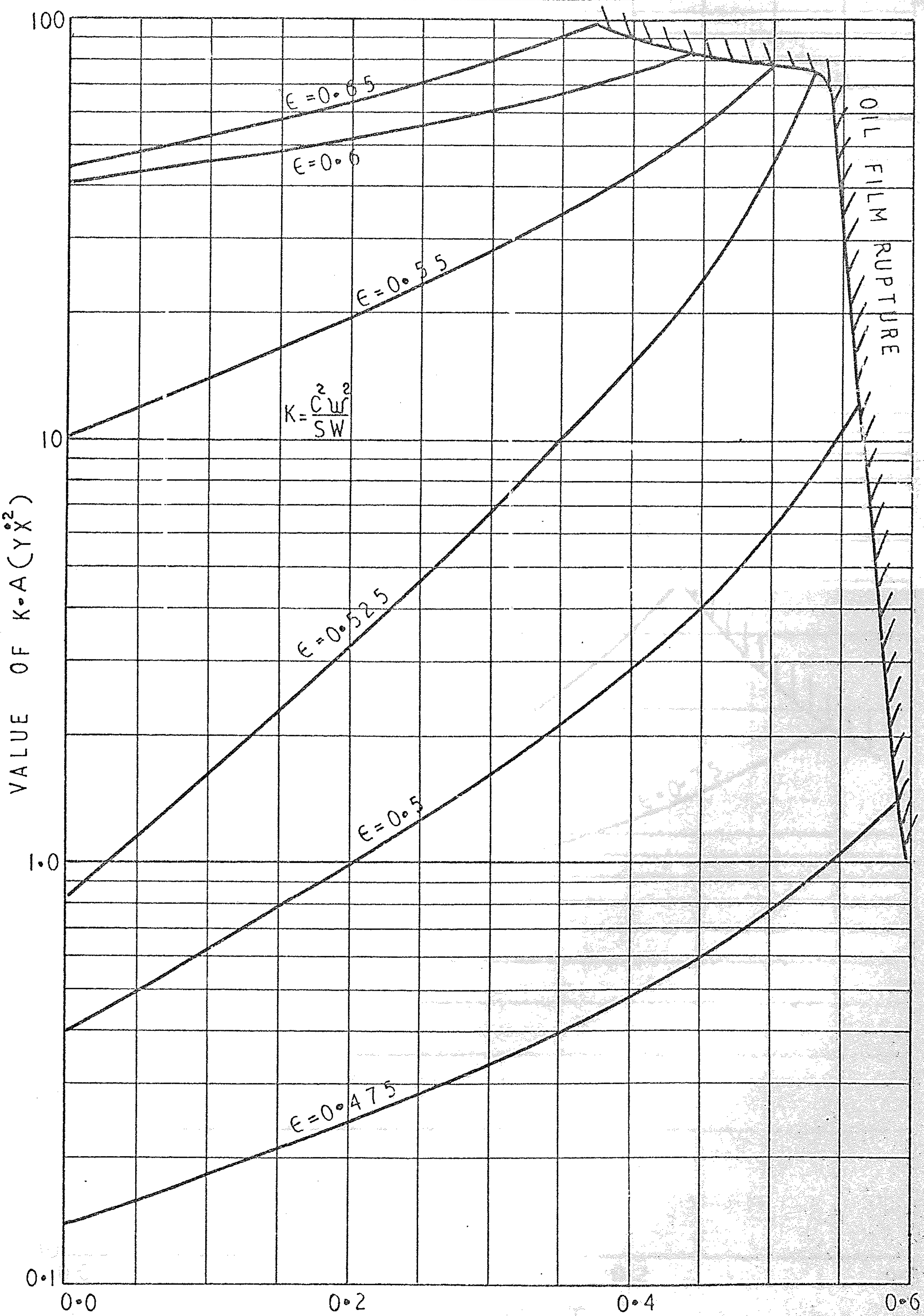
COEFFICIENT A(YXY) FIG. B-67.



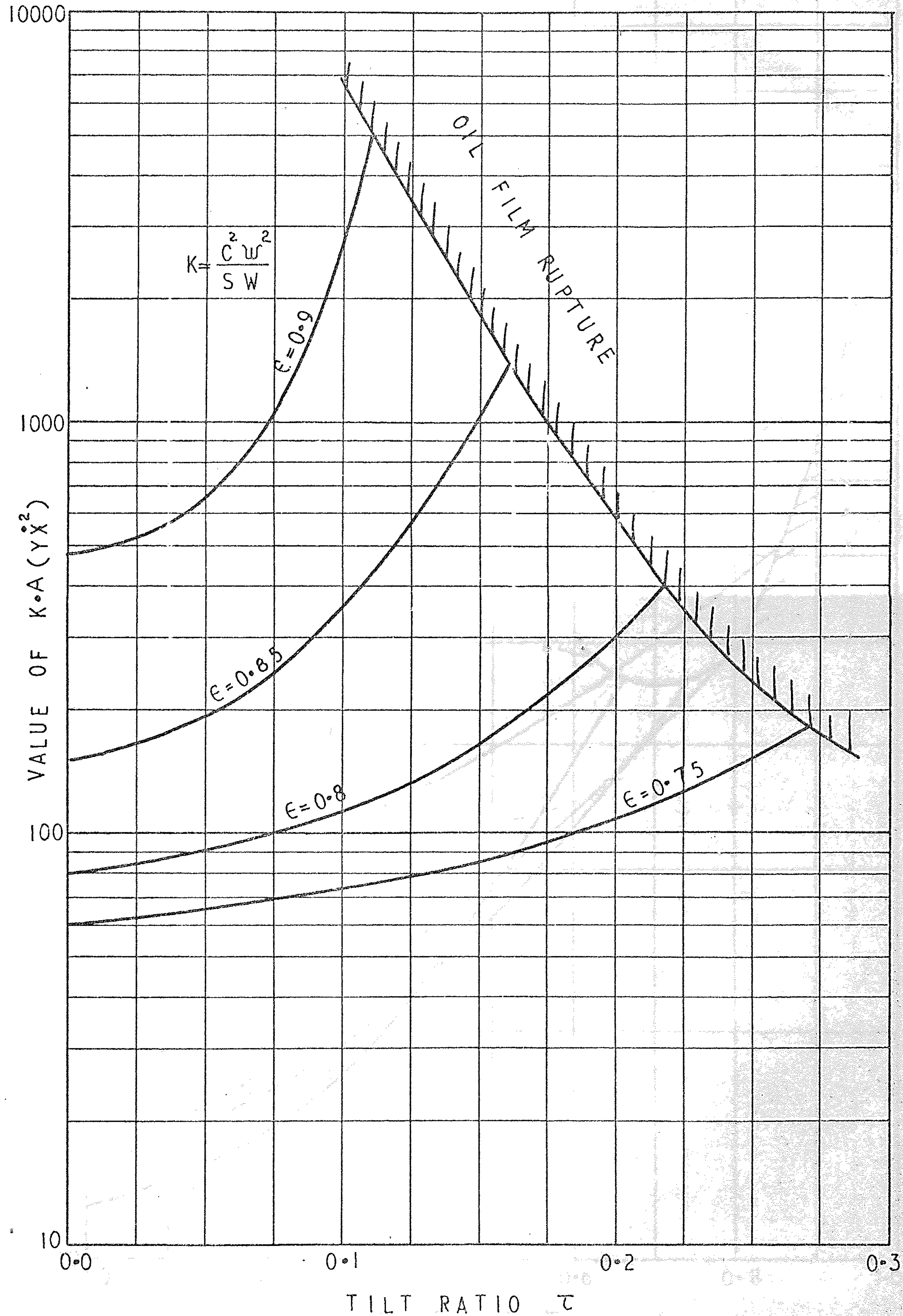
COEFFICIENT $A(Y\dot{X}^2)$ FIG. B-68.



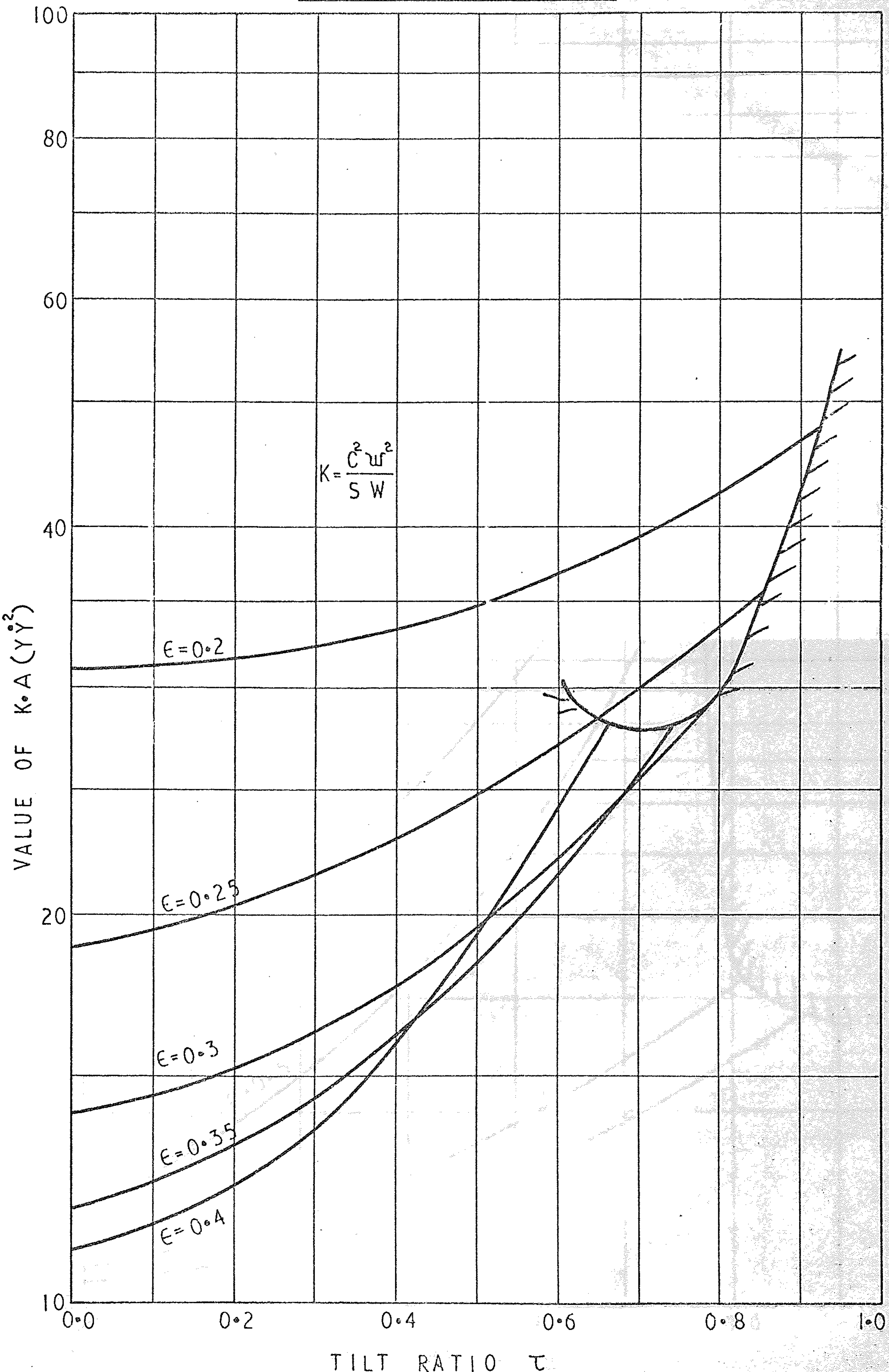
COEFFICIENT $A(Y\dot{X}^2)$ FIG. B-69.



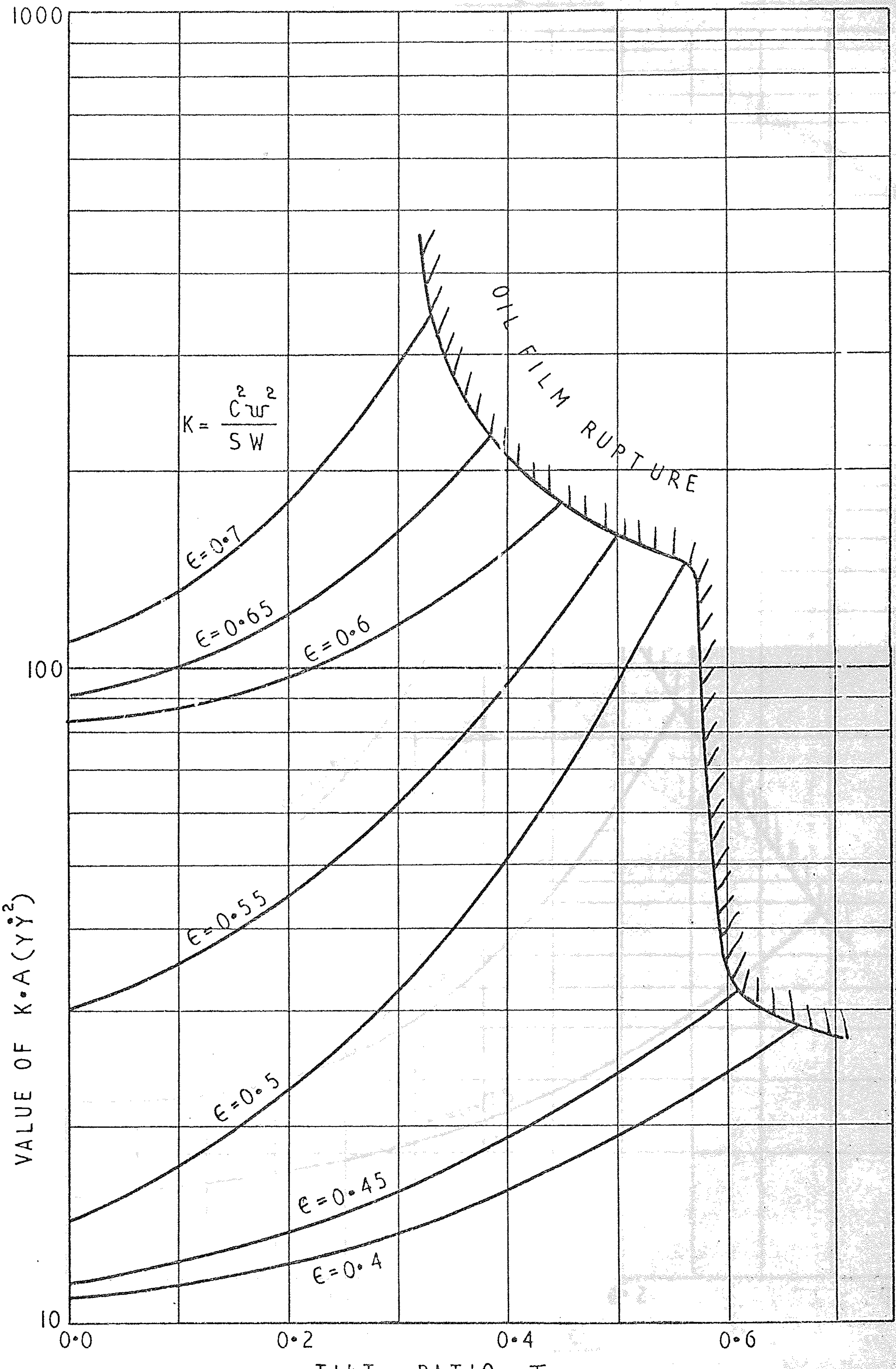
COEFFICIENT $A(Y\dot{X}^2)$ FIG. B-70.



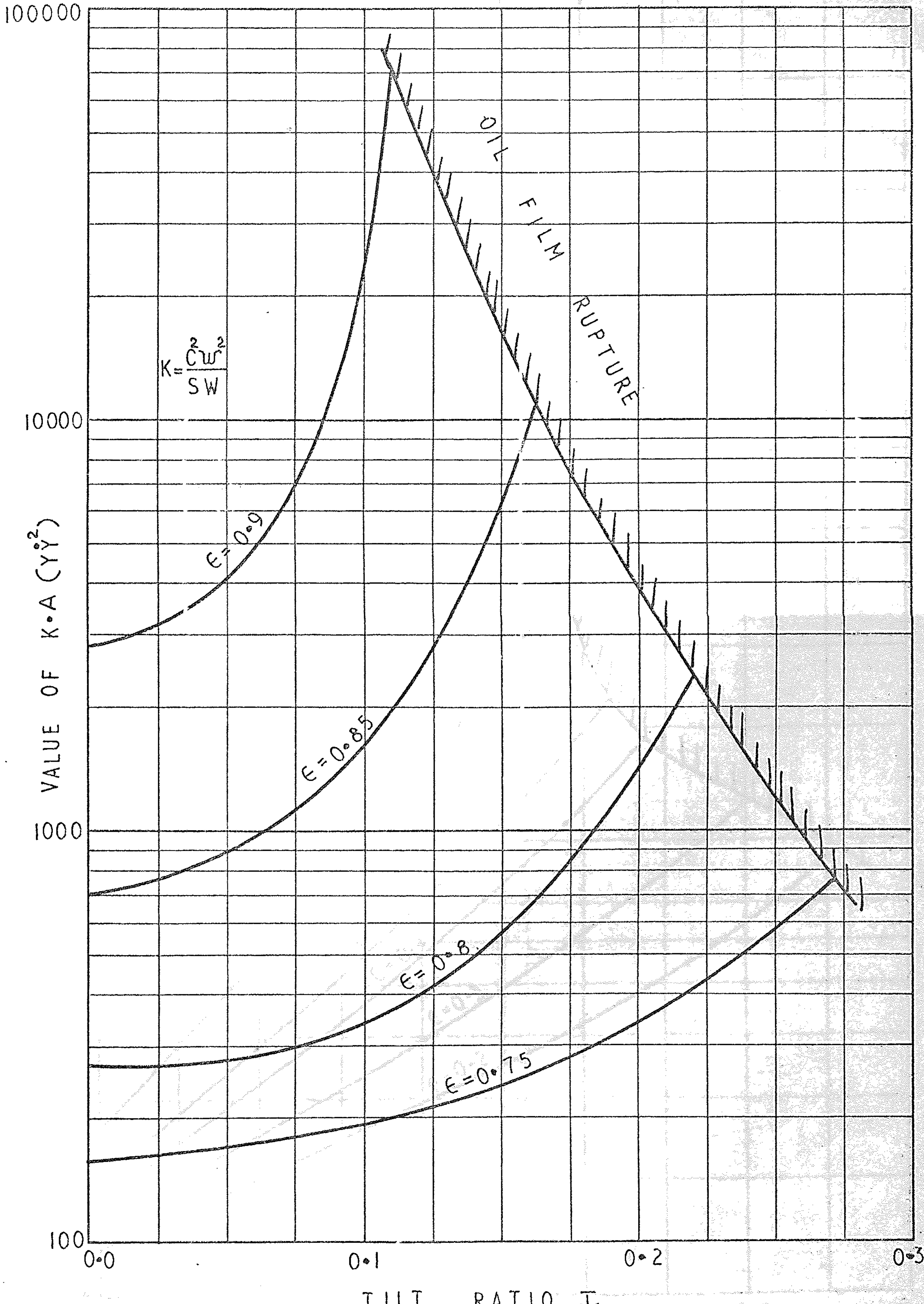
COEFFICIENT $A(Y\dot{Y}^2)$ FIG. B-71.



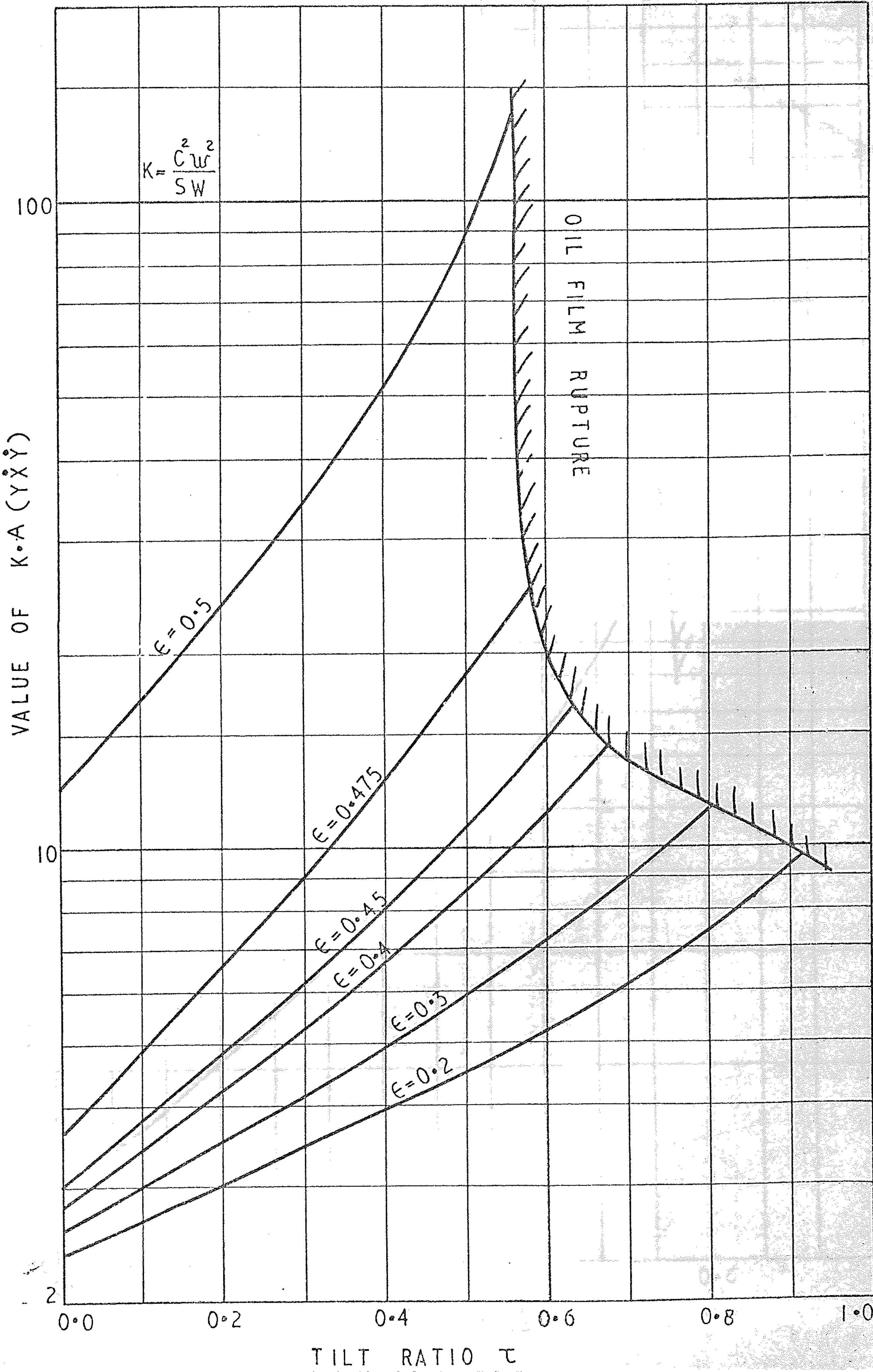
COEFFICIENT $A(Y\dot{Y}^2)$ FIG. B-72.



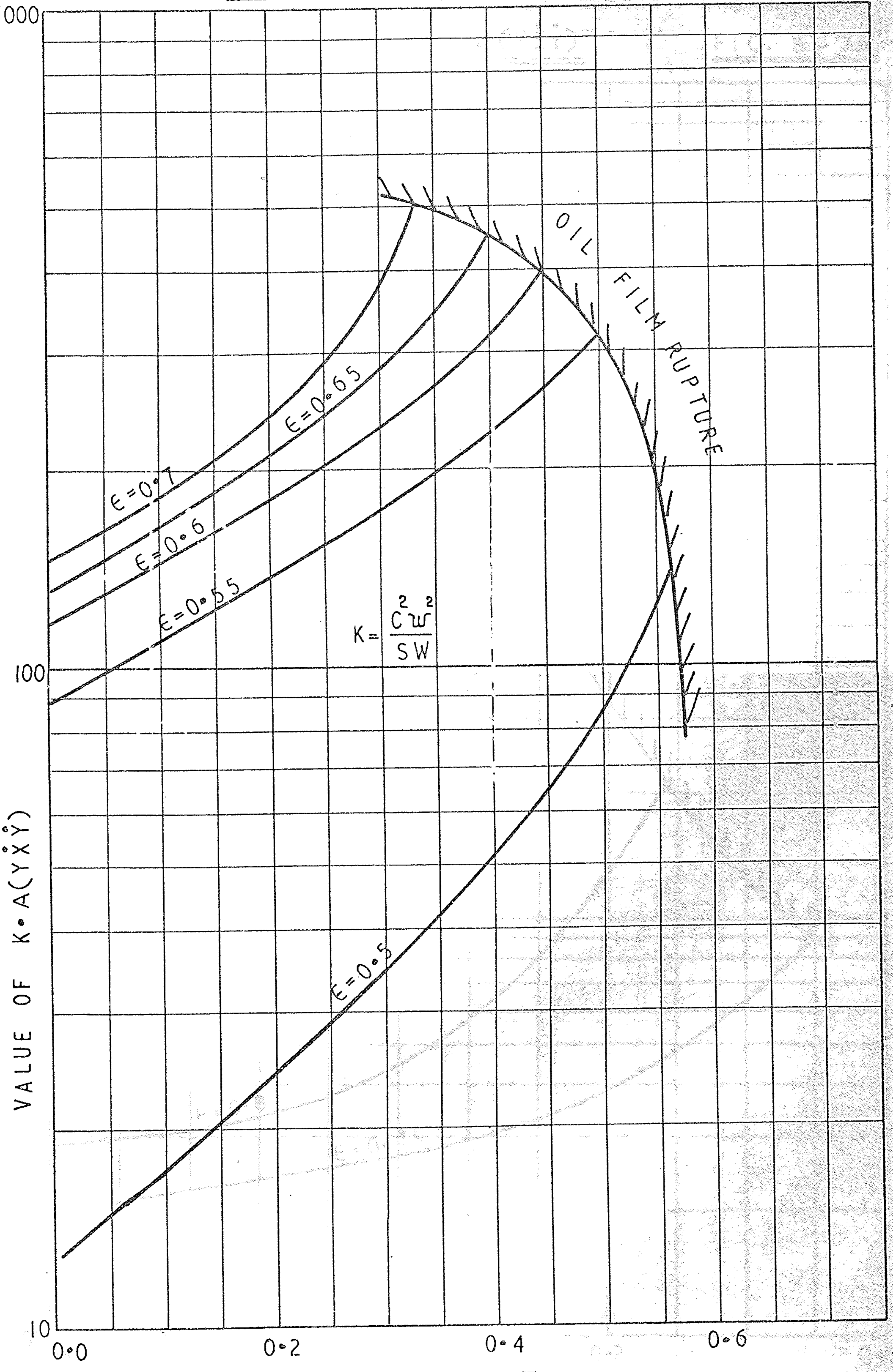
COEFFICIENT A ($Y\dot{Y}^2$) FIG. B-73.

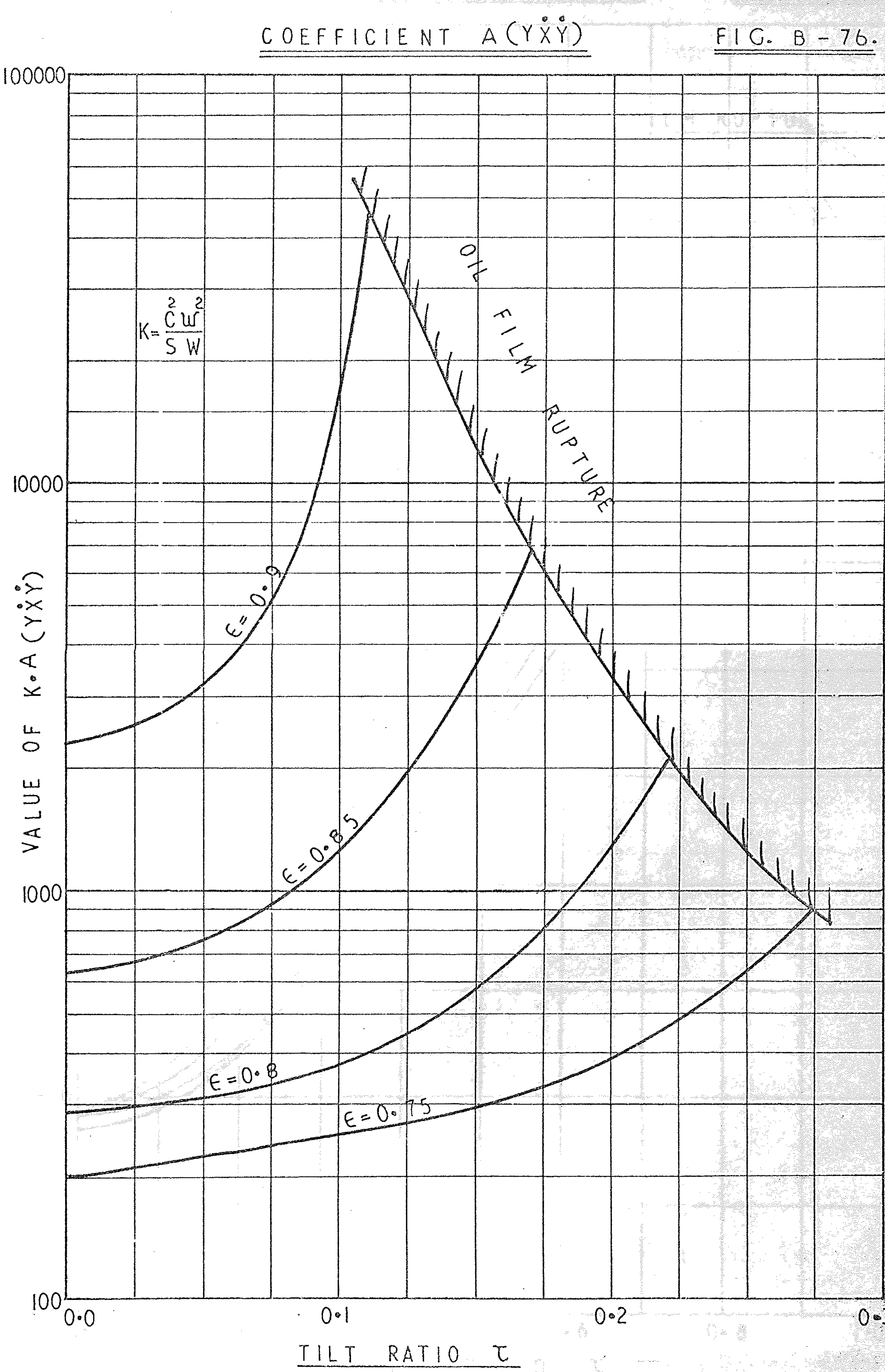


COEFFICIENT A (Y$\dot{X}\dot{Y}$) FIG. B-74.

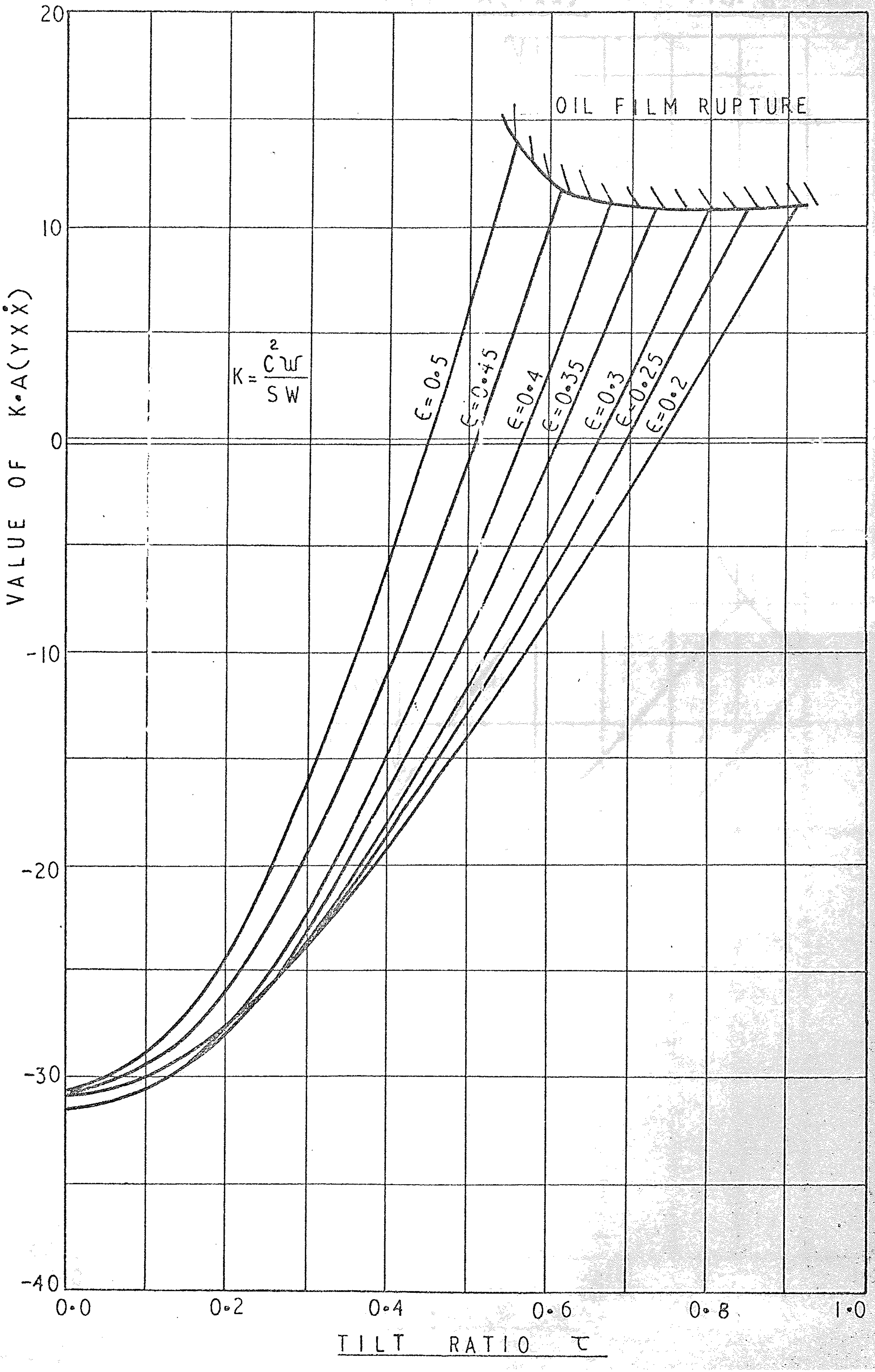


COEFFICIENT A(Y$\dot{X}$$\dot{Y}$) FIG. B-75.

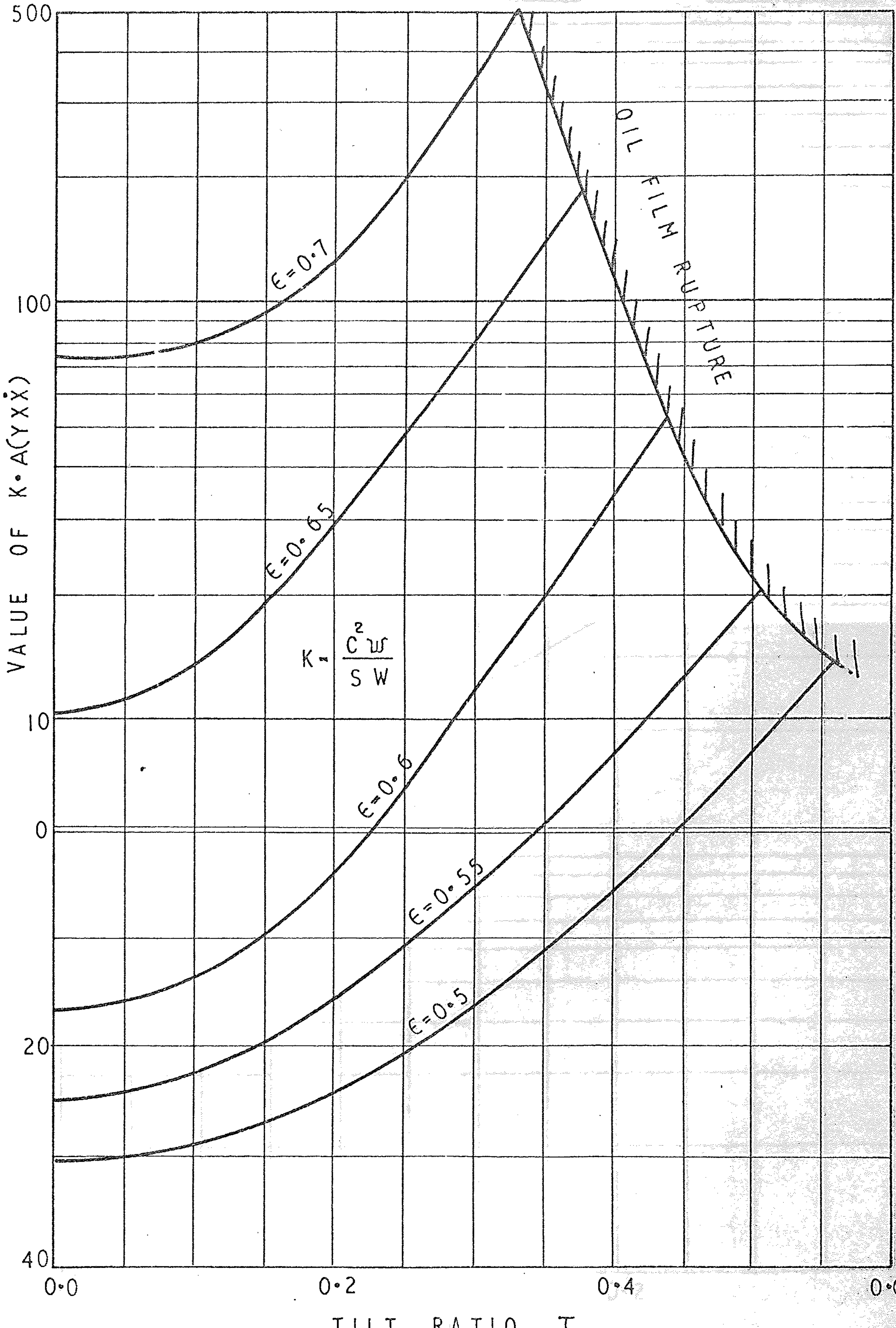



COEFFICIENT A(YẊẎ)
FIG. B-76.
100000
10000
1000
100
K= C²W² / S W
OIL FILM RUPTURE
∈=0·9
∈=0·85
∈=0·8
∈=0·75
VALUE OF K·A(YẊẎ)
0·0
0·1
0·2
0·3
TILT RATIO τ

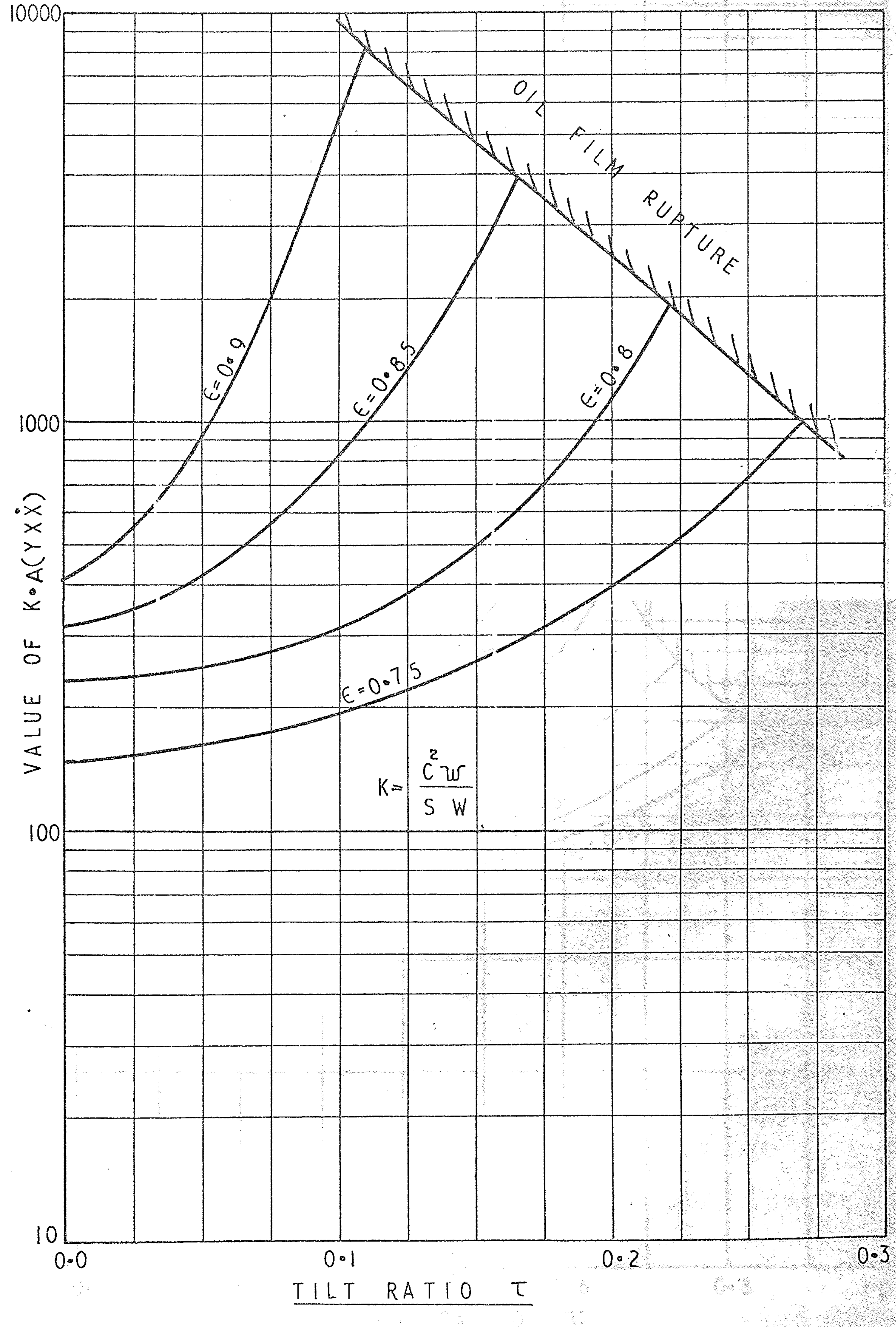
COEFFICIENT A(YXẊ) FIG. B-77.



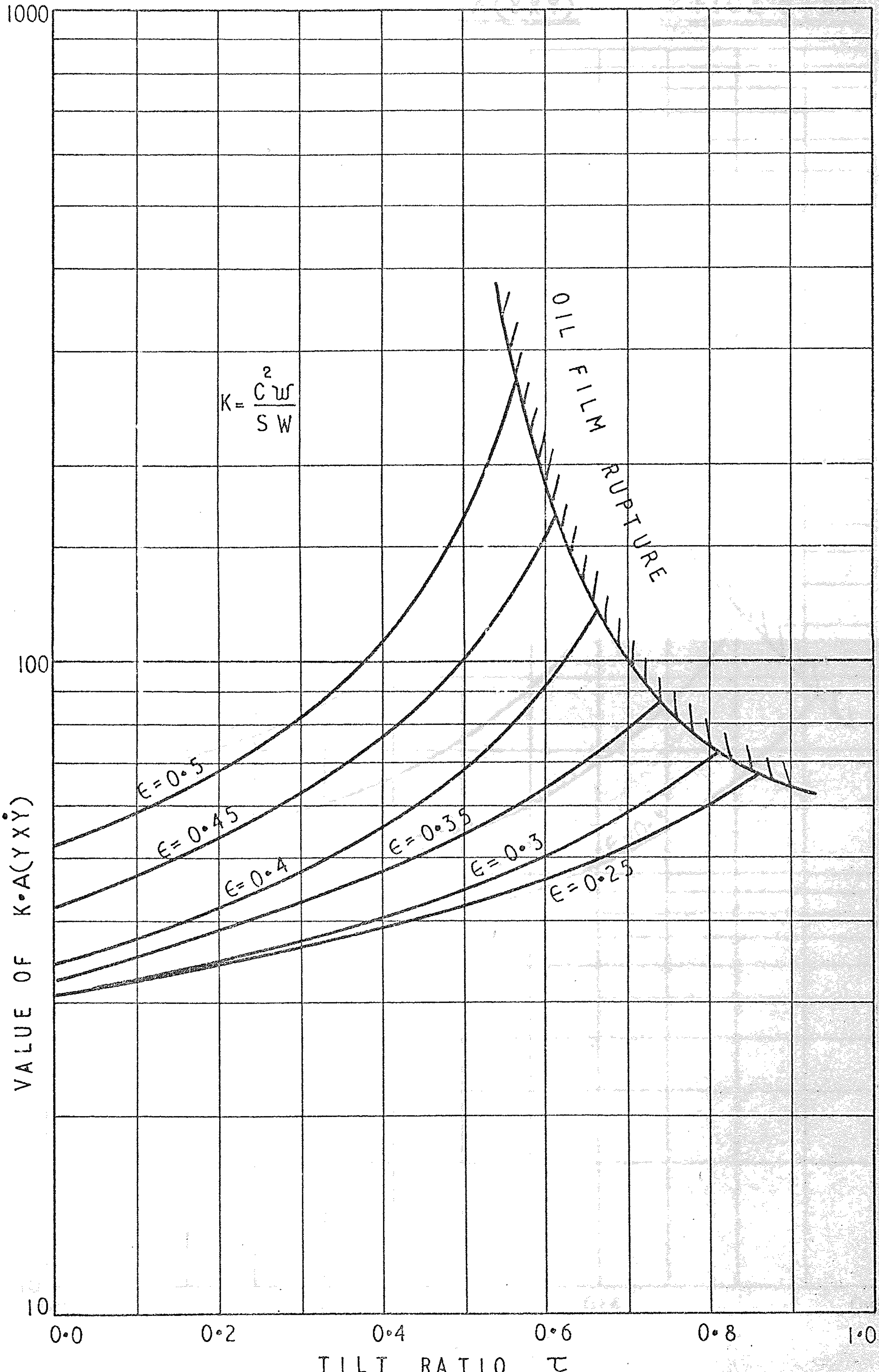
COEFFICIENT A(YXẊ) FIG. B-78.



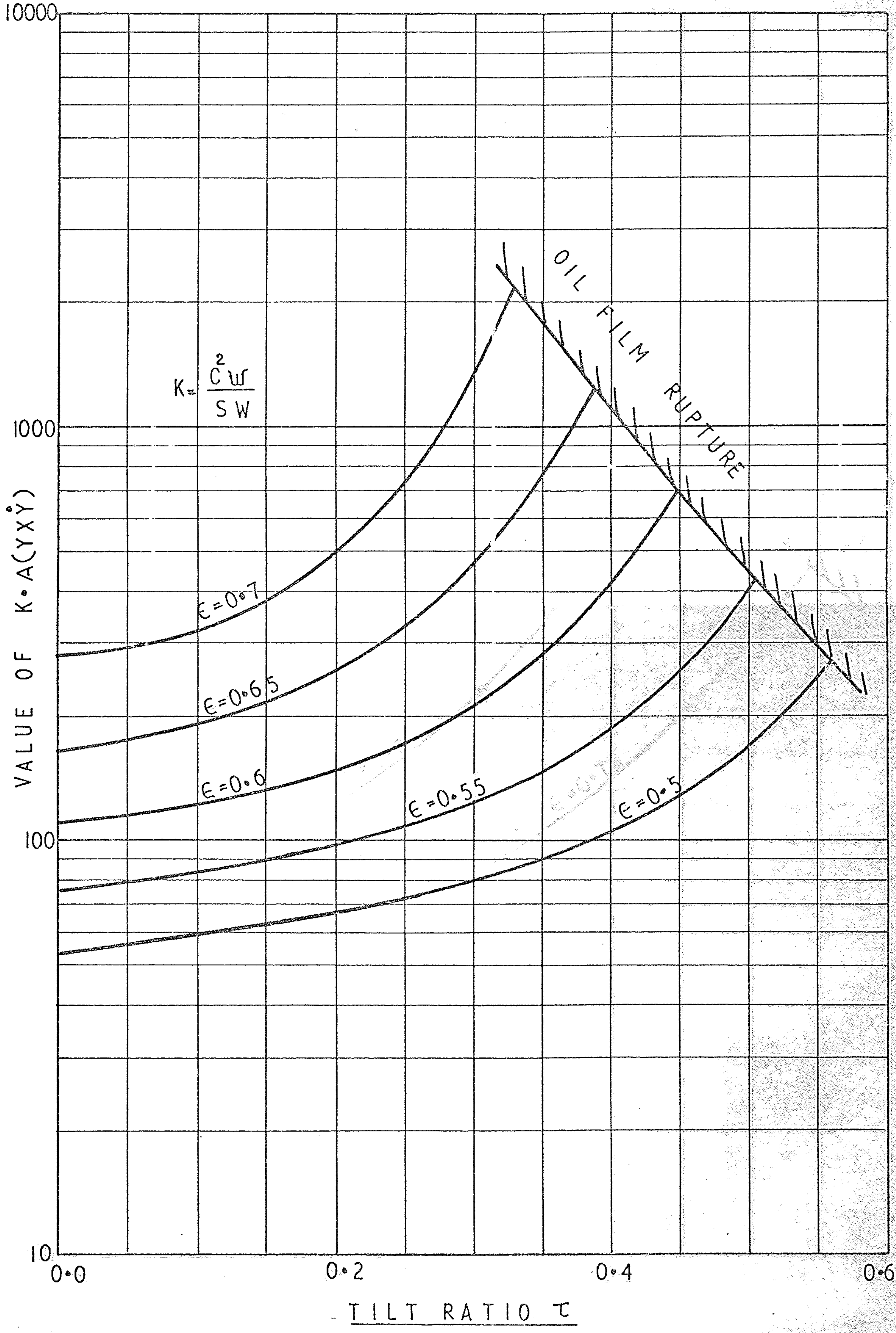
COEFFICIENT A(YXẊ) FIG. B-79.



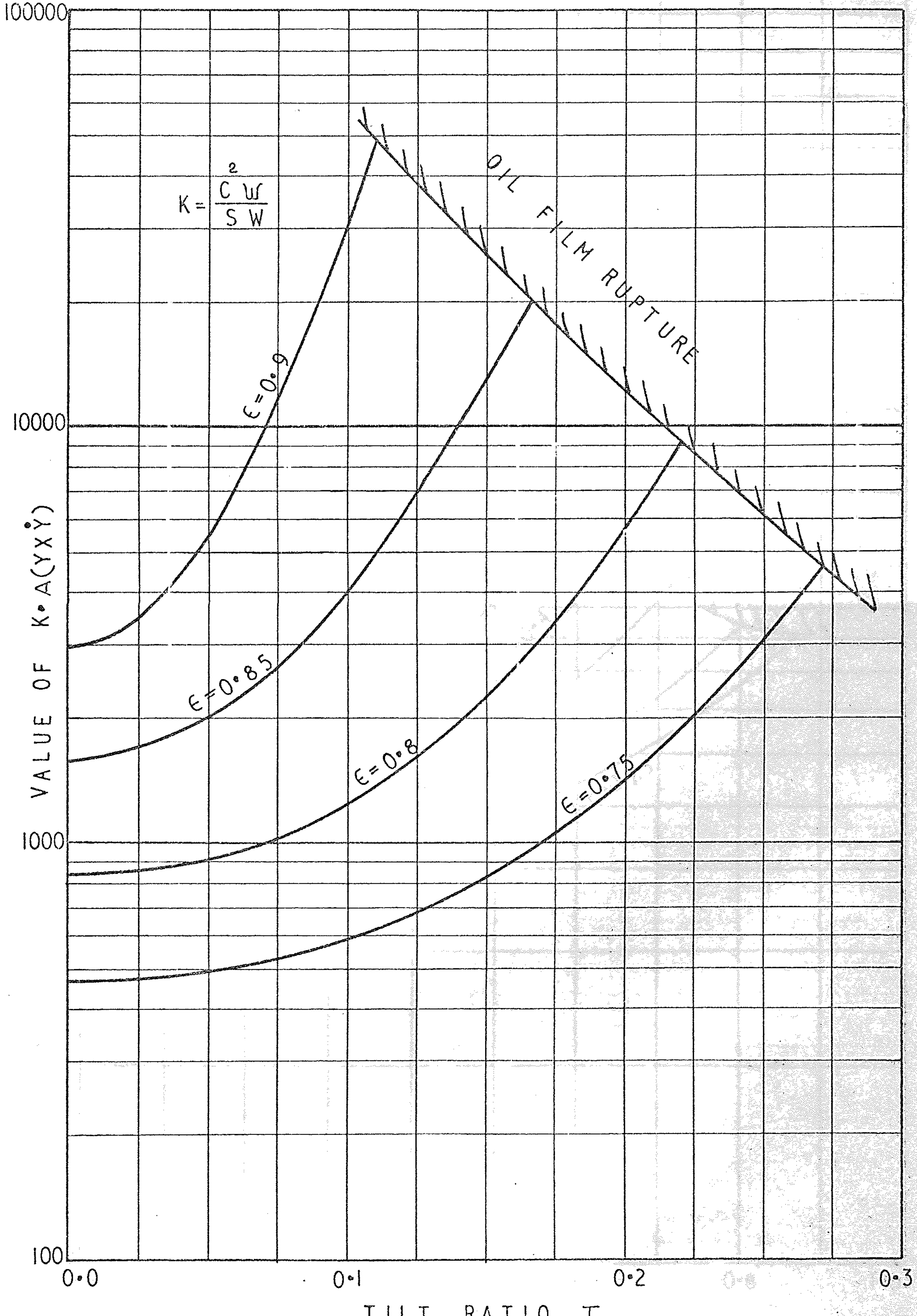
COEFFICIENT A(YXẎ) FIG. B-80.



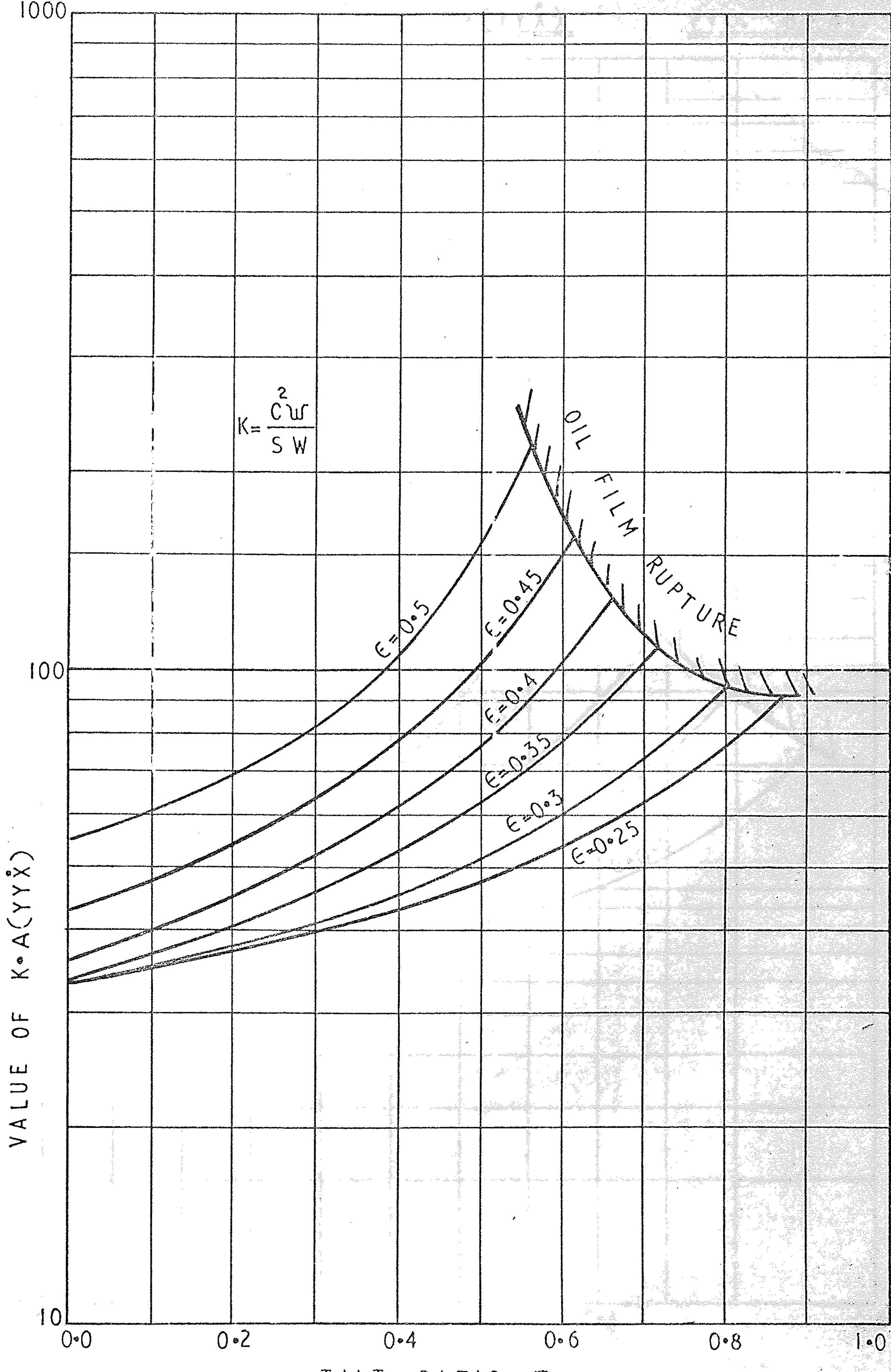
COEFFICIENT A(YXẎ) FIG. B-81.



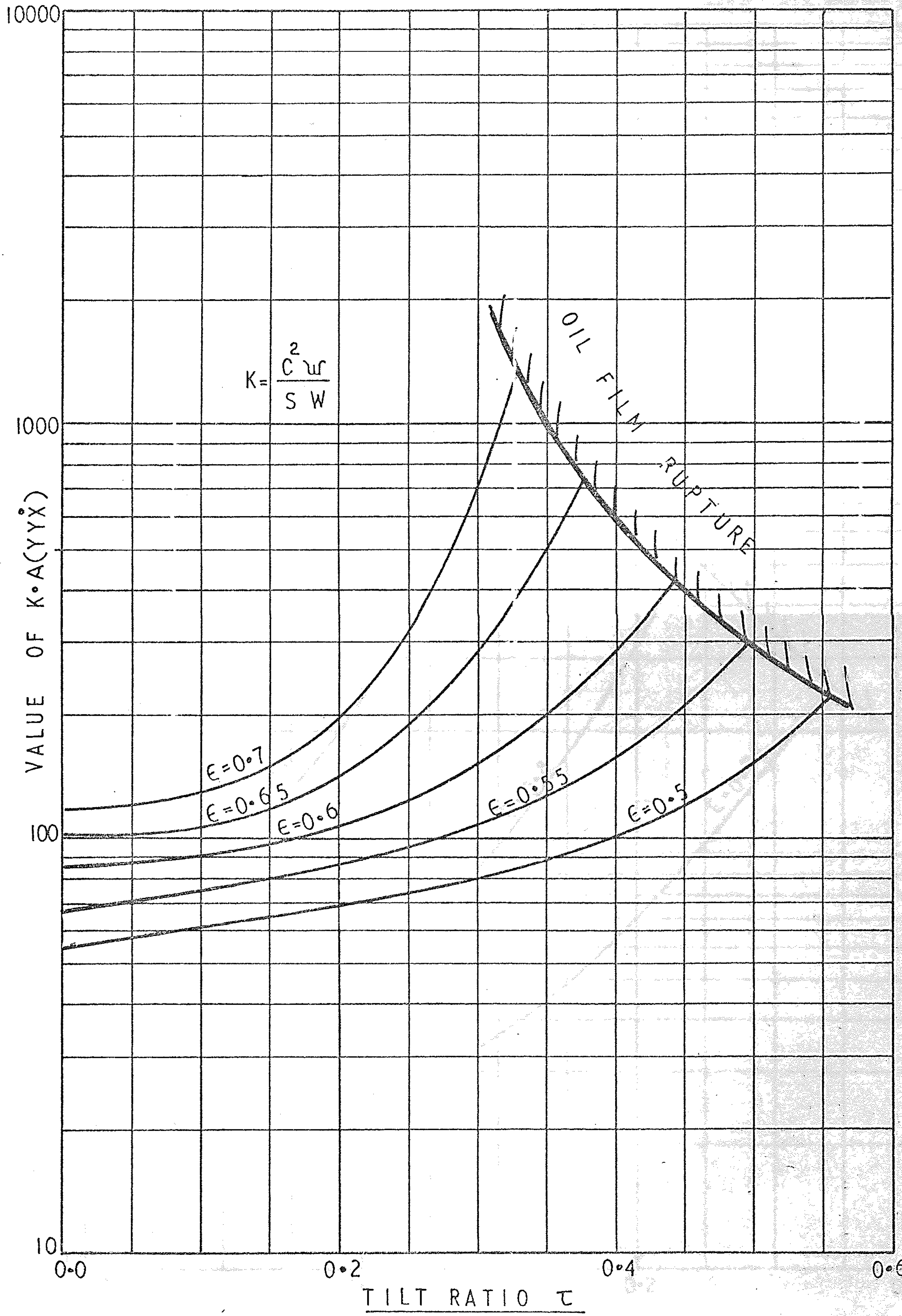
COEFFICIENT A(YXẎ) FIG. B-82.



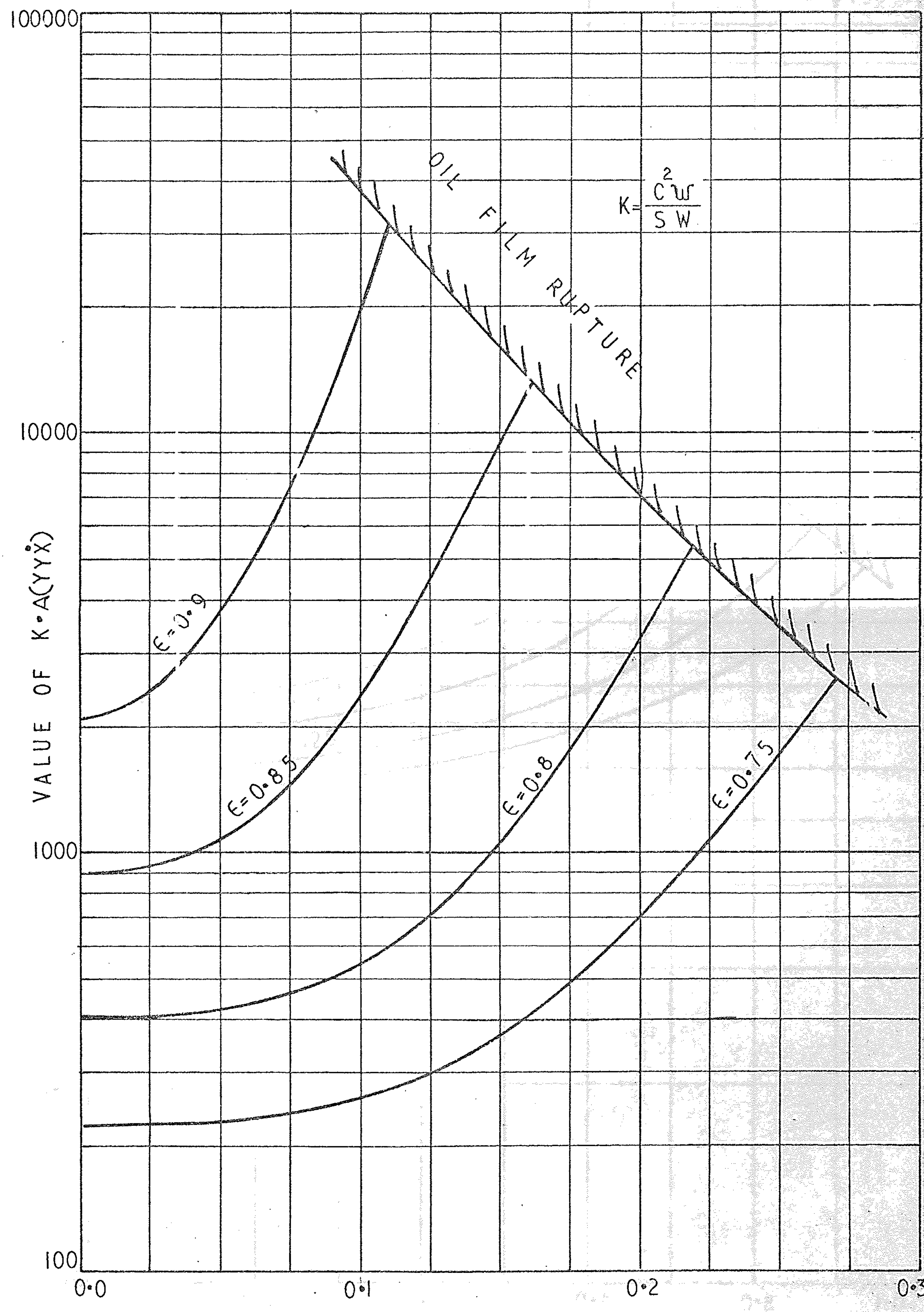
COEFFICIENT A(YYẊ) FIG. B-83.



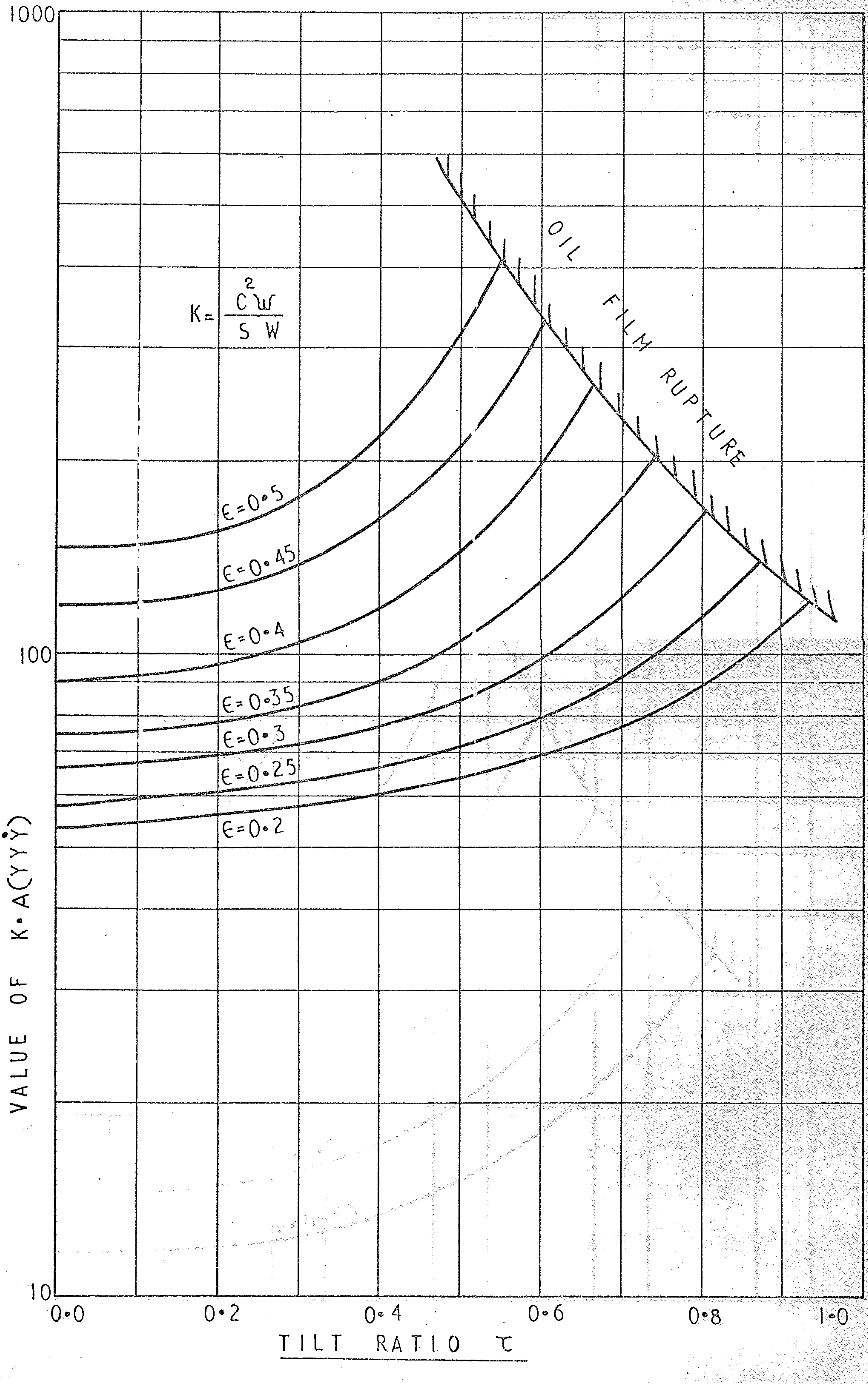
COEFFICIENT A(YYẊ) FIG. B-84.



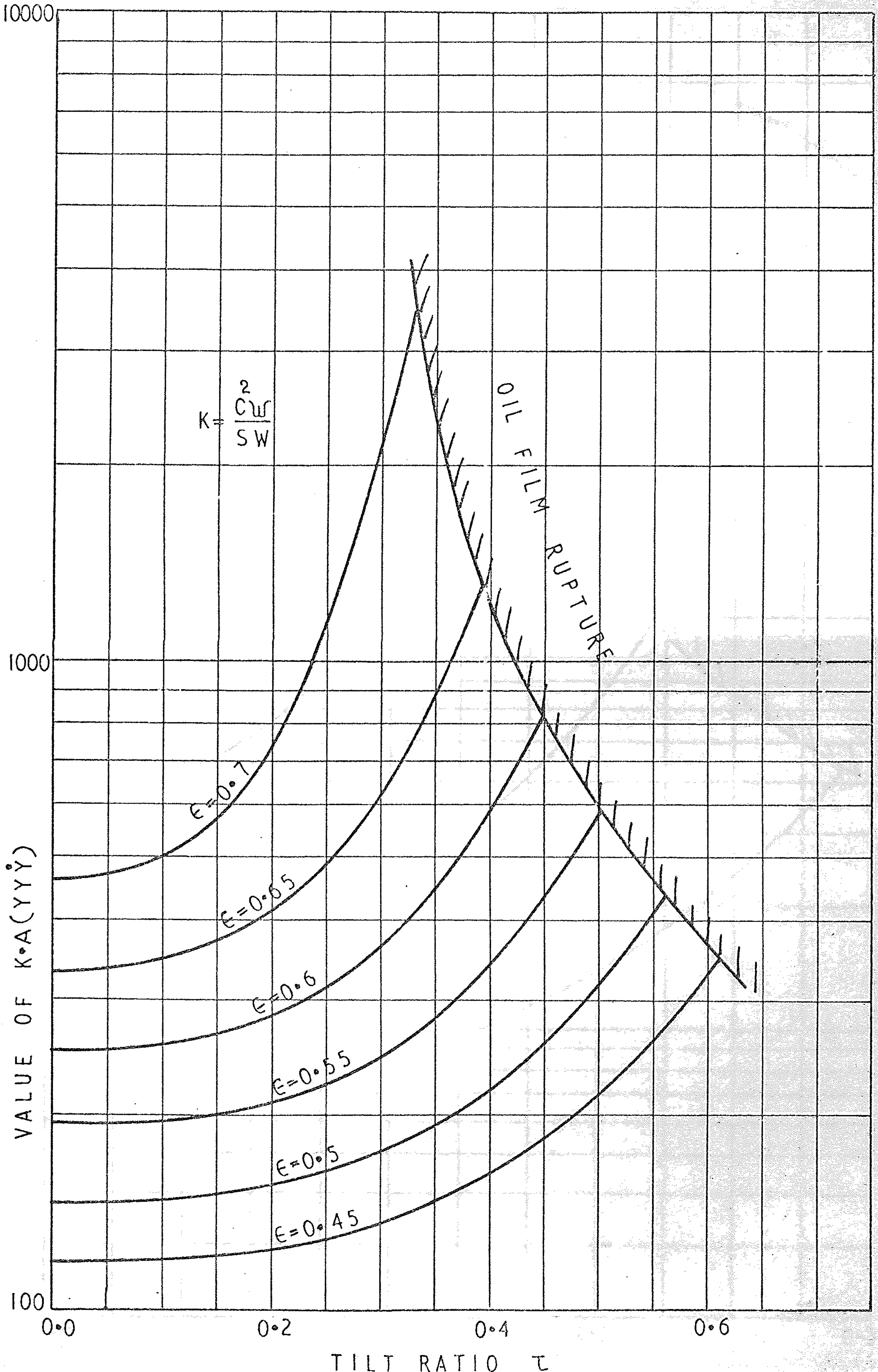
COEFFICIENT A(YYẊ) FIG. B-85.



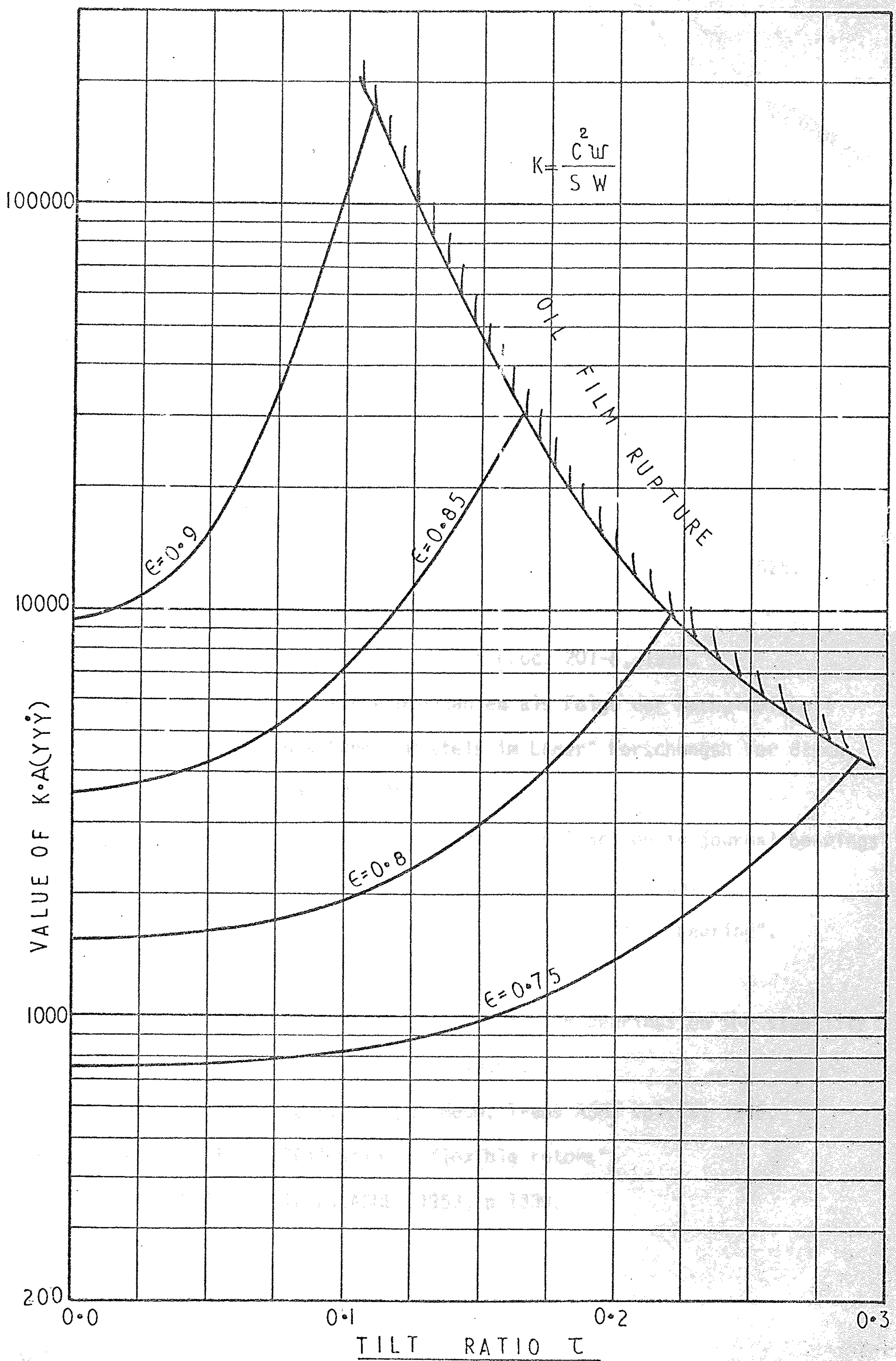
COEFFICIENT A(YYẎ) FIG. B-86.



COEFFICIENT A(YYẎ) FIG. B-87.



COEFFICIENT A(YYẎ) FIG. B-88.



APPENDIX C

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NOTATION

A(---) Force coefficient, the first subscript describes the direction along which the coefficient acts and the remaining single or double subscript describes the displacement or velocity causing a force to be generated from the coefficient.

C Radial clearance.

C_u Oil film damping along the major axis of vibration.

C_v Oil film damping along the minor axis of vibration.

D Diameter of journal.

e Journal eccentricity.

F_R Bearing force in the radial direction.

F_T Bearing force in the tangential direction.

f_R Non-dimensional oil film force, radial direction.

f_T Non-dimensional oil film force, tangential direction.

h Film thickness.

K_u Oil film stiffness along the major axis of vibration.

K_v Oil film stiffness along the minor axis of vibration.

L Length of bearing bush.

M_R Non-dimensional radial movement.

M_T Non-dimensional tangential moment.

M Mass of shaft acting on bush.

m Mass of unbalance.

N Speed of journal.

P Film pressure.

R Radius of bearing journal.

r Radius of unbalance mass.

S	Sommerfeld number $\frac{\mu N}{P} (\frac{R}{C})^2$
t	Time.
U	Peripheral velocity of bush.
u	Major axis of whirl trajectory.
V	Radial velocity of bush.
v	Minor axis of whirl trajectory.
W	Reactive load on journal.
X	Horizontal axis.
x	Displacement along the X axis.
x_p	Pedestal displacement along the X axis.
Δx	Circumferential step length of film grillage.
Y	Vertical axis.
y	Displacement along the Y axis.
y_p	Pedestal displacement along the Y axis.
Z	Axial length of bush to i^{th} node.
Δz	Axial step length of film grillage.
α	Phase of journal vibration.
β	Arc of bearing pad.
γ	Phase of pedestal vibration.
ε	Eccentricity ratio.
ε_o	Steady state eccentricity ratio.
θ	Circumferential angle of bush to i^{th} node.
λ	Bearing parameter SW/ω
μ	Viscosity.
ξ	Inclination of the major axis of vibration to the horizontal axis.
ρ	Mass density.
τ	Tilt ratio of journal.

ϕ	Angle of inclination.
ϕ_o	Steady state inclination angle.
ψ	Angle of tilt to the vertical axis.
Ω	Angular velocity of unbalance vibratory force.
ω	Angular velocity of journal.
$(\dot{\ })$	Denotes first derivative with respect to time.
$(\ddot{\ })$	Denotes second derivative with respect to time.