Analytical BER performance in differential $n$-PSK coherent transmission system influenced by equalization enhanced phase noise

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ABSTRACT

Long-haul high speed optical transmission systems are significantly distorted by the interplay between the electronic chromatic dispersion (CD) equalization and the local oscillator (LO) laser phase noise, which leads to an effect of equalization enhanced phase noise (EEPN). The EEPN degrades the performance of optical communication systems severely with the increment of fiber dispersion, LO laser linewidth, symbol rate, and modulation format. In this paper, we present an analytical model for evaluating the performance of bit-error-rate (BER) versus signal-to-noise ratio (SNR) in the $n$-level phase shift keying ($n$-PSK) coherent transmission system employing differential carrier phase estimation (CPE), where the influence of EEPN is considered. Theoretical results based on this model have been investigated for the differential quadrature phase shift keying (DQPSK), the differential 8-PSK (D8PSK), and the differential 16-PSK (D16PSK) coherent transmission systems. The influence of EEPN on the BER performance in term of the fiber dispersion, the LO phase noise, the symbol rate, and the modulation format are analyzed in detail. The BER behaviors based on this analytical model achieve a good agreement with previously reported BER floors influenced by EEPN. Further simulations have also been carried out in the differential CPE considering EEPN. The results indicate that this analytical model can give an accurate prediction for the DQPSK system, and a leading-order approximation for the D8PSK and the D16PSK systems.

Key words: coherent optical communication, phase shift keying, differential carrier phase estimation, equalization enhanced phase noise, bit-error-rate, signal-to-noise ratio

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1. Introduction

Coherent optical detection allows the significant equalization of transmission system impairments such as chromatic dispersion (CD), polarization mode dispersion (PMD), phase noise (PN), and fiber nonlinearities (FNLS) in the electrical domain by using the powerful digital signal processing (DSP) [1-7]. The carrier phase estimation (CPE) can be effectively implemented by employing the feedforward and the feedback algorithms [7-11]. As a conventional feedforward algorithm, the differential phase estimation has been validated as a simple and effective method for the phase noise compensation (PNC) in the coherent transmission system, which is also regarded as a benchmark for evaluating the CPE approaches [12-17]. However, the analysis of the phase noise in the transmitter (Tx) and the local oscillator (LO) lasers is often lumped together, and the interplay between the CD and the PN is not considered in the traditional evaluation of carrier phase estimation (including differential phase estimation). W. Shieh et al. have provided the theoretical assessment to evaluate the equalization enhanced phase noise (EEPN) from the interaction between the LO phase fluctuation and the fiber dispersion [18-21]. C. Xie has investigated the impacts of CD on both the LO phase noise to amplitude noise conversion and the fiber nonlinear effects [22,23]. I. Fatadin et al. have also studied the influence of the EEPN in quadrature phase shift keying (QPSK), 16-level quadrature amplitude modulation (16-QAM) and 64-QAM coherent transmission systems [24]. Meanwhile, the influence of EEPN on different CPE algorithms has also been investigated in detail by using numerical simulations [25-29]. Due to the EEPN, the requirement of laser
linewidth can not be generally relaxed for the transmission system with a higher symbol rate. It would be of importance to investigate the analytical estimation for the performance of bit-error-rate (BER) versus signal-to-noise ratio (SNR) in the carrier phase estimation in coherent optical transmission system considering the influence of EEPN.

In this paper, we present an analytical model for assessing the behavior of BER versus SNR in the differential phase estimation in the long-haul high speed $n$-level phase shift keying ($n$-PSK) coherent optical transmission system, where the impact of EEPN is considered. The additional noise variance induced by EEPN has been taken into account in the analytical estimation. Based on this model, we have investigated the performance of BER versus SNR in the differential QPSK (DQPSK), the differential 8-PSK (D8PSK), and the differential 16-PSK (D16PSK) coherent optical transmission systems in detail involving the influence of EEPN. Theoretical results demonstrate that the behaviors of BER versus SNR in the differential phase estimation are significantly distorted by EEPN, with the increment of the LO phase noise, the accumulated fiber dispersion, the symbol rate, and the modulation format. Meanwhile, the results from this model make a good agreement with our previously reported BER floors. Moreover, to investigate the accuracy of this analytical model, numerical simulations have also been implemented for the DQPSK, the D8PSK, and the D16PSK coherent transmission systems with differential carrier phase estimation, considering EEPN. The results indicate that this analytical model can give an accurate
prediction for the DQPSK system, and a leading-order approximation for the D8PSK and the D16PSK systems, when EEPN is considered.

2. Differential carrier phase estimation in coherent transmission system

The setup of $n$-PSK coherent optical transmission system is illustrated in Fig. 1, where the differential carrier phase estimation is employed. The pseudo random bit sequence (PRBS) data are differentially encoded and modulated into the $n$-PSK optical signals, and then fed into the transmission fiber channel. In the receiver end, the received optical signals are mixed with the LO laser and converted into the electrical signals by the photodiodes (PDs). Then the electrical signals are sampled by the analog-to-digital convertors (ADCs) at twice the symbol rate. The digitalized signals are processed by the DSP algorithms, including CD compensation, signal equalization, and differential carrier phase estimation, to mitigate the system impairments.

The differential carrier phase estimation is applied for the carrier phase recovery in the coherent optical detection, where the information is encoded in and extracted from the phase difference between two consecutive symbols [12-17]. In the differential CPE, the $k$-th symbol information is recovered from the phase difference between the current $k$-th symbol and the $(k+1)$-th symbol. Hence, the phase ambiguity $Δφ$ in the recovered data comes from the carrier phase fluctuation between the two adjacent symbols $Δφ=φ_k-φ_{(k+1)}$, during a symbol period [12-17]. Compared to other CPE algorithms such as the block-average (BA) and the Viterbi-Viterbi (VV) methods [7,9,14], differential phase estimation does not require any computational operations.
of \( n \)-power, averaging, and phase unwrapping, and can thus be efficiently implemented in the field-programmable gate array (FPGA) hardware [17]. In the following sections, we will investigate the BER performance in the differential carrier phase estimation considering the impact of EEPN, and our study can be employed as the benchmark and reference for evaluating the effects of EEPN in other CPE methods.

Fig. 1. Schematic of coherent optical transmission system with differential carrier phase estimation. \( N(t) \): additive white Gaussian noise (AWGN).

3. **Principle of equalization enhanced phase noise**

The schematic of EEPN in the coherent optical communication system employing electronic CD post-equalization and carrier phase estimation is depicted in Fig. 2. The Tx laser phase noise passes through both the transmission fiber and the digital CD equalization module, and therefore the net dispersion experienced by the transmitter PN is close to zero. However, the LO phase noise only goes through the digital CD equalization module, of which the transfer function is heavily dispersed in the transmission system without using any optical dispersion compensation (ODC). Consequently, the LO phase noise will interact with the CD equalization module, and will significantly influence the performance of the long-haul high speed coherent system.

Fig. 2. Schematic of EEPN in coherent transmission system. \( N(t) \): additive white Gaussian noise (AWGN), ADC: analog-to-digital convertor.
It has been demonstrated that the EEPN scales linearly with the accumulated CD, the linewidth of LO laser, and the symbol rate [18-20], and the variance of the additional noise due to the EEPN can be expressed as

$$\sigma_{EEPN}^2 = \frac{\pi \lambda^2}{2c} \cdot \frac{D \cdot L \cdot \Delta f_{LO}}{T_S}$$  \hspace{1cm} (1)$$

where $\lambda$ is the central wavelength of the optical wave, $c$ is the light speed in vacuum, $D$ is the CD coefficient of fiber, $L$ is the fiber length, $\Delta f_{LO}$ is the 3-dB linewidth of the LO laser, and $T_S$ is the symbol period of the coherent transmission system.

4. BER performance in differential CPE considering EEPN

The theoretical performance of BER versus SNR in the $n$-PSK coherent optical transmission system employing differential carrier phase estimation can be calculated according to the following expression [12-15],

$$BER \approx \frac{4}{\sqrt{2\pi n \sigma}} \log_2 n \int_{-\infty}^{\infty} \exp \left( -\frac{8e^2}{n^2 \sigma^2} \right) \cdot \text{erfc} \left( \sqrt{\text{SNR}} \right) de$$  \hspace{1cm} (2)$$

$$\Gamma = \sqrt{1 + \frac{\sin \pi n}{n} - \sqrt{1 - \sin \frac{\pi}{n}}}$$  \hspace{1cm} (3)$$

where $\sigma^2$ is the variance of the total phase noise in the coherent transmission system, $n$ is the level of the modulation format, and $SNR$ represents the signal-to-noise of the transmission system.

Considering the impact of equalization enhanced phase noise, the variance of the total noise in the $n$-PSK coherent transmission system can be expressed as [25,29],

$$\sigma^2 = \sigma_{Tx}^2 + \sigma_{LO}^2 + \sigma_{EEPN}^2 + 2\rho \cdot \sigma_{LO} \cdot \sigma_{EEPN}$$  \hspace{1cm} (4)$$
\[ \sigma_{tx}^2 = 2\pi\Delta f_{tx} \cdot T_s \]  
\[ \sigma_{lo}^2 = 2\pi\Delta f_{lo} \cdot T_s \]  
(5)  
(6)

where \( \sigma_{tx}^2 \) and \( \sigma_{lo}^2 \) are the intrinsic phase noise variance of the Tx and the LO lasers respectively, and \( \Delta f_{tx} \) is the 3-dB linewidth of the Tx laser, and \( \rho \) is the correlation coefficient between the LO phase noise and the EEPN. It has been demonstrated that we have \( \rho \approx 0 \) when the transmission fiber length is larger than 80 km [25].

Therefore, in the long-haul high speed \( n \)-PSK coherent optical transmission systems, the total noise variance in the differential carrier phase estimation can be described approximately as follows,

\[ \sigma^2 \approx \sigma_{tx}^2 + \sigma_{lo}^2 + \sigma_{EEPN}^2 \]  
(7)

By combining Eq. (2) and Eq. (7), we can obtain the analytical model for evaluating the performance of BER versus SNR in the differential \( n \)-PSK coherent optical transmission systems.

5. Theoretical results and analysis

In this section, we investigate the performance of BER versus SNR in the optical transmission system considering the influence of EEPN using the proposed theoretical model. The DQPSK, the D8PSK, and the D16PSK optical communication systems are applied as the examples for the analysis. Several transmission scenarios are considered for evaluating the BER performance: 1) different combination of the Tx laser and the LO laser linewidths, while the summation of linewidths are kept constant; 2) different LO laser linewidth; 3) different transmission distance (different fiber dispersion); 4) different symbol rate; 5) different modulation formats (DQPSK,
D8PSK, and D16PSK). The CD coefficient of 16 ps/nm/km is applied in all the evaluations.

Figure 3 shows the BER performance in the 2000 km 28-Gbaud DQPSK coherent transmission system with different combination of the Tx and the LO lasers linewidths while keeping the summation of linewidths constant. It can be seen that the BER behaviors degrade obviously with the increment of the LO laser linewidth. This demonstrates that the EEPN arises from the LO laser phase noise rather than the Tx phase noise, which is consistent with the reported analysis [18-22]. We can also find that the limits of the BER achieve a very good agreement with the BER floors (the dash curve) presented in our previous work [25,29]. Figure 4 shows the BER performance in the 1000 km 28-Gbaud D8PSK transmission system with different distribution of the Tx and the LO lasers linewidths. According to Eq. (1), the EEPN noise variance in the D8PSK transmission system in Fig. 4 is around 1/20 of the EEPN in the DQPSK system in Fig. 3. But the limits of BER behave moderately different in Fig. 3 and in Fig. 4. This indicates that the D8PSK system is more sensitive to the EEPN than the DQPSK system.

Fig. 3. BER performance in 2000 km 28-Gbaud DQPSK system with different distribution of Tx and LO lasers linewidths.

Fig. 4. BER performance in 1000 km 28-Gbaud D8PSK system with different distribution of Tx and LO lasers linewidths.

The performance of BER versus SNR in the 2000 km 28-Gbaud DQPSK coherent transmission system with different laser linewidth is shown in Fig. 5, where the Tx
laser linewidth is equal to the LO laser linewidth. We can see that the performance of the BER in the transmission system degrades obviously with the increment of the LO laser linewidth. It is also found that the performance of BER versus SNR in this model deviates significantly from the traditional analysis of the differential phase estimation [13,14], where the effect of EEPN was not considered. Figure 6 shows the BER performance in the 1000 km 28-Gbaud D8PSK coherent transmission system with different laser linewidth, where the Tx laser linewidth is also equal to the LO laser linewidth.

Fig. 5. BER performance in 2000 km 28-Gbaud DQPSK system with different LO laser linewidth.

Fig. 6. BER performance in 1000 km 28-Gbaud D8PSK system with different LO laser linewidth.

The performance of BER versus SNR in the 28-Gbaud DQPSK coherent transmission system with different fiber length is shown in Fig. 7, where both the Tx and the LO lasers linewidths are 5 MHz. We can find that the behaviors of the BER degrade obviously with the increment of the fiber length, which shows that the EEPN also depends on the accumulated fiber chromatic dispersion. It is also found that the BER floor for 2000 km fiber transmission is around $10^{-3}$, which corresponds to the BER threshold for the traditional 7% overhead hard-decision forward error correction (FEC). This means that the maximum reachable transmission distance is around 2000 km for the 28-Gbaud DQPSK coherent transmission system considering the EEPN, when both the Tx and the LO are using the distributed feedback (DFB) lasers with a linewidth of 5 MHz. As a comparison, the performance of BER
versus SNR in the 28-Gbaud D8PSK coherent transmission system with different fiber length is illustrated in Fig. 8, where both the Tx and the LO lasers linewidths are 0.5 MHz.

Fig. 7. BER performance in 28-Gbaud DQPSK system with different fiber length.

Fig. 8. BER performance in 28-Gbaud D8PSK system with different fiber length.

In Fig. 9, the behaviors of BER versus SNR in the 2000 km DQPSK coherent transmission system with different symbol rate are illustrated, where both the Tx and the LO lasers linewidths are also 5 MHz. Three symbol rates are considered for the evaluation: 14-Gbaud, 28-Gbaud, and 56-Gbaud. It is found that, with the increment of the transmission symbol rate, the BER behavior degrades significantly due to the severer EEPN. Therefore, the requirement of laser linewidth will not be relaxed in the long-haul transmission system with a higher symbol rate, due to the influence of EEPN.

Fig. 9. BER performance in DQPSK system with different symbol rate.

The performance of BER versus SNR in the coherent optical transmission system using different modulation formats is shown in Fig. 10, where the DQPSK, the D8PSK, and the D16PSK transmission systems are evaluated at 28-Gbaud. The linewidths of the Tx and the LO lasers are both 100 kHz. In order to study the impact
of the EEPN (rather than the intrinsic laser phase noise) in different modulation formats, we consider two cases of transmission distance (1000 km and 2000 km) to investigate the dispersion dependence of the EEPN. We find that the BER performance is distorted by EEPN more severely with the increment of the modulation formats, and the D16PSK system is the most sensitive to the EEPN.

Fig. 10. BER performance in 28-Gbaud transmission system with different modulation formats.

6. Numerical simulations and analysis

To investigate the accuracy of the proposed analytical model, numerical simulations have been carried out for the DQPSK, the D8PSK and the D16PSK coherent systems by using VPI and Matlab platforms [30,31]. The simulation setup of the 28-Gbaud \( n \)-PSK coherent transmission system employing differential CPE is illustrated in Fig.1. The central wavelength of the Tx and the LO lasers are both set as 1553.6 nm. The standard single mode fiber (SSMF) with a CD coefficient of 16 ps/nm/km is employed in all the simulation work. Fiber attenuation, polarization mode dispersion and nonlinear effects are neglected. The CD compensation is implemented by using a frequency domain equalizer (FDE) with appropriate parameters [4,5], and the signal equalization is carried out based on the constant modulus algorithm (CMA) [2]. The BER is estimated from a data sequence of \( 2^{17} \) bits. Several scenarios are considered in the simulation work: 1) DQPSK system with both Tx and LO laser linewidths of 5 MHz; 2) D8PSK system with both Tx and LO laser linewidths of 0.5 MHz; 3)
D16PSK system with both Tx and LO laser linewidths of 0.1 MHz. In the all the three systems, the transmission fiber lengths of 0 km and 1000 km are applied respectively, which corresponds to the cases of no EEPN and considerable EEPN.

The simulation results of the DQPSK, the D8PSK, and the D16PSK coherent transmission systems with and without EEPN (0 km and 1000 km fibers) are shown in Fig. 11, where the results are compared to their corresponding theoretical results (dashed lines for 0 km fiber, solid lines for 1000 km fiber) based on the analytical model. Here a BER range of $10^{-4}$ to 0.5 is considered, where we can ensure an accurate BER estimation in the simulations. It can be seen that, when no EEPN exists in the systems (0 km fiber case), the analytical evaluations are identical to the simulation results. When EEPN exists in the systems (1000 km fiber case), the analytical prediction is accurate for the DQPSK system, while some deviations can be found in the high SNR region for the D8PSK and the D16PSK systems. This is because that the influence of EEPN is actually a complicated combination of enhanced phase noise, enhanced amplitude noise and time jitter, etc. [18-20,24,28,29], and the leading order Taylor expansion ($e^x \approx 1 + x$), which ensures a conversion between the phase noise impact and the amplitude noise impact in $n$-PSK systems, is not accurate for a large EEPN and higher modulation formats. A higher order Taylor expansion in Eq. (2), Eq. (4) and Eq. (7) is required for further refinement, to give a more accurate prediction for the D8PSK, the D16PSK, and even the D64PSK coherent transmission systems.
Moreover, to be focused on the influence of EEPN in differential CPE, the impacts of fiber nonlinearities are neglected in both theoretical analysis and numerical simulations for evaluating the BER performance. Based on this analytical model, further analysis considering both the EEPN and the fiber nonlinear effects can also be investigated by involving the interference of fiber nonlinearities as an additional additive noise according to the Gaussian noise (GN) model [32].

Fig. 11. BER performance based on the analytical model (lines) and the numerical simulations (markers).

7. Discussions

It has been verified that the behavior of the feedback carrier phase estimation using the one-tap normalized least-mean-square (NLMS) algorithm resembles the differential phase estimation [5,10,25]. Therefore, the theoretical BER evaluation based on this model is also suitable for the one-tap NLMS carrier phase estimation. Besides, it is noted that this analytical model is applicable and available for both the single-polarization and the dual-polarization coherent optical transmission systems.

Several approaches have been investigated to mitigate the impact of EEPN in the long-haul coherent optical communication systems with electronic dispersion equalization, including dispersion pre-compensation, receiver extracted pilot tone,
partially modulated pilot carrier, and digital coherence enhancement, etc. [26-28,33-36]. Among these methods, the digital coherence enhancement shows the best performance [26,36], while it requires a complicated hardware implementation to measure the LO laser phase fluctuation. By contrast, EEPN does not exist in the coherent transmission systems using optical dispersion compensation, such as dispersion compensating fibers (DCF$s$) and dispersion compensating modules (DCM$s$) [18,19,22,25,37,38]. However, the management and the compensation of the fiber nonlinearities in such systems should be seriously considered. One promising method can be the optical back-propagation (OBP) which can compensate the fiber dispersion and the nonlinear interference simultaneously [39,40]. Both EEPN and fiber nonlinearities can be mitigated with a low complexity in the coherent transmission systems using OBP, which is a very promising approach for the real-time implementation of the high speed transmission system. Relevant research will be carried out in our future work.

8. Conclusion

In this paper, we present an analytical model for evaluating the BER performance in the differential $n$-PSK coherent transmission system considering the influence of equalization enhanced phase noise. Compared to the traditional analysis of differential phase estimation, this model has involved the additional noise variance induced by EEPN, which should be considered in the long-haul high speed transmission system. Theoretical results demonstrate that the BER performance degrades significantly with
the increment of LO laser linewidth, fiber dispersion, symbol rate, and modulation format, due to the influence of EEPN. The results from this theoretical model achieve a good agreement with the previously reported BER floors. Further simulations have also been carried out in the DQPSK, the D8PSK, and the D16PSK systems using differential CPE. The results indicate that this analytical model can give an accurate prediction for the DQPSK system, and a leading-order approximation for the D8PSK and the D16PSK systems. This model can be applied for predicting the BER behaviors in both the differential phase estimation and the one-tap NLMS feedback carrier phase estimation, and can also be employed as a reference for the evaluation of other CPE methods in the long-haul high speed coherent optical transmission systems, where the equalization enhanced phase noise can not be neglected.

Future work will incorporate improving the accuracy of this model using higher-order Taylor expansion and developing the analytical models for the BER evaluation in other carrier phase estimation approaches, such as the block average and the Viterbi-Viterbi algorithms [7,9,14], considering the impacts of EEPN. In addition, optical back-propagation will also be investigated for mitigating the EEPN and compensating the fiber nonlinearities in the long-haul transmission systems.

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Figure Captions

Fig. 1. Schematic of coherent optical transmission system with differential carrier phase estimation. N(t): additive white Gaussian noise (AWGN).

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